X

MATHEMATICS

COMPLETE QUESTION BANK

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ALGEBRA

- **POLYNOMIALS**

NCERT:

Example 3 : Find the zeroes of the polynomial $x^2 – 3$ and verify the relationship between the zeroes and the coefficients.

Example 9:Find all the zeroes of $2x^4 – 3x^3 – 3x^2 + 6x – 2$, if you know that two of its zeroes are root(2) and -root(2).

EXERCISE 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients. (i) $x^2 – 2x – 8$ (ii) $4s^2 – 4s + 1$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i)1/4, -1 (v)-1/4, 1/4

EXERCISE 2.3

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following : (i) $p(x) = x^3 – 3x^2 + 5x – 3$, $g(x) = x^2 – 2$
2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial: (iii) $x^3 – 3x + 1$, $x^5 – 4x^3 + x^2 + 3x + 1$

3. Obtain all other zeroes of $3x^4 + 6x^3 – 2x^2 – 10x – 5$, if two of its zeroes are

Root(5/3) and – root(5/3).

3. On dividing $x^3 – 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were x – 2 and –2x + 4, respectively. Find g(x).

EXERCISE 2.4

3.If the zeroes of the polynomial $x^3 – 3x^2 + x + 1$ are a – b, a, a + b, find a and b.

5. If the polynomial $x^4 – 6x^3 + 16x^2 – 25x + 10$ is divided by another polynomial $x^2 – 2x + k$, the remainder comes out to be x + a, find k and a.

- **PAIR OF LINEAR EQUATIONS IN TWO VARIABLES:**

NCERT:

Example 3 : Two rails are represented by the equations x + 2y – 4 = 0 and 2x + 4y – 12 = 0. Represent this situation geometrically.

Example 7 : Solve the following pair of equations by substitution method:

7x – 15y = 2 (1)

x + 2y = 3 (2)

Example 13 : The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

Example 15 : For which values of p does the pair of equations given below has unique solution?

4x + py + 8 = 0

2x + 2y + 2 = 0

Example 16 : For what values of k will the following pair of linear equations have infinitely many solutions?

kx + 3y – (k – 3) = 0

12x + ky – k =0

Example 19 : A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km down-stream. Determine the speed of the stream and that of the boat in still water.

EXERCISE 3.2

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen.

5.Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

7. Draw the graphs of the equations $x – y + 1 = 0$ and $3x + 2y – 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

EXERCISE 3.3

1. Solve the following pair of linear equations by the substitution method.

(iii) $3x – y = 3$ $9x – 3y = 9$

(vi) $3x/2-5y/3=-2$ $x/3+y/2=13/6$

2. Solve $2x + 3y = 11$ and $2x – 4y = – 24$ and hence find the value of 'm' for which $y = mx + 3$.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

EXERCISE 3.4

1. Solve the following pair of linear equations by the elimination method and the substitution method :

(ii) $3x + 4y = 10$ and $2x – 2y = 2$

(iii) $3x – 5y – 4 = 0$ and $9x = 2y + 7$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

EXERCISE 3.5

2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$2x + 3y = 7$

$(a – b) x + (a + b) y = 3a + b – 2$

(ii) For which value of k will the following pair of linear equations have no solution?

$3x + y = 1$

(2k – 1) x + (k – 1) y = 2k + 1

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods :

8x + 5y = 9

3x + 2y = 4

4.Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method :

(ii) A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes ¼ when 8 is added to its denominator. Find the fraction.

