X

MATHEMATICS

COMPLETE QUESTION BANK

DISCLAIMER- Please Watch Complete Video before solving this question bank.(Nuksaan tumhara hi hoga varna)

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CIRCLES+AREAS RELATED TO CIRCLES

- **CIRCLES-**

NCERT:

PROOF OF-

Theorem 10.1 : The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Theorem 10.2 : The lengths of tangents drawn from an external point to a circle are equal.

EXERCISE 10.1

2. Fill in the blanks (THESE CAN BE CONVERTED INTO MCQ IN BOARDS)

(i) A tangent to a circle intersects it in_________point (s).

(ii) A line intersecting a circle in two points is called a______.

(iii) A circle can have________parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called _________.

1. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

   (D)$\sqrt{119}$

4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

EXERCISE 10.2

n Q.1 to 3, choose the correct option and give justification.

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm

(C) 15 cm (D) 24.5 cm

2. In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to

(A) 60° (B) 70°

(C) 80° (D) 90°

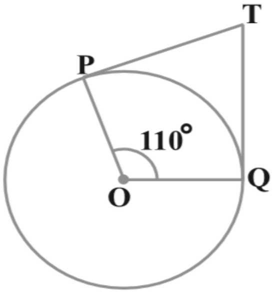


Fig. 10.11

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle POA$ is equal to

(A) 50° (B) 60°

(C) 70° (D) 80°

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

11. Prove that the parallelogram circumscribing a circle is a rhombus.

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

- **AREAS RELATED TO CIRCLES:**

NCERT:

Example 2 : Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector

Example 4 : In Fig. 12.15, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.

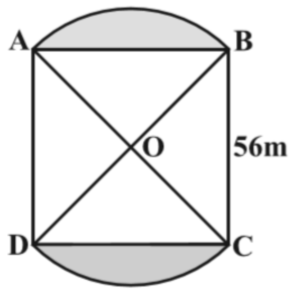
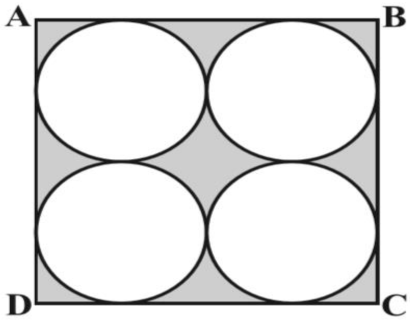


Fig. 12.15

Example 5 : Find the area of the shaded region in Fig. 12.16, where ABCD is a square of side 14 cm.



EXERCISE 12.1:

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

5. Tick the correct answer in the following and justify your choice :

If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(A) 2 units (B) pi units (C) 4 units (D) 7 units

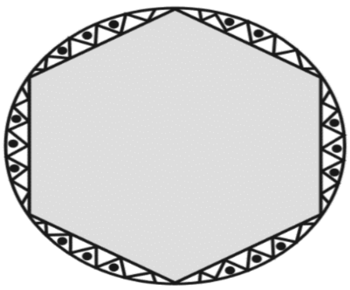
EXERCISE 12.2:

2. Find the area of a quadrant of a circle whose circumference is 22 cm.

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

13. A round table cover has six equal designs as shown in Fig. 12.14. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs 0.35 per cm2.



EXERCISE 12.3:

1. Find the area of the shaded region in Fig. 12.20, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and angle AOC=40degree.

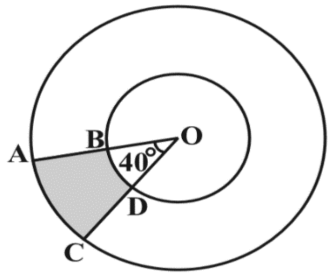
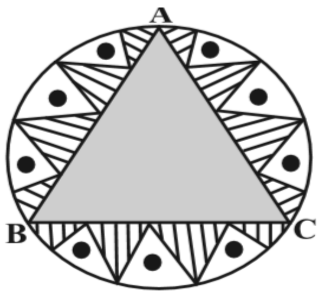


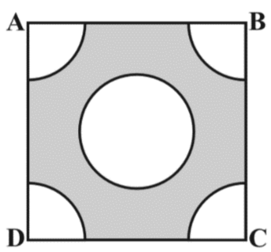
Fig. 12.20

portion of the square.

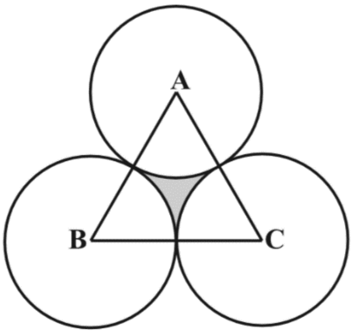
6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 12.24. Find the area of the design (shaded region).



5.From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 12.23. Find the area of the remaining portion of the square.



10. The area of an equilateral triangle ABC is 17320.5 cm2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 12.28). Find the area of the shaded region.



13. In Fig. 12.31, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3.14$)

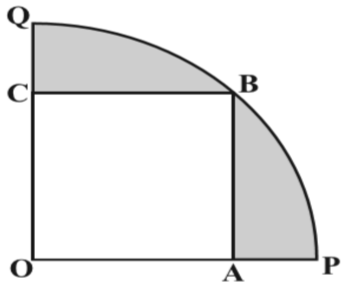


Fig. 12.31

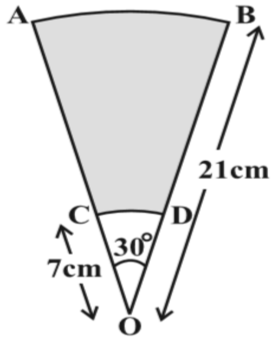


Fig. 12.32

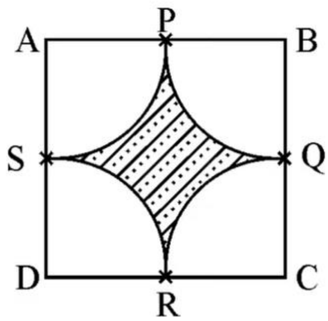
14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig. 12.32). If $\angle AOB = 30°$, find the area of the shaded region.

