Chapter 4 Geometry

Introduction to Euclid's Geometry

- **Euclid's Definitions, Axioms and Postulates:** A solid has shape, size, position, and can be moved from one place to another. A solid has three dimensions, a surface has two, a line has one and a point has none. Euclid summarised these statements as definitions.

  Euclid assumed certain properties, which were not to be proved. He divided them into two types: axioms and postulates.

  Euclid divided his axioms into two categories, calling the first five postulates and the next five "common notions." The distinction between postulates and common notions is that the postulates are geometric in character, whereas common notions were considered by Euclid to be true in general.

- **Common Notions or (axioms)**

  i. Things which equal the same thing also equal one another.

  ii. If equals are added to equals, then the wholes are equal.

  iii. If equals are subtracted from equals, then the remainders are equal.

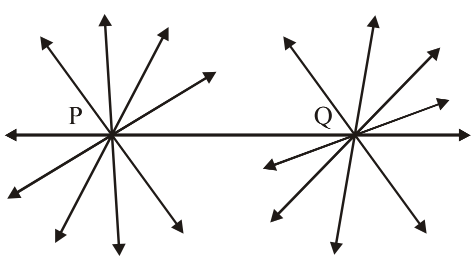
  iv. Things which coincide with one another equal one another.

  v. The whole is greater than the part.

- **Postulate 1 :** A straight line may be drawn from any one point to any other point

  We state this result in the form of an axiom as follows :

  Axiom 1 : Given two distinct points, there is a unique line that passes through them.



- **Postulate 2 :** A terminated line can be produced indefinitely.

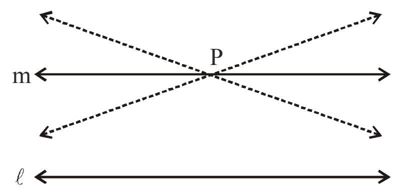
  The second postulate says that a line segment can be extended on either side to form a line.

- **Postulate 3 :** A circle can be drawn with any centre and any radius.

- **Postulate 4 :** All right angles are equal to one another.

- **Postulate 5 :** If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

- **Equivalent versions of Euclid's fifth Postulate :** Given a straight line, and a point P not on that line, there exists at most one straight line passing through P that is parallel to the given line



  This result can also be stated in the following form:

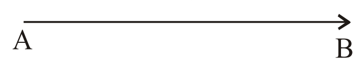
  Two distinct intersecting lines cannot be parallel to the same line.

- **Line Segment :**

  Given two points A and B on a line ℓ, the connected part (segment) of the line with end points at A and B, is called the line segment AB.

- **Ray :** A line segment AB when extended indefinitely in one direction is called the ray $\overrightarrow{AB}$. It has one end point A.

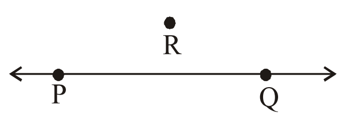
  A ray has no definite length. A ray cannot be drawn on a piece of paper, it can simply be represented.



- **Collinear points :** Three or more points are said to be collinear if there is a line which contains all to them.

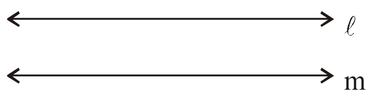
  P, Q and R in fig (i) are collinear while in fig. (ii) are non-collinear.

(i)



(ii)

- **Intersecting lines :** Two lines are intersecting if they have a common point. The common point is called the "point of intersection".
- **Parallel lines : (Non-Intersecting lines) :** The straight lines, which lie in the same plane and do not meet at any point on producing on either side, are called parallel lines. The distance between the two parallel lines always remains the same. If two straight lines are parallel to any other line, then they are parallel to each other also.



- **Angle :** An angle is the union of two non-collinear rays with a common initial point.

Types of Angles :

(i) **Right angle :** An angle whose measure is 90° is called a right angle.

(ii) **Acute angle :** An angle whose measuring is less than 90° is called an acute angle.

(iii) **Obtuse angle :** An angle whose measure is more than 90° but less than 180° is called an obtuse angle.

(iv) **Straight angle :** An angle whose measure is 180° is called a straight angle.

(v) **Reflex angle :** An angle whose measure is more that 180° is called a reflex angle.

(vi) **Complementary angles :** Two angles, whose sum measure is 90° are called complementary angles.

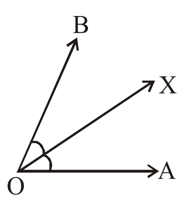
(vii) **Supplementary angles :** Two angles, the sum of whose measures is 180°, are called the supplementary angles.

(viii) **Adjacent angles :** Two angles are called adjacent angles, if

(a) they have the same vertex

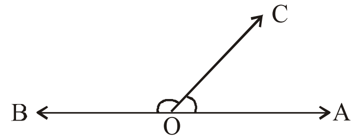
(b) they have a common arm and

(c) non common arms are on either side of the common arm.

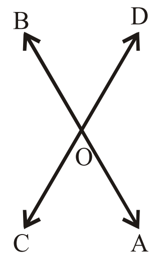


∠ AOX and ∠ BOX are adjacent angles, OX is common arm, OA and OB are common arms and lies on either side of OX.

(ix) **Linear pair of angles :** Two adjacent angles are said to form a linear pair of angles, if their non common arms are two opposite rays. If the sum of two adjacent angles is 180°, they are said to form a linear pair.

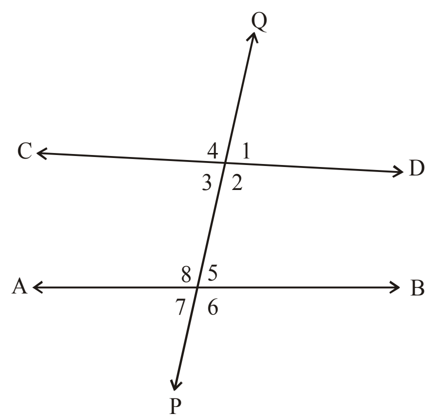


(x) **Vertically opposite angles :** Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays.



∠ AOC & ∠ BOD form a pair of vertically opposite angles. Also ∠ AOD and ∠ BOC form a pair of vertically opposite angles.

- **Transversal :** A line which intersects two or more given lines at distinct points is called a transversal of the given lines. Straight lines AB and CD are cut by transversal PQ.



- **Corresponding Angles :** Two angles on the same side of a transversal are known as the corresponding angles if both lie either above the two lines or below the two lines, in figure ∠ 1 & ∠ 5, ∠ 4 & ∠ 8, ∠ 2 & ∠ 6, ∠ 3 & ∠ 7 are the pairs of corresponding angles.
- **Alternate Interior Angles :** ∠ 3 & ∠ 5, ∠ 2 & ∠ 8, are the pairs of alternate interior angles.
- **Consecutive Interior Angles : (Allied or Co-interior Angles)**

  The pair of interior angles on the same side of the transversal are called pairs of consecutive interior angles. In figure ∠ 2 & ∠ 5, ∠ 3 & ∠ 8, are the pairs of consecutive interior angles.

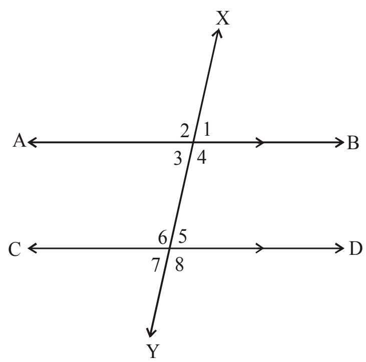
- **Corresponding Angles Axiom :** In a transversal intersects two parallel lines, then each pair of corresponding angles are equal. Conversely, if a transversal intersects two lines, making a pair of equal corresponding angles, then the lines are parallel.

