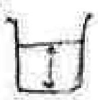
Capacitor

Article : 1 Capacitance

> It is a device which ~~gets~~ ~~stor~~ can store electric charge and electrostatic potential energy (EPE) (ie store energy in the form of charge)

> capacitor can be used to generate a puls of high power for a short duration such as in flash of camera and in defibrillator (device which measures heartbeat; used to give electric shock) while cells are used to generate low power for a long time.

> If charge of an object increases then its potential also increases with relation



$\boxed{Q \propto V} \Rightarrow \boxed{Q = CV}$

i.e. $\boxed{C = \dfrac{Q}{V}}$ capacity or capacitance

> Its SI unit is Farad (F) huge unit or col/volt

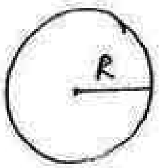
$1\mu F = 10^{-6}\ F \qquad 1pF = 10^{-12}\ F$

> capacity depends upon shape and size and surrounding of a capacitor but it is independant of charge given to capacitor.

~~capa~~

> we can measure but does not depends on Q

Capacity of a sphere:



$$V = \frac{KQ}{R} \qquad \therefore \quad \frac{Q}{V} = \frac{R}{K}$$

$$\therefore \; C = \frac{R}{K} = \boxed{4\pi\epsilon_0 R}$$

In a medium of ϵ_r

$$\boxed{C' = 4\pi\epsilon_0\epsilon_r R}$$

$$\boxed{C' = \epsilon_r C_{air}}$$

Q) Capacity of a capacitor is C. calculate work done in increasing its potential from V_1 to V_2

$Q \longrightarrow V$

$$\because dw = \Delta u = dq(\Delta V) = dq(V_f - V_i)$$

$$dw = dq(V - 0)$$

$$dW = dq\,V \quad \text{—— (1)}$$

$\because$ here $\quad q = CV$

$$\therefore \; dq = C\,dV$$

putting dq in eq (1)

$$dW = CV\,dV$$

$$W_{net} = \int_{V_1}^{V_2} CV\,dv = C\left(\frac{V^2}{2}\right)_{V_1}^{V_2}$$

* $\boxed{W_{net} = C\left[V_2^2 - V_1^2\right]}$ applicable for all type of

> If potential is raised from 0 to V, then work done is $\boxed{\frac{1}{2}CV^2}$ and it will be stored in the form of EPE b/w surface of sphere and infinity

$$\therefore \boxed{EPE = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV} \quad (\because Q = C\underset{\text{final}}{\underbrace{V}})$$

* To calculate work done, initial and final potentials are required but to calculate EPE, only final potential is required.

Article : 2 LOSS IN EPE OF SYSTEM

common potential

$$\therefore \quad Q_1 + Q_2 = Q_1' + Q_2' \quad \text{(charge conservation)}$$

$$C_1 V_1 + C_2 V_2 = C_1 V_c + C_2 V_c$$

$$\therefore \quad V_c = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{Q \text{ of system}}{C \text{ of system}}$$

> If after connection, their are some changes in C_1 and C_2 then new common potential

$$V_c' = \frac{Q \text{ of system}}{C_1' + C_2'} = \frac{C_1 V_1 + C_2 V_2}{C_1' + C_2'}$$

$$Q_1' = V_c C_1$$

$$Q_2' = V_c C_2$$

$$\therefore \quad \boxed{\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2} = \frac{R_1}{R_2} \text{ (if sphere)}}$$

loss in EPE of system

does not depends on [illegible]

loss $= U_i - U_f$ $(\because U_i > U_f \therefore \text{loss})$

$$= \left(\frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 \right) - \frac{1}{2} (C_1 + C_2) V_C^2$$

$$= \left(\frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 \right) - \frac{1}{2} \cancel{(C_1 + C_2)} \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)^{\cancel{2}}}$$

$$= \frac{1}{2} \left[\frac{\cancel{C_1^2 V_1^2} + \cancel{C_2^2 V_2^2} + C_1 C_2 V_1^2 + C_1 C_2 V_2^2 - (\cancel{C_1^2 V_1^2} + \cancel{C_2^2 V_2^2} + 2 C_1 C_2 V_1 V_2)}{C_1 + C_2} \right]$$

$$= \frac{C_1 C_2}{2(C_1 + C_2)} \left[V_1^2 + V_2^2 - 2 V_1 V_2 \right]$$

$$\therefore \boxed{\text{Loss} = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2}$$

Important note:

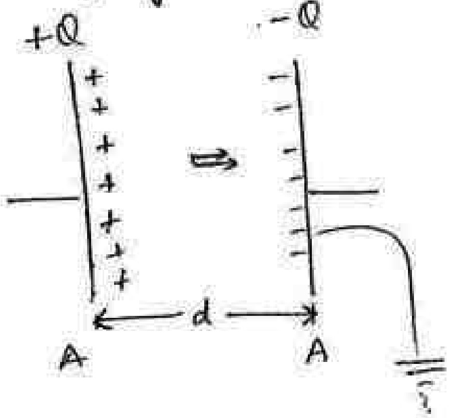
- This loss in EPE of system will appear in the form of heating of connecting wire and sphere.
- This loss is independent on value of Resistance of connecting wire.
- condition for no loss in EPE

$$V_1 = V_2$$

$$\frac{KQ_1}{R_1} = \frac{KQ_2}{R_2}$$

Article : 3 Parallel plate capacitor (PPC)

capacity of PPC :



$$E_{in} = \frac{\sigma}{\epsilon_0} \left(\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} \right)$$

$$\frac{\Delta V}{\Delta x} = \frac{Q}{A\,\epsilon_0} \quad \left(\because \sigma = \frac{Q}{A} \right)$$

$$\therefore \frac{V}{d} = \frac{Q}{A\,\epsilon_0}$$

Method to calculate C

$\therefore E \longrightarrow V \longrightarrow C$

$$\therefore C = \boxed{\frac{Q}{V} = \frac{A\,\epsilon_0}{d}} \;*$$

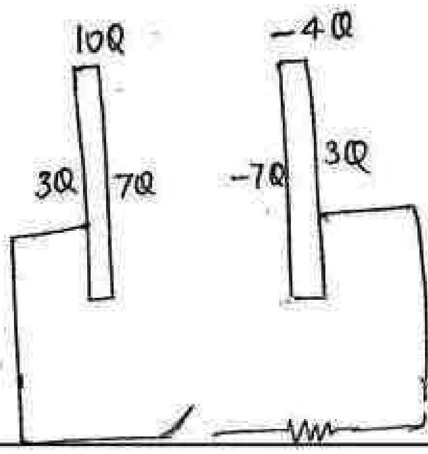
d = seperation b/w plates

A = overlapped area of plates

If space b/w plates filled by a medium of dielectric constant ϵ_r, then $C' = \dfrac{\epsilon_0 \epsilon_r A}{d}$

i.e. $\boxed{C' = \epsilon_r C_{air}}$

If both the plates are given unequal charge, then, use induction method

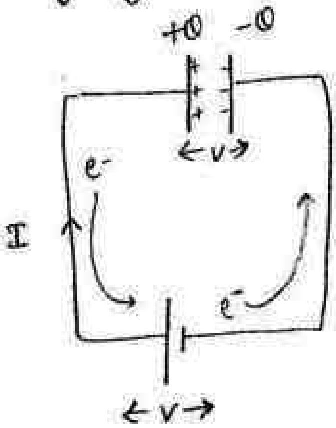


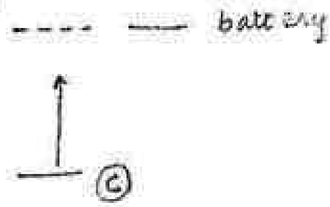
After closeing the switch, thermal energy general in R

within the two plates

$$= \frac{1}{2}\frac{((7Q))^2}{C}$$ ie it is independent of value of R

charging of a PPC by a battery.