PREVIOUS YEARS

2018

Prove that the lengths of tangents drawn from an external point to a circle are equal.

Find the area of the shaded region in Fig. 2, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD of side 12 cm. [Use $\pi = 3.14$]



Radius of each arc drawn = 6 cm

Area of one quadrant $= (3.14) \times \frac{36}{4}$

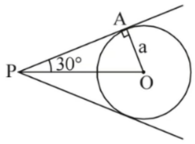
Area of four quadrants $= 3.14 \times 36 = 113.04 \text{ cm}^2$

Area of square ABCD $= 12 \times 12 = 144 \text{ cm}^2$

Hence Area of shaded region $= 144 - 113.04$

$= 30.96 \text{ cm}^2$

If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP.

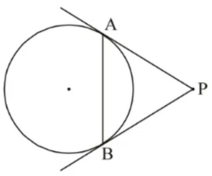


$\angle OPA = 30°$ $\frac{1}{2}$

$\sin 30° = \frac{a}{OP}$

$\Rightarrow$ OP = 2a $\frac{1}{2}$

Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.



Case I:

Correct Figure $\frac{1}{2}$

Since PA = PB

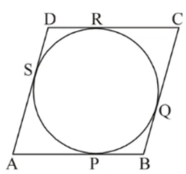
Therefore in ΔPAB $\frac{1}{2}$

$\angle PAB = \angle PBA$ $\frac{1}{2}$

Case II: If the tangents at A and B are parallel then each angle between chord and tangent = 90° $\frac{1}{2}$

A circle touches all the four sides of a quadrilateral ABCD. Prove that
AB + CD = BC + DA

8.



Here AP = AS

BP = BQ 1

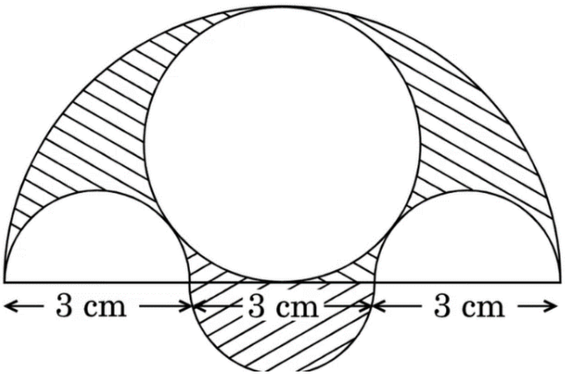
CR = CQ

DR = DS

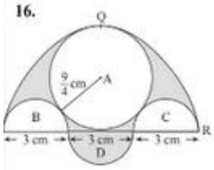
Adding (AP + PB) + (CR + RD) = (AS + SD) + (BQ + QC) $\frac{1}{2}$

$\Rightarrow$ AB + CD = AD + BC $\frac{1}{2}$

Three semicircles each of diameter 3 cm, a circle of diameter 4·5 cm and a semicircle of radius 4·5 cm are drawn in the given figure. Find the area of the shaded region.



16.



Area of semi-circle PQR $= \frac{\pi}{2}\left(\frac{9}{2}\right)^2 = \frac{81}{8}\pi\ \text{cm}^2$ $\frac{1}{2}$

Area of region A $= \pi\left(\frac{9}{4}\right)^2 = \frac{81}{16}\pi\ \text{cm}^2$ $\frac{1}{2}$

Area of region (B + C) $= \pi\left(\frac{3}{2}\right)^2 = \frac{9}{4}\pi\ \text{cm}^2$ $\frac{1}{2}$

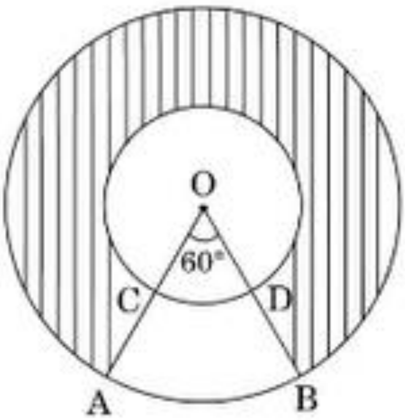
Area of region D $= \frac{\pi}{2}\left(\frac{3}{2}\right)^2 = \frac{9}{8}\pi\ \text{cm}^2$ $\frac{1}{2}$

Area of shaded region $= \left(\frac{81}{8}\pi - \frac{81}{16}\pi - \frac{9}{4}\pi + \frac{9}{8}\pi\right)\text{cm}^2$

$= \frac{63}{16}\pi\ \text{cm}^2$ or $\frac{99}{8}\text{cm}^2$ 1

In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If $\angle$ AOB = 60°, find the area of the shaded region.

[Use $\pi = \frac{22}{7}$]



Area of region ABDC $= \pi\frac{60}{360}\times(42^2 - 21^2)$

$= \frac{22}{7}\times\frac{1}{6}\times 63\times 21$

$= 693\ \text{cm}^2$

Area of shaded region $= \pi(42^2 - 21^2) -$ region ABDC

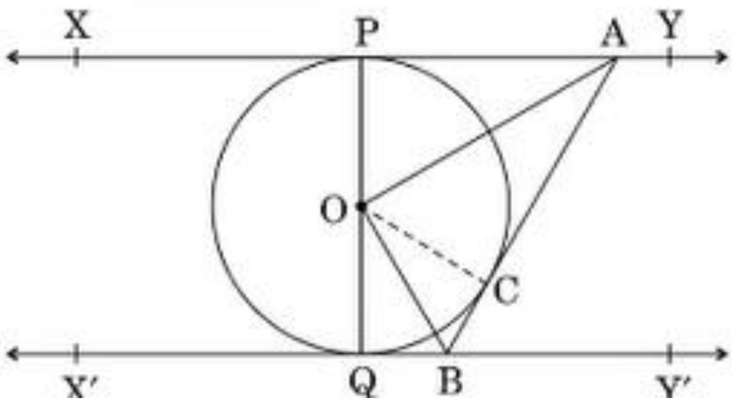
$= \frac{22}{7} \times 63 \times 21 - 693$

$= 4158 - 693$

$= 3465 \text{ cm}^2$

Prove that the lengths of two tangents drawn from an external point to a circle are equal.

