X

MATHEMATICS

COMPLETE QUESTION BANK

DISCLAIMER- Please Watch Complete Video before solving this question bank.(Nuksaan tumhara hi hoga varna)

Youtube.com/Shobhit Nirwan

TRIANGLES

NCERT:

PROOF OF-

Theorem 6.1 : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Theorem 6.4 : If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.

Theorem 6.6 : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Theorem 6.8 : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

EXERCISE 6.1

1. Fill in the blanks using the correct word given in brackets(THESE CAN BE ASKED IN FORM OF MCQ IN BOARDS)

(i) All circles are________ . (congruent, similar)

(ii) All squares are _________. (similar, congruent)

(iii) All_____________triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are ___________and (b) their corresponding sides are__________________ .(equal, proportional)

EXERCISE 6.2

2. E and F are points on the sides PQ and PR respectively of a triangle PQR. For each of the following cases, state whether EF || QR :

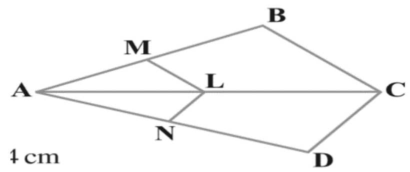


Fig. 6.18

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

1. In Fig. 6.19, DE || AC and DF || AE. Prove that BE/FE=BE/EC.

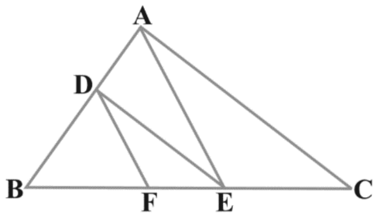


Fig. 6.19

9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that AO/BO=CO/DO.

EXERCISE 6.3

2. In Fig. 6.35, ΔODC ~ΔOBA angle BOC = 125° and angleCDO = 70°. Find angleDOC, angleDCO and angleOAB.

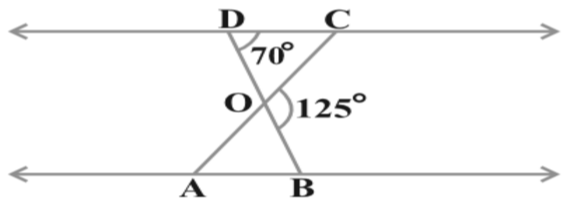
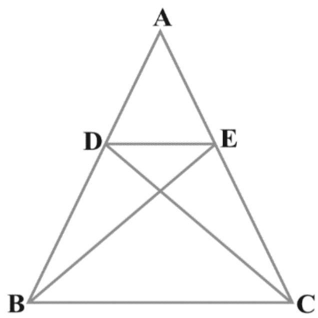


Fig. 6.35



6. In Fig. 6.37, if Δ ABE ≅ Δ ACD, show that Δ ADE ~ Δ ABC.

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ ABE ~ Δ CFB.

10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta FEG$, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$

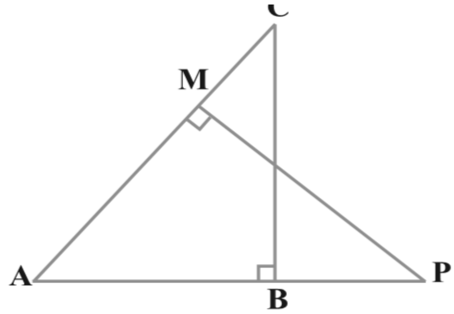


Fig. 6.39

13. D is a point on the side BC of a triangle ABC such that angleADC = angleBAC. Show that $CA^2 = CB.CD$.

15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

<u>EXERCISE 6.4</u>

1. Let ΔABC ~ ΔDEF and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4cm, find BC.

5.D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

9. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3 (B) 4 : 9 (C) 81 : 16 (D) 16 : 81

<u>EXERCISE 6.5</u>

4. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2\ AC^2$, prove that ABC is a right triangle.

6. ABC is an equilateral triangle of side 2a. Find each of its altitudes.

7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

15. In an equilateral triangle ABC, D is a point on side BC such that BD = 1/3BC. Prove that $9\ AD^2 = 7\ AB^2$.

17. Tick the correct answer and justify : In ΔABC, AB = 6√3 cm, AC = 12 cm and BC = 6 cm.

The angle B is :

(A) 120°　　(B) 60°

(C) 90°　　(D) 45°

PREVIOUS YEARS:

Given Δ ABC ~ Δ PQR, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{ar\ \Delta\ ABC}{ar\ \Delta\ PQR}$.

$$\frac{ar\ \Delta ABC}{ar\ \Delta PQR} = \frac{AB^2}{PQ^2}$$

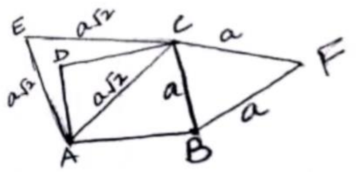
$$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

Prove that the area of an equilateral triangle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal.

