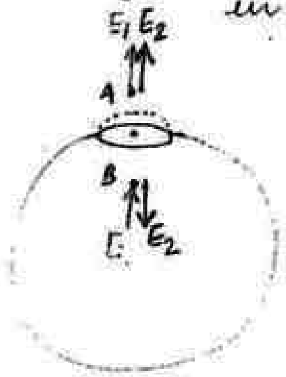
Apply G.T.

$$\oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{2\pi \rho_0 r^2}{\epsilon_0}$$

$$\therefore E = \frac{\rho_0}{2\epsilon_0}$$

Q. 29
NCERT A hollow charge conducting sphere has a tiny hole cut on its surface. Prove E in the hole is $\frac{\sigma}{2\epsilon_0}\hat{n}$




Consider a point A and B just above and below the cut portion

$\therefore$ here, $E_1$ remains same ($\because$ its a negligible change ~~as~~ for the remaining sphere)

while $E_2$ reverses its direction $\because$ it's a considerable change for the small cut portion)

$$\therefore E_A = E_1 + E_2 = \frac{\sigma}{\epsilon_0} \text{ (just outside the sphere)}$$

$$E_B = (+) \; E_1 - E_2 = 0 \text{ (opp. dir}^n\text{)}$$

$$\therefore 2E_1 = \frac{\sigma}{\epsilon_0} \quad \therefore \boxed{E_1 = \frac{\sigma}{2\epsilon_0}}\hat{n}$$

$\because$ $E_1$ = field of remaining part

$E_2$ = field of cut part

hence proved

## Article 5 — Electric Potential (V)

** $\boxed{W_{net} = \Delta K}$ ⇒ work energy theorem

** $\boxed{W_{c.f} = -\Delta U}$ c.f = conservative field

Always applicable

$$F_{c.f} = \frac{-dU}{dr}$$

$$\int \vec{F}_{c.f} \cdot \vec{dr} = \int -\frac{dU}{dr} \quad \therefore \quad W_{c.f} = -\Delta U$$

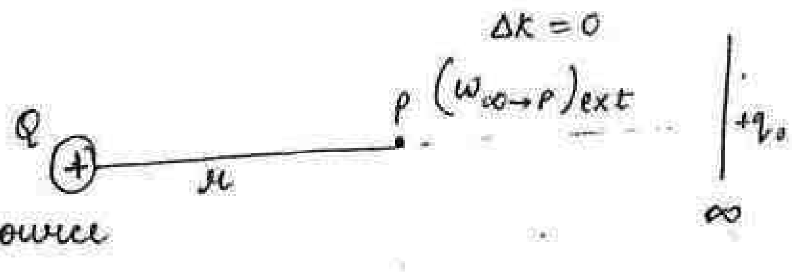
It means work done by conservative field is equal to loss in its potential energy

$$\therefore \; W_{ext} + W_{c.f} = \Delta K$$

$$W_{ext} - \Delta U = \Delta K$$

» If no external agent, then $\boxed{-\Delta U = +\Delta K}$

i.e loss in potential energy = gain in K.E.



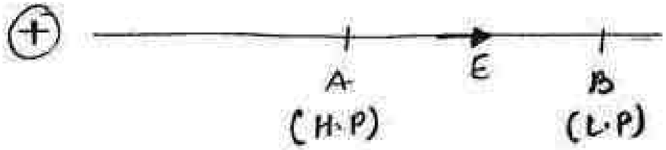

$$V_{P\infty} = V_P = \frac{(W_{\infty \to P})_{ext}}{q_0}, \; \Delta K = 0 \; = \frac{-(W_{\infty \to P})_{c.f}}{q_0} =$$

$$\frac{\Delta U}{q_0} = \frac{U_P - U_\infty}{q_0} = \frac{U_P}{q_0}$$

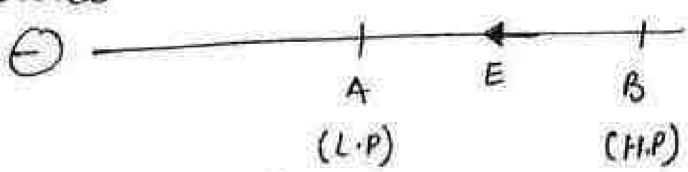
- Electric potential is defined as work done by external agent to bring a unit ⊕ve charge from infinity to point of study such that no change in its K.E.

- V →
  - scalar
  - Relative term
  - SI unit : $\boxed{\text{volt}} = \boxed{J/C}$
  - $[V] = \dfrac{M^1L^2T^{-2}}{A^1T^1} = [M^1L^2A^{-1}T^{-3}]$

- Source

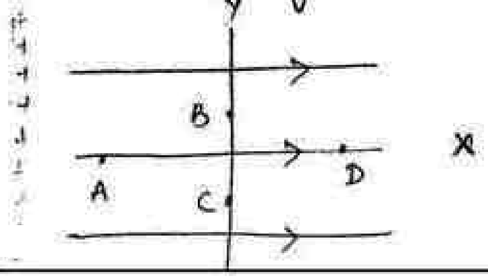



source



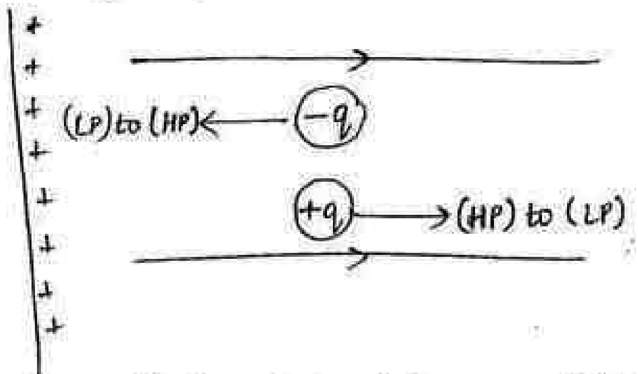

~~Electric~~ It means dir^n of electric field is always from high potential (H.P) to low potential (L.P) irrespective of nature of source charge.

Q)




$\therefore V_A > (V_B = V_C) > V_D$

- If a ⊕ve charge is released in an electric field then its motion will be towards ~~tow~~ low potential point but if released charged is ⊖ve charged, then its motion will be towards high potential point.




But in this example, ~~x~~ ⊖ve and ⊕ve both particles are moving towards their low potential energy.

Relation b/w E and V

$$F_{c.f} = \frac{-du}{dr}$$

$$\frac{F_{c.f}}{q_0} = \frac{-\left(\frac{du}{q_0}\right)}{dr}$$

$$\therefore \boxed{E = -\frac{dV}{dr}} = (-)\text{ Potential gradient}$$

$$dV = -\vec{E} \cdot \vec{dr}$$

$$\int^{V_f} dV = -\int^{r_2} \vec{E} \cdot \vec{dr}$$

$$\boxed{V_f - V_i = -\int_{r_1}^{r_2} \vec{E}\cdot\vec{dr}}$$

» where $\vec{dr}$ is always radially outward and limits will decide direction of displacement

$$W_{c.f} = -\Delta U$$
$$= -(U_f - U_i)$$
$$= -q_0(V_f - V_i)$$

$$W_{ext} = \Delta U \quad (\text{if } \Delta K = 0)$$
$$= q_0(V_f - V_i)$$

» All values should be ~~re~~placed with proper sign.

* If agent of doing work is not mentioned, then it will be considered as work done by external agent.

Q) A ($-10\,V$)      B ($-30\,V$)

calculate work done in moving an $e^-$ from A to B.

$$\therefore W_{ext} = \Delta U \quad (\text{if } \Delta K = 0)$$
$$= q_0(V_f - V_i)$$
$$= -e(-30+10)V = -20\,eV$$

Q)

A $\quad\quad\quad\quad\quad\quad$ B

$-10V$ $\quad\quad\quad\quad$ $-50V$

field + ext agent

calculate work done in <u>moving</u> an $\alpha$-particle from A to B

$$w_{ext} = q_0 (V_f - V_i)$$

$$= +2e(-50+10)V$$

$$= +2e(-40)V$$

$$= -80\,eV$$

If in the above que, $\alpha$-particle is <u>released</u> at point A, then at point B its KE will be 80 eV

(∵ released = absence of external field)

i.e.

$$-\Delta U = \Delta K$$

$$-[q_0 (V_f - V_i)] = K_f - K_i$$

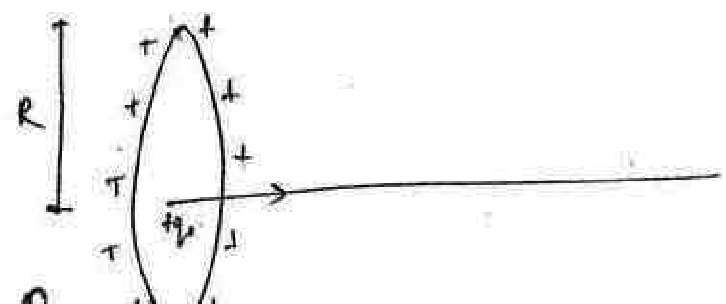
$$-2e[-40V] = K_f - 0$$

$$\therefore K_f = 80\,eV$$

* If particle is released / projected / thrown, then there is no external agent during motion.

so we can use $\boxed{-\Delta U = +\Delta K}$

Q)



Mass of $q_0$ is $m$, it is displaced slightly along the axis and released. Find its speed at very large distance away $\left( \because V_i = \frac{KQ}{R} \right)$

$$-\Delta U = \Delta K$$

$$-q_0 ( V_\infty - V_i ) = K_f - K_i$$

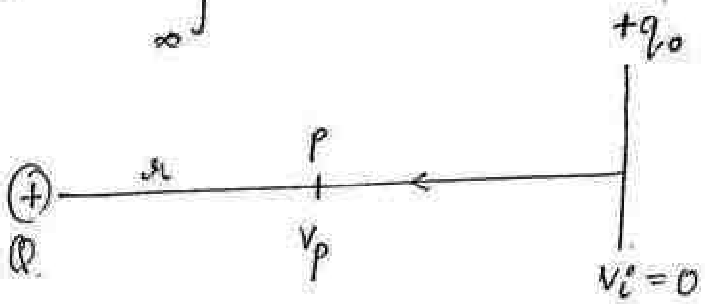
$$-q_0 \left( -\frac{KQ}{R} \right) = \frac{1}{2} m v^2 \qquad \because \begin{pmatrix} V_\infty = 0 \\ K_i = 0 \end{pmatrix}$$

$$\therefore \quad v^2 = \frac{2KQq_0}{Rm}$$

$$\therefore \quad v_{max} = \sqrt{\frac{2KQq_0}{mR}}$$

## Potential of a point charge:

$$V_f - V_i = -\int_{\infty}^{r} \vec{E} \cdot \vec{dr}$$



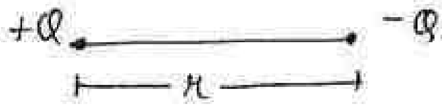

$\because$ $\vec{E} \parallel \vec{dr}$ (but $\vec{E}$ is not constant)

$$\therefore V_f = -\int_{\infty}^{r} \frac{KQ}{r^2} dr = -KQ \left( -\frac{1}{r} \right)_{\infty}^{r}$$

$$V_f = \frac{KQ}{r} - \frac{KQ}{\infty}$$

$$\therefore \boxed{V_p = \frac{KQ}{r}}$$

* Put Q with proper sign.