In the given figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$.



In right angled ΔPOA and ΔOCA

$\Delta OPA \cong \Delta OCA$

$\therefore \quad \angle POA = \angle AOC \quad ...(i)$

Also $\Delta OQB \cong \Delta OCB$

$\therefore \quad \angle QOB = \angle BOC \quad ...(ii)$

Therefore $\angle AOB = \angle AOC + \angle COB$

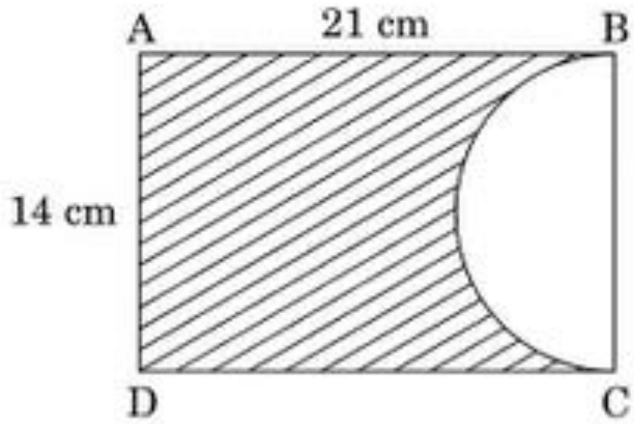
$= \frac{1}{2}\angle POC + \frac{1}{2}\angle COQ$

$= \frac{1}{2}(\angle POC + \angle COQ)$

$= \frac{1}{2} \times 180°$

$= 90°$

In the given figure, ABCD is a rectangle of dimensions 21 cm × 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.



Area of shaded region $= (21\times14) - \frac{1}{2}\times\pi\times7\times7$

$= 294 - \frac{1}{2}\times\frac{22}{7}\times7\times7$

$= 294 - 77$

$= 217\ cm^2$.

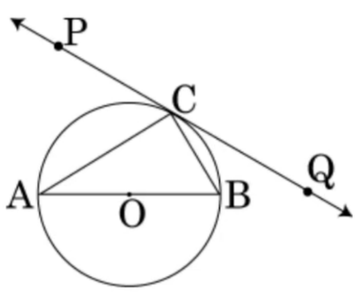
Perimeter of shaded region $= 21 + 14 + 21 + \frac{22}{7}\times7$

$= 56 + 22$

$= 78$ cm

2016

In fig.1, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30°$, find $\angle PCA$.



For $\angle ACB = 90°$

$\angle PCA = 60°$

In Fig.2, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that. AB + CD = BC + DA.

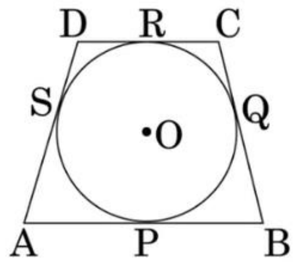


Figure 2

AP = AS, BP = BQ, CR = CQ and DR = DS

AP + BP + CR + DR = AS + BQ + CQ + DS ⇒ AB + CD = AD + BC

In Fig. 3, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If OP = 2r, show that $\angle OTS = \angle OST = 30°$.

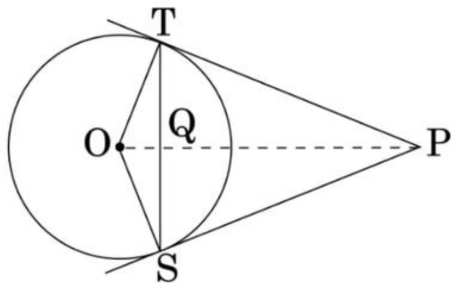


Figure 3

Let $\angle TOP = \theta \therefore \cos\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2} \therefore \theta = 60°$ Hence $\angle TOS = 120°$

In ΔOTS, $OT = OS \Rightarrow \angle OTS = \angle OST = 30°$

In fig.4, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. (Take π = 3.14)

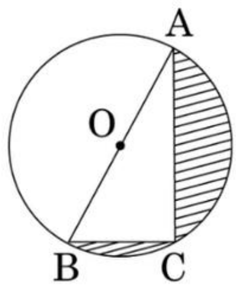


Figure 4

$BC^2 = AB^2 - AC^2 = 169 - 144 = 25 \therefore BC = 5cm$

Area of the shaded region = Area of semicircle – area of rt. ΔABC

$$= \frac{1}{2}(3.14)\left(\frac{13}{2}\right)^2 - \frac{1}{2}.12\times5$$

$$= 66.33 - 30 = 36.33 \text{ cm}^2$$

In fig. 6, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40°$. (Use $\pi = \frac{22}{7}$)

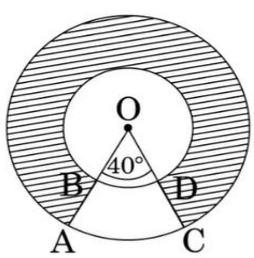


Figure 6

Shaded area $= \pi(14^2 - 7^2)\times\frac{320}{360}$

$$= \frac{22}{7}\times147\times\frac{8}{9}$$

$$= \frac{1232}{3} = 410.67 \text{ cm}^2$$

Prove that the lengths of the tangents drawn from an external point to a circle are equal.

In Fig. 7, two equal circles, with centres O and O', touch each other at X.OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.