OR

If the area of two similar triangles are equal, prove that they are congruent.

17.



Let the side of the square be 'a' units

$\therefore$ $AC^2 = a^2 + a^2 = 2a^2$

$\Rightarrow$ $AC = \sqrt{2}\,a$ units

Area of equilateral $\Delta BCF = \frac{\sqrt{3}}{4} a^2$ sq.u

Area of equilateral $\Delta ACE = \frac{\sqrt{3}}{4} (\sqrt{2}\,a)^2 = \frac{\sqrt{3}}{2} a^2$ sq.u

$\Rightarrow$ Area $\Delta BCF = \frac{1}{2}$ Ar ΔACE

OR

Let $\Delta ABC \sim \Delta PQR$.

$$\therefore \frac{ar\ \Delta ABC}{ar\ \Delta PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

Given ar ΔABC = ar ΔPQR

$$\Rightarrow \frac{AB^2}{PQ^2} = 1 = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

$\Rightarrow$ AB = PQ, BC = QR, AC = PR

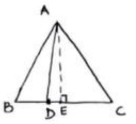
$\Rightarrow$ Therefore $\Delta ABC \cong \Delta PQR$. (sss congruence rule)

In an equilateral Δ ABC, D is a point on side BC such that $BD = \frac{1}{3} BC$. Prove that $9(AD)^2 = 7(AB)^2$

OR

Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

5.



Draw AE ⊥ BC

$\Delta AEB \cong \Delta AEC$ (RHS congruence rule)

$\therefore\ BE = EC = \frac{1}{2}BC = \frac{1}{2}AB$

Let AB = BC = AC = x

Now $BE = \frac{x}{2}$ and $DE = BE - BD$

$$= \frac{x}{2} - \frac{x}{3}$$

$$= \frac{x}{6}$$

Now $AB^2 = AE^2 + BE^2$ ...(1)

and $AD^2 = AE^2 + DE^2$ ...(2)

From (1) and (2) $AB^2 - AD^2 = BE^2 - DE^2$

$\Rightarrow\ x^2 - AD^2 = \left(\frac{x}{2}\right)^2 - \left(\frac{x}{6}\right)^2$

$\Rightarrow\ AD^2 = x^2 - \frac{x^2}{4} + \frac{x^2}{36}$

$\Rightarrow\ AD^2 = \frac{28}{36}x^2$

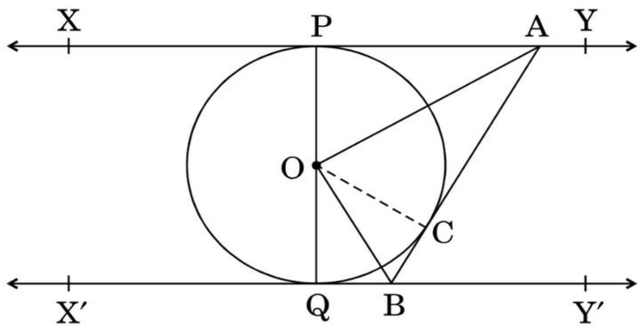
$\Rightarrow\ 9AD^2 = 7AB^2$

OR

Given, to Prove, Construction and Figure

Correct Proof

In the given figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and X′Y′ at B. Prove that ∠ AOB = 90°.



In right angled ΔPOA and ΔOCA

$\Delta OPA \cong \Delta OCA$

$\therefore\ \angle POA = \angle AOC$ ...(i)

Also $\Delta OQB \cong \Delta OCB$

$\therefore\ \angle QOB = \angle BOC$ ...(ii)

Therefore $\angle AOB = \angle AOC + \angle COB$

$$= \frac{1}{2}\angle POC + \frac{1}{2}\angle COQ$$

$$= \frac{1}{2}(\angle POC + \angle COQ)$$

$$= \frac{1}{2} \times 180°$$

$$= 90°$$

In fig. 8, the vertices of ΔABC are A(4, 6), B(1, 5) and C(7, 2). A line-segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC.

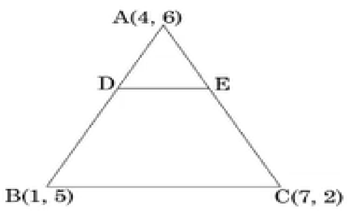


Figure 8

Coords of D are: $\left(\frac{1(1)+2(4)}{3}\right), \left(\frac{1(5)+2(6)}{3}\right)$ i.e. $\left(3, \frac{17}{3}\right)$

