Electro Statics

Article 1 : Electric charge and its properties.

Charge : It is a property by virtue of that particle experiences and generates electric and magnetic effects.

Elementary charge = e = $\boxed{\pm 1.6 \times 10^{-19}\ C}$ **
(Quanta of charge)

(attra or repulsion)
⊕/⊖ represents nature of charges. It doesn't decide value and direction.

SI unit : Coulomb. (C)

CGS or e.s.u : state coulomb (St C)

e.m.u : Absolute Coulomb (Ab C)

$1\ C = 3 \times 10^{9}\ St\ C$

$10\ C = 1\ Ab\ C$

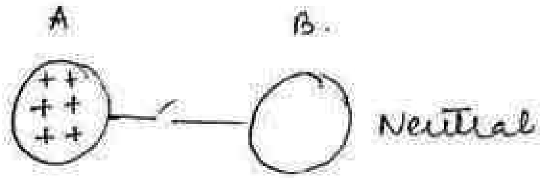
$1\ \text{Faraday} = N_A \times e = 96500\ C$

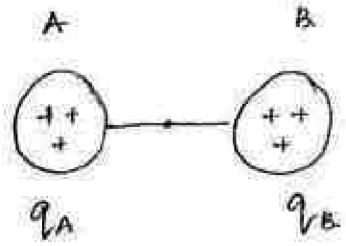
Method of Charging of a body :

i) By friction : when two bodies are rubbed against each other, then few e^-s are transferred from one to another and body becomes charge

*eg: charging of cloud is due to friction b/w its different layers and finally this charge appears in the form of lightening strokes.

ii) By Conduction (especially for metals) : When 2 metallic bodies are connected by a metallic wire then charge can be transferred from one to another.

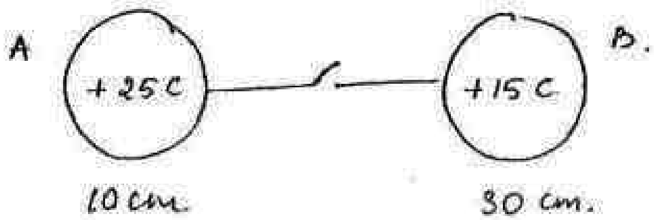




$\therefore$ After connection, $q \propto r$

$$\therefore \boxed{\frac{q_A}{q_B} = \frac{r_A}{r_B}}$$

Q-1



find charge on A and B after connection.

$$q_{system} = q_1 + q_2 = 25 + 15 = 40C$$

$$\therefore \frac{x}{40 - x} = \frac{1\cancel{0}}{3\cancel{0}}$$

$$\therefore \cancel{30x} \quad 3x = 40 - x$$

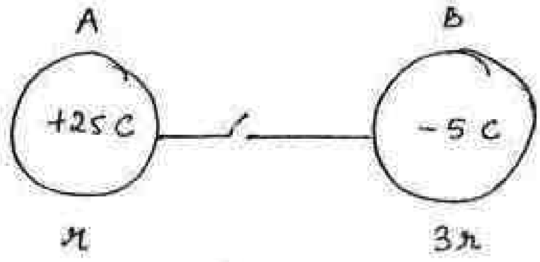
$$\therefore 4x = 40$$

$$x = \boxed{10\ C}$$

$$40 - x = 40 - 10 = \boxed{30\ C}$$

OR $\quad q_A = q_{system}\left(\dfrac{r_A}{r_A + r_B}\right)$

Q.2



After connection find charge on A and B.

$\therefore \; q_A = q_{system}\left(\dfrac{r_A}{r_A + r_B}\right)$

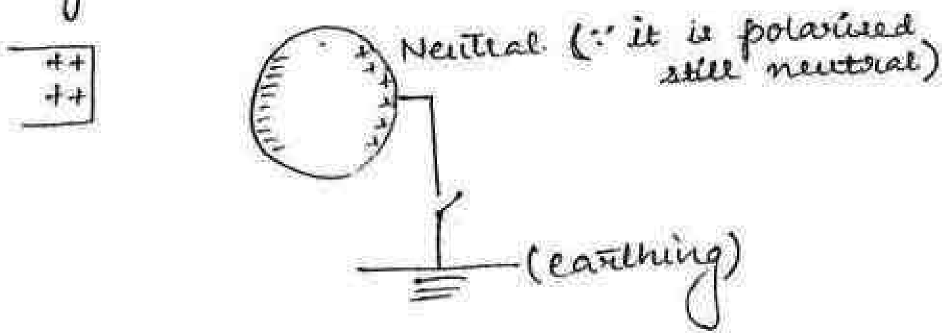
$= 20\left(\dfrac{r}{4r}\right) = \boxed{5\ C}$

$q_B = 20\left(\dfrac{3r}{4r}\right) = \boxed{15\ C}$

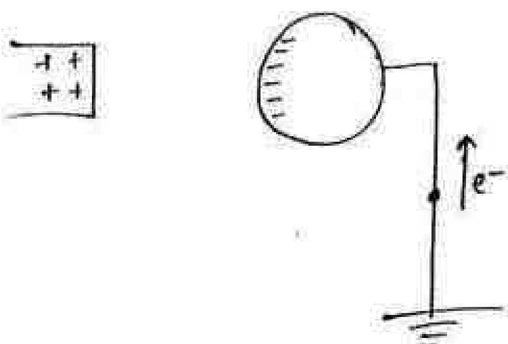
$\because \; q_{system} = +25 - 5 = 20\ C$

iii) Induction (only for metals): In this method, a charged body is brought near to a neutral metallic body but bodies are non-connected.

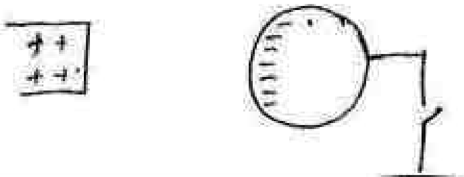
Step I



Step II



Step III



Removal of earth connection.

step IV Removable of source charge

Properties of Charge :

- It is a scaler
- It has additive nature ($\because$ its scaler)
- It is invariant (independant of frame of reference)

* Varient quantities : Mass, length and Time

It means charge is independant of speed. but mass is speed dependant.

given by $m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$

$\therefore$ As $v \uparrow$ $\therefore$ $m \uparrow$

$l \downarrow$

q $t \uparrow$

$t = \dfrac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

- conservation of charge (Net charge of an isolated system remains constant w.r.t Time)

If positive charge is created, then equal amount of $\ominus$ve charge will also appear so net charge remains unchanged.

$\therefore$ $\oplus$ve or $\ominus$ve charge can be created.

$\because$ $\cancel{\not}$ $n \longrightarrow p^+ + \bar{e}^-$

- Charge can appear only in integral multiple of electronic charge(e) it means each and every value of charge is not possible so charge has discrete value (non-continuous)

** $\boxed{Q = Ne}$

where N = integer (1, 2, 3, 4 ...)

but, in daily life, we experience that charge is not quantised because value of its quanta is very small.

Q.3 which charge is possible ?

i) 1.74 C ✓

ii) 1.56×10^{-9} C ✓

iii) 0.23×10^{-7} C ✓

* iv) 4/3 C ✗ $\because \frac{1.\overline{33}}{1.6 \times 10^{-19}}$ *

v) 2.5e ✗

vi) 1.89×10^{-18} C ✗

vii) 1.2×10^{-20} C ✗

viii) 3.2×10^{-18} C ✓

* Fraction of coulomb charge is possible but fraction of electronic charge(e) is not possible.

Very imp. notes:

some particles having charge $e/3$ or $2e/3$ have been discovered. These particles are called quark particle.

six types of quarks have been discovered but most common quarks are up quark $\longrightarrow u = +2e/3$

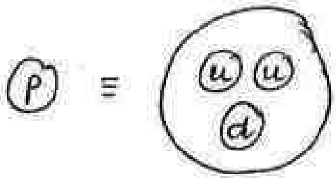
down quark $\longrightarrow d = -e/3$

but there is no free existence of quark particle.

These are found in groups therefore still we are using (e) as a quanta of charge.

Quarks are found in the composition of neutron and proton

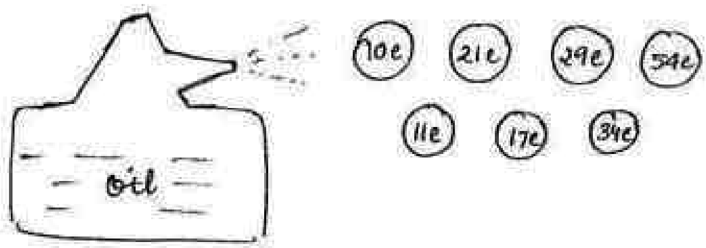
Proton



neutron

Free existence of quark will not spoil quantisation of charge.

* Experimental value of electronic charge was determined by Millikan oil drop experiment.



When fine droplets comes out from the container it becomes charged due to friction and Millikan obtained quanta of charge by taking $\boxed{\text{HCF}}$ of charges on the droplets and its value is e

where $e = 1.6 \times 10^{-19}$ C

Q. If there would be only 3 drops in Millikan experiment having charge 2e, 8e and 10e then what would be quanta of charge

HCF of 2e, 8e and 10e = 2e

Special Points.

> Charge can't exist without mass, but mass can exist without charge

> A neutral body can experience electrostatic attractio force but repulsion will appear when both the bodies are charged so confirmed test of electrification is Repulsion.

Q.5

100 e^-'s are transferred from A to B. find difference in their mass $(m_A - m_B)$

Ball B has gain of 100 e^- and A has loss of 100 e

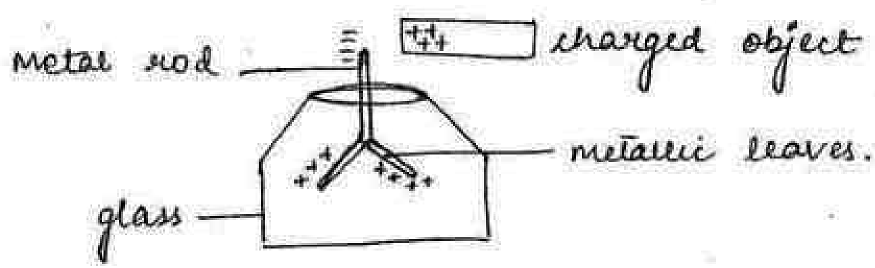
$$\therefore (m_A - m_B) = 200\, m_e$$

$$= 200 \times \boxed{9.1 \times 10^{-31}\ kg}$$

$$= 18.2 \times 10^{-29}\ kg$$

* If a soap bubble is given either a positive or negative charge, then its radius increases due to repulsion b/w various part.

<u>Goldleaf electroscope</u>



This instrument can be used to detect presence of charge and to compare value of charge.

Article 2 : Coulomb's law

$q_1 \bullet \xleftrightarrow{\; r \;} \bullet\, q_2$

$F_e \propto \frac{q_1 q_2}{r^2}$

force on any 1 particle (Electrostatic force)

$$\boxed{F_e = \frac{K q_1 q_2}{r^2}}$$

limitations of coulomb's law :

- q_1 and q_2 must be point charged particles
- If q_1 and q_2 are not point charged then their size be very very less than r

  (size <<< r) means huge radius as compared to size
- If charged is not point charged and r is not very large then use integration approach.
- K is not universal constant. Its value depends upon nature of medium placed b/w particles.

$\boxed{K = 9 \times 10^9 \text{ SI}}$ i.e $9 \times 10^9 \frac{Nm^2}{C^2}$ (vaccum/air)

$$K_{vaccum} = \frac{1}{4\pi \epsilon_0}$$

ϵ (epsilon) = Permittivity of medium

ϵ_0 = Permittivity of vaccum / free space

Permittivity expresses that how easily a medium can be polarised.

$\therefore \epsilon_0 = \frac{1}{4\pi K} = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$ $[\epsilon_{air} \simeq \epsilon_0]$

$[\epsilon_{metals} \simeq \infty]$

for other medium,

$$\therefore \epsilon_r = \frac{\epsilon_{med}}{\epsilon_0}$$

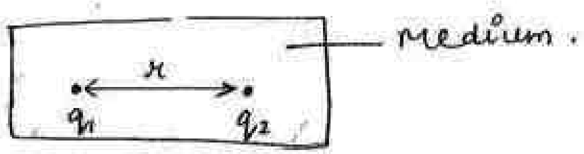
ϵ_r →
- Relative Permittivity
- Specific Permittivity
- dielectric constant

✓ $\epsilon_r = 1$ (for air and vaccum)

✓ $\epsilon_r = 81$ (water)

✓ $\epsilon_r = \infty$ (for all metals)

Force in the medium :



$$\therefore F_{med} = K_{med}\left(\frac{q_1 q_2}{r^2}\right)$$

$$= \frac{1}{4\pi \underbrace{\epsilon_0 \epsilon_r}_{\epsilon_{medium}}}\left(\frac{q_1 q_2}{r^2}\right)$$

$$= \frac{F_{vaccum}}{\epsilon_r}$$

$$\therefore ** \quad \boxed{K_{med} = \frac{K_{vacc}}{\epsilon_r}}$$

Q.6 Two charged particle are kept in the air with seperation r. Force on each particle is F now these particles are placed in a medium of dielectric constant ϵ_r. what should be distance b/w particles so that force on each particle is again F

$$\because F_{air} = F_{Med}$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r^2}\right) = \frac{1}{4\pi\epsilon_0\epsilon_r}\left(\frac{q_1 q_2}{r'^2}\right)$$

$$\frac{1}{4\pi\epsilon_0}\left(\frac{q_1q_2}{r^2}\right) = \frac{1}{4\pi\epsilon_0 \epsilon_r}\left(\frac{q_1q_2}{r'^2}\right)$$

$$\therefore \quad r'^2 = \frac{r^2}{\epsilon_r}$$

$$\therefore \quad \boxed{r' = \frac{r}{\sqrt{\epsilon_r}}}$$

Q.7.

Force on each particle is F. Now 50% charge is transferred from one to another then calculate force exerted on the particle.

$$\because \quad F = \frac{KQ\times Q}{r^2} = \frac{KQ^2}{r^2}$$

$$F' = \frac{K\left(\frac{Q}{2}\right)\left(\frac{3Q}{2}\right)}{r^2} = \frac{3}{4}\frac{KQ^2}{r^2}$$

$$\therefore \quad \frac{F}{F'} = \frac{K\cancel{Q^2}}{\cancel{r^2}}\,\frac{4}{3}\,\frac{\cancel{r^2}}{\cancel{KQ^2}} \qquad \therefore \boxed{F' = \frac{3}{4}F}$$

Q8

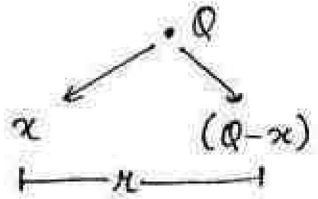
25% charge is transferred from one to another find force on the particle

$$\because \quad F = \frac{KQ\times Q}{r^2} = -\frac{KQ^2}{r^2}$$

$$\therefore \quad F' = \frac{K\left(\frac{3Q}{4}\right)\left(-\frac{3Q}{4}\right)}{r^2} = \frac{9}{16}\frac{KQ^2}{r^2}$$

$$\therefore \quad \frac{F'}{F} = \frac{9}{16}\,\frac{\cancel{KQ^2}}{\cancel{r^2}}\times\frac{\cancel{r^2}}{\cancel{KQ^2}} \qquad \therefore \boxed{F' = \frac{9}{16}F} \quad (-\text{ve})$$

Q)9 A point charge Q is divided into 2 point charges and kept at a fixed seperation. what should be value of each fraction so force b/w them becomes maximum.