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

EXERCISE 3.6

1. Solve the following pairs of equations by reducing them to a pair of linear equations:

   (iv)5/x-1 + 1/y-2=2 (vi)6x+3y=6xy

   6/x-1 – 3/y-2 =1 2x+4y=5xy

2. Formulate the following problems as a pair of equations, and hence find their solutions:

   (i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

IMPORTANT QUESTIONS

• Polynomials

Find all zeroes of the polynomial ($2x^4 – 9x^3 + 5x^2 + 3x – 1$) if two of its zeroes are (2 + root(3)) and (2 – root(3)). [2018,3]

$2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of p(x)

$\therefore$ $p(x) = (x-2-\sqrt{3})(x-2+\sqrt{3}) \times g(x)$

$= (x^2 - 4x + 1)\, g(x)$ 1

$(2x^4 - 9x^3 + 5x^2 + 3x - 1) \div (x^2 - 4x + 1) = 2x^2 - x - 1$ 1

$\therefore$ $g(x) = 2x^2 - x - 1$

$= (2x + 1)(x - 1)$

Therefore other zeroes are $x = -\frac{1}{2}$ and $x = 1$

$\therefore$ Therefore all zeroes are $2+\sqrt{3}, 2-\sqrt{3}, -\frac{1}{2}$ and 1 1

Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Answer:

α and β are the zeroes of the polynomial $2x^2 - 3x + 1$

$$\Rightarrow \quad \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$$

$$\alpha\beta = \frac{c}{a} = \frac{1}{2}$$

Now, zeroes of the required polynomial are 3α and 3β

$$\Rightarrow \quad S = 3\alpha + 3\beta = 3(\alpha + \beta) = 3\left(\frac{3}{2}\right) = \frac{9}{2}$$

$$\Rightarrow \quad P = (3\alpha)(3\beta) = 9(\alpha\beta) = 9 \times \frac{1}{2} = \frac{9}{2}$$

Now, required polynomial is $x^2 - Sx + p$

$$= x^2 - \frac{9}{2}x + \frac{9}{2} = \frac{k}{2}(2x^2 - 9x + 9), \text{ where } k \text{ be any constant.}$$

An NGO decided to distribute books and pencils to the students of a school running by some other NGO. For this they collected some amount from different people. The total amount collected is represented by $4x^4 + 2x^3 - 8x^2 + 3x - 7$. From this fund each student received an equal amount. The number of students, who received the amount, is represented by $x - 2 + 2x^2$. After distribution, $5x - 11$, amount is left with the NGO which they donated to school for their infrastructure. Find the amount received by each student from the NGO. What value has been depicted here?

Answer:

The total amount collected, $p(x) = 4x^4 + 2x^3 - 8x^2 + 3x - 7$

Number of students, $g(x) = x - 2 + 2x^2 = 2x^2 + x - 2$

Let amount received by each students, $q(x)$.

Amount left after distribution, $r(x) = 5x - 11$

By using division algorithm, we have

$$p(x) = g(x) \cdot q(x) + r(x)$$

$$4x^4 + 2x^3 - 8x^2 + 3x - 7 = (2x^2 + x - 2)\, q(x) + (5x - 11)$$

$$\frac{(4x^4 + 2x^3 - 8x^2 + 3x - 7) - (5x - 11)}{2x^2 + x - 2} = q(x)$$

$$q(x) = \frac{4x^4 + 2x^3 - 8x^2 - 2x + 4}{2x^2 + x - 2}$$

$$\begin{array}{r|l} & 2x^2 - 2 \\ \hline 2x^2 + x - 2 & 4x^4 + 2x^3 - 8x^2 - 2x + 4 \\ & 4x^4 + 2x^3 - 4x^2 \\ & - \quad - \quad + \\ \hline & -4x^2 - 2x + 4 \\ & -4x^2 - 2x + 4 \\ & + \quad + \quad - \\ \hline & 0 \end{array}$$

Amount received by each student, $q(x) = 2x^2 - 2$

Value: Humanity and socialism

If one zero of the quadratic polynomial $f(x) = 4x^2 - 8kx + 8x - 9$ is negative of the other, then find the zeroes of $kx^2 + 3kx + 2$

Answer:

$$f(x) = 4x^2 - 8kx + 8x - 9 = 4x^2 - (8k - 8)x - 9$$

Let one zero of $f(x)$ be α then other zero be $-\alpha$.

So, sum of zeroes $= 0$

$$\Rightarrow \quad \frac{8k - 8}{4} = 0 \Rightarrow 8k - 8 = 0 \Rightarrow k = 1$$

Now, other given polynomial is $p(x) = kx^2 + 3kx + 2$

$$= x^2 + 3x + 2$$

$$= (x + 2)(x + 1)$$

So, zeroes of $p(x)$ are -1 and -2.