Alternate Exterior Angles : (i) $\angle 4 = \angle 6$ (ii) $\angle 1 = \angle 7$

If AB parallel to CD and transversal XY cuts them (figure), then :

(i) Alternate angles are equal.
i.e. $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$.

(ii) Corresponding angles are equal.



i.e. $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$

(iii) Co-interior angles are supplementary
i.e. $\angle 3 + \angle 6 = 2$ rt. angles and $\angle 4 + \angle 5 = 2$ rt. angles.

- **Axiom-1 :** If a ray stands on a line, then the sum of two adjacent angles so formed is 180°.
- **Axiom-2 :** If the sum of two adjacent angles is 180°, then the non-common arms of the angles are in a straight line.
- **Axiom-3 :** If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.
- **Axiom-4 :** If a transversal intersects two lines, such that a pair of corresponding angles is equal, then the two lines are parallel to each other.
- **Theorem-1 :** If two lines intersect each other, then the vertically opposite angles are equal.
- **Theorem-2 :** If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal,.
- **Theorem-3 :** If a transversal intersects two lines, such that a pair of alternate interior angles is equal, then the two lines are parallel.
- **Theorem-4 :** If a transversal intersects two lines, then each pair of interior angles on the same side of the transversal is supplementary.
- **Theorem-5 :** If a transversal intersects two lines, such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.
- **Theorem-6 :** The sum of the angles of a triangle is 180°.
- **Theorem-7 :** If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Triangle

- **Criteria for Congruence of Triangles :**
  - **(i) Side-Angle-Side Rule (SAS Rule) :** Two triangles are congruent, if any two sides and the included angle of one triangle are equal to any two sides and the included angle of the other triangle.
  - **(ii) Angle-Side-Angle Rule (ASA Rule) :** Two triangles are congruent if any two angles and the included side of one triangle are equal of the two angles and the included side of the other triangle.
  - **(iii) Angle-Angle-Side Rule (AAS Rule) :** If any two angles and a side of one triangle are equal to two angles and the corresponding side of the other triangle, the two triangles are congruent.

    Statement 1 : (SSS Congruence Rule) : If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

    Statement 2 : (RHS Congruence Rule) : If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

Inequalities in a Triangle :

Statement 1: If two sides of a triangle are unequal, the angle opposite to the longer size is larger (or greater).

Statement 2: In any triangle, the side opposite to the larger (greater) angle is longer.

Statement 3 : The sum of any two sides of a triangle is greater than its third side.

- **Similarity of Triangles :** Two triangles are similar, if
  - (i) their corresponding angles are equal and
  - (ii) their corresponding sides are in the same ratio (or proportion)
- **Criteria for Similarity of Triangles**
  - (i) **AAA Similarity Criterion or (Equi - angular criterion)** The corresponding angles of two triangles are equal, then their corresponding sides are proportional and hence the triangles are similar.
  - (ii) **SSS Similarity Criterion :** In this criterion, the corresponding sides of two triangles are proportional, then their corresponding angles are equal. Hence the triangles are said to be similar.
  - (iii) **SAS Similarity Criterion :** In this case, if one angle of a triangle is equal to one angle of the other and the sides including these angles are proportional.
- **Basic Proportionality Theorem or Thale's Theorem :** "If a line is drawn parallel to one side of a triangle, it divides the other two sides in the same ratio"
- **Converse of B.P. Theorem** : "If a line divides any two sides of a triangle in the same ratio, the line parallel to the third side".
- **Areas of Two Similar Triangles :**

Theorem 1 : The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Theorem 2 : The areas of two similar triangles are in the ratio of the square of corresponding altitudes.

Theorem 3 : Pythagoras Theorem or Bodhayan Theorem : "In a right angled triangle, the square of the hypotenuse is equal to the sum of the other two sides".

Theorem 4 : Converse of Pythagoras Theorem : "In a triangle, the square of the longer side is equal to the sum of the squares of the other two sides then the triangle is right angled triangle".

Theorem 5 : Bisector of an Angle of Triangle by Application of B.P.T. : The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

Quadrilaterals

A quadrilateral is a closed figure obtained by joining four point (with no three points collinear) in an order.

Every quadrilateral has :

(i) Four vertices, (ii) Four sides,

(iii) Four angles and (iv) Two diagonals.

the sum of the angles of a quadrilateral is 360° (4-right angles)

- **Types of Quadrilaterals**

(i) Trapezium : It is quadrilateral in which one pair of opposite sides are parallel.

(ii) Parallelogram : It is a quadrilateral in which both the pairs of opposite sides are parallel.

(iii) Rectangle : It is a quadrilateral whose each angle is 90°. Rectangle ABCD is a parallelogram also.

(iv) Rhombus : It is a quadrilateral whose all the sides are equal.

(v) Square : It is a quadrilateral whose all the sides are equal and each angle is 90°.

(vi) Kite : It is a quadrilateral in which two pairs of adjacent sides are equal.

Note:

(i) Square, rectangle and rhombus are all parallelograms.

(ii) Kite and trapezium are not parallelograms.

(iii) A square is a rectangle.

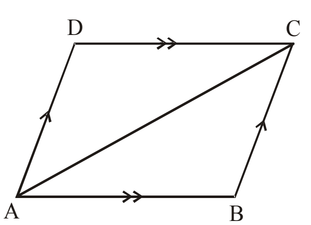
(iv) A square is a rhombus.

(v) A parallelogram is a trapezium.

- **Parallelogram :** A parallelogram is a quadrilateral in which opposite sides are parallel. It is denoted by ▱.

- **Properties of Parallelogram :**

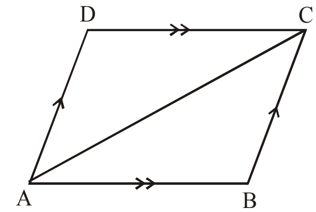
(i) A diagonal of a parallelogram divides it into two congruent triangles.



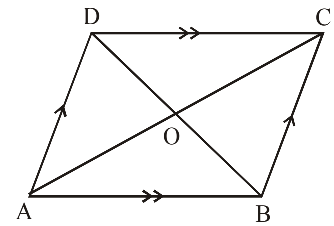
Thus, in the figure : $\Delta ABC \cong \Delta ADC$

(ii) The opposite sides of a parallelogram are equal. Thus, in the figure : AB = DC and AD = BC.

(iii) The opposite angles of a parallelogram are equal. Thus, in figure : $\angle A = \angle C$ and $\angle B = \angle D$.

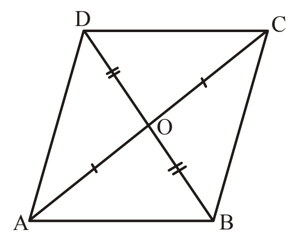


(iv) The diagonals of a parallelogram bisect each other.