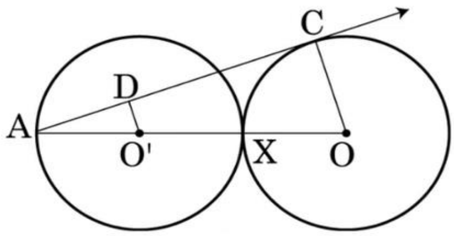


figure 7

AC is tangent to circle with centre 0,

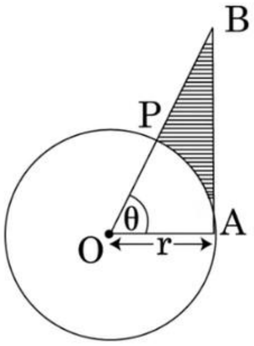
Thus $\angle ACO = 90°$

$\therefore \quad \Delta AO'D \sim \Delta AOC$

$\Rightarrow \quad \frac{AO'}{AO} = \frac{DO'}{CO}$

$\therefore \quad \frac{DO'}{CO} = \frac{r}{3r} = \frac{1}{3}$

In Fig. 9, is shown a sector OAP of a circle with centre O, containing $\angle\theta$. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$



Length of are $\widehat{AP} = 2\pi r\frac{\theta}{360}$ or $\frac{\pi r\theta}{180}$

$\frac{AB}{r} = \tan\theta \Rightarrow AB = r\tan\theta$

$\frac{OB}{r} = \sec\theta \Rightarrow OB = r\sec\theta$

$PB = OB - r = r\sec\theta - r$

Perimeter $= AB + PB + \widehat{AP}$

$= r\tan\theta + r\sec\theta - r + \frac{\pi r\theta}{180}$

or $\quad r\left[\tan\theta + \sec\theta - 1 + \frac{\pi\theta}{180}\right]$

2015

In Figure 2, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60°$, find $\angle PRQ$.

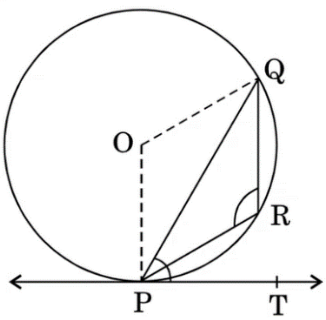


Figure 2

ans-120

In Figure 3, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle$ PRQ = 120°, then prove that OR = PR + RQ.

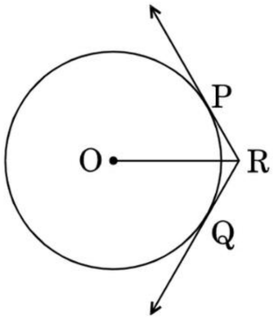
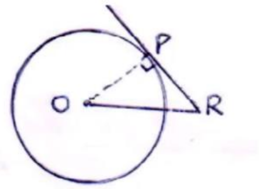


Figure 3



$\angle$ POR = 90 – 60 = 30° ½ m

$\frac{PR}{OR} = \sin 30° = \frac{1}{2} \Rightarrow OR = 2\,PR$

$= PR + QR$ ½ m

In Figure 4, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of Δ ABC is 54 cm^2, then find the lengths of sides AB and AC.

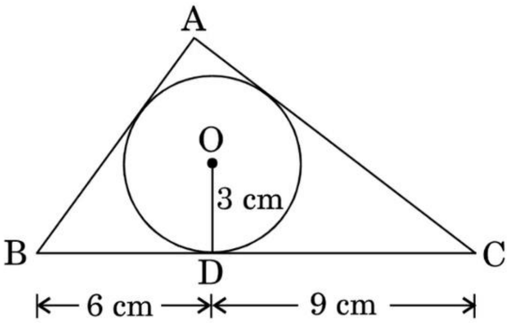
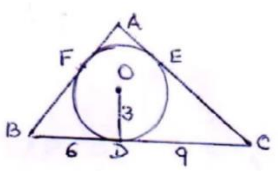


Figure 4



Let AF = AE = x

$\therefore$ AB = 6 + x, AC = 9 + x, BC = 15 ½ m

$\frac{1}{2}[15 + 6 + x + 9 + x] \cdot 3 = 54$ 1 m

$\Rightarrow x = 3 \;\therefore$ AB = 9 cm, AC = 12 cm ½ m

and BC = 15 cm

Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60°. Also find the area of the corresponding major segment. [Use $\pi = \frac{22}{7}$]

Area of minor segment $= \pi r^2 \frac{\theta}{360} - \frac{1}{2} r^2 \sin\theta$ ½ m

$= \frac{22}{7} \times 14 \times 14 \times \frac{60}{360} - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$ ½ m

$= \left(\frac{308}{3} - 49\sqrt{3}\right) cm^2$ or $17.89\ cm^2$ or $17.9\ cm^2$ Approx. 1 m

Area of Major segment

$= \pi r^2 - \left(\frac{308}{3} - 49\sqrt{3}\right)$ ½ m

$= \left(\frac{1540}{3} + 49\sqrt{3}\right) cm^2$ or $598.10\ cm^2$ ½ m

or $598\ cm^2$ Approx.

Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

In Figure 5, PQRS is a square lawn with side PQ = 42 metres. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).

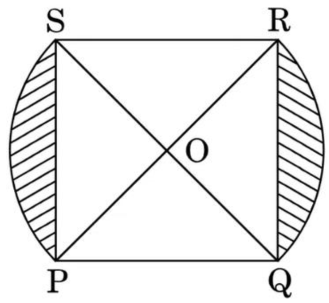


Figure 5

Radius of circle with centre O is OR

let OR = x $\therefore\ x^2 + x^2 = (42)^2 \Rightarrow x = 21\sqrt{2}$ m.