Q)) 


At midpoint, find E and V

$$E_{midpoint} = \frac{KQ}{\left(\frac{r}{2}\right)^2} \times 2 \quad (+ \text{ to } -)$$

$$= \frac{4KQ}{r^2} \times 2 = \frac{8KQ}{r^2}$$

$$V_M = V_1 + V_2$$

$$= \frac{KQ}{\left(\frac{r}{2}\right)} + \frac{K(-Q)}{\left(\frac{r}{2}\right)}$$

$$\frac{2KQ}{r} - \frac{2KQ}{r} = 0 \quad (\text{sum -ve} \therefore \text{sum +ve})$$

Q)) At a given point, which condition can be obtained

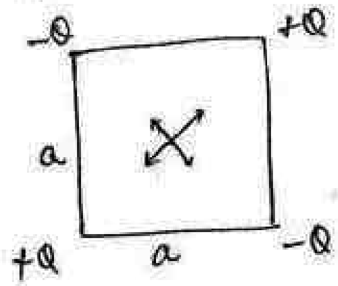
i) $E \neq 0$, $V \neq 0$

ii) $E \neq 0$, $V = 0$

iii) $E = 0$, $V \neq 0$

iv) $E = 0$, $V = 0$

✓ All of these

Q)) 

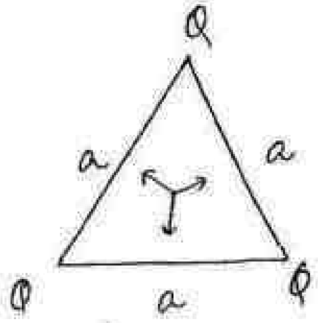

$\therefore E_c = 0$

$V_c = 0 \quad (\because V_c = V_1 + V_2 + V_3 + V_4)$

$= (-\cancel{V} - \cancel{V} + \cancel{V} + \cancel{V})$

$= 0$

Q)




Find E and V at centre

$\therefore E = 0$

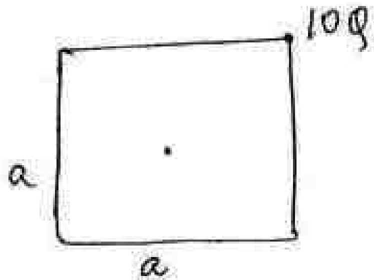
$\therefore V = 3V_1$

$= 3\left(\frac{KQ}{a/\sqrt{3}}\right)$

$= \frac{3\sqrt{3}\, KQ}{a}$

*** If all the charges are at equal distance from the point of study, then we can concentrate whole charge on one of the given point

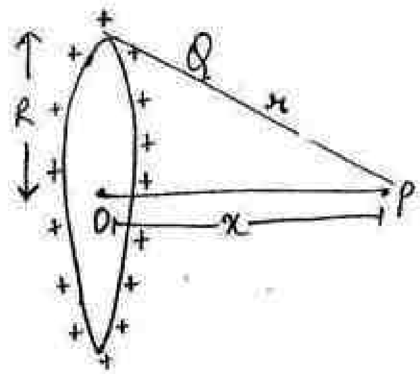
Q)




Find V ?

$V = \frac{10\sqrt{2}\, KQ}{a}$

Q)




Calculate V at centre and at point P.

$V_{centre} = \frac{KQ}{r} = \frac{KQ}{R}$

$V_P = \frac{KQ}{r}$

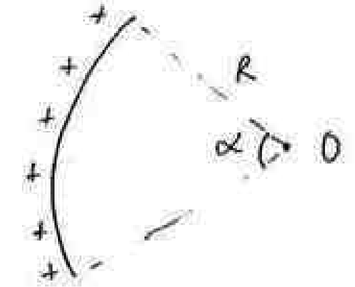
$= \frac{KQ}{\sqrt{R^2 + x^2}}$

$\therefore E = \frac{-dV}{dx}$

get d value of ring

Q)

$\angle CD = \lambda$




Find point at centre of the arc

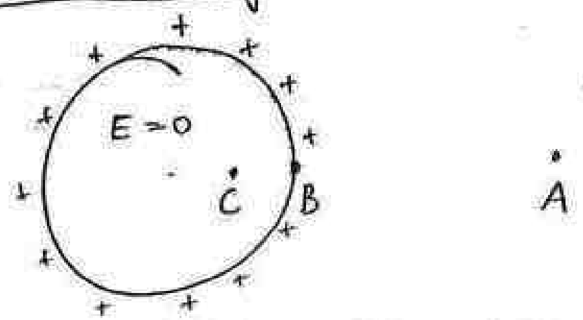
$$V_0 = \frac{KQ}{R}$$

$$= \frac{K(\lambda L)}{R}$$

$$= \frac{K}{R}(\lambda \alpha R)$$

$\alpha^*$ : must be in radian $\quad \therefore \boxed{K\lambda\alpha^*}$

## Potential of Metallic sphere :




$$\boxed{V_A = -\int_{\infty}^{A} \vec{E}\cdot\vec{dr} = \frac{KQ}{r}}$$

$$\boxed{V_B = -\int_{\infty}^{B} \vec{E}\cdot\vec{dr} = \frac{KQ}{R}}$$

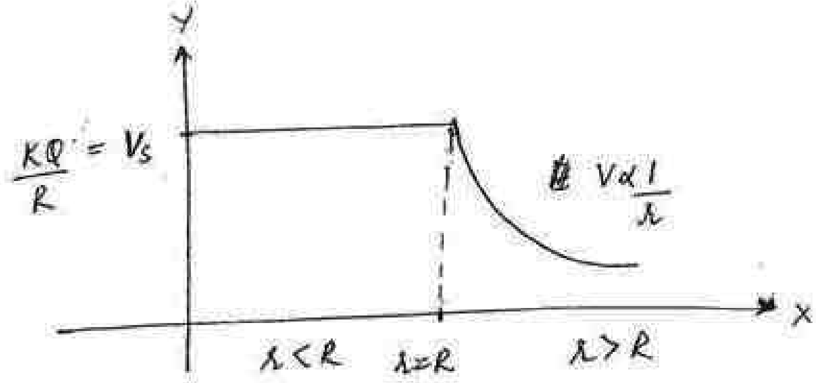
$$V_C = -\int_{\infty}^{B} \vec{E}\cdot\vec{dr} - \int_{B}^{C} \vec{E}\cdot\vec{dr}$$

$$\boxed{V_C = \frac{KQ}{R} - 0 = \frac{KQ}{R}}$$

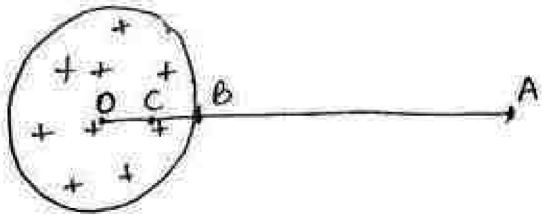
* For inside, $E = \frac{-dV}{dr}$ $\quad \therefore dV = 0$

$V$ = constant

$= \frac{KQ}{R} = V_{surface}$




Potential of uniformly charged sphere.




$r =$ distance from the centre

$$V_A = \boxed{\frac{KQ}{r_A}}$$

$$V_B = \boxed{\frac{KQ}{R}}$$

$$V_C = -\int_{\infty}^{R} \vec{E}_{out} \cdot \vec{dr} - \int_{R}^{r} \vec{E}_{in} \cdot \vec{dr}$$

$$= \frac{KQ}{R} - \int_{R}^{r} \frac{KQr}{R^3}\, dr$$

$$= \frac{KQ}{R} - \frac{KQ}{R^3}\left(\frac{r^2}{2}\right)_R^r$$

$$V_C = \frac{KQ}{R} - \frac{KQ}{2R^3}\left[r^2 - R^2\right] \Rightarrow \frac{3}{2}\frac{KQ}{R} - \frac{KQr^2}{2R^3}$$

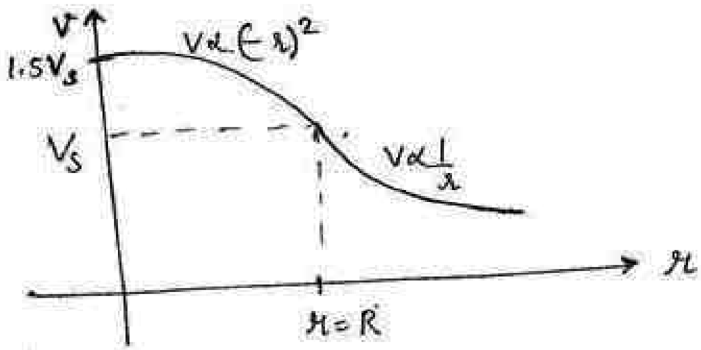
$$\boxed{V_C = \frac{KQ}{2R^3}\left[3R^2 - r^2\right]}$$

conclusion :

i) At the centre $r = 0$

$$\therefore V_{centre} = \frac{3}{2}\frac{KQ}{R} = 1.5\ V_{surface}$$

ii) graph of V v/s r




Q)


$V_A - V_B = ?$

$$\because \quad V_A = \frac{KQ_1}{x} + \frac{KQ_2}{x+d}$$

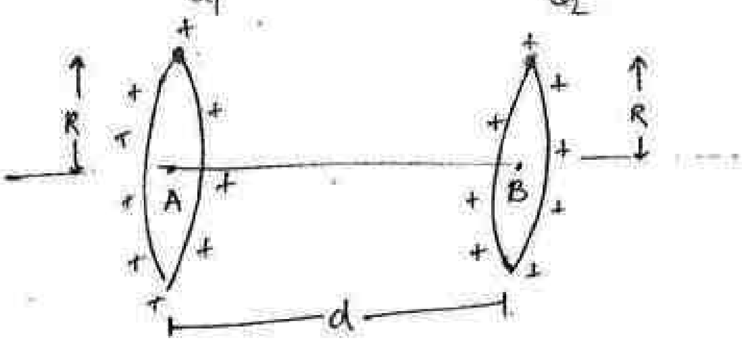
$$V_B = \frac{KQ_2}{x} + \frac{KQ_1}{x+d}$$

$$\therefore V_A - V_B = \frac{K}{x}(Q_1 - Q_2) + \frac{K}{x+d}(Q_2 - Q_1)$$

$$= K(Q_1 - Q_2)\left(\frac{1}{x} - \frac{1}{x+d}\right)$$

$$= \frac{K(Q_1 - Q_2)(d)}{x(d+x)}$$

Q)