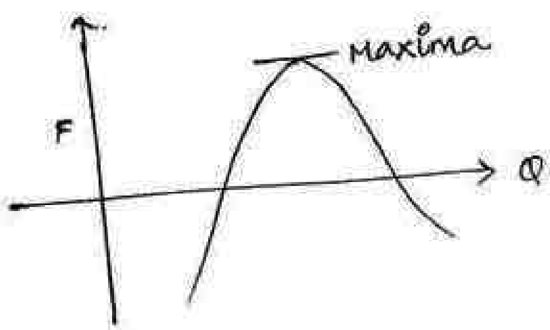


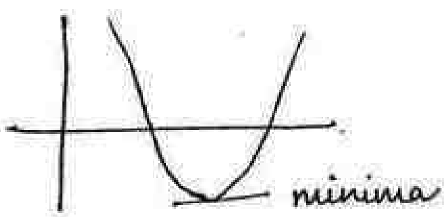
$\therefore \quad F = \dfrac{K x (Q-x)}{r^2} = \dfrac{K}{r^2}\left[Qx - x^2\right]$

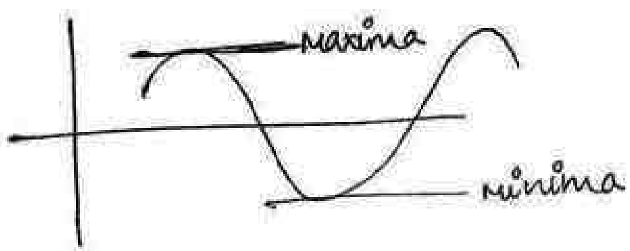
$\therefore \quad \dfrac{dF}{dx} = \dfrac{K}{r^2}\left[Q - 2x\right] = 0 \qquad \because \left(\dfrac{dy}{dx} = 0\right)$

$\therefore \quad Q - 2x = 0$

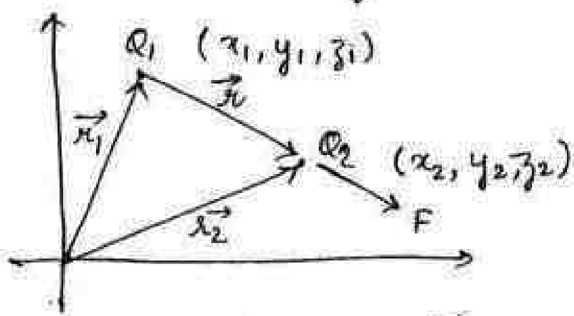
$\therefore \quad \cancel{Q=} \quad \boxed{x = \dfrac{Q}{2}}$







Force in vector form :



$$\because \vec{r}_1 + \vec{r} = \vec{r}_2$$

$$\therefore \cancel{\vec{r}_2 - r_1} \ \vec{r} = \vec{r}_2 - \vec{r}_1$$

$\therefore$ Force on Q_2

$$\vec{F} = \frac{K|Q_1||Q_2|}{r^2} (\hat{r})$$

$$= \frac{K|Q_1||Q_2|}{r^3} (\vec{r})$$

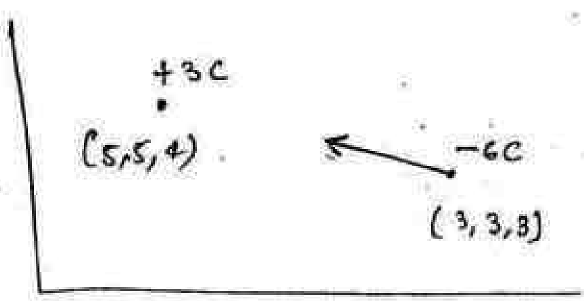
$$\vec{r} = \vec{r}_f - \vec{r}_i$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

✓ Put tail of all the vector on the point charge where net force has to be calculated.

Q) A point charge of +3 c is kept at (5, 5, 4) m and another point charge of −6 c is kept at pos^n (3, 3, 3) m . Find vector form of force excited on −6 C.



$$\vec{r} = 2\hat{i} + 2\hat{j} + \hat{k}$$

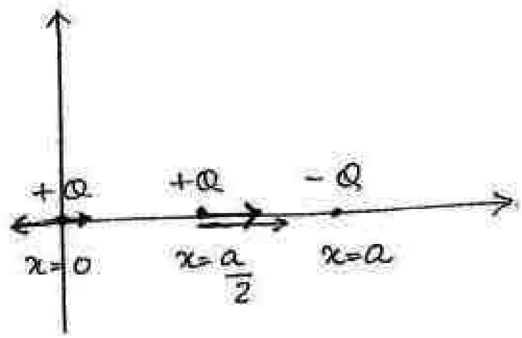
$$|\vec{r}| = \sqrt{4+4+1} = 3\,m.$$

$$\therefore \vec{F}_{on[-6C]} = \frac{k\,|Q_1|\,|Q_2|}{r^3}\,\vec{r}$$

$$= \frac{9\times10^9 \times 3\times 6}{3^3}\,(2\hat{i}+2\hat{j}+\hat{k})$$

$$= \boxed{10^9\,(12\hat{i}+12\hat{j}+6\hat{k})\quad N}$$

Q.11



Find force on particle kept at $x = a/2$ and $x = 0$

$x = \frac{a}{2}$ $\quad \because \vec{F}_{net} = \vec{F}_1 + \vec{F}_2$

$$= \frac{k(Q)(Q)}{\left(\frac{a}{2}\right)^2}\hat{i} + \frac{k(Q)(Q)}{\left(\frac{a}{2}\right)^2}\hat{i}$$

$$= \frac{8\,kQ^2}{a^2}(\hat{i})$$

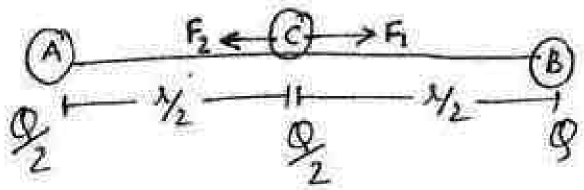
$x = 0$ $\quad \because \vec{F}_{net} = \vec{F}_1 + \vec{F}_2$

$$= \frac{k(Q)(Q)}{\left(\frac{a}{2}\right)^2}(-\hat{i}) + \frac{k(Q)(Q)}{a^2}(\hat{i})$$

$$= -4\frac{kQ^2}{a^2}\hat{i} + \frac{kQ^2}{a^2}\hat{i}$$

$$= -3\frac{kQ^2}{a^2}(\hat{i})$$

Q) A and B are two identical charged sphere which are kept at large separation. Force on each sphere is F. Now a third identical neutral sphere C is brought in contact with A and kept at midpoint of AB. Calculate force on the sphere C.



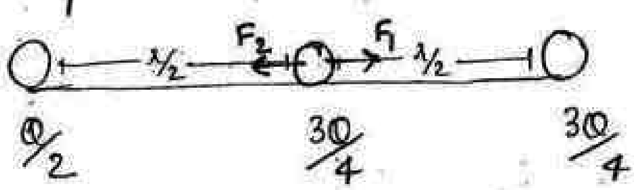
$\therefore \quad F = \dfrac{KQ^2}{r^2}$

$$F_1 = \frac{K\left(\frac{Q}{2}\right)\left(\frac{Q}{2}\right)}{\left(\frac{r}{2}\right)^2} = F \ (\hat{i})$$

$$\therefore \ F_2 = \frac{K\left(\frac{Q}{2}\right)(Q)}{\left(\frac{r}{2}\right)^2} = 2F \ (-\hat{i})$$

$\therefore \ F_{net} = 2F - F = \boxed{F} \ (-\hat{i})$

Q) In the above Q, after contact with A, C is brought at contact B. and finally kept at midpoint.

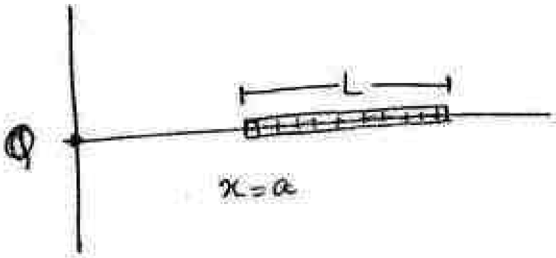


$$F_1 = \frac{K\left(\frac{Q}{2}\right)\left(\frac{3}{4}Q\right)}{\left(\frac{r}{2}\right)^2} = \frac{3}{2}F \ (\hat{i})$$

$$F_2 = \frac{K\left(\frac{3}{4}Q\right)\left(\frac{3}{4}Q\right)}{\left(\frac{r}{2}\right)^2} = \frac{9}{4}F(-\hat{i})$$

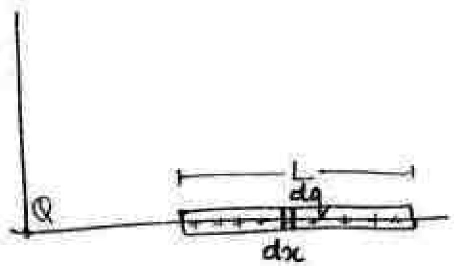
$$\therefore \quad F_{net} = \frac{9}{4}F - \frac{3}{2}F = \boxed{\frac{3}{4}F}\,(-\hat{i})$$

Q).



A thin rod is uniformly charged with total charge Q along its length. calculate F exerted on the rod

Force on the element



$$dF = \frac{K(Q)(dq)}{x^2}$$

$$\left(\because \frac{q}{L}dx = dq\right)$$

$$\left[\begin{array}{l} \because\ L \longrightarrow q \\ \quad 1 \longrightarrow \frac{q}{L} \\ \quad dx \longrightarrow \frac{q}{L}dx = dq \end{array}\right.$$

$$\therefore \quad dF = \frac{KQ}{x^2}\left(\frac{q}{L}dx\right)$$

$$\therefore \int dF = \frac{KQq}{L}\int_a^{a+L}\frac{1}{x^2}dx$$

$$\therefore \quad F_{net} = \frac{KQq}{L}\left[-\frac{1}{x}\right]_a^{a+L}$$

$$= \frac{KQq}{L}\left[\frac{1}{a} - \frac{1}{a+L}\right]$$

$$= \frac{KQq}{\cancel{L}}\left[\frac{\cancel{L}}{a(a+L)}\right] \Rightarrow \boxed{\frac{KQq}{a(a+L)}}$$

If $a >>>> L$, then rod behaves as a point charge

then $F = \frac{KQq}{a^2}$

*Q) A sphere of radius R is charged in its vol^m such that its volume - charge density (ρ)

$\rho = \rho_0 r$ $\quad$ ($\because \rho_0$ = constant, r = distance from the centre)

Calculate total charge on the sphere

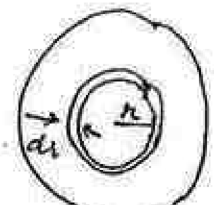
$$dq = \rho\,(dV) \qquad \left[\because \rho = \frac{q}{V} \quad \therefore dq = \rho\, dV\right]$$

$$= \rho_0 r\,(dV)$$

$$\left[\begin{array}{l} \because V = \frac{4}{3}\pi r^3 \\ \therefore \frac{dV}{dr} = \frac{4}{\cancel{3}}\pi(\cancel{3}r^2) \\ dV = 4\pi r^2\, dr \end{array}\right]$$

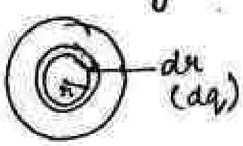
$$\int dq = \int_0^R \rho_0 r\,(4\pi r^2 dr)$$

$$Q_{net} = \rho_0(4\pi)\left[\frac{r^4}{4}\right]_0^R = \rho_0 \pi R^4$$



Q) A disc of negligible thickness and radius R. ~~q~~ its charge on its surface with surface charge density $\sigma_0 r$

$\sigma = \sigma_0 r$ $\quad$ (σ_0 = constant, r = dis from the centre)



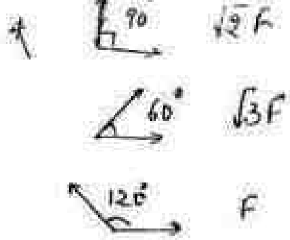
$$\therefore dq = \sigma(dA) \qquad \left[\because \sigma = \frac{q}{A} \quad \therefore dq = \sigma\, dA\right]$$

$$= \sigma_0 r(dA)$$

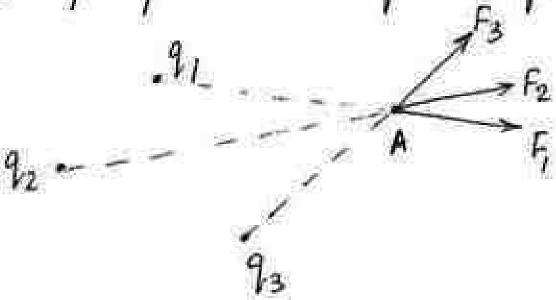
$$\int dq = \int_0^R \sigma_0 r\, 2\pi r\, dr$$

$$\left[\begin{array}{l} \because A = \pi r^2 \\ \therefore \frac{dA}{dr} = 2\pi r \\ \therefore dA = 2\pi r\, dr \end{array}\right]$$

$$\therefore q = \frac{2}{}\pi R^3 \sigma_0$$



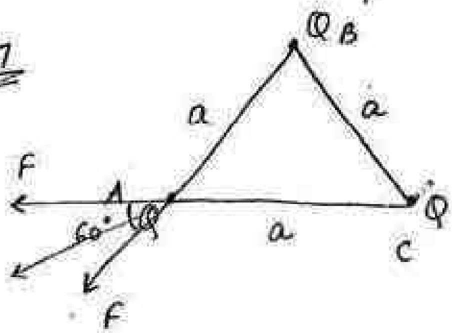
Superposition principle :



$$\therefore \quad \vec{F}_{net} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

Force b/w two particles remains unaffected with the presence of another particle so net force is the vector addition of all the forces exerted on the particle.

Q.17



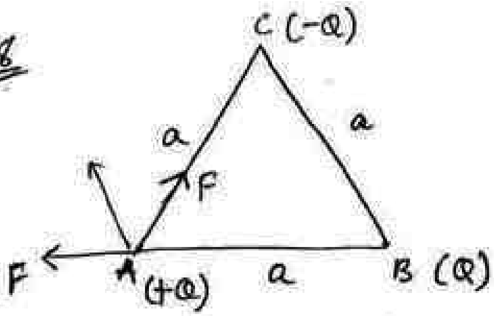
where ABC is an equilateral triangle. calculate $\vec{F_A}$

$$\because \quad R = 2a\cos\left(\frac{\theta}{2}\right)$$

$$= 2a\cos 30^\circ$$

$$= 2\cancel{F} \times \frac{\sqrt{3}}{2} = \sqrt{3}.F = \frac{\sqrt{3}\,KQ^2}{a^2}$$ (from centroid to A

Q 18

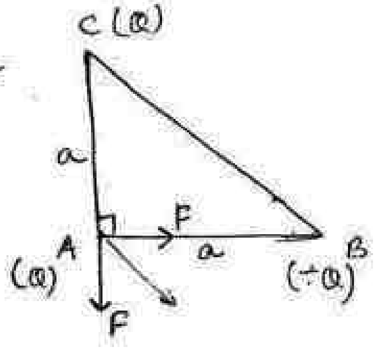


~~Cal~~ where ABC is an equilateral triangle ~~as~~ calculate $\vec{F_A}$

$$\because R = 2F\cos\left(\frac{120}{2}\right)$$

$$= \cancel{2}F\frac{1}{\cancel{2}} = F$$ (ll to BC)

Q.19

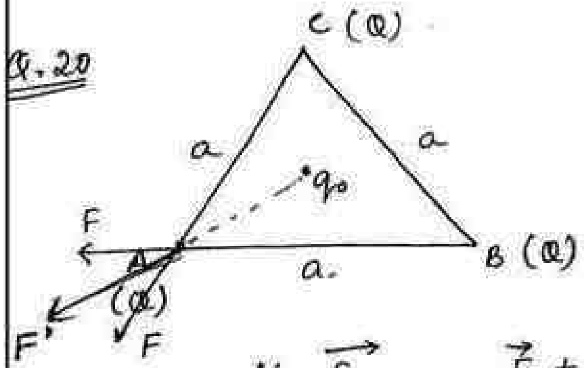


$$R = 2F\cos\frac{\theta}{2}$$

$$= 2F\frac{1}{\sqrt{2}} = \sqrt{2}F$$

$$= \sqrt{2}\left(\frac{KQ^2}{a^2}\right)$$, parallel to CB side

Q.20



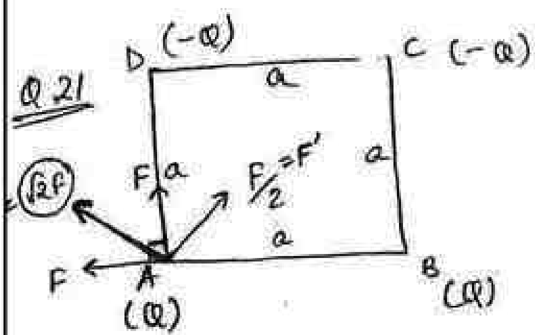
$$\because \overrightarrow{F_{net}} = \vec{F} + \vec{F} + \vec{F}'$$

$$= \sqrt{3}\vec{F} + \vec{F}'$$

$$= \sqrt{3}\left(\frac{KQ^2}{a^2}\right) + \frac{KQq_0}{\left(\frac{a}{\sqrt{3}}\right)^2}$$

$$= \sqrt{3}\frac{KQ^2}{a^2} + \frac{3KQq_0}{a^2}$$ (from centroid to A)