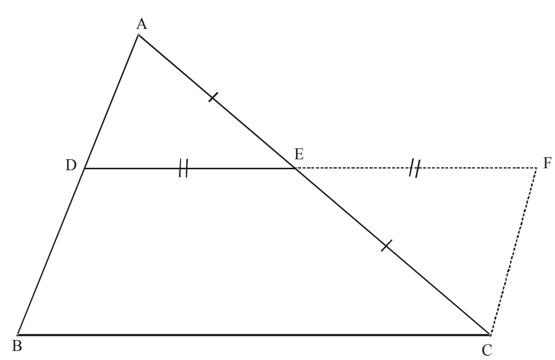
Thus, in the figure, AO = CO and BO = DO.

- **Theorem-1:** If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.
- **Theorem-2:** If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.
- **Theorem-3:** If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.



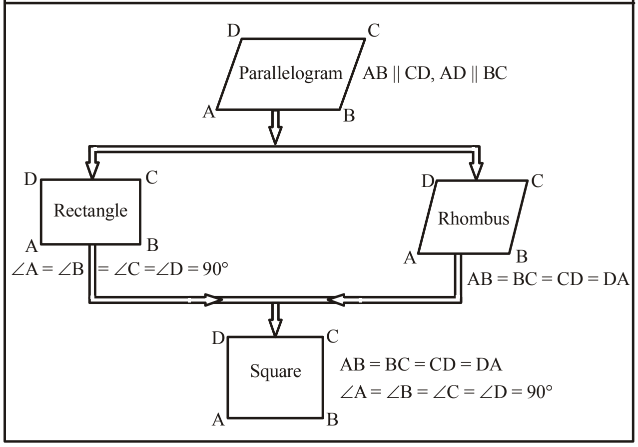
- **Mid Point Theorem (Basic Proportionality theorem)**

Statement 1: The line segment joining the mid-points of any two sides of a triangle is parallel to the third side



Statement 2 : The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

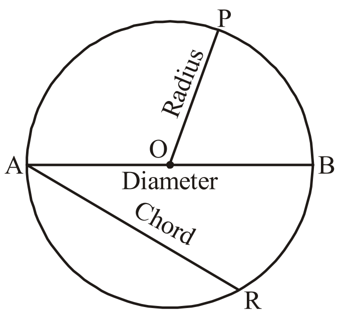
Rectangle, Rhombus and Square : At a glance



Circles

A circle is a simple closed curve, all the points of which are at the same distance from a given fixed point.

- **Parts of a Circle**



(i) Centre: The fixed point in the plane which is equidistant from every point on the boundary of the circle is called centre. In the adjoining figure, O is the centre of the circle.

(ii) Radius: The fixed distance between the centre and any point of the circle is called radius. In figure, $\overline{OP}$ is a radius.

(iii) Chord: A line segment joining any two points on a circle is called a chord of the circle. In figure. $\overline{AR}$ is a chord.

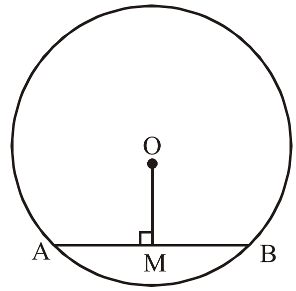
(iv) Diameter: A chord that passes through the centre of a circle is called diameter of the circle. In figure, $\overline{AB}$ is a diameter. The length of a diameter = 2 × radius. In a circle, diameter is the longest chord.

(v) Circumference: The distance around a circle is called the circumference. Circumference of a circle is the perimeter of that circle.

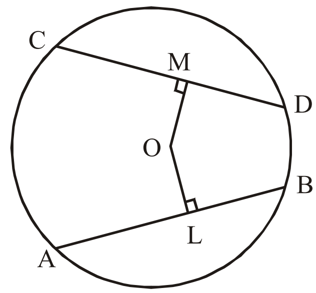
(vi) Arc: A part of a circumference is called an arc. In the above figure, the curve line AR is an arc of the circle. It is written as $\overset{\frown}{AR}$.

Properties of Circle

- In a circle perpendicular drawn from the centre to a chord bisects the chord.
- If M is the mid-point of AB $\Rightarrow$ $OM \perp AB$.
- In a circle; if a line joining mid-point of a chord to the centre is perpendicular to the chord.



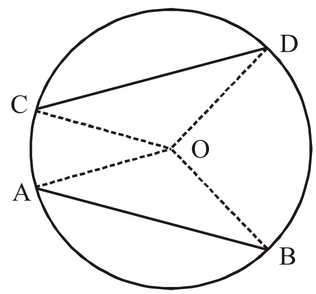
- If $OM \perp AB \Rightarrow AM = MB$.
- Equal chords of a circle are equidistant from the centre.
- Chords equidistant from the centre are equal in length.



If $AB = CD \Rightarrow OL = OM$

If $OL = OM \Rightarrow AB = CD$

- Equal chords of a circle subtend equal angles at the centre.
- Chords of a circle which subtend equal angles at the centre are equal.

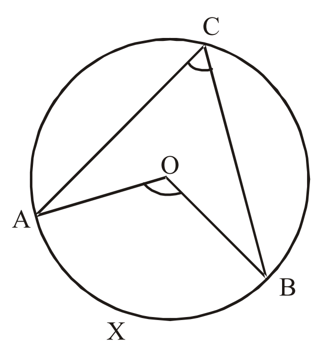


If $AB = CD \Rightarrow \angle AOB = \angle COD$

If $\angle AOB = \angle COD \Rightarrow AB = CD$

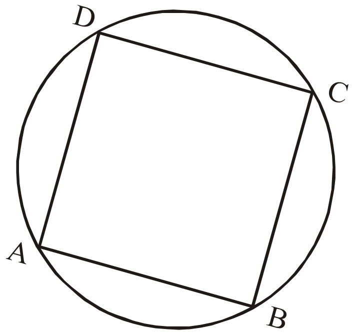
- Angle subtended by an arc at the centre is double the angle subtended by it on the remaining part of the circle.

$$\angle AOB = 2\angle ACB$$



Angle subtended by the arc AB at the centre is $\angle AOB$ and the angle subtended by the same arc to the remaining part of the circle is $\angle ACB$.

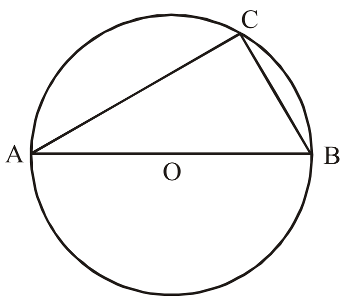
- A quadrilateral is called cyclic if all the four vertices lie on a circle. And the four vertices are called the concyclic points.



- Sum of opposite internal angles of a cyclic quadrilateral is 180°.

  $\square$ ABCD is a cyclic quadrilateral.

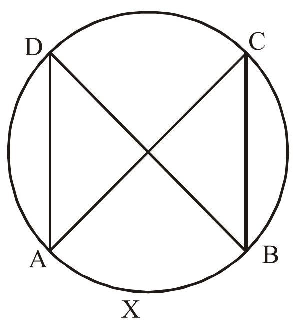
  $\Rightarrow$ $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$.
- Angle made in semi-circle is equal to 90°



$\Rightarrow$ $\angle ACB$ is angle of semicircle.

$\Rightarrow$ $\angle ACB = 90°$

- Angles in the same segment of a circle are equal. Arc AB subtends $\angle ACB$ and $\angle ADB$ is the same segment



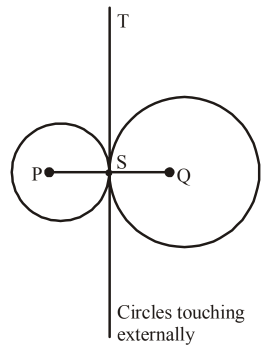
$\Rightarrow$ $\angle ACB = \angle ADB$.

