X

MATHEMATICS

COMPLETE QUESTION BANK

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Youtube.com/Shobhit Nirwan

QUADRATIC EQUATIONS

NCERT:

Example 2 : Check whether the following are quadratic equations:

(i) (x – 2)2 + 1 = 2x – 3 (ii) x(x + 1) + 8 = (x + 2) (x – 2)

Example 5 : Find the roots of the quadratic equation $3x^2$ -2root(6)x +2=0

Example 8: Find the roots of the equation $5x^2 - 6x - 2 = 0$ by the method of completing the square.

Example 11 : Find two consecutive odd positive integers, sum of whose squares is 290.

EXERCISE 4.1

1. Check whether the following are quadratic equations :

   (iii) (x – 2)(x + 1) = (x – 1)(x + 3) (iv) (x – 3)(2x +1) = x(x + 5)

2. Represent the following situations in the form of quadratic equations :

   (i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

   (ii) The product of two consecutive positive integers is 306. We need to find the integers.

EXERCISE 4.2

1. Find the roots of the following quadratic equations by factorisation:

   (ii) $2x^2 + x - 6 = 0$ (v) $100x^2 - 20x + 1 = 0$

3.Find two numbers whose sum is 27 and product is 182.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

EXERCISE 4.3

1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

   (i) $2x^2 + x - 4 = 0$

6.The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

8. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

EXERCISE 4.4

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

   (ii)$3x^2$-4root(3)x+4=0

5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m2? If so, find its length and breadth.

PREVIOUS YEARS:

- ✓ If x = 3 is one root of the quadratic equation x2 – 2kx – 6 = 0, then find the value of k. [2018,1]

1. x = 3 is one root of the equation

$\therefore \quad 9 - 6k - 6 = 0$ ½

$\Rightarrow \quad k = \frac{1}{2}$ ½

- ✓ A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed. [2018,3]

Let the usual speed of the plane be x km/hr.

$\therefore \quad \frac{1500}{x} - \frac{1500}{x+100} = \frac{30}{60}$ 1

$\Rightarrow \quad x^2 + 100x - 300000 = 0$

$\Rightarrow \quad x^2 + 600x - 500x - 300000 = 0$

$\Rightarrow \quad (x + 600)(x - 500) = 0$ 1

$x \neq -600, \quad \therefore x = 500$ ½

Speed of plane = 500 km/hr ½

✓ A motor boat whose speed is 18 km/hr in still water takes 1hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Let the speed of stream be x km/hr.

$\therefore$ The speed of the boat upstream = (18 − x) km/hr
and Speed of the boat downstream = (18 + x) km/hr 1

As given in the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1 \qquad 1$$

$\Rightarrow \quad x^2 + 48x - 324 = 0$ ½

$\Rightarrow \quad (x + 54)(x - 6) = 0$

$x \neq -54, \quad \therefore x = 6$ 1

$\therefore$ Speed of the stream = 6 km/hr. ½

OR

A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed ? [2018,4]

Let the original average speed of train be x km/hr.

Therefore $\frac{63}{x} + \frac{72}{x+6} = 3$ 1½

$\Rightarrow \quad x^2 - 39x - 126 = 0$ 1

$\Rightarrow \quad (x - 42)(x + 3) = 0$

$x \neq -3 \quad \therefore x = 42$ 1

Original speed of train is 42 km/hr. ½

✓ Find the value of p, for which one root of the quadratic equation $px^2-14x + 8 = 0$ is 6 times the other. [2017,2]

Let the roots of the given equation be α and 6α.

Thus the quadratic equation is $(x-\alpha)(x-6\alpha) = 0$

$\Rightarrow x^2 - 7\alpha x + 6\alpha^2 = 0 \quad ...(i)$

Given equation can be written as $x^2 - \frac{14}{p}x + \frac{8}{p} = 0 \quad ...(ii)$

Comparing the co-efficients in (i) & (ii) $7\alpha = \frac{14}{p}$ and $6\alpha^2 = \frac{8}{p}$

Solving to get $p = 3$

✓ If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots. [2017,3]

$D = 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$

$= -4(a^2d^2 + b^2c^2 - 2abcd)$

$= -4(ad - bc)^2$

Since $ad \neq bc$

Therefore $D < 0$

The equation has no real roots

- [2017,4]

Solve for x :

$$\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4},\ x \neq -1,\ -\frac{1}{5},\ -4$$

Here $[(5x + 1) + (x + 1)3](x + 4) = 5(x + 1)(5x + 1)$

$\Rightarrow \quad (8x + 4)(x + 4) = 5(5x^2 + 6x + 1)$

$\Rightarrow \quad 17x^2 - 6x - 11 = 0$

$\Rightarrow \quad (17x + 11)(x - 1) = 0$

$\Rightarrow \quad x = \frac{-11}{17}, x = 1$

- [2017,4]

Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank ?

