EMI

Magnetic flux :

i) $\phi = \int \vec{B} \cdot \vec{ds}$

ii) If B is uniform, $\phi = \vec{B} \cdot \vec{S}$ $\therefore$ $\phi = BS\cos\theta$

where θ is angle b/w $\vec{B}$ and area vector

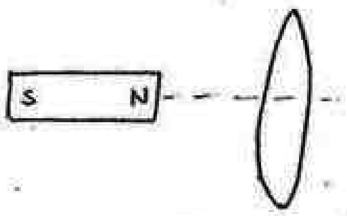
iii) Unit: SI $\Rightarrow$ $T\text{-}m^2$, weber

CGS $\Rightarrow$ $G\text{-}cm^2$, maxwell

$1\,Wb = 10^8\,Mx$

Faraday's experiment

i)



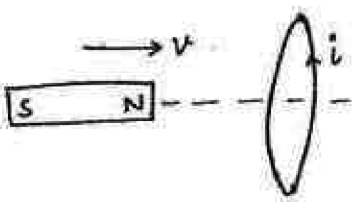
$\phi \neq 0$

$\Delta\phi = 0$

$i = 0$

conductor ≠ current carrying loop

ii)

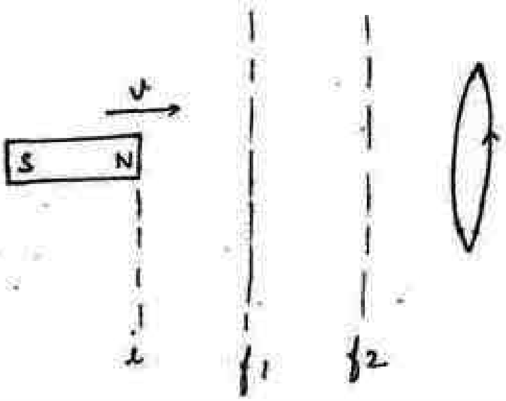


$\phi \neq 0$

$\Delta\phi \neq 0$

$i_1 \neq 0$

iii)



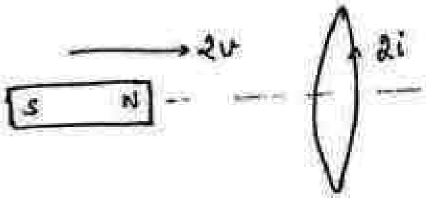
$\Delta\phi_2 > \Delta\phi_1$

$i_2 > i_1$

$e_2 > e_1$

$e \propto \Delta\phi$

iv)



$v_2 = 2v$

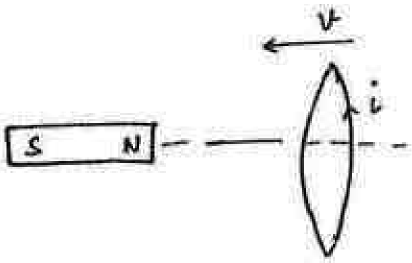
$t_2 = t_1/2$

$i_2 = 2i_1$

$e_2 > e_1 \quad (e_2 = 2e_1)$

$$\boxed{e \propto \frac{1}{\Delta t}}$$

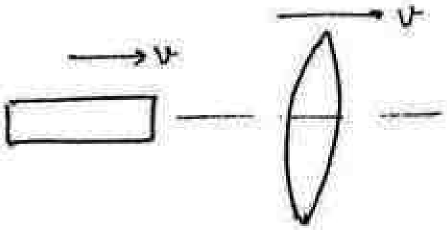
v)



$\phi = 0 \qquad \Delta\phi \neq 0$

$i \neq 0$

vi)



$\Delta\phi = 0$

$\because$ relative flux $= 0$

Faraday's law of induction

According to this law, magnitude of induced emf in circuit is equal to time rate of change of magnetic flux through that circuit

a) $$|e| = \frac{d\phi}{dt} \quad \therefore \boxed{e = -\frac{d\phi}{dt}} = -\text{slope}_{(\phi - t)}$$

b) $$e = \frac{-d}{dt}(BS\cos\theta) = -S\cos\theta\frac{dB}{dt}$$

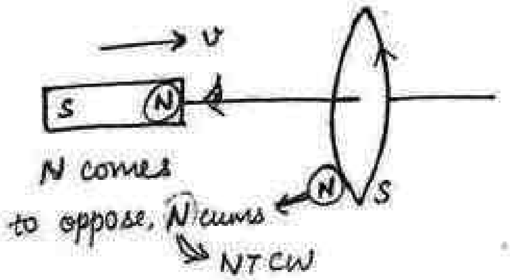
$$= -B\cos\theta\frac{dS}{dt} = \frac{-BS\,d(\cos\theta)}{dt}$$

c) $$e_{coil} = N\left(\frac{-d\phi}{dt}\right)$$

d) $e_{avg} = \Delta\phi \quad = -(\phi_2 - \phi_1)$ e) $e_{avg} = -[(NBS\cos\theta)_2 - (NBS\cos\theta)_1]$

Lenz's Law :

According to this law, polarity of induced emf is such that, it tends to produce current which <u>opposes the cause</u> which is responsible to produce it.

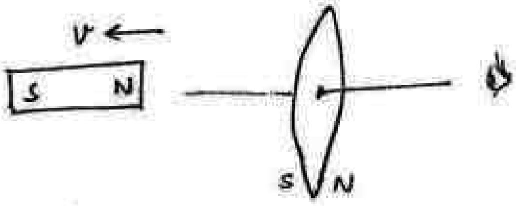
a) 

ACW

NANA

N →

→

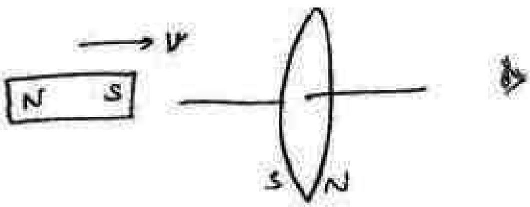
b) 

ACW

← N

→ S

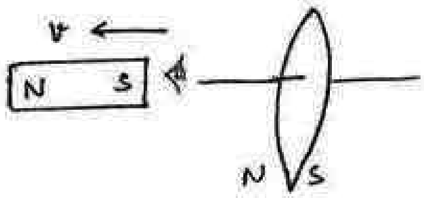
SAS

c) 

ACW

→ S

→

d) 

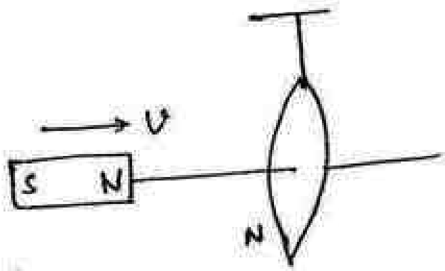
ACW

S ←

→ N

In expression $e = \frac{-\Delta\phi}{dt}$, $-ve$ sign indicates opposition to the change in flux in accordance to the lenz law

Q) In the given diagram, magnet is moving towards the loop. then induced emf, induced current and induced charge are e, i and q respectively. If speed of the magnet is doubled then find dirⁿ of induced current and new values of e, i and q and find whether the loop will move towards or away from the magnet



i) Anticlockwise

ii) $v' = 2v$ $\left(\because e \propto \frac{\Delta\phi}{\Delta t}\right)$

$t' = t/2$

$e' = 2e$

$i' = \frac{V}{R} = \frac{e'}{R}$ $(\therefore i' = 2i)$

$\left(\because i = \frac{q}{\Delta t}\right)$ $q = i\Delta t = \frac{e}{R}\Delta t = \frac{-\Delta\phi}{\cancel{\Delta t}\,R}\cancel{\Delta t} = \frac{-\Delta\phi}{R}$

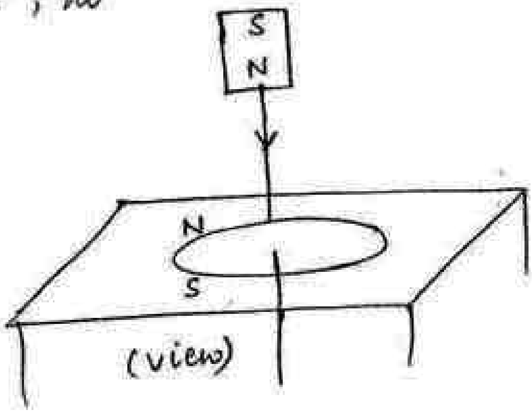
$\therefore \boxed{q = \frac{-\Delta\phi}{R}}$ $\boxed{q = \frac{\phi_1 - \phi_2}{R}}$ $\therefore q \propto t^0$ $\therefore q' = q$

iii) loop will move away (Repulsive force)

Q) In the given diagram, magnet is released along the axis of the circular loop, then find

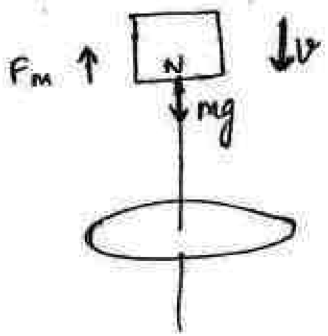
i) dirⁿ of induced current

ii) Find whether the magnet will fall freely or not; no



i) Clockwise

ii)

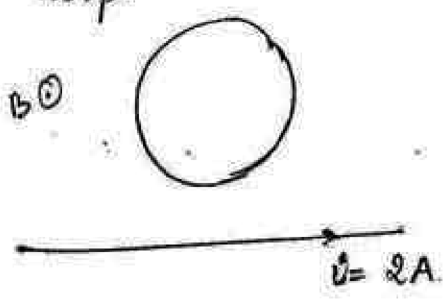


$$a = \frac{mg - F_M}{m} \quad \left(\because a = \frac{F_{net}}{m}\right)$$

$$= g - \frac{F_M}{m}$$

$$\therefore \boxed{a < g}$$

Q) Find the dirⁿ of induced current in the circular loop.



i_{wire} = constant

B_{wire} ⊙ = constant

ϕ_{loop} = constant

$\therefore \Delta\phi = 0$

$\therefore e = 0 \quad \therefore \boxed{i = 0}$

Q) If $i = (2t + 1)$ A in above que, then find the dir^n of induced current.

$$\underbrace{\cancel{\text{inducing}}} \quad \cancel{\text{induced}}$$

$i_w = 2t + 1 \uparrow$ ($\because t \uparrow$ always and can't $\downarrow$)

$\odot B_w \uparrow$ $\rightarrow$ i_{loop} = C.W.

