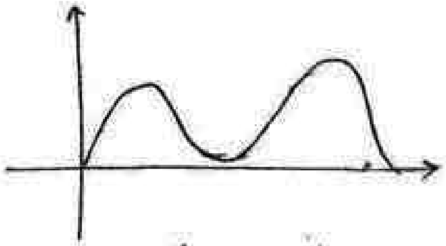
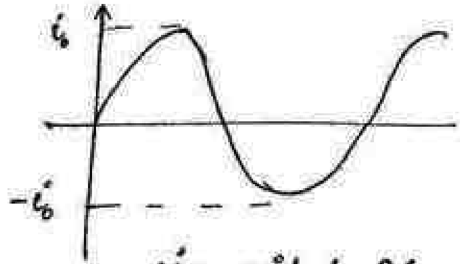
$\boxed{V_{rms} = \frac{V_0}{\sqrt{2}}}$

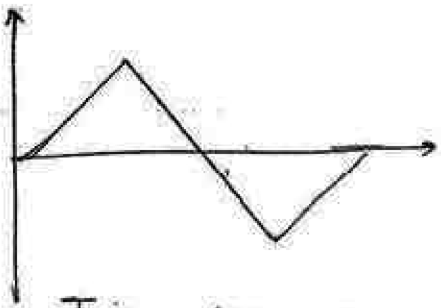
ALTERNATING CURRENT

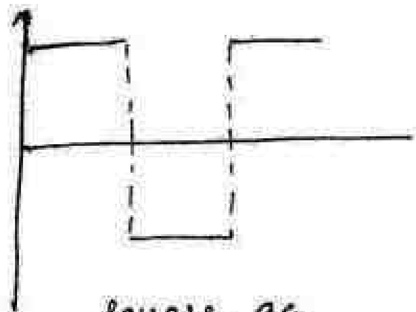
A current in a circuit is called A.C. if –

- It changes its dirⁿ after every half cycle
- It amplitude remains constant with time









Different values in ac:

for $i = i_0 \sin \omega t$

$V = V_0 \sin \omega t$

i) instantaneous value

i = instantaneous value

no instrument is designed on this concept cz it gives zero value

ii) max / Peak value:

i_0 = max value

it gives max value

iii) average value of ac

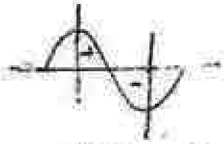
average value of any current for any interval is given by

0 1 2 −2 −1 0 ; ∴ avg = 0

$$\boxed{i_{avg} = \bar{i} = \langle i \rangle = \frac{\int_{t_1}^{t_2} i\,dt}{t_2 - t_1} = \frac{\text{area}}{\text{interval}}}$$

a) avg. value for full cycle:  $: \frac{\text{area}}{\text{interval}} = 0$

$$\bar{i}_T = \bar{i_0} \, \overline{\sin \omega t} \qquad \boxed{\bar{i}_T = 0}$$

b) avg value for half cycle:  

area = 0

$$T = \frac{\int_0^{T/2} i\,dt}{\frac{T}{2} - 0} = \frac{2 i_0 \int_0^{T/2} \sin \omega t}{T}$$

$$= -\frac{2 i_0}{T} \left(\frac{\cos \omega t}{\omega} \right)_0^{T/2} = \frac{-2 i_0}{\omega T} \left(\cos \frac{\omega T}{2} - \cos 0 \right)$$

$$= \frac{-2 i_0}{2\pi} \left(\cos \pi - \cos 0 \right) \qquad \{ \omega T = 2\pi \}$$

$$\therefore \boxed{\bar{i} = \frac{2 i_0}{\pi}} \qquad \bar{i} = 0.636\, i_0$$

$\rightarrow$ I_{dc} for full wave rectifier in electronics

c) rms value / apparant value / virtual value / effective:

rms value of ac for any interval is given by

$$i_{rms} = \sqrt{\overline{i^2}} = \sqrt{\frac{\int_{t_1}^{t_2} i^2\,dt}{t_2 - t_1}}$$

square
↓
mean
↓
root

$$\text{so, } \sqrt{\bar{i}} = \sqrt{\frac{\int_0^T i_0^2 \sin^2 \omega t\,dt}{T - 0}}$$

$$= i_0 \sqrt{\frac{\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt}{T}}$$

$\because \cos 2\omega t = 1 - 2\sin^2 \omega t$

$\therefore \sin^2 \omega t = \frac{1-\cos 2\omega t}{2}$

* so all the ac instrument measures rms value of ac

Q) A current in circuit is given by 2A, 50 Hz then find

i) after what time, current will attain its max and rms value

ii) max, avg, rms, and instantenous value of given current.

i) a) $i = i_0 \sin \omega t$

$\cancel{i_0} = \cancel{i_0} \sin \omega t$

$\therefore \sin \omega t = \sin \frac{\pi}{2}$

$$\therefore \quad \omega t = \frac{\pi}{2}$$

$$2\pi f t = \frac{\pi}{2}$$

$$\therefore \quad t = \frac{1}{4f} = \frac{1}{4\times 50} = \frac{1}{200} \text{ sec}$$

b).

$$\frac{\cancel{i_0}}{\sqrt{2}} = \cancel{i_0} \sin \omega t \qquad \{ i_{rms} \}$$

$$\sin \frac{\pi}{4} = \sin \omega t$$

$$\therefore \quad \frac{\pi}{4} = \omega t$$

$$\frac{\cancel{\pi}}{4} = 2\cancel{\pi} f t \qquad \therefore \quad t = \frac{1}{8f} = \frac{1}{400} \text{ sec}$$

ii) $\Rightarrow$ $i_{rms} = 2A$ [i_{rms}]

$\Rightarrow$ $i_0 = \sqrt{2}\, i_{rms} = 2\sqrt{2}$ [i_{max}] $\left\{ \because i_{rms} = \frac{i_0}{\sqrt{2}} \right\}$

$\Rightarrow$ $i = i_0 \sin(2\pi f t) = 2\sqrt{2} \sin(100\pi t)$ [$i_{instant}$]

$\because$ $i = i_0 \sin \omega t$

$\Rightarrow$ $\overline{i} = \overline{i_0}\, \overline{\sin \omega t} = 0$ [i_{avg}]

* MRT

$$\left\{ \begin{aligned} \overline{\sin \theta} &= \overline{\cos \theta} = 0 \\ \overline{\sin^2 \theta} &= \overline{\cos^2 \theta} = \frac{1}{2} \\ \omega T &= 2\pi \end{aligned} \right\}$$