» At $t=0$, capacitor is uncharged so a heavy current will flow in the circuit. it means capacitor behaves as a short circuit (R = 0) but after charging of capacitor, current becomes zero, it means capacitor behaves as an open circuit (infinite resistance)

» charging state is called <u>Transient state</u> and final state is called <u>steady state</u>

» If state is not mentioned then consider stedy ~~stage~~ state ie final state

» capacitor blocks <u>DC current</u> (constant voltage) ie in stedy state and capacitor allows to flow DC current during charging and both statements are correct.

$$W_{battery} = W_b = Q(\Delta V)$$

$$W_b = Q(V) = CV(V)$$

$$\therefore \boxed{W_b = CV^2}$$

and $EPE = \frac{1}{2} CV^2$

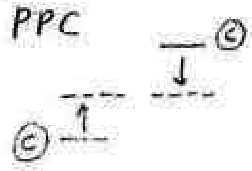
so loss in wire $= CV^2 - \frac{1}{2}CV^2 = \boxed{\frac{1}{2}CV^2}$

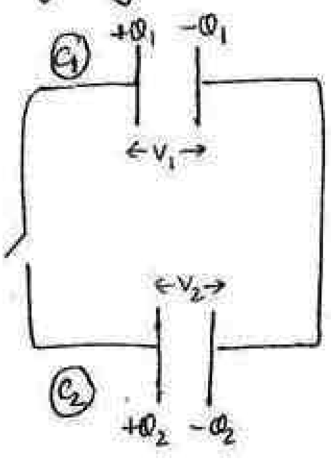
<u>Note</u> :

If battery is not given, then work done in charging a capacitor is $\frac{1}{2}CV^2$

work done in charging of a capacitor <u>by battery</u> is CV^2

charging of a PPC by an another PPC





after closing ~~on~~ the switch

$$V_c = \frac{Q \text{ of } \cancel{\text{sttm}} \text{ system}}{C \text{ of system}}$$

$$= \frac{Q_1 + Q_2}{C_1 + C_2} \Rightarrow \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \Rightarrow \frac{C_1V_1 + C_2V_2}{C_1' + C_2'}$$

if c is changed

$Q_1' = V_c C_1$

$Q_2' = V_c C_2$

$$\therefore \boxed{\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}}$$

loss of EPE of system $= \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$

if opposite polarity is connected. (only when i is mentione

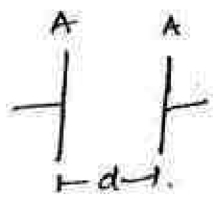
$$V_c = \frac{Q \text{ of system}}{C \text{ of system}} = \frac{Q_1 - Q_2}{C_1 + C_2} = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

$$= \frac{C_1V_1 - C_2V_2}{C_1' + C_2'} \text{ (if } C \text{ is change}$$

$$\boxed{\text{loss of EPE of system} = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 + V_2)^2}$$

after connection, the polarity will be same

Energy density in capacitor



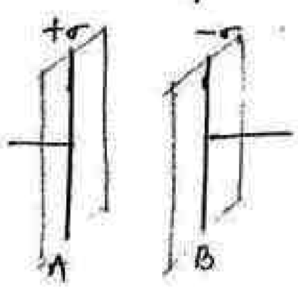
$$\text{Energy density} = \frac{EPE}{vol^m}$$

$$\text{energy density} = \frac{\frac{1}{2}CV^2}{vol^m}$$

$$= \frac{1}{2}\frac{\epsilon_0 A}{d}\frac{V^2}{A \times d} = \frac{1}{2}\epsilon_0\left(\frac{V}{d}\right)^2$$

$$** = \boxed{\frac{1}{2}\epsilon_0 E^2}$$ It is applicable for all type of capacitor.

Electric pressure:



$$F = qE$$

$$F = q_B \; E \text{ of } A$$

$$F = (\sigma A)\left(\frac{\sigma}{2\epsilon_0}\right)$$

$$\therefore \quad \frac{F}{A} = \frac{1}{2}\frac{\sigma^2}{\epsilon_0}$$

$$\therefore \quad \boxed{\text{pressure} = \frac{\sigma^2}{2\epsilon_0}}$$

* Energy density: $\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\frac{\sigma}{\epsilon_0}\right)^2 = \frac{\sigma^2}{2\epsilon_0}$

Article : 4 Study of Q; V; C; E; u (E.PE) in a capacitor.

Case : I During changes, if battery remains connected then potential of capacitor remains constant (V = constant)

Case II : During changes, if capacitor is isolated then its charge remains constant (Q = constant)

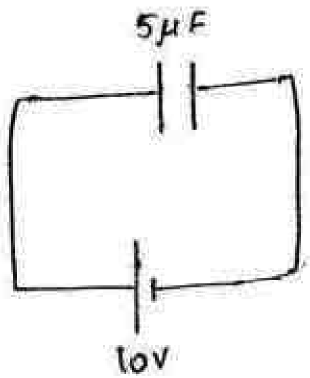
Case : I battery connected

Changes	$Q = CV$ i.e. $Q \propto C$ (III)	$C' = \left(\frac{A\epsilon_0}{d}\right)\epsilon_r$ (II)	V (I)	E ($\frac{V}{d}$; $\frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0\epsilon_r}$) (IV)	EPE ($\frac{1}{2}CV^2$; $\frac{1}{2}\frac{Q^2}{C}$) (V)
i) $A\uparrow$	$\uparrow$	$\uparrow$	—	— ($\frac{V}{d}$)	$\uparrow$ [EPE $\propto$ C when V is constant]
ii) $d\uparrow$	$\downarrow$	$\downarrow$	—	$\downarrow$	$\downarrow$
iii) $\epsilon_r\uparrow$	$\uparrow$	$\uparrow$	—	—	$\uparrow$

Case II: battery disconnected

Change	Q	$C' = \frac{A\epsilon_0\epsilon_r}{d}$	$V = Q/C$ $\therefore V \propto 1/C$	E	$EPE = \frac{1}{2}\frac{Q^2}{C}$ $\therefore EPE \propto 1/C$
$A \uparrow$	—	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
$d \uparrow$	—	$\downarrow$	$\uparrow$	—	$\uparrow$
$\epsilon_r \uparrow$	—	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$

Q)



If space b/w plates filled with dielectric constant 3, then calculate

i) additional charge passing through the battery

ii) Effect on E and EPE

i) $Q_1 = C_1 V_1 \Rightarrow 50\mu C$

$Q_2 = C_2 V_2 \Rightarrow 3(50\mu C) = 150\mu C$

$\therefore \Delta Q = Q_2 - Q_1 = 100\mu C$ (additional charge)

ii) $E = \frac{V}{d}$ ie remains unchanged ($\because V =$ constant)

$EPE = \frac{1}{2}CV^2 \quad EPE' = \frac{1}{2}(3C)V^2 = 3EPE$

final value of EPE is 300% of initial value and % change in EPE is 200%.

Q) A capacitor of capacity c is charged to a potential V, now it is connected across a capacitor of same capacity which is initially uncharged. calculate after connection

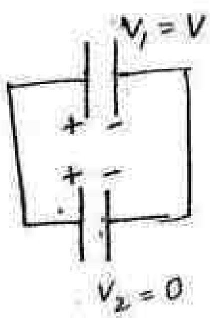
i) Potential on each capacitor

ii) Charge on each capacitor

iii) Loss in EPE of system

iv) Loss $(u_i - u_f)$ in EPE of 1st capacitor

v) final EPE of system



i) $V_c = \dfrac{CV + C\times 0}{2C} = \dfrac{V}{2} = V_1' = V_2'$

ii) $Q_1' = C\left(\dfrac{V}{2}\right) = \dfrac{CV}{2} = Q_2'$

iii) Loss (system) $= \dfrac{1}{2}\dfrac{C\times C}{2C}(V-0)^2 = \dfrac{CV^2}{4}$

iv) $(u_i)_I = \dfrac{1}{2}CV^2 \qquad (u_f)_I = \dfrac{1}{2}C\left(\dfrac{V}{2}\right)^2$

$\therefore$ loss $= \dfrac{1}{2}CV^2\left(1 - \dfrac{1}{4}\right) \Rightarrow \dfrac{3}{8}CV^2$