Let one tap fill the tank in x hrs.

Therefore, other tap fills the tank in $(x + 3)$ hrs.

Work done by both the taps in one hour is

$\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$

$\Rightarrow \quad (2x + 3)\,40 = 13(x^2 + 3x)$

$\Rightarrow \quad 13x^2 - 41x - 120 = 0$

$\Rightarrow (13x + 24)(x - 5) = 0$

$\Rightarrow \quad x = 5$

(rejecting the negative value)

Hence one tap takes 5 hrs and another 8 hrs separately to fill the tank

- If -5 is a root of the quadratic equation $2x^2 + px – 15 = 0$ and the quadratic equation p(x2 + x)k = 0 has equal roots, find the value of k. [2016,2]

$2(-5)^2 + p(-5) - 15 = 0 \Rightarrow p = 7$

$7x^2 + 7x + k = 0$ gives $49 - 28k = 0 \Rightarrow k = \frac{7}{4}$

- [2016,3]

Solve for x: $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1, 2, 3$

Here $3(x - 3 + x - 1) = 2(x - 1)(x - 2)(x - 3)$

$\Rightarrow \quad 3(2x - 4) = 2(x - 1)(x - 2)(x - 3)$

$\Rightarrow \quad 3 = (x - 1)(x - 3)$ i.e. $x^2 - 4x = 0$

$\therefore \quad x = 0, x = 4$

- [2016,4]

Solve for x: $\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}, x\neq -1,-2,-4$

$(x + 4)(x + 2 + 2x + 2) = 4(x + 1)(x + 2)$

$(x + 4)(3x + 4) = 4(x^2 + 3x + 2)$

$\Rightarrow \quad x^2 - 4x - 8 = 0$

$\Rightarrow \quad x = \frac{4\pm\sqrt{16+32}}{2} = 2\pm 2\sqrt{3}$

- ✓ A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream. [2016,4]

let x km/h be the speed of the stream

$\therefore \quad \frac{32}{24-x}-\frac{32}{24+x} = 1$

$\Rightarrow \quad 32(2x) = (24 - x)(24 + x)$

$x^2 + 64x - 576 = 0$

$(x + 72)(x - 8) = 0 \Rightarrow x = 8$

$\therefore$ Speed of stream = 8 km/h.

- ✓ If the quadratic equation px^2-2root(5)+15=0 has two equal roots then find the value of p. [2015,1]

P=3

- ✓ Solve the following quadratic equation for x: 4x^2+4bx-(a^2-b^2)=0. [2015,2]

$4x^2 + 4bx + b^2 - a^2 = 0 \quad\Rightarrow\quad (2x + b)^2 - (a)^2 = 0$

$\Rightarrow \quad (2x + b + a)(2x + b - a) = 0$

$\Rightarrow \quad x = -\frac{a+b}{2},\ x = \frac{a-b}{2}$

- ✓ [2015,3]

Solve for x: $\sqrt{3x^2}-2\sqrt{2x}-2\sqrt{3}=0$

$$\sqrt{3}\,x^2 - 2\sqrt{2}\,x - 2\sqrt{3} = 0$$

$$\Rightarrow \sqrt{3}\,x^2 - 3\sqrt{2}\,x + \sqrt{2}\,x - 2\sqrt{3} = 0 \Rightarrow \left(x - \sqrt{6}\right)\left(\sqrt{3}x + \sqrt{2}\right) = 0$$

$$\Rightarrow x = \sqrt{6},\quad x = -\sqrt{\frac{2}{3}}$$

- ✓ A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed? [2015,4]

Let the original average speed of (first) train be x km./h.

$$\therefore \frac{54}{x} + \frac{63}{x+6} = 3$$

$$\Rightarrow 54x + 324 + 63x = 3x(x+6)$$

$$\Rightarrow x^2 - 33x - 108 = 0$$

Solving to get $x = 36$

$\therefore$ First speed of train = 36 km/h.