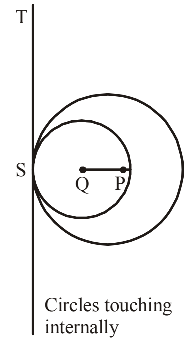
- **Secant of a Circle :** A line which intersects a circle in two distinct points is called a secant of the circle.
- **Tangent to a circle :** A line which meets a circle exactly at one point is called a tangent to the circle.

  The point where the line touches the circle is called its point of contact. In the figure 'A' is the point of contact.

- **Common Tangent :** A line touching two circles is called a common tangent.

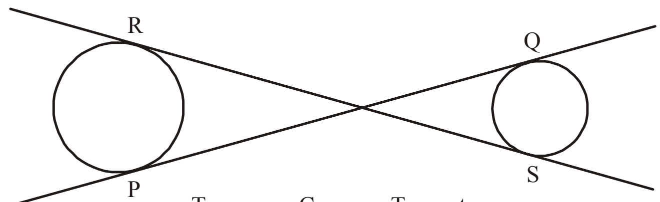
  Two circles may touch internally or externally.





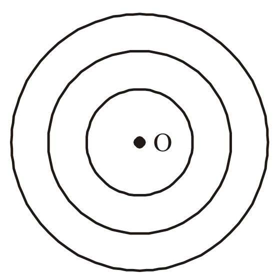
(i) ST is a common tangent.
(ii) PQ is the line of centres.
(iii) S is the point of contact.

- **Transverse Common Tangent :** If the circles lie on opposite sides of the common tangent, the tangent is called a transverse common tangent.



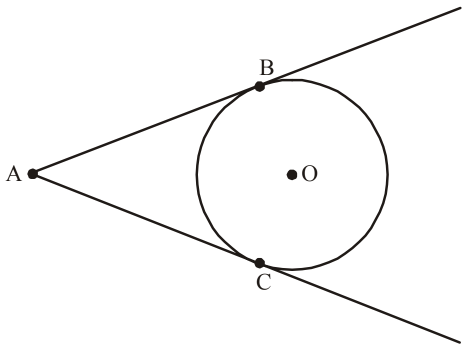
Transverse Common Tangents

- **Concentric Circles :** Two or more circles are said to be concentric if and only if they have the same centre and different radii.



Remarks :

1. All points other than the point of contact of a tangent to a circle lie outside the circle.
2. No tangent can be drawn to a circle through a point inside the circle.
3. Not more than one tangent can be drawn to a circle at a point on the circumference of the circle.
4. Two tangents can be drawn to a circle from a point outside the circle.



AB and AC are two tangents drawn from the point A to the circle with centre O.

Note : The distance AB (or AC) is called the length of tangent.

- **Theorems Related to Tangent to a Circle**

  Theorem 1 : The tangent at any point of a circle is perpendicular to the radius through the point of contact.

  The line containing the radius through the point of contact is also sometimes called the 'normal' to the circle at the point.

  Theorem 2 : The lengths of tangents drawn from an external point to a circle are equal.

Area of Parallelogram and triangles

- **Parallelograms on the same base and between the same parallels**

  Statement : Parallelograms on the same base and between the same parallels are equal in area.
  Remark : Area of a parallelogram is the product of its any side and the corresponding altitude.
  Statement : If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

- **Triangles on the same base and between the same parallels**

  Two triangles on the same base (or equal bases) and between the same parallels are equal in area. Area of a triangle is half the product of its base (or any side) and the corresponding altitude (or height).
  Remark : Two triangles with same base (or equal bases) and equal areas will have equal corresponding altitudes.
  Theorem : Two triangles having the same base (or equal bases) and equal areas lie between the same parallels.

Exercise 1

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) out of which only one is correct.*

1. If two lines intersected by a transversal, then each pair of corresponding angles so formed is –

(1) Equal (2) Complementary
(3) Supplementary (4) None of these

2. An angle is 14° more than its complementary angle then angle is –

(1) 38° (2) 52°
(3) 50° (4) None of these

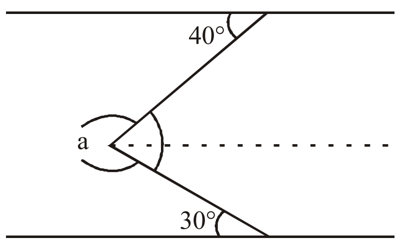
3. If one angle of triangle is equal to the sum of the other two then triangle is –

(1) acute triangle (2) obtuse triangle
(3) right triangle (4) None

4. If the supplement of an angle is three times its complement, then angle is –

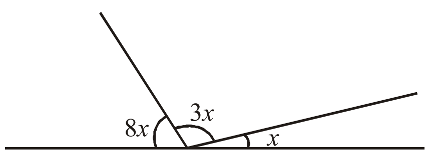
(1) 40° (2) 35°
(3) 50° (4) 45°

5. In the figure, angle a is



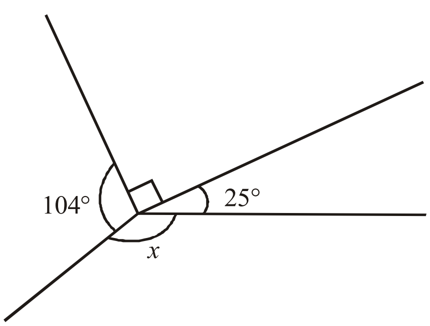
(1) 290° (2) 70°
(3) 105° (4) 45°

6. Value of $\angle x =$



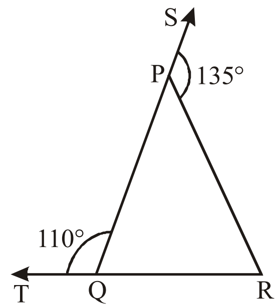
(1) 270° (2) 70°
(3) 15° (4) 45°

7. Value of $\angle x =$



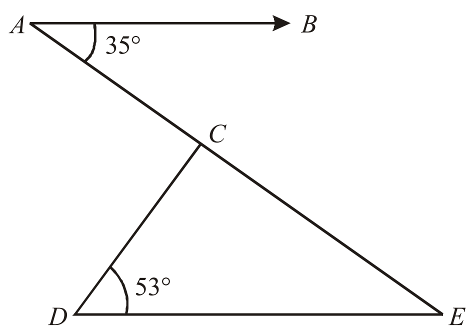
(1) 141° (2) 70°
(3) 105° (4) 45°

8. In figure, sides QP and RQ of $\Delta\ PQR$ are produced to points S and T respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°$, then the value of $\angle PRQ$ is



