X

MATHEMATICS

COMPLETE QUESTION BANK

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ARITHMETIC PROGRESSION

(PHAADU NOTES ALSO AVAILABLE!, Please Visit my Channel)

NCERT:

Example 3 : Find the 10th term of the AP : 2, 7, 12, . . .

Example 4 : Which term of the AP : 21, 18, 15, . . . is – 81? Also, is any term 0? Give reason for your answer.

Example 5 : Determine the AP whose 3rd term is 5 and the 7th term is 9.

Example 11 : Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

Example 12 : If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.

Example 15 : Find the sum of first 24 terms of the list of numbers whose nth term is given by $a_n = 3 + 2n$

EXERCISE 5.1

2. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) a = 10, d = 10 (ii) a = –2, d = 0

3. For the following APs, write the first term and the common difference:

(i) 3, 1, – 1, – 3, . . . (ii) – 5, – 1, 3, 7, . . .

EXERCISE 5.2

2. Choose the correct choice in the following and justify : (i) 30th term of the AP: 10, 7, 4, . . . , is

(A)97 (B) 77 (C) –77 (D) – 87

7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

12. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

13. How many three-digit numbers are divisible by 7?

16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

EXERCISE 5.3

1. Find the sum of the following APs:

(i) 2, 7, 12, . . ., to 10 terms. (ii) –37, –33, –29, . . ., to 12 terms.

3. In an AP:

(i) given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

(iv) given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

(vi) given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.

(ix) given $a = 3$, $n = 8$, $S = 192$, find d.

(x) given $l = 28$, $S = 144$, and there are total 9 terms. Find a.

4. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?

7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

10. Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP where an is defined as below : (i) $a_n = 3 + 4n$

11. If the sum of the first n terms of an AP is 4n – n2, what is the first term (that is S1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

13. Find the sum of the first 15 multiples of 8.

PREVIOUS YEAR

✓ In an AP, if the common difference (d) = –4, and the seventh term (a_7) is 4, then find the first term. [2018,1]

4. $a + 6(-4) = 4$ ½

$\Rightarrow \quad a = 28$ ½

✓ Find the sum of first 8 multiples of 3. [2018,2]

9. $S = 3 + 6 + 9 + 12 + \dots + 24$

$= 3(1 + 2 + 3 + \dots + 8)$ ½

$= 3 \times \frac{8 \times 9}{2}$ 1

$= 108$ ½

✓ The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers. [2018,4]

Let the four consecutive terms of the A.P. be

$a - 3d, a - d, a + d, a + 3d.$ ½

By given conditions

$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

$\Rightarrow \quad 4a = 32$

$\Rightarrow \quad a = 8$ 1

and $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$ 1

$\Rightarrow \quad 8a^2 = 128d^2$

(9) 30/1

30/1

$\Rightarrow \quad d^2 = 4$ ½

$\Rightarrow \quad d = \pm 2$ 1

∴ Numbers are 2, 6, 10, 14 or 14, 10, 6, 2.

✓ What is the common difference of an A.P. in which $a_{21} - a_7 = 84$?[2017,1]

$$a_{21} - a_7 = 84 \Rightarrow (a + 20d) - (a + 6d) = 84$$

$$\Rightarrow \quad 14d = 84$$

$$\Rightarrow \quad d = 6$$

✓ Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots$ is the first negative term ?

Here $d = \frac{-3}{4}$

Let the nth term be first negative term

$\therefore \quad 20 + (n-1)\left(\frac{-3}{4}\right) < 0$

$\Rightarrow \quad 3n > 83$

$\Rightarrow \quad n > 27\frac{2}{3}$

Hence 28^{th} term is first negative term.

✓ The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P.

[2017,3]

Here $a = 5$, $l = 45$ and $S_n = 400$

$\therefore \frac{n}{2}(a + l) = 400$ or $\frac{n}{2}(5 + 45) = 400$

$\Rightarrow \quad n = 16$

Also $5 + 15d = 45$

$\Rightarrow \quad d = \frac{8}{3}$

✓ If the ratio of the sum of the first n terms of two A.Ps is (7n + 1) : (4n + 27), then find the ratio of their 9th terms. [2017,4]

Let the first terms be a and a' and d and d' be their respective common differences

$$\frac{S_n}{S_n'} = \frac{\frac{n}{2}(2a + (n-1)d)}{\frac{n}{2}(2a' + (n-1)d')} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{a + \left(\frac{n-1}{2}\right)d}{a' + \left(\frac{n-1}{2}\right)d'} = \frac{7n+1}{4n+27}$$

To get ratio of 9^{th} terms, replacing $\frac{n-1}{2} = 8$

$\Rightarrow \quad n = 17$

Hence $\frac{t_9}{t_9'} = \frac{a + 8d}{a' + 8d'} = \frac{120}{95}$ or $\frac{24}{19}$

✓ For what value of k will k + 9, 2k – 1 and 2k + 7 are the consecutive terms of an A.P?