Ring find $V_A - V_B$

$$V_A = \left(\frac{KQ_1}{R} + \frac{KQ_2}{r}\right)$$

$$V_B = \left(\frac{KQ_2}{R} + \frac{KQ_1}{r}\right)$$

$$\therefore V_A - V_B = \frac{K}{R}(Q_1 - Q_2) + \frac{K}{r}(Q_2 - Q_1)$$

$$= K(Q_1 - Q_2)\left(\frac{1}{R} - \frac{1}{r}\right)$$

$$= K(Q_1 - Q_2)\left(\frac{r - R}{Rr}\right)$$

$$= K(Q_1 - Q_2)\left(\frac{\sqrt{d^2 + R^2} - R}{R\sqrt{d^2 + R^2}}\right)$$

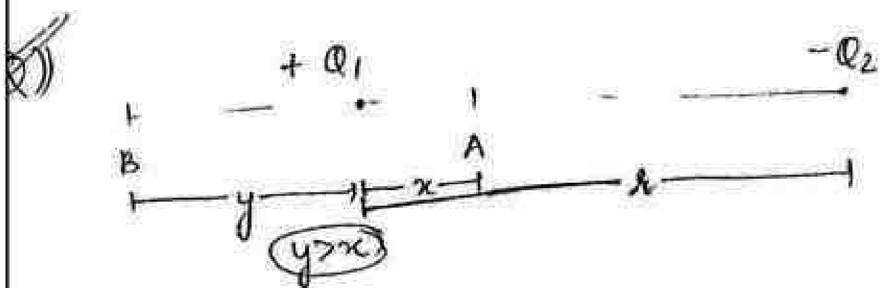
Q) In the above Que calculate work done in moving a point charge from B to A

$$\because \boxed{W = q\Delta V}$$

$$\therefore W = qK(Q_1 - Q_2)\left(\frac{\sqrt{d^2 + R^2} - R}{R\sqrt{d^2 + R^2}}\right)$$

Zero potential point on the line joining $Q_1$ and $Q_2$

- If $Q_1$ and $Q_2$ are like charges, then there will be no zero potential point.
- If $Q_1$ and $Q_2$ are equal and opposite charges then the plane passing through the midpoint and bisect the given line perpendicularly has zero potential at all point.
- $|Q_1| > |Q_2|$ and opposite in nature, then on both sides of small charge, there will be zero potential point.




If point A and B are zero then find value of $x$ and $y$

$$V_A = \frac{KQ_1}{x} - \frac{KQ_2}{(r-x)} = 0 \quad \text{—— (I)}$$

$$V_B = \frac{KQ_1}{y} - \frac{KQ_2}{(r+y)} = 0 \quad \text{—— (II)}$$

from (I) $\quad \frac{KQ_1}{x} = \frac{KQ_2}{r-x}$

$$Q_1(r-x) = xQ_2$$

$$Q_1 r - Q_1 x = Q_2 x$$

$$Q_1 r = Q_2 x + Q_1 x = x(Q_1 + Q_2)$$

$$\therefore \quad x = \frac{Q_1 r}{Q_1 + Q_2}$$

i.e. $$\boxed{x = \frac{|Q_1|\, r}{|Q_1| + |Q_2|}}$$

from eq (11)

$$\frac{KQ_1}{y} = \frac{KQ_2}{r+y}$$

$$\therefore \quad Q_1 r + Q_1 y = Q_2 y$$

$$\therefore Q_1 r = Q_2 y - Q_1 y = y(Q_2 - Q_1)$$

$$\therefore \quad y = \frac{Q_1 r}{Q_2 - Q_1}$$

i.e. $$\boxed{y = \frac{|Q_1|\, r}{|Q_2| - |Q_1|}}$$

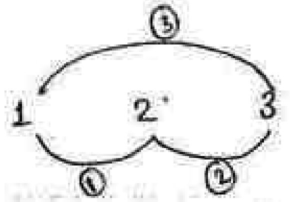
Electrostatic

## Potential energy of the system (EPE)


$\Delta U = -W_{c.f.}$

$\Delta U = -\int_{\infty}^{r} \vec{F}_{cf} \cdot \vec{dr} = \frac{KQ_1Q_2}{r}$ (with sign)

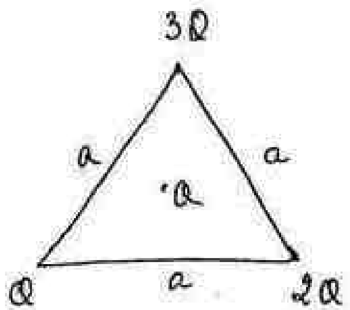
no. of combination




$n$ = no of particles

no of pairs = $\boxed{\frac{n(n-1)}{2}}$

Q)




Find:

i) $U_{system}$

ii) work done in formation of system

iii) work done in putting a point charge $+Q$ at the centroid

i) $U_f = U_1 + U_2 + U_3$

$= \frac{KQ(2Q)}{a} + \frac{K(2Q)(3Q)}{a} + \frac{K(3Q)(Q)}{a}$

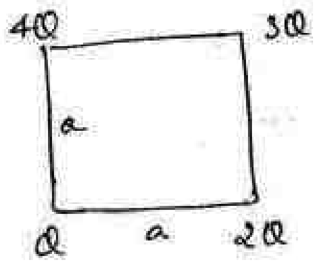
$\frac{11KQ^2}{a}$

ii) $\Delta U \Rightarrow U_f - U_i \Rightarrow W$

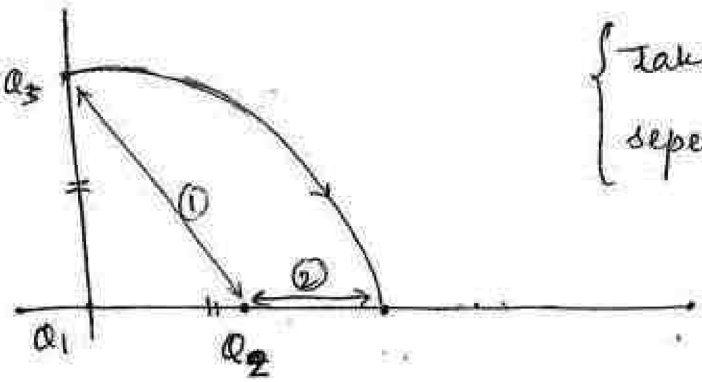
$$\frac{11KQ^2}{a} - 0 = +\frac{11KQ^2}{a}$$

iii) $W = U_f - U_i$

Q)



Q)




{Take pairs in which separation is changed}

① ≠ ②

∵ here ~~po~~ $Q_3$ is shifted on the x-axis in the circular path.

∵ $\boxed{n=3}$ ∴ no of pairs ⇒ $\frac{n(n-1)}{2}$

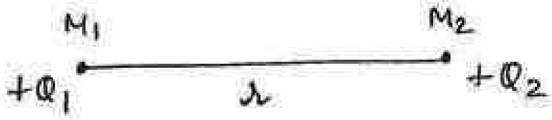
$$= \frac{3(3-1)}{2} = 3$$

here, $Q_1$ and $Q_2$ remains same (before and after shifting).

∴ Relative positions of $Q_1$ and $Q_3$ also remains same (∵ it follows a circular path, $r$ = constant)

∴ Only position of $Q_2$ w.r.t $Q_3$ changes. (considered)

Q)




If both the particles are <u>released</u> then find KE of each particle at ∞.

$$-\Delta U = +\Delta K$$

$$-[U_f - U_i] = K_f - K_i$$

$$-[0 - U_i] = K_f - 0$$

$$\therefore \frac{KQ_1Q_2}{r} = K_1 + K_2$$

$\because K = \frac{p^2}{2m}$ where $p$ is constant

$$\therefore \frac{K_1}{K_2} = \frac{m_2}{m_1} \qquad \therefore K_1 = \frac{m_2 \times K_2}{m_1}$$

$$\cancel{\therefore \frac{K}{2} = m} \qquad K_1 = \frac{m_2}{m_1+m_2} U_i = \frac{m_2}{m_1+m_2}\left(\frac{KQ_1Q_2}{r}\right)$$

$$\therefore K_2 = \frac{m_1}{m_1+m_2} U_i = \frac{m_1}{m_1+m_2}\left(\frac{KQ_1Q_2}{r}\right)$$

Partial Differentiation ie Relation b/w E and V

$$E = \frac{-dV}{dr} = -\text{gradiant } (V)$$

in vector form) : $\vec{E} = -\vec{\nabla} V$

del operator

$$\vec{E} = -\left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right] V$$

Q) $V = (4 - 2x^2)$ volt. calculate E at $x = 2m$

$$\because E = \frac{-dV}{dx} = \frac{-d(4-2x^2)}{dx} = -(-4x) = 4x$$

$$= 4 \times 2 = 8 \frac{V}{m}$$

Q) $V = (xy + 2yz + 3zx)$ volt. Find $|\vec{E}|$ at $(0, 1, 2)$m point.

$$\vec{E} = \frac{-\partial(V)}{dx}$$

$$\vec{E} = -\left[(y+3z)\hat{i} + (x+2z)\hat{j} + (2y+3x)\hat{k}\right]$$

$$= -\left[(7\hat{i}) + (4\hat{j}) + (2\hat{k})\right]$$

$$\therefore |\vec{E}| = \sqrt{7^2 + 4^2 + 2^2}$$

$$= \sqrt{49+16+4} = \sqrt{69}\ \frac{V}{m}$$

Q) $V = (x^2yz + xy^2z + xyz^2)$ volt

If in the above que, a point charge 3C is kept at given point, then find force on the point charge

$$F = q_0 E$$
$$= 3(2) = 6\,N$$

Q) $V = Axy$ (where $A$ = constant, then and $r$ = distance from origin)

~~i)~~ $E \propto r$

ii) $E \propto r^2$

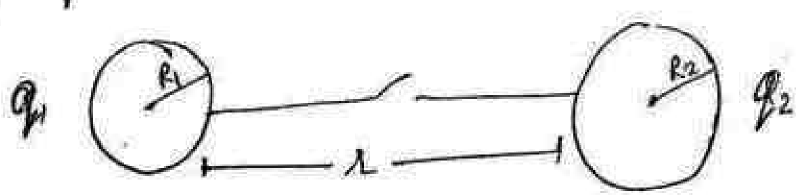
iii) $E \propto 1/r$

iv) $E \propto 1/r^2$

$$\vec{E} = -A\left[(y)\hat{i} + (x)\hat{j}\right]$$

$$|\vec{E}| = A\sqrt{y^2+x^2} = Ar \quad \therefore E \propto r$$

Q) Two metallic spheres are given. Calculate ratio of their ~~p~~. V, Q, E and $\sigma$ (surface charge density after connection).




