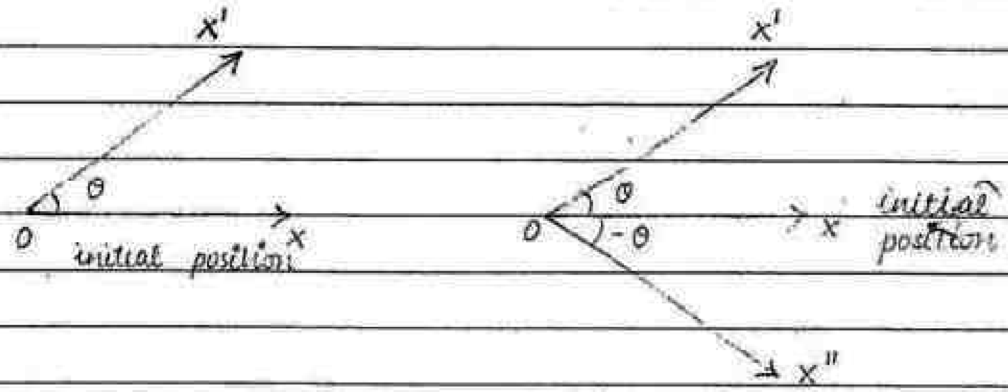
Physics: It is a branch of science which deals with study of nature and natural Phenomena

BASIC MATHEMATICS

TRIGNOMETRY

Angle: It is defined as the amount of revolution which the revolving line makes with the initial position.



* If angle is measured in anticlock wise direction, it is taken as positive.
* If angle is measured in clock wise direction, it is taken as negative.

Units of angle:

i) degree
ii) radian (SI unit of angle)

Relations of degree:

- 1 right angle = 90°
- 1° = 60 min = $60'$
- $1'$ = 60 sec = $60''$

- $1 \text{ degree} = 60 \times 60'' = 3600''$

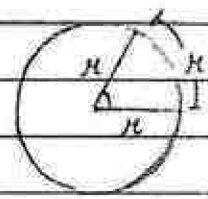
Relation of Radian

- Angle at centre of circle = $\frac{\text{Arc}}{\text{Radius}}$ (in radian)

* If length of the arc = radius of circle

$\text{Angle} = \frac{r}{r} = 1 \text{ radian} = 1 \text{ rad}$

1 Radian : It is the angle substended by an arc at the centre of the circle whose length of the arc is equal to the radius of the circle



- $\boxed{\pi \text{ radian} = 180^\circ \quad 1 \text{ radian} = \frac{180^\circ}{\pi}}$

$1 \text{ radian} = \frac{180^\circ}{3.14} \quad \{\pi = 3.14\}$

$1 \text{ radian} \simeq 57.3^\circ$

For one complete revolution of circle

$\text{Angle} = \frac{\text{Arc}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi \text{ radian}$

$2\pi \text{ radian} = 360^\circ$

$\pi \text{ rad} = 180^\circ$

$180^\circ = \pi \text{ radian}$

$\boxed{1^\circ = \frac{\pi}{180^\circ} \text{ rad}}$

Q.1 convert degree to radian.

i) 45°

$1^\circ = \frac{\pi}{180}$ rad.

$\therefore 45^\circ = \frac{\pi}{180} \times 45$ rad $= \frac{\pi}{4}$ rad.

ii) 120°

$1^\circ = \frac{\pi}{180}$ rad.

$\therefore 120^\circ = \frac{\pi}{180} \times 120 = \frac{2\pi}{3}$ rad

iii) $150^\circ =$

$1^\circ = \frac{\pi}{180}$ rad.

$\therefore 150^\circ = \frac{\pi}{180} \times 150^\circ = \frac{5\pi}{6}$ rad.

iv) 270°

$1^\circ = \frac{\pi}{180}$ rad.

$\therefore$ ~~18~~ $270^\circ = \frac{\pi}{180} \times 270^\circ = \frac{3\pi}{2}$ rad.

v) 330°

$1^\circ = \frac{\pi}{180}$ rad.

$\therefore 330^\circ = \frac{\pi}{180} \times 330^\circ = \frac{11\pi}{6}$ rad.