Q.21



$$\vec{F}_A = ?$$

$$R = 2a\cos\theta/2 = 2F\cos\left(\frac{90}{2}\right)$$

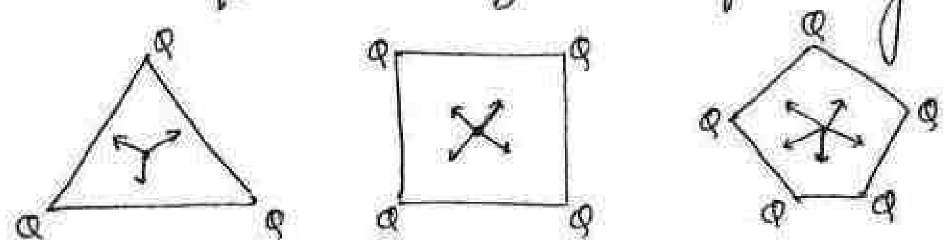
$$= 2F\frac{1}{\sqrt{2}} = \sqrt{2}F$$

$$\therefore F' = \frac{KQQ}{(\sqrt{2}a)^2} = \frac{KQ^2}{2a^2} = \frac{F}{2}$$

$$\therefore \vec{F}_A = \sqrt{(\sqrt{2}F)^2 + \left(\frac{F}{2}\right)^2} = \sqrt{2F^2 + \frac{F^2}{4}} = \sqrt{\frac{9}{4}F^2}$$

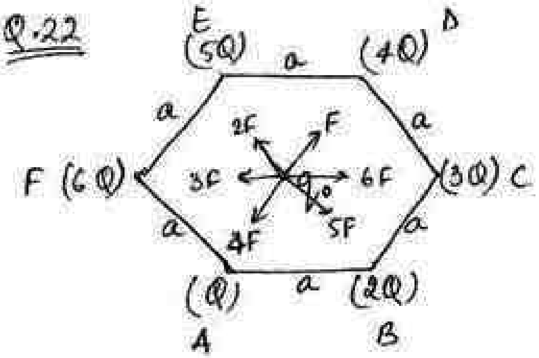
$$= \frac{3}{2}F = \frac{3}{2}\left(\frac{KQ^2}{a^2}\right)$$

* Forces on q_0 will be zero in following cases :



$F_{on\, q_0} = 0$

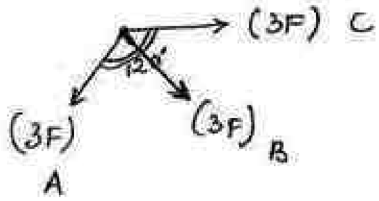
Q.22



$\vec{F}_{q_0} = ?$

$R_{(A\,and\,C)} = 2a\cos\frac{\theta}{2} = 2(3F)\cos 60$

$= 2\times 3F\times\frac{1}{2} = 3F$

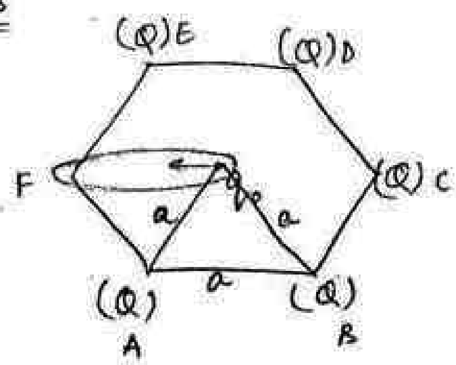


$\therefore\ \vec{F}_{q_0} = R + 3F$

$= 3F + 3F = 6F$

i.e. $6F$ (towards B)

Q.23



$\vec{F}_{on\, q_0} = ?$

$\underbrace{\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \vec{F_5}} + \vec{F_6} = 0$

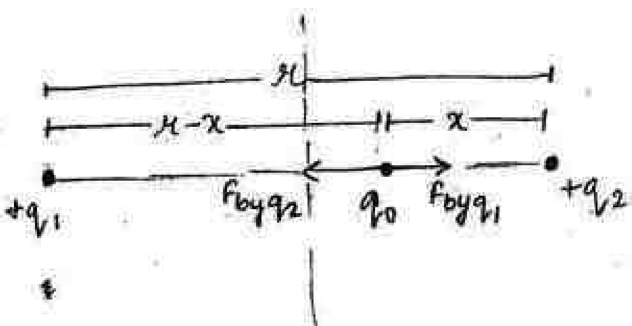
$\therefore\ \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \vec{F_5} = -\vec{F_6}$

$= -\left(\frac{KQq_0}{a^2}\right)$ F to centroid

Equilibrium of a test charge (q_0) on the line joining two fixed point charges (q_1 and q_2)

- If q_1 and q_2 are like charges with equal magnitude then neutral point will be at midpoint.
- Position of neutral point is independent of nature and magnitude of test charge. (i.e. $q_0/2q_0/3q_0$)
- If q_1 and q_2 are like charges with unequal magnitude, then neutral point will be b/w q_1 and q_2 but closer to smaller charge
- If q_1 and q_2 are unlike charges, with unequal magnitude then neutral point will be outside of smaller charge.
- If q_1 and q_2 are unlike nature with equal magnitude, then there will be no neutral/null point of the system.

Case I: q_1 and q_2 are like charges and $|q_1| > |q_2|$



For equilibrium of q_0,

$$F_{by\, q_1} = F_{by\, q_2}$$

$$\frac{\cancel{K} q_1 \cancel{q_0}}{(r-x)^2} = \frac{K q_2 \cancel{q_0}}{x^2}$$

$$\frac{\sqrt{q_1}}{r-x} = \frac{\sqrt{q_2}}{x}$$

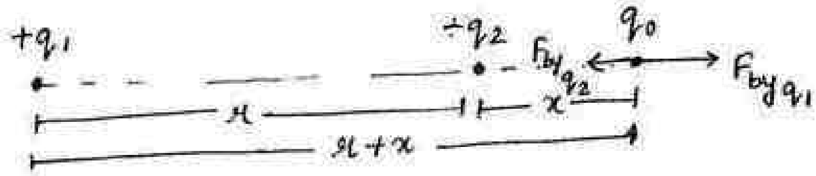
$$\therefore (\sqrt{q_1})x = (\sqrt{q_2})\, r-x = \sqrt{q_2}\, r - \sqrt{q_2}\, x$$

$$\therefore \sqrt{q_1}\, x + \sqrt{q_2}\, x = \sqrt{q_2}\, r$$

(from q_2) $\therefore$ $$x = \left(\frac{\sqrt{q_2}}{\sqrt{q_1}+\sqrt{q_2}}\right) r$$

$$\therefore r - x = r - \left(\frac{\sqrt{q_2}}{\sqrt{q_1}+\sqrt{q_2}}\right) r$$

<u>case II</u> : q_1 and q_2 are unlike charges and $|q_1| > |q_2|$



For equilibrium of q_0,

$$F_{by\, q_1} = F_{by\, q_2}$$

$$\frac{K q_1 \cancel{q_0}}{(r+x)^2} = \frac{K q_2 \cancel{q_0}}{x^2}$$

$$\frac{\sqrt{q_1}}{r+x} = \frac{\sqrt{q_2}}{x}$$

$$(\sqrt{q_1})x = (\sqrt{q_2})\, r + x = \sqrt{q_2}\, r + \sqrt{q_2}\, x$$

$$\sqrt{q_1}\, x - \sqrt{q_2}\, x = \sqrt{q_2}\, r$$

$$\therefore\ x = \left(\frac{\sqrt{q_2}}{\sqrt{q_1} - \sqrt{q_2}}\right) r$$

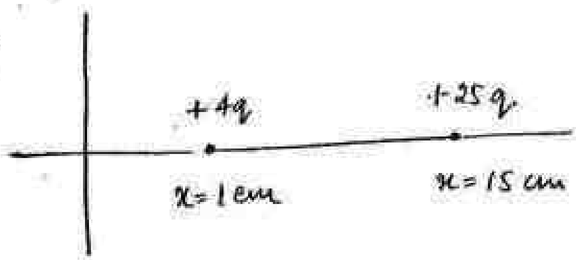
$$r + x = r + \left(\frac{\sqrt{q_2}}{\sqrt{q_1} - \sqrt{q_2}}\right) r$$

Conclusion :

$$\boxed{\text{distance from } q_* = \frac{\sqrt{q_*}\quad r}{\sqrt{q_{bigger}} \pm \sqrt{q_{smaller}}}}$$

[⊕ like and ⊖ unlike]

Q.24



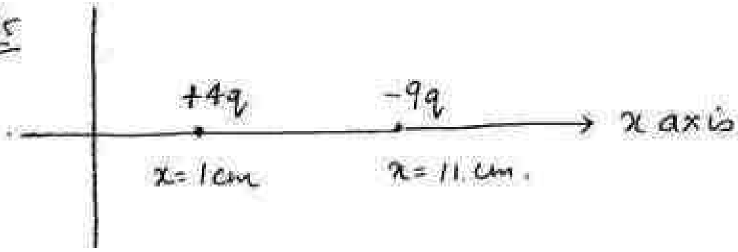
Find position of third charge so that third charge remains in equilibrium

$$\therefore\ \text{distance from } 4q = \frac{\sqrt{4q}}{\sqrt{4q} + \sqrt{25q}} \times 14$$

$$= \frac{2}{7} \times 14 = 4\text{ cm}$$

$$\therefore\ \boxed{x = 4 + 1 = 5\text{ cm.}}$$

Q.25



Find position of third charge so third charge remains in equilibrium

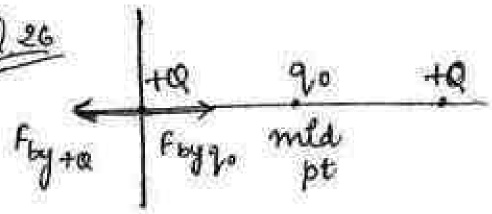
distance from 4q = d = $\dfrac{\sqrt{4q}}{\sqrt{9q} - \sqrt{4q}} \times (11-1)$

$= \dfrac{2}{1} \times 10 = 20$ cm

(in left side of 4q)

$\boxed{\therefore x = -19 \text{ cm}}$

Q.26



What should be value and nature of q_0 charge so that F on any one corner charge becomes 0.

$$\frac{\cancel{K}(\cancel{Q})(Q)}{r^2} = \frac{\cancel{K}(\cancel{Q})(q_0)}{\left(\frac{x}{2}\right)^2}$$

$Q = 4q_0$

$\therefore q_0 = \dfrac{Q}{4}$

with nature $\boxed{q_0 = -Q/4}$

Equilibrium of point charge~~st~~ system

- In this case, force on each particle must be zero
- Test charge must be kept at neutral point.
- To find nature and magnitude of test charge.

Select any one corner charge and make its net force = 0

Q. 27 ***
Find position, nature and magnitude of third charge so whole system remains in equilibrium

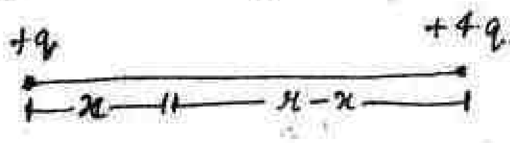
Position → midpoint

Nature → opposite of + Q i.e. (−)

$$\therefore \frac{KQq_0}{\left(\frac{r}{2}\right)^2} = \frac{KQQ}{r^2} \quad (\because |F| = |F'|)$$

$$\therefore q_0 = \frac{Q}{4} \quad \text{i.e.} \quad \boxed{q_0 = -\frac{Q}{4}}$$

Q.28 Find position, nature and magnitude of third charge so whole system remains in equilibrium



$$x = \frac{\sqrt{q}\, r}{\sqrt{q} + \sqrt{4q}} = \frac{q r}{q + 2q} = \frac{r}{3} \quad \text{i.e position}$$

$\therefore$ nature = (-ve).

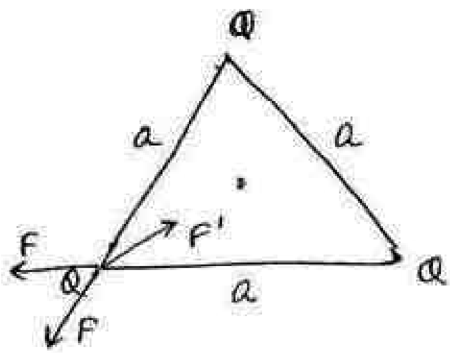
$$|F| = |F'|$$

$$\frac{Kqq_0}{\left(\frac{r}{3}\right)^2} = \frac{4Kqq}{r^2}$$

$$\therefore 9\frac{Kqq_0}{r^2} = 4\frac{Kqq}{r^2}$$

$$\therefore q_0 = \frac{4}{9}q \quad \text{i.e} \quad \boxed{-\frac{4q}{9}}$$

Q.29



Find position, nature and magnitude of forth charge so whole system remains in equilibrium.

Position $\rightarrow$ centroid

nature $\rightarrow$ (-ve)

$$\because \sqrt{3}F = F'$$

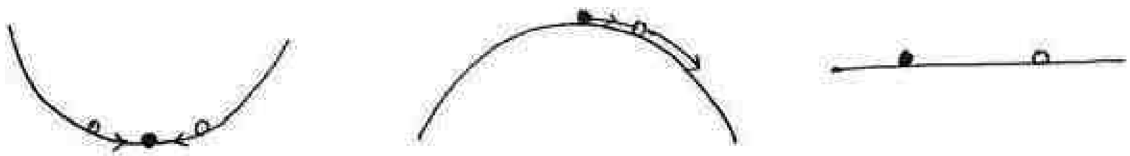
$$\therefore \frac{\sqrt{3}KQQ}{a^2} = \frac{KQq_0}{\left(\frac{a}{\sqrt{3}}\right)^2}$$

$$\therefore \frac{\sqrt{3}Q}{a^2} = \frac{3q_0}{a^2}$$

$$\therefore q_0 = \frac{\sqrt{3}Q}{3} = \frac{Q}{\sqrt{3}}$$

$$\therefore \boxed{q_0 = -\frac{Q}{\sqrt{3}}}$$

Types of Equilibrium :



$F_{net} = 0$ Stable equilibrium

$F_{net} = 0$ unstable equilibrium

$F_{net} = 0$ Neutral equilibrium

> In all the cases, Net force must be zero.

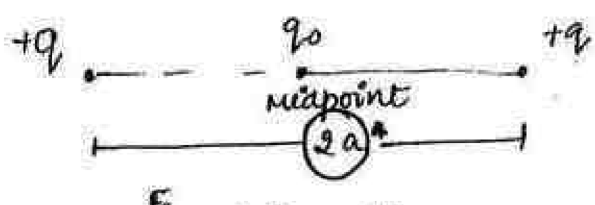
> In case of stable equilibrium, there is possibility of oscillation and SHM

> To find type of equilibrium, displace the particle slightly and consider the motion

> Overall equi of electric system is always unstable system

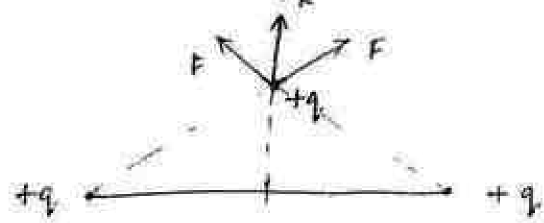
Q. 30 Find i) Nature of equilibrium along the axis and along the ~~equilib~~ equator

ii) If q_0 is given small displacement along the axis and then released, find angular frequency of its SHM provided that mass of q_0 is m.



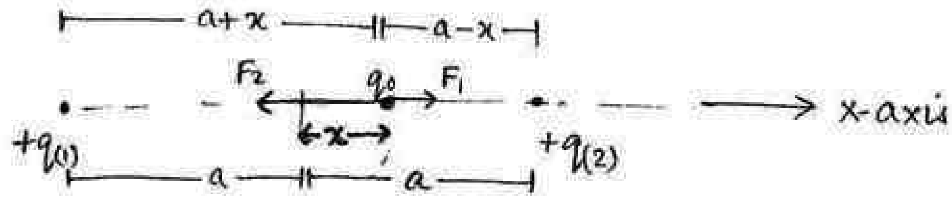
i)

stable equilibrium along the axis.