Coords of E are: $\left(\frac{1(7)+2(4)}{3}, \frac{1(2)+2(6)}{3}\right)$ i.e. $\left(5, \frac{14}{3}\right)$

ar. $\Delta ADE = \frac{1}{2}\left[4(1)+3\left(\frac{14}{3}-6\right)+5\left(6-\frac{17}{3}\right)\right]=\frac{5}{6}$

ar. $\Delta ABC = \frac{1}{2}[4(3)+1(-4)+7(1)]=\frac{15}{2}$

ar. ΔADE: ar. $\Delta ABC = \frac{5}{6}:\frac{15}{2}$ or $1:9$

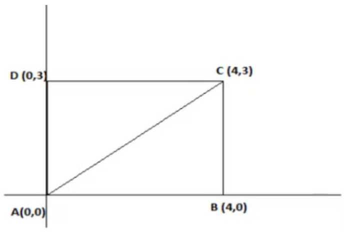
. ABCD is a rectangle whose three vertices are B (4, 0), C(4, 3) and D (0, 3). The length of one of its diagonals is

(A) 5

(B) 4

(C) 3

(D) 25



We see that AB = 4 units and BC = 3 units

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

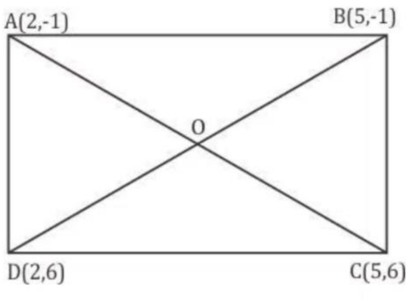
$= 4^2 + 3^2$

$AC^2 = 25$

Thus AC = 5 units

Hence length of the diagonal of the rectangle is 5 un

Prove that the diagonals of a rectangle ABCD, with vertices A(2, -1), B(5, -1) C(5, 6) and D(2, 6), are equal and bisect each other.



$AC^2 = (5-2)^2 + (6+1)^2 = 9 + 49 = 58$ sq. unit

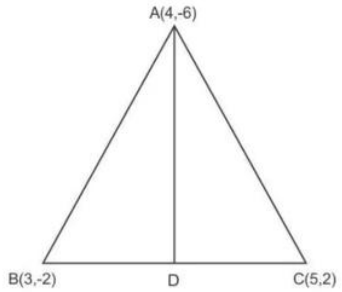
$BD^2 = (5-2)^2 + (-1-6)^2 = 9 + 49 = 58$ sq. unit

Diagonals of parallelogram are equal so rectangle

A(4, - 6), B(3,- 2) and C(5, 2) are the vertices of a ΔABC and AD is its median. Prove that the median AD divides ΔABC into two triangles of equal areas.

Let co – ordinate of D (x, y) and D is midpoint of BC

x = (3 + 5)/2 = 4; y = (2 - 2)/2 = 0



Now Area of triangle ABD = ½ {4(-2 – 0) + 3[(0 – (-6)] + 4 [(-6) – (-2)]}

= 0.5 × [-8 + 18 - 16] = 3 sq unit

and Area of triangle ACD = ½ [5(-6 - 0) + 4(0 - 2) + 4(2 + 6)] = 3 sq unit

Hence, the median AD divides triangle ABC into two triangle of equal area.

IMPORTANT QUESTIONS

R and S are points on the sides DE and EF respectively of a ΔDEF such that ER = 5 cm, RD = 2.5 cm, SE = 1.5 cm and FS = 3.5 cm. Find whether RS || DF or not.

Solution:

Construction: Join RS

To find: RS || DF or not

Proof: We have RE = 5 cm

and RD = 2.5 cm

Now $\frac{RE}{RD} = \frac{5}{2.5} = \frac{2}{1}$

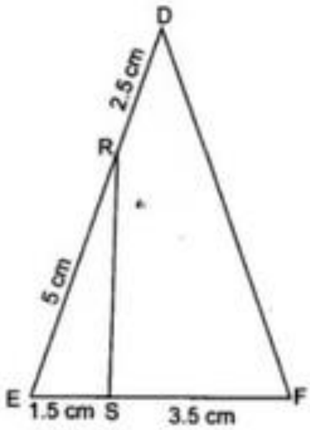
Similarly, we have, ES = 1.5 cm

and SF = 3.5 cm

Now, $\frac{SF}{ES} = \frac{3.5}{1.5} = \frac{7}{3}$

Here $\frac{RE}{RD} \neq \frac{SF}{ES}$

⇒ RS is not parallel to DF.



From airport two aeroplanes start at the same time. If the speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h, then find the distance between two aeroplanes after 2 hours.

Solution:

Speed of aeroplane along north = 500 km/h

Speed of aeroplane along east = 650 km/h

Distance travelled by aeroplane in 2 hours in North direction.

$= OB = 500 \times 2$

$= 1000$ km.

Distance travelled by aeroplane in 2 hours in East direction.

$= OA = 650 \times 2$

$= 1300$ km

Distance between both aeroplanes after 2 hours = AB.