$(\odot = ACW)$

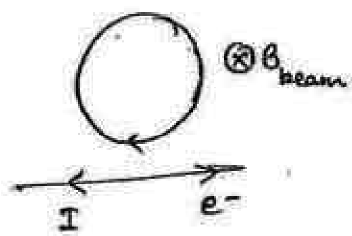
inducing		induced	force
$\otimes$	$\uparrow$	$\odot$	Repulsion
$\odot$		$\otimes$	$\uparrow$ (excess of anything is not good)
ACW		CW	
CW		ACW	
$\otimes$	$\downarrow$	$\otimes$	Attraction
$\odot$		$\odot$	
ACW		ACW	
CW		CW	

Q) If in above que wire is replaced by an e^- beam, then find dir^n of induced current

a) v_{beam} = constant

i_{beam} = constant $\therefore \Delta\phi = 0$ &

$i_{net} = 0$

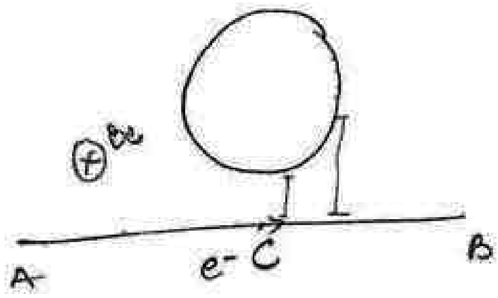


b) $v_{beam} \uparrow$

$i_{beam} \uparrow$

$\otimes B_{beam} \uparrow \rightarrow \therefore i_{net}$ = ACW

Q) In the given diagram, e^- is moving from A to then find dir^n of induced current in the loo



$\because d\vec{B} = \dfrac{\mu_0 i (d\vec{l} \times \vec{r})}{4\pi r^3}$

$B \propto \dfrac{1}{r^3}$

i) A to C

$\otimes$ $B_e \uparrow$ $\rightarrow$ i_{loop} = ACW

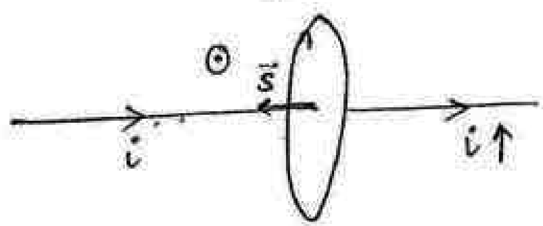
$(\because \otimes = CW)$

ii) C to B

$\otimes$ $B_e \downarrow$ $\rightarrow$ $\therefore$ i_{loop} = CW

$(\because \otimes = CW)$

Q) Find the dir^n of induced current in the loop

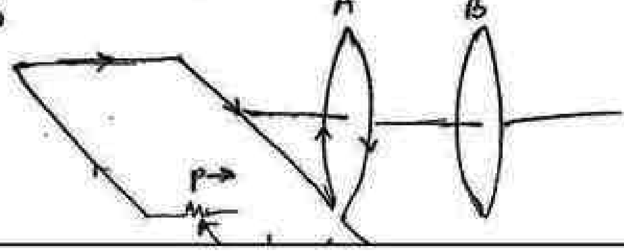


$\odot \perp (\leftarrow)$

$\phi = B_W S_{loop} \cos\theta$ $\left\{ \because \vec{B_W} \perp \vec{S}_{loop} \quad \theta = 90 \right\}$

$\phi = 0$ $\quad \Delta\phi = 0$ $\quad e = 0$ $\quad i_{indu} = 0$

Q) In the given diagram, pointer P is taken towards right then find dir^n of induced current in the loop B



P shifts towards right

$\therefore$ Resistance $\uparrow$ $\therefore i_A (CW) \downarrow$

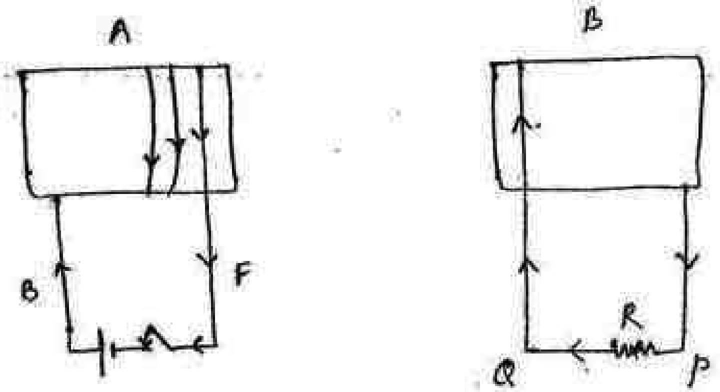
$\therefore i_B$ (CW)

Q) In the given diagram, find the dirⁿ of induced current through the resistance R

i) just after switch is closed

ii) long after switch is closed

iii) just after switch is opened

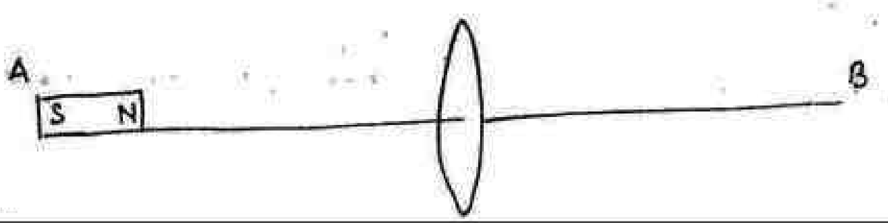


i) $i_A \uparrow \quad \therefore i_R \Rightarrow$ (P to Q)

ii) i_A - constant $\quad \therefore i_R = 0$

iii) $i_A \downarrow \quad \therefore i_R \Rightarrow$ Q to P

Q) Draw the variations in induced e.m.f in the ring when a bar magnet passes through the ring from distant point A to distant point B



i) At m...d C

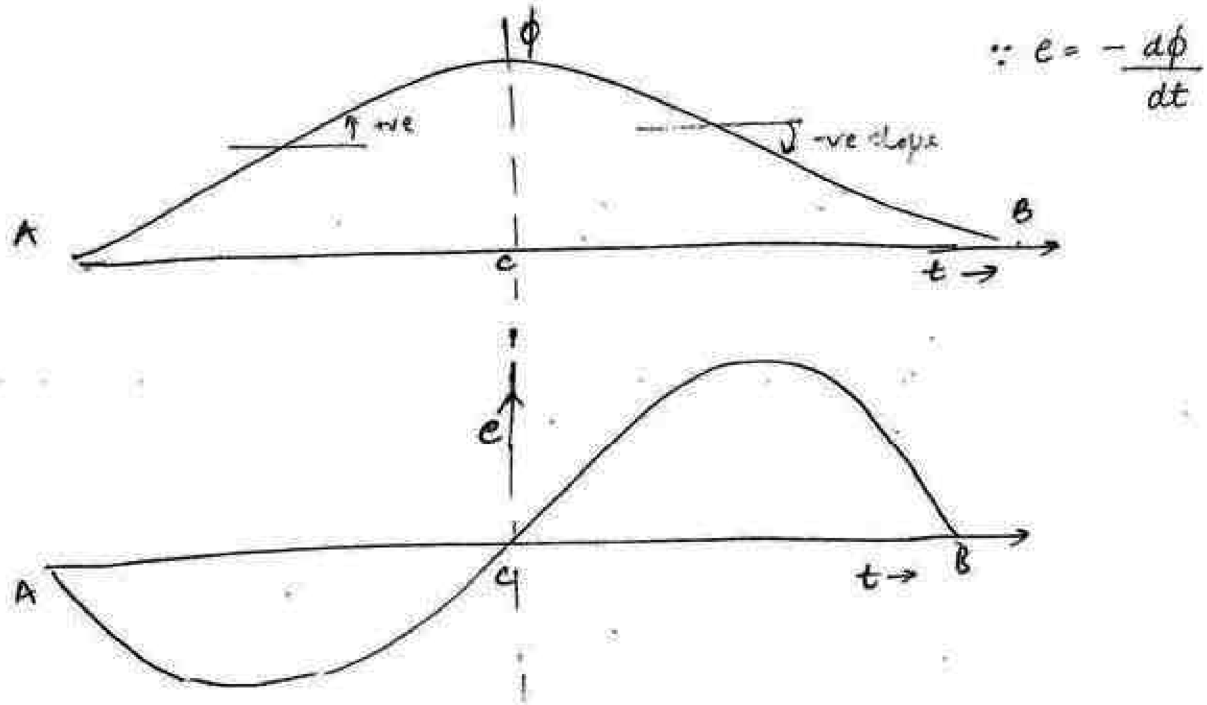
slope = 0 $\quad\therefore\quad e = -\text{slope} = 0$

ii) from A to C

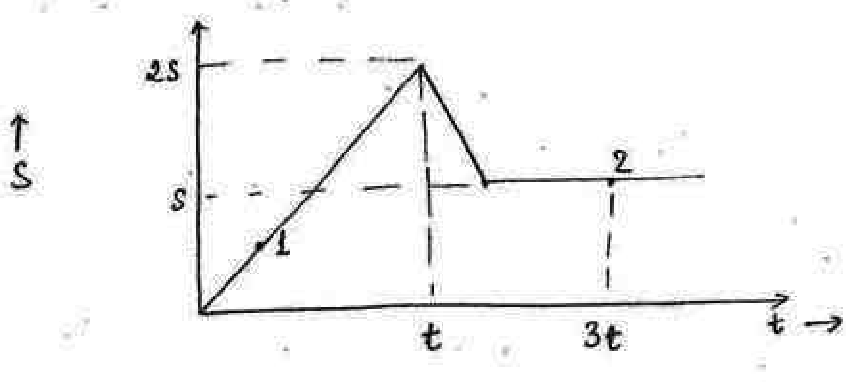
ϕ is $\uparrow$ing $\quad\therefore\quad$ slope : +ve and $e = -$ve

iii) from C to B

ϕ is $\downarrow$ing $\quad\therefore\quad$ slope : $-$ve and $e = +$ve



Q) Find the induced emf in the ring for which area is changing with time as shown in diagram.



