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CLASS-10

CHAPTER-4

Quadratic Equations

→ An equation of the form $ax^2+bx+c=0$ having ($a \neq 0$) → ये, दो होती cond^n है इस

eg
$2x^2+4x+2=0$ ✓
$4x^2+2=0$ ✓
$2x+2=0$ ✗
$2x^2=0$ ✓
$4x^2+2x=0$ ✓

Methods of Finding solution of Quad. Eqn.

① Factorisation Method
② Method of completing square
③ Quadratic formula method

K³B (Kuch kaam ki Baat) ⇒ If α & β are the solutions/roots/zeroes of a quadric equation $ax^2+bx+c=0$ the α & β will satisfy that quadratic equation.

हिन्दी मे :- x कि जगह, α या β रखोगे तो equation $=0$ होगी।

** ① Factorisation Method :-

steps :-

(i) Write the given equation in the form ax^2+bx+c
(ii) Find two numbers α and β such that sum of α and β is equal to b and product of α and β

is equals to ac.

(iii) Write the middle term bx as $\alpha x + \beta x$ and factorise it by splitting the middle term.
Let the factors are $(x+p)$ and $(x+q)$

i.e. $ax^2+bx+c=0 \Rightarrow (x+p)(x+q)=0$

(iv) Now equate each factor to zero and find the values of x.

(v) These values of x are the required roots/solutions of the given quadratic equation.

eg $6x^2 - x - 2 = 0$

Working Rule:-
→ $6 \times 2 = 12$
→ 12 के ऐसे दो factors जिनको add/substract करके 12 ही आए
→ 4 & 3 ✓

answer

$\Rightarrow 6x^2 - 4x + 3x - 2 = 0$

कुछ common निकालो | कुछ common नहीं दिया

$\Rightarrow 2x(3x-2) + (3x-2) = 0$

$\Rightarrow (3x-2)[2x+1] = 0$ [∵ (3x-2) common आ रहा था]

$\Rightarrow 3x-2=0$ | $2x+1=0$

$\boxed{x = 2/3}$ | $\boxed{x = -1/2}$

eg $3x^2 - 2\sqrt{6}x + 2 = 0$

Solutions:

we know $\sqrt{6} = (\sqrt{3})(\sqrt{2})$

$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow (\sqrt{3}x - \sqrt{2})[\sqrt{3}x - \sqrt{2}] = 0$

$\Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}}, \; x = \frac{\sqrt{2}}{\sqrt{3}}$

Situation based problems:-

Q. The area of a rectangular plot is $528 m^2$. The length of the plot (in metres) is one more than twice its breadth. Find length and breadth of the plot.

solution: Concept: जो दो quantities निकालनी हैं, उनमें से किसी एक को "x" मान लो, दूसरी quantity को "x" के ही term में लिखो (with the help of given condition).

Let ~~length~~ breadth$(b) = x$

ATQ, length$(l) = 2b + 1$

(or) $= 2x + 1$

Now, Area $= 528$

$$l \times b = 528$$
$$(2x+1)\,x = 528$$
$$2x^2 + x - 528 = 0$$
$$2x^2 - 32x + 33x - 528 = 0$$
$$2x(x-16) + 33(x-16) = 0$$
$$(2x+33)(x-16) = 0$$

$x = \frac{-33}{2}$ ✗ | $x = 16$ ✓

[Value of breadth cannot be -ve]

$\therefore\ b = x = 16$

$l = 2x + 1 = 33$

(2) Method of Completing Square:

Let us understand it by one example:

eg $5x^2-6x-2=0$

steps:

(i) make the coefficient of x^2 as 1, by dividing both sides by coeff. of x^2

$\div$ by 5 here,

$$x^2-\frac{6}{5}x-\frac{2}{5}=0$$

(ii) Shift the constant on RHS and add square of half of the coefficient of x ie. here $\left(\frac{6/5}{2}\right)^2$ both sides.