Unstable equilibrium along equator

ii)



$\therefore \; \vec{F}_{net} = F_1 + F_2$

$$= \frac{Kqq_0}{(a+x)^2} - \frac{Kqq_0}{(a-x)^2} \quad (\because \text{ opposite dir}^n)$$

$$= Kqq_0 \left[\frac{(a-x)^2 - (a+x)^2}{(a+x)^2 (a-x)^2} \right]$$

$$\therefore \; F_{net} = Kqq_0 \left[\frac{-4ax}{(a^2-x^2)^2} \right] \quad (\text{oscillation})$$

here $x <<< a$

$$\therefore \; F_{net} = Kqq_0 \left[\frac{-4ax}{a^4} \right] \Rightarrow \frac{4Kqq_0}{a^3}(-x)$$

here, $F_{net} \propto (-x)$ (SHM)

$$a_{net} = \frac{4Kqq_0}{ma^3}(-x) = \omega^2(-x)$$

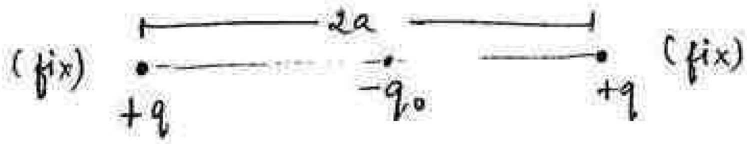
so $\boxed{\omega = \sqrt{\frac{4kqq_0}{ma^3}}}$ *

conclusion:

- If displacement is not negligible then oscillatory motion but if displacement is negligiable then SHM
- If seperation b/w fixed charges would be (a) instead of (2a), then $\omega = \sqrt{\dfrac{4 q q_0 K}{m\left(\frac{a}{2}\right)^3}}$

$$= \sqrt{\frac{32 K q q_0}{m a^3}}$$

<u>Q 31</u>
(not for boards)



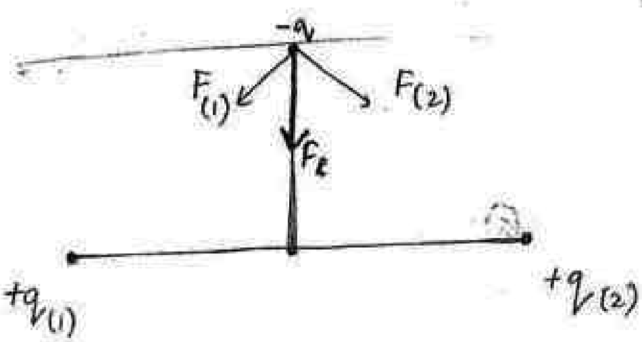
Find i) Nature of equilibrium of q_0 along axis and equator.

ii) If q_0 is given a small displacement along the equator and released, then find angular frequency of its SHM. consider mass of q_0 to be m

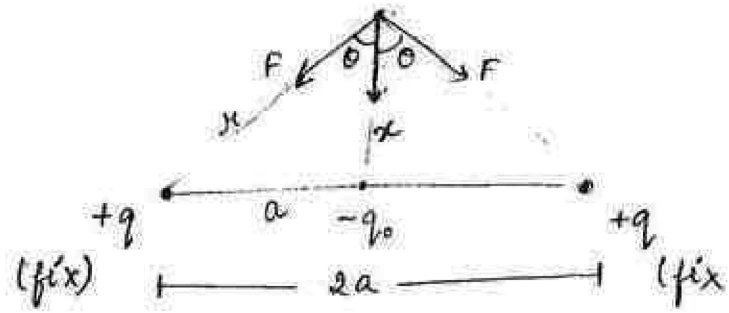
i)

∴ <u>unstable</u> equilibrium along axis.

ii)



∴ stable equilibrium along ~~axe~~ equator ii)



↑ ⊕ve dir^n

$$F_{net} = -2F\cos\theta$$

$$= -2\left(\frac{Kqq_0}{r^2}\right)\left(\frac{x}{r}\right) = \frac{-2Kqq_0 x}{r^3}$$

$$\therefore \vec{F}_{net} = \frac{-2Kqq_0 x}{(a^2+x^2)^{3/2}}$$ (oscillations).

$\left(\because r = (a^2+x^2)^{1/2}\right)$

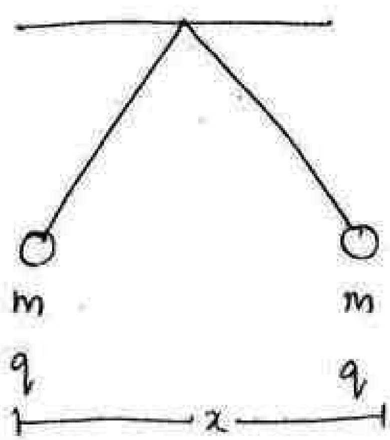
If $x << a$

then $F_{net} = \dfrac{-2Kqq_0 x}{a^3}$

$$\therefore a_{net} = \frac{2Kqq_0}{ma^3}(-x) = \omega^2(-x)$$

$$\therefore \boxed{\omega = \sqrt{\frac{2Kqq_0}{ma^3}}}$$

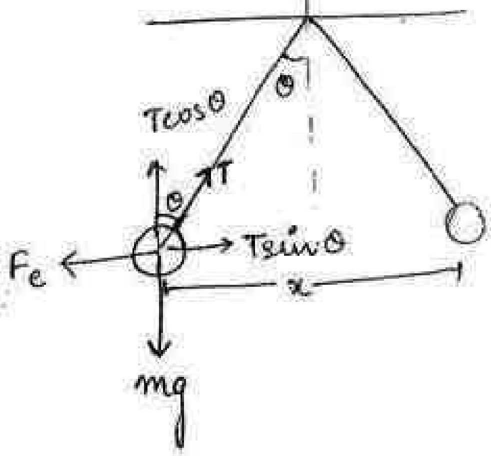
Q.32



System is in equilibrium, find

i) Tension in each string

ii) Angle of string with the vertical



$\because \; T\cos\theta = mg \quad — (I)$

$T\sin\theta = F_e = \dfrac{Kq^2}{x^2} \quad — (II)$

On dividing $\dfrac{(II)}{(I)}$, we get,

$$\frac{T\sin\theta}{T\cos\theta} = \frac{Kq^2}{x^2 mg}$$

$$\therefore \; \tan\theta = \frac{Kq^2}{x^2 mg}$$

$$\therefore \; \theta = \tan^{-1}\left(\frac{Kq^2}{x^2 mg}\right)$$

On adding $(I)^2 + (II)^2$, we get,

$$T^2(\sin^2\theta + \cos^2\theta) = \left(\frac{Kq^2}{x^2}\right)^2 + (mg)^2$$

$$\therefore T = \sqrt{\left(\frac{Kq^2}{x^2}\right)^2 + (mg)^2}$$

Q.33 In the above que, length of each string is l and angle b/w string is small, then find value of x.

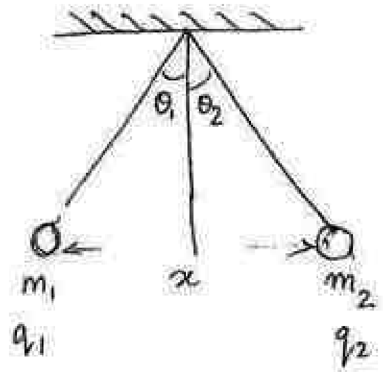
$\because$ Angle $= \dfrac{\text{Arc}}{\text{Radius}}$

$\therefore \; 2\theta = \dfrac{x}{l}$

$\therefore \; x = (2\theta)l = 2l\left(\dfrac{Kq^2}{mgx^2}\right)$

$$\left[\begin{array}{l} \because \theta <<< l \\ \therefore \tan\theta \simeq \theta \\ \quad = \left(\dfrac{Kq^2}{x^2 mg}\right) \end{array}\right]$$

$\therefore x^3 = 2l\left(\dfrac{Kq^2}{mg}\right) \qquad \therefore \; x = \left[2l\left(\dfrac{Kq^2}{mg}\right)\right]^{1/3}$



Case I : If $m_1 = m_2$

$q_1 > q_2$

then $\theta_1 = \theta_2$

$\because \tan\theta = \dfrac{F_e}{mg}$ $\begin{matrix} \rightarrow \text{constant} \\ \rightarrow \text{constant} \end{matrix}$

Case II : If $m_1 > m_2$

and $q_1 > q_2$

then $\theta_2 > \theta_1$

$\tan\theta \propto \dfrac{1}{m}$

$\therefore m \uparrow \therefore \theta \downarrow$

Article-3 Intensity of Electric field ($\vec{E}$)

Electric field is defined as force exerted on unit positive test charge.

- test charge should have negligiable value, so it will not change original electric field.

$$\vec{E} = \lim_{q_0 \to 0} \left(\frac{\vec{F}}{q_0} \right)$$

- Electric field is independent of nature of q_0. Actually, electric field is the property of source charge.

$$\therefore \boxed{\vec{F} = q_0 \vec{E}}$$

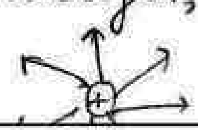
here, source of $\vec{E}$ may be point charge or some other configuration of charge but q_0 must be point charge.

<u>$\vec{E}$ of point charge :</u>

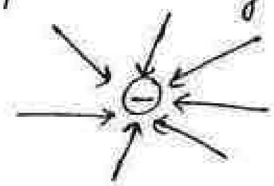
$$F = \frac{Kqq_0}{r^2}$$

$$\because E = \frac{F}{q_0} = \frac{Kq}{r^2} \qquad \left(\therefore E \propto \frac{1}{r^2} \right)$$

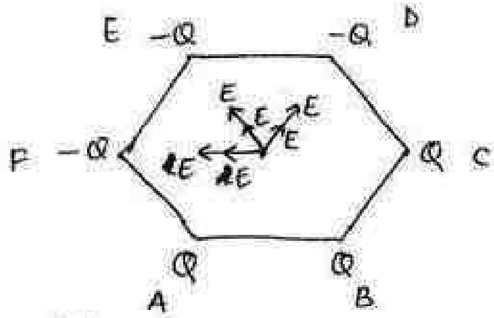
(+)ve point charged, radially outward



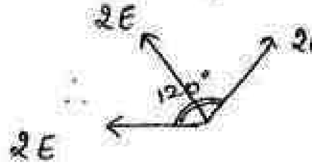
(−)ve point charge, radially inward.



Q.1



find $\vec{E}$ at centre



$\therefore$ $E_{net} = 2E + 2E = 4E$

$= 4\left(\frac{KQ}{a^2}\right)$ towards E

* At equilibrium or neutral point, force as well as electric field becomes zero, if a charged particle is kept at a neutral point, then it will experience zero force.

** Electric field on the position of a point charged cannot be defined, its value remains uncertain.

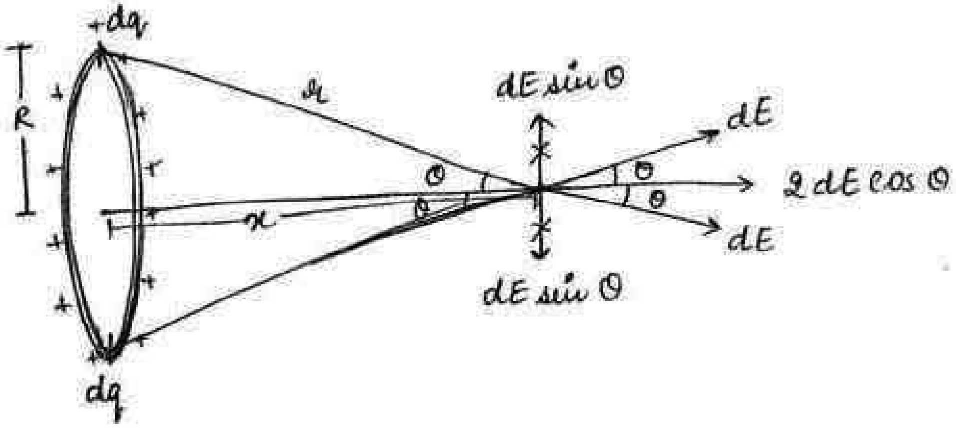
Q.2

If at point A, field of −3Q is $\vec{E}$, then find field at point B, due to charge +Q

$$\underset{(\text{of } -3Q)}{\vec{E}_A} = \frac{K(3Q)}{r^2}(\hat{i}) = \vec{E}$$

$$\underset{(\text{of } +Q)}{\vec{E}_B} = \frac{KQ}{r^2}(\hat{i}) = \frac{\vec{E}}{3}$$

* Electric field of uniformly charged ring on its axis at x distance away from its centre, provided that total charge on the ring $= Q$ and Radius $= R$



contribution of each element in net electric field $= dE\cos\theta$

$$\therefore \quad E_{net} = \int_{Ring} dE\cos\theta$$

$$= \int_{Ring} \frac{K\,dq}{r^2} \times \frac{x}{r} \qquad \left(\because \cos\theta = \frac{x}{r} = \text{constant}\right)$$

$$= \frac{Kx}{r^3}\int_{Ring} dq = \frac{Kx}{r^3}(Q)$$

$$= \frac{KQx}{(x^2+R^2)^{3/2}}$$

MRI

$$\boxed{\therefore \quad \vec{E} = \frac{KQx}{(x^2+R^2)^{3/2}}\,(\hat{i})}$$

Ring: $\vec{E}$ at ~~centre~~ x distance from centre

conclusion :

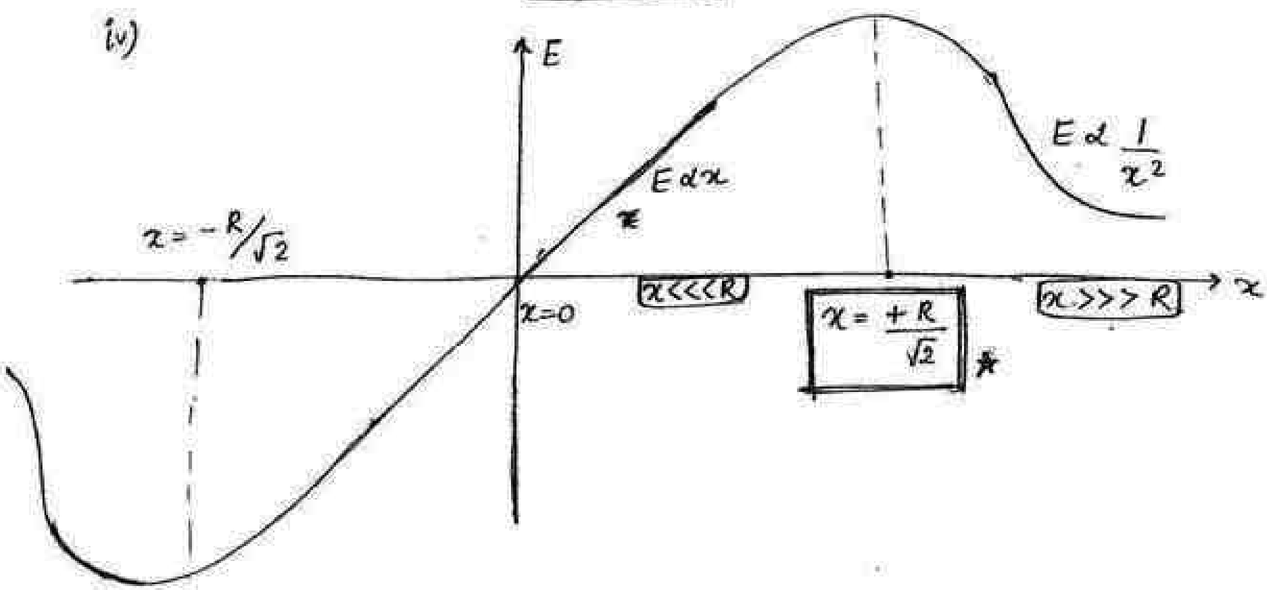
i) at the centre $\underline{x=0}$ so $\boxed{E=0}$

ii) For $\underline{x <<< R}$, $E = \frac{KQx}{R^3}$ i.e. $\boxed{E \propto x}$

iii) For $\underline{x >>> R}$, $E = \frac{KQx}{x^3} = \frac{KQ}{x^2}$ (behaves as a point charge)

