X

MATHEMATICS

COMPLETE QUESTION BANK

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STATISTICS AND PROBABILITY

PREVIOUS YEARS

2018

Two different dice are tossed together. Find the probability :

(i) of getting a doublet

(ii) of getting a sum 10, of the numbers on the two dice.

Total number of possible outcomes = 36

(i) Doublets are (1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)

Total number of doublets = 6

$\therefore$ Prob (getting a doublet) = $\frac{6}{36}$ or $\frac{1}{6}$

(ii) Favourable outcomes are (4, 6) (5, 5) (6, 4) i.e., 3

$\therefore$ Prob (getting a sum 10) = $\frac{3}{36}$ or $\frac{1}{12}$

An integer is chosen at random between 1 and 100. Find the probability that it is :

(i) divisible by 8.

(ii) not divisible by 8.

Total number of outcomes = 98

(i) Favourable outcomes are 8, 16, 24, ..., 96 i.e., 12

$\therefore$ Prob (integer is divisible by 8) = $\frac{12}{98}$ or $\frac{6}{49}$

(ii) Prob (integer is not divisible by 8) $= 1 - \frac{6}{49}$

$$= \frac{43}{49}$$

The table below shows the salaries of 280 persons :

Salary (In thousand ₹)	No. of Persons
5 – 10	49
10 – 15	133
15 – 20	63
20 – 25	15
25 – 30	6
30 – 35	7
35 – 40	4
40 – 45	2
45 – 50	1

Calculate the median salary of the data.

Salary (in thousand Rs)	No. of persons (f)	cf
5–10	49	49
10–15	133	182
15–20	63	245
20–25	15	260
25–30	6	266
30–35	7	273
35–40	4	277
40–45	2	279
45–50	1	280

$\frac{N}{2} = \frac{280}{2} = 140$

Median class is 10–15

$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - C\right)$

$= 10 + \frac{5}{133}(140 - 49)$

$= 10 + \frac{5 \times 91}{133}$

$= 13.42$

Median salary is Rs 13.42 thousand or Rs 13420 (approx)

The mean of the following distribution is 18. Find the frequency f of the class 19 – 21.

Class	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

OR

The following distribution gives the daily income of 50 workers of a factory :

Daily Income (in ₹)	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

Class	x	f	fx	
11–13	12	3	36	
13–15	14	6	84	
15–17	16	9	144	
17–19	18	13	234	
19–21	20	f	20f	
21–23	22	5	110	For x
23–25	24	4	96	Σf
		40 + f	704 + 20f	Σfx

$$\text{Mean} = 18 = \frac{704 + 20f}{40 + f}$$

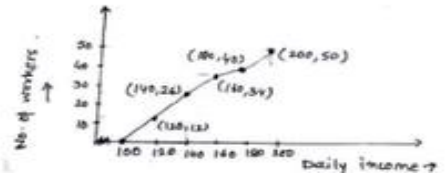
$\Rightarrow \quad 720 + 18f = 704 + 20f$

$\Rightarrow \quad f = 8$

OR

Cumulative frequency distribution table of less than type is

Daily income	Cumulative frequency
Less than 100	0
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50



2017

The probability of selecting a rotten apple randomly from a heap of 900 apples is 0·18. What is the number of rotten apples in the heap ?

Let the number of rotten apples in the heap be n.

$$\therefore \quad \frac{n}{900} = 0.18$$

$$\Rightarrow \quad n = 162$$

A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag.

Let the number of black balls in the bag be n.

$\therefore$ Total number of balls are $15 + n$

Prob(Black ball) = 3 × Prob(White ball)

$$\Rightarrow \quad \frac{n}{15+n} = 3 \times \frac{15}{15+n}$$

$$\Rightarrow \quad n = 45$$

Two different dice are thrown together. Find the probability that the numbers obtained have

(i) even sum, and

(ii) even product.

Total number of outcomes = 36

(i) $P(\text{even sum}) = \frac{18}{36} = \frac{1}{2}$

(ii) $P(\text{even product}) = \frac{27}{36} = \frac{3}{4}$

2016

A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen.

No. of red cards and queens: 28

Required Probability: $\frac{24}{52}$ or $\frac{6}{13}$

Three different coins are tossed together. Find the probability of getting (i) exactly two heads (ii) at least two heads (iii) at least two tails.