* $\left\{ \begin{aligned} 2 \sin\theta \cos\theta &= \sin 2\theta \\ \sin^2\theta + \cos^2\theta &= 1 \end{aligned} \right\}$ $\qquad \cos\theta = \sin\left(\theta + \frac{\pi}{2}\right) = \sin\left(\theta - \frac{\pi}{2}\right)$

Q) If ac voltage is given by $V = 622 \sin(50\pi t)\cos(50\pi t)$ then find max value, rms value and average value for ~~the~~ half cycle of voltage.

$$V = 311\left[2\sin(50\pi t)\cos(50\pi t)\right]$$

$$\boxed{= 311 \sin(100\pi t) \text{ volt}}$$ ⇒ eq. ghar ghar ki :P

$V_0 = \boxed{311}$, $\quad V_{rms} = \dfrac{V_0}{\sqrt{2}} = \dfrac{311}{\sqrt{2}} = \boxed{220 \text{ volt}}$

$$\overline{V_{T/2}} = \frac{2V_0}{\pi} = \boxed{198 \text{ volt}}$$

Q) A 3A dc is superimposed on $4\sin\omega t$ ac, then find max, avg, and rms values of the resultant current

$\Rightarrow \quad i_{max} = i_{ac} + i_{dc}$

$= 4\sin\omega t + 3 = 4 + 3 = \boxed{7A}$

$\Rightarrow \quad \overline{i_{avg}} = \overline{3} + \overline{4\sin\omega t}$

$= 3 + 4 \times 0 = \boxed{3A}$

$\Rightarrow \quad i_{rms} \Rightarrow \quad \because i = 3 + 4\sin\omega t$

$\therefore \overline{i^2} = \overline{9} + \overline{16\sin^2\omega t} + \overline{24\sin\omega t}$

$\overline{i^2} = 9 + 16 \times \frac{1}{2} + 0 = 17$

$\therefore i_{rms} = \sqrt{\overline{i^2}} = \boxed{\sqrt{17}}$

Q) An ac of $3 \sin wt$ is superimposed on another of $4 \cos wt$. Then find peak, ~~max~~ avg, rms, of resultant.

$$\Rightarrow \because\ i = 3\sin wt + 4 \cancel{\sin wt} \cos wt$$

$$i_0 = \sqrt{3^2 + 4^2 + 2\times3\times4\cos\frac{\pi}{2}} = \boxed{5A = i_{max}}$$

$$\Rightarrow \cancel{max}\ \bar{i} = \bar{3}\ \overline{\sin wt} + \bar{4}\ \overline{\cos wt} = \boxed{0 = i_{avg}}$$

$$\Rightarrow \overline{i^2} = \bar{9}\ \overline{\sin^2 wt} + \overline{16}\ \overline{\cos^2 wt} + \overline{24}\ \overline{\sin wt \cos wt}$$

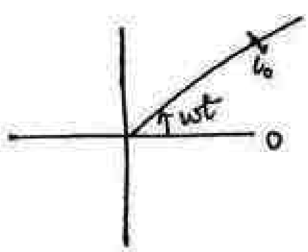
$$= 9\times\frac{1}{2} + 16\times\frac{1}{2} + 0 = \frac{25}{2}$$

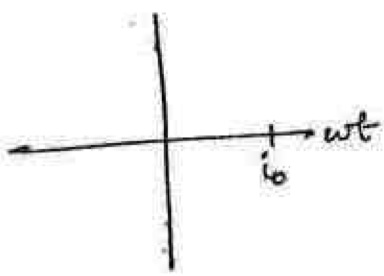
$$\therefore \sqrt{\overline{i^2}} = \boxed{\frac{5}{\sqrt{2}} = i_{rms}}$$

Phaser Quantities :

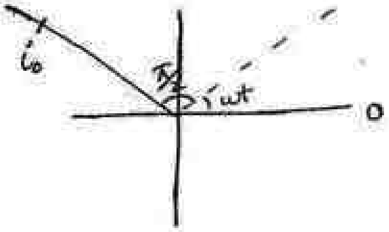
These are the quantities which behaves like an vector due to phase angle also called rotating vectors

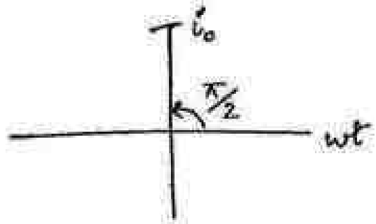
i) $i = i_0 \sin wt$



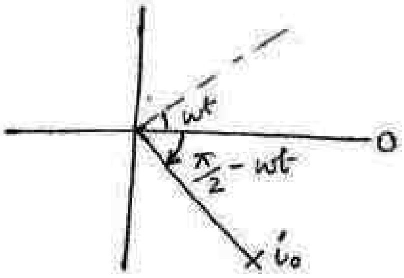


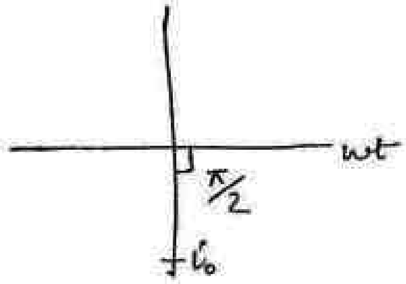
ii) $i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$





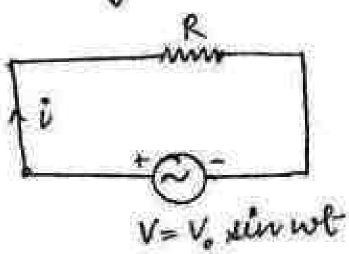
iii) $i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$





different element in ac circuit :

i) <u>If only resistance (R) is connected in circuit.</u>



By KVL,

$-iR + V = 0$

$\therefore V_0 \sin \omega t = iR$

$\therefore i = \frac{V_0}{R} \sin \omega t$

$\therefore \boxed{i = i_0 \sin \omega t}$

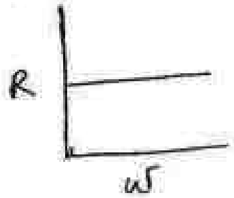
$\because V = V_0 \sin \omega t$ and $i = i_0 \sin \omega t$

> Phase diff^n b/w V and i

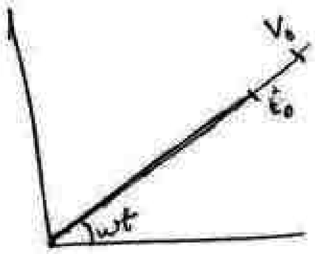
phase diff^n $\phi = \omega t - \omega t = 0$ $\therefore \boxed{\phi = 0}$

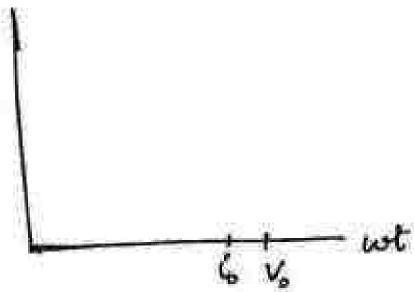
so across the resistance, V and i are in same phase

- as $R \propto \omega^0$, so behaviour of Resistance is same in ac as well as dc.

