X

MATHEMATICS

COMPLETE QUESTION BANK

DISCLAIMER- Please Watch Complete Video before solving this question bank.(Nuksaan tumhara hi hoga varna)

Youtube.com/Shobhit Nirwan

Introduction to Trigonometry+Applications of Trigonometry

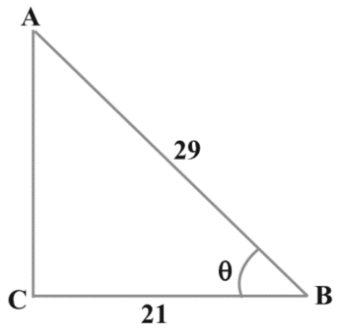
- **Introduction to trigonometry**

NCERT:

Example 3 : Consider Δ ACB, right-angled at C, in which AB = 29 units, BC = 21 units and $\angle ABC = \theta$ (see Fig. 8.10). Determine the values of

(i) $\cos^2\theta + \sin^2\theta$,

(ii) $\cos^2\theta - \sin^2\theta$.



Example 4 : In a right triangle ABC, right-angled at B, if tan A = 1, then verify that

2 sin A cos A = 1.

Example 8 : If $\sin(A - B) = \frac{1}{2}$, $\cos(A + B) = \frac{1}{2}$, $0° < A + B \leq 90°$, $A > B$, find A and B.

Example 10 : If sin 3A = cos (A – 26°), where 3A is an acute angle, find the value of A.

Example 13 : Prove that sec A (1 – sin A)(sec A + tan A) = 1.

Example 14 : Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

Example 15 : Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$, using the identity $\sec^2\theta = 1 + \tan^2\theta$.

EXERCISE 8.1:

4.Given 15 cot A = 8, find sin A and sec A.

6. If angleA and angleB are acute angles such that cos A = cos B, then show that angleA = angleB.

If 3 cot A = 4, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

8. 1

10. In ΔPQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

EXERCISE 8.2:

Full exercise is important except true/false😉

EXERCISE 8.3:

1. Evaluate (i)sin18/cos72

4.If tan A = cot B, prove that A + B = 90°.

6. If A, B and C are interior angles of a triangle ABC, then show that

$$\sin\left(\frac{B + C}{2}\right) = \cos\frac{A}{2}.$$

EXERCISE 8.4:

MOST IMPORTANT (ALL QUESTIONS)(Don't skip any question from this exercise)

- **Some Applications of Trigonometry:**

NCERT:

Example 3 : An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°. What is the height of the chimney?

Example 5 : The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.

Example 6 : The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

EXERCISE 9.1:

6.A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

8.A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

16. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

PREVIOUS YEARS:

$(\cos^2 67° - \sin^2 23°)$ का मान क्या है ?

What is the value of $(\cos^2 67° - \sin^2 23°)$?

$\because \quad \cos 67^\circ = \sin 23^\circ$

$\therefore \quad \cos^2 67^\circ - \sin^2 23 = 0$

If $4 \tan \theta = 3$, evaluate $\left(\frac{4 \sin \theta - \cos \theta + 1}{4 \sin \theta + \cos \theta - 1} \right)$

OR

If $\tan 2A = \cot (A - 18^\circ)$, where 2A is an acute angle, find the value of A.

$4 \tan \theta = 3$

$\Rightarrow \quad \tan \theta = \frac{3}{4}$

$\Rightarrow \quad \sin \theta = \frac{3}{5}$ and $\cos \theta = \frac{4}{5}$

$\therefore \quad \frac{4 \sin \theta - \cos \theta + 1}{4 \sin \theta + \cos \theta - 1} = \frac{4 \times \frac{3}{5} - \frac{4}{5} + 1}{4 \times \frac{3}{5} + \frac{4}{5} - 1}$

$= \frac{13}{11}$

OR

$\tan 2A = \cot (A - 18^\circ)$

$\Rightarrow \quad 90^\circ - 2A = A - 18^\circ$

$\Rightarrow \quad 3A = 108^\circ$

$\Rightarrow \quad A = 36^\circ$

Prove that : $\frac{\sin A - 2 \sin^3 A}{2 \cos^3 A - \cos A} = \tan A.$

$$\text{LHS} = \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$

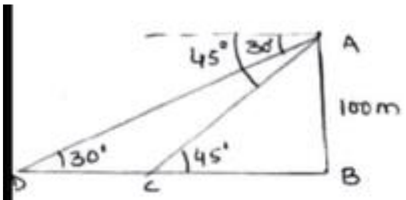
$$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \frac{\sin A(1 - 2(1 - \cos^2 A))}{\cos A(2\cos^2 A - 1)}$$

$$= \tan A \frac{(2\cos^2 A - 1)}{(2\cos^2 A - 1)}$$

$$= \tan A = \text{RHS}$$

As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$]



Let AB be the tower and ships are at points C and D.