$\because V_1 = V_2$

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \quad \therefore \boxed{\frac{q_1}{q_2} = \frac{R_1}{R_2}}$$

$\because E = \frac{KQ}{R^2}$ ✗ $= \frac{V}{R}$

$\therefore E \propto \frac{1}{R}$ $\therefore$ $\boxed{\frac{E_1}{E_2} = \frac{R_2}{R_1}}$

$\sigma = \frac{Q}{4\pi R^2}$ ✗ $= \frac{\sigma}{\epsilon_0}$

$$\therefore \boxed{\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} = \frac{d_2}{d_1}}$$

## Equipotential Surface :

i) A surface having equal potential at all the point is an equipotential surface (EPS)

ii) dirⁿ of net electric field must be ⊥ to the surface of EPS

iii) component of electric field ∥ EPS (tangential component) must be zero, It means potential gradiant along the surface must be zero.

iv) 2 EPS never intersect each other because 2 values of potential are not possible at same point.

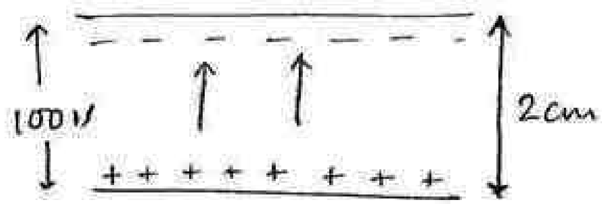
v) $E = -\frac{dV}{dr}$ $dr \rightarrow$ min

~~dirⁿ of net~~

vi) Dirⁿ of net electric field is always along the dirⁿ of greatest potential gradiant and toward ↓ing value of potential

vii) $E = -\left(\frac{\Delta V}{\Delta r}\right)$ can only be used, if field is uniform.

Q) Find E b/w given plates:




$\because$ $E = \frac{\Delta V}{\Delta r} = \frac{100}{2} = 50\ V/cm = 5000\ V/m$

(+ve $\longrightarrow$ −ve)

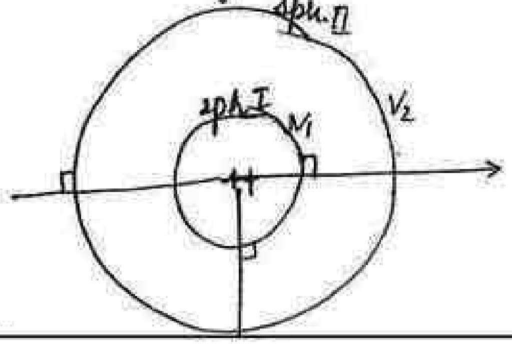
* If a particle is in equilibrium b/w plates then we can use, $mg = qE$

$$\left(\frac{4}{3}\pi r^3 d\right) g = q\left(\frac{\Delta V}{\Delta r}\right)$$

$g = 10\ m/s^2$

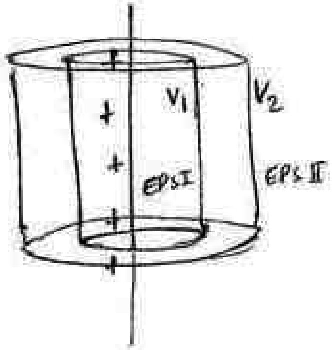
density, $d = 1000\ kg/m^3$

i) EPS of a point charge.



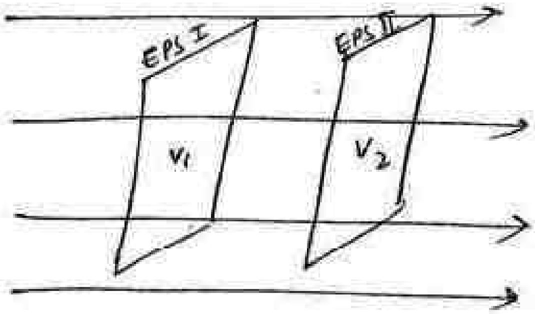

$V_I > V_{II}$

ii) EPS of infinite wire




$V_1 > V_{II}$

iii) EPS of uniform electric field.

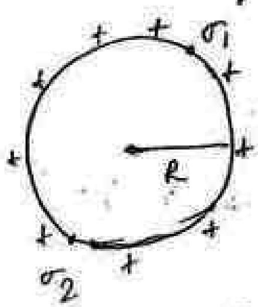


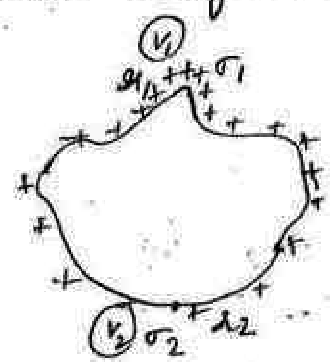

$V_I > V_{II}$

Work done in moving a point charge b/w 2 points on same EPS must be zero.

A charged metallic surface is an ~~eqp~~ example of equipotential surface because field outside the metallic surface is $\perp$ to the surface.

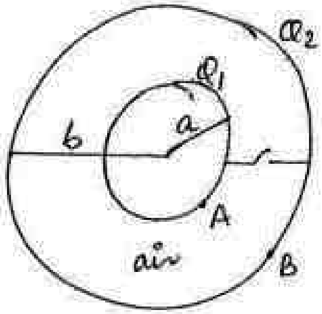
eg




$r_2 > r_1$ // $\sigma_1 = r_2$

Q) Two concentric metallic shells are given.



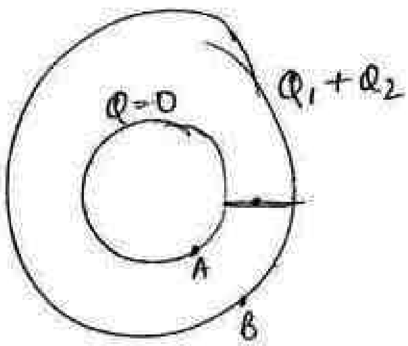

After closing the switch, find charge on inner and outer shell.

$\therefore \quad V_A > V_B$

so charge will flow from inner shell to outer shell, finally, charge on inner shell $\cancel{Q_1} = 0$ and charge on outer shell $= Q_1 +$

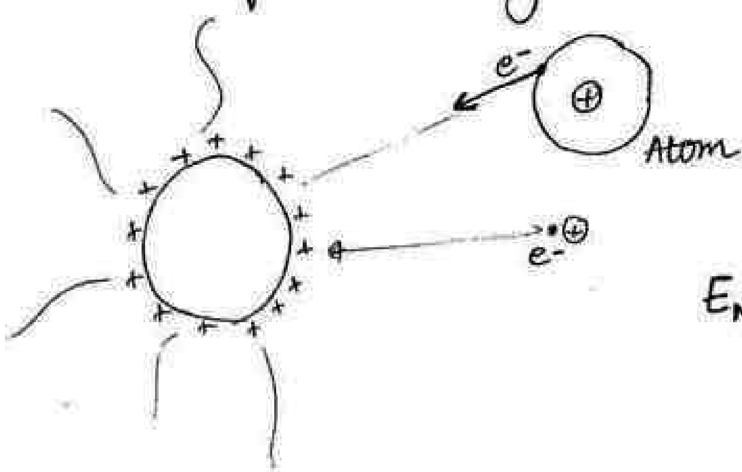
Now, both the shells are at equal potential

* In this case, 100% charge can be transferred from one to another shell (only case).




$\therefore \quad V_A = V_B$

Breakdown of insulating properties of air




$E_{MAX} = 3 \times 10^6 \ \frac{V}{m}$

$(3 \times 10^6 \; V/m)$

> If very <u>high electric field</u> is created in the air, then ionisation of air takes place and air becomes conducting. This high value is called breakdown strength of air.

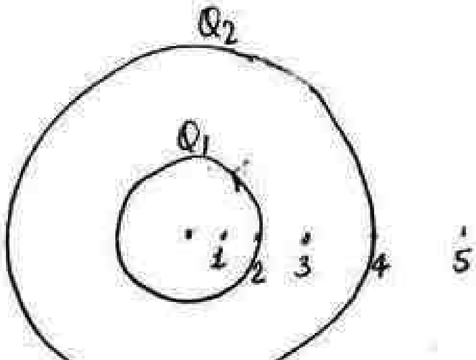
Q) Radius of a metallic sphere is 1 m. Find value of max. charge that can be supplied to this sphere.

$$\because \quad E_{Max} = \frac{KQ}{R^2}$$

$$3 \times 10^6 = \frac{9 \times 10^9 \times Q}{1^2}$$

$$\therefore \quad Q = \frac{1}{3 \times 10^3} = \frac{1}{3} \times 10^{-3} \; C$$

Questions based on ~~metallic~~ concentric <u>metallic</u> sphere




distances from centre are $r_1, r_2, r_3, r_4$ and $r_5$ respectively. Find E and V of each point

remains same when induction is considered

$$E_1 = 0 + 0$$

$$V_1 = \frac{KQ_1}{r_2} + \frac{KQ_2}{r_4}$$

$$E_2 = \frac{KQ_1}{r_2^2} + 0$$

$$V_2 = V_1 = \frac{KQ_1}{r_2} + \frac{KQ_2}{r_4}$$

$$E_3 = \frac{KQ_1}{r_3^2} + 0$$

$$V_3 = \frac{KQ_1}{r_3} + \frac{KQ_2}{r_4}$$

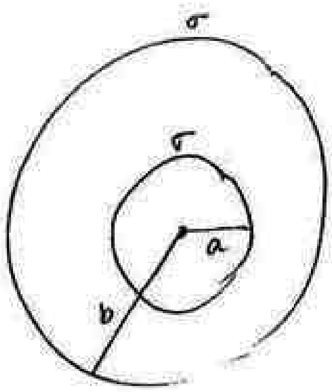
$$E_4 = \frac{KQ_1}{r_4^2} + \frac{KQ_2}{r_4^2}$$

$$V_4 = \frac{KQ_1}{r_4} + \frac{KQ_2}{r_4}$$

$$E_5 = \frac{KQ_1}{r_5^2} + \frac{KQ_2}{r_5^2}$$

$$V_5 = \frac{KQ_1}{r_5} + \frac{KQ_2}{r_5}$$

Q)