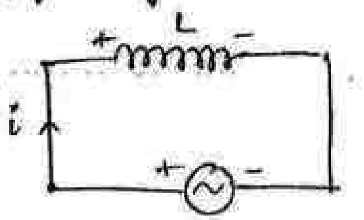


- Phasor diagram:





ii) <u>If only inductor is connected in circuit</u>



$V = V_0 \sin\omega t$

By KVL,

$$-V_L + V = 0$$

$$\frac{L\,di}{dt} = V_0 \sin\omega t$$

definite integer ≠
∵ at t = 0, $i \neq 0$ (eq unknown)

$$\therefore \int di = \frac{V_0}{L}\int \sin\omega t\, dt$$

$$i = \frac{V_0}{L}\left(\frac{-\cos\omega t}{\omega}\right) + C$$

here C must be 0 cz there will be no dc component.

$$i = \frac{V_0}{\omega L} - \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$\because \cos\theta = \sin\left(\theta + \frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2} - \theta\right) = \sin\left(\theta - \frac{\pi}{2}\right)$$

$$\therefore \boxed{i = \frac{V_0}{[\omega L]} \sin\left(\omega t - \frac{\pi}{2}\right)}$$

$$\boxed{i = \frac{V_0}{[\ ]} \sin\left(\omega t - \frac{\pi}{2}\right)} \quad \because X_L = \omega L$$

> Phase difference

$$\phi = \underbrace{\omega t}_{\text{reference}} - \left(\omega t - \frac{\pi}{2}\right) = +\pi/2$$

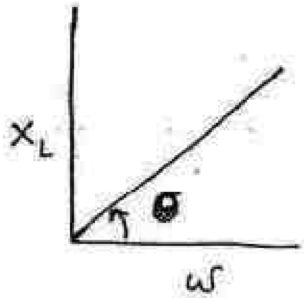
So, voltage leads the current by a phase angle $+\pi/2$ in an inductor

> X_L = Inductive reactance

It represents the resistance offered by the inductor due to its self inductance.

$\boxed{X_L = \omega L}$ ∴ $\boxed{X_L = 2\pi f L}$ ∴ $\boxed{X_L \propto \omega}$ & $\boxed{X_L \propto L}$

∴



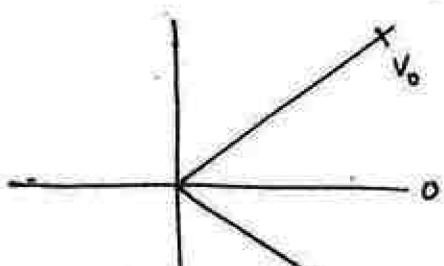
slope $= \frac{X_L}{\omega} = L$

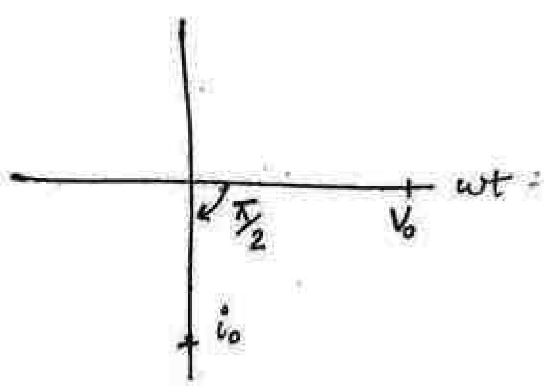
> Behaviour of inductor   inductor dc ko jane dega if

in dc: $\omega = 0$ ∴ $X_L = 0$ ∴ Behaviour ⇒ connecting wire

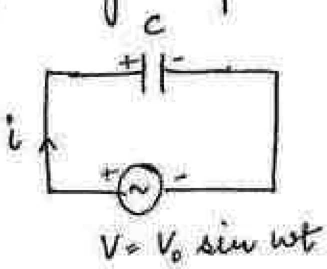
in ac: $X_L \propto \omega$ ∴ $\omega\uparrow$ $X_L\uparrow$ $i\downarrow$

> $V = V_0 \sin \omega t$ and $i = i_0 \sin\left(\omega t - \pi/2\right)$





iii) If only capacitor is connected in circuit



current in circuit

By KVL,

$$-V_c + V = 0$$

$$\frac{q}{C} = V_0 \sin \omega t$$

$\because Q = CV$ $\therefore V = q/C$

$$\therefore q = CV_0 \sin \omega t$$

$$\therefore \frac{dq}{dt} = CV_0 \omega \cos \omega t$$

$$\therefore i = \frac{V_0}{\left[\frac{1}{C\omega}\right]} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore i = \frac{V_0}{[X_c]} \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \left\{\because X_c = \frac{1}{C\omega}\right\}$$

> Phase difference:

$$\phi = \omega t - \left(\omega t + \frac{\pi}{2}\right) = -\frac{\pi}{2}$$

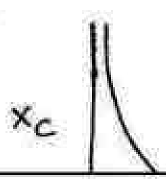
$\therefore$ Voltage lags behind the current by a phase angle $\frac{\pi}{2}$

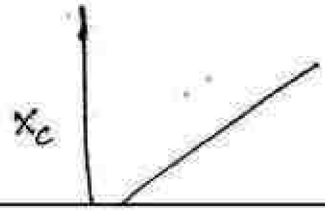
capacitor & CC → current $\therefore$ In capacitor, current leads by $\frac{\pi}{2}$

> X_c = capacitive reactance

It represents the resistance offered by the capacitor cz existing charge always repels upcoming charge.

> $\boxed{X_c = \frac{1}{\omega C}}$ $\boxed{X_c = \frac{1}{2\pi f C}}$ $\therefore$ $\boxed{X_c \propto \frac{1}{\omega}}$





Behaviour of capacitor.