$\therefore \boxed{E \propto \frac{1}{x^2}}$

iv)



v) For what value of $x \longrightarrow E_{max}$

$\therefore \quad E = \frac{KQx}{(R^2+x^2)^{3/2}} \qquad \therefore \quad \frac{dE}{dx} = 0$

$$\left(\because \frac{d}{dx}\frac{U}{V} = \frac{V\left(\frac{dU}{dx}\right) - U\left(\frac{dV}{dx}\right)}{V^2} \right)$$

$$KQ \left[\frac{(R^2+x^2)^{3/2}(1) - x\left(\frac{3}{2}(R^2+x^2)^{1/2} \times (2x)\right)}{(R^2+x^2)^3} \right] = 0$$

$$(R^2+x^2)^{1/2} \left[(R^2+x^2)^1 - 3x^2 \right] = 0$$

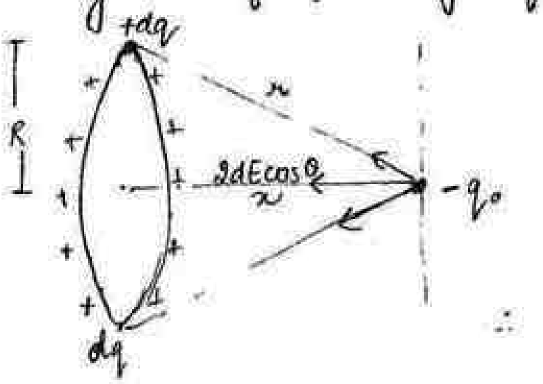
$R^2 - 2x^2 = 0$

$R^2 = 2x^2$

$\therefore x^2 = \dfrac{R^2}{2}$ *

$\therefore x = \pm \dfrac{R}{\sqrt{2}}$

Q.2 A ring of radius R is ~~positively~~ uniformly charged with total charge +Q. A particle of charge −q and mass m is kept on its centre. If −q is given a small displacement along the axis and released then find angular frequency of its SHM.



contribution of each element in net electric field $\vec{E} = dE\cos\theta$

$\therefore E_{net} = \int \dfrac{Kdq}{r^2}\left(\dfrac{-x}{r}\right)$

$\therefore E_{net} = \dfrac{-Kx}{r^3}(Q) = \dfrac{-KQx}{(R^2+x^2)^{3/2}}$

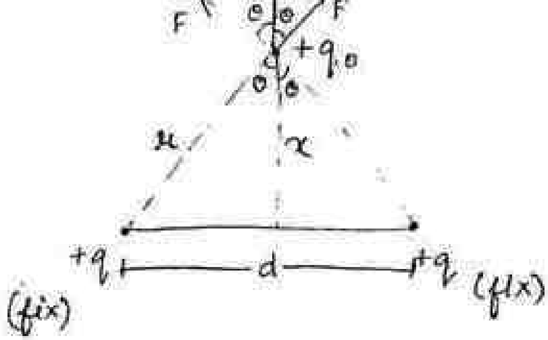
$\therefore F = q_0 \dfrac{(-KQx)}{(R^2+x^2)^{3/2}}$

$\therefore a = \dfrac{KQq_0}{m(R^2+x^2)^{3/2}}(-x) = \omega^2(-x)$ i.e. SHM

$\therefore \omega^2 = \dfrac{KQq_0}{m(R^2+x^2)^{3/2}}$ $\quad \therefore \omega = \sqrt{\dfrac{KQq_0}{m(R^2+x^2)^{3/2}}}$

If $x <<< R$ $\quad \therefore$ $\boxed{\omega = \sqrt{\dfrac{KQq}{\quad}}}$

Q) For what value of x, force on the particle q_0 will be max



$$\therefore \; F_{net} = 2F\cos\theta = 2\left(\frac{kqq_0}{r^2}\right)\cos\theta$$

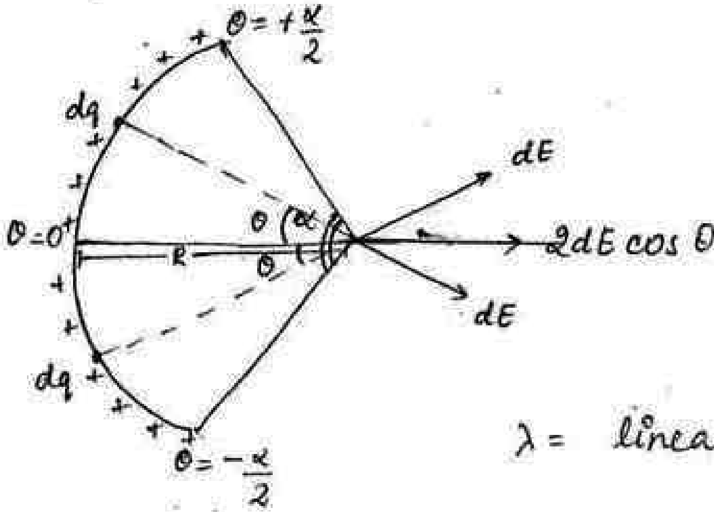
$$= 2\left(\frac{kqq_0}{r^2}\right)\left(\frac{x}{r}\right) = \frac{2kqq_0x}{r^3}$$

$$\Rightarrow \frac{2kqq_0x}{\cancel{k}\left[x^2+\left(\frac{d}{2}\right)^2\right]^{3/2}}$$

$$\therefore \text{ For } F_{Max} \Rightarrow \frac{dF}{dx} = 0$$

$$\therefore \boxed{x = \pm\frac{d}{2\sqrt{2}}}$$

Q) An arc of radius R is uniformly charged with linear charge density λ. Angle subtend by R at the centre is α, then find electric field at the centre.



$\lambda =$ linear charged density

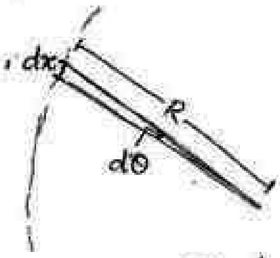
Effective contribution of each element : $dE\cos\theta$

$$\therefore \quad E_{net} = \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} dE\cos\theta$$

$$= \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} \frac{K\,dq}{R^2}\cos\theta = \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} \frac{K\lambda\,dx}{R^2}\cos\theta$$

$(\because dq = \lambda dx)$

$$\therefore \quad = \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} \frac{K\lambda R\,d\theta}{R^2}\cos\theta$$



$$\left(\begin{array}{l} \because d\theta = \frac{dx}{R} \\ \therefore dx = R\,d\theta \end{array}\right)$$

$$= \frac{K\lambda}{R}\int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} \cos\theta\,d\theta \Rightarrow \frac{K\lambda}{R}\left(\sin\theta\right)_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}}$$

$$= \frac{K\lambda}{R}\left[\sin\frac{\alpha}{2} - \sin\left(-\frac{\alpha}{2}\right)\right]$$

$$= \frac{K\lambda}{R}\left[2\sin\left(\frac{\alpha}{2}\right)\right] \Rightarrow$$

$$\therefore \quad \boxed{E_{net} = \frac{2K\lambda}{R}\sin\left(\frac{\alpha}{2}\right)}$$

Arc :

Conclusion :

i) $\lambda = \frac{Q}{L} = \frac{Q}{R\alpha^*}$ (charge per unit length)
(* : in radian)

ii) Full ring ($\alpha = 360^\circ$ or 2π rad)

$$\therefore \quad E = \frac{2K\lambda}{R}\sin(180^\circ) = 0$$

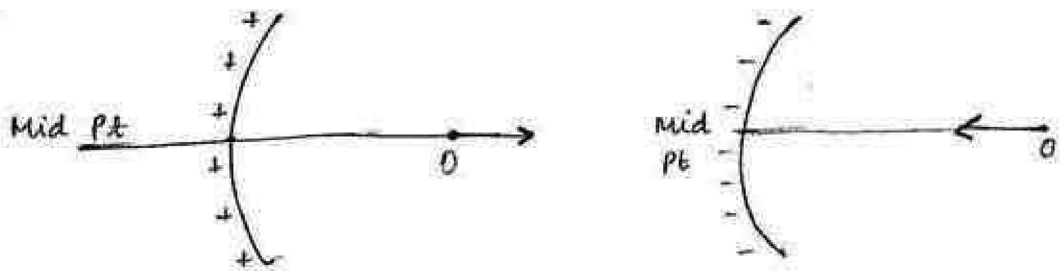
iii) $\frac{1}{2}$ ring $\quad \alpha = 180^\circ$

$$\therefore E = \frac{2k\lambda}{R} \sin(90^\circ) \Rightarrow \frac{2k\lambda}{R}$$

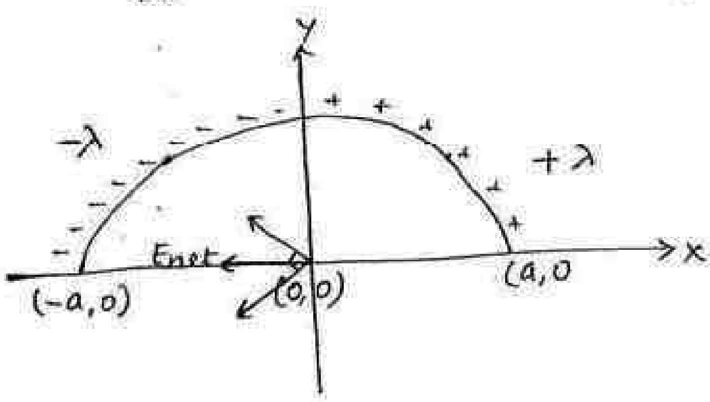
iv) $\frac{1}{4}$ ring $\quad \alpha = 90^\circ$

$$\therefore E = \frac{2k\lambda}{R} \sin(45^\circ) \Rightarrow \frac{\sqrt{2}k\lambda}{R}$$

v)



Q)



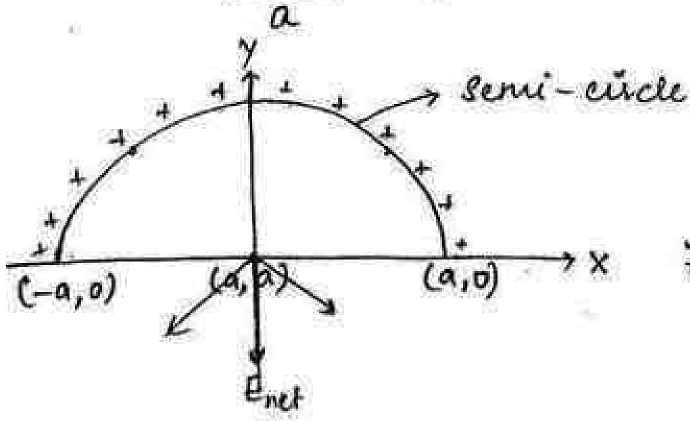
Find $\vec{E}$ at the centre in this semicircle.

$$\therefore E_{net} = \sqrt{2}E$$

$$= \sqrt{2}\left(\frac{\sqrt{2}k\lambda}{a}\right)$$

$$= \frac{2k\lambda}{a}(-\hat{i})$$

Q)



Total charge = Q

Find E at centre

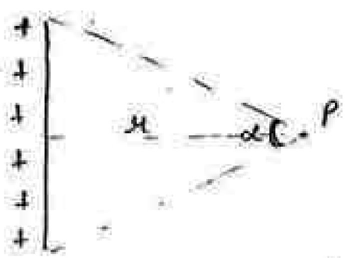
$$\lambda = \frac{Q}{L} = \frac{Q}{R\alpha} = \frac{Q}{\pi R}$$

$$\therefore \vec{E} = \frac{2K\lambda}{a}(-\hat{j})$$

$$= \frac{2}{a}\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{Q}{\pi a}\right)(-\hat{j})$$

$$= \frac{Q}{2\pi^2 a^2 \epsilon_0}(-\hat{j})$$

* For straight wire



λ = linear charge density

$$\boxed{\therefore E_p = \frac{2K\lambda}{r}\sin\left(\frac{\alpha}{2}\right)}$$ (for finite length)

Article 4A — Lines of Electric field (LEF)

Properties of Electric field : (Straight or parabolic)

> These are imaginary lines

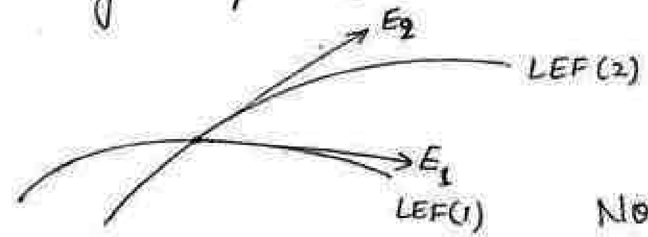
> concept of field lines is given by Faraday

> These lines originates either from ⊕ve charge or infinity and terminates at ⊖ve charge or infinity

> maybe straight line or curve line

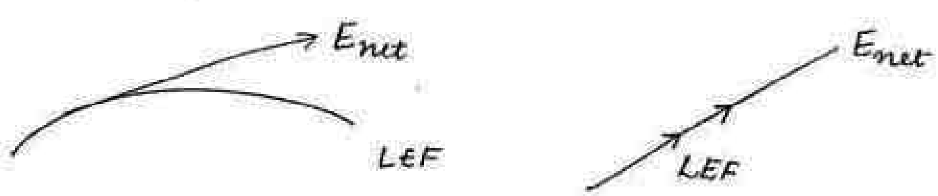
> must be drawn in accordance with net electric field.

> Two field lines never intersect each other because two values of electric field is not possible at a given points.

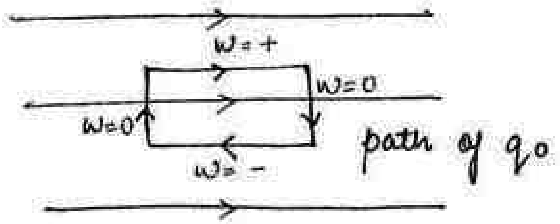


Not possible

> A tangent drawn on the field lines must be in the direction of net electric field.



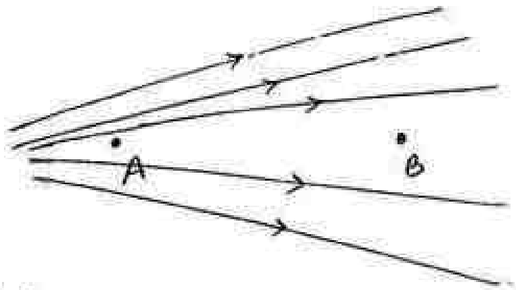
> Lines of electrostatistics field never form closed loop. This properties shows that nature of electrostatistics field is $\boxed{\text{conservative}}$



$W_{net} = 0$

$$\boxed{|\vec{E}| = \frac{1}{\text{seperation b/w lines}}}$$

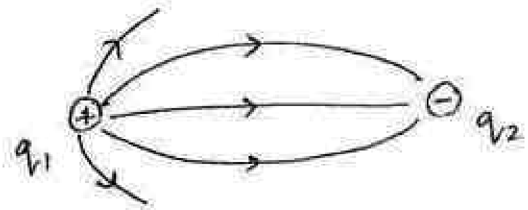
∴

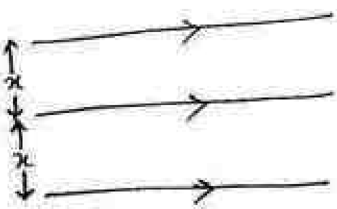


$\therefore \; E_A > E_B$

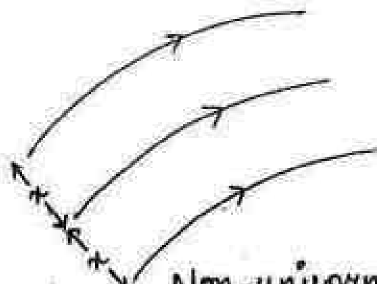
$$\boxed{|q| \propto \text{no of LEF}}$$

Q). Find $\frac{|q_1|}{|q_2|} = \frac{5}{3}$

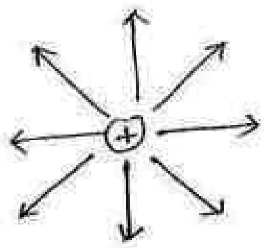




Uniform



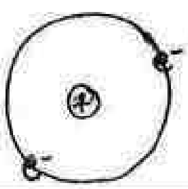
Non-uniform

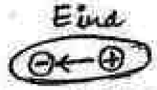


Non-uniform

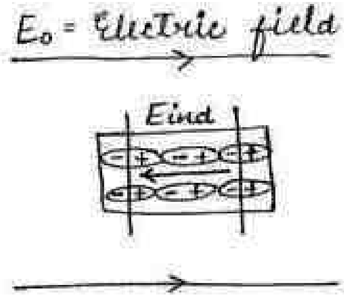
Polarisation of slab in electric field:

E_0 = electric field →





Atom (dipole)



$$E_{slab} = E_0 - E_{ind} = \boxed{\frac{E_0}{\epsilon_r}}$$

$$\therefore \; E_{ind} = E_0 \left(1 - \frac{1}{\epsilon_r}\right)$$

$$\text{and } q_{ind} = q_0 \left(1 - \frac{1}{\epsilon_r}\right)$$

For Metals :

$\because \; \epsilon_r = \infty \quad \therefore \; E_{metals} = \frac{E_0}{\epsilon_r} = 0$

$\therefore$ Inside the metals, $E_0 = E_{ind}$

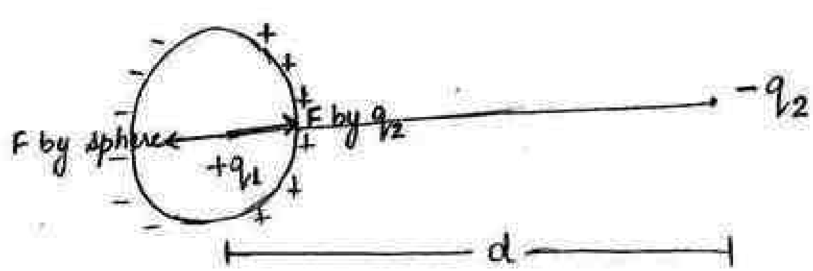
> It means net electric field inside the metals becomes zero. This property of metals can be used in electrostatistics shielding.