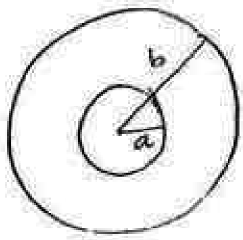

$$V_{centre} = \frac{KQ}{R} = \frac{Q}{4\pi R}$$

$$= \frac{\sigma}{\epsilon_0} R$$

$$V_c = V_1 + V_2 = \frac{\sigma a}{\epsilon_0} + \frac{\sigma b}{\epsilon_0}$$

$$= \frac{\sigma}{\epsilon_0}(a+b)$$

$$E = E_1 + E_2$$
$$= 0$$

Q)) 

A total charge Q is divided on these shells such that there surface charge densities are equal. Find potential (V) at the centre

$$V = \frac{\sigma}{\epsilon_0}(a+b)$$

$$\because \quad Q = Q_1 + Q_2$$

$$= \sigma(4\pi a^2) + \sigma(4\pi b^2) = Q.$$

$$4\pi\sigma(a^2+b^2)$$

$$\therefore \quad \sigma = \frac{Q}{4\pi(a^2+b^2)}$$

$$\therefore \quad V = \frac{Q(a+b)}{4\pi\epsilon_0(a^2+b^2)}$$

OR

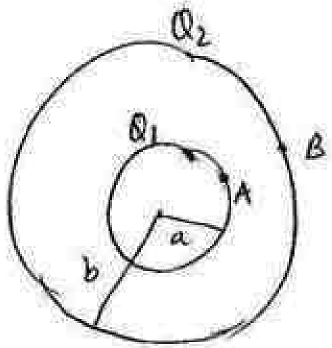
By ratio:
$$\frac{Q_1}{Q_1} = \frac{\sigma A_1}{\sigma A_2} \Rightarrow \frac{a^2}{b^2}$$

$$\therefore \quad Q_1 = \frac{a^2}{a^2+b^2} \times Q$$

$$Q_2 = \frac{b^2}{a^2+b^2} \times Q.$$

$$\therefore \quad V = \frac{KQ_1}{a} + \frac{KQ_2}{b}$$

Q)




$$V_A = \frac{KQ_1}{a} + \cancel{\frac{KQ_2}{b}}$$

$$V_B = \frac{KQ_1}{b} + \frac{KQ_2}{b}$$

$$(-) \qquad (-)$$

---

$$\therefore V_A - V_B = KQ_1\left(\frac{1}{a} - \frac{1}{b}\right)$$

** Potential difference is independant on charge of outershell. If there is any change in the charge of outershell. Potential will change but Potential difference remains same.

Induction in concentric metallic shells:

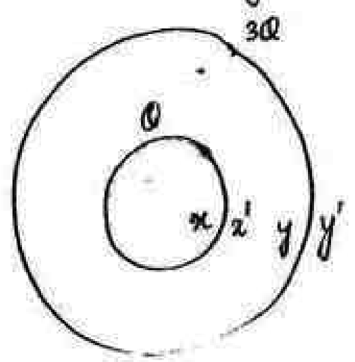
> Net electric field in metallic material must be zero

> Opposite faces of 2 shells must have equal and opposite charges

> Net charge on each shell remains unchanged

> calculate charges from centre to outward.

Q)

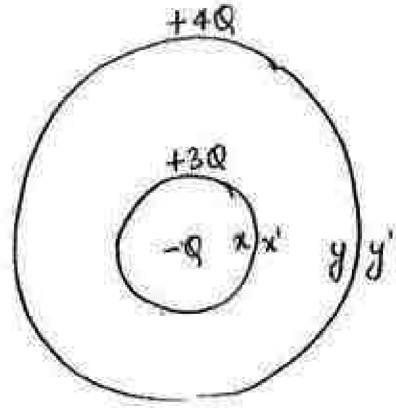



Find

$$\left.\begin{aligned} x &= 0 \\ x' &= Q \end{aligned}\right\} Q$$

$$\left.\begin{aligned} y &= -Q \\ y' &= 4Q \end{aligned}\right\} 3Q$$

Q)




Find

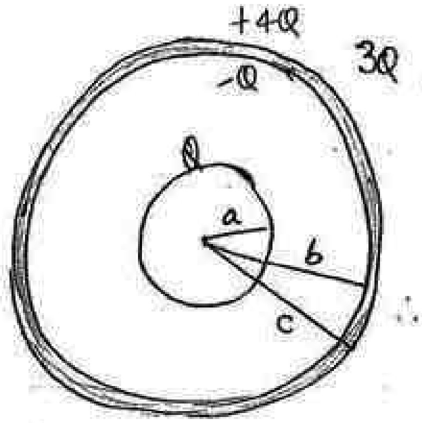
$x = +Q$

$x' = +2Q$

$y = -2Q$

$y' = 6Q$

Q)

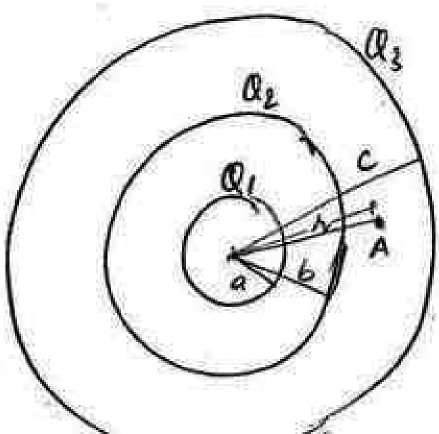



3Q total

$V_{centre} = ?$

$\therefore V_{centre} = \frac{KQ}{a} - \frac{KQ}{b} + \frac{4KQ}{c}$

* If thickness of shell is negligiable, then there is no use of induction for calculation of field and potential.
(Induction applicable only for thick cell)

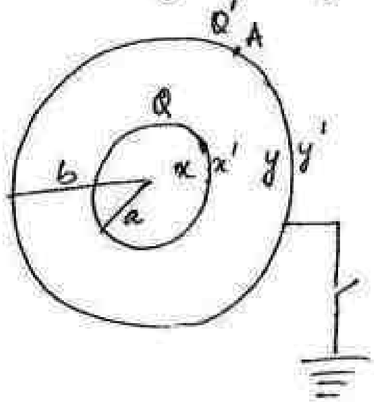



Find $E_A$ and $V_A$ at point A.

Questions based on earthing of metallic sl

If a metallic object is connected through the earth, then its potential becomes zero but its charge may or may not be zero.

Q)




Find value of $x, x'$, $y$ and $y'$ after earthing.

Let the charge of outer surface be $Q'$

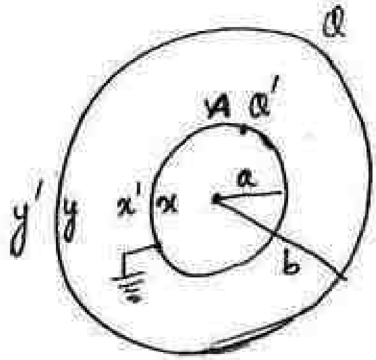
$$\therefore V_A = \frac{KQ}{b} + \frac{KQ'}{b} = 0$$

$$\therefore Q' = -Q$$

$$\therefore \left.\begin{array}{l} x = 0 \\ x' = Q \end{array}\right] Q$$

$$\left.\begin{array}{l} y = -Q \\ y' = 0 \end{array}\right] -Q$$

Q)




Total charge on outer shell is Q.

Find $x$, $x'$, $y$, $y'$

$$\therefore V_A = \frac{KQ'}{a} + \frac{KQ}{b} = 0$$

$$\therefore Q' = \frac{-a}{b} Q$$

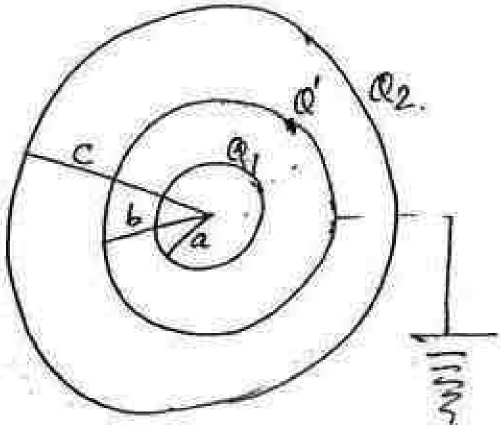
$x = 0$

$x' = Q' = -\frac{a}{b}Q$

$y = +\frac{a}{b}Q$

$y' = Q - \frac{a}{b}Q.$

Q))




Find net charge on earth sphere and distribution of charge on all surfaces.

$$\frac{KQ_1}{b} + \frac{KQ'}{b} + \frac{KQ_2}{c} = 0$$

$$\therefore \frac{KQ'}{b} = -K\left(\frac{Q_1}{b} + \frac{Q_2}{c}\right).$$

$$\therefore \frac{Q'}{b} = -\frac{Q_1}{b} - \frac{Q_2}{c}$$

$$\therefore Q' = -Q_1 - \frac{b}{c}Q_2$$

$\therefore x = 0 \quad , \quad x' = Q$

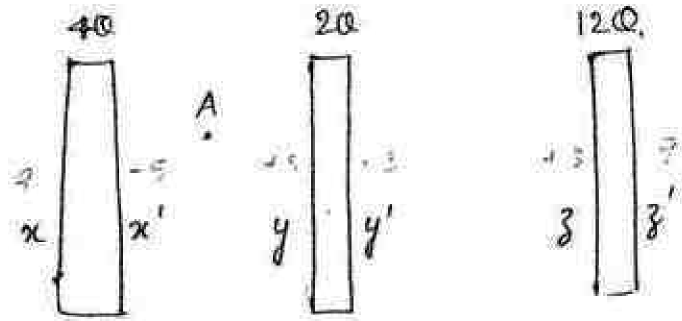
$y = -Q \quad , \quad y' = -\frac{b}{c}Q_2$

$z = \frac{b}{c}Q_2 \qquad z' = Q_2 - \frac{b}{c}Q_2$

Induction in parallel metallic plates:

> half of total charge of system will appear on each outermost surface of system

a))