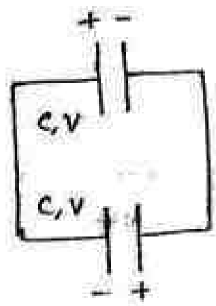
v) EPE of system,

$$= \frac{1}{2}(C_1 + C_2) V_c^2$$

$$= \frac{1}{2}(2C)\left(\frac{V}{2}\right)^2 = \frac{CV^2}{4}$$

Q) There are 2 identical capacitor, each of capacity C and potential V, now, there opposite polarity plates are connected. Calculate all above 5 Que again

i)



i) $V_c = \frac{CV - CV}{2C} = \frac{0}{2C} = 0$

ii) $Q_1' = CV_c = C \times 0 = 0$

iii) Loss (system) : $\frac{1}{2} \frac{C \times C}{2C} [V - (-V)]^2 = \frac{1}{2} \frac{C}{2} 4V^2 = CV^2$

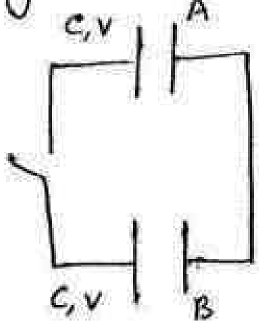
iv) $(U_i)_I = \frac{1}{2} CV^2 \quad (U_f)_I = \frac{1}{2} C (0)^2 = 0$

$\therefore$ loss $= \frac{1}{2} CV^2$

v) EPE of system,

$\frac{1}{2}(C + C) 0^2 = 0$

Q) There are 2 identical capacitors each of capacity C and potential V



after closing the switch, capacitor A is filled with dielectric constant K and B is filled with dielectric constant 5. Calculate

i) their common potential

ii) new value of charge on each capacitor

iii) Loss in EPE of system

i) $V_c = \dfrac{CV + CV}{5C + KC} = \dfrac{2V}{5+K}$

ii) $\cancel{Q_f} = \cancel{\dfrac{2CV}{5+K}} = Q_B$. $\quad Q_1' = C_1' V_c = KC\left(\dfrac{2V}{5+K}\right)$

$Q_2' = C_2' V_c = 5C\left(\dfrac{2V}{5+K}\right)$

iii) Loss $= (U_i)_{system} - (U_f)_{system}$

$$\frac{1}{2}CV^2 \times 2 - \frac{1}{2}(C_1' + C_2')V_c^2$$

we could use $\frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$

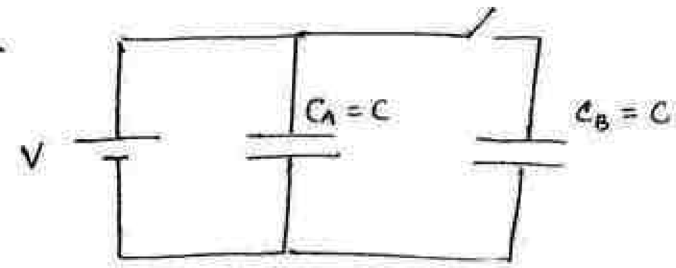
$$\frac{CV^2}{2} \times 2 - \frac{1}{2}(5C + KC)\left(\frac{2V}{5+K}\right)^2$$

∵ it is filled with dielectric constant ...

$$\therefore CV^2 - \frac{C}{2}\left(\frac{4V^2}{5+K}\right)$$

$$\therefore \text{Loss} = CV^2\left[1 - \frac{2}{(5+K)}\right] \Rightarrow CV^2\left(\frac{3+K}{5+K}\right)$$

Q)



Initially switch is close and after a long time switch is open. now space b/w both capacitor is filled with dielectric constant 3, calculate ratio of EPE of system before and after filling of capacitors.

$$(U_i)_{system} = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2 \qquad V = const.$$

$$(U_A)' = \frac{1}{2}(3C)V^2$$

For B $\longrightarrow$ $\underline{\underline{Q = constant}}$ $\quad \therefore V' = \frac{V}{3}$ $\left[\because Q = CV \; \therefore V \propto \frac{1}{C}\right]$

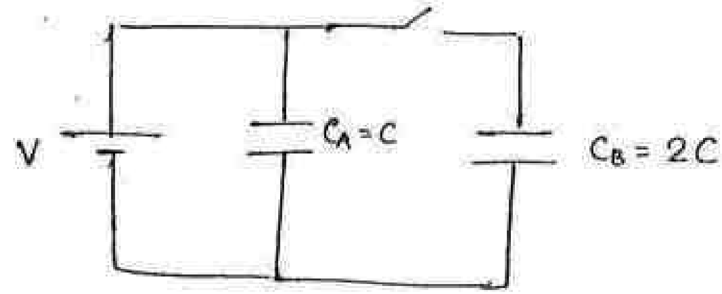
$$(U_B)' = \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2 = \frac{1}{2}\frac{CV^2}{3}$$

$$(U_f)_{system} = \frac{1}{2}(3C)V^2 + \frac{1}{2}\left(\frac{CV^2}{3}\right)$$

$$= \frac{1}{2}\left(\frac{10}{3}CV^2\right) = \frac{5}{3}CV^2$$

$$\therefore \frac{(U_i)_{system}}{(U_f)_{system}} = \frac{3}{5}$$

Q)



Initially switch is closed and after a long time it is opened now capacitor B is filled with ~~d~~ $\epsilon_r = 5$.

calculate ratio of EPE of system before and after filling

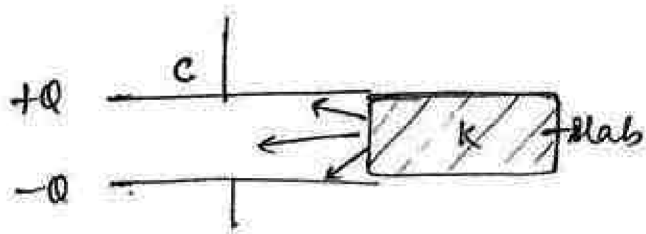
(U_i) system $\Rightarrow \frac{1}{2}CV^2 + \frac{1}{2}(2C)V^2 = \frac{3}{2}CV^2$

(U_f) system $\Rightarrow \frac{1}{2}CV^2 + \frac{1}{2}(10C)\left(\frac{V}{5}\right)^2$ [∵ Q remains same when opened]

$= \frac{1}{2}CV^2 + \frac{CV^2}{5} = \frac{7}{10}CV^2$

$\frac{(U_i)\text{ system}}{(U_f)\text{ system}} = \frac{3}{2} \times \frac{10}{7} \Rightarrow \frac{15}{7}$

Q)



calculate work done by capacitor to put the ~~slag~~ slab b/w plates

$$\boxed{W_f = -\Delta U} = -(U_f - U_i)$$

$$= U_i - U_f$$

$$= \frac{1}{2}\frac{Q^2}{C} - \frac{1}{2}\frac{Q^2}{(KC)}$$

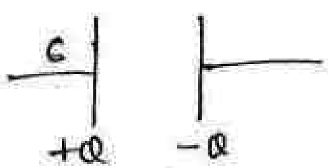
$$= \frac{1}{2}\frac{Q^2}{C}\left[1 - \frac{1}{K}\right] = \boxed{\text{loss in EPE}}$$ (always +ve) $\because K > 1$

* If surfaces are frictionless then this loss will appear in the form of KE of the slab and slab will oscillate b/w plates.

* but if there is friction b/w surfaces, then finally KE will appear in the form of heating of surfaces and slab comes to rest b/w plates.

* If battery is also connected then all the required energy are supplied by battery

Q) calculate work done by external agent to double the seperation b/w plates.