Set of possible outcomes is

{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

(i) P(exactly 2 heads) = 3/8

(ii) P(at least 2 heads) = 4/8 or 1/2

(iii) P(at least 2 tails) = 4/8 or 1/2

A number x is selected at random from the numbers 1, 2, 3 and 4. Another number y is selected at random from the numbers 1, 4, 9 and 16. Find the probability that product of x and y is less than 16.

x can be any one of 1, 2, 3 or 4.

y can be any one of 1, 4, 9 of 16

Total number of cases of xy = 16

Number of cases, where product is less than 16 = 8

{1, 4, 9, 2, 8, 3, 12, 4}

$\therefore$ Required Probability = $\frac{8}{16}$ or $\frac{1}{2}$

2015

Two different dice are tossed together. Find the probability that t product of the two numbers on the top of the dice is 6.

Ans-1/9

The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$. If the jar contains 10 orange balls, find the total number of balls in the jar.

$P(\text{Red}) = \frac{1}{4}$, $P(\text{blue}) = \frac{1}{3}$

$\Rightarrow P(\text{orange}) = 1 - \frac{1}{4} - \frac{1}{3} = \frac{5}{12}$

$\Rightarrow \frac{5}{12}(\text{Total no. of balls}) = 10$

$\Rightarrow$ Total no. of balls $= \frac{10 \times 12}{5} = 24$

A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is

(i) a card of spade or an ace.

(ii) a black king.

(iii) neither a jack nor a king.

(iv) either a king or a queen.

(i) P(spade or an ace) $= \frac{13+3}{52} = \frac{4}{13}$

(ii) P(a black king) $= \frac{2}{52} = \frac{1}{26}$

(iii) P(neither a jack nor a king) $= \frac{52-8}{52} = \frac{44}{52} = \frac{11}{13}$

(iv) P(either a king or a queen) $= \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$

2014

1. The probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4, is

(A) $\frac{4}{15}$

(B) $\frac{2}{15}$

(C) $\frac{1}{5}$

(D) $\frac{1}{3}$

Correct answer: C

The multiples of 4 between 1 and 15 are 4, 8 and 12. The probability of getting a multiple of 4 $= \frac{3}{15} = \frac{1}{5}$

Two different dice are tossed together. Find the probability

(i) That the number on each die is even.

(ii) That the sum of numbers appearing on the two dice is 5.

Solution:

The total number of outcomes when two dice are tossed together is 36.

The sample space is as follows

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2, 2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

i. Favourable outcomes = { (2,2) (2,4) (2,6) (4,2) (4,4) (4,6) (6,2) (6,4) (6,6) }

Probability that the number on each dice is even

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{9}{36} = \frac{1}{4}$$

ii. Favouable outcomes = { (1,4) (2,3) (3,2) (4,1) }

Probability that the sum of the numbers appearing on the two dice is 5

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{4}{36} = \frac{1}{9}$$

All the red face cards are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards, after reshuffling them. Find the probability that the drawn card is

(i) of red colour

(ii) a queen

(iii) an ace

(iv) a face card

Solution:

(i) face card are removed from a pack of 52 playing card = 6

Total favorable outcomes = 52 – 6 = 46

Number of all possible outcomes = 26 - 6= 20

P[E] = 20/46 = 0.43

(ii) Number of all possible outcomes a queen = 2

P[E] = 2/46= 1/23

(iii) Number of all possible outcomes an ace = 2

P[E] = 2/46= 1/23

(iv) Number of all possible outcomes = 6

P[E] = 6/46= 3/23

HOTS

Find the unknown values in the following table:

Class interval	frequency	Cumulative frequency
0 – 10	5	5
10 – 20	7	x_1
20 – 30	x_2	18
30 – 40	5	x_3
40 – 50	x_4	30

Solution:

Class interval	frequency	Cumulative frequency
0 – 10	5	5
10 – 20	7	x_1
20 – 30	x_2	18
30 – 40	5	x_3
40 – 50	x_4	30

From table:

$x_1 = 7 + 5 = 12$

$18 = x_1 + x_2 = 12 + x_2 \Rightarrow x_2 = 18 - 12 = 6$

$x_3 = 18 + 5 = 23$

$30 = x_3 + x_4 \Rightarrow x_4 = 30 - x_3 = 30 - 23 = 7.$

The average score of boys in the examination of a school is 71 and that of the the girls is 73. The average score of the school in the examination is 71.8. Find the ratio of the number of boys to the number of girls who appeared in the examination.