> If temperature of the slab is increased then random motion of dipole increases so, strength of the poles decreases. It means permittivity and dielectric constant also decreases.

Properties of Metals :

i) Electrostatic Shielding :

To protect a device or a charge from external electric field, it means it should be kept in a metallic chamber



force on q_1

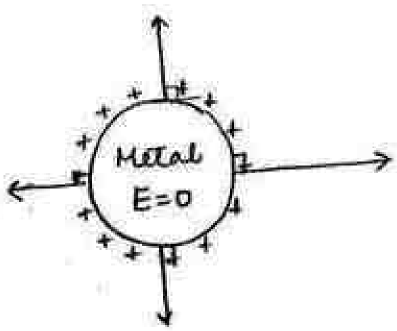
i) by q_2, $F = \frac{Kq_1q_2}{d^2} \neq 0$

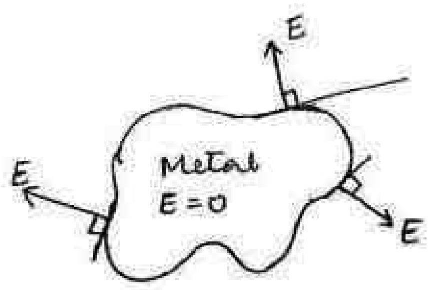
ii) by sphere, $F = \frac{Kq_1q_2}{d^2} \neq 0$

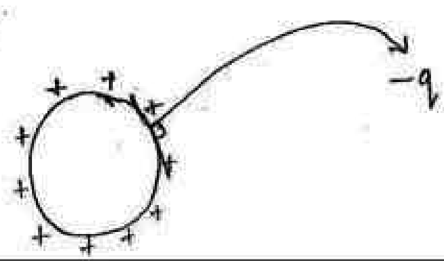
iii) Net force on $q_1 = 0$

ii) charge given to metallic sphere always appear on the surface of metals such that net electric field in metallic volume becomes zero.

iii) Net electric field just outside the metal must be perpendicular to local surface of metals so that there will be no electric field parallel ~~perpendicular~~ to the surface and there will be no motion in free e's







Article 48 Electric flux (ϕ_e)

> It is defined as dot product of intensity of electric field and area vector

$$\boxed{\phi_e = \vec{E} \cdot \vec{\Delta S}}$$

$\vec{\Delta S}$ = Area vector

> It is a scalar quantity.

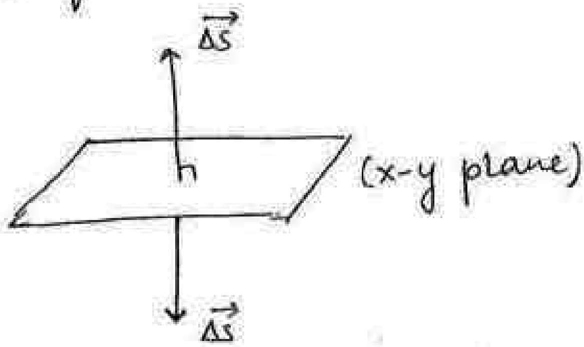
> SI unit = $\frac{N}{C} m^2$ = V-m

> Actually, it represent the effect of an electric field which passes through an given ar

Area vector ($\vec{\Delta S}$)

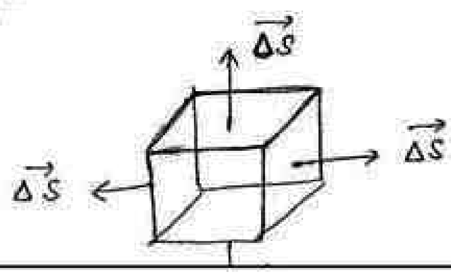
> Its direction is taken perpendicular to plane of given surface

- 2-D surface or open surface

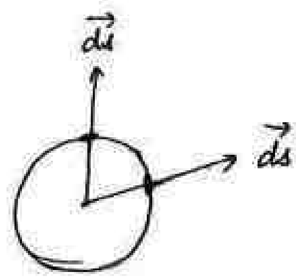


⊕ve is preferred

- 3-D surface : Its area vector ($\vec{\Delta S}$) must be outward from the vol^m.



or



* 2D surfaces are always open surfaces but 3D surfaces may be open or closed surface.

Q) A electric field $\vec{E} = (20\hat{i} + 30\hat{j})\,V/m$ is passing through a plain surface of area $40\,m^2$. calculate electric flux passing through this surface if surface lies in

i) X-Y plane

ii) Y-Z plane

iii) Z-X plane

i) $\phi_e = \vec{E} \cdot \vec{\Delta S}$

$= (20\hat{i} + 30\hat{j}) \cdot (40\hat{k}) = 0$

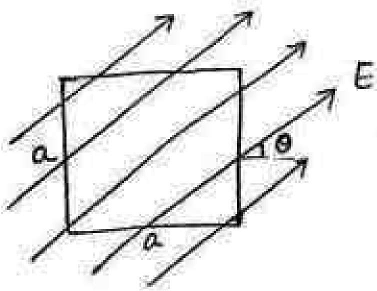
ii) $\phi_e = \vec{E} \cdot \vec{\Delta S}$

$= (20\hat{i} + 30\hat{j}) \cdot (40\hat{i}) = 800$ V-m

iii) $\phi_e = \vec{E} \cdot \vec{\Delta S}$

$= (20\hat{i} + 30\hat{j}) \cdot (40\hat{j}) = 1200$ V-m

Q)



Plane of square and electric field, both lies in the plane of paper, then find flux passing through the square

i) Ea^2 $\qquad \because \vec{E} \perp \vec{\Delta s}$

ii) $Ea^2 \cos\theta$

iii) $Ea^2 \sin\theta$

iv) zero

$$\phi_e = \int \vec{E} \cdot \vec{ds} \quad \left(\begin{array}{c} \text{for non-uniform } \vec{E} \\ \text{and} \\ \text{curved surface} \end{array} \right)$$

> In case of 3-D surface, there may be incoming flux and outgoing flux but are meaningless for 2-D surfaces.

Gauss's Theorem (or Gauss law)

> Total electric flux passing through a closed surface is the ratio of charge enclosed by the surface and permittivity of free space. it means,

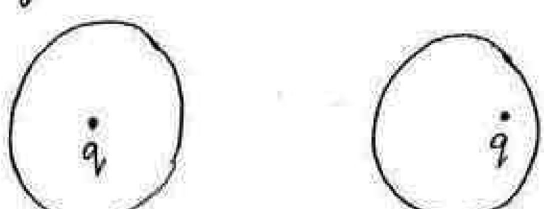
$$\boxed{\phi = \frac{q_{enclosed}}{\epsilon_0} = \frac{q_{en}}{\epsilon_0}}$$

$$\underset{\text{surface}}{\text{closed}} \longrightarrow \oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0}$$

<u>special Points</u> :

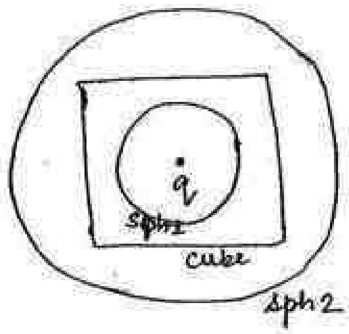
Total flux is independent of pos^n of charge within the closed surface (closed surface is an imaginary surface)

$\phi_{sph} = \dfrac{q}{\epsilon_0}$



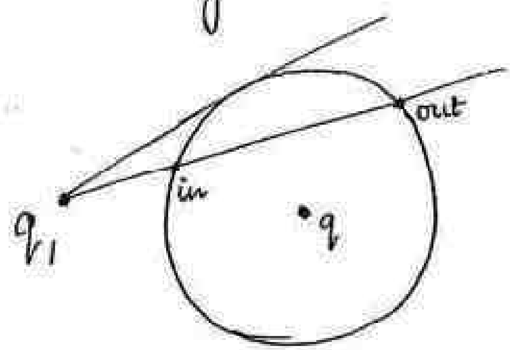
$\phi_{sph} = \dfrac{q}{\epsilon_0}$

> Total flux is independent of shape and size of closed surface



$$\phi_{sph1} = \phi_{cube} = \phi_{sph2} = \frac{q}{\epsilon_0}$$

> Total flux is independent of presence of outside charges.



$$\phi = \frac{q}{\epsilon_0} \quad \text{(sphere)}$$

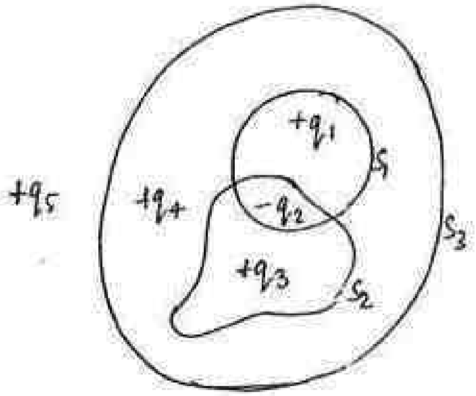
> $$\oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0}$$

here ~~ø~~ $\vec{E}$ is net electric field of all inside and outside charges.

* Gauss law is a fundamental law which does not require any prove.

Q) S_1, ~~ø~~ S_2, S_3 are closed surfaces.

Find ϕ_1, ϕ_2 and ϕ_3

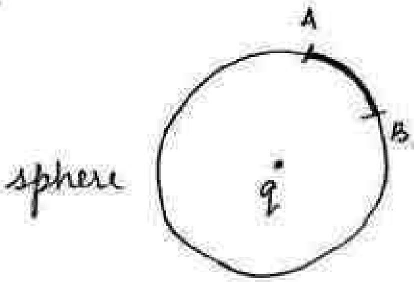


i) $\phi_1 = \frac{q_1 - q_2}{\epsilon_0}$

ii) $\phi_2 = \frac{q_3 - q_2}{\epsilon_0}$

iii) $\phi_3 = \frac{+q_1 + q_3 + q_4 - q_2}{\epsilon_0}$

Q)

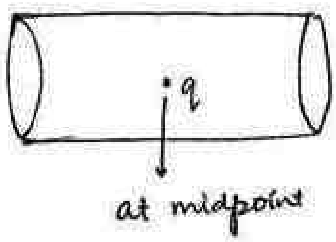


flux passing through part AB is ϕ, then find flux passing througt remaining part of spher

$\therefore$ q at centre $\quad \phi' = \frac{q}{\epsilon_0} - \phi$

$\because \left(\phi + \phi' = \frac{q}{\epsilon_0} \right)$

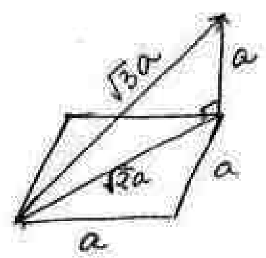
Q)



Flux passing through any one circular path is ϕ, then find flux passing through the curved area

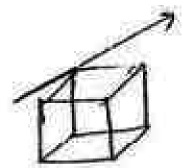
$\phi_{curved} = \frac{q}{\epsilon_0} - 2\phi$

Q) A long straight wire can be passed through imaginary cube of side length a. wire is uniformly charged with linear charged density λ. Find max. possible flux that can be passed through the cube



$$q_{en} = \lambda (L_{max})$$

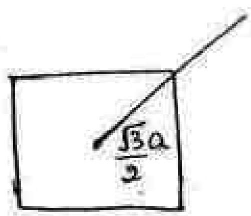
$$\lambda \sqrt{3} a$$



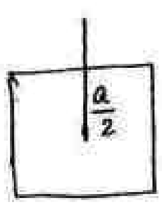
$$\therefore \phi_{max} = \frac{\sqrt{3} a \lambda}{\epsilon_0} \quad \text{and} \quad \phi_{min} = 0$$

$$dq \to 0$$

2)) In the above que if one end is fixed at the centre, then find ϕ_{max} and ϕ_{min}.

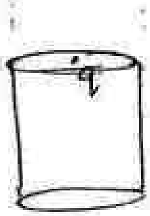


$$\therefore \phi_{max} = \frac{\sqrt{3} a \lambda}{2 \epsilon_0}$$



$$\phi_{min} = \frac{\lambda a}{2 \epsilon_0}$$

Q)

~~cyclo~~ 

cylinder.

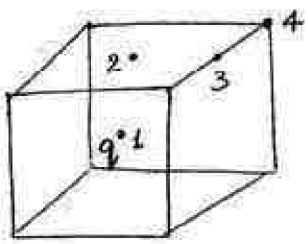
Calculate flux passing through cylinder

$\therefore \phi = \dfrac{q}{2\epsilon_0}$ $\qquad \phi_{\text{upper circle}} = 0$

* <u>Note</u>

If a charge is kept on a plane surface, then its flux passing through the surface will be

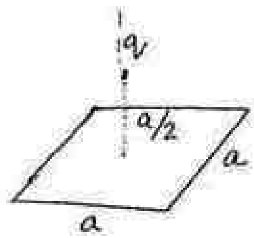
<u>Flux from the cube</u> :



Position of charge	Flux from the cube
1 → Body centre	q/ϵ_0
2 → Face centre	$q/2\epsilon_0$
3 → Edge centre	$q/4\epsilon_0$
4 → vertex	$q/8\epsilon_0$

1)) Flux from each face $= q/6\epsilon_0$

Q))

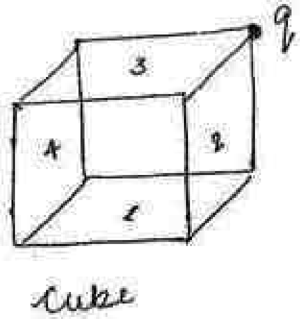


calculate flux from the square.