$\because C = \frac{A\epsilon_0}{d}$ $\therefore$ $d\uparrow$ $\therefore C\downarrow$

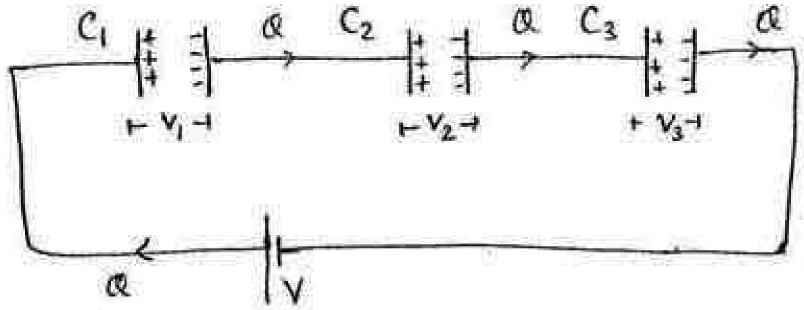
$$\boxed{W_{ext} = +\Delta U} \Rightarrow U_f - U_i$$

$$= \frac{1}{2}\frac{Q^2}{(C/2)} - \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{1}{2}\frac{Q^2}{C}(2-1) = \frac{1}{2}\frac{Q^2}{C} = \boxed{\text{gain in EPE}}$$

Article : 5 combination of capacitor

i) series combination



$\because \quad V = V_1 + V_2 + V_3$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

i) $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$

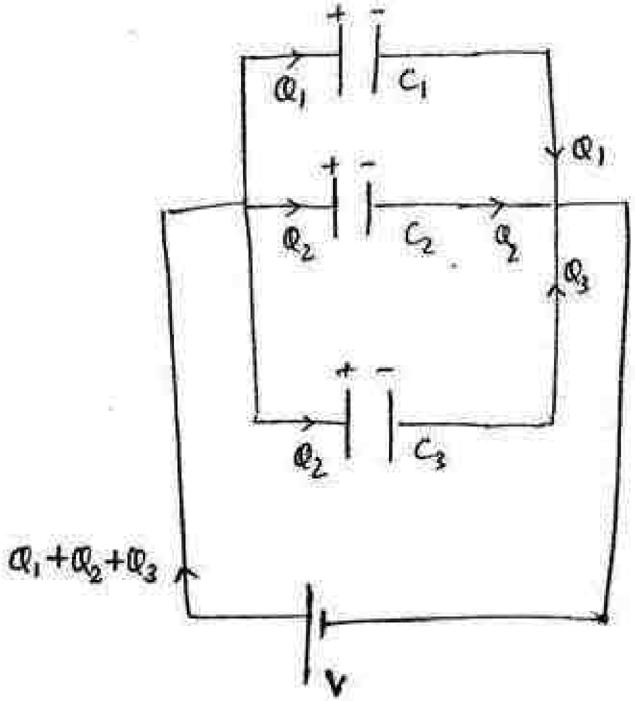
ii) $C_{eq} = \dfrac{C}{N}$ [for N capacitor of identical capacity]

iii) C_{eq}, $[Q_{eq} = C_{eq} V_b]$ → $V_1 = Q/C_1$, $V_2 = Q/C_2$, $V_3 = Q/C_3$

iv) EPE of system ⇒ $\frac{1}{2} C_{eq} V_b^2$

v) $V_{gc} \propto \frac{1}{C}$

ii) Parallel combination



$$Q_{eq} = Q_1 + Q_2 + Q_3$$

$$C_{eq} V_b = C_1 V_b + C_2 V_b + C_3 V_b$$

$$\therefore \; C_{eq} = C_1 + C_2 + C_3$$

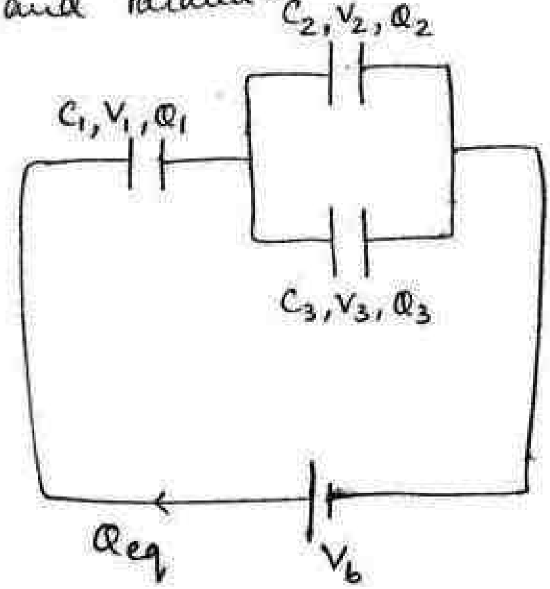
$$\text{EPE of system} = \frac{C_{eq}}{2}\left(V_b^2\right)$$

$Q_1 = C_1 V_b$

$Q_2 = C_2 V_b$ $\quad \therefore \boxed{Q_{q_c} \propto C}$

$Q_3 = C_3 V_b$

Combination of series and parallel:



Important ~~capacitors~~ equations:

i) $C_{eq} = \dfrac{C_1 C'}{C_1 + C'}$ $[C' = C_3 + C_2$

ii) $Q_{eq} = \cancel{}\, C_{eq} V_b$

iii) $Q_{eq} = Q_1 = Q_2 + Q_3$

iv) $V_2 = V_3$ and $V_b = V_1 + V_2 = V_1 + V_3$

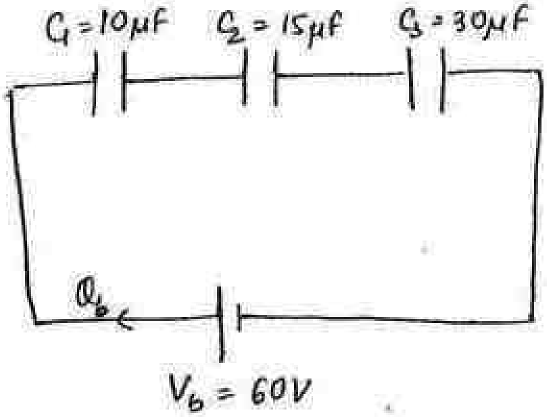
v) $Q_2 = \left(\dfrac{C_2}{C_2 + C_3}\right) Q_{eq}$

$Q_3 = \left(\dfrac{C_3}{C_2 + C_3}\right) Q_{eq}$

vi) $\dfrac{Q_2}{Q_3} = \dfrac{C_2}{C_3}$

vii) $EPE = \dfrac{1}{2} C_{eq} V_b^2$

Q)



Find:

i) charge passing through the battery

ii) charge on each capacitor

iii) Potential of each capacitor

iv) EPE of system

i) $Q_b = C_{eq} V_b$

$$\because \frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{15} + \frac{1}{30} = \frac{3+2+1}{30} = \frac{6}{30}$$

$$\therefore C_{eq} = \frac{30}{6} = 5 \quad ; \; V_b = 60$$

$$\therefore Q_b = 5 \times 60 = 300\,\mu C$$

ii) $\because$ in series $\therefore Q_1 = Q_2 = Q_3 = 300\,\mu C$

iii)
$$\therefore V_1 = \frac{Q_{eq}}{C_1} = \frac{300}{10} = 30V$$

$$V_2 = \frac{Q_{eq}}{C_2} = \frac{300}{15} = 20V$$

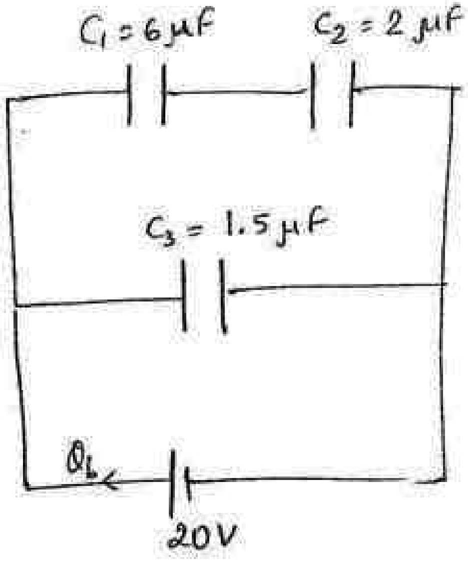
$$V_3 = \frac{Q_{eq}}{C_3} = \frac{300}{30} = 10V$$

verify: $30 + 20 + 10 = 60V$

iv) $EPE = \frac{1}{2} C_{eq} V_b^2$

$$= \frac{1}{2} \times \cancel{300} 5 \times 60^2 = \frac{1}{2} \times 5 \times 60 \times 60 = 9000\,\mu J$$