Solution:

Let number of boys in the school be x

Average score of boys = 71

Total score of boys in the examination of the school = 71xx = 71x

Let number of girls in the school be y

Average score of girls = 73

Total score of girls in the examination of the school = 73 x y = 73y

Now,

average score of the school in examination = 71.8

$$\therefore \frac{\text{Total score of boys} + \text{Total score of girls}}{\text{Total number of boys and girls}} = 71.8$$

$$\frac{71x + 73y}{x + y} = 71.8$$

$$71x + 73y = 71.8x + 71.8y$$

$$73y - 71.8y = 71.8x - 71x$$

$$1.2y = 0.8x$$

$$\frac{1.2}{0.8} = \frac{x}{y}$$

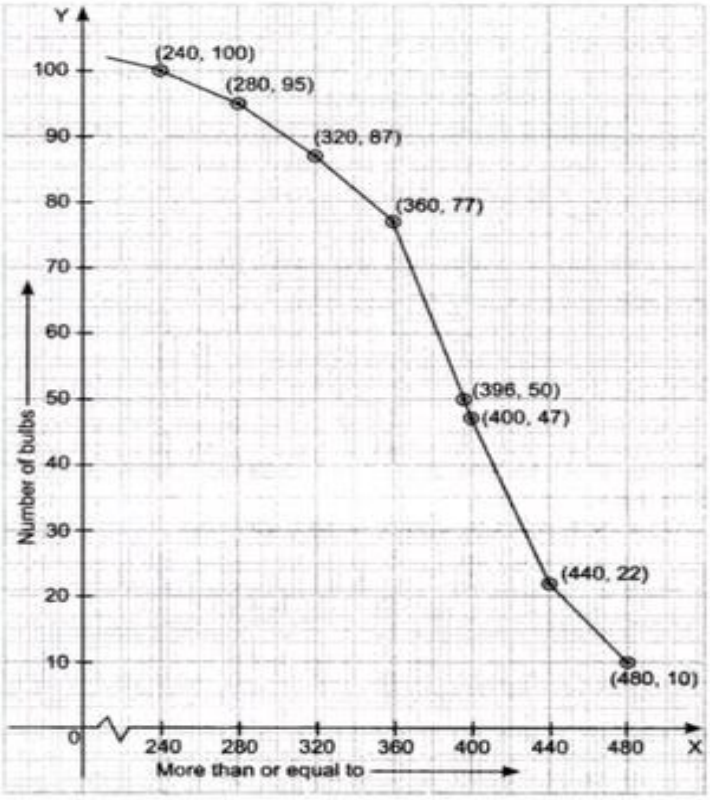
$$\frac{12}{8} = \frac{x}{y} \Rightarrow x : y = 3 : 2$$

Life time (in hours)	More than or equal to 240	More than or equal to 280	More than or equal to 320	More than or equal to 360	More than or equal to 400	More than or equal to 440	More than or equal to 480
Number of bulbs	100	95	87	77	47	22	10

Draw a 'more than type' ogive and from it, find median. Verify it by actual calculations.

Solution:

More than or equal to	Number of bulbs
240	100
280	95
320	87
360	77
400	47
440	22
480	10



Median from curve is 396.

Class interval	Frequency	Cumulative frequency	
240 – 280	5	5	
280 – 320	8	13	
320 – 360	10	23	
360 – 400	30	53	Median class
400 – 440	25	78	
440 – 480	12	90	
480 – 520	10	100	
	100		