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \frac{AB}{BC} = 1$$

$$\Rightarrow AB = BC$$

Also $\tan 30° = \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{AB + CD}$$

$$\Rightarrow AB + CD = \sqrt{3}AB$$

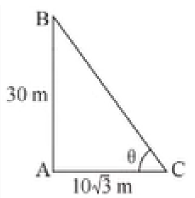
$$\Rightarrow CD = AB(\sqrt{3} - 1)$$

$$= 100 \times (1.732 - 1)$$

$$= 73.2 \text{ m.}$$

(11) 30/1

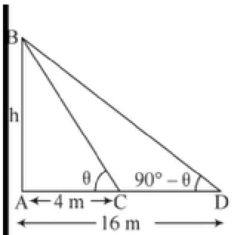
If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun ?



$$\tan \theta = \frac{30}{10\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \theta = 60°$$

On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.



Correct Figure

$$\tan \theta = \frac{h}{4} \quad \text{...(i)}$$

$$\tan (90 - \theta) = \frac{h}{16}$$

$$\Rightarrow \cot \theta = \frac{h}{16} \quad \text{...(ii)}$$

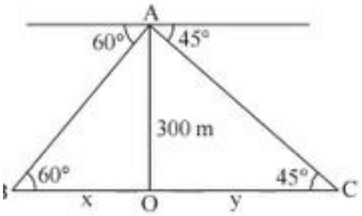
Solving (i) and (ii) to get

$$h^2 = 64$$

$$\Rightarrow h = 8\text{m}$$

An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° and 60° respectively. Find the width of the river. [Use $\sqrt{3}$ = 1·732]

27.



Correct Figure

$$\tan 45° = \frac{300}{y}$$

$$\Rightarrow \quad 1 = \frac{300}{y} \text{ or } y = 300$$

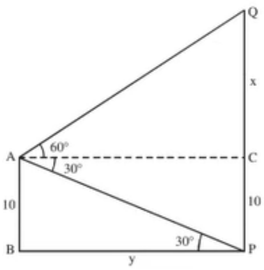
$$\tan 60° = \frac{300}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{300}{x} \text{ or } x = \frac{300}{\sqrt{3}} = 100\sqrt{3}$$

$$\text{Width of river} = 300 + 100\sqrt{3} = 300 + 173.2$$

$$= 473.2 \text{ m}$$

A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.



Correct Figure

In ΔABP, $\frac{y}{10} = \cot 30° = \sqrt{3}$

$\therefore$ $y = 10\sqrt{3}$ m

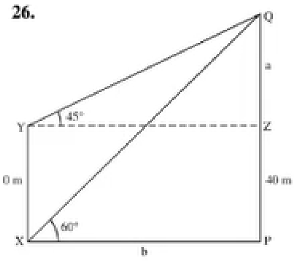
In ΔACQ, $\frac{x}{y} = \tan 60° = \sqrt{3}$

$x = \sqrt{3}(10\sqrt{3}) = 30$ m

$\therefore$ Height of hill = 30 + 10 = 40 m

The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. (Use $\sqrt{3}$ = 1.73)

26.



Correct Figure

In ΔYZQ, $\frac{a}{YZ} = \tan 45° = 1$

$\Rightarrow$ $YZ = a$ i.e. $a = b$

In ΔQPX, $\frac{a+40}{b} = \frac{a+40}{a} = \tan 60° = \sqrt{3}$

$\therefore$ $(\sqrt{3}-1)a = 40$ or $a = \frac{40}{\sqrt{3}-1} = 20(\sqrt{3}+1)$

$= 20(2.73) = 54.60$ m

$\therefore$ $PX = 54.6$ m

$PQ = 54.6 + 40 = 94.6$m

In Figure 1, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the Sun's altitude.