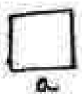
$|e| = B\cos\theta \dfrac{ds}{dt}$ — $\Rightarrow$ Slope$_{(s\text{-}t)}$

$\therefore e_1 = B \times \dfrac{2S}{t}$ $\qquad e_2 = B \times 0 = 0$

Q) A square loop of side a is placed perpendicular to the magnetic field B. It is converted into an ~~the~~ equilateral Δ loop in time t then find induced emf in the loop

$$e = \frac{\phi_1 - \phi_2}{\Delta t} = \frac{\phi_{square} - \phi_{\Delta}}{\Delta t}$$

$= \phi$



perimeter $= 4a$ $\qquad \therefore 3x \quad \therefore x = \dfrac{4}{3}a$

$$\therefore A_{tri} = \frac{\sqrt{3}}{4}\left(\frac{a4}{3}\right)^2 = \frac{4a^2}{3\sqrt{3}} \qquad \because \left(Area_{\Delta} = \frac{\sqrt{3}x^2}{4}\right)$$

$$e = \frac{Ba^2 - B\dfrac{4a^2}{3\sqrt{3}}}{t} = \frac{Ba^2}{t}\left(1 - \frac{4}{3\sqrt{3}}\right)$$

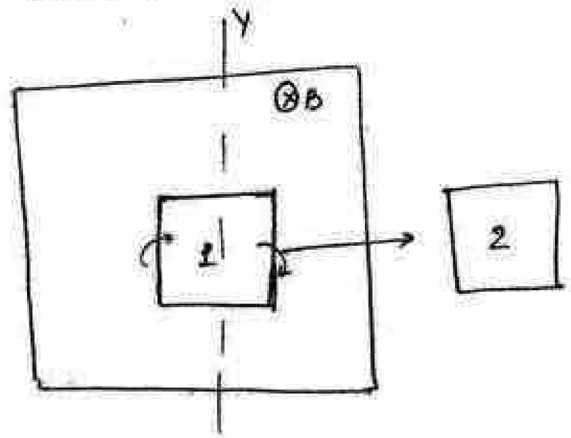
Q) A circular coil of radius 10 cm having 100 turns is placed in a magnetic field which is varying at a rate 2T/sec. Find induced emf in the coil

$$|e|_{coil} = N\frac{d\phi}{dt} = NS\cos\theta\frac{dB}{dt}$$

Q) In the given diagram, a square loop of side 'a' is placed in ⊥ magnetic field B. Find the induced emf in the loop if

i) It is taken out of the magnetic field in time t

ii) It is rotated about an axis y-y through an angle 180° in time t



i) induced current : CW (⊗ $\phi \downarrow$)

$$e_{avg} = \frac{\phi_1 - \phi_2}{\Delta t} = \frac{Ba^2 - 0}{t} = \frac{Ba^2}{t} \left(\because \phi_2 = 0\right)$$

$$q_{ind.} = \frac{\phi_1 - \phi_2}{R} = \frac{Ba^2}{R}$$

By CCAD

ii) a) Initially ⊗ ϕ ↓es

induced current = CW [clockwise ⇒ cross]

$\vec{S}$ ⊗ ∴ $\vec{B}$ ⊗ ∴ $\theta = 0$

$\phi_1 = BS\cos\theta$

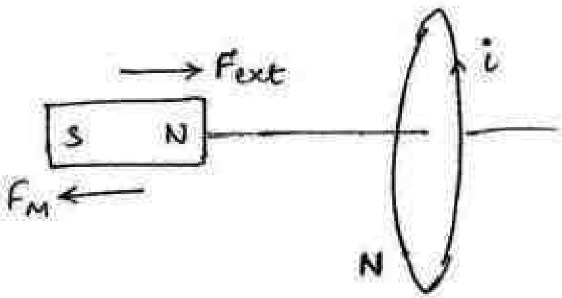
b) finally, ϕ ⊗ ↑ ∴ induced current ACW

$\vec{S}$ ⊙ ; $\vec{B}$ ⊗ ∴ $\theta = \pi$ [Anticlockwise ⇒ dot] By CC(AD)

$\theta_2 = BS\cos\pi$ ∴ $e = \dfrac{\phi_1 - \phi_2}{\Delta t} = \dfrac{BS\cos 0 - BS\cos\pi}{t}$

$= \dfrac{2a^2B}{t}$

Lenz law and energy conservation:



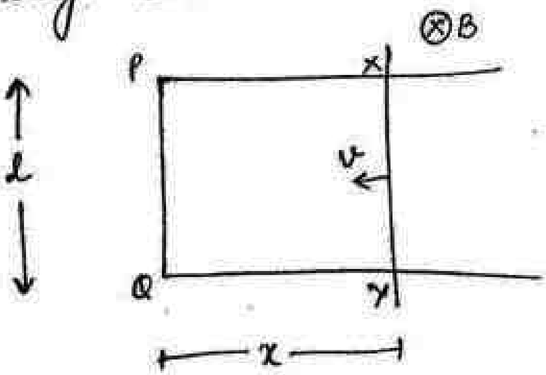
when magnet is taken towards the loop, then similar pole appears in the loop which opposes the motion of the magnet.

External agent does some work against this repelling force which is converted into electrical energy in the loop and then further lost in form of heat energy

So, lenz law is based on law of conservation of energy

Motional EMF

i) Moving conductor:



let at any instant 't' conductor ~~to~~ XY is moving with the velocity v, then flux through the area bounded by the conductor

$$\phi = BS\cos 0^\circ$$

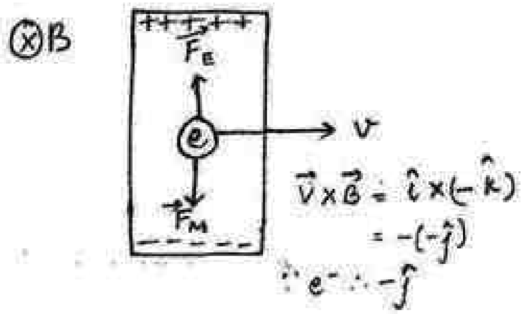
So induced emf in the loop

$$e = -\frac{d\phi}{dt} = -B\ell\left(\frac{dx}{dt}\right) = B\ell\left(\frac{-dx}{dt}\right)$$

$$\therefore \boxed{e = v\ell B} \quad \left\{ \text{as } x \downarrow \text{ so } v = \frac{-dx}{dt} \right\}$$

Important Points :

> General expression for induced emf due to moving conductor



In equilibrium,

$$\vec{F_E} = -\vec{F_M}$$

$$qE = -q(\vec{V}\times\vec{B})$$

$$E = -(\vec{V}\times\vec{B})$$

so pd. across ends or induced emf

$$e = -\int \vec{E}\cdot\vec{dl} = -\int -(\vec{V}\times\vec{B})\cdot\vec{dl}$$

$$\boxed{e = \int (\vec{V}\times\vec{B})\cdot\vec{dl}}$$

if $\vec{V}\times\vec{B}$ is constant

$$\boxed{e = (\vec{V}\times\vec{B})\cdot\vec{\ell}} \boxed{= [VB\ell]} \text{ — box product}$$

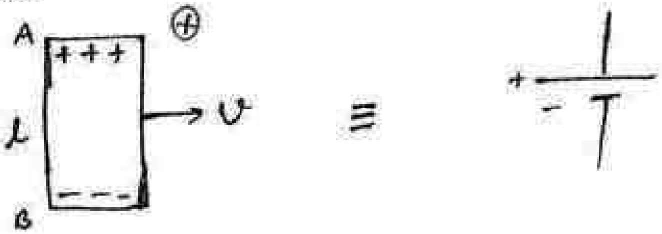
> If any of the two vectors in box product are in same direction, then its result is always zero

> If all three vectors are mutually ⊥ to each other in box product, then

$$[VB\ell] = (v\hat{i}\times B\hat{j})\cdot \ell\hat{k} = vB\hat{k}\cdot\ell\hat{k}$$

$$[VB\ell] = V\ell B$$

- A moving conductor can be replaced by a battery or a cell



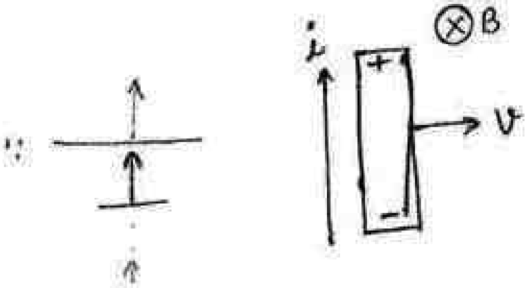
$e = vlB$
$r = R$

Direction of induced current :

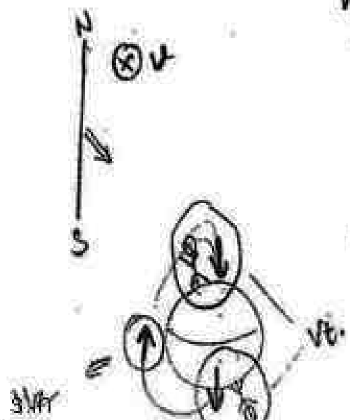
- It can be obtained by using right hand flemming rule

If index finger = $\vec{B}$
thumb = v
middle = i

CUL



Q) A conductor of length 'l' is placed horizontally in N-S dir^n and released, then find, then find induced emf in the conductor



$e_V = [v B_V l]$ $\quad \because B_V = \pm\hat{k}$
$= -\hat{k}\ \pm\hat{k}\ \hat{j} \quad = 0$

$e_H = [v B_H l] \quad = 0 \quad \because B_H = \hat{j}$ (always)
$-\hat{k}\ \hat{j}\ \hat{j}$

Q) If in above que. conductor is oriented in E-W dirⁿ and released then find induced emf across it as a funᶜ of time

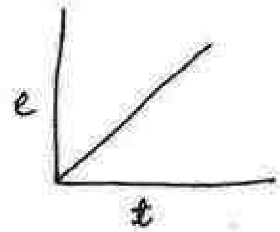
$$e_V = [\underset{-\hat{k}}{v}\ \underset{\pm\hat{k}}{B_V}\ \underset{\hat{j}}{l}] = 0$$

$$e_H = [\underset{-\hat{k}}{v}\ \underset{\hat{j}}{B_H}\ \underset{\hat{i}}{l}] = vlB_H$$

$$= (u + gt)\,lB_H = gtlB_H \quad (\because u = 0)$$

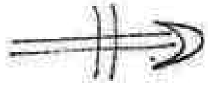
$$\boxed{e = gt\,lB_H}$$

$\therefore e \propto t$



Q) An aeroplane is moving ~~due east~~ towards due east with the velocity v, then find induced emf across its
i) length, l ii) wings ~~span = b~~ (if wings span = b)

