X

MATHEMATICS

COMPLETE QUESTION BANK

DISCLAIMER- Please Watch Complete Video before solving this question bank.(Nuksaan tumhara hi hoga varna)

Youtube.com/Shobhit Nirwan

COORDINATE GEOMETRY

NCERT:

Example 2 : Show that the points (1, 7), (4, 2), (–1, –1) and (– 4, 4) are the vertices of a square.

Example 4 : Find a relation between x and y such that the point (x , y) is equidistant from the points (7, 1) and (3, 5).

Example 6 : Find the coordinates of the point which divides the line segment joining the points (4, – 3) and (8, 5) in the ratio 3 : 1 internally.

Example 7 : In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?

Example 10 : If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.

Example 13 : Find the area of the triangle formed by the points P(–1.5, 3), Q(6, –2) and R(–3, 4).

Example 14 : Find the value of k if the points A(2, 3), B(4, k) and C(6, –3) are collinear.

Example 15 : If A(–5, 7), B(– 4, –5), C(–1, –6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

EXERCISE 7.1

1. Find the distance between the following pairs of points :

(i) (a, b), (– a, – b)

3.Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.

4. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

8. Find the values of y for which the distance between the points P(2, – 3) and Q(10, y) is 10 units.

9. If Q(0, 1) is equidistant from P(5, –3) and R(x, 6), find the values of x. Also find the distances QR and PR.

10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (– 3, 4).

EXERCISE 7.2

1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.
2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

6.If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

8. If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that AP = 3/7 AB and P lies on the line segment AB.

9. Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in

order.

EXERCISE 7.3

1. Find the area of the triangle whose vertices are :

   (ii) (–5, –1), (3, –5), (5, 2)

2. In each of the following find the value of ‘k’, for which the points are collinear.

(i) (7, –2), (5, 1), (3, k)

3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

4. Find the area of the quadrilateral whose vertices, taken in order, are (– 4, – 2), (– 3, – 5), (3, – 2) and (2, 3).

PREVIOUS YEARS

2018

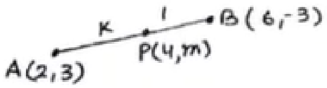
Find the distance of a point P(x, y) from the origin.

$OP = \sqrt{x^2 + y^2}$

Find the ratio in which P(4, m) divides the line segment joining the points A(2, 3) and B(6, –3). Hence find m.

Let AP : PB = k : 1

$\therefore \quad \frac{6k+2}{k+1} = 4$



$\Rightarrow$ k = 1, ratio is 1 : 1

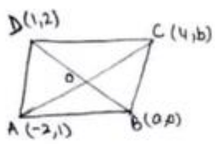
Hence $m = \frac{-3+3}{2} = 0$

If A(–2, 1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Hence find the lengths of its sides.

OR

If A(–5, 7), B(–4, –5), C(–1, –6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

15.



ABCD is a parallelogram

$\therefore$ diagonals AC and BD bisect each other

Therefore

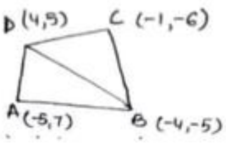
Mid point of BD is same as mid point of AC

$\Rightarrow \left(\frac{a+1}{2}, \frac{2}{2}\right) = \left(\frac{-2+4}{2}, \frac{b+1}{2}\right)$

$\Rightarrow \frac{a+1}{2} = 1$ and $\frac{b+1}{2} = 1$

$\Rightarrow$ a = 1, b = 1. Therefore length of sides are $\sqrt{10}$ units each

OR



Area of quad ABCD = Ar ΔABD + Ar ΔBCD

Area of $\Delta ABD = \frac{1}{2}|(-5)(-5-5) + (-4)(5-7) + (4)(7+5)|$

= 53 sq units

Area of $\Delta BCD = \frac{1}{2}|(-4)(-6-5) + (-1)(5+5) + 4(-5+6)|$

= 19 sq units

Hence area of quad. ABCD = 53 + 19 = 72 sq units

A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, – 5) is the mid-point of PQ, then find the coordinates of P and Q.

Let the coordinates of points P and Q be (0, b) and (a, 0) resp.

$\therefore \quad \frac{a}{2} = 2 \Rightarrow a = 4$

$\frac{b}{2} = -5 \Rightarrow b = -10$

$\therefore \quad P(0, -10)$ and $Q(4, 0)$

2016

If the distances of P(x, y) from A(5, 1) and B(– 1, 5) are equal, then prove that $3x = 2y$.