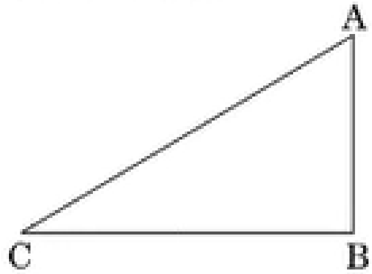
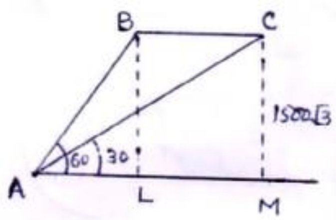


Figure 1

Ans-30degree

The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr.



Let $AL = x$ $\therefore$ $\frac{BL}{x} = \tan 60^0$ Fig.

$\Rightarrow \frac{1500\sqrt{3}}{x} = \sqrt{3} \Rightarrow x = 1500$ m.

$\frac{CM}{AL+LM} = \tan 30^0 = \frac{1}{\sqrt{3}}$

$\Rightarrow 1500 + LM = 1500\,(3) = 4500$

$\Rightarrow LM = 3000$ m.

$\therefore$ Speed $= \frac{3000}{15} = 200$ m./s. $= 720$ Km/hr.

2. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. The distance of the car from the tower (in metres) is

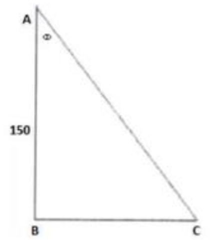
(A) $50\sqrt{3}$

(B) $150\sqrt{3}$

(C) $150\sqrt{2}$

(D) 75

Correct answer: A



Let AB be the tower and BC be distance between tower and car. Let θ be the angle of depression of the car.

According to the given information,

In ΔABC,

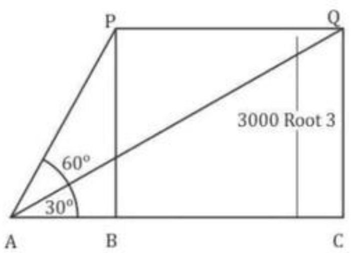
$\tan\theta = \frac{BC}{AB}$ [Using (1)] and $\tan 30° = \frac{1}{\sqrt{3}}$

$\therefore BC = \frac{150}{\sqrt{3}} = \frac{150\sqrt{3}}{3} = 50\sqrt{3}$

Hence, distance between the tower and car is $50\sqrt{3}$.

The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30^0. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.

olution:



Let P and Q be the two positions of the plane and A be the point of observation. Let ABC be the horizontal line through A.

It is given that angles of elevation of the plane in two positions P and Q from a point A are 60° and 30° respectively.

$\therefore \angle PAB = 60°$, $\angle QAB = 30°$. It is also given that PB = $3000\sqrt{3}$ m meters

In ΔABP, we have

Tan 60 = BP/AB

Root 3 = $3000\sqrt{3}$/ AB

AB = 3000 m

In ΔACQ, we have

tan30 = CQ/AC

$1/\sqrt{3} = 3000\sqrt{3}$/ AC

AC = 9000 m

$\therefore$ Distance = BC = AC – AB = 9000m – 3000m = 6000m

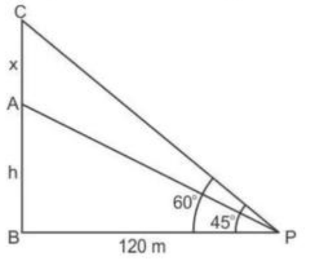
Thus, the plane travels 6km in 30 seconds

Hence speed of plane = 6000/30 = 200 m/sec = 720km/h

The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground is 45°. If the angle of elevation of the top of a flagstaff fixed at the top of the tower, at A is 60°, then find the height of the flagstaff. [Use $\sqrt{3} = 1.73$]

Solution:

Let AB is the tower of height h meter and AC is flagstaff of height x meter.



$\angle APB = 45^\circ$ and $\angle BPC = 60^\circ$

Tan 60 = (x + h)/120

$\sqrt{3} = (x + h)/120$

$(x + h) = 120\sqrt{3}$

$x = 120\sqrt{3} - h$

Tan 45° = h/120

1 = h/120

h = 120

Therefore height of the flagstaff = $120\sqrt{3} - 120$

$= 120(\sqrt{3} - 1)$ m

= 87.6 m

The angle of depression of a car, standing on the ground, from the top of a 75 m high tower, is 30°. The distance of the car from the base of the tower (in m.) is :

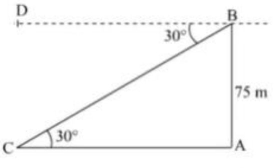
(A) $25\sqrt{3}$

(B) $50\sqrt{3}$

(C) $75\sqrt{3}$

(D) 150

Correct answer: C



Let AB be the tower of height 75 m and C be the position of the car.