Q)



calculate all the four parts of the previous Que

* All the Tricks remains same just the $\boxed{C \propto \frac{1}{R}}$

$\therefore$ series in parallel inverse

i) $Q_b = C_{eq} V_b$

$\because \quad C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{6 \times 2}{6+2} = \frac{12}{8} = \frac{3}{2} = 1.5$

$\therefore \quad C_{eq} = 1.5 + 1.5 = 3 \quad ; \quad V_b = 20$

$\therefore \quad Q_b = 20 \times 3 = 60 \mu C$

ii) $(Q)_{on\, C_3} = C_3 V_b \Rightarrow 1.5 \times 20 = 30 \mu C$

$\because C_1$ and C_2 are in series $\therefore C_1 = C_2 = 30 \mu C$

i.e $(60 - 30) = 30 \mu C$

30+30 = 60μ

iii) $\cancel{C_1}$ $V_3 = \frac{Q_3}{C_3} = \frac{30}{1.5} = 20 V$

? $V_1 = \frac{Q_1}{\frac{C_1 C_2}{C_1 + C_2}} = \frac{30}{C'} = \frac{30}{1.5} = 20 V = V_2$

$V_1 + V_2 = 20$

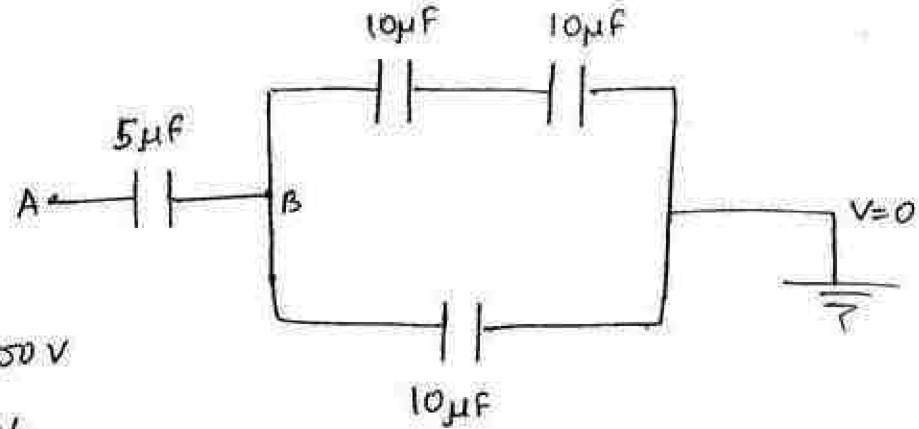
$V_1 = \frac{30}{6} = 5 \quad V_2 = \frac{30}{2} = 15 \quad = 20$

iv) $EPE = \frac{1}{2} C_{eq} V_b^2$

$= \frac{1}{2} \times 3 \times 20^2 = \frac{1}{2} \times 3 \times 20 \times 20$

$= 600 \mu J$

Q1)

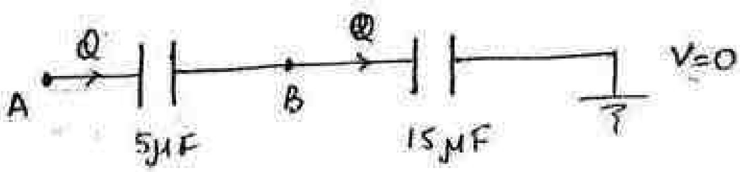


$V_A = 1200\,V$

Find V_B

$$C' = \frac{10 \times 10}{10+10} = \frac{100}{20} = 5\,\mu F$$

$$\therefore C_{eq} = C' + C = 5\mu F + 10\mu F = 15\mu F$$



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow \frac{5 \times 15}{5+15} = \frac{5 \times 15}{20} = \frac{15}{4}$$

$$Q = C_{eq} V_A$$

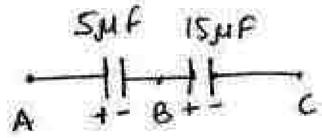
$$= \frac{15}{4} \times (1200) = 4500\,\mu C$$

$$\therefore V_1 = \frac{Q}{C_1} = \frac{4500}{5} = 900 \text{ (it's the drop)}$$

$$\therefore V_A - V_1 = V_B$$

$$1200 - 900 = V_B = 300$$

$$\therefore \boxed{V_B = 300}$$



KVL in A to C

$$1200 - \frac{Q}{5} - \frac{Q}{15} = 0$$

$$1200 = \frac{4Q}{15} \qquad \therefore \quad Q = 4500\,\mu C$$

KVL A to B,

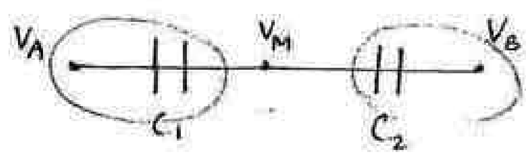
$$1200 - \frac{Q}{5\mu} = V_B. \qquad \left[\because V = \frac{Q}{C}\right]$$

$$1200 - \frac{4500}{5} = V_B$$

$$1200 - 900 = V_B$$

$$\therefore \quad V_B = 300$$

* Short trick :

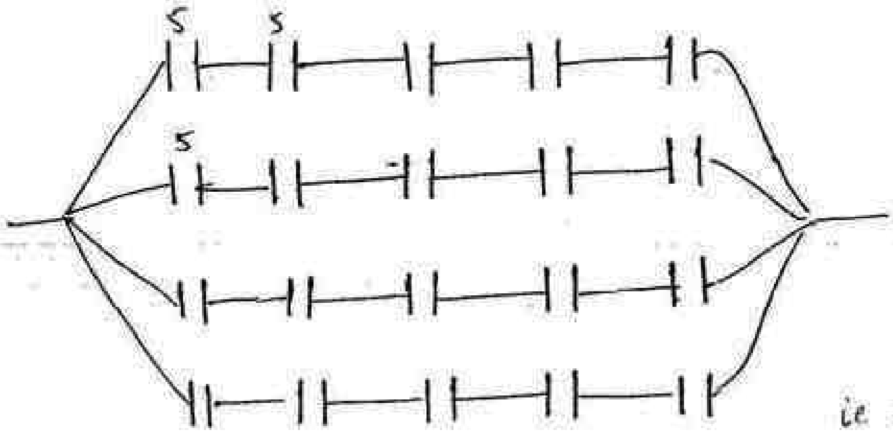


$$\boxed{V_M = \frac{C_1 V_A + C_2 V_B}{C_1 + C_2}} \quad \text{ie } \frac{\text{Q of system}}{\text{C of system}}$$

$$\Rightarrow \frac{5 \times 1200 + 15 \times 0}{5 + 15}$$

$$= \frac{5 \times 1200}{20} = 300 = V_M$$

Q) There are several identical capacitor each of capacity 5μF and breakdown voltage is 200 V and we want to form a system of capacity 4μF and breakdown voltage 1000 V. calculate no of min. capacitor required.