Area of one flower bed = Area of segment of circle with centre angle 90^0

$= \frac{22}{7} \times 21\sqrt{2} \times 21\sqrt{2} \times \frac{90}{360} - \frac{1}{2} \times 21\sqrt{2} \times 21\sqrt{2}$

$= 693 - 441 = 252\ m^2$

$\therefore$ Area of two flower beds $= 2 \times 252 = 504\ m^2$

2014

Two circles touch each other externally at P. AB is a common tangent to the circles touching them at A and B. The value of $\angle APB$ is

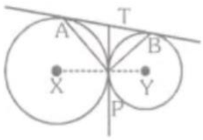
(A) 30°

(B) 45°

(C) 60°

(D) 90°

Correct answer: D



$TA = TP \Rightarrow \angle TAP = \angle TPA$

$TB = TP \Rightarrow \angle TBP = \angle TPB$

$\therefore \angle TAP + \angle TBP = \angle TPA + \angle TPB = \angle APB$

$\underline{\angle TAP + \angle TBP} + \angle APB = 180°$ $[\because \text{sum of.....} 180°]$

$\therefore \angle APB + \angle APB = 180°$

$\therefore 2\angle APB = 180°$

$\therefore \angle APB = 90°$

A chord of a circle of radius 10 cm subtends a right angle at its centre. The length of the chord (in cm) is

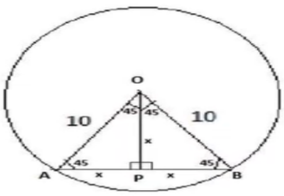
(A) $5\sqrt{2}$

(B) $10\sqrt{2}$

(C) $\frac{5}{\sqrt{2}}$

(D) $10\sqrt{3}$

Correct answer: B



Given $\angle AOB$ is given as 90°

ΔAOB is an isosceles triangle since OA = OB

Therefore $\angle OAB = \angle OBA = 45°$

Thus $\angle AOP = 45°$ and $\angle BOP = 45°$

Hence ΔAOP and ΔBOP also are isosceles triangles

Thus let AP = PB = OP = x

Using Pythagoras theorem

$x^2 + x^2 = 10^2$

Thus $2x^2 = 100$

$x = 5\sqrt{2}$

Hence length of chord AB = 2 x = $10\sqrt{2}$

In Figure 1, common tangents AB and CD to the two circles with centres O_1 and O_2 intersect at E. Prove that AB = CD.

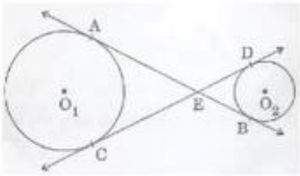
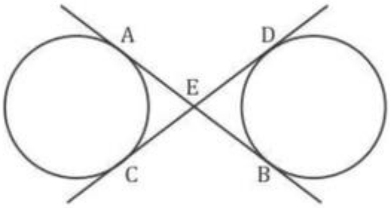


Figure 1

Solution:



Given: AB and CD are common tangents to both the circles.

To prove: AB = CD

Proof:

We know that two tangents drawn to a circle for the same exterior point are equal.

Thus we get AE = EC

Similarly ED = EB

AB = AE + EB and CD = ED + EC

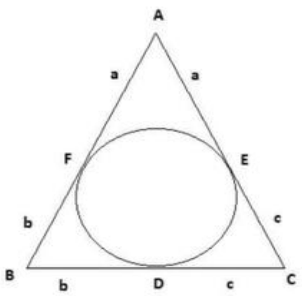
Since AE = EC we can write AB = EC + EB

And since ED = EB we get CD = EB + EC

Therefore AB = CD

Hence proved.

The incircle of an isosceles triangle ABC, in which AB = AC, touches the sides BC, CA and AB at D, E and F respectively. Prove that BD = DC.



Given: ΔABC is an isosceles triangle with a circle inscribed in the triangle.

To prove: BD = DC

Proof:

AF and AE are tangents drawn to the circle from point A.

Since two tangents drawn to a circle from the same exterior point are equal,
AF = AE = a

Similarly BF = BD = b and CD = CE = c

We also know that ΔABC is an isosceles triangle

Thus AB = AC,

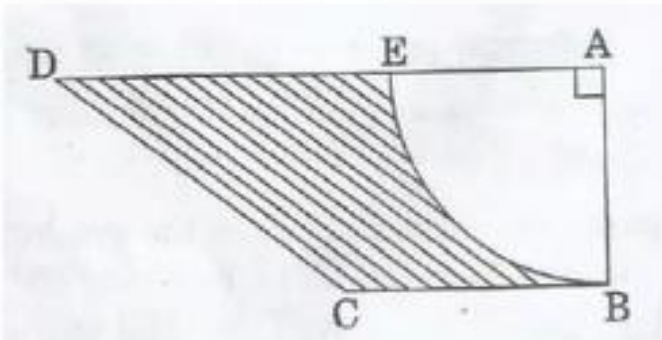
a + b = a + c

Thus b = c

Therefore, BD = DC

Hence proved.

. In Figure 2, ABCD is a trapezium of area 24.5 sq. cm. In it, AD|| BC, $\angle DAB = 90^0$, AD = 10 cm and BC = 4 cm. If ABE is a quadrant of a circle, find the area of the shaded region. $\left[\text{Take } \pi = \frac{22}{7}\right]$



Solution:

Area of trapezium = 24.5 cm^2

½ [AD + BC] × AB = 24.5 cm^2

½ [10+4] × AB = 24.5

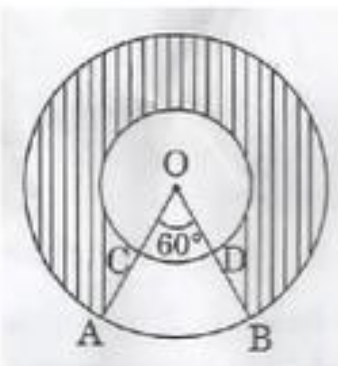
AB = 3.5 cm

r = 3.5 cm

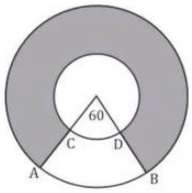
Area of quadrant = ¼ × pi × r^2 = 0.25 × 22/7 × 3.5 × 3.5 = 9.625 cm^2

The area of shaded region = 24.5 - 9.625 = 14.875 cm^2

. In Figure 3, two concentric circles with centre O, have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Solution:



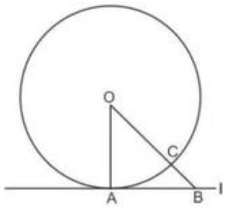
The area of shaded region = Area of ring – Area of ABCD

$= [\pi(R^2 - r^2)] - [\pi (R^2 - r^2)]\times(\theta/360)$

$= [\pi (R^2 - r^2)] [1 - (\theta/360)]$

$= [22/7 (422 - 212)] [1 - (60/360)] = 3465 \text{ cm}^2$

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.