As velocity of an aeroplane is always along its length, so their will be no induced emf along its length



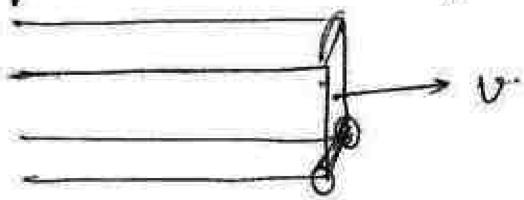
i) $$e_V = [\underset{+\hat{i}}{v}\ \underset{\pm\hat{k}}{B_V}\ \underset{\hat{i}}{l}] = 0$$

$$e_H = [\underset{+\hat{i}}{v}\ \underset{\hat{j}}{B_H}\ \underset{\hat{j}}{l}] = 0$$

ii) $$e_V = [\underset{\hat{i}}{v}\ \underset{\pm\hat{k}}{B_V}\ \underset{\hat{j}}{l}] \neq 0$$

$$e_H = [\underset{\hat{i}}{v}\ \underset{\hat{j}}{B_H}\ \underset{\hat{j}}{l}] = 0$$

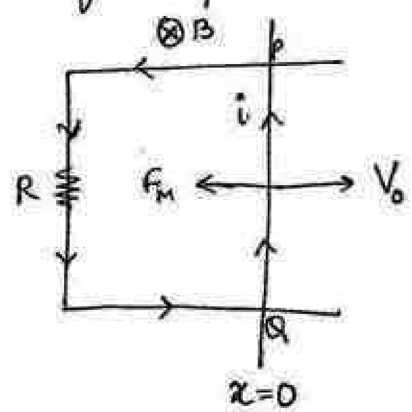
Q) A train is moving on the north pole towards east with the speed 108 km/hr on meter gauge. If magnetic field of the earth is 0.5 gauss. Then find induced emf across its axle.



$$e_v = \underset{\hat{i} \pm \hat{k}}{[v B_v l]} = v B_v l = \left(108 \times \frac{5}{18}\right)(0.5 \times 10^{-4}) 1$$

$$= 1.5 \text{ mV}$$

Q) In the given diagram, conductor PQ is given initial velocity V_0, then find i) dirⁿ of induced current ii) The speed of the conductor will ↑, ↓ or remain constant iii) velocity of the conductor as a funᶜ of displacement



i) $\because \phi \otimes \uparrow$ $\therefore$ NB⊗ ↑es $\therefore$ reverse i.e ⊙ induced current = ACW

ii) just after giving the velocity to the conductor PQ, It experiences a magnetic force in opp^s dirⁿ to the velocity due to induced current $\therefore$ its velocity decreases (CVL)

iii) $F_m = Bil = B\frac{e}{R}l = \frac{BVlBl}{R} = \boxed{\frac{B^2l^2V}{R}}$

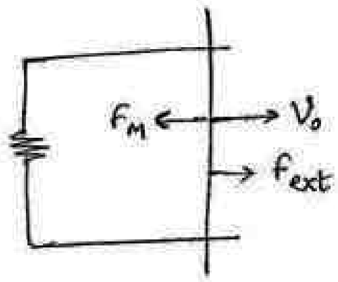
$\therefore -a = \frac{F}{m}$ ($\because v$ is $\downarrow$ing)

$$-\frac{\cancel{v}\,dv}{dx} = \frac{\cancel{v}l^2B^2}{mR} \quad \therefore -\int_{v_0}^{v} dv = \frac{l^2B^2}{mR}\int_0^x dx$$

$$\therefore v_0 - v = \frac{l^2B^2}{mR}(x-0)$$

$$\therefore v = v_0 - \frac{l^2B^2x}{mR}$$

*Q) In above que, initial velocity is kept constant by applying some external force, then find
i) external force
ii) work done by the external force in time t
iii) Power delivered by the external force to the conductor



i) $\vec{v}$ = constant

$\therefore \vec{a} = 0$, $-F_{ext} = 0$ $\therefore -\vec{F}_{ext} = -\vec{F}_M$

$$\therefore F_{ext} = \frac{V_0 l^2 B^2}{R}$$

ii) $W_{ext} = F_{ext} \times x = \frac{V_0 l^2B^2}{R} \times (V_0 t)$ $\quad \left\{ \because V_0 = \frac{x}{t} \right.$

$$= \frac{V_0^2 l^2 B^2 t}{R}$$

$W_{ext} \neq 0 \quad \therefore \quad W_{total} = 0 \quad (\because V = \text{constant})$

$W_{ext} + W_M = 0 \quad \therefore \quad W_M = -W_{ext}$

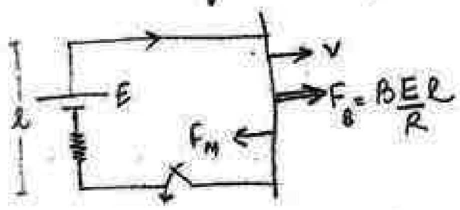
iii) $P_{ext} = \vec{F}_{ext} \cdot \vec{V_0} = \dfrac{V_0 l^2 B^2}{R} V_0 = \dfrac{V_0^2 l^2 B^2}{R}$

$= \dfrac{(V_0 l B)^2}{R} = \dfrac{e^2}{R} = P_{elect}$

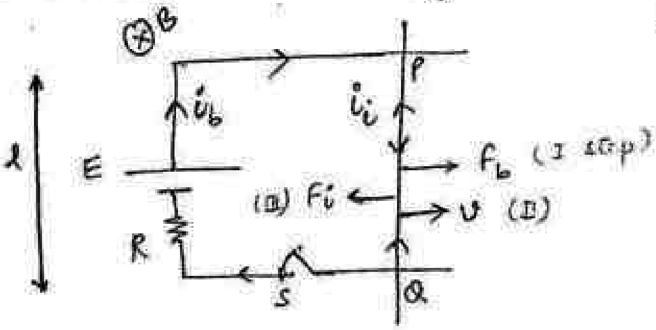
Q) In the given diagram ~~at~~ ~~t~~ switch is closed at $t=0$, then find

i) acc^n of the conductor PQ ~~with~~ as the fun^c of its velocity

ii) It's terminal velocity



$\therefore F_M = \dfrac{VB^2l^2}{R}$



i) $F_b = B i_b l = B \dfrac{E}{R} l = \text{constant}$ (CUL)

$F_i = B i_i l = B \dfrac{e}{R} l = B \dfrac{VlB}{R} l = \dfrac{Vl^2B^2}{R}$ (CUL)

$a = \dfrac{F_b - F_i}{m} = \dfrac{\dfrac{BEl}{R} - \dfrac{Vl^2B^2}{R}}{m}$

$$\boxed{a = \frac{Bl}{mR}(E - VlB)}$$

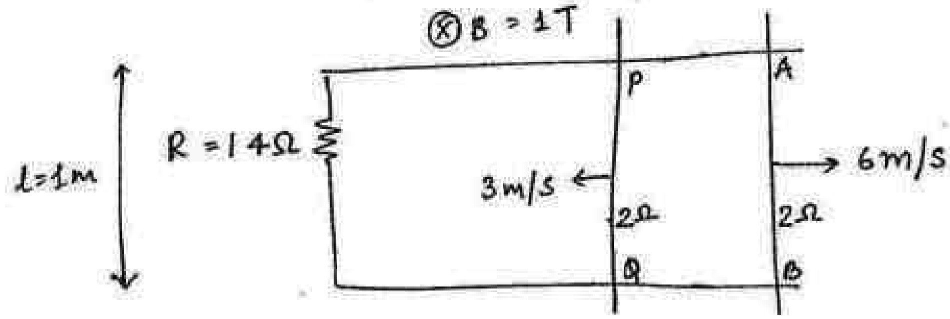
ii) when $F_b = F_i$

$$F_{net} = 0 \quad \therefore \quad \vec{a} = 0 \quad \therefore v = \text{const} = V_t$$

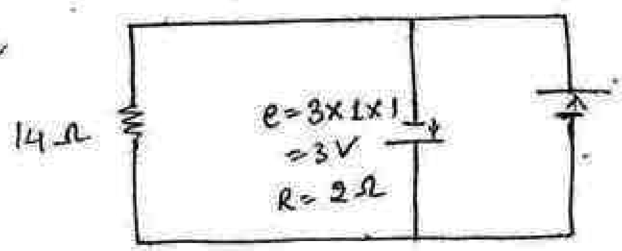
$$a = \frac{Bl}{MR}\left(E - V_t l B\right) \qquad \therefore \quad V_t = \frac{E}{lB}$$

emf

Q) In the given diagram. Find current through ~~a~~ resistance, R



find dir^n of current in ckt



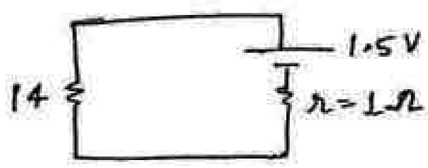
$e = VlB$

$= 6 \times 1 \times 1 = 6V$

$\therefore R = 2\Omega$

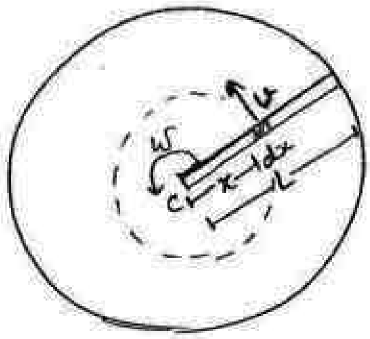
$$\therefore e_{eq} = \cancel{6x} \; \frac{E_1 r_2 - E_2 r_1}{r_1 + r_2}$$

$$= \frac{6\times 2 - 3\times 2}{2+2} = 1.5 \text{ V}$$

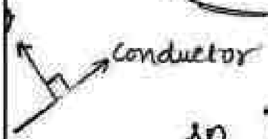


$$\therefore i = \frac{1.5}{14+1} = 0.1 \text{ A}$$

ii) Rotating conductor :



Considering an elemental conductor dx, moving with the linear velocity v, then induced emf across it is $de = v\,dx\,B$