[2016,1]

$2(2k - 1) = k + 9 + 2k + 7$

$k = 18$

✓ The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term. [2016,2]

$a + 3d = 0 \Rightarrow a = -3d$

$a_{25} = a + 24d = 21d$

$3a_{11} = 3(a + 10d) = 3(7d) = 21d$

✓ If the ratio of the sum of first n terms of two A.P's is (7n +1): (4n + 27), find the ratio of their mth terms. [2016,3]

$$\frac{S_n}{S_n'} = \frac{n/2\,(2a + (n-1)d)}{n/2\,(2a' + (n-1)\,d')} = \frac{7n+1}{4n+27}$$

$$= \frac{a + \frac{n-1}{2}d}{a' + \frac{n-1}{2}d'} = \frac{7n+1}{An+27} \qquad \text{...(i)}$$

Since $\frac{t_m}{t_m'} = \frac{a + (m-1)\,d}{a + (m-1)\,d'}$, So replacing $\frac{n-1}{2}$ by $m - 1$ i.e. $n = 2m - 1$ in (i)

$$\frac{t_m}{t_m'} = \frac{a + (m-1)\,d}{a' + (m-1)\,d'} = \frac{7\,(2m-1)+1}{4\,(2m-1)+27} = \frac{14m-6}{8m+23}$$

✓ In an AP, if $S_5+S_7=167$, then find the A.P., where S_n denotes the sum of its first n terms. [2015,2]

$S_5 + S_7 = 167 \Rightarrow \frac{5}{2}[2a+4d] + \frac{7}{2}[2a+6d] = 167$

$24a + 62d = 334$ or $12a + 31d = 167$ ....(i)

$S_{10} = 235 \Rightarrow 5[2a+9d] = 235$ or $2a + 9d = 47$ ....(ii)

Solving (i) and (ii) to get $a = 1,\ d = 5$. Hence AP is $1, 6, 11,$ ....

✓ The 14th term of an A.P. is twice its 8th term. If its 6th term is -8, then find the sum of its first 20 terms. [2015,3]

$a_{14} = 2\,a_8 \Rightarrow a + 13d = 2\,(a + 7d) \Rightarrow a = -d$

$a_6 = -8 \Rightarrow a + 5d = -8$

solving to get $a = 2,\ d = -2$

$$S_{20} = 10\,(2a + 19d) = 10\,(4 - 38) = -340$$

✓ Find the 60th term of the AP 8, 10, 12, ……., if it has a total of 60 terms and hence find the sum of its last 10 terms. [2015,4]

$t_{60} = 8 + 59(2) = 126$

sum of last 10 terms $= (t_{51} + t_{52} + \ldots\ldots\ldots + t_{60})$

$t_{51} = 8 + 50(2) = 108$

$\therefore$ Sum of last 10 terms $= 5[108 + 126]$

$= 1170$

✓ If nth term of an A.P. is (2n+1), then the sum of its first three terms is?[2012,1]

Ans- 15

✓ Find the sum of all multiples of 7 lying between 500 and 900. [2012,3]

The numbers are 504, 511, 518, ---, 896

$\Rightarrow 896 = 504 + (n-1)7$

$\Rightarrow n = 57$

$$S_{57} = \frac{57}{2} \cdot [504 + 896] = \frac{57}{2}[1400]$$

$$= 57 \times 700 = 39900$$

SOME MORE IMPORTANT CONCEPTUAL QUESTIONS

How many terms of the A.P. 18, 16, 14, … be taken so that their sum is zero?

Solution:

Let the number of terms taken for sum to be zero be n.