∴ $AB^2 = OB^2 + ON^2$

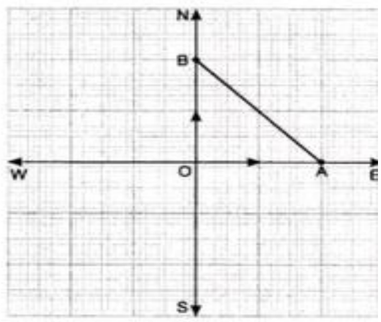
[By Pythagoras theorem in ΔAOB]

$= (1000)^2 + (1300)^2$

$= 1000000 + 1690000$

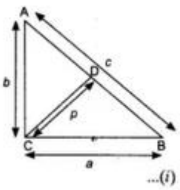
$= 2690000$

$AB = 100\sqrt{269}$ km



ΔABC, is right angled at C. If p is the length of the perpendicular from C to AB and a, b, c are the lengths of the sides opposite ∠A, ∠B, ∠C respectively then prove that

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}.$$



Given: In ΔACB, ∠C = 90°, and CD ⊥ AB.

Also, AB = c, BC = a, CA = b and CD = p

To prove: $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Proof: In ΔABC, ∠C = 90°

Apply Pythagoras theorem

$$AB^2 = BC^2 + AC^2$$

⇒ $$c^2 = a^2 + b^2 \quad ...(i)$$

Now, area of $\Delta ABC = \frac{1}{2} \times AB \times CD = \frac{1}{2} \times p \times c$ ...(ii)

Also area of $\Delta ABC = \frac{1}{2} \times AC \times BC = \frac{1}{2} \times b \times a$ ...(iii)

Equating (iii) and (ii), we have

$$\frac{1}{2}pc = \frac{1}{2}ab$$

$$pc = ab$$

$$c = \frac{ab}{p} \text{ and } c^2 = \frac{a^2b^2}{p^2}$$

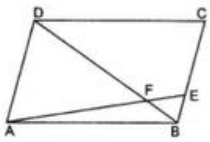
Put value of c^2 in equation (i)

$$\frac{a^2b^2}{p^2} = a^2 + b^2$$

$$\frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2} \quad \text{(dividing both sides by } a^2b^2)$$

$$\frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

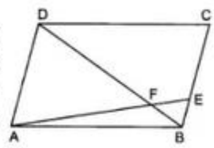
In the figure, ABCD is a parallelogram and E divides BC in the ratio 1: 3. DB and AE intersect at F. Show that DF = 4FB and AF = 4FE



Given: ABCD is a parallelogram and BE : EC : : 1 : 3

To show: DF = 4FB and AF = 4FE

Proof: In ΔADF and ΔEBF



∠ADF = ∠EBF (Alternate angles)

∠AFD = ∠EFB (V.O.A.)

ΔADF ~ ΔEBF (by AA)

$$\frac{DF}{BF} = \frac{AF}{EF} = \frac{AD}{BE} \quad ...(i)$$

as $\frac{BE}{EC} = \frac{1}{3} \Rightarrow EC = 3BE$

∴ BC = BE + CE = BE + 3BE

⇒ BC = 4BE

as AD = BC

∴ AD = 4BE

Put in (i), we get

$$\frac{DF}{BF} = \frac{AF}{EF} = \frac{4}{1}$$

DF = 4BF

and AF = 4EF

In figure, if ∠CAB = ∠CED, then prove that AB X DC = ED X BC.

Solution:

Given: $\angle CAB = \angle CED$

i.e. $\angle 1 = \angle 2$

To prove: $AB \times DC = ED \times BC$

Proof: In ΔCAB and ΔCED

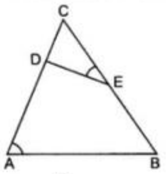
$\angle 1 = \angle 2$ (Given)

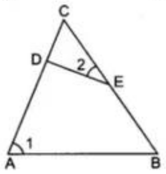
$\angle C = \angle C$ (Common)

So, $\Delta CAB \sim \Delta CED$ (By AA similarity)

$\Rightarrow \dfrac{BC}{DC} = \dfrac{AB}{ED}$

$\Rightarrow AB \times DC = BC \times ED$ Hence Proved





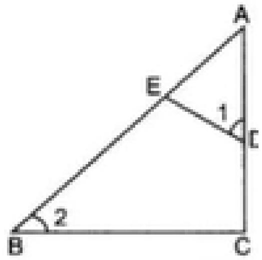
In ΔABC, if ∠ADE = ∠B, then prove that ΔADE ~ ΔABC. Also, if AD = 7.6 cm, AE = 7.2 cm, BE = 4.2 cm and BC = 8.4 cm, then find DE.