$PA^2 = PB^2$

$\Rightarrow \quad (x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$

$\Rightarrow \quad 12x = 8y$

$\Rightarrow \quad 3x = 2y$

If the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 1, 5k) are collinear, then find the value of k.

Points A, B and C are collinear

Therefore $\frac{1}{2}[(k+1)(2k+3-5k) + 3k(5k-2k) + (5k-1)(2k-2k-3)] = 0$

$= (k+1)(3-3k) + 9k^2 - 3(5k-1) = 0$

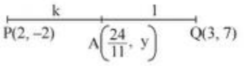
$= 2k^2 - 5k + 2 = 0$

$= (k-2)(2k-1) = 0$

$\Rightarrow \quad k = 2, \frac{1}{2}$

In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points P(2, – 2) and Q(3, 7) ? Also find the value of y.

15.



Let PA : AQ = k : 1

$\therefore \quad \frac{2+3k}{k+1} = \frac{24}{11}$

$\Rightarrow \quad k = \frac{2}{9}$

Hence the ratio is 2 : 9.

Therefore $y = \frac{-18+14}{11} = \frac{-4}{11}$

2016

Let P and Q be the points of trisection of the line segment joining the points A(2, –2) and B(–7, 4) such that P is nearer to A. Find the coordinates of P and Q.

P divides AB in 1 : 2

$\therefore$ Coords of P are: (–1, 0)

Q is mid-point of PB

$\therefore$ Coords of Q are: (–4, 2)

If the point P(x, y) is equidistant from the points A(a + b, b – a) and B(a – b, a + b). Prove that bx = ay.

PA = PB or $(PA)^2 = (PB)^2$

$(a + b - x)^2 + (b - a - y)^2 = (a - b - x)^2 + (a + b - y)^2$

$(a + b)^2 + x^2 - 2ax - 2bx + (b - a)^2 + y^2 - 2by + 2ay$

$= (a - b)^2 + x^2 - 2ax + 2bx + (a + b)^2 + y^2 - 2ay - 2by$

$\Rightarrow \quad 4ay = 4bx$ or $bx = ay$

2015

Find the relation between x and y if the points A(x, y), B(– 5, 7) and C(– 4, 5) are collinear.

Using ar (Δ ABC) = 0

$\Rightarrow \; x(7-5) - 5(5-y) - 4(y-7) = 0$

$2x - 25 + 5y - 4y + 28 = 0$

$2x + y + 3 = 0$

If the coordinates of points A and B are (– 2, – 2) and (2, – 4) respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$, where P lies on the line segment AB.

$AP = \frac{3}{7}AB \Rightarrow AP : PB = 3:4$

A (–2, –2) —— P (x, y) 3:4 —— B (2, –4)

$\therefore \ x = \frac{6-8}{7} = -2/7$

$y = \frac{-12-8}{7} = -\frac{20}{7}$

$P\left(-\frac{2}{7}, -\frac{20}{7}\right)$

Find the values of k so that the area of the triangle with vertices (1, – 1), (– 4, 2k) and (– k, – 5) is 24 sq. units.

$\frac{1}{2}[1(2k+5)-4(-5+1)-k(-1-2k)] = 24$

$\Rightarrow \ 2k^2 + 3k - 27 = 0$

Solving to get $k = 3, \ k = -\frac{9}{2}$

Some More Questions

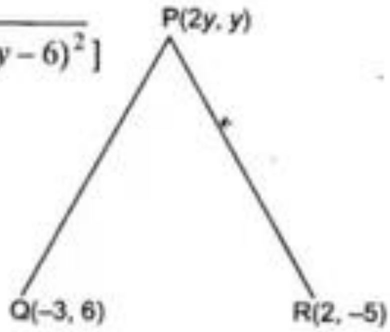
The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, -5) and R(-3, 6), find the coordinates of P.

Solution:

Let the required point be (2y, y). Let Q(2, – 5) and R(–3, 6) are given points.

Now, $PQ = PR \Rightarrow \sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$

[$\because$ using Distance formula, $\sqrt{(x-2)^2+(y+5)^2} = \sqrt{(x+3)^2+(y-6)^2}$]



Squaring both sides we get

$4y^2 + 4 - 8y + y^2 + 10y + 25 = 4y^2 + 9 + 12y + y^2 - 12y + 36$

$\Rightarrow \quad 2y + 29 = 45$

$\Rightarrow \quad 2y = 45 - 29 = 16$

$\Rightarrow \quad y = 8$

$\Rightarrow \quad 2y = 16$

Hence coordinates of P are (16, 8)

In figure ABC is a triangle coordinates of whose vertex A are (0, -1). D and E respectively are the mid-points of the sides AB and AC and their coordinates are (1, 0) and (0,1) respectively. If F is the mid-point of BC, find the areas of ΔABC and ΔDEF.