Q.2. Convert radian to degree.

i) $\frac{\pi}{2}$ radian

$1 \text{ rad} = \frac{180^\circ}{\pi}$

$\frac{\pi}{2} \text{ rad} = \frac{180^\circ}{\pi} \times \frac{\pi}{2} = 90^\circ$

ii) $\frac{\pi}{3}$ radian

$= \frac{180^\circ}{3} = 60^\circ$

iii) $\frac{3\pi}{2}$ radian

$\frac{3 \times 180}{2} = 270^\circ$

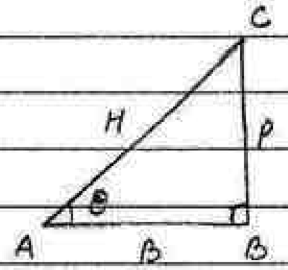
iv) $\frac{5\pi}{4}$ radian

$\frac{5 \times 180}{4} = 225^\circ$

v) $\frac{5\pi}{3}$ radian

$\frac{5 \times 180}{3} = 300^\circ$

Trignometrical ratio : (T- ratio) :



i) $\sin\theta = \frac{P}{H}$ i) $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

ii) $\cos\theta = \frac{B}{H}$ ii) $\sin\theta \times \operatorname{cosec}\theta = 1$

iii) $\tan\theta = \frac{P}{B}$ iii) $\cos\theta = \frac{1}{\sec\theta}$

iv) $\cot\theta = \frac{B}{P}$ iv) $\cos\theta \times \sec\theta = 1$

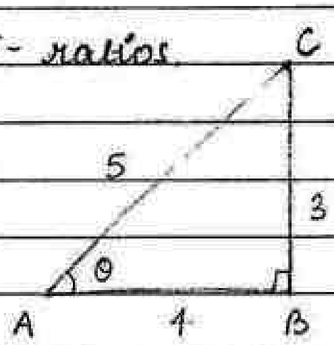
v) $\sec\theta = \frac{H}{B}$ v) $\tan\theta = \frac{1}{\cot\theta}$

vi) $\operatorname{cosec}\theta = \frac{H}{P}$ vi) $\tan\theta \times \cot\theta = 1$

vii) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

viii) $\cot\theta = \frac{\cos\theta}{\sin\theta}$.

Q-3. If $\cos\theta = \frac{4}{5}$, find other T- ratios.



$\cos\theta = \frac{4}{5} = \frac{B}{H}$

$P = \sqrt{5^2 - 4^2} = \sqrt{25-16}$

$= \sqrt{9} = 3.$

$\sin\theta = \frac{P}{H} = \frac{3}{5}$

$\cos\theta = \frac{B}{H} = \frac{4}{5}$

$\tan\theta = \frac{P}{B} = \frac{3}{4}$

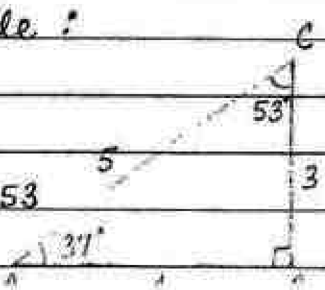
$\cot\theta = \frac{B}{P} = \frac{4}{3}$

$\sec\theta = \frac{H}{B} = \frac{5}{4}$

$\operatorname{cosec}\theta = \frac{H}{P} = \frac{5}{3}$.

	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Standard right angled triangle :



* In 3, 4 and 5 triplets,
the angle between 5 and 3 is always 53
and the remaining angle is 37

i) $\sin 37^\circ = \frac{P}{H} = \frac{3}{5}$

ii) $\cos 37^\circ = \frac{B}{H} = \frac{4}{5}$

iii) $\tan 37^\circ = \frac{P}{B} = \frac{3}{4}$

iv) $\sin 53^\circ = \frac{P}{H} = \frac{4}{5}$

v) $\cos 53^\circ = \frac{B}{H} = \frac{3}{5}$

vi) $\tan 53^\circ = \frac{P}{B} = \frac{4}{3}$

Q.4. If $\tan \theta = \sqrt{3}$, find θ

$\tan \theta = \sqrt{3} = \tan 60^\circ$

$\therefore \theta = 60^\circ$

Q.5 If $\sin \theta = \frac{4}{5}$, find θ

$\sin \theta = \frac{4}{5} = \sin 53^\circ$

$\therefore \theta = 53^\circ$

Q.6. If $\tan \theta = \frac{3}{4}$, find θ

$\tan \theta = \frac{3}{4} = \tan 37^\circ$

$\therefore \theta = 37^\circ$

Some other right angle triplets:

i) 3, 4, 5

ii) 6, 8, 10

iii) 7, 24, 25

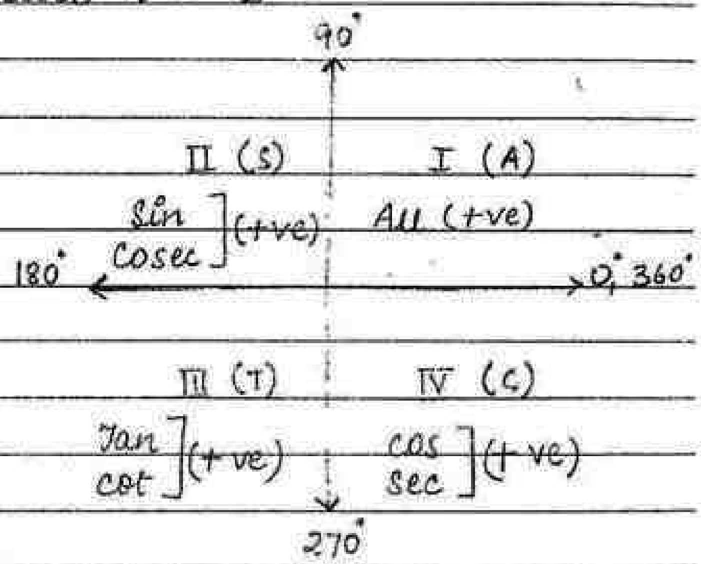
iv) 5, 12, 13.

Q.7. Find the value of $\sin 11^\circ \times \operatorname{cosec} 11^\circ$

$\because \sin \theta \times \operatorname{cosec} \theta = 1.$

$\therefore \sin 11^\circ \times \operatorname{cosec} 11^\circ = 1$

Quadrant Theory:



Q-8 Give the sign of given trignometrical function

i) $\sin 120^\circ = +$ ve

ii) $\cos 120^\circ = -$ ve

iii) $\tan 240^\circ = +$ ve

iv) $\sin 330^\circ = -$ ve

RULE 1 :

When $(90^\circ \pm \theta)$ or $(270^\circ \pm \theta)$,
function will be changed and remove 90° or 270°, where θ is an acute angle (including 0° and 90°)

Changes in function:

sin $\rightleftharpoons$ cos
tan $\rightleftharpoons$ cot
cosec $\rightleftharpoons$ sec

i) for $(90^\circ - \theta)$

$\sin(90^\circ - \theta) = \cos\theta$
$\cos(90^\circ - \theta) = \sin\theta$
$\tan(90^\circ - \theta) = \cot\theta$
$\cot(90^\circ - \theta) = \tan\theta$
$\operatorname{cosec}(90^\circ - \theta) = \sec\theta$
$\sec(90^\circ - \theta) = \operatorname{cosec}\theta$

ii) for $(90^\circ + \theta)$

$\sin(90^\circ + \theta) = \cos\theta$
$\cos(90^\circ + \theta) = -\sin\theta$
$\tan(90^\circ + \theta) = -\cot\theta$
$\cot(90^\circ + \theta) = -\tan\theta$
$\operatorname{cosec}(90^\circ + \theta) = \sec\theta$
$\sec(90^\circ + \theta) = -\operatorname{cosec}\theta$

iii) for $(270^\circ - \theta)$

$\sin(270^\circ - \theta) = -\cos\theta$
$\cos(270^\circ - \theta) = -\sin\theta$
$\tan(270^\circ - \theta) = \cot\theta$
$\cot(270^\circ - \theta) = \tan\theta$
$\operatorname{cosec}(270^\circ - \theta) = -\sec\theta$
$\sec(270^\circ - \theta) = -\operatorname{cosec}\theta$

iv) for $(270^\circ + \theta)$

$\sin(270° + \theta) = -\cos\theta$
$\cos(270° + \theta) = \sin\theta$
$\tan(270° + \theta) = -\cot\theta$
$\cot(270° + \theta) = -\tan\theta$
$\operatorname{cosec}(270° + \theta) = -\sec\theta$
$\sec(270° + \theta) = \operatorname{cosec}\theta$

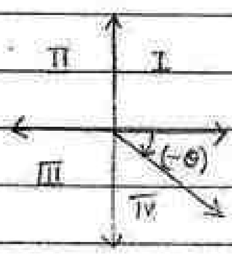
RULE 2 :
When $(180° \pm \theta)$ or $(360° \pm \theta)$ function does not change and remove $180°$ and $360°$ where θ is an acute angle (including $0°$ and $90°$)

i) for $(180° - \theta)$

$\sin(180° - \theta) = \sin\theta$
$\cos(180° - \theta) = -\cos\theta$
$\tan(180° - \theta) = -\tan\theta$
$\cot(180° - \theta) = -\cot\theta$
$\operatorname{cosec}(180° - \theta) = \operatorname{cosec}\theta$
$\sec(180° - \theta) = -\sec\theta$