ie = 20

OR

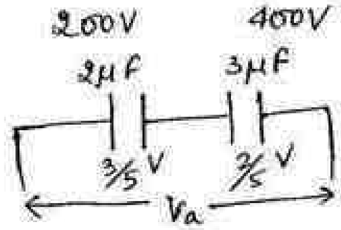
$$N\left(\frac{1}{2} C_1 V_1^2\right) = \frac{1}{2} C_{eq} V_{eq}^2$$

$$N \ \frac{1}{2} \times 5 \times (200)^2 = \frac{1}{2} \ 4 \times (1000)^2$$

$$N \times \frac{1}{\cancel{2}} \times 5 \times \overset{2}{\cancel{40000}} = \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}} \times 1000000$$

$$N = \frac{200}{10} = 20$$

Q)



$$\frac{3}{5}V = 200 \qquad \therefore \; V = \frac{1000}{3}$$

OR

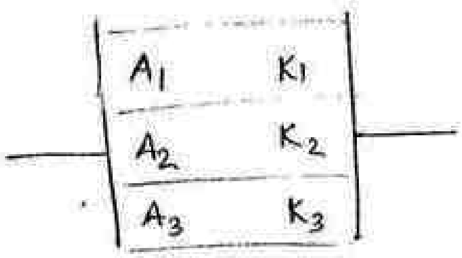
$$Q_{max} = 400\mu C$$

$$\therefore V_a = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} \Rightarrow \frac{400}{2} + \frac{400}{3}$$

$$= \frac{1200 + 800}{6} = \frac{2000}{6} = \frac{1000}{3}$$

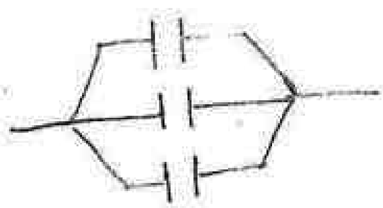
$\boxed{\text{Article : 6}}$ Formation of cap's with diff. medium

Case I: Area division:



$$C_{eq} = C_1 + C_2 + C_3$$

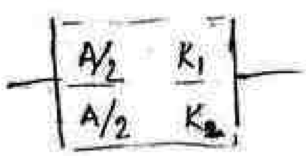
$$\therefore C_{eq} = \frac{\epsilon_0 A_1 K_1}{d} + \frac{\epsilon_0 A_2 K_2}{d} + \frac{\epsilon_0 A_3 K_3}{d}$$



$$\frac{\epsilon_0 A_{eq} K_{eq}}{d} = \frac{\epsilon_0}{d}\left[A_1 K_1 + A_2 K_2 + A_3 K_3\right.$$

$$\therefore \boxed{K_{eq} = \frac{A_1 K_1 + A_2 K_2 + A_3 K_3}{A}}$$

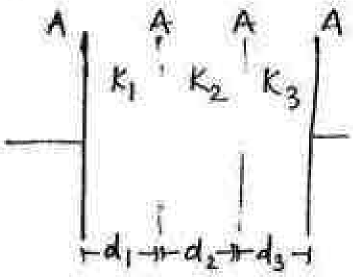
*



$$\therefore K_{eq} = \frac{\frac{A}{2} K_1 + \frac{A}{2} K_2}{A} \Rightarrow \boxed{\frac{K_1 + K_2}{2}}$$

Arithmatic

Case II : distance division



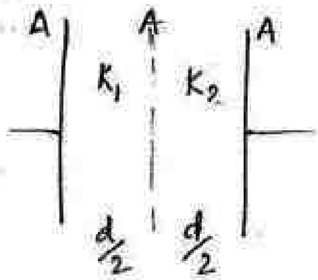
$$\therefore \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{d_1}{\epsilon_0 K_1 A} + \frac{d_2}{\epsilon_0 K_2 A} + \frac{d_3}{\epsilon_0 K_3 A}$$

$$= \frac{1}{\epsilon_0 A}\left[\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}\right]$$

$$\therefore \boxed{C_{eq} = \frac{\epsilon_0 A}{\left[\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3} + \cdots\right]}}$$

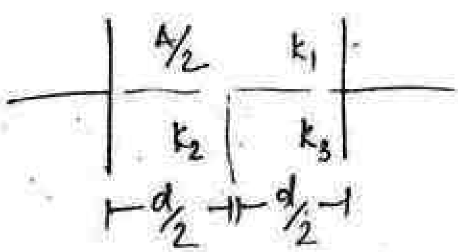
*



$$\therefore C_{eq} = \frac{\epsilon_0 A}{\left(\frac{d}{2K_1} + \frac{d}{2K_2}\right)} = \frac{\epsilon_0 K_{eq} A}{d}$$

$$\therefore K_{eq} = \boxed{\frac{2K_1 K_2}{K_1 + K_2}} = \cancel{K_{eq}}$$ Harmonic mean of K_1 and K_2

Q)



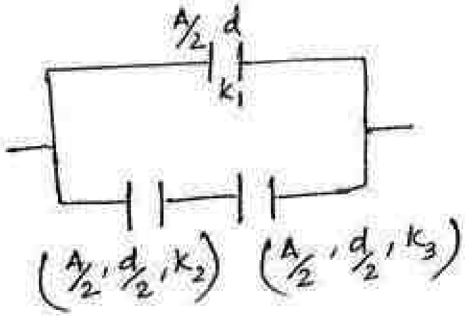
Find K_{eq} and C_{eq}

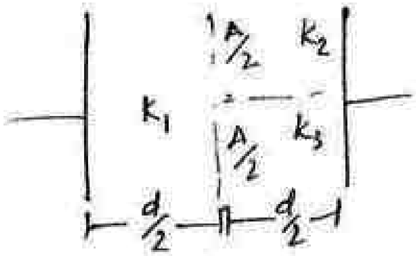
$$K' = \frac{2K_2 K_3}{K_2 + K_3} \qquad \therefore K_{eq} = \frac{K_1 + \frac{2K_2K_3}{K_2+K_3}}{2}$$

$$\Rightarrow \cancel{2} \quad K_1(K_2 + K_3) + 2K_2K_3$$

$\therefore \quad C_{eq} = K_{eq}\, C$

OR



Q) 

Find keq and Ceq

$K' =$ area division,

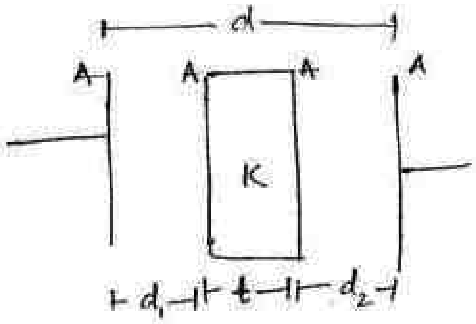
$$\frac{K_2 + K_3}{2}$$

$\therefore \quad K_{eq} =$ distance division,

$$\frac{2K_1\left(\frac{K_2+K_3}{2}\right)}{K_1 + \left(\frac{K_2+K_3}{2}\right)} = \frac{K_1\,(K_2+K_3)}{\frac{2K_1+K_2+K_3}{2}}$$

$$\therefore \quad K_{eq} = \frac{2K_1\,(K_2+K_3)}{2K_1+K_2+K_3}$$

Q)



$C' = ?$

$$C' = \frac{\epsilon_0 A}{\left(\frac{d_1}{1} + \frac{t}{K} + \frac{d_2}{1}\right)} = \frac{\epsilon_0 A}{(d_1+d_2) + \frac{t}{K}} = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}}$$

$$= \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

* In the above Que, if slab is made up of metal ($K = \infty$) $\therefore$ $C' = \frac{\epsilon_0 A}{d-t}$ so c increases

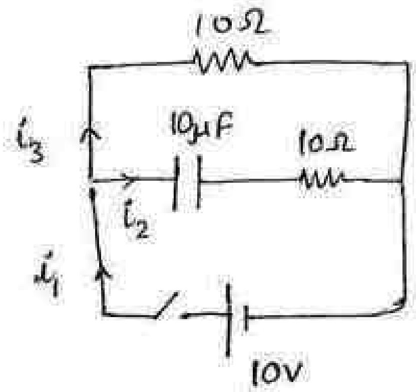
* If space b/w plates is filled with metals then $C' = K_{eq} C = \infty$, it means we can't raise its potential so it is practically useless.