In $\triangle ABC$,

$\cot 30^\circ = \frac{AC}{AB}$

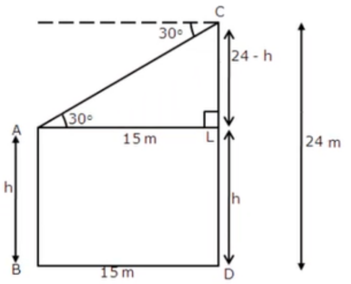
$\Rightarrow AC = AB \cot 30^\circ$

$\Rightarrow AC = 75m \times \sqrt{3}$

$\Rightarrow AC = 75\sqrt{3}m$

Thus, the distance of the car from the base of the tower is $75\sqrt{3}$ m.

The horizontal distance between two poles is 15 m. The angle of depression of the top of first pole as seen from the top of second pole is 30°. If the height of the second pole is 24 m, find the height of the first pole. [Use $\sqrt{3} = 1.732$]



Let AB and CD be the two poles, where CD (the second pole) = 24 m.

BD = 15 m

Let the height of pole AB be h m.

AL = BD = 15 m and AB = LD = h

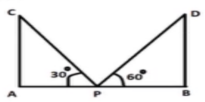
So, CL = CD - LD = 24 – h

In Δ ACL,

$$\tan 30^\circ = \frac{CL}{AL}$$
$$\Rightarrow \tan 30^\circ = \frac{24-h}{15}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{24-h}{15}$$
$$\Rightarrow 24 - h = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$
$$\Rightarrow h = 24 - 5\sqrt{3}$$
$$\Rightarrow h = 24 - 5 \times 1.732 \quad [\text{Taking} \sqrt{3} = 1.732]$$
$$\Rightarrow h = 15.34$$

Thus, height of the first pole is 15.34 m.

Two poles of equal heights are standing opposite each other on either side of the roads, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.



Let AC and BD be the two poles of the same height h m.

Given AB = 80 m

Let AP = x m, therefore, PB = (80 - x) m

In Δ APC,

$$\tan 30^\circ = \frac{AC}{AP}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{x} \quad \ldots (1)$$

In Δ BPD,

$$\tan 60^\circ = \frac{BD}{AB}$$
$$\sqrt{3} = \frac{h}{80-x} \quad \ldots (2)$$

Dividing (1) by (2),

$$\frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{\frac{h}{x}}{\frac{h}{80-x}}$$
$$\Rightarrow \frac{1}{3} = \frac{80-x}{x}$$
$$\Rightarrow x = 240 - 3x$$
$$\Rightarrow 4x = 240$$
$$\Rightarrow x = 60$$

From (1),

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$
$$\Rightarrow h = \frac{60}{\sqrt{3}} = 20\sqrt{3}\text{m}$$

Thus, the height of both the poles is $20\sqrt{3}$m and the distances of the point from the poles are 60 m and 20 m.

The length of shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. The angle of elevation of sun is :

(A) 45°

(B) 30°

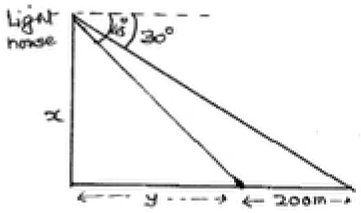
(C) 60°

(D) 90°

B

The angles of depression of two ships from the top of a light house and on the same side of it are found to be 45° and 30°. If the ships are 200 m apart, find the height of the light house.

A D K



Writing the trigonometric equations

(i) $\frac{x}{y} = \tan 45° = 1 \Rightarrow x = y$ ½ m

(ii) $\frac{x}{y+200} = \tan 30° \Rightarrow \frac{x}{x+200} = \frac{1}{\sqrt{3}}$ 1 m

$$\left.\begin{aligned} \Rightarrow x(\sqrt{3}-1) = 200 \Rightarrow x &= \frac{200(\sqrt{3}+1)}{2} \\ &= 100(\sqrt{3}+1)\text{ m} \end{aligned}\right\} 1\text{ m}$$