X

MATHEMATICS

COMPLETE QUESTION BANK

DISCLAIMER- Please Watch Complete Video before solving this question bank.(Nuksaan tumhara hi hoga varna)

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REAL NUMBERS

NCERT:

Example 1 : Use Euclid's algorithm to find the HCF of 4052 and 12576.

Example 3 : Show that any positive odd integer is of the form 4q + 1 or 4q + 3, where q is some integer.

Example 7 : Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

Example 8 : Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

Example 11 : Show that 32 is irrational.

EXERCISE 1.1

1. Use Euclid's division algorithm to find the HCF of : (i) 135 and 225
2. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

EXERCISE 1.2

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers. (i) 26 and 91 (ii) 510 and 92
3. Find the LCM and HCF of the following integers by applying the prime factorisation method. (i) 12, 15 and 21

4. Check whether 6n can end with the digit 0 for any natural number n.

EXERCISE 1.3

1. Prove that 5 is irrational.

EXERCISE 1.4

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(iii)64/455 (v)29/343

PREVIOUS YEARS(NUMBER SYSTEM):

- ✓ What is the HCF of smallest prime number and the smallest composite number ? [2018,1]

The required numbers are 2 and 4.

HCF of 2 and 4 is 2.

- ✓ Given that root(2) is irrational, prove that root(5 + 3 2) is an irrational number. [2018,2]

Let us assume $5+3\sqrt{2}$ is a rational number.

$\therefore \quad 5+3\sqrt{2}=\frac{p}{q}$ where $q \neq 0$ and p and q are integers.

$\Rightarrow \quad \sqrt{2}=\frac{p-5q}{3q}$

$\Rightarrow \quad \sqrt{2}$ is a rational number as RHS is rational

This contradicts the given fact that $\sqrt{2}$ is irrational.

Hence $5+3\sqrt{2}$ is an irrational number.

- ✓ Find HCF and LCM of 404 and 96 and verify that HCF x LCM = Product of the two given numbers [2018,3]

$404 = 2 \times 2 \times 101 = 2^2 \times 101$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$\therefore$ HCF of 404 and 96 $= 2^2 = 4$

LCM of 404 and 96 $= 101 \times 2^5 \times 3 = 9696$

HCF $\times$ LCM $= 4 \times 9696 = 38784$

Also $404 \times 96 = 38784$

Hence HCF $\times$ LCM = Product of 404 and 96.

- ✓ How many 3 digit natural numbers are divisible by 7?

[2013,2]

Three digit numbers divisible by 7 are

105, 112, 119, ... 994

This is an AP with first term (a) = 105 and common difference (d) = 7

Let a_n be the last term.

$a_n = a + (n - 1)d$

$994 = 105 + (n - 1)(7)$

$7(n - 1) = 889$

$n - 1 = 127$

$n = 128$

Thus, there are 128 three-digit natural numbers that are divisible by 7.

- ✓ How many 2 digit numbers are divisible by 3? [2012,2]

The numbers are 12, 15, 18, ---, 99

$\therefore \quad 99 = 12 + (n-1)\,3$

$\Rightarrow n = 30$

- ✓ Find the sum of all multiples of 7 lying between 500 and 900.[2012,3]

The numbers are 504, 511, 518, ---, 896 — 1 m

$\Rightarrow 896 = 504 + (n-1)\,7$ — ½ m

$\Rightarrow n = 57$ — ½ m

$S_{57} = \frac{57}{2}\cdot[504+896] = \frac{57}{2}[1400]$ — ½ m

$= 57 \times 700 = 39900$ — ½ m

- ✓ The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by 1/15. Find the fraction. [2012,4]

let the denominator be x, so numerator is $x-3$

Hence, fraction is $\frac{x-3}{x}$ — ½ m

$\therefore \frac{x-3}{x} - \frac{x-3}{x+1} = \frac{1}{15}$ — 1 m

$\Rightarrow 15\,[(x+1)(x-3)-(x-3)(x)] = x\,(x+1)$

$\Rightarrow 15\,(x-3)\,(1) = x^2 + x$

or $x^2 - 14x + 45 = 0$ — 1 m

$\Rightarrow (x-5)(x-9) = 0$ or $x = 5, 9$ — ½ m

$\therefore$ Fraction is $= \frac{2}{5}$ or $\frac{6}{9}$ — 1 m

- ✓ how many 2 digit numbers are divisible by 6? [2011,2]
- ✓ Has the rational number $441/2^2.5^7.7^2$ a terminating or non-terminating decimal representation? [2010,1]

terminating

- ✓ Prove that 2-3root(5) is an irrational number. [2010,3]

Let $2-3\sqrt{5} = x$, where x is a rational number ½ m

$\therefore\ 2-x = 3\sqrt{5}$ or $\frac{2-x}{3} = \sqrt{5}$ ....(i) ½ m

As x is a rational number, so is $\frac{2-x}{3}$ 1 m

$\therefore$ LHS of (i) is rational but RHS of (i) is irrational

$\therefore$ Our supposition that x is rational is wrong

$\Rightarrow\ 2-3\sqrt{5}$ is irrational 1 m

- ✓ Three consecutive positive integers are such that the sum of the square of the first and the product of the other two is 46, find the integers.