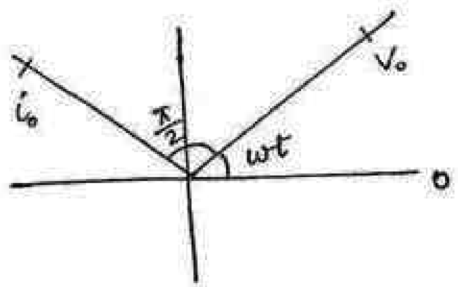
<u>In dc</u> $\omega = 0 \quad \therefore X_c = \infty \quad \therefore i = 0$

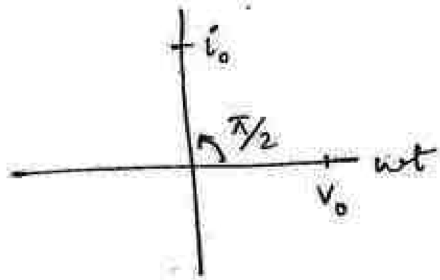
<u>In ac</u> $X_c \propto \frac{1}{\omega} \quad \therefore \omega \uparrow \; X_c \downarrow \; \therefore i \uparrow$

* So, inductor opposes high frequency ac. but capacitor blocks dc

effect of dc → on capacitor ⇒ blocks
effect of dc → on inductor ⇒ no effect

> $V = V_0 \sin \omega t \qquad i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$





Q) An inductor of 1 H is connected ~~accro~~ across ac source of 200 V, $100/\pi$ Hz. ~~th~~ Find current through inductor.

$$i = \frac{V_0}{X_L} = \frac{V_0}{\omega L} = \frac{V_0}{2\pi f L}$$

$$= \frac{200}{2\pi \frac{100}{\pi} \times 1} = 1 \text{ A}$$

Q) An ac voltage is given by $V = 100\sqrt{2} \sin(100t)$ volt then find current through the following element

i) Resistance of $10\,\Omega$

ii) Inductance of 10 H and

iii) capacitance of $10\,\mu F$

i) $i = \frac{V_{rms}}{R} = \left(\frac{100\sqrt{2}}{\sqrt{2}}\right) \Big/ 10 = 10\ A$

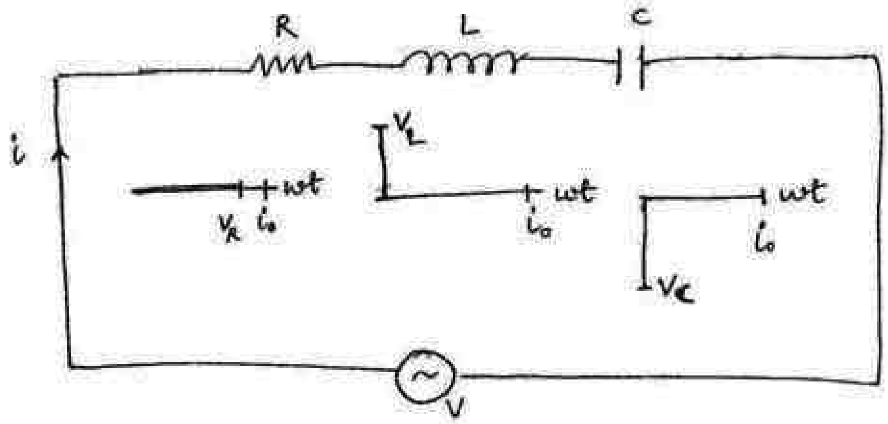
ii) $i = \frac{V_{rms}}{\omega L} = \frac{100}{100 \times 10} = 0.1\ A$

iii) $i = \frac{V_{rms}}{\frac{1}{\omega C}} = V_{rms}\,\omega C = 100 \times 100 \times (10 \times 10^{-6})$

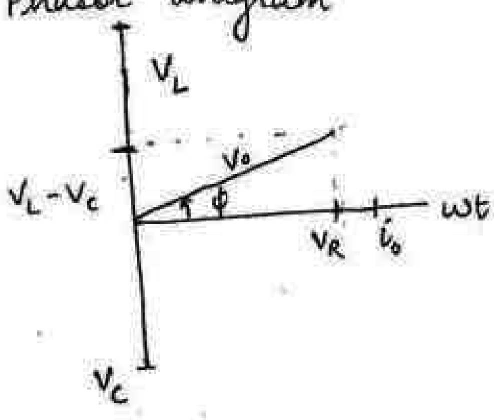
$= 0.1\ A$

Series R-L-C combination

Let a resistance R, inductance L, and capacitance C are connected in series, then current through each element. $i = i_0 \sin \omega t$ will remain same



Phasor diagram

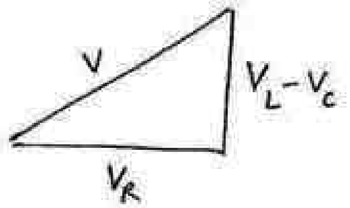


$V = V_0 \sin(\omega t \pm \phi)$ $\quad V_L > V_C \ (+\phi)$ $\quad V_L < V_C \ (-\phi)$

here,

$$\boxed{V_0 = \sqrt{V_{0R}^2 + (V_{0L} - V_{0C})^2}}$$

> Voltage triangle



$$\therefore \boxed{V = \sqrt{V_R^2 + (V_L - V_C)^2}} \quad \text{—①}$$

> $V_R = iR$

$V_L = iX_L$

$V_C = iX_C$

$V = iZ \quad \because Z = \text{impedence}$

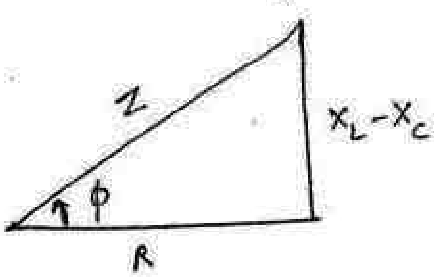
here, impedence represents total resistance offered by all the element in circuit

From eq ①

$$iZ = \sqrt{(iR)^2 + (iX_L - iX_C)^2}$$

$$\therefore \boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

> Impedence triangle

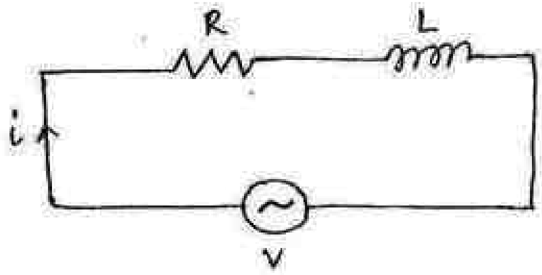


> Phase difference b/w voltage and current.