Face area of each plate = A

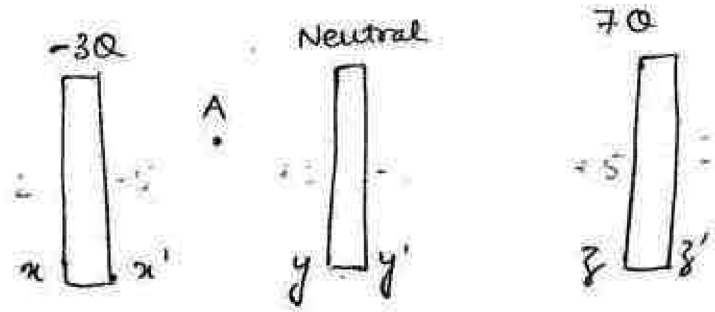
Find $x, x', y, y', z, z'$ and field at point A [E,

$x = 9Q$ $\quad$ $y = +5Q$ $\quad$ $z = +3Q$

$x' = -5Q$ $\quad$ $y' = -3Q$ $\quad$ $z' = +9Q$

$$\therefore E_A = \frac{\sigma}{\epsilon_0} = \frac{5Q}{A\epsilon_0} \qquad \left(\because \sigma = \frac{5Q}{A}\right)$$

c))

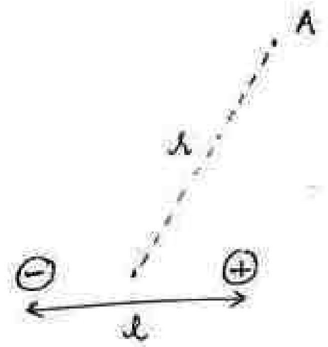



$x = 2Q$ $\quad$ $y = +5Q$ $\quad$ $z = +5Q$

$x' = -5Q$ $\quad$ $y' = -5Q$ $\quad$ $z' = 2Q$

$$\therefore E_A = \frac{\sigma}{\epsilon_0} = \frac{5Q}{A\epsilon_0}$$

## Article : 6 Electric dipole :




Conditions for a dipole

i) $|-q| = |+q|$

ii) separation << distance

$l << r$

$r >> l$

Dipole moment ($\vec{p}$)

$|\vec{p}| = \underline{\text{charge} \times \text{seperation}}$

$= q \times l$

Dir$^n$ is from ⊖ to ⊕ pole

SI unit : Cm

* $\boxed{1 \text{ Debye} = 10^{-18} \text{ esu}}$

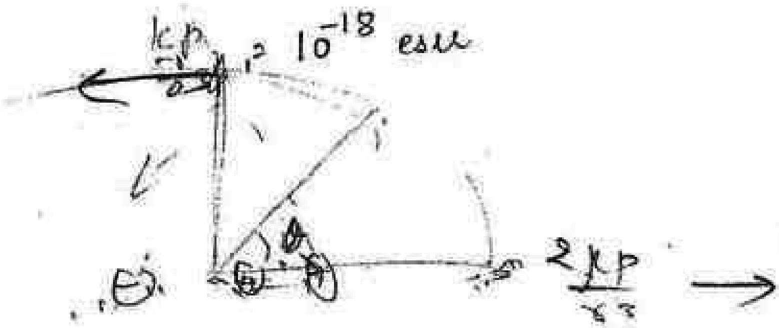
$-10^{-10}$ esu ——— $+10^{-10}$ esu

$A^\circ$

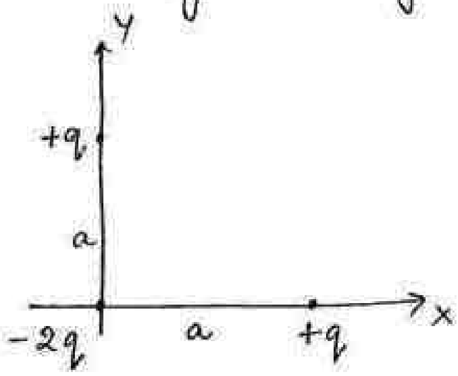
$\therefore$ 1 Debye $= q \times l$

$= 10^{-10} (\text{st C}) \times 10^{-8}$ cm

$= 10^{-18}$ esu

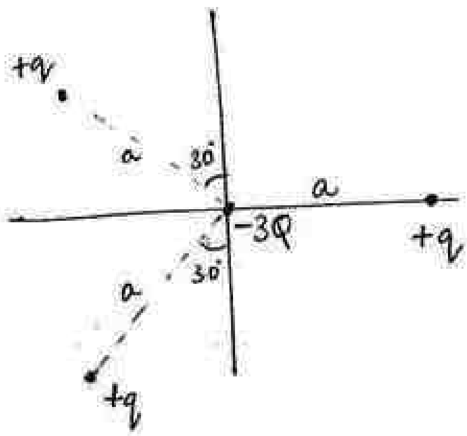


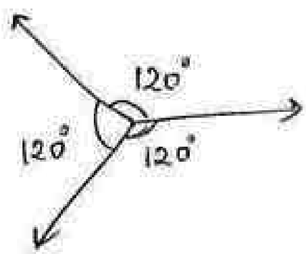
Q) Find magnitude of dipole moment of system




$$|\vec{p}| = \sqrt{2}p$$
$$= \sqrt{2}qa$$

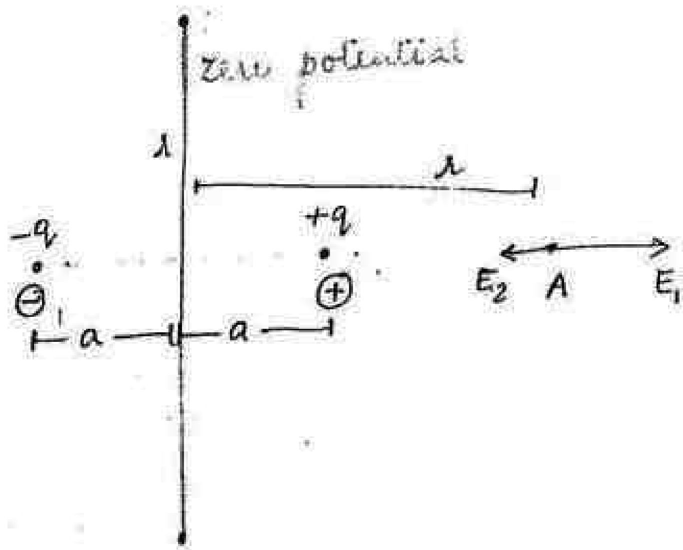
Q)




$\therefore p_{system} = 0$

Field of electric dipole on its axis and on its equator.




$$p = q(2a)$$

At point A,

$$E_A = E_1 - E_2$$

$$= \frac{Kq}{(r-a)^2} - \frac{Kq}{(r+a)^2}$$

$$= Kq\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

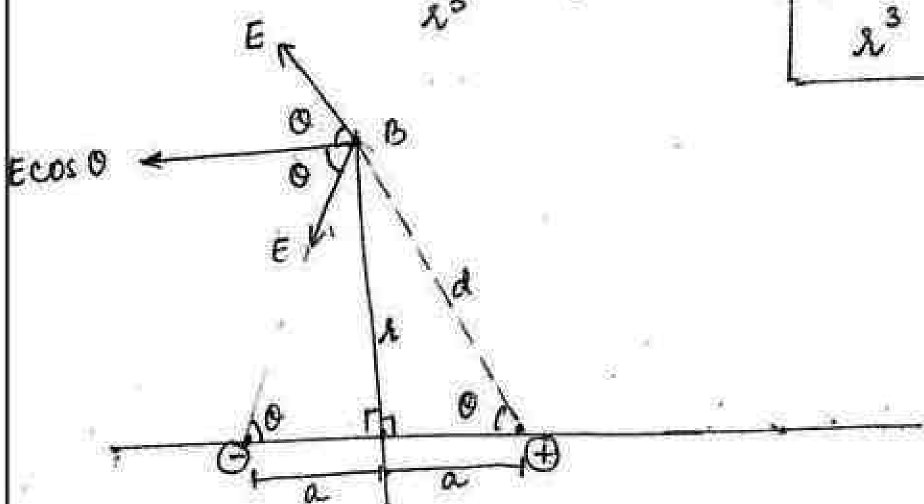
$$= Kq\left[\frac{(r+a)^2 - (r-a)^2}{(r^2-a^2)^2}\right]$$

$$= \frac{Kq(4ra)}{(r^2-a^2)^2}$$

For a dipole : $r >>> a$

$$E_A = \frac{2K(2qra)}{r^4} = \boxed{\frac{2K(p)}{r^3}}$$

$$\therefore \vec{E}_A = \frac{2Kp}{r^3}(\hat{p}) = \boxed{\frac{2K\vec{p}}{r^3}}$$ *** in vector form



At point B,

$$E_B = 2E\cos\theta$$

$$= 2\left(\frac{Kq}{d^2}\right)\left(\frac{a}{d}\right)$$

$$E_B = \frac{K(2qa)}{d^3} = \frac{KP}{(r^2+a^2)^{3/2}}$$

For a dipole, $r >>> a$

$$E_B = \frac{Kp}{r^3}$$

Vector form,

$$\vec{E_B} = \frac{kp}{r^3}(-\hat{p}) = \frac{-kp\hat{p}}{r^3} = \boxed{-\frac{k\vec{p}}{r^3}}$$

Find potential at point A and B.

$$V_A = V_1 + V_B$$

$$= \frac{Kq}{(r-a)} + \frac{K(-q)}{(r+a)} = \frac{Kq}{r-a} - \frac{Kq}{r+a}$$

$$= Kq\left[\frac{1}{r+a} - \frac{1}{r+a}\right]$$

$$= Kq\left[\frac{(r+a)-(r-a)}{r^2-a^2}\right]$$

For a dipole, $r \gg a$

$$V_A = \frac{kq(2a)}{r^2} = \boxed{\frac{kp}{r^2}}$$

$$V_B = V_1 + V_2$$

$$= \frac{KQ}{d} - \frac{KQ}{d} = \boxed{0}$$

Conclusion:

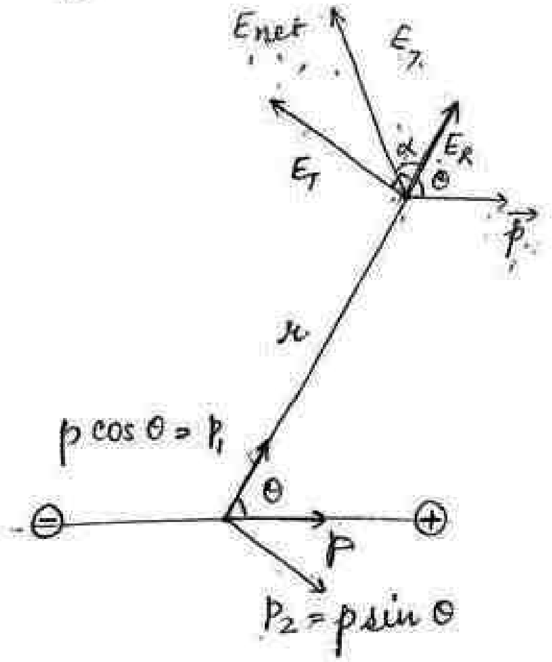
i) Point charge → $E \propto \frac{1}{r^2}$
→ $V \propto \frac{1}{r}$

ii) dipole → $E \propto \frac{1}{r^3}$
→ $V \propto \frac{1}{r^2}$

Field and potential of a dipole decreases rapidly as compared to point charge.