- ✓ [2013,2]

Solve the following quadratic equation for x :
$4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

$$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$\Rightarrow 4x\ \sqrt{3}x + 2\ - \sqrt{3}\ \sqrt{3}x + 2\ = 0$$

$$\Rightarrow\ 4x - \sqrt{3}\ \ \sqrt{3}x + 2\ = 0$$

$$\therefore x = \frac{\sqrt{3}}{4} \text{ or } x = -\frac{2}{\sqrt{3}}$$

- ✓ For what value of k, are the roots of the quadratic equation equation kx(x-2)+6=0 equal? [2013,3]

$k + 4)x^2 + (k + 1)x + 1 = 0$

$a = k + 4,\ b = k + 1,\ c = 1$

For equal roots, discriminant, $D = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (k + 1)^2 - 4(k + 4) \times 1 = 0$

$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$

$\Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow k(k - 5) + 3(k - 5) = 0$

$\Rightarrow (k - 5)(k + 3) = 0$

$\Rightarrow k = 5$ or $k = -3$

Thus, for $k = 5$ or $k = -3$, the given quadratic equation has equal roots.

✓ [2013,4]

Solve the following for x :

$$\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$$

$$\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$$

$$\Rightarrow \frac{1}{2a+b+2x}-\frac{1}{2x}=\frac{1}{2a}+\frac{1}{b}$$

$$\Rightarrow \frac{2x-2a-b-2x}{2x\ \ 2a+b+2x}=\frac{b+2a}{2ab}$$

$$\Rightarrow \frac{-\ 2a+b}{2x\ \ 2a+b+2x}=\frac{b+2a}{2ab}$$

$$\Rightarrow \frac{-1}{x\ \ 2a+b+2x}=\frac{1}{ab}$$

$$\Rightarrow 2x^2+2ax+bx+ab=0$$

$$\Rightarrow 2x\ \ x+a\ \ +b\ \ x+a\ \ =0$$

$$\Rightarrow\ \ x+a\ \ 2x+b\ \ =0$$

$$\Rightarrow x+a=0 \text{ or } 2x+b=0$$

$$\Rightarrow x=-a, \text{ or } x=\frac{-b}{2}$$

✓ Sum of the areas of two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of the two squares. [2013,4]

. Let the sides of the two squares be x cm and y cm where x > y.

Then, their areas are x^2 and y^2 and their perimeters are 4x and 4y.

By the given condition:

$x^2 + y^2 = 400$ ... (1)

and $4x - 4y = 16$

$\Rightarrow 4(x - y) = 16 \Rightarrow x - y = 4$

$\Rightarrow x = y + 4$ ... (2)

Substituting the value of x from (2) in (1), we get:

$(y + 4)^2 + y^2 = 400$

$\Rightarrow y^2 + 16 + 8y + y^2 = 400$

$\Rightarrow 2y^2 + 16 + 8y = 400$

$\Rightarrow y^2 + 4y - 192 = 0$

$\Rightarrow y^2 + 16y - 12y - 192 = 0$

$\Rightarrow y(y + 16) - 12 (y + 16) = 0$

$\Rightarrow (y + 16) (y - 12) = 0$

$\Rightarrow y = -16$ or $y = 12$

Since, y cannot be negative, y = 12.

So, x = y + 4 = 12 + 4 = 16

Thus, the sides of the two squares are 16 cm and 12 cm.

✓ The roots of the quadratic equation $2x^2-x-6=0$ are? [2012,1]

Ans- 2,-3/2

✓ Find the value of p for which the roots of the equation $px(x-2)+6=0$, are equal. [2012,2]

$px(x-2)+6=0 \Rightarrow px^2-2px+6=0$

For equal roots $b^2-4ac=0$

$\Rightarrow (-2p)^2-4(p)(6)=0 \Rightarrow 4p^2-24p=0$

$\Rightarrow p=0$ or $p=6$

but $p=0$ is rejected

$\therefore\ p=6$

- ✓ Solve for x: $x^2-4ax-b^2+4a^2=0$

$$x = \frac{4a \pm \sqrt{(4a)^2 - 4(1)(-b^2+4a^2)}}{2}$$

$$= \frac{4a \pm 2b}{2}$$

$$= 2a+b,\ \ 2a-b$$