∴ emf generated

so, induced emf across the conductor,

$$e = \int_0^L v\,dx\,B = \int_0^L \omega x\,dx\,B$$

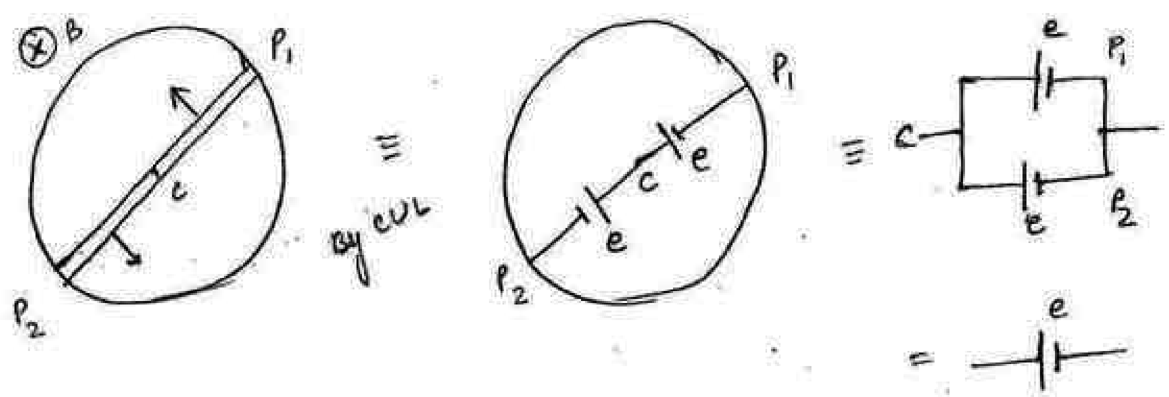
$$\omega B \left(\frac{x^2}{2}\right)^L \qquad \therefore \boxed{e = \frac{1}{2} B\omega L^2}$$

IMP

induced $i \longrightarrow$ P to C

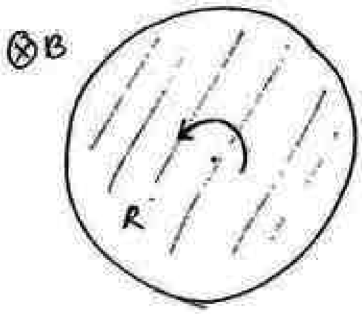
P = −ve and C = +ve

If more than one conductor are rotated then,



iii) Rotating disc / Faraday disc generator:

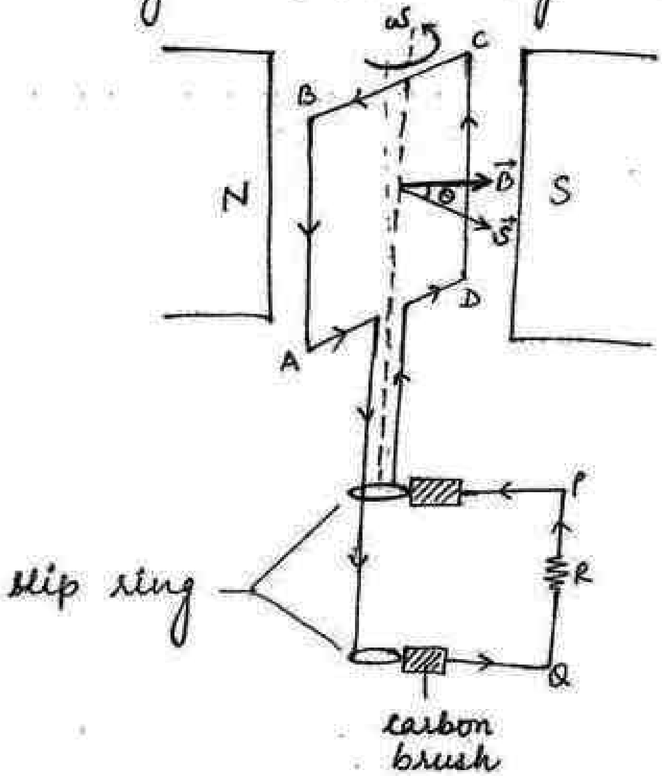
A disc can be assumed to be consist of no. of rotating conductor producing emf in parallel combination then,



$$e_{disc} = e_{conductor}$$

$$e = \frac{1}{2} B\omega R^2$$

iv) Rotating coil / ~~Ro~~ AC generator / Dynamo:



Flux through the coil at any instant t,

$$\phi = NBS\cos\theta = NBS\cos(\omega t) \quad [\because \theta = \omega t]$$

so, emf induced in the coil $e = \frac{-d\phi}{dt}$

$$= -NBS(-\sin\omega t)\omega$$

$\therefore \boxed{e = NBS\omega \sin(\omega t)}$ $\therefore \boxed{e = e_0 \sin\omega t}$

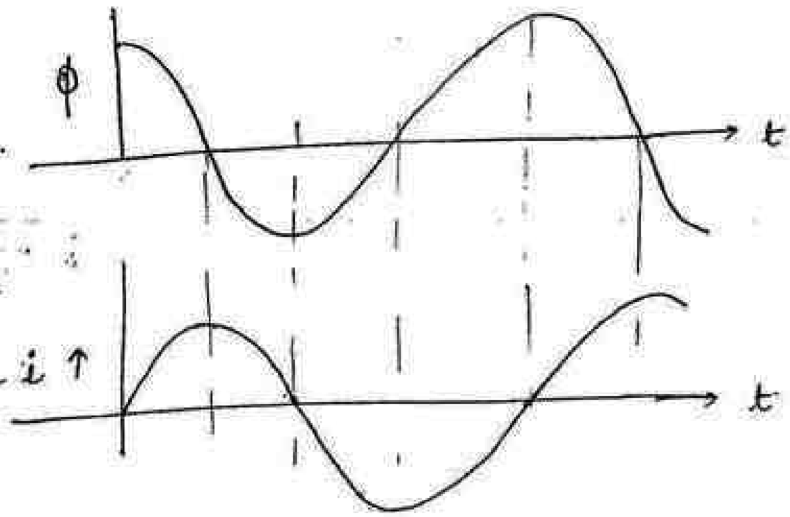
e_0 = max/ peak / amplitude emf $\boxed{e_0 = NBS\omega}$

$\therefore \boxed{e_0 = NBS\, 2\pi f}$ $(\because \omega = 2\pi f)$

induced current / a.c., $i = \dfrac{e}{R} = \dfrac{NBS\omega \sin(\omega t)}{R}$

$\therefore \boxed{i = i_0 \sin(\omega t)}$ $\left\{ i_0 = \dfrac{NBS\omega}{R} \right\}$

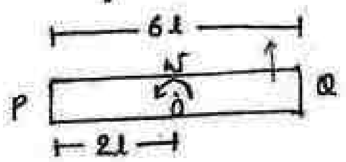
Graph:

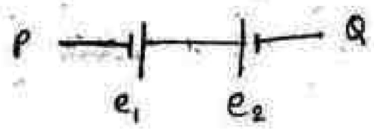


a) when ϕ = max, emf = 0

b) When ϕ = 0, emf = max/min

Q) In the given diagram, conductor is rotated abt point O, then find induced emf across it.





$\therefore V_P + e_1 - e_2 = V_Q$

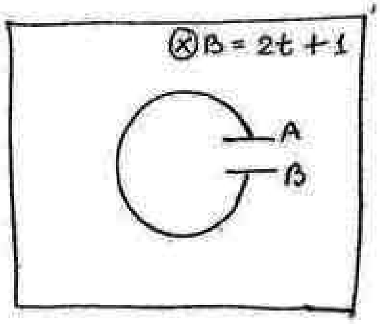
$$V_P - V_Q = e_2 - e_1$$

$$= \frac{1}{2}B\omega l_2^2 - \frac{1}{2}B\omega l_1^2$$

$$= \frac{1}{2}B\omega (l_2^2 - l_1^2)$$

$$= \frac{1}{2}B\omega \left[(4l)^2 - (2l)^2\right] = 6B\omega l^2$$

Q) In the given diagram, find polarity of the plates of the capacitor.



$\because \phi \otimes \uparrow$ $\therefore$ induced current $\Rightarrow$ A CW

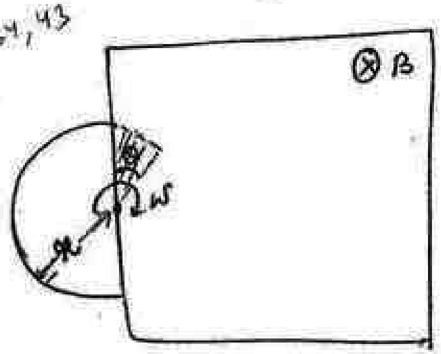
i.e. (A to B)

e^- (B to A) $\therefore$ B is +ve and A is −ve

Q) In the given diagram, semicircular loop is rotated ~~abt~~ an axis passing through O and $\perp$ to the plane. Then find induced emf across its centre and circumference

1. 4, 19, 22, 25, 31, 34, 43, 49, 54, 59, 66, 67

II 38 to 49



$2\pi \longrightarrow \pi r^2$

$\longrightarrow \frac{\pi r^2}{2\pi}$

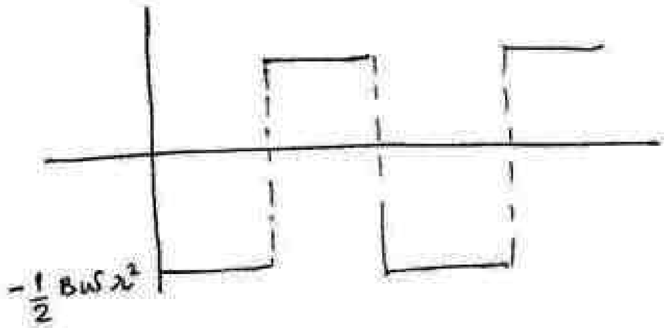
$\theta \longrightarrow \frac{r^2\theta}{2}$

$$\phi = BS\cos 0^\circ$$

$$= \frac{B\cancel{\pi}r^2}{2\cancel{\pi}} = \frac{Br^2}{2}$$

$\because \; e = \frac{-d\phi}{dt} = -\frac{Br^2}{2}\frac{d\theta}{dt}$ $\quad \therefore \; e = -\frac{1}{2}B\omega r^2$

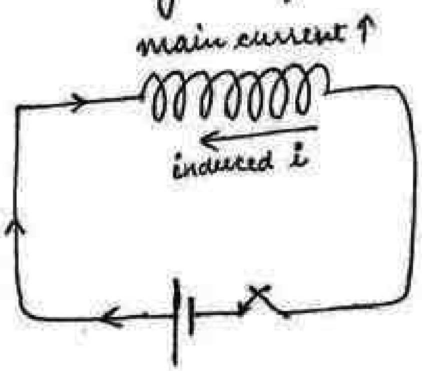
$$e = \pm \frac{1}{2}B\omega r^2$$



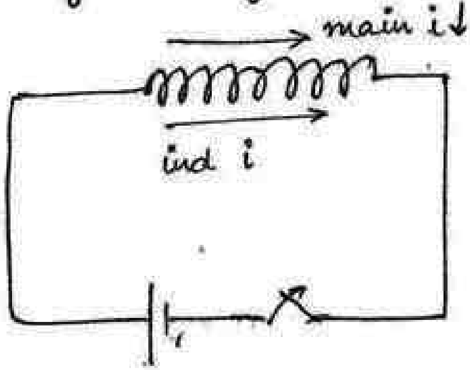
Self Induction:

It is a phenomena in which current is induced in the coil itself due to the changes in flux through it.