- Field on the axis and field on the equator are anti-parallel vectors.

- If dipole is kept along the x-axis, then y-axis and z-axis will be zero potential line, so work done in moving a point charge field and potential at angle $\theta$ :




$$E_r = \text{radial } \vec{E} = \frac{2kp_1}{r^3} = \boxed{\frac{2Kp\cos\theta}{r^3}}$$

$$E_T = \text{transverse } \vec{E} = \frac{kp_2}{r^3} = \boxed{\frac{kp\sin\theta}{r^3}}$$

$$\vec{E_r} \perp \vec{E_T}$$

$$\therefore E_{net} = \sqrt{E_r^2 + E_T^2} = \frac{kp}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\boxed{= \frac{Kp}{r^3}\sqrt{3\cos^2\theta + 1}}$$

$$\tan\alpha = \frac{E_T}{E_R} = \frac{\sin\theta}{2\cos\theta} = \frac{1}{2}\tan\theta$$

$$\therefore \alpha = \tan^{-1}\left[\frac{1}{2}\tan\theta\right]$$

$$\angle \vec{p} \text{ and } \vec{E} = \theta + \alpha = \theta + \tan^{-1}\left[\frac{1}{2}\tan\theta\right]$$

Potential at point A

$$V_A = V_{p_1} + V_{p_2}$$

$$= \frac{K p_1}{r^2} + 0$$

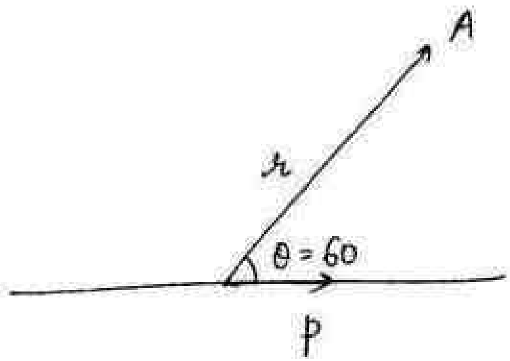
$$\therefore \boxed{V_A = \frac{Kp\cos\theta}{r^2}} \quad *$$

$$\therefore V_A = \frac{K\vec{p}\cdot\hat{r}}{r^2} = \frac{K\vec{p}\cdot\vec{r}}{r^3} \quad [\text{dot product}]$$

$$\vec{A}\cdot\hat{B} = A(1)\cos\theta$$

$$\vec{p}\cdot\hat{r} = p(1)\cos\theta$$

Q)




At point A, what is the angle b/w dipole moment and electric field.

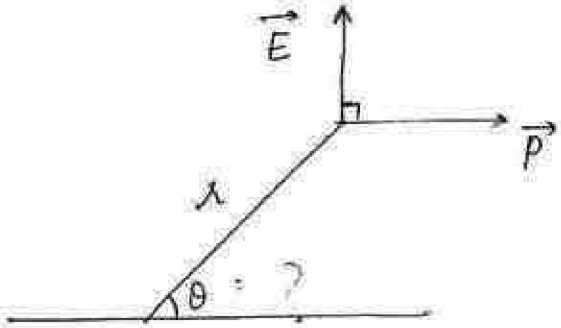
$$\angle \vec{p} \text{ and } \vec{E} = \theta + \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$$

$$= 60^\circ + \tan^{-1}\left(\frac{1}{2}\tan 60^\circ\right)$$

$$= 60^\circ + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\pi}{3} + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Q)




For what value of $\theta$, $\vec{E}$ and $\vec{p}$ are mutually $\perp$

$\because \angle \vec{p}$ and $\vec{E} = \theta + \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$

$$90 = \theta + \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$$

$$\tan(90-\theta) = \frac{1}{2}\tan\theta$$

$$\cot\theta = \frac{1}{2}\tan\theta$$

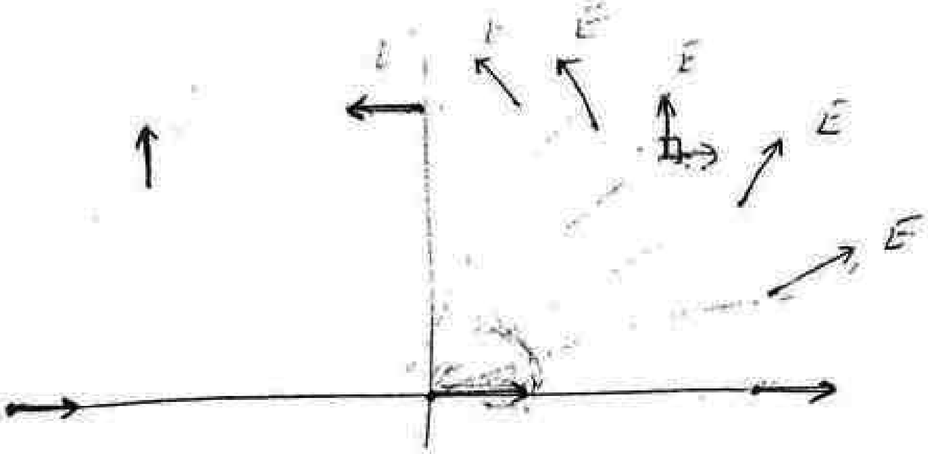
$$\frac{1}{\tan\theta} = \frac{1}{2}\tan\theta$$

$$2 = \tan^2\theta$$

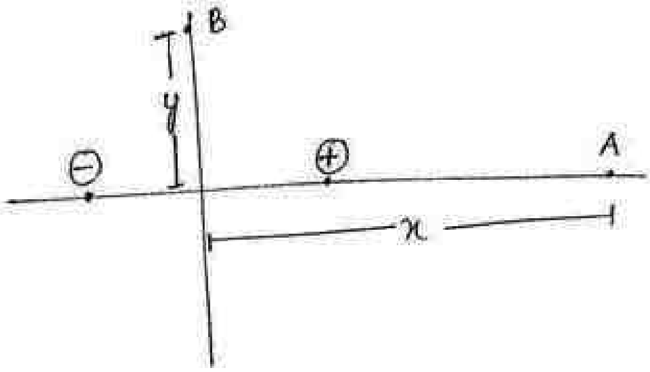
$$\therefore \tan\theta = \pm\sqrt{2}$$

$$\boxed{\therefore \theta = \tan^{-1}(\pm\sqrt{2}) \simeq 56^\circ}$$

Q)



Q)




i) If $x = y$ then $\frac{|E_A|}{|E_B|} = ?$

ii) If $|E_A| = |E_B|$, then $\frac{x}{y} = ?$

i) $\frac{E_A}{E_B} = \frac{2\cancel{Kp}/\cancel{x^3}}{\cancel{Kp}/\cancel{y^3}} = \frac{2}{1}$

ii) $E_A = E_B$

$\frac{2K\cancel{p}}{x^3} = \frac{K\cancel{p}}{y^3}$

$\left(\frac{x}{y}\right)^3 = 2 \quad \therefore \quad \frac{x}{y} = 2^{1/3}$

Q) Force exerted by a dipole on a point charge is F if distance b/w them is doubled, then find force on the point charge.

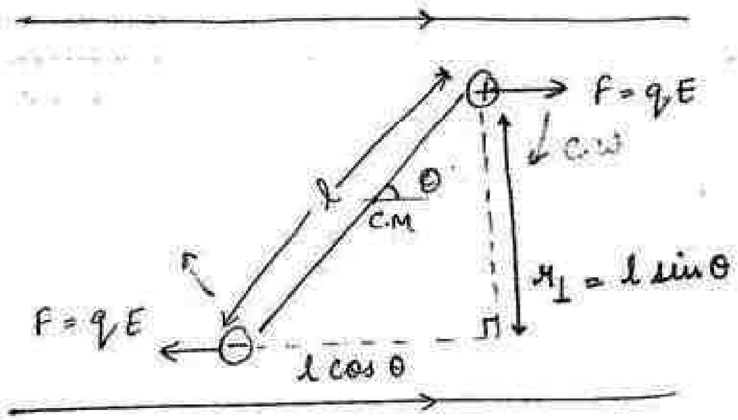
$F = q_0 \, E_{dipole}$

$$q_0 \left( \frac{kp}{r^3} \sqrt{3\cos^2\theta + 1} \right)$$

$\therefore \boxed{F \propto \frac{1}{r^3}}$ $\therefore \boxed{F' = \frac{F}{8}}$

# Dipole is kept in external uniform electric field :

i) Force and Torque on the dipole




$F_{net} = 0$

$\because \boxed{\tau = F \times r_\perp = F \, r \sin\theta}$

$= qE\,(l \sin\theta)$

$= pE \sin\theta$

Conclusion:

> In uniform field, force on the dipole <u>must</u> be zero but torque may or may not be zero so their will be no translatory motion but their may be rotatory motion.

> If $\theta = 0^\circ$
$\tau = 0$ , $F = 0$ i.e. stable Equilibrium.

If $\theta = 180^\circ$
$\tau = 0$ , $F = 0$ i.e Unstable Equilibrium.