If a polynomial $x^4 - 3x^3 - 6x^2 + kx - 16$ is exactly divisible by $x^2 - 3x + 2$, then find the vaue of A.

Answer:

Let $p(x) = x^4 - 3x^3 - 6x^2 + kx - 16$ and $g(x) = x^2 - 3x + 2 = (x - 1)(x - 2)$

$\because$ $p(x)$ is divisible by $g(x)$

So, 1 and 2 are zeroes of $p(x)$.

$\therefore p(1) = 0$

$$\Rightarrow \quad (1)^4 - 3(1)^3 - 6(1)^2 + k(1) - 16 = 0$$

$$\Rightarrow \quad 1 - 3 - 6 + k - 16 = 0$$

$$\Rightarrow \quad k - 24 = 0$$

$$\Rightarrow \quad k = 24$$

Thus, the value of k is 24.

What must be subtracted or added to $p(x) = 8x^4 + 14x^3 - 2x^2 + 8x - 12$ so that $4x^2 + 3x - 2$ is a factor of $p(x)$?

Answer:

$$\begin{array}{r} 2x^2 + 2x - 1 \\ 4x^2 + 3x - 2 \overline{)\,8x^4 + 14x^3 - 2x^2 + 8x - 12} \\ 8x^4 + 6x^3 - 4x^2 \\ - \quad - \quad + \\ \hline 8x^3 + 2x^2 + 8x - 12 \\ 8x^3 + 6x^2 - 4x \\ - \quad - \quad + \\ \hline - 4x^2 + 12x - 12 \\ - 4x^2 - 3x + 2 \\ + \quad + \quad - \\ \hline 15x - 14 \end{array}$$

Remainder $= 15x - 14$

$\therefore$ If we subtract $15x - 14$ or add $-15x + 14$ then remainder will be 0.

Then $4x^2 + 3x - 2$ will be a factor of given polynomial.

- **Pair of linear equations in two variables**

A part of monthly Hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay 13000 as hostel charges whereas, Mansi who takes food for 25 days pays ? 3500 as hostel charges. Find the fixed charges and the cost of food per day.

Solution:

Let fixed hostel charges be ₹ x

Charge per day is ₹ y

Charge paid by Swati = ₹ 3000

$\therefore$ 1st condition is $x + 20y = 3000$

Charge paid by Mansi = ₹ 3500

$\therefore$ 2nd condition is $x + 25y = 3500$

Subtracting (ii) from (i).

$$\begin{array}{r} x + 20y = 3000 \\ x + 25y = 3500 \\ - \quad - \quad - \\ \hline -5y = -500 \\ \hline \end{array}$$

$$\Rightarrow \quad y = ₹\ 100$$

Put value of $y = 100$ in equation (i), we get

$$\Rightarrow \quad x + 20\,(100) = 3000$$

$$x = 3000 - 2000 = ₹\ 1000$$

Hence, fixed charges is ₹ 1000 and charges per day is ₹ 100.

4 chairs and 3 tables cost ? 2100 and 5 chairs and 2 tables cost ? 1750. Find the cost of one chair and one table separately

Solution:

Let the cost of one chair and one table is ₹ x and ₹ y respectively.

According to question, $4x + 3y = 2100$

$5x + 2y = 1750$

Multiplying equation (i) by 2 and equation (ii) by 3, we get

$8x + 6y = 4200$

$15x + 6y = 5250$

Subtracting equation (iii) from (iv), we get

$$\begin{array}{r} 15x + 6y = 5250 \\ 8x + 6y = 4200 \\ - \quad - \quad - \\ \hline 7x = 1050 \Rightarrow x = 150 \\ \hline \end{array}$$

Putting $x = 150$ in equation (i), we get

$4 \times 150 + 3y = 2100$

$\Rightarrow \quad 3y = 2100 - 600 \Rightarrow 3y = 1500 \Rightarrow y = 500$

Hence, cost of one chair = ₹ 150 and cost of one table = ₹ 500.