This square can be considered as a face of a cube having charge on its centre.

i.e. $q/6\epsilon_0$

Q))



Find:

$$\phi_1 = \phi/3 = \frac{q}{8\times 3\epsilon_0} = q/24\epsilon_0$$

$$\phi_2 = 0$$

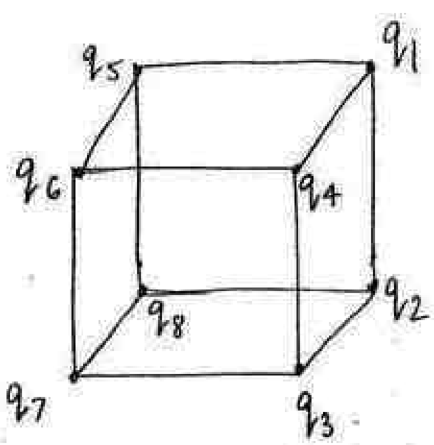
$$\phi_3 = 0$$

$$\phi_4 = q/24\epsilon_0$$

$$\phi_{front} = q/24\epsilon_0$$

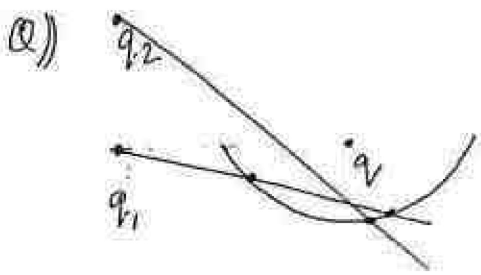
$$\phi_{back} = 0$$

Q))



calculate ϕ from the cube

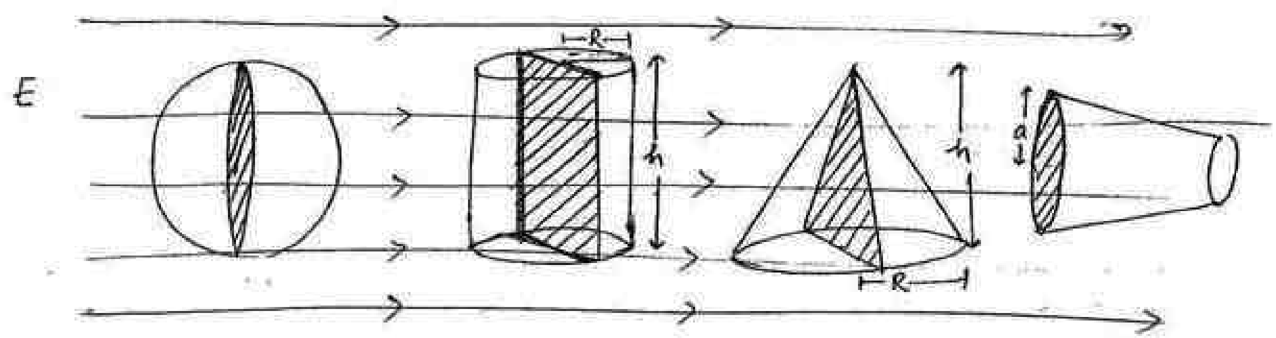
$$\frac{1}{8\epsilon_0}(q_1 + q_2 + q_3 \ldots\ldots + q_8)$$

Q) 

A charge q is placed in ~~open~~ the centre of open hemisphere which can change the flux of the hemisphere (q_1 or q_2)?

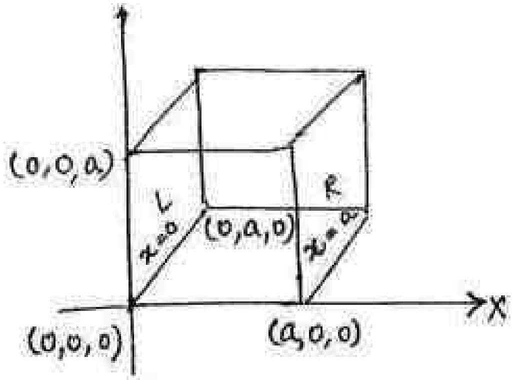
∵ line of q_2 will intersect at only one point ∴ it can change the flux whereas q_1 will intersect at ~~only~~ ~~one~~ two point ∵ its placed on the same level.

* If a closed surface having 0 charge is kept in a uniform electric field, then its flux must be zero.



ϕ_{in} :	$E(\pi R^2)$	$E(2Rh)$	$E\left(\frac{1}{2}\times 2R\times h\right)$	$E(\pi a^2)$
	$=\phi_{out}$	$=\phi_{out}$	$=\phi_{out}$	$=\phi_{out}$
ϕ_{net} :	0	0	0	0

Flux in non-uniform plane:

Q) 

given cube is kept in an electric field

$$\vec{E} = \alpha x \, (\hat{i})$$

here, α is constant. Find net flux passing through the cube and charge enclosed by the cube

$$\vec{E} = \alpha x \, (\hat{i})$$

$\therefore$ For left face, $x = 0$

so, $E = 0$ $\quad \therefore \phi_L = 0$

For right face, $x = a$

so $\vec{E} = \alpha(a)\,\hat{i}$ $\quad \therefore \phi_R = \vec{E} \, \vec{\Delta S}$

$$= \alpha a \, (\hat{i}) \cdot [a^2 (\hat{i})]$$

$$= \alpha a^3$$

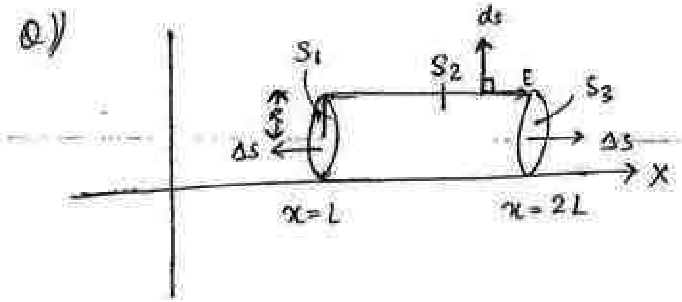
$\phi_{\text{remaining faces}} = 0 \quad \because (\vec{E} \perp \vec{\Delta S})$

$\phi_{net} = \alpha a^3$

$\because \phi_{net} = \dfrac{q_{en}}{\epsilon_0}$ $\quad \therefore q_{en} = \phi_{net} \, \epsilon_0$

$$= \alpha a^3 \epsilon_0$$

Q)



$\vec{E} = \alpha \cdot x^2 (\hat{i})$

$\alpha =$ constant

Find charge enclosed by the cylinder

On S_1 surface, $x = L$

$\vec{E} = \alpha (L^2) \hat{i}$

$\vec{\Delta S} = \pi R^2 (-\hat{i})$

$\phi_{S1} = \vec{E} \cdot \vec{\Delta S} = -\alpha L^2 (\pi R^2)$

On S_2 (curved) surface, ~~x=~~
on each point $\vec{E} \perp \vec{ds}$

$\therefore \phi_{S2} = 0$

On S_3 surface. $x = 2L$

$\vec{E} = \alpha (2L)^2 \hat{i} = 4L^2 \alpha \hat{i}$

$\vec{\Delta S} = \pi R^2 (\hat{i})$

$\therefore \phi_{S3} = 4\pi R^2 \alpha L^2$

$\therefore \phi_{net} = \phi_{S1} + \phi_{S2} + \phi_{S3}$

$= 3\alpha L^2 \pi R^2$

Now, $\phi = \dfrac{q_{en}}{\epsilon_0}$

$\therefore q_{en} = \epsilon_0 (3\alpha L^2 \pi R^2)$

Article 4C Application of Gauss's law

> Draw a Gaussian surface such that point of study lies on Gaussian surface

> Select shape of Gaussian surface such that angle b/w $\vec{E}$ and $\vec{ds}$ is 0°, 90° or 180°.

> Calculate charge enclosed by Gaussian surface

> Apply G.T. i.e. $\oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0}$

> Solve the dot product firstly, then integrate it

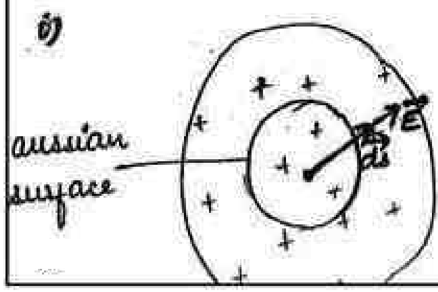
Electric field of Uniformly charged sphere

(Non-conducting, Non-metal, insulators, dielectrics) having charge in its volume.

Q) A sphere of radius R is uniformly charged in its vol^m with total charge Q. Find its electric field at r distance away from the centre for

i) $r < R$ (inside) ii) $r = R$ (on the sphere) iii) $r > R$ (outside)

i)



charge enclosed by G.S. $= q_{en} = \left(\dfrac{Q}{\frac{4}{3}\pi R^3}\right) \times \frac{4}{3}\pi r^3$

$$= \frac{Qr^3}{R^3}$$

Apply gaussian theorem,

$$\oint \vec{E}\cdot\vec{ds} = \frac{q_{en}}{\epsilon_0}$$

On each point of gaussian surface, $\vec{E}$ is parallel to $\vec{ds}$ and $|\vec{E}|$ = constant.

$$E\oint ds = \frac{q_{en}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Qr^3}{R^3\epsilon_0}$$

$$\therefore \quad E = \frac{Qr}{4\pi\epsilon_0 R^3} = \boxed{\frac{KQr}{R^3}} \quad r < R$$

$$\boxed{E \propto r}$$

In term of vol^m, (ρ)

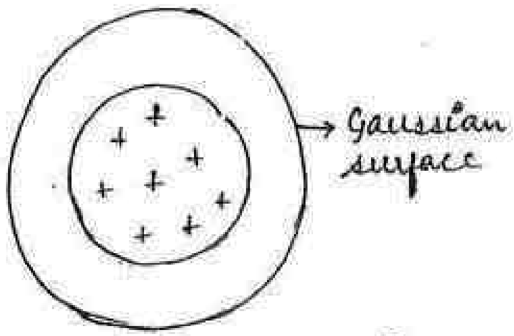
$$E = \frac{Q/3}{\left(\frac{4\pi R^3}{3}\right)}\,\frac{r}{\epsilon_0} \Rightarrow \quad \therefore \boxed{E = \frac{\rho r}{3\epsilon_0}}$$

divide by 3

ii) $r = R$

$$\boxed{E = \frac{KQ}{R^2}} \quad r = R$$

iii) $r > R$



$q_{en} = Q.$

(On each point of gaussian surface, $\vec{E}$ is parallel to $\vec{ds}$ and $|\vec{E}|$ = constant)

Apply G.T

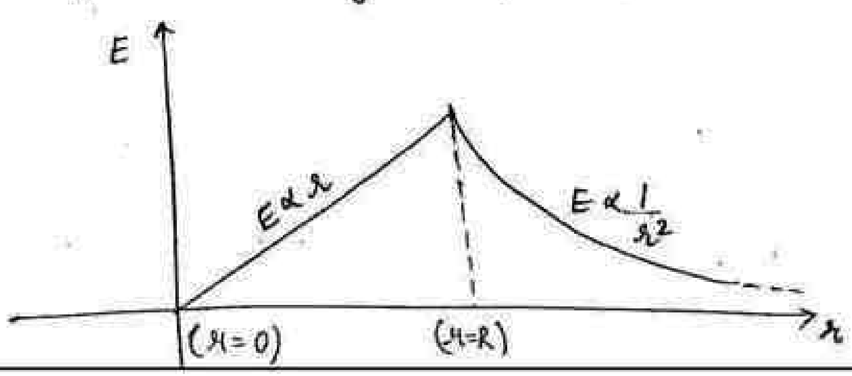
$$E \oint ds = \frac{Q}{\epsilon_0}$$

$$E\,(4\pi r^2) = \frac{Q}{\epsilon_0} = \frac{\cancel{KQ}}{\cancel{r^2}} \therefore E = \frac{Q}{4\pi r^2 \epsilon_0} = \boxed{\frac{KQ}{r^2}}$$

$$\therefore \boxed{E \propto \frac{1}{r^2}}$$

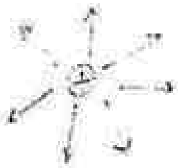
* Conclusion :

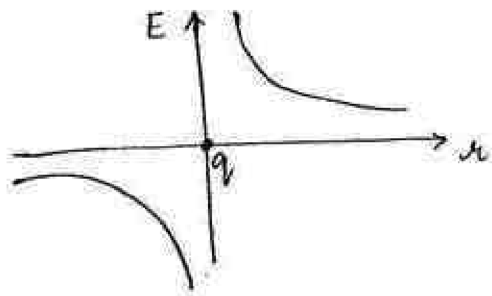
> If point of study is on the sphere or outside the sphere then results are similar to electric field of point charge. It means for these points, sphere can be considered as a point charge kept on its centre.



? Field of uniformly charged sphere is continuous at all points but field of point charge is discontinuous on its position.

(for point charge)





> Gauss Theorem can be applied for gravitation field also.

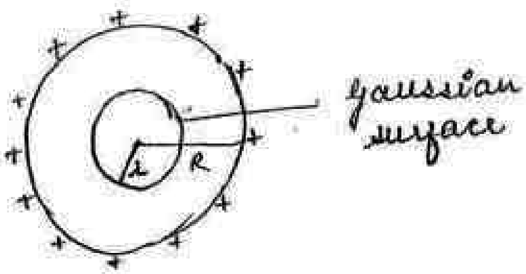
$$\oint \vec{g} \cdot \vec{ds} = M_{eq}\,(4\pi G)$$

$\vec{g}$ → gravitational field

> Coulomb's law follow inverse square law, which makes Gauss theorem applicable in electrostatic field.

E of Metallic sphere

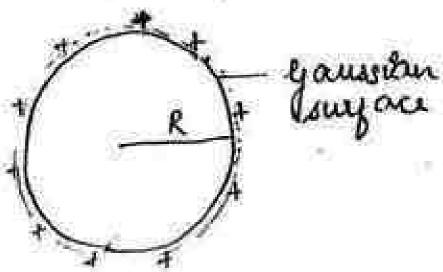
i) $r < R$ (inside)



$q_{en} = 0$

$\therefore E = 0$

ii) $r = R$



$q_{en} = Q$

Apply G.T.

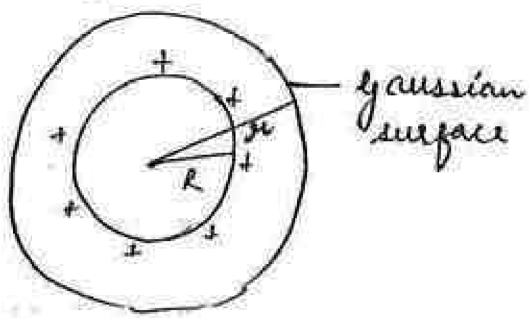
$$\oint \vec{E}\,\vec{ds} = \frac{q_{en}}{\epsilon_0}$$

(On every point of Gaussian surface, $\vec{E} \parallel \vec{ds}$ and $|\vec{E}|$ = constant)

$$E \oint ds = \frac{Q}{\epsilon_0} \qquad \therefore E = \frac{Q}{4\pi R^2 \epsilon_0}$$

$$\therefore E = \frac{KQ}{R^2} = \frac{Q}{4\pi R^2 \epsilon_0} = \frac{\sigma}{\epsilon_0}$$

iii) $r > R$



$q_{en} = Q$

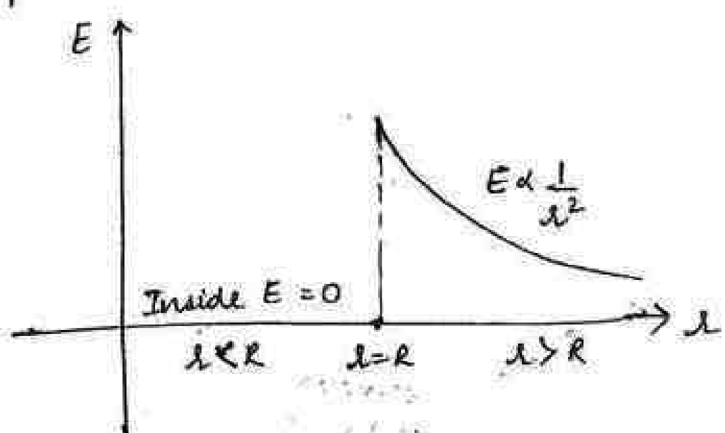
$$E \oint ds = \frac{Q}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$\therefore E = \frac{KQ}{r^2}$$

Conclusion :

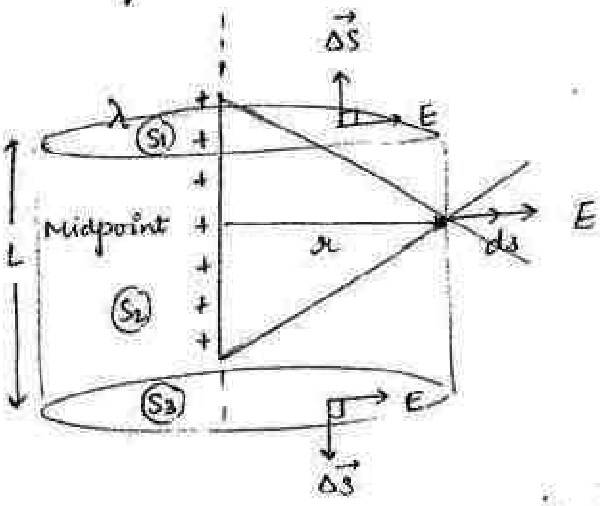
> These results are applicable for hollow metallic sphere (metallic shell), ~~and~~ solid metallic sphere and hollow non-metallic sphere