Solution:

. Let coordinates of B are (x, y). Then using mid point formula we

$$\frac{x+0}{2} = 1 \quad \Rightarrow \quad x = 2$$

$$\frac{y-1}{2} = 0 \quad \Rightarrow \quad y = 1$$

Coordinates of B are (2,1)

Let coordinates of C are (p, q)

Similarly coordinates of C we have

$$\frac{p+0}{2} = 0 \quad \Rightarrow \quad p = 0$$

$$\frac{q-1}{2} = 1 \quad \Rightarrow \quad q = 3$$

Coordinates of C are (0, 3)

Area of ΔABC

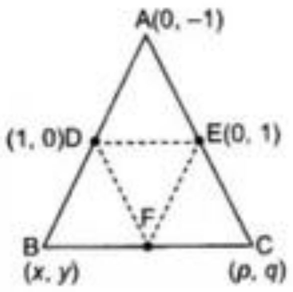
$$\Rightarrow \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = \frac{1}{2}[0(1-3) + 2(3+1) + 0(-1-1)]$$

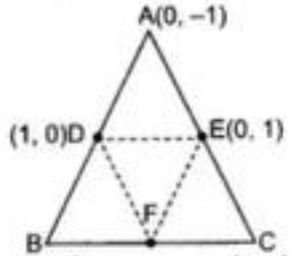
$$= \frac{1}{2} \times 8 = 4 \text{ sq. units}$$

Coordinates of F are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ i.e. (1, 2) [$\because$ Using mid-point formula]

$$\text{Area of } \Delta\text{DEF} = \frac{1}{2}[1(1-2) + 0(2-0) + 1(0-1)] = \frac{1}{2}[-1+0-1]$$

$$= \frac{1}{2} \times (-2) = [-1] = 1 \text{ sq. units} \quad [\because \text{Area cannot be negative}]$$





The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are (0, -3). The origin is the mid-point of the base. Find the coordinates of the points A and B Also find the coordinates of another point D such that BACD is a rhombus.

Solution:

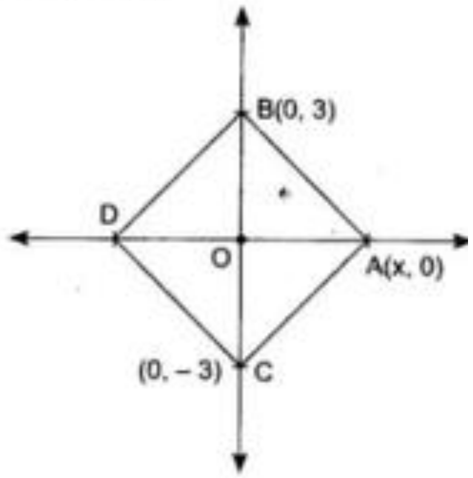
Given that, $\because$ O is mid point of BC and coordinates of C are $(0, -3)$

$\therefore$ coordinate of B are $(0, 3)$

Now AO will be the perpendicular bisector of BC. Therefore A will lie on x-axis. let coordinates of A are $(x, 0)$

Now, in equilateral ΔABC, AB = BC

Using distance formula,



$$\Rightarrow \quad \sqrt{(x-0)^2+(0-3)^2} = 6$$

$$\sqrt{x^2+9} = 6$$

$$x^2 + 9 = 36 \Rightarrow x^2 = 27$$

$$x = \pm 3\sqrt{3}$$

$\therefore$ coordinates of A are $(3\sqrt{3}, 0)$ or $(-3\sqrt{3}, 0)$

When A is $(3\sqrt{3}, 0)$ then D will be $(-3\sqrt{3}, 0)$ so that BACD is a rhombus, since opposite sides are equal.

Question [illegible]

Points A(-l, y) and B(5,7) lie on a circle with centre 0(2, -3y). Find the values ofy. Hence, find the radius of the circle.

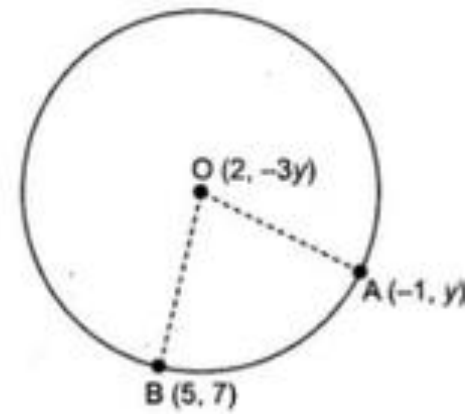
Solution:

Given, O is the centre of the circle and the points A and B lie on the circle.