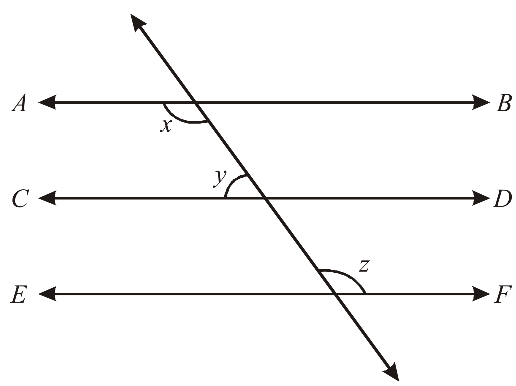
(1) 65° (2) 35°
(3) 75° (4) 30°

9. In figure, if $AB \parallel DE$, $\angle BAC = 35°$ and $\angle CDE = 53°$, $\angle DCE = ?$



(1) 102° (2) 92°
(3) 80° (4) 72°

10. In figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, $x = ?$



(1) 112° (2) 116°
(3) 96° (4) 126°

11. In a isosceles triangle AB = AC and BA is produced to D, such that AB = AD then $\angle$ BCD is

(1) 70° (2) 90°

(3) 60° (4) 45°

12. If two sides of a triangle are unequal then opposite angle of larger side is –

(1) greater (2) less

(3) equal (4) half

13. The sum of altitudes of a triangle is .... then the perimeter of the triangle

(1) greater (2) equal

(3) half (4) less

14. In ΔABC, BD $\perp$ AC and CE $\perp$ AB. If BD and CE intersect at O, then $\angle$ BOC =

(1) $\angle$ A (2) 90 + A

(3) 180 + $\angle$ A (4) 180 – $\angle$ A

15. If the three altitudes of a Δ are equal then triangle is

(1) isosceles (2) equilateral

(3) right angled (4) none

16. If D is any point on the side BC of a Δ ABC, then

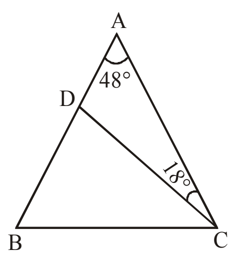
(1) AB + BC + CA > 2 AD

(2) AB+ BC + CA < 2AD

(3) AB + BC + CA > 3AD

(4) None

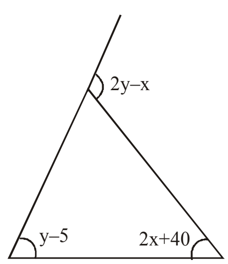
17. In the given figure, AB = AC, $\angle$ A = 48 and $\angle$ ACD =18°. BC equal to –



(1) AC (2) CD

(3) BD (4) AB

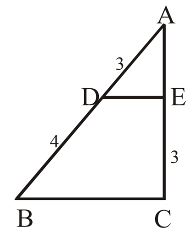
18. In figure value y, if x = 5° is –



(1) 50° (2) 60°

(3) 65° (4) 45°

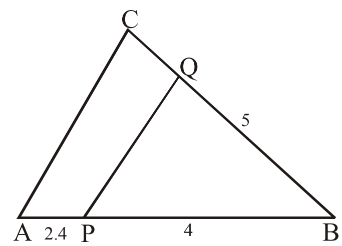
19. In the adjoining figure,
DE $\parallel$ BC and all measurements are
in centimetres. The length of AE is



(1) 2 cm (2) 2.25 cm

(3) 3.5 cm (4) 4 cm

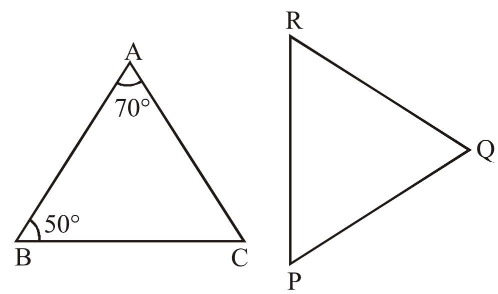
20. In the adjoining figure, PQ $\parallel$ CA and all lengths are given in centimetres. The length of BC is



(1) 6.4 cm (2) 7.4 cm

(3) 8 cm (4) 9 cm

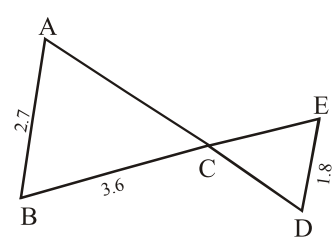
21. In the adjoining figure, ΔABC ~ ΔQPR. Then $\angle$R is



(1) 60° (2) 50°

(3) 70° (4) 80°

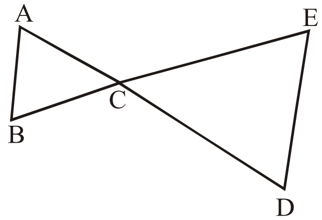
22. In the adjoining figure, ΔABC ~ ΔDEC and all measurements are in centimetres. The length of CE is



(1) 2.4 cm (2) 3 cm

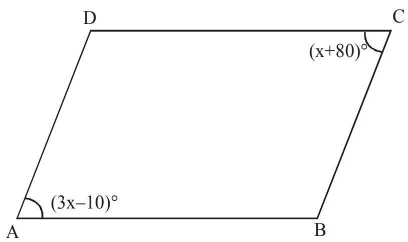
(3) 4.2 cm (4) 4.8 cm

23. In the adjoining figure, if AB || ED, then the two triangles ABC and DEC are



(1) similar
(2) congruent
(3) both similar and congruent
(4) neither similar nor congruent

24. In figure, ABCD is a parallelogram then the value of x is



(1) 25° (2) 60°
(3) 75° (4) 45°

25. The ratio of a side and a diagonal of a square is

(1) $1:\sqrt{2}$ (2) $3:\sqrt{2}$
(3) $\sqrt{2}:1$ (4) $\sqrt{2}:3$

26. ABCD is quadrilateral. If AC and BD are its diagonals then the

(1) sum of the squares of the sides of the quadrilateral is equal to the sum of the squares of its diagonals.
(2) perimeter of the quadrilateral is equal to the sum of the diagonals.
(3) perimeter of the quadrilateral is less than the sum of the diagonals.
(4) perimeter of the quadrilateral is greater than the sum of the diagonals.

27. Quadrilateral whose four sides are equal but angles are not equal is

(1) square (2) quadrilateral
(3) rectangle (4) parallelogram

28. Two diagonals of a parallelogram are equal then it is

(1) quadrilateral (2) a rectangle
(3) a rhombus (4) a trapezium

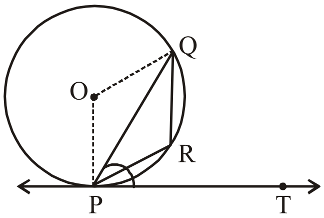
29. Two diagonals of parallelogram are equal and perpendicular to each other then it is

(1) rectangle (2) rhombus
(3) trapezium (4) square

30. *ABCD* is a rhombus with $\angle ABC = 56°$, then $\angle ACD$ will be

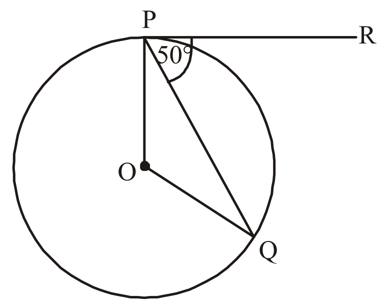
(1) 56° (2) 124°
(3) 62° (4) 34°

31. In Fig. PQ is a chord of a circle and PT is the tangent at P such that ∠QPT = 60°. Then ∠ PRQ is equal to



(1) 135° (2) 150°
(3) 120° (4) 110°

32. In Fig. if O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then ∠ POQ is equal to



(1) 100° (2) 80°
(3) 90° (4) 75°

33. C (O, r_1) and C (O, r_2) are two concentric circles with $r_1 > r_2$. AB is a chord of C (O, r_1) touching C (O, r_2) at C then