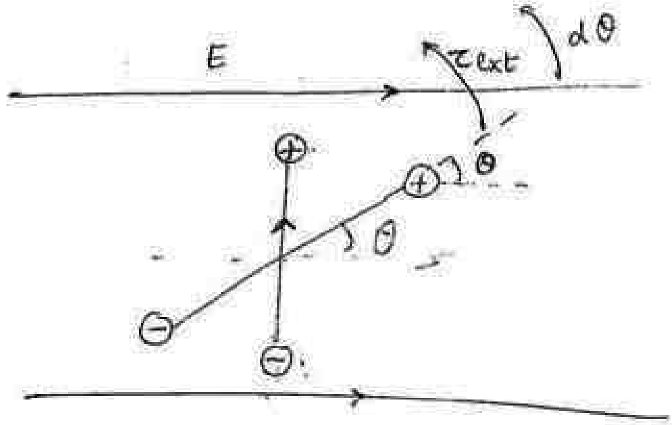
> If $\theta = 90^\circ$, then
$\tau_{max} = pE$

# Potential energy of Dipole:

When dipole and field are $\perp$, then potential energy of dipole is zero so $\theta = 90^\circ$ can be taken as reference position to find P.E at some other angle

$$\theta = 90^\circ$$

$$U_{system} = +qV - qV + \frac{K(q)(-q)}{l} = 0$$




$$W_{ext} = \Delta U$$

$$\int_{90^\circ}^{\theta} \vec{\tau}_{ext} \cdot \vec{d\theta} = U_f - U_i$$

$$\vec{\tau}_{ext} \parallel \vec{d\theta}$$

$$\int_{90^\circ}^{\theta} \tau_{ext}\, d\theta = U_\theta - U_{90^\circ}$$

$$\int_{90^\circ}^{\theta} (pE \sin\theta \, d\theta) = U_\theta$$

$$U_\theta = -pE\,(\cos\theta)_{90}^{\theta}$$

$$\therefore \boxed{U_\theta = -pE\cos\theta}$$

$$\boxed{U_\theta = -\vec{p}\cdot\vec{E}}$$

Conclusion:

> $\theta = 0^\circ$
$U = -pE$ (minimum)
so stable equilibrium.

> $\theta = 90^\circ$
$U = 0$ , $\tau$ max

> $\theta = 180^\circ$
$U = +pE$ (maximum).
so unstable equilibrium.

> work done in rotating dipole from $\theta_1$ to $\theta_2$

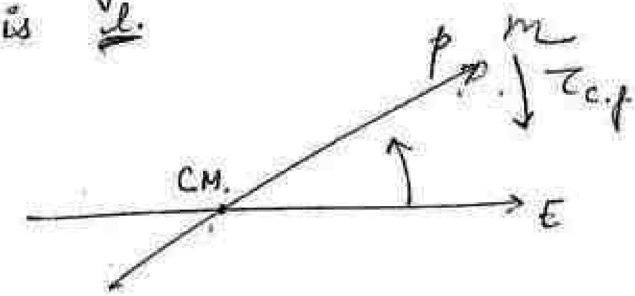
$$W_{ext} = \Delta U = U_f - U_i$$
$$= -pE\cos\theta_2 - (-pE\cos\theta_1)$$
$$= pE(\cos\theta_1 - \cos\theta_2)$$

* direction of $d\theta$ is always anticlockwise and limits will decide dir$^n$ of motion.

* If $\theta_1$ is not given, then consider ~~$\theta$~~ $0^\circ$

Q) A dipole of moment $p$ is kept in uniform field $E$ at angle $= 0^\circ$. Calculate work done in rotating it by an angle
i) $45^\circ$ ii) $90^\circ$ iii) $180^\circ$

Q) A dipole of moment pE is kept in uniform electric field E at angle 0°. Now dipole is given a small angular displacement and released. Find angular frequency of its SH consider mass of each pole is 'm' and length of dipole is $\underline{l}$.




$$\tau = pE \sin\theta$$

$$\frac{\tau}{I_{CM}} = \frac{pE}{I_{CM}} \cancel{\sin} (-\theta)$$

$$\alpha = \frac{pE}{I_{CM.}}(-\theta) = \omega^2(-\theta)$$

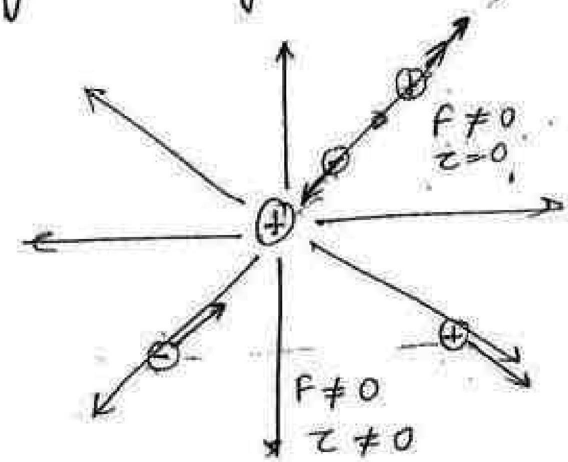
$$\therefore \omega = \sqrt{\frac{pE}{I_{CM}}} = \frac{2\pi}{T}$$

$$I_{CM} = 2m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{2}$$

- Time taken from extreme position to become parallel with the field is $T/4$ and extreme to other extreme position = $T/2$

* If dipole is kept in non uniform electric field, then force and torque may ~~are~~ or may not be zero.

* If dipole is kept in the field of Point Charge, then F can't be zero but torque may or may not be zero.



## Article - 7 Motion of charge particle in uniform $\vec{E}$.

> If electrostatic field is uniform and particle is released / projected, then we can use equation of motion because acceleration is also uniform.

> If type of electrostatic field is not mentioned and if particle is released / projecte then do not use eq of motion. here, we can use $\boxed{-\Delta U = +\Delta K}$

Q) A particle of charge q and mass m is released in an uniform electric field E calculate distance travelled in 1st time t and its KE at the end of time t.

$$\because \quad S = ut + \frac{1}{2}at^2$$

$$= \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

$$\because \quad V = u + at$$

$$= \left(\frac{qE}{m}\right)t$$

$$\therefore \quad KE = \frac{1}{2}mV^2 = \frac{1}{2}m.\left(\frac{qE}{m}\right)^2 t^2$$

(OR)

$-\Delta U = +\Delta K$

$W_{g} = K_f - K_i$

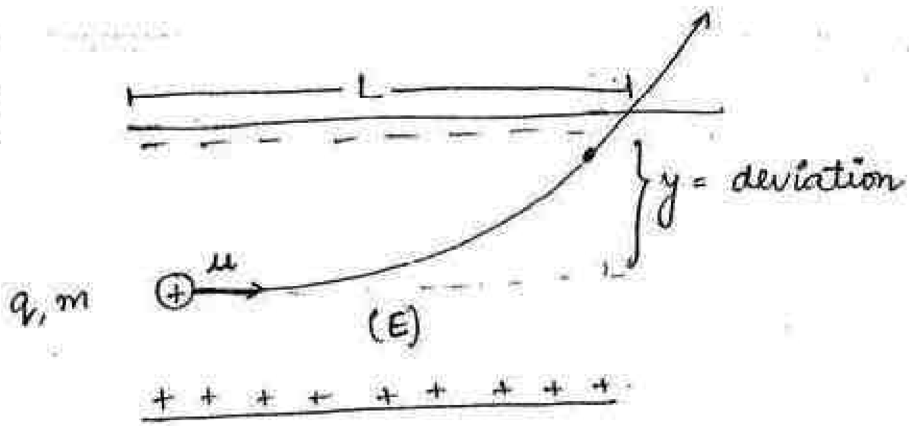
$F \cdot s = K_f - 0 \quad (\because K_i = 0)$

$(qE) \cdot s = K_f$

$qE \cdot \frac{1}{2}\left(\frac{qE}{m}\right)t^2 = K_f$

$\frac{1}{2}\frac{q^2E^2t^2}{m} = K_f$

Q)




calculate deviation in the path of the particle when it is leaving the field.

$y = u_y t + \frac{1}{2}a_y t^2 \quad$ (In y-dirn)

$y = \frac{1}{2}\left(\frac{qE}{m}\right)t^2 \quad (\because u_y = 0)$

(In x-dirn)

$x = u_x t + \frac{1}{2}a_x t^2$

$x = u_x t \quad (\because a_x = 0$ because $u_x =$ constant

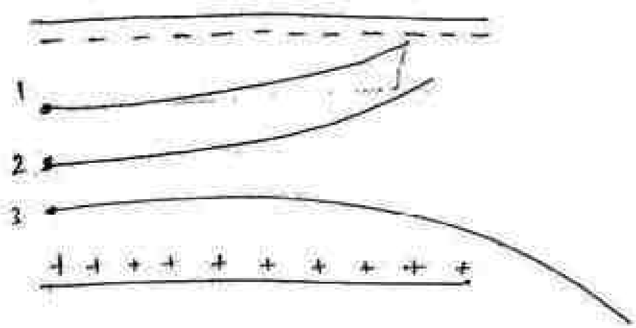
$L = u(t)$

$\therefore t = \frac{L}{u}$

$\therefore y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{u}\right)^2$

i.e. $\boxed{y \propto \frac{q}{m}}$

Q)

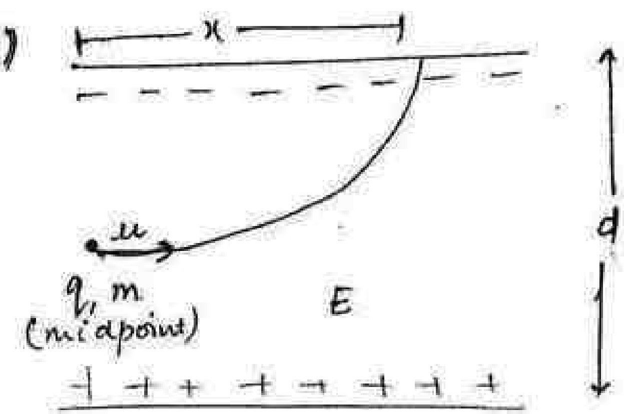



which one has greatest $q/m$?

$\therefore \boxed{y \propto \frac{q}{m}}$

3rd particle has greatest $q/m$ because its deviation is highest.

Q)




Calculate value of $x$

$\because \; x = ut$

$\because \; y = u_y t + \frac{1}{2} a_y t^2$

$$\frac{d}{\cancel{2}} = \frac{1}{\cancel{2}}\left(\frac{qE}{m}\right)t^2$$

$$\therefore \; t = \sqrt{\frac{dm}{qE}}$$

$$\therefore \; x = u\sqrt{\frac{dm}{qE}}$$

OR

$$R = u \sqrt{\frac{2H}{a}} = u\sqrt{\frac{\cancel{2} d/\cancel{2}}{\left(\frac{qE}{m}\right)}}$$

$$\therefore \; u = \sqrt{\frac{dm}{(qE}}$$

Q) Electric field and electric potential on the surface of a small charge drop are E and V respectively. Now, such n drops are combined to form a big drop. calculate value of field and potential on its surface

$V_B = nV_s$

$\because \; R_{big} = N^{1/3} r \qquad \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 n$

$$E_{big} = \frac{K(Nq)}{R^2} = \frac{KNq}{N^{2/3} r^2}$$

$$= N^{1/3} \frac{Kq}{r^2} = N^{1/3} \; E_{small}$$

$$V_{big} = \frac{K(Nq)}{R} = \frac{KNq}{N^{1/3} r}$$

$$= N^{2/3} \frac{Kq}{r} = N^{2/3} \; V_{small}.$$