Solution:

Given: $\angle ADE = \angle B$, i.e. $\angle 1 = \angle 2$

To prove: $\Delta ADE \sim \Delta ABC$

Proof: In ΔADE and ΔABC



$\angle 1 = \angle 2$ [Given]

$\angle A = \angle A$ [Common]

So, $\Delta ADE \sim \Delta ABC$ [By AA similarity]

$\Rightarrow \dfrac{AD}{AB} = \dfrac{DE}{BC}$

$\Rightarrow \dfrac{7.6}{7.2+4.2} = \dfrac{DE}{8.4}$ $\{\because AB = AE + BE = 7.2 + 4.2$

$\Rightarrow \dfrac{7.6}{11.4} = \dfrac{DE}{8.4} \Rightarrow DE = \dfrac{7.6 \times 8.4}{11.4} = 5.6$

Hence, $DE = 5.6$ cm.

In the figure, ABC is a triangle and BD ⊥ AC. Prove that $AB^2 + CD^2 = AD^2 + BC^2$

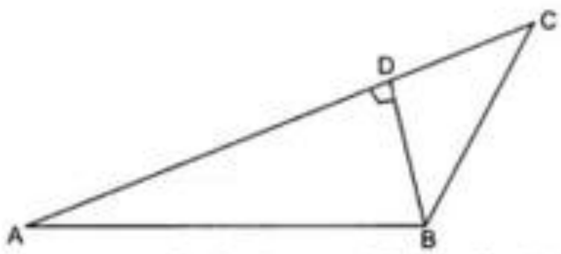
Solution:

Given: A triangle ABC in which $BD \perp AC$.

To prove: $AB^2 + CD^2 = AD^2 + BC^2$

Proof: In right ΔADB,

$AB^2 = AD^2 + BD^2$ [By Pythagoras Theorem] ...(*i*)



In right ΔCDB,

$BC^2 = CD^2 + BD^2$ [By Pythagoras Theorem] ...(*ii*)

Subtracting equation (*ii*) from (*i*), we get,

$$AB^2 - BC^2 = AD^2 + BD^2 - CD^2 - BD^2$$

$$AB^2 - BC^2 = AD^2 - CD^2$$

$\therefore$ $AB^2 + CD^2 = AD^2 + BC^2$ Hence proved.

Equiangular triangles are drawn on sides of right angled triangle in which perpendicular is double of its base. Show that area of triangle on the hypotenuse is the sum of areas of the other two triangles?

Solution:

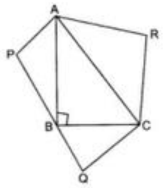
Given: A right angled triangle ABC with right angled at B.

Equiangular triangles PAB, QBC and RAC are described on sides AB, BC and CA respectively.

Let BC = x and AB = $2x$

To prove: ar(ΔPAB) + ar(ΔQBC) = ar(ΔRAC)

Proof: ∵ Equiangular triangles are equilateral also,



∴ Area of ΔPAB $= \frac{\sqrt{3}}{4} \times (2x)^2$

$= \sqrt{3}x^2$ ...(*i*)

Area of ΔQBC $= \frac{\sqrt{3}}{4} \times (x)^2$

$= \frac{\sqrt{3}}{4}x^2$ ...(*ii*)

Area of ΔRAC $= \frac{\sqrt{3}}{4} \times (AC)^2$

$= \frac{\sqrt{3}}{4} \times 5x^2$ [∵ In ΔABC, $AB^2 + BC^2 = AC^2$
$AC^2 = (2x)^2 + x^2 = 5x^2$]

$= \frac{5\sqrt{3}x^2}{4}$ ...(*iii*)

Adding (*i*) and (*ii*)

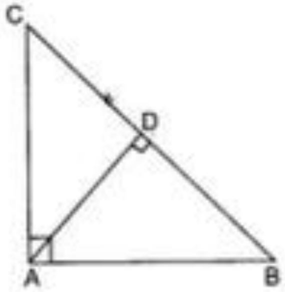
ar(ΔPAB) + ar(ΔQBC) $= \sqrt{3}x^2 + \frac{\sqrt{3}x^2}{4} = \frac{4\sqrt{3}x^2 + \sqrt{3}x^2}{4} = \frac{5\sqrt{3}x^2}{4}$

= ar(ΔRAC) [From (*iii*)]

∴ ar(ΔPAB) + ar(ΔQBC) = ar(ΔRAC) Hence proved.

If in a right angle AABC, right angled at A, AD ⊥ BC, then prove that $AB^2 + CD^2 = BD^2 + AC^2$

Solution:



In right ΔADB, ∠D = 90°

$AB^2 = AD^2 + BD^2$ ...(*i*) [By Pythagoras Theorem]

In right ΔADC, ∠D = 90°

$AC^2 = AD^2 + CD^2$ ...(*ii*) [By Pythagoras Theorem]

Subtracting equation (*ii*) from (*i*), we get

$AB^2 - AC^2 = AD^2 + BD^2 - AD^2 - CD^2$

$AB^2 + CD^2 = BD^2 + AC^2$ Hence proved.