> It means field of metal is discontinuous on its surface

> $E_{surface} = \frac{KQ}{R^2} = \boxed{\frac{\sigma}{\epsilon_0}}$ for metals

Q) An infinite long wire is uniformly charged with linear charged density λ. Find its electric field, $\vec{E}$ at r distance away from the wire



$q_{en} = \lambda L$

Apply G.T.

$$\oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0}$$

$$\therefore \int_{S_1} \vec{E} \cdot \vec{ds} + \int_{S_2} \vec{E} \cdot \vec{ds} + \int_{S_3} \vec{E} \cdot \vec{ds} = \frac{q_e}{\epsilon_0}$$

$\because$ On surface S_1 and S_3 $\vec{E} \perp \vec{\Delta s}$

On S_2, $\vec{E} \parallel \vec{ds}$ $|\vec{E}| =$ constant

$$E \int_{S_2} ds = \frac{\lambda L}{\epsilon_0}$$

$$E\,(2\pi r L) = \frac{\lambda L}{\epsilon_0}$$

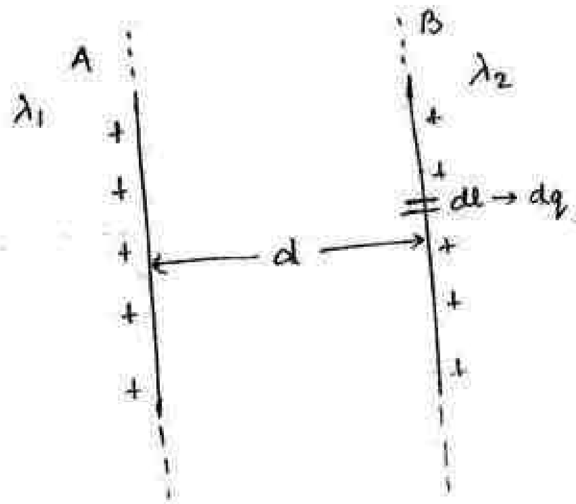
$$\therefore E = \frac{\lambda}{2\pi r \epsilon_0} = \boxed{\frac{2K\lambda}{r}} \quad \therefore \boxed{E \propto \frac{1}{r}}$$

Conclusion :

> Field of line charge $\boxed{E \propto \frac{1}{r}}$ and

Field of point charge $\boxed{E \propto \frac{1}{r^2}}$

It means, field of line charge decreases to lower rate w.r.t distance

Q) 

Find force exerted on unit length of a wire.

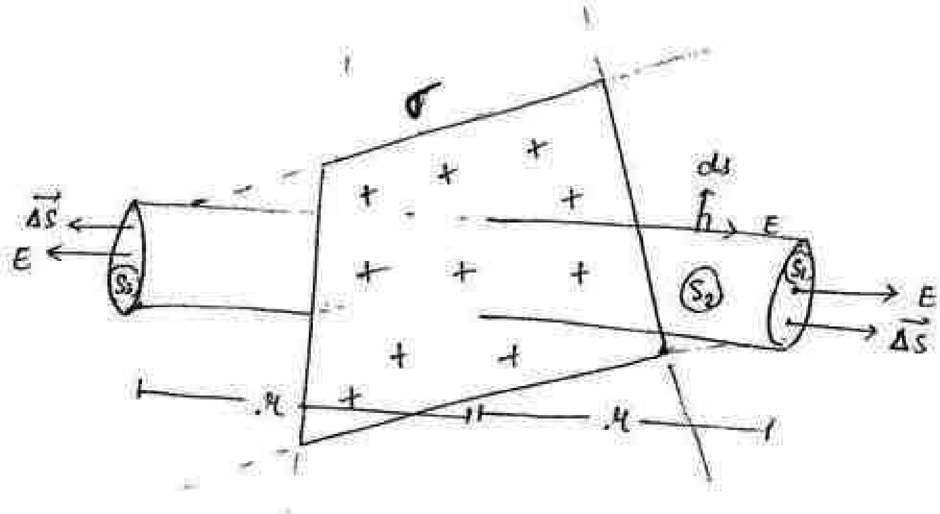
$$\because \ F = qE \quad \left(\because E_{line} = \frac{2K\lambda}{r}\right)$$

$$\therefore F = (\lambda_2 dl)\left(\frac{2K\lambda_1}{d}\right) = \text{force on } dl \text{ length}$$

$$\therefore \boxed{\frac{F}{dl} = \frac{2K\lambda_1\lambda_2}{d}}$$

> Electric field of a uniformly charged surface/ layer/ sheet/ Plane / thin plate.

on next page ⇒



$$q_{en} = \sigma \, \Delta S$$

Apply G.T.

$$\oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0}$$

$$\int_{S1} \vec{E} \cdot \vec{ds} + \int_{S2} \vec{E} \cdot \vec{ds} + \int_{S3} \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0}$$

On S_2 $\quad \vec{E} \perp ds$

$\therefore$ On S_1 and S_3 $\quad |\vec{E}| = \text{constant}$

$$E \int_{S1} ds + E \int_{S3} ds = \frac{\sigma \, \Delta S}{\epsilon_0}$$

$$E(\Delta S) + E(\Delta S) = \frac{\sigma \, \Delta S}{\epsilon_0}$$

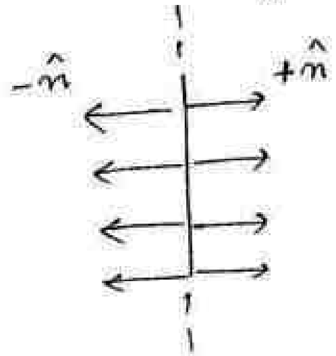
$$\therefore 2E = \frac{\sigma \cancel{\Delta S}}{\epsilon_0}$$

$$\therefore \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

Conclusion :

> Electric field of infinite plane/very large plane is independant of distance., it means its electric field is uniform.

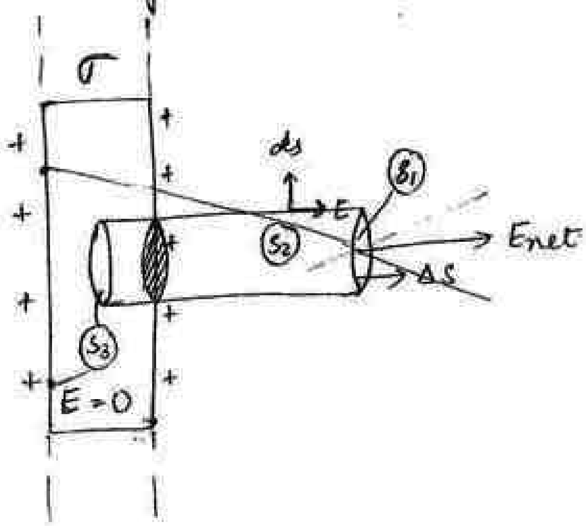
> Direction of electric field :



$$\vec{E} = \frac{\sigma}{2\epsilon_0} (\pm \hat{n})$$

$\hat{n} = \perp$ to the plane

Electric field of large metallic plate :



$q_{en} = \sigma \Delta S$

Apply G.T.

$$\oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0}$$

$$\int \vec{E} \vec{ds} + \int \vec{E} \cdot \vec{ds} + \int \vec{E} \cdot \vec{ds} = \frac{q_{en}}{}$$

on S_3 , $\vec{E} = 0$

on S_2, $\vec{E} \perp \vec{ds}$

on S_1, $\vec{E} \parallel \vec{ds}$, $|\vec{E}|$ = constant

$$E \int_{S_1} ds = \frac{\sigma \Delta S}{\epsilon_0}$$

$$E(\Delta S) = \frac{\sigma \Delta S}{\epsilon_0}$$

$$\therefore \boxed{E = \frac{\sigma}{\epsilon_0}}$$ Applicable for all metals.

conclusion :

> Electric field of metallic plate is double of electric field of single layer of charge ~~density~~ having same density

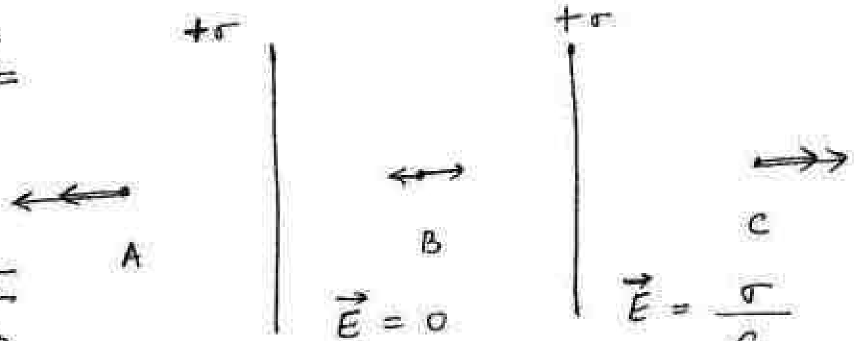
> If metal word is not mentioned then plate should be considered as single layer of charge.

> $$\oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0}$$

here $\vec{E}$ is net electric field of all inside and outside charges.

Q) ∞ planes are given, Find E_A, E_B and E_C.

Case I



i.e. $\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$

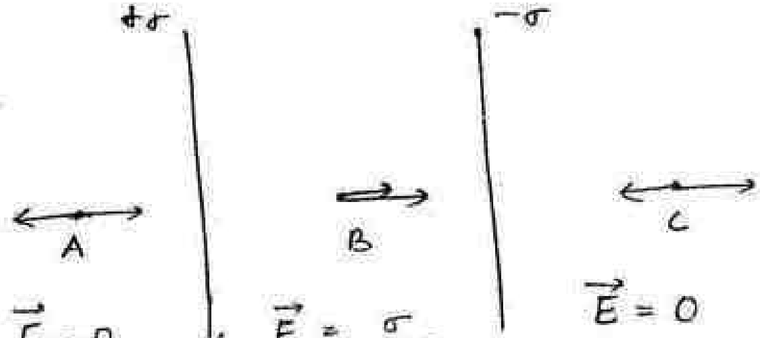
$\frac{\sigma}{\epsilon_0} + \frac{\sigma}{2\epsilon_0}$

$\Rightarrow \frac{\sigma}{\epsilon_0}$

$\vec{E} = 0$

$\vec{E} = \frac{\sigma}{\epsilon_0}$

Case II

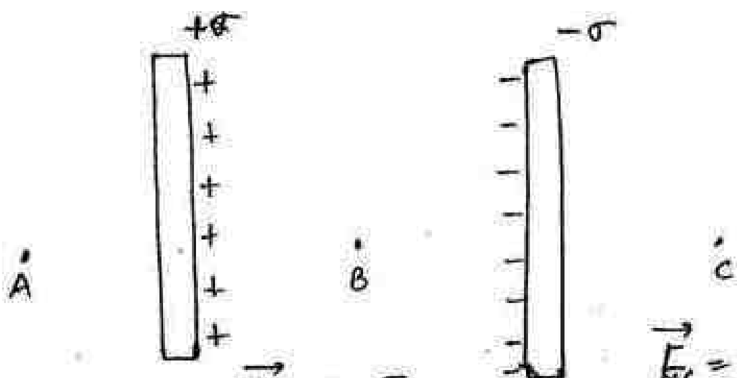


$\vec{E} = 0$

$\vec{E} = \frac{\sigma}{\epsilon_0}$

$\vec{E} = 0$

Case III



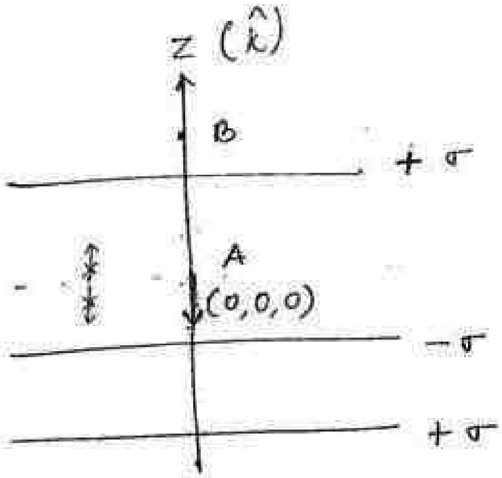
$\vec{E}_A = 0$

$\vec{E}_B = \frac{\sigma}{\epsilon_0}$

$\vec{E}_C = 0$

here each plate behaves as single layer of charge.

Q)

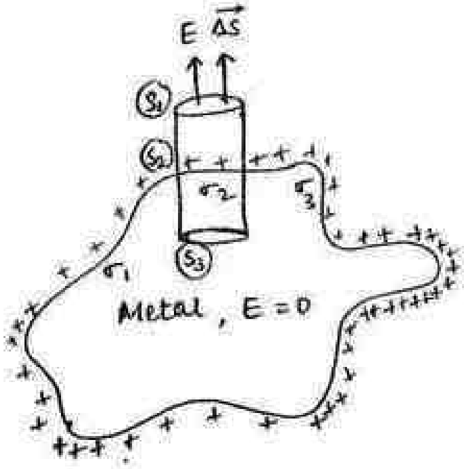


Find E_A and E_B

$$\therefore \quad E_A = \frac{\sigma}{2\epsilon_0}(-\hat{k})$$

$$E_B = \frac{\sigma}{2\epsilon_0}(\hat{k})$$

Q) Prove that net electric field just outside any type of metallic surface is always $\frac{\sigma}{\epsilon_0}$ here σ is local surface charge density.



$$q_{en} = \sigma_2 (\Delta S)$$

Apply G.T.

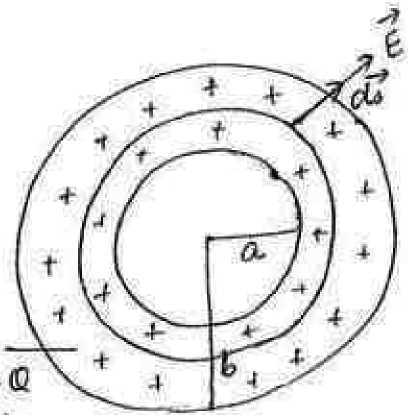
$$\oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0}$$

$$E \int_{S_1} \vec{ds} = \frac{\sigma_2 \Delta s}{\epsilon_0}$$

Here $\vec{E}$ is net electric field of all inside and outside charges

$$\therefore \boxed{\vec{E} = \frac{\sigma_2}{\epsilon_0}}$$

Q)



Total charge = Q (uniform)

Find electric field at r distance away from the centre.

$a < r < b$.

$$\boxed{q_{en} = \rho_1 V_1 + \rho_2 V_2}$$

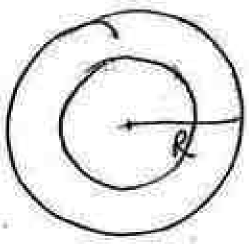
$$q_{en} = (0)\,V_1 + \frac{Q}{\left(\frac{4}{3}\pi(b^3 - a^3)\right)} \times \frac{4}{3}\pi(r^3 - a^3)$$

By Gauss law:

$$E(4\pi r^2) = \frac{q_{en}}{\epsilon_0}$$

$$\therefore E = \frac{Q(r^3 - a^3)}{(b^3 - a^3)\,4\pi r^2 \epsilon_0}$$

Q) A sphere of radius R is non-uniformly charged with volm charged density $\rho = \frac{\rho_0}{r}$, ρ_0 = constant and r = distance from the centre. Find its electric field for $r < R$.



$$\left\{\begin{aligned} &\because V = \tfrac{4}{3}\pi r^3 \\ &\therefore dV = \tfrac{4}{3}\pi 3r^2 \\ &= \boxed{4\pi r^2} \end{aligned}\right\}$$

$$dq = \rho\, dV$$

$$= \rho\,(4\pi r^2\, dr)$$

$$= \frac{\rho_0}{r}(4\pi r^2)\, dr$$

$$q_{en} = \int_0^r \rho_0 (4\pi r)\, dr$$

$$= \cancel{4\pi\rho} \; 4\pi\rho_0 \left(\frac{r^2}{2}\right)_0^r = 2\pi\rho_0 r^2$$