*

$C' = K_{eq} C = \infty$

* If a metallic foil of negligiable thickness is kept b/w plates, then its capacitance remains almost unchanged

Article : 7   R – C (d.c. circuits)



At $t = 0$, switch is close

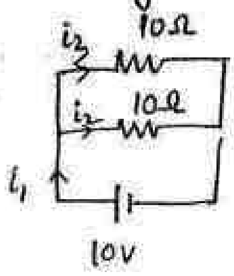
find i) ~~i~~ i_1, i_2 and i_3 at $t = 0$

ii) i_1, i_2, i_3 after a long time ($t \to \infty$)

iii) Final charge on the capacitor.

i) initially, capacitor behaves as a short circuit

∴



∴

$i_3 = \frac{V}{R} = \frac{10}{10} = 1A$

$i_2 = \frac{V}{R} = \frac{10}{10} = 1A$

$i_1 = i_2 + i_3 = 1 + 1 = 2$

ii) In steady state, capacitor behaves as an open circuit so $i_2 = 0$

$\therefore\ i_1 = i_3 = \frac{10}{10} = 1A$

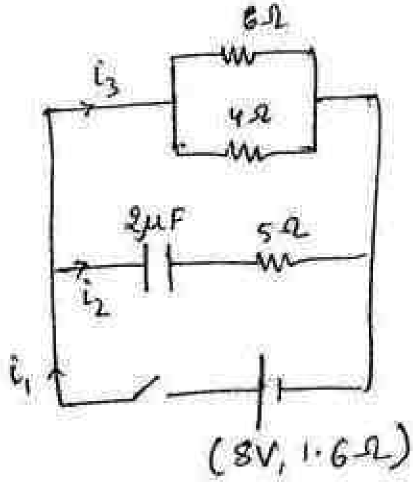
iii) current in 10Ω resistance which is in series of capacitor is 0, so its potential is also 0 and 10 volt will appear on the capacitor $\therefore\ V_c = 10V$

$Q = CV$

$= 10\mu(10)$

$= 100\mu C$

Q)



Find all three parts of previous que.

i) $\frac{1}{R} = \frac{1}{5} + \frac{1}{4} + \frac{1}{6} = \frac{12+15+10}{60} = \frac{37}{60}$

$\therefore R = \frac{60}{37}$ $\quad \therefore R_{eq} = \frac{60}{37} + 1.6 = 3.2$

$i_1 = \frac{8}{3.2} = 2.5A \quad i_2 = \frac{2.5}{3} \quad i_3 = \frac{2}{3}(2.5)$

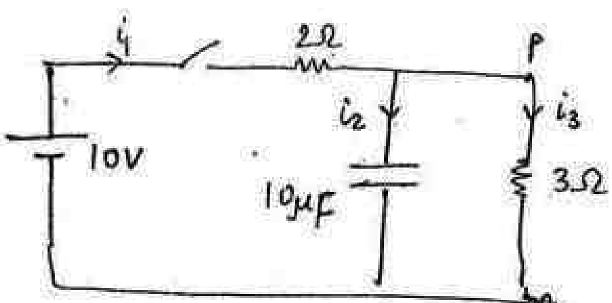
ii) $R_{net} = 2.4 + 1.6 = 4\Omega$

$\therefore i_1 = \frac{8}{4} = 2A \quad i_2 = 0 \quad i_3 = i_1 = 2A$

iii) $TPD = E - I_1 r \Rightarrow 8 - 2 \times 1.6 = 8 - 3.2 \Rightarrow 4.8V$

$Q = CV_c = 4.8 \times 2 = 9.6 \mu C$

Q)

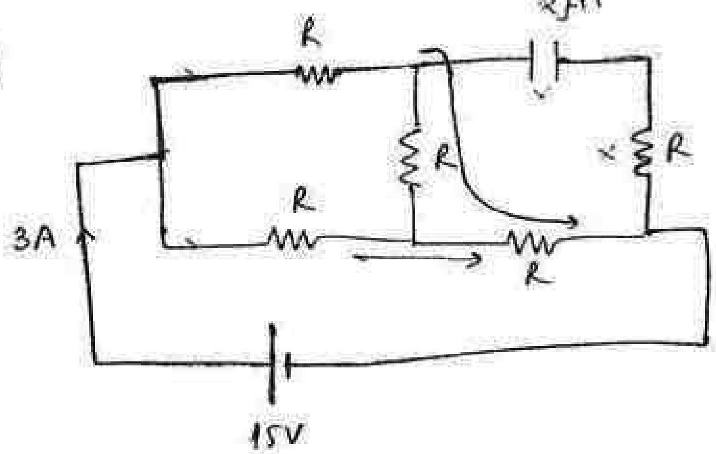


t=0

$i_1 = \frac{V}{R} = \frac{10}{2} = 5A = i_2 \quad i_3 = 0$ [cap behaves as short circuiting]

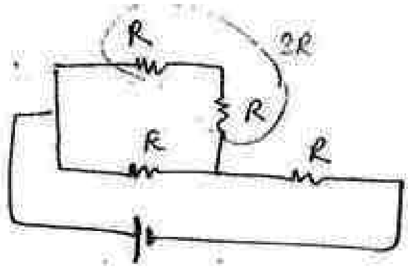
t→∞

$i_1 = \frac{V}{R} = \frac{10}{2+5} = 2A = i_3 \quad i_2 = 0$

Q) 

$R = 3\Omega$
Find Q on the capacitor

In steady state,

$R_{net} \Rightarrow$ 

$\therefore R_{net} = 5\Omega$

$$\therefore \frac{2R \times R}{3R} + R = 5\Omega$$

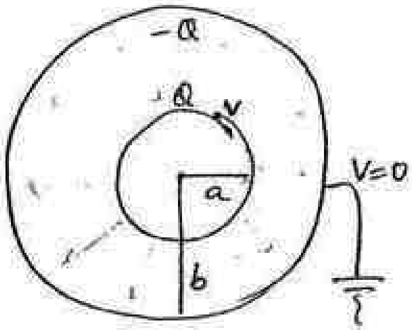
$$\therefore I = \frac{V}{R} = \frac{15}{5} = 3A$$

By KVL,

$$\Delta V_c = 15V - (3 \times 1)V = 15 - 3 = 12V$$

$$Q = CV = 2 \times 12 = 24\mu C$$

Spherical capacitor :



$$V = \frac{KQ}{a} - \frac{KQ}{b}$$

$$= KQ\left(\frac{b-a}{ab}\right)$$

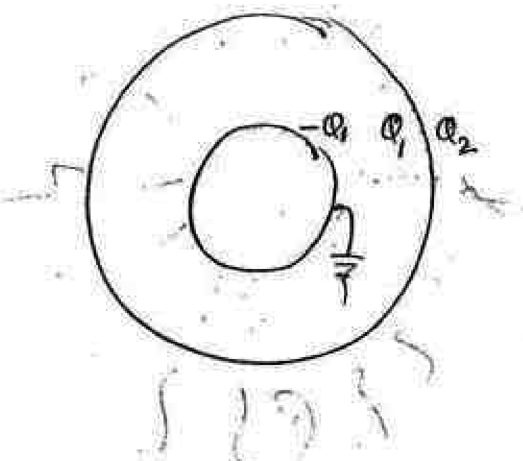
$$\therefore \frac{Q}{V} = \frac{ab}{K(b-a)}$$

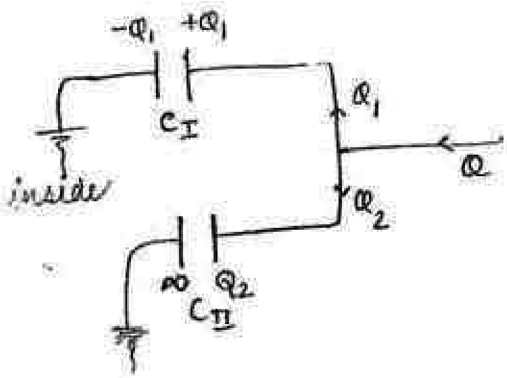
$$\therefore \boxed{C = \frac{4\pi\epsilon_0 ab}{(b-a)}}$$

Important points :

i) If there is a single metallic sphere, it can be treated as a spherical capacitor and radius of its bigger shell is at ∞ i.e $(b = \infty)$

ii) If inner shell is earthed, then its capacity is greater than usual spherical capacitor