Find those integral values of m for which the c-coordinate of the point of intersection of lines represented by y = mx + 1 and 3x + 4y = 9 is an integer.

Solution:

Given equations are

$$y = mx + 1 \quad ...(i)$$

$$3x + 4y = 9 \quad ...(ii)$$

Substitute the value of y from (i) in equation (ii), we get

$$3x + 4(mx + 1) = 9$$

$$\Rightarrow \quad 3x + 4mx + 4 = 9$$

$$\Rightarrow \quad (3 + 4m)x = 5 \Rightarrow x = \frac{5}{3+4m}$$

If $m = -2$ then $x = -1$

Hence, for $m = -2$ the x-coordinate is an integral value equal to -1.

The owner of a taxi company decides to run all the taxi on CNG fuels instead of petrol/ diesel. The taxi charges in city comprises of fixed charges together with the charge for the distance covered.

For a journey of 13 km, the charge paid is ? 129 and for journey of 22 km, the charge paid is ^ 210.

(i) What will a person have to pay for travelling a distance of 32 km?

(ii) Why did he decide to use CNG for his taxi as a fuel?

Solution:

(i) Let the fixed charges be ₹ x and the charge for per km be ₹ y.

According to Ist condition, $x + 13y = 129$ ...(i)

According to IInd condition, $x + 22y = 210$ ...(ii)

On solving equations (i) and (ii) we get, $x = 12$ and $y = 9$

Thus, for travelling a distance of 32 km, a person has to pay ₹ $(12 + 32 \times 9)$ i.e. ₹ 300.

(ii) He decided to use CNG as it is pollution free. It is good for environment and also cheaper in comparison to petrol/diesel.

At a certain time in a zoo, the number of heads and the number of legs of tiger and peacocks were counted and it was found that there were 47 heads and 152 legs. Find the number of tigers and peacocks in the zoo:

Why it is necessary to conserve these animals?

Solution:

Let x be the number of tigers and y be the number of peacocks.

According to conditions given,

$$x + y = 47 \quad ...(i)$$

$$4x + 2y = 152 \quad ...(ii)$$

On solving the above equations, we get

$$x = 29 \text{ and } y = 18$$

Hence, number of tigers = 29 and number of peacocks = 18

It is necessary to conserve each species of animals because all the animals play an important role in balancing the eco-system.

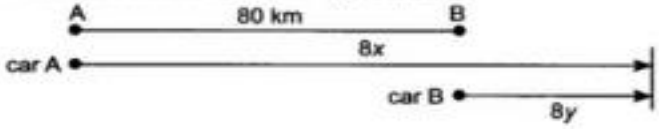
Places A and B are 80 km apart from each other on a highway. A car starts from A and another from B at the same time. If they move in same direction they meet in 8 hrs and if they move in opposite directions they meet in 1 hr 20 minutes. Find speeds of the cars.

Solution:

Let the speed of car starts from A or car A = x km/hr

and the speed of car starts from B or car B = y km/hr

Case I:



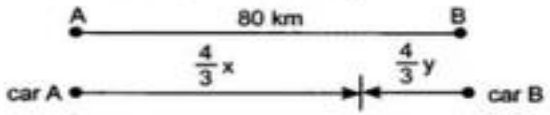
After 8 hours,

distance covered by car A = $8x$ $\left[\text{as speed} = \frac{\text{distance}}{\text{time}}\right]$

and distance covered by car B = $8y$

So, $8x - 8y = 80 \Rightarrow x - y = 10$ ...(*i*)

Case II:



After 1 hr 20 minutes, i.e. $\frac{4}{3}$hrs, distance covered by car A = $\frac{4}{3}x$

and distance covered by car B = $\frac{4}{3}y$

So, $\frac{4}{3}x + \frac{4}{3}y = 80$

$\Rightarrow$ $x + y = 60$ ...(*ii*)

On solving equations (*i*) and (*ii*), we get

$x = 35$ and $y = 25$

Hence, speed of car A = 35 km/hr and speed of car B = 25 km/hr.