Long Answer Type Questions [4 Marks]

In ΔABC, $AX \perp BC$ and Y is middle point of BC. Then prove that,

(*i*) $AB^2 = AY^2 + \frac{BC^2}{4} - BC.XY$

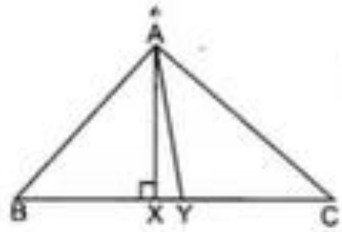
(*ii*) $AC^2 = AY^2 + \frac{BC^2}{4} + BC.XY$

Solution:

In ΔABC, $AX \perp BC$ and Y is middle point of BC.
Then prove that,

(*i*) $AB^2 = AY^2 + \frac{BC^2}{4} - BC.XY$

(*ii*) $AC^2 = AY^2 + \frac{BC^2}{4} + BC.XY$



Given: A ΔABC in which $AX \perp BC$ and Y is mid-point of BC.

To prove: (*i*) $AB^2 = AY^2 + \frac{BC^2}{4} - BC.XY$

(*ii*) $AC^2 = AY^2 + \frac{BC^2}{4} + BC.XY$

Proof: (*i*) In ΔABX.

$AB^2 = AX^2 + BX^2$ [By Pythagoras Theorem]

$AB^2 = AX^2 + (BY - XY)^2$

$AB^2 = AX^2 + \left(\frac{BC}{2} - XY\right)^2$ [$\because$ Y is mid point of BC]

$AB^2 = AX^2 + \frac{BC^2}{4} + XY^2 - 2\left(\frac{BC}{2}\right)(XY)$

$AB^2 = (AX^2 + XY^2) + \frac{BC^2}{4} - \frac{2BC}{2}.XY$

$AB^2 = AY^2 + \frac{BC^2}{4} - BC.XY$ [$\because$ In ΔAXY, $AX^2 + XY^2 = AY^2$]

Hence proved.

(*ii*) In ΔAXC,

$AC^2 = AX^2 + XC^2$ [By Pythagoras Theorem]

$AC^2 = AX^2 + (XY + YC)^2$

$AC^2 = AX^2 + \left(XY + \frac{BC}{2}\right)^2$ [$\because$ In Y is mid-point of BC]

$AC^2 = (AX^2 + XY^2) + \frac{BC^2}{4} + 2(XY).\left(\frac{BC}{2}\right)$

$AC^2 = AY^2 + \frac{BC^2}{4} + BC.XY$ [$\because$ In ΔAXY, $AX^2 + XY^2 = AY^2$]

Hence proved.

In AABC, X is any point on AC. If Y, Z , U and Y are the middle points of AX, XC, AB and BC respectively, then prove that UY || VZ and UV|| YZ.

Solution:

Given: In ΔABC, Y, Z, U and V are mid-points of AX, XC, AB and BC.

To prove: UY || VZ and UV || YZ

Construction: Join BX

Proof: In ΔABX,

$$AU = UB, \text{ i.e. } \frac{AU}{UB} = \frac{1}{1}$$

and $$AY = YX, \text{ i.e. } \frac{AY}{YX} = \frac{1}{1}$$

$$\therefore \quad \frac{AU}{UB} = \frac{AY}{YX}$$

So, UY || BX

Similarly, in ΔBCX,

$$BX \parallel VZ$$

∴ From (*i*) and (*ii*)

$$UY \parallel VZ$$

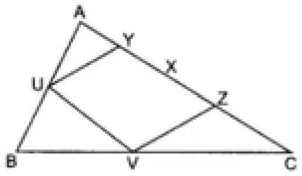
In ΔABC,

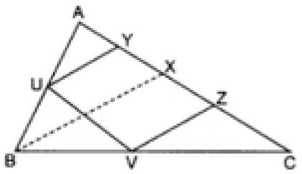
$$\frac{BU}{UA} = \frac{BV}{VC} = \frac{1}{1}$$

∴ By converse of B.P.T.,

$$UV \parallel YZ$$

From equation (*iii*) and (*iv*), UY || VZ and UV || YZ.