ii) for $(180° + \theta)$

$\sin(180° + \theta) = -\sin\theta$
$\cos(180° + \theta) = -\cos\theta$
$\tan(180° + \theta) = \tan\theta$
$\cot(180° + \theta) = \cot\theta$
$\operatorname{cosec}(180° + \theta) = -\operatorname{cosec}\theta$
$\sec(180° + \theta) = -\sec\theta$



iii) for $(360° - \theta)$ or $(-\theta)$

$\sin(360° - \theta) = -\sin\theta$
$\cos(360° - \theta) = \cos\theta$

$\tan(360° - \theta) = -\tan\theta$
$\cot(360° - \theta) = -\cot\theta$
$\operatorname{cosec}(360° - \theta) = -\operatorname{cosec}\theta$
$\sec(360° - \theta) = \sec\theta$

iv) for $(360° + \theta)$ or $(+\theta)$
$\sin(360° + \theta) = \sin\theta$
$\cos(360° + \theta) = \cos\theta$
$\tan(360° + \theta) = \tan\theta$
$\cot(360° + \theta) = \cot\theta$
$\operatorname{cosec}(360° + \theta) = \operatorname{cosec}\theta$
$\sec(360° + \theta) = \sec\theta$

To find the value of T-ratios for angle greater then 90°.

- we should split the given angle in two part

→ 1st part should be multiple of 90°.
$(90° \times 1,\ 90° \times 2,\ 90° \times 3 \ldots)$

→ 2nd part should be acute angle.

Q-9. Find value of T-ratios greater then 90°

i) $\cos 120°$

1st method:

$$\cos(90° + 30°) = -\sin 30° = -\frac{1}{2}$$

2nd method:

$$\cos(180° - 60°) = -\cos 60° = -\frac{1}{2}$$

ii) $\sin 150^\circ$

$\sin(90^\circ + 60^\circ) = \cos 60$

$= \frac{1}{2}$

iii) $\cos 225^\circ$

$\cos(180^\circ + 45^\circ) = -\cos 45^\circ$

$= -\frac{1}{\sqrt{2}}$

iv) $\tan 240^\circ$

$\tan(180^\circ + 60^\circ) = \tan 60^\circ$

$= \sqrt{3}$

v) $\sin 330^\circ$

$\sin(270^\circ + 60^\circ) = -\cos 60^\circ$

$-\frac{1}{2}$

vi) $\sin(-60^\circ)$

$-\sin 60^\circ = -\frac{\sqrt{3}}{2}$

vii) $\cos(-45^\circ)$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$

viii) $\tan(-60^\circ)$

$-\tan 60^\circ = -\sqrt{3}$

ix) $\cos 180^\circ$

$\cos(180^\circ + 0^\circ) = -\cos 0^\circ = -1$

x) $\sin 270^\circ$

$\sin(180^\circ + 90^\circ) = -\sin 90^\circ$

$= -1$

Trignometrical Identities :

i) $\sin^2\theta + \cos^2\theta = 1$

ii) $1 + \tan^2\theta = \sec^2\theta$

iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

iv) $\sin(A+B) = \sin A\cos B + \sin B\cos A$

v) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

vi) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

vii) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

viii) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

ix) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

x) $\sin(2\theta) = 2\sin\theta\cos\theta$

* $\sin\theta = 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$ (half of the angle)

xi) $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

* $\cos\theta = \cos^2\dfrac{\theta}{2} - \sin^2\dfrac{\theta}{2}$ (half of the angle

$= 2\cos^2\dfrac{\theta}{2} - 1$

$= 1 - 2\sin^2\dfrac{\theta}{2}$

xii) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Q.10 Choose the correct answer :

A) $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$

B) $\sin(20A) = 2\sin(10A)\cos(10A)$

C) $\sin(120^\circ) = 2\sin 60^\circ \cos 60^\circ$

D) All of the above

Ans: D

Q-11 Find the value of $\sin 15^\circ$

$\sin 15^\circ = \sin(60^\circ - 45^\circ)$

$\sin(A-B) = \sin A\cos B - \cos A\sin B.$

$= \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

Q-12. Find the value of $\cos 105^\circ$

$\cos(105^\circ) = \cos(60^\circ + 45^\circ)$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

Q.13. Find the value of $\left(\frac{3\pi}{2}\right)$ cos :

$$\cos\left(\frac{3\pi}{2}\right) = \cos\left(\frac{3\times180}{2}\right)$$

$$= \cos 270^\circ$$

$$= \cos(270^\circ+0) = \sin 0^\circ = 0.$$

Q.14. Find the value of $\cos 15^\circ$

$$\cos 15^\circ = \cos(60^\circ - 45^\circ)$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{1}}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Q.15. Find the value of $\sin 105^\circ$

$$\sin 105^\circ = \sin(60^\circ + 45^\circ)$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$