Construction: Take a point B, other than A, on the tangent l. Join OB. Suppose OB meets the circle in C.

Proof: We know that, among all line segment joining the point O to a point on l, the perpendicular is shortest to l.

OA = OC (Radius of the same circle)

Now, OB = OC + BC.

$\therefore$ OB > OC

$\Rightarrow$ OB > OA

$\Rightarrow$ OA < OB

. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

2013

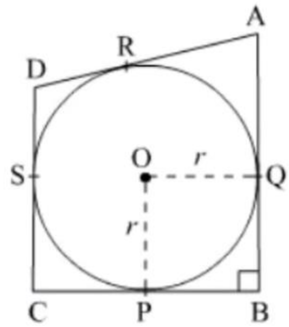
In fig., a circle with centre O is inscribed in a quadrilateral ABCD such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively, If AB = 29 cm, AD = 23 cm, $\angle B = 90^\circ$ and DS = 5 cm, then the radius of the circle (in cm) is

(A) 11

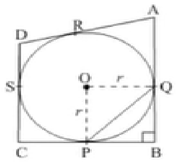
(B) 18

(C) 6

(D) 15



Correct answer: A



Given: AB, BC, CD and AD are tangents to the circle with centre O at Q, P, S and R respectively.

AB = 29 cm, AD = 23, DS = 5 cm and $\angle B = 90^\circ$

Construction: Join PQ.

We know that, the lengths of the tangents drawn from an external point to a circle are equal.

DS = DR = 5 cm

$\therefore$ AR = AD - DR = 23 cm - 5 cm = 18 cm

AQ = AR = 18 cm

$\therefore$ QB = AB - AQ = 29 cm - 18 cm = 11 cm

QB = BP = 11 cm

In $\triangle$ PQB,

$PQ^2 = QB^2 + BP^2 = (11\text{ cm})^2 + (11\text{ cm})^2 = 2 \times (11\text{ cm})^2$

$PQ = 11\sqrt{2}$ cm ... (1)

In $\triangle$ OPQ,

$PQ^2 = OQ^2 + OP^2 = r^2 + r^2 = 2r^2$

$(11\sqrt{2})^2 = 2r^2$

$121 = r^2$

r = 11

Thus, the radius of the circle is 11 cm.

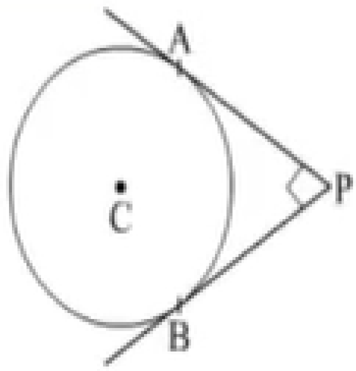
In fig., PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA $\perp$ PB, then the length of each tangent is

(A) 3 cm

(B) 4 cm

(C) 5 cm

(D) 6 cm



Correct answer: B

AP $\perp$ PB (Given)

CA $\perp$ AP, CB $\perp$ BP (Since radius is perpendicular to tangent)

AC = CB = radius of the circle

Therefore, APBC is a square having side equal to 4 cm.

Therefore, length of each tangent is 4 cm.

. If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = \frac{22}{7}$, the circumference (in cm) of the circle is:

(A) 154

(B) 44

(C) 14

(D) 7

Correct answer: B

Let r be the radius of the circle.

From the given information, we have

$2\pi r - r = 37$ cm

$\Rightarrow r\ 2\pi - 1\ = 37$ cm

$\Rightarrow r\left(2\times\frac{22}{7}-1\right) = 37$ cm

$\Rightarrow r\times\frac{37}{7} = 37$ cm

$\Rightarrow r = 7$ cm

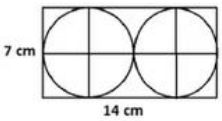
$\therefore$ Circumference of the circle $= 2\pi r = 2\times\frac{22}{7}\times 7$ cm $= 44$ cm

Prove that the parallelogram circumscribing a circle is a rhombus.

Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular card board of dimensions 14 cm $\times$ 7 cm. Find the area of the remaining card board. $\left[\text{Use}\pi = \frac{22}{7}\right]$

Dimension of the rectangular card board = 14 cm $\times$ 7 cm

Since, two circular pieces of equal radii and maximum area touching each other are cut from the rectangular card board, therefore, the diameter of each of each circular piece is $\frac{14}{2}$ = 7 cm.



Radius of each circular piece = $\frac{7}{2}$ cm.

$\therefore$ Sum of area of two circular pieces = $2\times\pi\left(\frac{7}{2}\right)^2 = 2\times\frac{22}{7}\times\frac{49}{4} = 77\text{cm}^2$

Area of the remaining card board

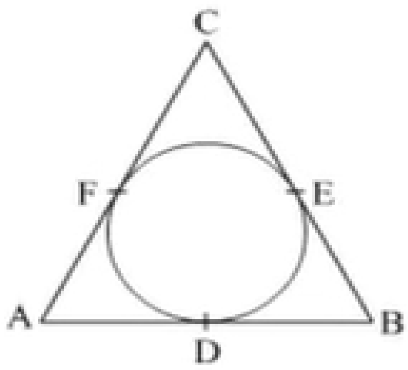
= Area of the card board - Area of two circular pieces

= 14 cm $\times$ 7 cm - 77 cm^2

= 98 cm^2 - 77 cm^2

= 21 cm^2

In fig., a circle is inscribed in triangle ABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the length of AD, BE and CF.