$\therefore$ Self inductance is the property of a coil due to which it ~~po.~~ opposes the changes in strength of current flowing through it.



just after switch is closed



just after switch

Important Points

» As flux through the coil is directly proportional to the current flowing through it.

$$\phi \propto i \quad \therefore \quad \boxed{\phi = Li} \quad \therefore \quad L = \frac{\phi}{i}$$

$$\boxed{L = \frac{NBS}{i}}$$

» $e = -\dfrac{d\phi}{dt} = -\dfrac{d(Li)}{dt}$

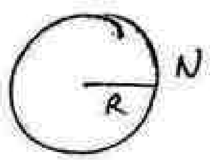
» units : henry (H), $\dfrac{Wb}{A}$, $\dfrac{V\text{-}sec}{A}$, $\Omega\text{-}sec$

Resistor	Conductor	Inductor
		
$V \propto I$	$q \propto V$	$\phi \propto i$
$V = iR$	$C = \dfrac{q}{V}$	$L = \dfrac{\phi}{i}$
$R = \dfrac{\rho l}{A}$	$C = \dfrac{A\epsilon_0}{d}$	$e = -L\dfrac{di}{dt}$

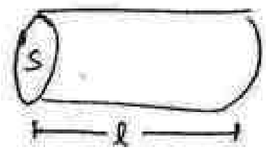
Self inductance of a coil

$$\because L = \frac{NB_cS}{i} = N\frac{\mu_0 N i}{2R}\,\frac{\pi R}{i}$$

$$\therefore \boxed{L = \frac{\mu_0 N^2 \pi R}{2}} \qquad \boxed{L \propto N^2} \qquad \boxed{L \propto R}$$



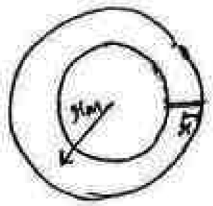
Self inductance of a solenoid:



$\because \; L = \dfrac{NB_sS}{i} = \dfrac{N}{l}\dfrac{\mu_0 n i}{i} Sl$

$\boxed{L = \mu_0 n^2 V}$ $\left\{ \because \; n = \dfrac{N}{l}, \; V = Sl \right\}$

Self inductance of a toroid:

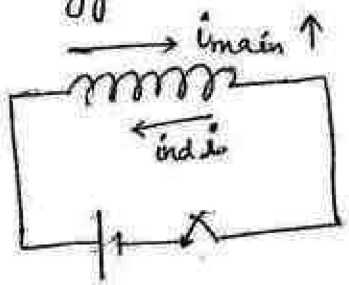


$\because \; L = \dfrac{NB_tS}{i}$

$= \dfrac{N\mu_0 N i \pi r^2}{2\pi r_m i}$

$\therefore \; \boxed{L = \dfrac{\mu_0 N^2 r^2}{2r_m}}$

Magnetic energy stored in an inductor



When an inductor / coil is connected to the battery then battery does some work against induced current which is stored as magnetic energy in the coil

Power delivered by the battery to the inductor at any instant

$$P = |e|\, i$$

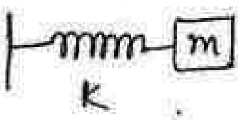
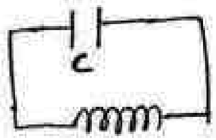
$$\frac{dW}{dt} = \left| -\frac{L\,di}{dt} \right| i$$

$$\therefore \int_0^W dW = L \int_0^i i\,di \qquad \therefore W = \frac{1}{2} L i^2$$

$$\because U_M = W \qquad \therefore \boxed{U_M = \frac{1}{2} L i^2}$$

* Magnetic energy density is given by (u_m)

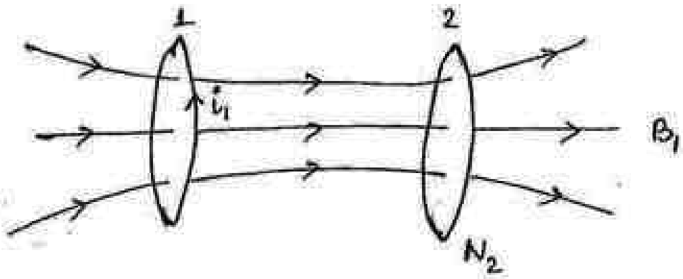
$$\boxed{u_m = \frac{B^2}{2\mu_0}}$$

<u>Mechanical</u>	<u>Electrical</u>
	
$\frac{1}{2} K x^2$	$\frac{q^2}{2C}$ i.e. $\frac{1}{2} C V^2$
$\frac{1}{2} m v^2$	$\frac{1}{2} L i^2$
K	$1/C$
m	L (electrical inertia)
$\omega = \sqrt{\frac{K}{m}}$	$\omega = \frac{1}{\sqrt{LC}}$

Mutual Inductance

It is the property of two coils due to which one coil opposes the changes in strength ~~of~~ in current of other coil and this phenomena is called mutual induction

As flux through the one coil is directly proportional to the current in other coil, then



$$\phi_2 \propto i_1 \qquad \therefore \boxed{\phi_2 = M i_1}$$

$$\therefore \boxed{M = \frac{\phi_2}{i}} \qquad \therefore \boxed{M = \frac{N_2 B_1 S_2 \cos\theta}{i_1}}$$

$\therefore$ induced emf in secondary coil, $e_2 = \dfrac{-d\phi_2}{dt}$

$$= -\frac{d(M i_1)}{dt} \qquad \boxed{e_2 = \frac{-M di_1}{dt}}$$

- unit : same as L

- mutual inductance of two concentric coil.

$$M = \frac{N_2 B_1 S_2}{i_1} = N_2 \frac{\mu_0 N_1 i_1}{2R_1} \frac{\pi R_2^2}{i_1} = \boxed{\frac{\mu_0 N_1 N_2 \pi r_2^2}{2R_1}}$$

mutual inductance of two solenoids

$$M = \frac{N_2 B_1 S_2}{i_1} = \frac{N_2}{l_2} \frac{\mu_0 n_1 i_1}{i_1} S_2 l_2$$

$$\therefore \boxed{M = \mu_0 n_1 n_2 V_2}$$

Dependency of L and M

a) Presence of medium $\boxed{L \propto \mu}$

* If iron rod is inserted $\therefore$ $\boxed{\mu \uparrow \ L \uparrow \ B \uparrow \ \phi \uparrow}$

b) no of turns

$\boxed{L \propto N^2}$ $\boxed{M \propto N_1 N_2}$

c) dimensions / size

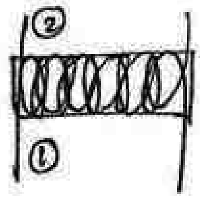
$\boxed{L \propto V}$

d) mutual inductance also depends on the arrangement of the 2 coils which is mathematically given by a no. called coefficient of coupling (k) is defined as $\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$

where $K = 0$ to 1 i.e $\boxed{0 \leq K \leq 1}$

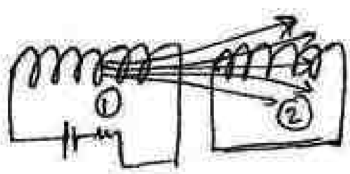
So, 'k' represent % of flux of one coil passing through the other coil.

⇒ <u>Tight coupling</u>



$K = 1$ $\therefore M = \sqrt{L_1 L_2}$

⇒ <u>Loose coupling</u>



$0 < K < 1$

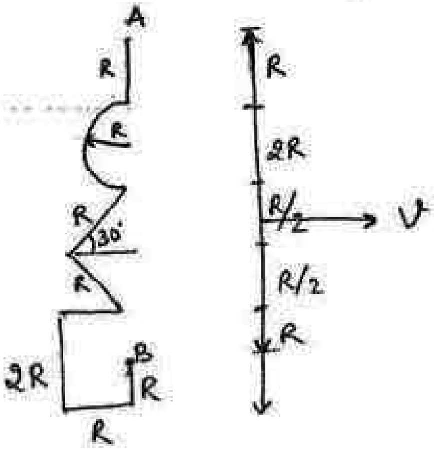
$\therefore M = K\sqrt{L_1 L_2}$

⇒ no coupling :

K

	R	L	M
—	✓	✗	✗
O	✓	✓	✗
OO	✓	✓	✓

Q) Find the induced emf across the moving



$\therefore\ l = 5R$

$\therefore\ e = v(5R)B$

Q) A coil of self inductance 2mH is carrying a current 2A. Now, this current in 4 sec

i) becomes zero

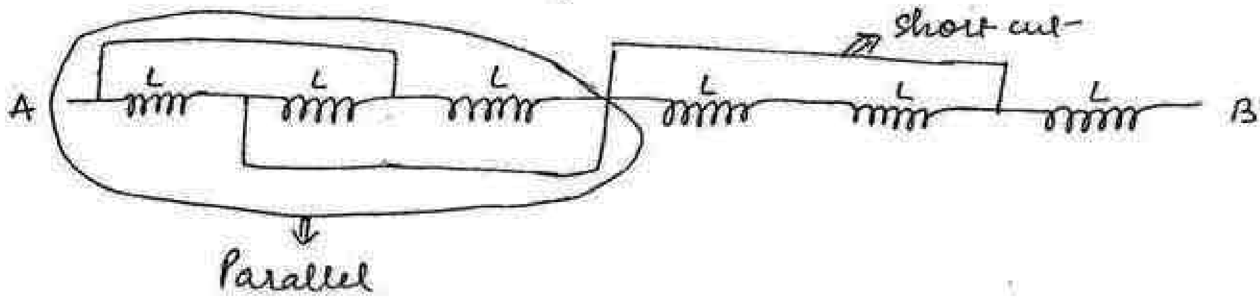
ii) is reversed

then find induced emf in the coil

i) $e = -L\dfrac{di}{dt} = -2\times10^{-3}\dfrac{(0-2)}{4} = 10^{-3} = 1\text{mV}$

ii) $e = -2\times10^{-3}\dfrac{(-2-2)}{4} = 2\text{mV}$

Q) Find equivalent self inductance b/w A and B



$$\therefore L_{AB} = \frac{L}{3} + L = \frac{4L}{3}$$

Q) Mutual inductance of two coil of self inductances 9H & 4H is 2H. Find % loss in flux

$$\because K = \frac{M}{\sqrt{L_1 L_2}} = \frac{2}{\sqrt{9 \times 4}} = \frac{1}{3} = 33\%$$