~~here,~~ $\cancel{x^2-\frac{6}{5}x+\left(\frac{6}{2\times5}\right)^2}$

ie. here, $x^2-\frac{6}{5}x+\left(\frac{6}{2\times5}\right)^2=\frac{2}{5}+\left(\frac{6}{2\times5}\right)^2$

दोनों sides add किया है तो कोई change नही हुआ है equation में।

observe करे तो LHS मे a^2+b^2-2ab बन रहा ie. $(a-b)^2$

$$\Rightarrow \left(x-\frac{6}{10}\right)^2=\frac{2}{5}+\left(\frac{6}{10}\right)^2$$

$$\left(x-\frac{6}{10}\right)^2=\frac{76}{100}$$

$$\left(x-\frac{6}{10}\right)=\pm\sqrt{\frac{76}{100}}$$

$$x=\frac{6}{10}\pm\frac{\sqrt{76}}{10}$$

$$x=\frac{6\pm2\sqrt{19}}{10}$$

$$x=\frac{\cancel{2}(3\pm\sqrt{19})}{\cancel{10}\,5}$$

$\therefore$ roots are, $x=\dfrac{3+\sqrt{19}}{5}$ & $\dfrac{3-\sqrt{19}}{5}$

(3) Quadratic Formula Method:

Steps:-

(i) firstly, write the given quadratic equation by quadratic formula method.

(ii) Write values of a, b and c by comparing the given equation with standard form.

(iii) find Discriminant by

$$\boxed{D=b^2-4ac}$$

If D is negative, then is no real solution i.e. solution does not exist. If value of $D\geq 0$, then solution exists follow the next step.

(iv) put the value of a, b and D in quadratic formula, $\boxed{x=\dfrac{-b\pm\sqrt{D}}{2a}}$ and get the required roots.

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Q $x^2+4x+5=0$

$\underline{\text{sol}^n}$: $a=1,\ b=4,\ c=5$

$$D = b^2-4ac$$
$$= 16-4(1)(5) \Rightarrow -4 < 0$$

~~Since square of neg~~ No real roots for this eq.

(*) Q $\dfrac{1}{x} - \dfrac{1}{x-2} = 3$, find roots.

<u>Concept</u>: जुगाड़ करके $ax^2+bx+c=0$ के type में लाओ

<u>Sol^n</u> LCM ले लो : $\dfrac{(x-2)-x}{x(x-2)} = 3$

$$\Rightarrow \cancel{x}-2-\cancel{x} = 3x(x-2)$$
$$\Rightarrow -2 = 3x^2-6x$$
$$\Rightarrow \boxed{3x^2-6x+2=0}$$

Now easily find the roots by any method you want.

*** KB ⇒ जब भी factorisation Method में दिक्कत आए तब Quadratic formula Method use करना !

<u>NATURE OF ROOTS</u>:

We know, $D=b^2-4ac$.

and roots are $x = \dfrac{-b \pm \sqrt{D}}{2a}$

PTO

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$\boxed{D}$

$D > 0$	$D = 0$	$D < 0$
2 distinct roots	2 equal roots	No real roots exists.
i.e. $\frac{-b+\sqrt{D}}{2a}$ & $\frac{-b-\sqrt{D}}{2a}$	i.e. $\frac{-b}{2a}$ & $\frac{-b}{2a}$	

K3B ⇒ जब भी question में "possible"/"is it possible" दिखे तो Nature of Roots वाला concept ही लगेगा। बस eqn लिखो और $\boxed{D}$ check करो।

Q Is it possible to design a rectangular park of perimeter 80 m and area 400 m²? If so, find its length and breadth.

Solution:- ~~Concept~~ Concept ⇒ पहली बात तो "possible" है तो $\boxed{D}$ वाला concept लगेगा।
Secondly, situation based है तो उसका भी concept लगेगा।

let length $(l) = x$.

ATQ, perimeter $= 80$

$$2(l+b) = 80$$

$$x + b = 40$$

$$\boxed{b = 40 - x}$$

variable 2nd in form of 1st.

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Now,

$$\text{Area} = 400$$
$$l \times b = 400$$
$$x(40-x) = 400$$
$$40x - x^2 = 400$$
$$\boxed{x^2 - 40x + 400 = 0}$$

Now check D → +, −, 0.

—— X ——