Then, sum of n terms $(S_n) = 0$

First term $(a) = 18$

Common difference $(d) = -2$

Therefore, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 0 = \frac{n}{2}[2 \times 18 + (n-1)(-2)] \Rightarrow 0 = 38 - 2n$

$\Rightarrow n = 19$

$\therefore$ **Hence, sum of 19 terms is 0.**

If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P

Solution:

Given: $S_7 = 49$, where $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \frac{7}{2}[2a + (7-1)d] = 49$

$\Rightarrow 2a + 6d = 14 \Rightarrow a + 3d = 7$ ...(*i*)

Similarly, $S_{17} = 289$

$\frac{17}{2}[2a + (17-1)d] = 289$

$\Rightarrow 2a + 16d = 34 \Rightarrow a + 8d = 17$ ...(*ii*)

Solving (*i*) and (*ii*), we get

$a = 1$ and $d = 2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)2] = \frac{n}{2}[2 + 2n - 2] = n \times n = n^2$

Divide 56 in four parts in A.P. such that the ratio of the product of their extremes (1st and 4th) to the product of means (2nd and 3rd) is 5 : 6.

Solution:

Let the four parts of the A.P. are $a-3d, a-d, a+d, a+3d$

Now, $a-3d+a-d+a+d+a+3d = 56$ $\quad [\because \text{Sum} = 56]$

$\Rightarrow \quad 4a = 56 \Rightarrow a = 14$

According to question,

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$$

$$\Rightarrow \quad \frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6} \quad [\because \text{Putting } a = 14]$$

$$\Rightarrow \quad \frac{196-9d^2}{196-d^2} = \frac{5}{6}$$

$$\Rightarrow \quad 1176 - 54d^2 = 980 - 5d^2$$

$$\Rightarrow \quad 49d^2 = 196 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$$

Thus, 4 parts are $a-3d, a-d, a+d, a+3d$, i.e. 8, 12, 16, 20.

The pth, 9th and rth terms of an A.P. are a, b and c respectively. Show that a(q – r) + b(r – p) + c(p – q) = 0

Solution:

Let A and d be the first term and common difference of the given A.P., then

$$a_p = A + (p-1)d = a \quad \text{...(i}$$

$$a_q = A + (q-1)d = b \quad \text{...(ii}$$

$$a_r = A + (r-1)d = c \quad \text{...(iii}$$

Now, subtracting (*i*) and (*ii*), we get

$$(p-q)d = a-b$$

$$p-q = \frac{a}{d} - \frac{b}{d}$$

Multiplying by 'c' both sides,

$$c(p-q) = \frac{ca}{d} - \frac{cb}{d} \quad \text{...(iv}$$

Now, (*ii*) – (*iii*), we get

$$(q-r)d = b-c$$

$$q-r = \frac{b}{d} - \frac{c}{d}$$

Multiplying by 'a' both sides,

$$a(q-r) = \frac{ab}{d} - \frac{ac}{d} \quad \text{...(v}$$

Now, (*iii*) – (*i*), we get

$$(r-p)d = c-a$$

$$(r-p) = \frac{c}{d} - \frac{a}{d}$$

Multiplying by 'b' both sides,

$$(r-p)b = \frac{bc}{d} - \frac{ba}{d} \quad \text{...(vi}$$

Adding (*iv*), (*v*) and (*vi*), we get

$$a(q-r) + b(r-p) + c(p-q) = \frac{ab}{d} - \frac{ac}{d} + \frac{bc}{d} - \frac{ba}{d} + \frac{ca}{d} - \frac{cb}{d} = 0$$

A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief.

Solution:

Let total time be $(n-1)$ minutes in which the police catch the thief.

Since thief ran 1 minute before police start running.

$\because$ Time taken by thief before he was caught $= (n-1+1) = n$ minute

Then total distance covered by thief $= (100 \times n)$ metres

Total distance covered by policeman in $(n-1)$ minute

$$= 100 + 110 + 120 + \dots + (n-1) \text{ terms}$$
$$= \frac{(n-1)}{2}[2000 + (n-2)10] \quad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$$

According to question,

Total distance covered by thief in 'n' minute

$$= \text{total distance covered by policeman in } (n-1) \text{ minute}$$
$$100n = \frac{(n-1)}{2}[200 + (n-2)10]$$
$$\Rightarrow \quad 200n = (n-1)[200 + 10n - 20]$$
$$\Rightarrow \quad 200n = (n-1)(10n + 180)$$
$$\Rightarrow \quad 200n = 10n^2 + 180n - 10n - 180$$
$$\Rightarrow \quad 10n^2 - 30n - 180 = 0$$
$$\Rightarrow \quad n^2 - 3n - 18 = 0 \Rightarrow n^2 - 6n + 3n - 18 = 0$$
$$\Rightarrow \quad n(n-6) + 3(n-6) = 0 \Rightarrow (n-6)(n+3) = 0$$
$$\Rightarrow \quad n = 6 \text{ or } n = -3 \text{ (rejected)}$$

Hence, time taken by policeman to catch the thief is $(6-1)$, i.e. 5 minutes.