(1) $AB = r_1$ (2) $AB = r_2$
(3) $AC = BC$ (4) $AB = r_1 + r_2$

34. Number of tangents to a circle which are parallel to a secant is

(1) 1 (2) 2
(3) 3 (4) infinite

35. A circle is inscribed in a triangle with sides 8, 15 and 17 cm. The radius of the circle is

(1) 6 cm (2) 5 cm
(3) 4 cm (4) 3 cm

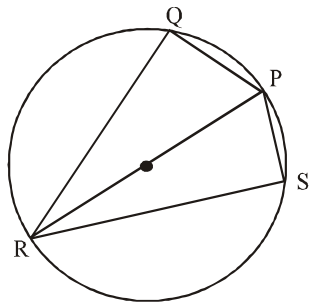
36. Number of tangents that can be drawn through a point which is inside the circle is

(1) 3 (2) 2
(3) 1 (4) 0

37. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is

(1) 3 r (2) 6 r
(3) 9 r (4) 12 r

38. In the figure if $\angle QPR = 67°$ and $\angle SPR = 72°$ and RP is a diameter of the circle, then $\angle QRS$ is equal to

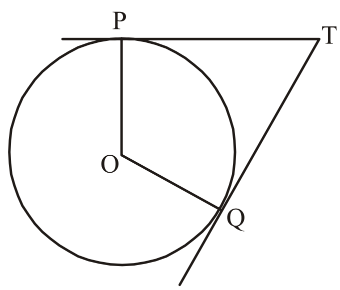


(1) 18° (2) 23°
(3) 41° (4) 67°

39. Two circles of radii 20 cm and 37 cm intersect in A and B. If O_1 and O_2 are their centres and AB = 24 cm, then the distance O_1O_2 is equal to

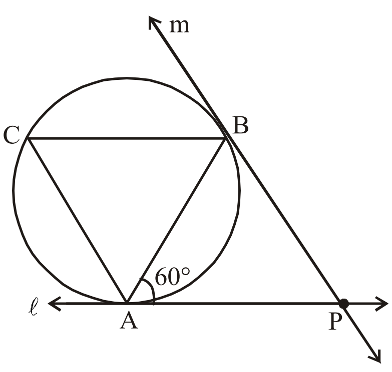
(1) 44 cm (2) 51 cm
(3) 40.5 cm (4) 45 cm

40. In the adjoining figure, TP and TQ are the two tangents to a circle with centre O. If $\angle POQ = 110°$, then $\angle PTQ$ is



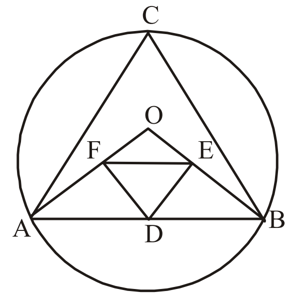
(1) 60° (2) 70°
(3) 80° (4) 90°

41. In the diagram below , if ℓ and m are two tangents and AB is a chord making an angle of 60° with the tangent ℓ , then the angle between ℓ and m is



(1) 45° (2) 30°
(3) 60° (4) 90°

42. In the diagram, O is the centre of the circle and D, E and F are mid points of AB, BO and OA respectively. If $\angle DEF = 30°$, then $\angle ACB$ is



(1) 30° (2) 60°
(3) 90° (4) 120°

43. Area of the quadrilateral *ABCD* whose diagonal $AC = 15$ cm. and sides $AB = 7$ cm, $BC = 12$ cm, $CD = 12$ cm and $DA = 9$ cm is

(1) 25.9 cm^2 (2) 29.3 cm^2
(3) 95.2 cm^2 (4) 92.5 cm^2

44. The lengths of two adjacent sides of a parallelogram are 5 cm and 3.5 cm. One of its diagonals is 6.5 cm long. Area of the parallelogram is

(1) $13\sqrt{10}$ cm^2 (2) $23\sqrt{5}$ cm^2
(3) $10\sqrt{5}$ cm^2 (4) $10\sqrt{3}$ cm^2

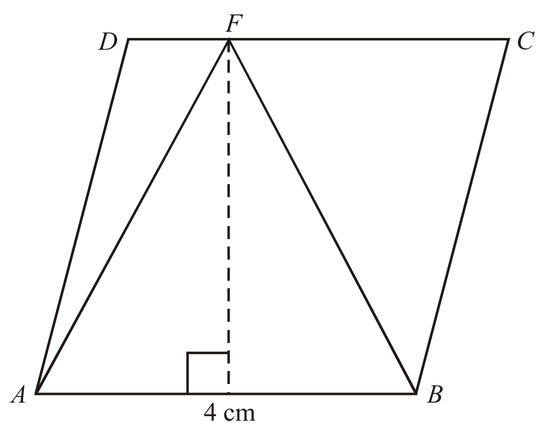
45. Area of a trapezium whose parallel sides are 55 cm, 40 cm and non-parallel sides are 20 cm and 25 cm respectively is

(1) 590 cm^2 (2) 950 cm^2
(3) 595 cm^2 (4) 940 cm^2

46. The base of a right triangle is 8 cm and hypotenuse is 10 cm. Its area will be

(1) 24 cm^2 (2) 40 cm^2
(3) 48 cm^2 (4) 80 cm^2

47. In the given figure, *ABCD* is a parallelogram then ar(ΔAFB) is



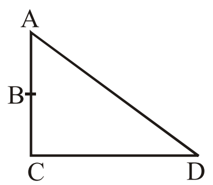
(1) 16 cm^2 (2) 8 cm^2
(3) 4 cm^2 (4) 6 cm^2

Exercise 2

MCQ Based Questions

DIRECTIONS (Qs. 1 to 3) : *This section contains multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) out of which only one is correct.*

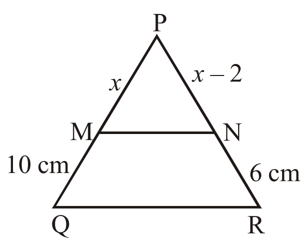
1. In a right triangle shown below, AB + AD = BC + CD, if AB = x , BC = h and CD = d, then x equal.



(1) $\frac{hd}{2h+d}$ (2) $d - h$

(3) $h + d$ (4) $\frac{1}{2}h$

2. In given figure, MN || QR. If PM = x cm, MQ = 10 cm, PN = x – 2 cm, NR = 6 cm, then x =

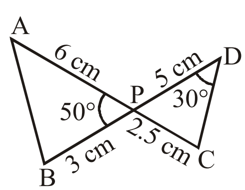


(1) 4 cm (2) 5 cm

(3) 6 cm (4) 8 cm

3. In Fig. two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, $\angle APB = 50°$ and $\angle CDP = 30°$. Then,

$\angle PBA$ is equal to



(1) 50° (2) 30°

(3) 60° (4) 100°

Matching Based Questions

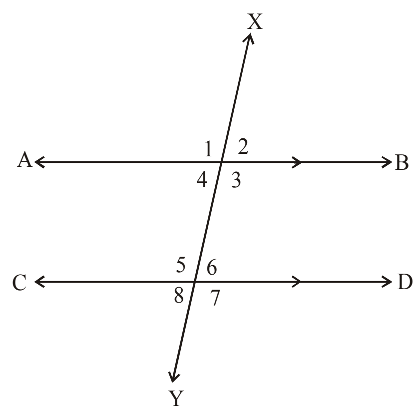
DIRECTIONS (Qs. 4 to 8) : *Match the Column-I with Column-II and select the correct answer given below the columns.*

4. Column-I gives postulate number given by Euclid for statement in column-II match them.

	Column - I		**Column - II**
(A)	Postulated 1	(p)	A terminated line can be produced indefinitely.
(B)	Postulated 2	(q)	All right angles are equal to one another.
(C)	Postulated 3	(r)	A straight line may be drawn from any one point to any other point.
(D)	Postulated 4	(s)	A circle can be drawn with any centre and any radius.