Let the three consecutive numbers be $x, x+1, x+2$ 1 m

According to the question

$$x^2 + (x+1)(x+2) = 46$$

or $2x^2 + 3x - 44 = 0 \Rightarrow 2x^2 + 11x - 8x - 44 = 0$ 2 m

$\Rightarrow (x-4)(2x+11) = 0$ 1 m

As x is positive $\Rightarrow x = 4 \left(x = \frac{-11}{2} \text{ rejected}\right)$ 1 m

$\therefore$ The numbers are 4, 5, 6 1 m

Or

The difference of squares of two numbers is 88. If the larger number is 5 less than twice the smaller number, then find the two number.

Ans-x=-29/3 y=-7/3

HOTS

Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number times.

Solution:

Maximum capacity of a container, which can measure the petrol in exact number of times.

= HCF of (850 and 680)

2	850
5	425
5	85
	17

2	680
2	340
2	170
5	85
	17

$850 = 2 \times 5 \times 5 \times 17$

$680 = 2 \times 2 \times 2 \times 5 \times 17$

HCF = (850 and 680) = $2 \times 5 \times 17 = 170$ litres.

Find the value of: $(-1) + (-1)^{2n} + (-l)^{2n+1} + (-l)^{4n+1}$, where n is any positive odd integer

Solution:

To find $(-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2}$

as 'n' is any positive odd integer

$\Rightarrow$ $2n$ and $4n + 2$ are even positive integers.

Now

$$(-1)^n = -1$$

$$(-1)^{2n} = +1$$

$$(-1)^{2n+1} = (-1)^{2n}(-1)^1 = 1(-1) = -1$$

$$(-1)^{4n+2} = (-1)^{4n}(-1)^2 = 1(1) = 1$$

Put all values in the above expression (i)

We have $-1 + 1 - 1 + 1 = 0$

Prove that the product of any three consecutive positive integers is divisible by 6.

Solution: Let three consecutive numbers are n, n + 1, n + 2

Solution:

1st Case: If n is even

This means n + 2 is also even.

Hence n and n + 2 are divisible by 2

Also, product of n and (n + 2) is divisible by 2.

∴ n(n + 2) is divisible by 2.

This conclude n(n + 2) (n + 1) is divisible by 2 ...(i)

As, n, n + 1, n + 2 are three consecutive numbers. n(n + 1) (n + 2) is a multiple of 3.

This shows n(n + 1) (n + 2) is divisible by 3. ...(ii)

By equating (i) and (ii) we can say

n(n + 1) (n + 2) is divisible by 2 and 3 both.

Hence, n(n + 1) (n + 2) is divisible by 6.

2nd Case: When n is odd.

This show (n + 1) is even

Hence (n + 1) is divisible by 2. ...(iii)

This conclude n(n + 1) (n + 2) is an even number and divisible by 2.

Also product of three consecutive number is a multiple of 3.

n(n + 1)(n + 2) is divisible by 3. ...(iv)

Equating (iii) and (iv) we can say

n(n + 1) (n + 2) is divisible by both 2 and 3 Hence, n(n + 1)(n + 2) is divisible by 6.

Three bells toll at intervals of 12 minutes, 15 minutes and 18 minutes respectively. If they start tolling together, after what time will they next toll together?

Solution:

LCM of 12, 15, 18 = $2^2 \times 3^2 \times 5$

$= 4 \times 9 \times 5 = 180$

So, next time the bells will ring together after 180 minutes.

2	12, 15, 18
2	6, 15, 9
3	3, 15, 9
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

Show that reciprocal of 3+2√2 is an irrational number

Solution:

We have to prove that $\frac{1}{3+2\sqrt{2}} = 3 - 2\sqrt{2}$ is an irrational number.

Let us assume that $3 - 2\sqrt{2}$ is rational.

$\therefore$ There exists coprime integers a and b $(b \neq 0)$ such that

$$3 - 2\sqrt{2} = \frac{a}{b} \Rightarrow 2\sqrt{2} = 3 - \frac{a}{b}$$

$$\Rightarrow \quad 2\sqrt{2} = \frac{3b-a}{b}$$

$$\Rightarrow \quad \sqrt{2} = \frac{3b-a}{2b} = \frac{3}{2} - \frac{a}{2b}$$

Since a and b are integers, we get $\frac{3}{2} - \frac{a}{2b}$ is rational and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3 - 2\sqrt{2}$ is rational.

Hence, we conclude that $3 - 2\sqrt{2}$ is irrational.

If $\frac{241}{4000} = \frac{241}{2^m 5^n}$, find the values of m and n where m and n are non-negative integers.Hence write its decimal expansion without actual division.

Solution:

$$\frac{241}{4000} = \frac{241}{2^m \cdot 5^n} \Rightarrow 4000 = 2^m.5^n$$

$$2^5 \times 5^3 = 2^m.5^n \Rightarrow m = 5, n = 3$$

$$\frac{241}{4 \times 1000} = \frac{1}{4} \times 0.241 = 0.06025 \text{ (Terminating)}.$$

Short Answer Type Questions II [3 Marks]

Has the rational number $\frac{441}{2^2 \cdot 5^7 \cdot 7^2}$, a terminating or a non-terminating decimal repressentation

Solution:

$\frac{441}{2^2 \cdot 5^7 \cdot 7^2}$ is non-terminating decimal.

Since $q = 2^2 \times 5^7 \times 7^2$ is not of the form $2^m \times 5^n$.

Question 31.

Write whether $\frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}}$ on simplification gives a rational or an irrational

Solution:

$$\frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{9 \times 5} + 3\sqrt{4 \times 5}}{2\sqrt{5}} = \frac{6\sqrt{5} + 6\sqrt{5}}{2\sqrt{5}} = \frac{12\sqrt{5}}{2\sqrt{5}}$$

$= 6$ which is rational number.