Reshma wanted to save at least ? 6,500 for sending her daughter to school next year (after 12 months). She saved ? 450 in the first month and raised her savings by ? 20 every next month. How much will she be able to save in next 12 months? Will she be able to send her daughter to the school next year?

Solution:

The amounts saved form an A.P. 450, 470, 490, ... in which

first term $(a) =$ ₹ 450

Common difference $(d) =$ ₹ 20

Total terms $(n) = 12$ (number of months)

Then,
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{12} = \frac{12}{2}[2 \times 450 + (12-1)(20)] = 6[900 + 220] = 6 \times 1120 = 6720$$

Now, $6720 > 6500$

$\therefore$ Reshma will be able to send her daughter to school as she has saved more than ₹ 6500.

Now, Reshma is very much concerned about her daughter's education. She is awared and dedicated towards her daughter is education.

Find the middle term of the sequence formed by all numbers between 9 and 95, which leave a remainder 1 when divided by 3. Also find the sum of the numbers on both sides of the middle term separately.

Solution:

List of number between 9 and 95 leaving remainder 1, when divided by 3 are 10, 13, 16, ... 94

These numbers are in AP with

$$a = 10, d = 3$$

$\because$ number of terms in AP $= n$,

$$a_n = 94 \Rightarrow a + (n-1)d = 94$$
$$10 + (n-1)3 = 94$$
$$(n-1)3 = 84 \Rightarrow n - 1 = 28$$
$$n = 29$$

Since number of terms is odd, it has only one middle term.

$\therefore$ Now, Middle term $= \frac{29+1}{2} = 15^{\text{th}}$ term $= \left(\frac{n+1}{2}\right)^{\text{th}}$

$$a_{15} = a + 14d = 10 + 14 \times 3 = 52$$

Number of terms before 15th term = 14

$\therefore$ Sum of first 14 terms, $S_{14} = \frac{14}{2}(2 \times 10 + 13 \times 3)$ $\left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$$= \frac{14}{2}(20 + 39) = 7 \times 59 = 413$$

$$a_{29} = 94$$

$\therefore$ $S_{29} = \frac{29}{2}[a + a_{29}] = 1508$

$\therefore$ Sum of last 14 terms $= S_{29} - [S_{14} + a_{15}] = 1043$

The sum of the first n terms of an AP is $5n - n^2$. Find the nth term of this AP.

Solution:

Given, sum of first 'n' terms of AP is

$$S_n = 5n - n^2$$

Replacing 'n' by $(n-1)$

So, $S_{n-1} = 5(n-1) - (n-1)^2 = 5n - 5 - (n^2 - 2n + 1)$

$$= 5n - 5 - n^2 + 2n - 1 = 7n - n^2 - 6$$

Now, $a_n = S_n - S_{n-1} = n^{\text{th}}$ term $= 5n - n^2 - 7n + n^2 + 6$

$$a_n = 6 - 2n$$

The 19th term of an AP is equal to three times its 6th term. If its 9th term is 19, find the AP.

Solution:

Let Ist term of the AP = a and common difference = d.

A.T.Q., $a_{19} = 3 \times a_6$, where $a_n = a + (n-1)d$

$\Rightarrow$ $a + 18d = 3(a + 5d) \Rightarrow a = \frac{3}{2}d$ ...(*i*)

Also, given that $a_9 = 19$

$\Rightarrow$ $a + 8d = 19$

$\Rightarrow$ $\frac{3}{2}d + 8d = 19$ [Using eq. (*i*)]

$\Rightarrow$ $19d = 38 \Rightarrow d = 2$

Putting $d = 2$, equation (*i*), we get

$$a = \frac{3}{2} \times 2 = 3$$

$\therefore$ Required AP is 3, 5, 7, 9, ...

Students of a school thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. Find the total number of trees planted by the students of the school.

Pollution control is necessary for everybody's health. Suggest one more role of students in it.

Solution:

Number of trees planted by class I $= 3 \times 1 = 3$

Number of trees planted by class II $= 3 \times 2 = 6$

Number of trees planted by class III $= 3 \times 3 = 9$

Number of trees planted by class XII $= 3 \times 12 = 36$

Total number of trees planted by students

$$= 3 + 6 + 9 + \cdots + 36 \qquad [12 \text{ terms}]$$

$$= \frac{12}{2}(3 + 36) = 234 \qquad \left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$$

Role of students for everybody's health. To provide safety and pollution-free environment.