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

$$\sin\phi = \frac{V_L - V_C}{V} = \frac{X_L - X_C}{Z}$$

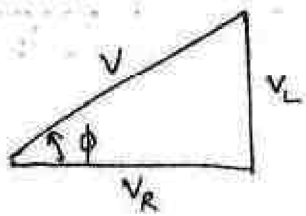
** Series R-L circuit



Phasor:

$$V = V_0 \sin(\omega t + \phi)$$

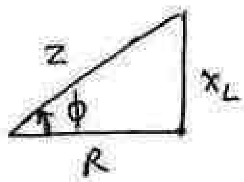
> Voltage triangle



$$\boxed{V = \sqrt{V_R^2 + V_L^2}}$$

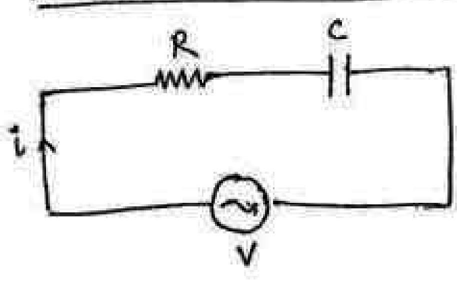
> Impedence triangle

$$iZ = \sqrt{(iR)^2 + (iX_L)^2} \quad \therefore \quad Z = \sqrt{R^2 + X_L^2}$$

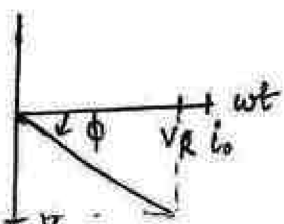


$$\tan \phi = \frac{X_L}{R} = \frac{V_L}{V_R}$$

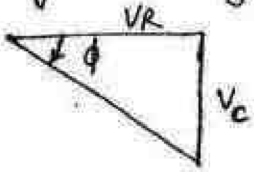
Series R-C circuit



Phasor:



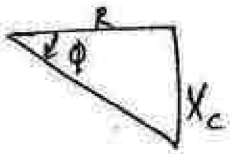
- Voltage Triangle



$\therefore \boxed{V = \sqrt{V_R^2 + V_C^2}}$

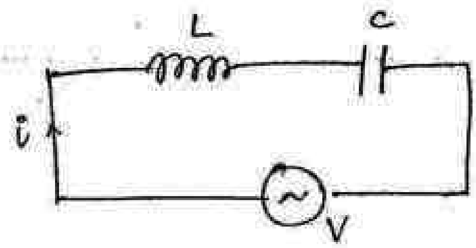
- Impedence triangle

$iZ = \sqrt{(iR)^2 + (iX_c)^2} \quad \therefore \boxed{Z = \sqrt{R^2 + X_c^2}}$

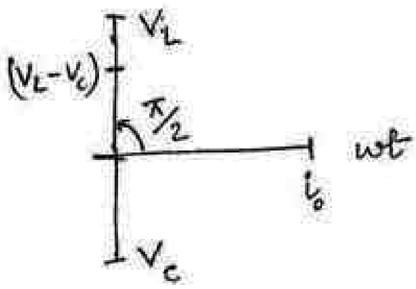


$\therefore \tan\phi = \dfrac{V_c}{V_R} = \dfrac{X_c}{R}$

Series L-C circuit



Phasor:



$\therefore V = V_0 \sin wt \pm \dfrac{\pi}{2}$

$V_L > V_c \quad \left(+\frac{\pi}{2}\right)$

$V_L < V_c \quad \left(-\frac{\pi}{2}\right)$

$V = V_L - V_c$

$Z = X_L - X_c$

$\phi = \pm\dfrac{\pi}{2}$

Q) In series R-L-C circuit, voltages across resistance, inductance and capacitance are 80 V, 110 V, and 170 V, then find applied instantaneous voltage

$$V = \sqrt{80^2 + (110 - 170)^2} = 100\,V = V_{rms}$$

$$\therefore \quad V_0 = V_{rms}\sqrt{2} = 100\sqrt{2}$$

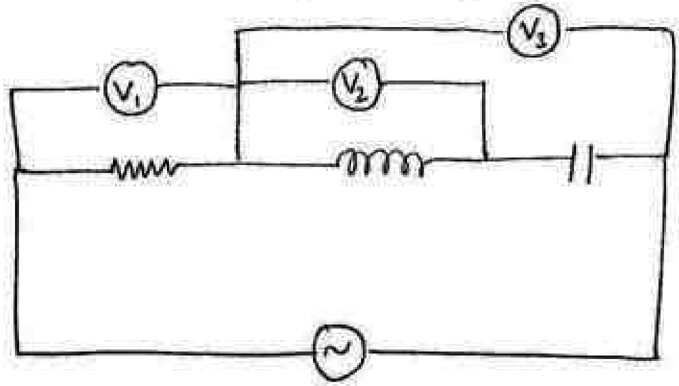
$$\because \; V = V_0 \sin(\omega t \pm \phi) \qquad \therefore \; \tan\phi = \frac{V_L - V_C}{V_R}$$

$$\tan\phi = \frac{110 - 170}{80} = \frac{\ominus 3}{4} \qquad \therefore \; \phi = 37^\circ$$

$\because$ it's its -ve

$$\therefore \; V = 100\sqrt{2}\,\sin(\omega t - 37^\circ) \qquad \therefore \; V_c > V_L$$

Q) In the given diagram, resistance, inductance resistance and capacitance resistance are are 30 Ω, 100 Ω, 60 Ω respectively, then find reading of voltmeter V_1, V_2, V_3 if current in the circuit is $i = 2\sqrt{2}\sin(10t)$



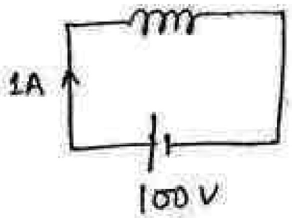
$$i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2}} = 2$$

$$\therefore \; V_1 = V_R = iR = 2 \times 30 = 60\,V$$

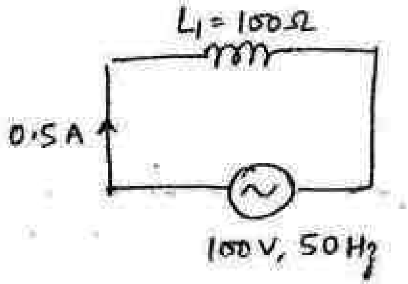
$$V_2 = iX_L = 2 \times 100 = 200\,V$$

$$\therefore \; V_3 = V_2 - V_1 = 200 - 120$$

Q) A choke coil when connected to dc source of 100 V then 1A current flows through it when it is connected to an ac source of 100 V, 50 Hz, then 0.5 A current flows through. Find its inductance



$$R = \frac{V}{i} = \frac{100}{1} = 100\,\Omega$$



$$\therefore\ i = \frac{V}{\left[\sqrt{R^2 + X_L^2}\right]}$$

$\because$ R and X_L are always perpendicular

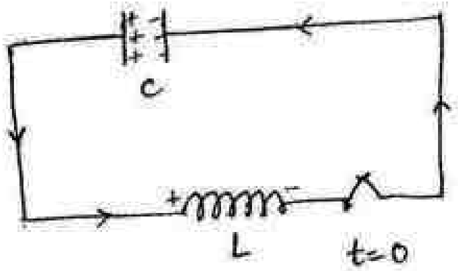
$$0.5 = \frac{100}{\sqrt{R^2 + X_L^2}} \qquad \therefore\ R^2 + X_L^2 = 40{,}000$$