In ΔABC, ZB = 90°, BD ⊥ AC, ar (ΔABC) = A and BC = a, then prove that

$$BD = \frac{2Aa}{\sqrt{4A^2 + a^4}}$$

Solution:

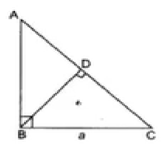
In ΔABC, ∠B = 90°, BD ⊥ AC, ar (ΔABC) = A and BC = *a*, then prove that

$$BD = \frac{2Aa}{\sqrt{4A^2 + a^4}}$$

Given: Area of ΔABC = A

BC = *a* and BD ⊥ AC

To prove: $BD = \frac{2Aa}{\sqrt{4A^2+a^4}}$



Proof: A = Area of ΔABC

$$A = \frac{1}{2} BC \times AB = \frac{1}{2} \times a \times AB$$

$$AB = \frac{2A}{a} \qquad ...(i)$$

In ΔADB and ΔABC

$$\angle ADB = \angle ABC = 90^\circ$$

$$\angle BAD = \angle BAC \qquad \text{[Common]}$$

∴ By AA similarity criterion, ΔADB ~ ΔABC

$$\therefore \quad \frac{AB}{AC} = \frac{BD}{BC} \qquad ...(ii)$$

In ΔABC, $AB^2 + BC^2 = AC^2$ [By Pythagoras Theorem]

$$\frac{4A^2}{a^2} + a^2 = AC^2$$

$$AC = \sqrt{\frac{4A^2+a^4}{a^2}} = \frac{\sqrt{4A^2+a^4}}{a}$$

From (*i*) and (*ii*), we get

$$\frac{2A}{a \times AC} = \frac{BD}{a}$$

$$BD = \frac{2A}{AC} = \frac{2Aa}{\sqrt{4A^2+a^4}}$$

Hence proved.

2013

Short Answer Type Questions I [2 Marks]

In the figure, DB⊥BC, DE⊥AB and AC⊥BC prove that

$\frac{BE}{DE} = \frac{AC}{BC}$

Solution:

Given DB ⊥ BC, i.e. $\angle 1 + \angle 2 = 90°$

DE ⊥ AB, i.e. $\angle E = 90°$

AC ⊥ BC, i.e. $\angle C = 90°$

To prove: $\frac{BE}{DE} = \frac{AC}{BC}$

Proof: In ΔABC, $\angle C = 90°$

$$\angle A + \angle 2 = 90° \quad ...(i)$$

Also, $$\angle 1 + \angle 2 = 90° \text{(given)} \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$\angle 1 + \angle 2 = \angle 2 + \angle A$$

$\Rightarrow$ $$\angle 1 = \angle A$$

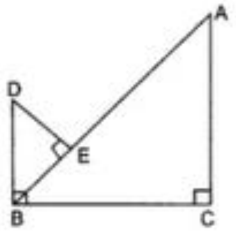
Now, in ΔDEB and ΔBCA

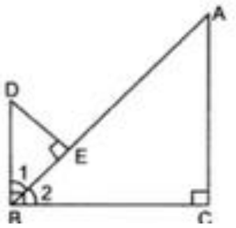
$$\angle E = \angle C \quad \text{(Each 90°)}$$

$$\angle 1 = \angle A \quad \text{(Proved above)}$$

$\therefore$ $$\Delta DEB \sim \Delta BCA$$

$\Rightarrow$ $$\frac{BE}{AC} = \frac{DE}{BC} \Rightarrow \frac{BE}{DE} = \frac{AC}{BC}$$

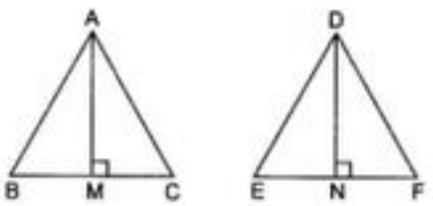




The area of two similar triangles are 49 cm^2 and 64 cm^2 respectively. If the difference of the corresponding altitudes is 10 cm, then find the lengths of altitudes (in centimetres).

Solution:

$\Delta ABC \sim \Delta DEF$ (Given)



$\therefore$ $$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$\Rightarrow$ $$\frac{49}{64} = \frac{BC^2}{EF^2} \Rightarrow \frac{BC}{EF} = \frac{7}{8}$$

Also $$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\frac{1}{2}BC \times AM}{\frac{1}{2}EF \times DN} \Rightarrow \frac{49}{64} = \frac{BC}{EF} \times \frac{AM}{DN}$$

$\Rightarrow$ $$\frac{49}{64} = \frac{7}{8} \times \frac{AM}{DN} \Rightarrow \frac{7}{8} = \frac{AM}{DN} \Rightarrow DN = \frac{8}{7}AM$$

Also $$DN - AM = 10 \quad \text{(Given)}$$

$\Rightarrow$ $$\frac{8}{7}AM - AM = 10 \Rightarrow \frac{1}{7}AM = 10$$

$$AM = 70 \text{ cm}$$

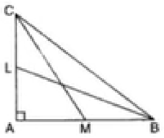
$\therefore$ $$DN = 80 \text{ cm}$$

If BL and CM are medians of a triangle ABC right angled at A, then prove that $4(BL^2 + CM^2) = 5BC^2$.

Solution:

Given: A right angled triangle ABC, right angled at A.
BL and CM are the medians.