Given: AB = 12 cm, BC = 8 cm and AC = 10 cm.

Let, AD = AF = x cm, BD = BE = y cm and CE = CF = z cm

(Tangents drawn from an external point to the circle are equal in length)

$\Rightarrow$ 2(x + y + z) = AB + BC + AC = AD + DB + BE + EC + AF + FC = 30 cm

$\Rightarrow$ x + y + z = 15 cm

AB = AD + DB = x + y = 12 cm

$\therefore$ z = CF = 15 - 12 = 3 cm

AC = AF + FC = x + z = 10 cm

$\therefore$ y = BE = 15 - 10 = 5 cm

$\therefore$ x = AD = x + y + z - z - y = 15 - 3 - 5 = 7 cm

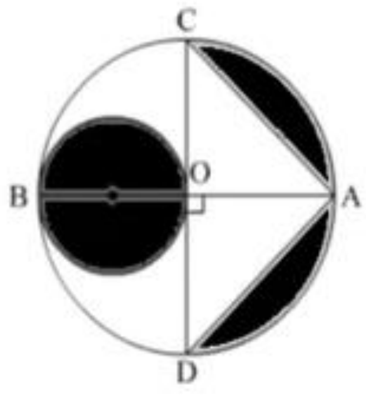
In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find: (i) the length of the arc (ii) area of the sector formed by the arc. $\left[\text{Use } \pi = \frac{22}{7}\right]$

The arc subtends an angle of 60° at the centre.

(i) $l = \frac{\theta}{360^\circ} \times 2\pi r = \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21 \text{ cm} = 22 \text{ cm}$

(ii) Area of the sector $= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 2$

In Fig., AB and CD are two diameters of a circle with centre O, which perpendicular to each other. OB is the diameter of the smaller circle. If = 7 cm, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



AB and CD are the diameters of a circle with centre O.

$\therefore$ OA = OB = OC = OD = 7 cm (Radius of the circle)

Area of the shaded region

= Area of the circle with diameter OB + (Area of the semi-circle ACDA - Area of $\triangle$ ACD)

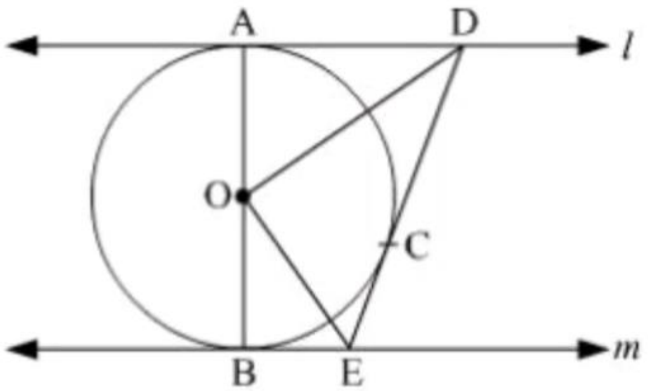
$= \pi\left(\frac{7}{2}\right)^2 + \left(\frac{1}{2} \times \pi \times 7^2 - \frac{1}{2} \times CD \times OA\right)$

$= \frac{22}{7} \times \frac{49}{4} + \frac{1}{2} \times \frac{22}{7} \times 49 - \frac{1}{2} \times 14 \times 7$

$= \frac{77}{2} + 77 - 49$

$= 66.5 \text{ cm}^2$

In fig., l and m are two parallel tangents to a circle with centre O, touching the circle at A and B respectively. Another tangent at C intersects the line l at D and m at E. Prove that $\angle DOE = 90^o$

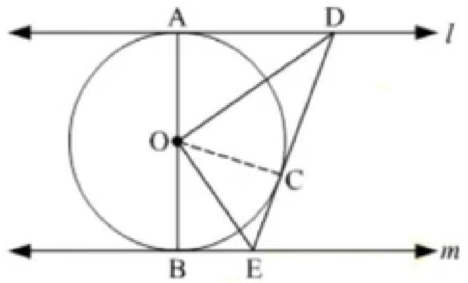


Given: *l* and m are two parallel tangents to the circle with centre O touching the circle at A and B respectively. DE is a tangent at the point C, which intersects *l* at D and m at E.

To prove: $\angle DOE = 90^o$

Construction: Join OC.

Proof:



In ΔODA and ΔODC,

OA = OC (Radii of the same circle)

AD = DC (Length of tangents drawn from an external point to a circle are equal)

DO = OD (Common side)

$\Delta ODA \cong \Delta ODC$ (SSS congruence criterion)

$\therefore \angle DOA = \angle COD$ ... (1)

Similarly, $\Delta OEB \cong \Delta OEC$

$\therefore \angle EOB = \angle COE$ ... (2)

Now, AOB is a diameter of the circle. Hence, it is a straight line.

$\angle DOA + \angle COD + \angle COE + \angle EOB = 180^o$

From (1) and (2), we have:

$2\angle COD + 2\angle COE = 180^\circ$

$\Rightarrow \angle COD + \angle COE = 90^o$

$\Rightarrow \angle DOE = 90^o$

Hence, proved.

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

2012

In Fig. 1, the sides AB, BC and CA of a triangle ABC, touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11 cm, then the length of BC (in cm) is :

(A) 11

(B) 10

(C) 14

(D) 15

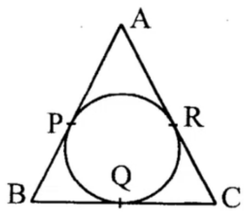


Fig. 1

ans-B

In Fig 2, a circle touches the side DF of Δ EDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then the perimeter of Δ EDF (in cm) is :

(A) 18

(B) 13.5

(C) 12

(D) 9

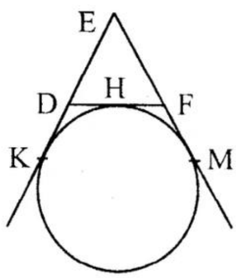


Fig. 2

ans-A

Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii 8 cm and 5 cm respectively, as shown in Fig. 3. If AP = 15 cm, then find the length of BP.