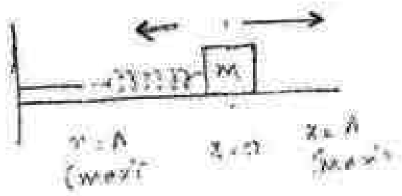
$$10{,}000 + X_L^2 = 40{,}000$$

$$\omega L = X_L = 100\sqrt{3} \quad ; \quad 2\pi f L = 100\sqrt{3} \qquad (\because f = 50)$$

$$\therefore\ \cancel{100}\,\pi L = \sqrt{3} \times \cancel{100}\ ;\ L = \frac{\sqrt{3}}{\pi}\ \text{H}$$

L-C oscillations





When an charged capacitor is connected to an inductor then due to discharging of capacitor. current ↑es in circuit which is opposed by the inductor.

The potential energy of the capacitor continuously converted into magnetic energy in the inductor and then magnetic energy starts converting into potential energy in the capacitor. The process continuous to repeat itself considering no heat loss in connecting wire. This phenomenon is called L-C oscillations

Applying KVL for the above circuit

$$-V_L + V_C = 0$$

$$-L\frac{di}{dt} + \frac{q}{C} = 0$$

$$-L\frac{d}{dt}\left(-\frac{dq}{dt}\right) + \frac{q}{C} = 0 \quad \text{(as } q \text{ is decreasing)}$$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \quad -(1) \quad \text{(on dividing by } L\text{)}$$

comparing eq ① with stardard equation of SHM

$$\frac{d^2x}{dt^2} + w^2x = 0 \quad —②$$

then $\boxed{w = \frac{1}{\sqrt{LC}}} = w_n$ = natural frequency of LC oscillati

as sol^n of eq ② is $x = A\cos(wt + \phi)$

so, sol^n of eq ①

$$q = q_0 \cos(wt + \phi)$$

initially at $t=0$, $q = q_0$

$\therefore q_0 = q_0 \cos\phi \qquad \therefore \phi = 0$

so $\boxed{q = q_0 \cos wt}$ $\qquad$ ~~Then~~ $\because i = \frac{dq}{dt}$

$\therefore \boxed{i = -q_0 w \sin wt}$

$\because U = \frac{1}{2} L i^2 \qquad \therefore \boxed{U = \frac{1}{2} L q_0^2 w^2 \sin^2 wt}$

Series R-L-C resonance

when applied frequency in series R-L-C circuit becomes equal to natural frequency of R-L oscillation then, this condition is called resonance.

so, $w = w_n = w_r$ where w_r = resonating frequency

Regarding series R-L-C resonance

$w_r = \frac{1}{\sqrt{LC}} \qquad \therefore w_r^2 = \frac{1}{LC} \qquad \therefore w_r L = \frac{1}{w_r C}$

$$\therefore \boxed{X_L = X_C}$$

$$iX_L = iX_C \quad \therefore \quad \boxed{V_L = V_C}$$

$$\because \quad V = \sqrt{V_R^2 + (V_L - V_C)^2} \quad \therefore \quad \cancel{V=P} \quad \boxed{V = V_R} \quad (\because V_L = V_C)$$

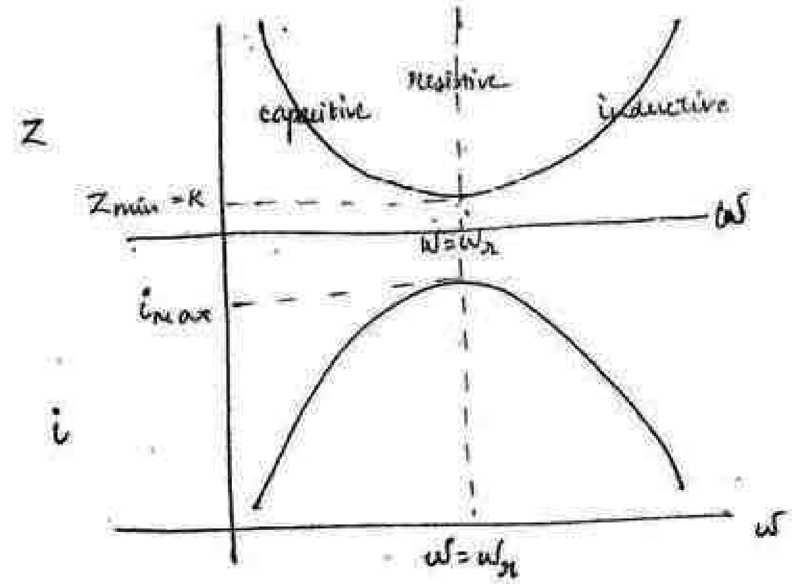
$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \therefore \quad \boxed{Z = R = min}$$

$$i = \frac{V}{Z_{min}} \quad \therefore \quad \boxed{i = \frac{V}{R} = max}$$

$$\tan\phi = \frac{X_L - X_C}{R} = 0 \quad \therefore \quad \phi = 0 \quad (\because X_L = X_C)$$

$\therefore$ V and i are in same phase in resonating condition.