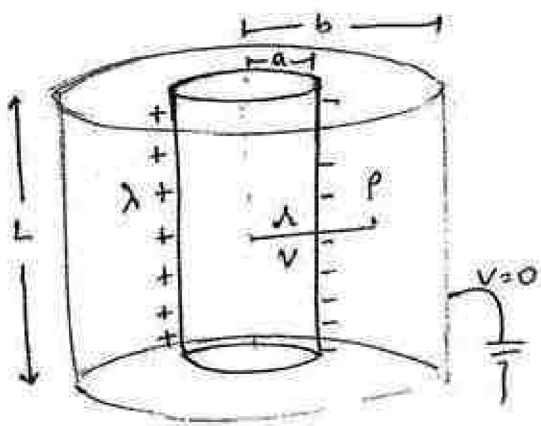
$$C = C_I + C_{II}$$

$$\frac{4\pi\epsilon_0\, ab}{(b-a)} + 4\pi\epsilon_0 b$$

$$= 4\pi\epsilon_0 \left[\frac{ab + b^2 - ab}{b-a}\right]$$

$$= \boxed{\frac{4\pi\epsilon_0\, b^2}{b-a}}$$

Cylindrical capacitor



$E \longrightarrow V \longrightarrow C$ (method

$$\therefore \boxed{E_p = \frac{2K\lambda}{r}} = \frac{-dV}{dr}$$

$$\int_0^V dV = -\int_b^a \frac{2K\lambda}{r}\, dr$$

$$\therefore V = -2K\lambda\,(\ln r)_b^a$$

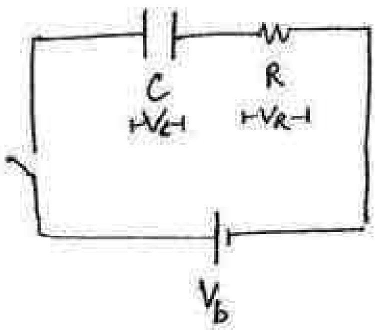
$$\therefore V = 2K\lambda\,[\ln b - \ln a]$$

$$V = 2K\lambda\left(\ln\frac{b}{a}\right)$$

$$V = 2\left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q}{L}\left(\ln\frac{b}{a}\right)$$

$$\boxed{C = \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}}$$

i) charging of a capacitor :



At $t=0$, switch is closed

Time	Q	V_c	i	V_R
$t=0$	0	0	$i_{max} = \frac{V_b}{R}$	$V_R = V_b$ (max)
$t\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
$t \to \infty$	$Q_{max} = (V_b)C$	$(V_c)_{max} = V_b$	0	0

equation of i

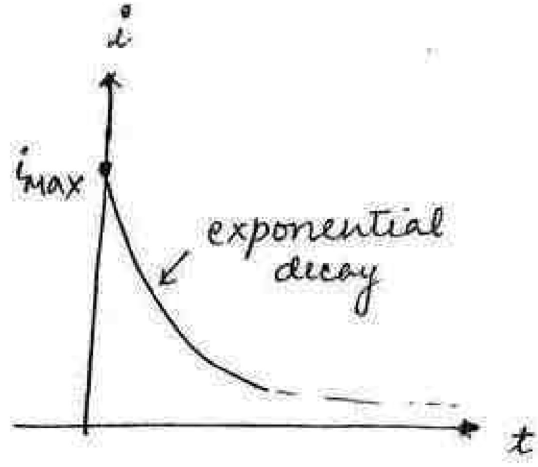
$$\boxed{i = i_{max}\, e^{-t/RC}}$$

RC is called time constant of the circuit

$$\boxed{e = 2.718}$$

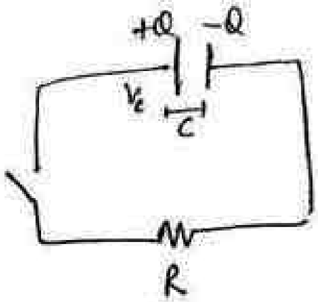
RC $\longrightarrow$ unit (second)
$\Downarrow$
(time constant)

$$\begin{tikzcd} i \arrow[d] \\ V_R = i_R \times R \arrow[d] \\ V_c = V_b - V_R \arrow[d] \\ Q = V_c \times C \arrow[uuu, bend left, "\frac{dQ}{dt}"] \end{tikzcd}$$



ii) discharging of a capacitor

Time	V_c	Q	V_R	i
$t = 0$	max	max	max	max $= \frac{V_c}{R}$
$t \uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$t \to \infty$	0	0	0	0



$$\boxed{x = x_0 e^{-t/RC}}$$

→ V_R
→ V_c
→ i
→ Q

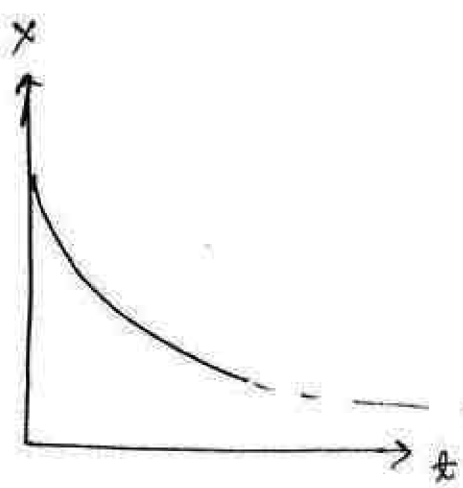
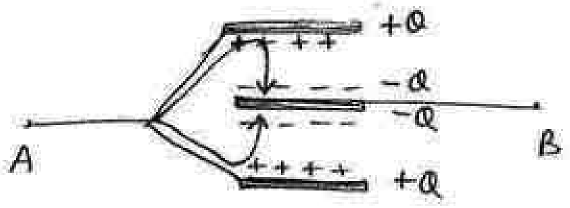
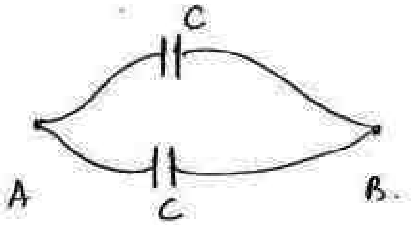


Plate connection

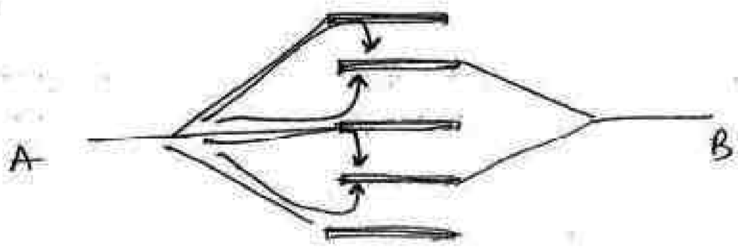


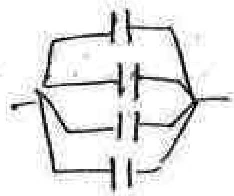
find C_{AB}



$$\therefore\ C_{AB} = 2C = \frac{2A\epsilon_0}{d}$$

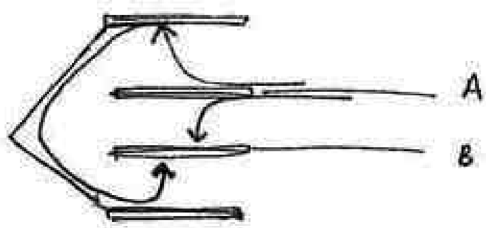
Q)

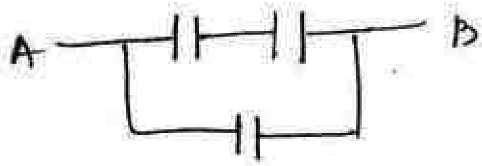




$$C_{AB} = 4C = \frac{4\epsilon_0 A}{d}$$

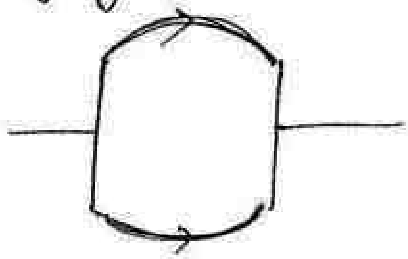
Q)





$$= \frac{3}{2}C$$

Fringing effect :



due to finite size of plate, its electric field spread out on boundary and becomes non-uniform. This effect is called fringing effect