So, $\quad OA = OB\ (= r) \quad$ [$\because$ radius of same circle]

$$\Rightarrow \quad OA^2 = OB^2$$

Using distance formula,



$$\Rightarrow \quad (2+1)^2 + (-3y-y)^2 = (2-5)^2 + (-3y-7)^2$$

$$\Rightarrow \quad 9 + 16y^2 = 9 + 9y^2 + 42y + 49$$

$$\Rightarrow \quad 7y^2 - 42y - 49 = 0$$

$$\Rightarrow \quad y^2 - 6y - 7 = 0$$

$$\Rightarrow \quad (y-7)(y+1) = 0$$

$$\Rightarrow \quad y = -1 \text{ or } 7$$

When $y = -1$, then co-ordinates are: O(2, 3) and A(–1, –1)

$$\text{Radius of circle, } r = OA = \sqrt{(2+1)^2+(3+1)^2} = \sqrt{9+16} = 5 \text{ units}$$

When $y = 7$, then coordinates are: O(2, – 21) and A(–1, 7)

$$\text{Radius of circle, } r = OA = \sqrt{(2+1)^2+(-21-7)^2} = \sqrt{9+784} = \sqrt{793} \text{ units.}$$

If the point A (x, y) is equidistant from two points P (6, -1) and Q (2,3), prove that y = x – 3.

Solution:

Point A(x, y) is equidistant from P(6, –1) and Q(2, 3). Using distance formula,

$$PA = AQ$$

$$\Rightarrow \quad \sqrt{(6-x)^2+(-1-y)^2} = \sqrt{(2-x)^2+(3-y)^2}$$

Squaring both sides, we get

$$(6-x)^2 + (-1-y)^2 = (2-x)^2 + (3-y)^2$$

$$\Rightarrow \quad 36 + x^2 - 12x + 1 + y^2 + 2y = 4 + x^2 - 4x + 9 + y^2 - 6y$$

$$\Rightarrow \quad -12x + 2y + 37 = -4x - 6y + 13$$

$$\Rightarrow \quad 2y + 6y = 13 - 4x + 12x - 37$$

$$\Rightarrow \quad 8y = 8x - 24$$

$$\Rightarrow \quad y = x - 3$$

Hence, proved.

Point P divides the line segment joining the points A(2,1) and B(5, -8) such that AP /AB=1/3. If P lies on the line 2x – y + k = 0, find the value of k.

Solution:

P is the point of intersection of line segment AB and line $2x - y + k = 0$.

Here, given that, $\dfrac{AP}{AB} = \dfrac{1}{3} \Rightarrow 3AP = AB$

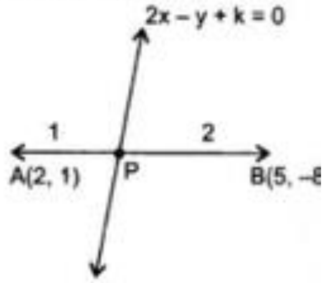
$$\Rightarrow \quad 3AP = AP + PB$$

$$\Rightarrow \quad 2AP = PB$$

$$\Rightarrow \quad \frac{AP}{PB} = \frac{1}{2} \Rightarrow AP : PB = 1 : 2$$

⇒ P divides the line segment joining A(2, 1) and B(5, – 8) in the ratio 1 : 2.

∴ Coordinates of point P are



$$x = \frac{1\times5+2\times2}{1+2} = 3 \quad [\because \text{Using section formula}]$$

$$y = \frac{1\times(-8)+2\times1}{1+2} = -2$$

i.e. P (3, – 2)

As point P lies on the line $2x - y + k = 0$, P must satisfy it.

$$\Rightarrow \quad 6 + 2 + k = 0 \quad \Rightarrow k = -8$$

If the points (p, q); (m, n) and (p-m,q-n) are collinear, show that pn = qm

Solution:

If $P(p, q)$, $Q(m, n)$, $R(p-m, q-n)$ are collinear then area of triangle formed by them is zero.

Hence, $\text{ar}\,\Delta PQR = 0$

$$[\because \text{Area of triangle} = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]]$$

$$\frac{1}{2}|pn - qm + mq - mn - pn + mn + pq - mq - qp + pn| = 0$$

$$\Rightarrow \quad |pn - qm| = 0$$

$$\Rightarrow \quad pn - qm = 0$$

$$\Rightarrow \quad pn = qm$$

Hence, proved.