Resonance curve



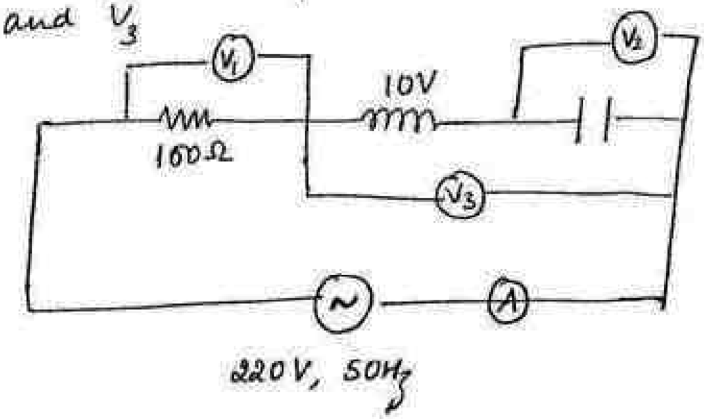
nature of circuit

a) if $\omega < \omega_r \rightarrow$ capacitive

b) if $\omega = \omega_r \rightarrow$ resistive

c) if $\omega > \omega_r \rightarrow$ Inductive

Q) In the given diagram, find readings of voltmeter V_1, V_2 and V_3



Reading of ammeter is 2.2 A

$V_1 = iR = 2.2 \times 100 = 220V = V_R$

$\therefore V_R = V$ ($\because$ circuit is in resonance)

$V_2 = V_C = V_L = 10V$ ($V_L = V_C$ in resonance)

$V_3 = V_L - V_C = 10 - 10 = 0$

Q) In series R-L-C circuit, resistance and capacitance are 10Ω and $1\mu F$ connected to an ac source 100 V and $\frac{10}{\pi}$ Hz, then find voltage across inductor if circuit is in resonance / max current flows through the circuit / Impedence is min / power consumed is max / phase diff = 0

$R = 10\Omega$

$C = 1\mu F$

$V = 100V$

$f = \frac{10}{\pi}$ Hz

$\because Z = $ min (reso.)

$\because V_L = V_c$ (i.e resonance)

$$V_L = \left(V_c = i_r X_c\right) = \frac{V}{R} \times \frac{1}{2\pi f C}$$

$$= \frac{100}{10} \times \frac{1}{2\pi \times \frac{10}{\pi} \times 10^{-6}}$$

$$= 5 \times 10^{5} V$$

Q) In L-C oscillating circuit, capacitance becomes 4 times and inductance becomes 2 ~~main~~ times then find by what times natural ~~no~~ frequency will ↓

$$\omega = \frac{1}{\sqrt{LC}} \qquad \omega' = \frac{1}{\sqrt{4C \times 2L}} = \frac{\omega}{2\sqrt{2}}$$

$$= 0.35\,\omega \qquad \left[\because \frac{1}{\sqrt{2}} = 0.7\right]$$

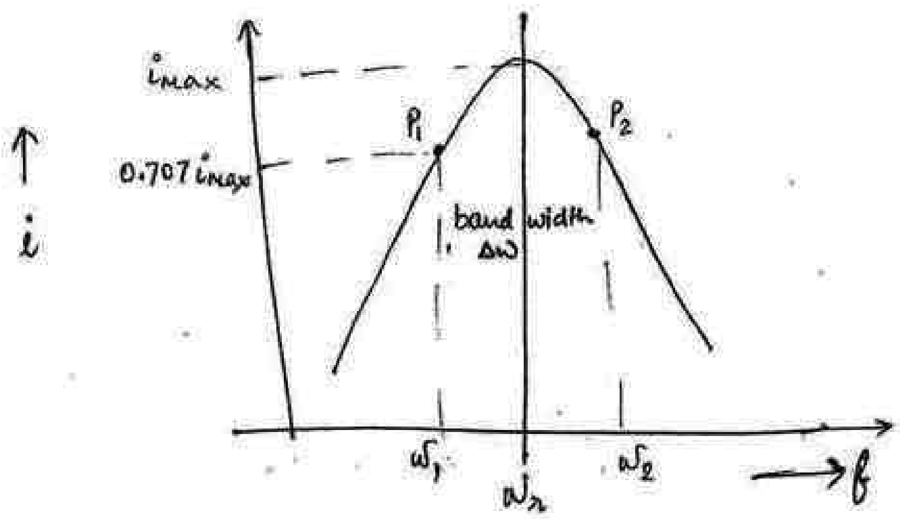
$\therefore$ ω decreases by $(1 - 0.35)\,\omega = 0.65\,\omega$

Half Power Point

These are the points in resonance curve where power consumed in circuit is half of the max power consumed at the time of resonance.

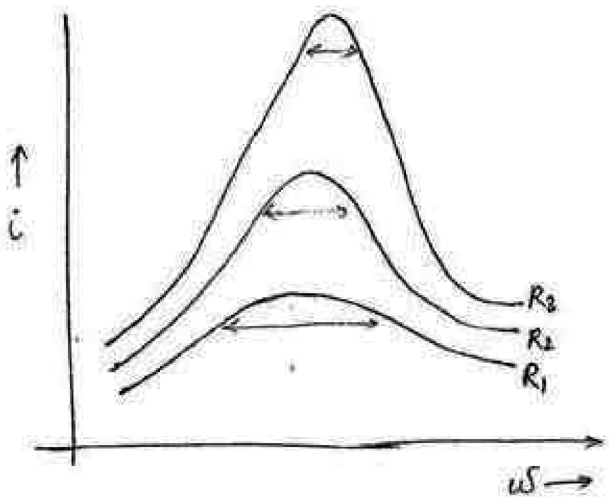
$$P = \frac{P_{max}}{2} \quad \text{i.e} \quad i^2 R = \frac{i_{max}^2 R}{2} \qquad \left[\because P = I^2 R\right]$$

$$\boxed{i = \frac{i_{max}}{\sqrt{2}} = 0.707\, i_{max}}$$



Half Power points (HPP) ; ω_1, ω_2 = HPP frequency

Sharpness of resonance curve :



$\boxed{R_1 > R_2 > R_3}$

Quality factor :

It represents the quality / selectivity of an electrical signal and given by the ratio of resonating frequency to band width.

$$\boxed{Q = \frac{\omega_r}{\Delta\omega}} \qquad Q = \frac{\omega_r L}{R} \quad (\because \Delta\omega L = R) \qquad \therefore \boxed{Q = \frac{X_L}{R} = \frac{X_C}{R}}$$

$\because$ its in resonance $\therefore X_L = X_C$

$$Q = \frac{1}{\sqrt{LC}} \times \frac{L}{R} \Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$\therefore$ $\boxed{R \downarrow \quad i_{max} \uparrow \quad BW \downarrow \quad \text{sharpness} \uparrow \quad \text{selectivity} \uparrow \quad Q \uparrow}$

(Band width)

Power in ac circuit

let instantaneous Voltage and current are given by

$$V = V_0 \sin \omega t \qquad i = i_0 \sin(\omega t \pm \phi)$$

the instantaneous Power is given by

$$P = Vi = V_0 \sin \omega t \left[i_0 \sin(\omega t \pm \phi) \right]$$

$$\therefore \quad P = V_0 I_0 \sin\omega t \left[\sin\omega t \cos\phi \pm \cos\omega t \sin\phi \right]$$

$$= V_0 I_0 \left[\sin^2\omega t \cos\phi \pm \cos\omega t \sin\omega t \sin\phi \right]$$