$N = 10$

$\frac{N}{2} = \frac{100}{2} = 50$

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 360 + \left(\frac{50 - 23}{30}\right) \times 40 = 360 + \left(\frac{27}{30}\right) \times 40$$

$$= 360 + 36 = 396$$

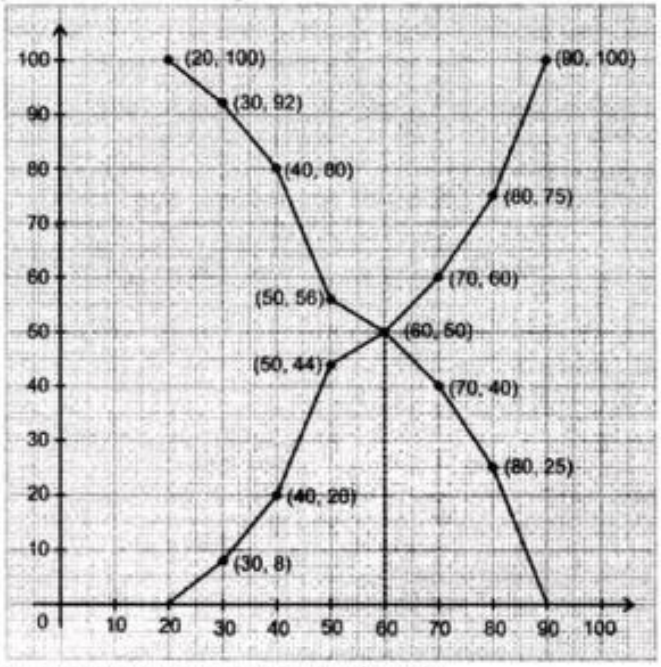
Draw 'less than ogive' and 'more than ogive' for the following distribution and hence find its median

Class	Frequency
20 – 30	8
30 – 40	12
40 – 50	24
50 – 60	6
60 – 70	10
70 – 80	15
80 – 90	25

Solution:

C.I.	For 'less than' Ogive				For 'more than' Ogive		
	f	C.I. (less than)	*c.f*	Point	C.I. (more than)	*c.f*	Point
20 – 30	8	30	8	(30, 8)	20	100	(20, 100)
30 – 40	12	40	20	(40, 20)	30	92	(30, 92)
40 – 50	24	50	44	(50, 44)	40	80	(40, 80)
50 – 60	6	60	50	(60, 50)	50	56	(50, 56)
60 – 70	10	70	60	(70, 60)	60	50	(60, 50)
70 – 80	15	80	75	(80, 75)	70	40	(70, 40)
80 – 90	25	90	100	(90, 100)	80	25	(80, 25)

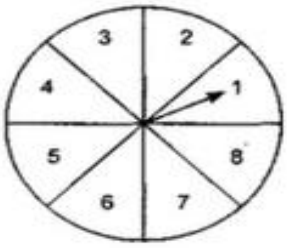
'Less than' Ogive and 'more than' Ogive curves



we notice both curves intersect at (60,50)
median =60

A game of chance consists of spinning an arrow on a circular board, divided into 8 equal parts, which comes to rest pointing at one of the numbers 1,2,3,..., 8 which are equally likely outcomes. What is the probability that the arrow will point at

1. an odd number
2. a number greater than 3
3. a number less than 9.



Solution:

1. Total possible outcomes when the arrow points at one of the numbers are 8.
   Favourable outcomes when the required number is odd are 1, 3, 5, 7, i.e. 4 outcomes.
   $$\therefore \text{P(an odd number)} = \frac{\text{Favourable outcomes}}{\text{Total possible outcomes}} = \frac{4}{8} = \frac{1}{2}$$
2. Favourable outcomes when the required number is more than 3 are 4,5,6,7, 8, i.e. 5 outcomes.
   $$\text{P(a number is more than 3)} = \frac{\text{Favourable outcomes}}{\text{Total possible outcome}} = \frac{5}{8}$$
3. Favourable outcomes when the required number is less than 9 are 1,2,3,4,5,6,7,8 i.e 8 outcomes
   $$\text{P(number is less than 9)} = \frac{\text{Favourable outcomes}}{\text{Total possible outcomes}} = \frac{8}{8} = 1$$

A number JC is selected at random from the numbers 1,2,3 and 4. Another number y is selected at random from the numbers 1, 4, 9 and 16. Find the probability that product of JC and y is less than 16.