$\therefore$ loss in flux $\simeq 67\%$

Q) current in a coil is given by $i = \sqrt{2}\sin(50t)$ find induced emf in secondary coil when current in this coil is 1A ~~x~~. Mutual inductance b/w the coil is 1mH

when $i = 1A$

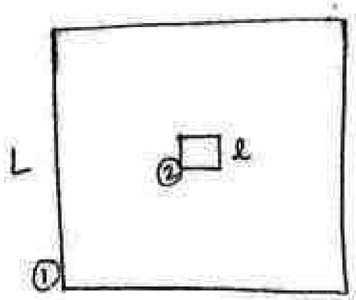
$$1 = \sqrt{2}\sin(50t)$$

$$\therefore 50t = \frac{\pi}{4}$$

$$\because e_2 = \frac{-M\,di}{dt} = -M\left[\sqrt{2}\cos 50t \times 50\right]$$

$$= \cancel{-50\,mV} \; -50\,mV$$

Q) In the given diagram. A small square loop of side l is placed at the centre of large square loop of side L ($L >> l$) then find mutual inductance b/w them.

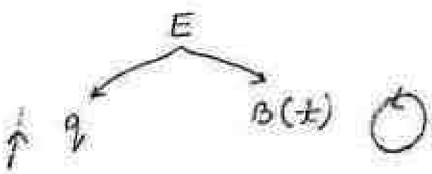


$$\because \quad M = \frac{B_1 N_2 S_2}{i_1}$$

$$\because \left\{ B_{centre} = \frac{2\sqrt{2}\mu_0 i}{\pi a} \right\}$$

$$\therefore \quad 1 \times \frac{2\sqrt{2}\mu_0 i_1}{\pi L} \times \frac{l^2}{i_1} = \frac{2\sqrt{2}\mu_0 l^2}{\pi L}$$

$$\therefore \quad \boxed{M \propto \frac{l^2}{L}}$$



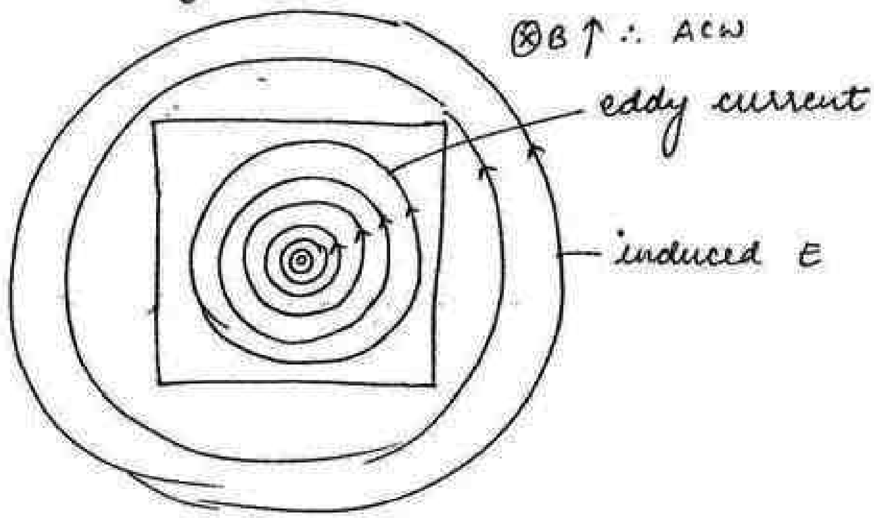
Induced Electric field :

- Whenever there exist time dependant magnetic field in certain region of space, then their appear concentric circular electric field lines called induced electric field
- Induced electric field lines forms a closed loop so these fields are <u>non-conservative</u> in nature
- From Faraday's law, $\because e = \frac{-d\phi}{dt}$

Eddy current :

> when a bulk piece of metal is placed in time dependant magnetic field, then their appears concentric circular currents called eddy currents.

> direction of these eddy current can be obtained by using lenz's law.



> Plane of the eddy current is always ⊥ to the dir^{n} of magnetic field.

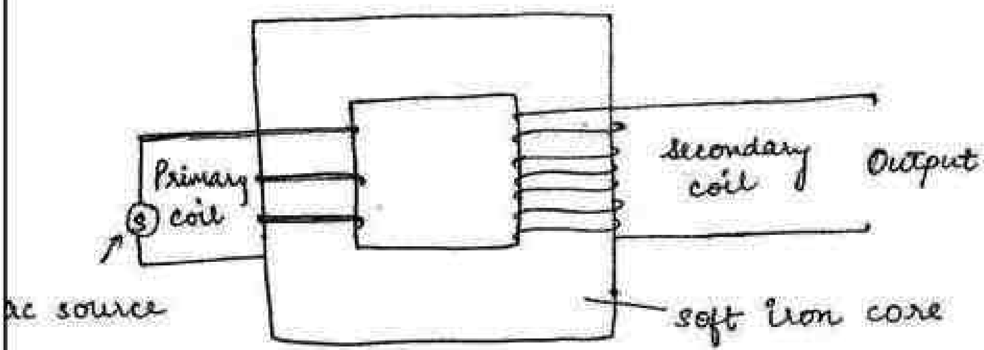
> when a bar magnet is released in long copper tube, then initially its velocity ↑es and becomes constant after certain times called Terminal velocity

TRANSFORMER

i) Power is transmitted to remote places at very high voltages to minimise the power losses in transmission line so transformer is require to regulate (↑ or ↓) the voltage

> Its working is based on EMI / mutual inductance

> Types of Transformer: i) step up



a) $V_s > V_p$

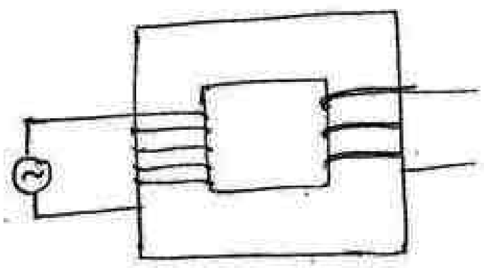
b) $N_s > N_p$

c) $i_s < i_p$

d) $t_s < t_p$ (thickness of wire)

e) $f_s = f_p$

ii) step down



a) $V_s < V_p$

b) $N_s < N_p$

c) $i_s > i_p$

d) $t_s > t_p$

e) $f_s = f_p$

Considering negligible flux losses in the coils then flux through one turn will be same in primary and secondary winding. then $\phi_1 = \phi_2$

$$\frac{d\phi_1}{dt} = \frac{d\phi_2}{dt}$$

let N_p and N_s be no of turns in primary and secondary coil respectively then emf induced in each coil

$$\therefore \quad e_p = -N_p \frac{d\phi}{dt} \quad \text{—①}$$

$$e_s = -N_s \frac{d\phi}{dt} \quad \text{—⑪}$$

If resistances of both the coils are negligible then $V_p = e_p$ —③ and $V_s = e_s$ —④

from ① ⑪, ③ & ④ $\boxed{\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p}}$ for all kinds of Transformer

$$\boxed{V \propto N}$$

for ideal Transformer,
their will be no power losses.

$\therefore \quad P_{in} = P_{out} \qquad \therefore \quad V_p i_p = V_s i_s$

$$\therefore \boxed{\frac{V_s}{V_p} = \frac{i_p}{i_s} = \frac{N_s}{N_p}}$$ only for ideal transformer

efficiency of Transformer

$$\eta = \frac{P_{out}}{P_{in}} \qquad \therefore \boxed{\eta = \frac{V_s i_s}{V_p i_p} = \frac{N_s i_s}{N_p i_p}}$$

$\eta \Rightarrow 90 - 99\%$

This efficiency is so high because their is no relative motion b/w any 2 parts of the transformer

losses in Transformer:

i) Copper losses

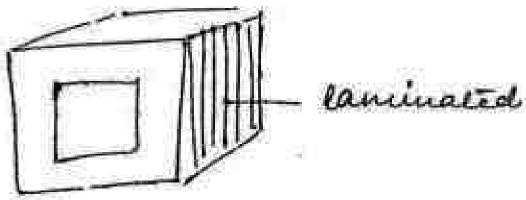
<u>Reason</u>: loss of energy in form of heat due to flow of current in primary & secondary winding

sol^n: copper is selected to form coil due to its low resistivity

ii) iron losses

Reason: Due to the application of time dependent magnetic field. strong eddy's are induced in core of the transformer results in heat losses.

sol^n: Instead of taking bulk piece of the core, it is taken laminated. In which it is cut into thin strip and each strip having a non-conducting liquid film



iii) hysteresis losses

Reason: Due to the application of cyclic magnetic field, hysteresis losses occur in core of transformer

sol^n: soft iron is selected for the core due to its low area under hysteresis curve

iv) leakage of flux:

Reason: Due to some seperation b/w the coil. flux is lost in surrounding

sol^n: Single soft iron core is selected due to its high permeability

a) In a transformer, transformation ~~received~~ ratio is 1/20. If line voltage and current are 100 V and 1 A respectively, then find load voltage and current transformation ratio $= \frac{N_s}{N_p} =$ turn ratio

line → input → primary

load → output → sec.

$\frac{V_s}{V_p} = \frac{N_s}{N_p}$ $\therefore \frac{V_s}{100} = \frac{1}{20}$ $\therefore V_s = 5V$

$\frac{i_p}{i_s} = \frac{N_s}{N_p}$

Q) A transformer is used to light up a bulb of 180 W & 100 V. If AC mains is 200 V, 1 A. then find efficiency of transformer

$\eta = \frac{P_o}{P_i} = \frac{180}{200 \times 1} = 0.9 = (90\%)$

Q) Flux through primary coil is given by $\phi = 2 + 2t$ weber. , then find induced emf in secondary of the transformer if no of turns in primary and secondary coils are 100 & 1000 respectively.