To prove: $4(BL^2 + CM^2) = 5BC^2$



Proof: In right angled triangle CAB,

$BC^2 = AC^2 + AB^2$ [By Pythagoras theorem] ...(*i*)

In right-angled triangle CAM,

$CM^2 = AC^2 + AM^2$

Also, $AM = \frac{1}{2}AB$ [As CM is median]

$\therefore$ $CM^2 = AC^2 + \left(\frac{1}{2}AB\right)^2$

$\Rightarrow$ $4CM^2 = 4AC^2 + AB^2$ ...(*ii*)

In right-angled triangle LAB,

$BL^2 = AL^2 + AB^2$

Also, $AL = \frac{1}{2}AC$ [As BL is median]

$\therefore$ $BL^2 = \left(\frac{1}{2}AC\right)^2 + AB^2$

$\Rightarrow$ $4BL^2 = AC^2 + 4AB^2$ ...(*iii*)

Adding (*ii*) and (*iii*), we get

$4BL^2 + 4CM^2 = 5AC^2 + 5AB^2$

$\Rightarrow$ $4(BL^2 + CM^2) = 5(AC^2 + AB^2)$

$\Rightarrow$ $4(BL^2 + CM^2) = 5BC^2$ [From (*i*)] Hence proved.

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
Using the above, prove the following:
If the areas of two similar triangles are equal, then prove that the triangles are congruent.

Solution:

To prove: $\Delta ABC \cong \Delta PQR$

Proof: Using the above result, we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2}$$

Also $ar(\Delta ABC) = ar(\Delta PQR)$ [Given]

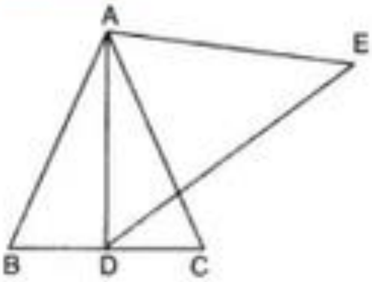
$\therefore$ $1 = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2}$

$\Rightarrow$ $AB = PQ, AC = PR, BC = QR$

$\Rightarrow$ $\Delta ABC \cong \Delta PQR$ [SSS]

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. Using the above, do the following:
AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that, area (ΔADE): area (ΔABC) =3:4

Solution:



ΔABC and ΔADE are equilateral triangles.

$$\frac{ar\,\Delta ABC}{ar\,\Delta ADE} = \frac{(AB)^2}{(AD)^2} \qquad ...(i)$$

where $$AD = \frac{\sqrt{3}}{2}AB$$

Putting in equation (*i*)

$$\frac{ar\,\Delta ABC}{ar\,\Delta ADE} = \left|\frac{(AB)^2}{\left(\frac{\sqrt{3}}{2}AB\right)^2}\right| = \left(\frac{2}{\sqrt{3}}\cdot\frac{AB}{AB}\right)^2$$

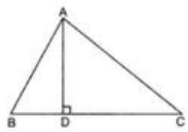
$$\frac{ar\,\Delta ABC}{ar\,\Delta ADE} = \frac{4}{3}$$

In figure, AD ⊥ BC and BD = 1/3 CD. Prove that $2CA^2 = 2AB^2 + BC^2$.

Solution:

In figure, AD ⊥ BC and BD = $\frac{1}{3}$CD.

Prove that $2CA^2 = 2AB^2 + BC^2$. [All India]



Given: In ΔABC, AD ⊥ BC and BD = $\frac{1}{3}$CD

To prove: $2CA^2 = 2AB^2 + BC^2$

Proof: ∵ BC = BD + CD and BD = $\frac{1}{3}$CD [Given]

∴ $BC = \frac{1}{3}CD + CD = \frac{4}{3}CD$

⇒ $CD = \frac{3}{4}BC$ ...(*i*)

In right angled ΔADC,

$AC^2 = CD^2 + AD^2$ [By Pythagoras theorem] ...(*ii*)

In right angled ΔADB,

$AB^2 = AD^2 + BD^2$

⇒ $AD^2 = AB^2 - BD^2$

Substituting in (*ii*), we get

⇒ $AC^2 = CD^2 + AB^2 - BD^2$

⇒ $AC^2 = CD^2 + AB^2 - \left(\frac{1}{3}CD\right)^2$ [Put BD = $\frac{1}{3}$CD]

⇒ $AC^2 = CD^2 - \frac{1}{9}CD^2 + AB^2$

⇒ $AC^2 = \frac{8}{9}CD^2 + AB^2$

⇒ $AC^2 = \frac{8}{9}\left(\frac{3}{4}BC\right)^2 + AB^2$ [Using (*i*)]

⇒ $AC^2 = \frac{8}{9}\times\frac{9}{16}BC^2 + AB^2$

⇒ $AC^2 = \frac{1}{2}BC^2 + AB^2$

⇒ $2AC^2 = BC^2 + 2AB^2$ Hence proved.