Solution:

x can be any one of 1, 2, 3 or 4 andy can be any one of 1, 4, 9 or 16.
Total number of cases of xy = 16
Number of cases when product is less than 16 are 1 x 1,1 x 4,1 x 9, 2 x 1, 2 x 4, 3 x 1, 3 x 4,4 x 1, i.e. 8 cases

$$\text{Required probability} = \frac{\text{Number of favourable cases}}{\text{Total number of cases}} = \frac{8}{16} = \frac{1}{2}$$

The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is 1/4. The probability of selecting a blue ball at random from the same jar is 1/3. If the jar contains 10 orange balls, find the total number of balls in the jar.

Solution:

Let number of red balls in the jar = x
number of blue balls in the jar = y
Total number of balls in the jar = x + y + 10
Probability of selecting red ball = 1/4

$$\Rightarrow \quad \frac{x}{x+y+10} = \frac{1}{4} \quad \Rightarrow \quad 4x = x + y + 10$$

$$3x - y = 10 \qquad \ldots(i)$$

Also

$$\frac{1}{3} = \frac{y}{x+y+10}$$

$$\Rightarrow \quad x + y + 10 = 3y \quad \Rightarrow \quad x = 2y - 10 \qquad \ldots(ii)$$

From (i) and (ii), we have

$$3(2y - 10) - y = 10 \quad \Rightarrow \quad 6y - 30 - y = 10$$

$$\Rightarrow \quad 5y = 40 \quad \Rightarrow \quad y = 8$$

Putting $y = 8$ in (ii), we get

$$x = 2 \times 8 - 10 = 16 - 10 = 6 \quad \Rightarrow \quad x = 6$$

Hence, total number of balls in the jar = 6 + 8 + 10 = 24

A bag contains 25 cards numbered from 1 to 25. A card is drawn at random from the bag. Find the probability that the number on the drawn card is:

1. divisible by 3 or 5.
2. a perfect square number.

Solution:

1. Number from 1 to 25 which are divisible by 3 or 5 are 3, 6, 9,12,15,18, 21, 24, 5,10,20,25.
   P(a number divisible by 3 or 5) = 12/25
2. Number from 1 to 25 which are perfect square are 1, 4, 9,16, 25
   P(a perfect square number) = ^5/25= 1/5.

Red queens and blackjacks are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards, after reshuffling them. Find the probability that the drawn card is

1. a king.
2. of red colour.
3. a face card.
4. a queen.

Solution:
Red queens and blackjacks, i.e. 2 + 2 = 4 cards are removed from a pack of 52 playing cards. Remaining cards = 52 – 4 = 48.
∴ Possible outcomes of drawing one card from 48 cards is 48.

1. Favourable outcomes for drawing a king are 4.
   ∴ Probability of drawing a queen=4/48=1/12
2. Favourable outcomes for a card of red colour are 24 as 2 red queens have been removed.
   ∴ Probability of drawing a queen=24/48=1/2
3. Favourable outcomes for a face card (4 kings, 2 queens, 2 jacks) = 8
   ∴ Probability of drawing a queen=8/48=1/6
4. Favourable outcomes for drawing a queen are 2, as 2 red queens have been removed.
   ∴ Probability of drawing a queen = 2/48=1/24

A piggy bank contains hundred 50 p coins, fifty 1 coins, twenty 2 coins and ten 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, find the probability that the coin which fell

1. will be a 50 p coin
2. will be of value more than 1
3. will be of value less than 5
4. will be a 1 or 2 coin

Solution:
Total coins in piggy bank are:
50 p coins = 100; 1 coins = 50; 2 coins = 20; 5 coins = 10
Total coins = 100 + 50 + 20 + 10 = 180
Therefore, total possible outcomes that one coin will fall = 180

1. Favourable outcomes for falling a 50 p coin = 100
   ∴Probability that a 50 p coin falls =100/180=5/9
2. Favourable outcomes for a coin of value more than 1 are 30 (20 + 10).
   ∴Probability of falling a coin of value more than 1=30/180=1/6
3. Favourable outcomes for falling a coin of value less than ? 5 are 170 (100 + 50 + 20)
   ∴Probability of falling a coin of value less than 5 = 170/180=17/18
4. Favourable outcomes for falling a coin of value 1 or 2 = 70(50 + 20)
   ∴ Probability of falling a coin of value 1 or 2 =7/180=7/18