$\therefore V_p = \frac{-d\phi}{dt} = -2$ volt

$\because \frac{V_s}{V_p} = \frac{N_s}{N_p}$ $\therefore \frac{V_s}{2} = \frac{1000}{100} = \therefore V_s = 20$ volt

Q) 1.1 kW power is transmitted by using transmission lines of resistance 10 Ω at 22000 V find power lost in transmission line

$i = \frac{P}{V} = \frac{1.1 \times 10^3}{22000} = \frac{1}{20}$ A

$$\boxed{P_{lost} = i^2R} = \left(\frac{1}{20}\right)^2 \times 10 = 0.025 \text{ watt}$$

Q) A conducting ring is placed in perpendicular magnetic field which is oscillating with max value 0.1 T and frequency = 50 Hz. Find max value of induced electric field in the ring. $r = 1\,m$

$B_0 = 0.1$ T $\quad f = 50$ Hz

$\therefore B = B_0 \sin(2\pi f t) = 0.1 \sin(100\pi t)$

$$\therefore \frac{dB}{dt} = 0.1 \cancel{\sin} \cos(100\pi t)\, 100\pi$$

$$= 10\pi \cos(100\pi t)$$

$$\therefore \oint \vec{E} \cdot \vec{dr} = S \frac{dB}{dt}$$

$$\therefore E \times 2\pi r = \pi r^2\, 10\pi \cos(100\pi t)$$

$$\therefore E \times 2 = 10\pi \times 1 \cos(100\pi t)$$

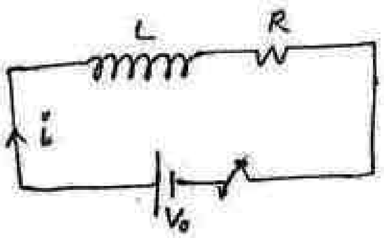
$$E = 5\pi \cos(100\pi t)$$

$$\therefore E_0 = 5\pi \text{ V/m}$$

Growth and decay of current

a) Growth of current:

i) circuit



ii) eqⁿ ⇒ $i = i_0 \left(1 - e^{-Rt/L}\right)$ i.e $i \uparrow$ exponentially

iii) voltage across the inductor / emf induced :

$$e = -L\frac{di}{dt} = -L\left[i_0\left(0 - e^{-Rt/L} \times \left(-\frac{R}{L}\right)\right)\right]$$

$$= -i_0 R e^{-Rt/L}$$

$$\boxed{e = -V_0 e^{-Rt/L}} \qquad \{V_0 = i_0 R\}$$

iv) Just after switch is closed.

> at $t = 0$ $\therefore i = i_0(1 - e^0)$ $\therefore i = 0$ (min) [Initial state]

> $V = -V_0$ (max) and

It behaves like an open circuit / Broken wire

> at $t = \tau = \frac{L}{R}$ = time constant [Transient state]

$$\because i = i_0\left(1 - e^{-\frac{R}{L}\times\frac{L}{R}}\right) = i_0(1 - e^{-1})$$

$$i_0(1 - 0.37) = \boxed{0.63\, i_0}$$

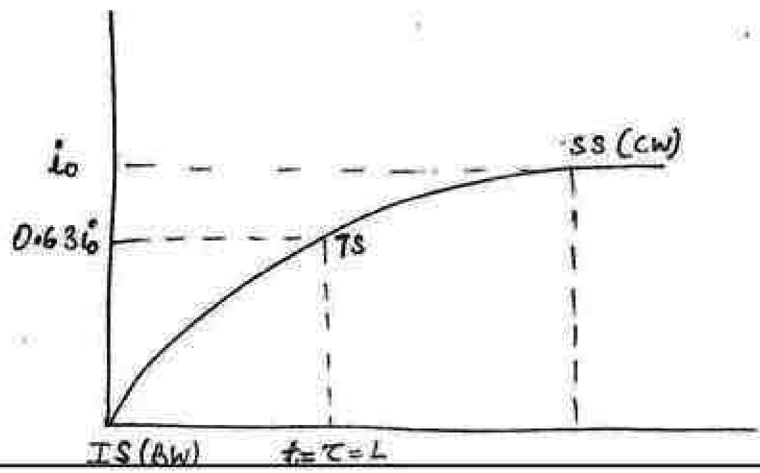
$$\left\{ \frac{1}{e} = 0.37, \quad \frac{1}{e^2} = 0.13 \right.$$

$V = -0.37\, V_0$

> long after S is closed [steady state / final state]

$t = \infty$, $i = i_0 = \frac{V_0}{R}$ $\therefore V = 0 = e$

Behaves like connecting wire (CW)



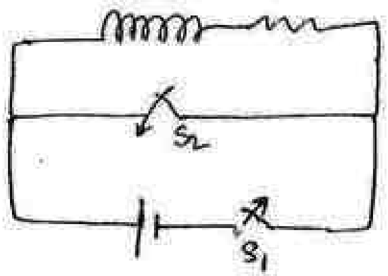
BW = broken wire

CW = connecting wire

IS = initial state

SS = steady state

B) Decay of current:

i) 

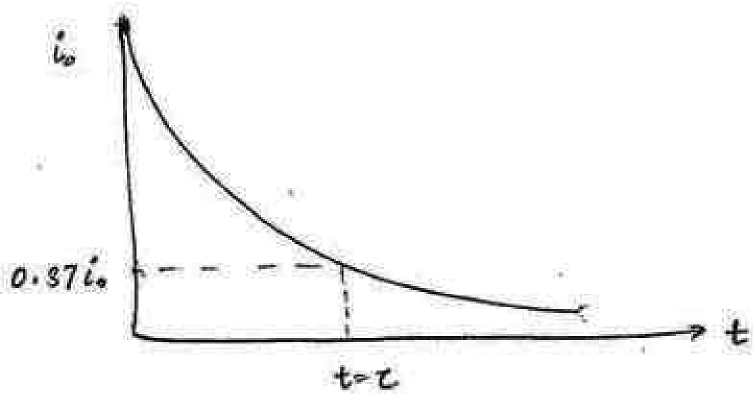
when switch S_1 is opened and S_2 is closed then due to continuous conversion of magnetic energy into electrical energy and then heat energy. current ↓es exponentially in the circuit

> $i = i_0 e^{-Rt/L}$

> $V = e = -L\frac{di}{dt}$

> $V = V_0 e^{-Rt/L}$

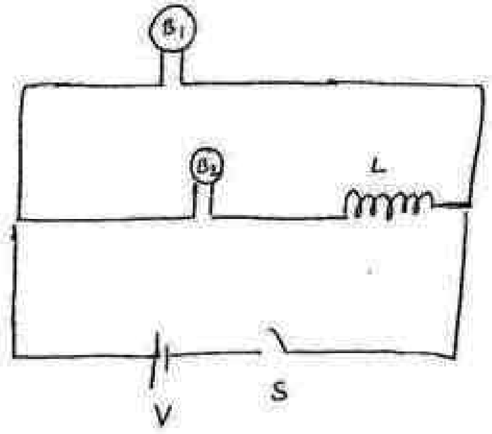
when $t = \tau = \frac{L}{R}$; $i = 0.37\, i_0$



Q) In the given diagram, discuss the brightness of identical bulb.

i) when switch is closed

ii) when switch is open



i) when S is closed,

at $t=0$, $L \rightarrow$ ~~BW~~ Broken wire

$i_{B_1} \neq 0$ $\quad i_{B_2} = 0$

then brightness and becomes same as bulb B_1

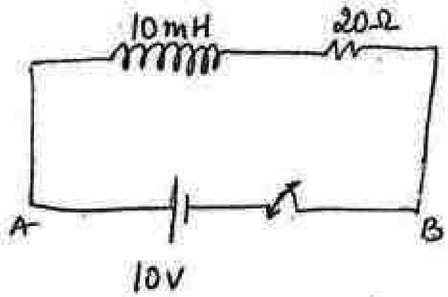
ii) when S is opened

Brightness of both of bulb $\downarrow$es with the same intensity due to decay of current

iii) when iron core is inserted, then $\mu \uparrow \therefore L \uparrow$

$\boxed{\tau = \frac{L}{R}} \uparrow$. so Bulb B_2 will have same max brightness but will take longer time to attain it

2) In the given diagram, find the time after which current becomes $\frac{1}{n}$ times of max value when switch is closed.



$$i = i_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\frac{i_0}{n} = i_0\left(1 - e^{-t/\tau}\right)$$

$$e^{-t/\tau} = 1 - \frac{1}{n} = \frac{n-1}{n}$$

$$\therefore -\frac{t}{\tau} = \log_e\left(\frac{n-1}{n}\right)$$

$$\therefore \boxed{t = \tau \ln \frac{n}{n-1}}$$

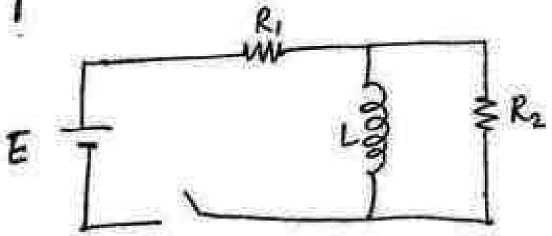
Q) In above que Point A and B are directly connected by a connecting wire then find current in the circuit after 1 ms

$$i = i_0 e^{-Rt/L} \quad (\text{in decay})$$

$$= \frac{V_0}{R} e^{-Rt/L} = \frac{10}{20} e^{-\frac{20 \times 1 \times 10^{-3}}{10 \times 10^{-3}}} = \frac{1}{2} e^{-2}$$

$$\therefore \frac{1}{2} \times 0.13 = 0.065 \text{ A}$$

Q) Find current through the battery in the given circuit. i) just after switch is closed and ii) long after switch is closed



I - 10, 15, 42, 47, 61

II - 15, 16, 21, 22 (14-30). 32, 64

i) $t = 0$, L = broken wire

$$\therefore i = \frac{E}{R_1 + R_2}$$

ii) $t = \infty$, L = connecting wir

R_2 = short circuit

$$\therefore i = \frac{E}{R_1}$$