(1) A → q; B → p; C → s; D → r
(2) A → p; B → q; C → s; D → r
(3) A → q; B → p; C → r; D → s
(4) A → p; B → q; C → r; D → s

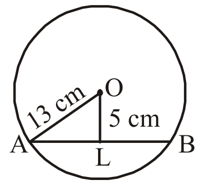
5. For the figure shown, Match column I and II correctly



	Column I		**Column II**
(A)	corresponding angles	(p)	1 and 5
(B)	alternate interior angles	(q)	4 and 6
(C)	alternate exterior angles	(r)	1 and 7
(D)	interior angles on same side of the transversal	(s)	4 and 5

(1) A → q; B → p; C → s; D → r
(2) A → p; B → q; C → r; D → s
(3) A → p; B → r; C → q; D → s
(4) A → q; B → p; C → r; D → s

6.



	Column-I		Column-II
(A)	A quadrilateral ABCD is drawn to circumscribe a circle. If AB = 5 cm, CD = 9 cm, BC = 8 cm then AD is	(p)	24 cm
(B)	The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Then the radius of the circle is	(q)	8 cm
(C)	Two concentric circles are of radii 5 cm and 3 cm. Then the length of the chord of the larger circle which touches the smaller circle is	(r)	6 cm
(D)	If OL = 5 cm, OA = 13 cm then AB is	(s)	3 cm

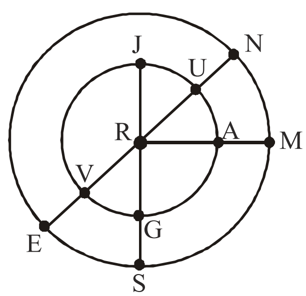
(1) A $\rightarrow$ r; B $\rightarrow$ s; C $\rightarrow$ q; D $\rightarrow$ p
(2) A $\rightarrow$ s; B $\rightarrow$ r; C $\rightarrow$ p; D $\rightarrow$ q
(3) A $\rightarrow$ r; B $\rightarrow$ s; C $\rightarrow$ p; D $\rightarrow$ q
(4) A $\rightarrow$ s; B $\rightarrow$ r; C $\rightarrow$ q; D $\rightarrow$ p

7.

	Column-I		Column-II
(A)	The tangent at any point of a circle is	(p)	known as a tangent to the circle.
(B)	The line containing the radius through the point of contact is	(q)	perpendicular to the radius through the point of contact
(C)	The lengths of tangents drawn from an external point to a circle are	(r)	called the 'normal' to the circle
(D)	When two end points of the corresponding chord of a secant coincide, it is	(s)	equal

(1) A $\rightarrow$ r; B $\rightarrow$ q; C $\rightarrow$ p; D $\rightarrow$ s
(2) A $\rightarrow$ r; B $\rightarrow$ p; C $\rightarrow$ q; D $\rightarrow$ s
(3) A $\rightarrow$ q; B $\rightarrow$ r; C $\rightarrow$ s; D $\rightarrow$ p
(4) A $\rightarrow$ q; B $\rightarrow$ s; C $\rightarrow$ r; D $\rightarrow$ p

8. Consider the figure and match the columns correctly



	Column I		Column II
(A)	Radius of interior concentric circle	(p)	segment RE
(B)	Chord of exterior concentric circle	(q)	segment RA
(C)	Radius of exterior concentric circle	(r)	segment MS
(D)	An acute central angle	(s)	angle NRM

(1) A $\rightarrow$ q; B $\rightarrow$ r; C $\rightarrow$ p; D $\rightarrow$ s
(2) A $\rightarrow$ r; B $\rightarrow$ q; C $\rightarrow$ s; D $\rightarrow$ p
(3) A $\rightarrow$ q; B $\rightarrow$ r; C $\rightarrow$ s; D $\rightarrow$ p
(4) A $\rightarrow$ r; B $\rightarrow$ q; C $\rightarrow$ p; D $\rightarrow$ s

Statement Based Questions

9. Consider the following statements :

I. Axioms are the assumptions which are obvious universal truths.

II. Things which are equal to the same thing are equal to one another.

III. If equals are subtracted from equals, the remainder are equal.

IV. Things which are double of the same things are equal one another.

Which of the statements is/are correct?

(1) I and II (2) II and III

(3) III and IV (4) All are correct

10. Consider the following statements:

I. A plane surface is a surface which lies evenly with the straight line on itself.

II. A surface is that which has length and breadth only.

III. A line is breadthless length.

IV. A point is that which has no part.

Which of the statements is/are correct?

(1) I and II (2) II and III

(3) III and IV (4) All are correct

11. Consider the following statements:

I. Things which are equal to the same thing are never equal to one another.

II. If equals are added to equals, the wholes are equal.

III. The whole is greater than the part.

IV. Things which are halves of the same things are equal to one another.

Which of the statements are Euclid's axioms?

(1) I, II and III (2) II, III and IV

(3) I, III and IV (4) None of the above

12. Consider the following statements:

I. A straight line may be drawn from any one point to any other point.

II. A circle can be drawn with any centre and any radius.

III. A terminated line cannot be produced indefinitely.

IV. All right angles are never equal to one another.

Which of the statements are Euclid's postulates?

(1) I and II (2) II and III

(3) III and IV (4) I and IV

13. Consider the following statements:

I. A triangle whose sides are equal, is called a scalene triangle.

II. A triangle, each of whose angle is less than 90°, is called an acute triangle.

III. If all sides of a polygon are different, it is called a regular polygon.

IV. A triangle with one of its angles greater than 90°, is known as an obtuse triangle.

Which of the statements given above is/are incorrect ?

(1) I and II (2) I and III

(3) I and IV (4) All above

14. Consider the following statements :

I. Only one line can pass through a single point.

II. If two circles are equal, then their radii are equal.

III. A terminated line can be produced indefinitely on both the sides.

IV. There are an infinite number of lines which pass through two distinct points.

Which of the statements given above is/are incorrect?

(1) I and II (2) I and III

(3) I and IV (4) All above

15. Consider the following statements :

I. If two angles forming a linear pair, then each of these angles is of measure 90°.

II. angles forming a linear pair can both be acute angles

III. one of the angles forming a linear pair can be obtuse angles.

IV. bisectors of the adjacent angles form a right angle.

Which of the statement(s) given above is/are not correct?

(1) Both I and II (2) Only I

(3) Only III (4) All above

16. Consider the following statements :

I. The difference of any two sides of a Δ is less than the third side.

II. A triangle cannot have two obtuse angles.

III. A triangle cannot have an obtuse angle and a right angle.

IV. All the above

Which of the statement is/arecorrect?

(1) Both I and II (2) Both II and III

(3) Both I and III (4) All above

17. Consider the following statements :

I. Two line segments having the same length are congruent.

II. Two squares having the same side length are congruent.

III. Two circles having the same radius are congruent.

IV. All the above

Which of the statement(s) given above is/are correct?

(1) I and II (2) neither I nor II

(3) II and III (4) All I, II, III are correct

18. Consider the following statements :

I. Two triangles having same area are congruent.

II. If two sides and one angle of a triangle are equal to the corresponding two sides and the angle of another triangle, then the two triangles are congruent.