So, average power consumed in circuit

$$\overline{P} = \overline{V_0 I_0} \left[\overline{\sin^2\omega t}\ \overline{\cos\phi} \pm \overline{\cos\omega t}\ \overline{\sin\omega t}\ \overline{\sin\phi} \right]$$

$$= V_0 I_0 \left[\frac{1}{2} \cos\phi \pm 0 \right]$$

$$\therefore \boxed{\overline{P} = \frac{V_0 i_0 \cos\phi}{2}}$$

$$\overline{P} = \frac{V_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \cos\phi \qquad \overline{P} = V_{rms}\, i_{rms} \cos\phi$$

$$\therefore \quad = (i_{rms}) Z \left(i_{rms} \frac{R}{Z} \right)$$

$$\therefore \boxed{\overline{P} = i_{rms}^2 R}$$

$$\therefore \quad \overline{P} = \left(\frac{V_{rms}}{Z} \right)^2 R \qquad \therefore \boxed{\overline{P} = \frac{V_{rms}^2 R}{R^2 + X^2}}$$

$$\because (X = X_L - X_C)$$

Power factor (PF)

In expression of <u>avg. power</u>, the multiplying factor $\cos\phi$ is called power factor

$$\boxed{P.f = \cos\phi = \frac{\overline{P}}{P_{rms}} = \left(\frac{R}{Z}\right) = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}} \qquad 0 \leq P.F \leq 1$$

Power consumption in diff element:

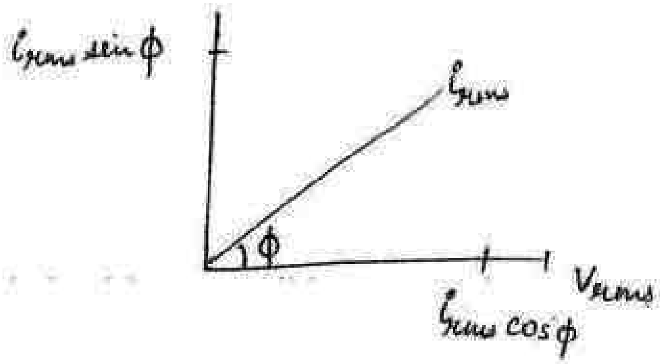
i) only R $\therefore \phi = 0$ $\therefore$ P.F $= \cos\phi = 1 =$ Max

ii) series R-L-C resonance $\phi = 0$ $\therefore$ PF $= \cos\phi = 1 =$ Max

iii) if only L or C or LC both

$\therefore \phi = \pm \pi/2$ $\therefore$ P.F $= 0$

Wattless and working current



The component $i_{rms}\sin\phi$ which is having a phase diff of $\frac{\pi}{2}$ with the voltage consumes no power in circuit called wattless current

a) $i_{rms}\sin\phi =$ wattless current

b) $i_{rms}\cos\phi =$ working current

Choke coil

> It consist of a copper wire which is bend to form a helix 

> Its working is based on wattless current

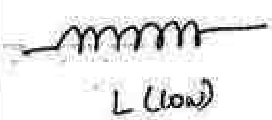
> It is used to control / ↓ the current in circuit

- A practical choke coil will have very small resistance and high inductance

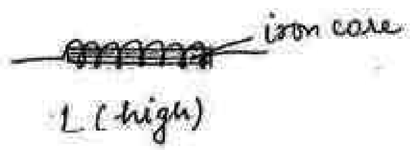
Resistance of ideal choke coil is assumed to be zero also called pure inductor.

choke coil are of 2 types:

air cored



Iron cored



AC / DC instruments

AC instruments are based on heating effects of current cz heat released is independant of dirn of current.

These instruments are also called hot wire instruments

DC instruments can't be used in AC circuit but AC instruments can be used in DC circuit

In DC instruments, scale is linear ($\theta \propto i$) but in AC instrument, scale is non-linear ~~to~~.

$(i\uparrow, H\uparrow, i^2Rt\uparrow, T\uparrow, l\uparrow \therefore \theta\uparrow) \therefore \theta \propto i^2$

Q) In series R-L-C circuit, resistance, inductive rectance and capacitive reactance are 2Ω, 6Ω, 4Ω respectively connected to an ac.

$i = 2\sqrt{2}\sin(314t)$ A then find

i) reactance of circuit.

ii) Phase diff^n b/w voltage n current

iii) Power factor

iv) Power consumed

i) $X = X_L - X_C = 6-4 = 2\Omega$

ii) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{6-4}{2} = 1 \quad \therefore \phi = 45 = \dfrac{\pi}{4}$

iii) $P.F = \cos\phi = \dfrac{1}{\sqrt{2}}$

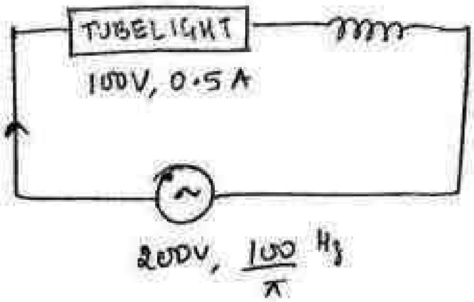
iv) $P = i^2R = 2^2 \times 2 = 8W$

Q) In AC circuit, voltage and current are given by $V = V_0 \sin\omega t$ and $I = I_0$ ~~sin~~ $\cos\omega t$ then find Power consumed in the circuit.

$\because \phi = \pi/2 \quad \therefore P = 0$

$\because \cos\dfrac{\pi}{2} = \cos 90^\circ = 0$

Q) A tubelight operates at 100V, 0.5A connected to an AC source of 200V, $\frac{100}{\pi}$ Hz, then find inductance required to be connected in series with tubelight.



$$\therefore \quad i = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$$

$$0.5 = \frac{200}{\sqrt{200^2 + \left(2\pi \frac{100}{\pi} L\right)^2}} \qquad \therefore \quad L = \sqrt{3} \text{ H}$$

Q) In AC circuit, voltage and current are $\overset{rms}{\overline{200\text{ V}, 2\text{A}}}$ and power consumption is $\underset{avg}{\underbrace{300 \text{ Watt}}}$. Then find Power factor of the circuit

$$P.F = \frac{\bar{P}}{P_{rms}} = \frac{300}{200 \times 2} = \frac{3}{4} = 0.75$$