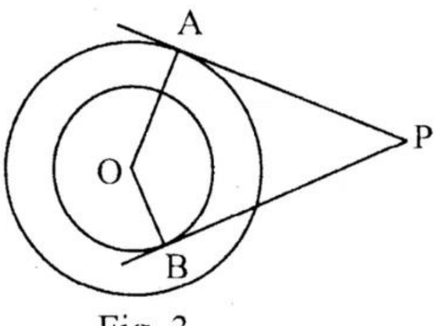


Fig. 3

Join OP. In right Δ OAP, $OP^2 = AP^2 + OA^2 = 15^2 + 8^2 = 289 = 17^2$

$\Rightarrow$ OP = 17 cm

Again, $BP^2 = OP^2 - OB^2 = 17^2 - 5^2 = 22 \times 12 = 264$

$\therefore$ $BP = \sqrt{264}$ or $2\sqrt{66}$

In Fig. 4, an isosceles triangle ABC, with AB = AC, circumscribes a circle. Prove that the point of contact P bisects the base BC.

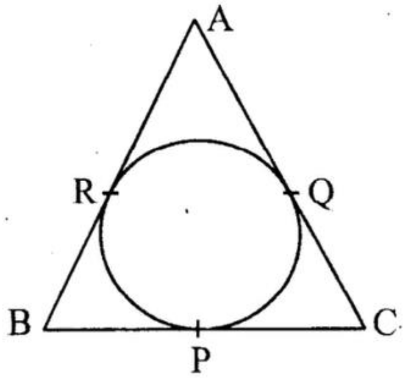


Fig. 4

or

In Fig. 5, the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that AC = CB.

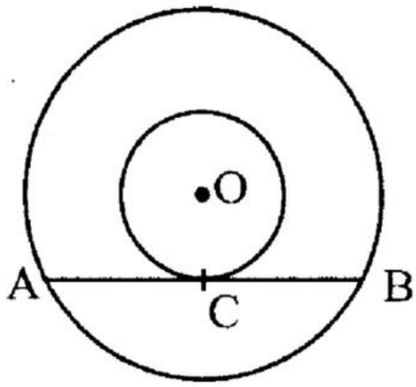


Fig. 5

AR = AQ, BR = BP and QC = PC (lengths of tangents from an external point are equal)

$\therefore$ AR + BR + PC = AQ + QC + BP or AB + PC = AC + BP

But AB = AC (Given) $\Rightarrow$ BP = PC or P bisects base BC

OR

Join OC. OC $\perp$ AB (A tangent is $\perp$ to the radius at the point of contact)

Also, perpendicular from the centre of a circle to a chord bisects it

$\therefore$ AC = CB

In Fig. 6, OABC is a square of side 7 cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region. [Use $\pi = \frac{22}{7}$]

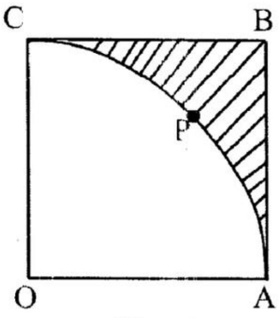


Fig. 6

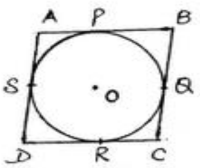
Shaded area = (Area of square of side 7 cm) – (Area of Quadrant of circle of radius 7 cm)

$$= \left(7^2 - \frac{1}{4} \cdot \frac{22}{7} \times 7 \times 7\right) \text{cm}^2 = 10.5 \text{ cm}^2$$

Prove that the parallelogram circumscribing a circle is a rhombus.

OR

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



ABCD is a parallelogram

circumscribing a circle with centre O

AP = AS, DS = DR, CR = CQ, PB = BQ

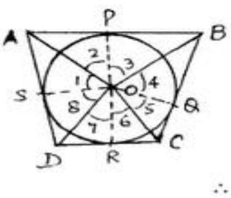
$\therefore$ (AP + PB) + (DR + CR) = (AS + DS) + (BQ + QC)

$\Rightarrow$ AB + DC = AD + BC or 2AB = 2AD

$\Rightarrow$ AB = AD

$\therefore$ A BCD is a rhombus

OR



Figure

Tangents drawn to a circle from an external point subtend equal angles at the centre

$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6$ and $\angle 7 = \angle 8$ .....(i)

Also $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$ .....(ii)

From (i) and (ii)

$$2(\angle 2 + \angle 3) + 2(\angle 6 + \angle 7) = 360^\circ$$

$\therefore (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^\circ$ or $\angle AOB + \angle DOC = 180^\circ$

Similarly, $\angle AOD + \angle BOC = 180^\circ$

In Fig. 7, PQ and AB are respectively the arcs of two concentric circles of radii 7 cm and 3.5 cm and centre O. If $\angle$ POQ = 30°, then find the area of the shaded region. [Use $\pi = \frac{22}{7}$]

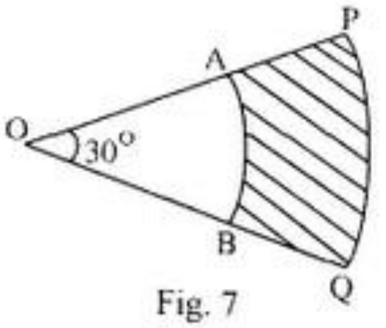


Fig. 7

Area of shaded region $= \frac{22}{7}\times\frac{30^\circ}{360^\circ}\left(7^2-\left(\frac{7}{2}\right)^2\right)$ cm^2

$= \left(\frac{22}{7}\times\frac{1}{12}\times\frac{147}{4}\right)$ cm^2

$= \frac{77}{8}$ or $9\frac{5}{8}$ cm^2

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

OR

A quadrilateral ABCD is drawn to circumscribe a circle. Prove that
AB + CD = AD + BC.