III. If the hypotenuse of one right triangle is equal to the hypotenuse of another triangle, then the triangles are congruent.

IV. All the above

Which of the statement(s) given above is/are incorrect?

(1) I and II (2) neither I nor II

(3) II and III (4) All I, II, III are correct

19. Consider the following statements :

I. The centroid of an acute angled triangle lies in the interior of the triangle.

II. The orthocenter of an acute angled triangle lies in the interior of the triangle

III. The circumcenter of an acute angled triangle lies in the interior of the triangle

IV. All the above

Which of the statement(s) given above is/are correct?

(1) I and II (2) neither I nor II

(3) II and III (4) All I, II, III are correct

20. Consider the following statements :

I. The orthocenter of a right angled triangle is the vertex containing right angle.

II. The circumcenter of a right angled triangle is the mid-point of its hypotenuse.

III. The centroid of a right angled triangle lies in the interior of the triangle.

IV. All the above.

Which of the statement(s) given above is/are correct?

(1) I and II (2) neither I nor II

(3) II and III (4) All I, II, III are correct

21. Consider the following statements :

I. The centroid of an obtuse angled triangle lies in the interior of the triangle

II. Orthocentre of an obtuse angled triangle lies in the exterior of the triangle

III. Circumcentre of an obtuse angled triangle lies in the exterior of the triangle

IV. All the above

Which of the statement(s) given above is/are correct?

(1) I and II (2) neither I nor II

(3) II and III (4) All I, II, III are correct

22. Consider the following statements :

I. Each diagonal of a quadrilateral divides it into two triangles

II. Each side of a quadrilateral is less than the sum of the remaining three sides

III. A quadrilateral can utmost have three obtuse angles

IV. A quadrilateral has four diagonals

Which of the statement(s) given above is/are incorrect?

(1) I and II (2) neither I nor II

(3) II and III (4) only IV

23. Consider the following statements :

I. A parallelogram in which two adjacent angles are equal is a rectangle.

II. A quadrilateral in which both pairs of opposite angles are equal is parallelogram.

III. In a parallelogram the number of acute angles is zero (or) two

IV. All the above

Which of the statement(s) given above is/are correct?

(1) I and II (2) neither I nor II

(3) II and III (4) All I, II, III are correct

24. Consider the following statements :

I. Sides of triangle are 3 cm, 4 cm, 6 cm

II. Sides of triangle are 4 cm, 5 cm, 6 cm

III. Sides of triangle are 7 cm, 24 cm, 25 cm

IV. Sides of triangle are 5 cm, 12 cm, 14 cm.

Which of these is right triangle ?

(1) I (2) II

(3) III (4) IV

25. Consider the following statements :

I. The ratios of the areas of two similar triangles is equal to the ratio of their corresponding sides.

II. The areas of two similar triangles are in the ratio of the corresponding altitudes.

III. The ratio of area of two similar triangles are in the ratio of the corresponding medians.

IV. If the areas of two similar triangles are equal, then the triangles are congruent.

Which among the above is/are not correct?

(1) I, II and III (2) I, III and IV

(3) II, III and IV (4) None

26. Consider the following statements :

I. A tangent to a circle is a line that intersects the circle in exactly one point.

II. The point of contact is the only point which is common to the tangent and the circle.

III. A line intersecting a circle in two points is called a tangent.

IV. The common point of a tangent and the circle is called point of contact.

Which of the statement(s) given above is/are incorrect?

(1) Only I (2) Only II

(3) Only III (4) Only IV

27. Consider the following statements :

I. Diameter is the longest chord of the circle.

II. A diameter of a circle divides the circular region into two parts, each part is called a semi-circular region.

III. Every circle has unique centre and it lies inside the circle.

IV. From a given point in the exterior of a circle, two tangents can be drawn to it and these two tangents are equal in length.

Which of the statement(s) given above is/are correct?

(1) Only I (2) Neither I nor II

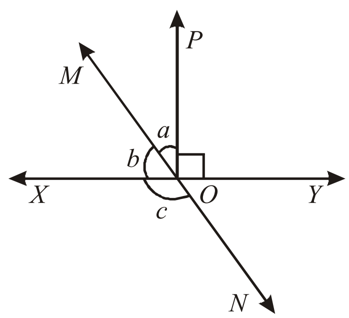
(3) Only II (4) All are correct

Passage Based Questions

DIRECTIONS (Qs. 28 to 35) : *Read the passage(s) given below and answer the questions that follow.*

Passage - 1

In figure, lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$, then



28. Measure of angle 'a' is

(1) 54° (2) 36°

(3) 126° (4) none of these

29. Measure of angle 'b' is

(1) 126° (2) 36°

(3) 54° (4) none of these

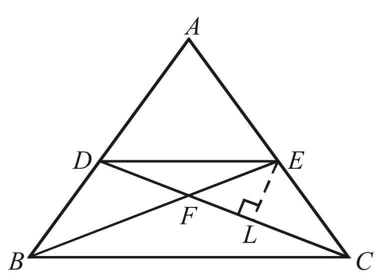
30. Measure of angle 'c' is

(1) 36° (2) 126°

(3) 54° (4) none of these

Passage - 2

In the adjoining figure, $DE \parallel BC$ and $AD : DB = 4 : 3$



31. Value of $\frac{AD}{AB}$ is

(1) $\frac{3}{7}$ (2) $\frac{4}{7}$

(3) $\frac{4}{3}$ (4) none of these

32. Value of $\frac{DE}{BC}$ is

(1) $\frac{4}{7}$ (2) $\frac{3}{7}$

(3) $\frac{3}{4}$ (4) none of these

33. $\frac{\text{area}(\Delta\, DEF)}{\text{area}(\Delta\, DEC)} =$

(1) $\frac{11}{4}$ (2) $\frac{4}{7}$

(3) $\frac{4}{11}$ (4) none of these

Passage - 3

The lengths of two parallel chords of a circle are 6 cm and 8cm. The smaller chord is at a distance of 4 cm from the centre.

34. The radius of the circle is.

(1) 10 cm (2) 5 cm

(3) 3 cm (4) none of these

35. The distance of the other chord from the centre is.

(1) 3 cm (2) 6 cm

(3) 4 cm (4) none of these

Assertion Reason Based Questions

DIRECTIONS (Qs. 36 to 45) : *Following questions consist of two statements, one labelled as the **'Assertion' (A)** and the other as **'Reason' (R)**. You are to examine these two statements carefully and select the answer to these items using the code given below.*

Code :

(1) Both A and R are individually true and R is the correct explanation of A:

(2) Both A and R are individually true but R is not the correct explanation of A.

(3) A is true but R is false

(4) A is false but R is true.

36. **Assertion :** Sum of the pair of angles (like 120°, 60°) is supplementary.

Reason : Two angles, the sum of whose measures is 180°, are called supplementary angles.

37. **Assertion :** If an angle formed by two intersecting lines is 60°, then its vertically opposite angle is 60°

Reason : If two lines intersect each other, then the vertically opposite angles are equal.

38. **Assertion :** If angles 'a' and 'b' form a linear pair of angles and $a = 40°$, then b = 150°.

Reason : Sum of linear pair of angles is always 180°

39. **Assertion :** If the angles of a quadrilateral are in the ratio 2 : 3 : 7 : 6, then the measure of angles are 40º 60º,140º,120º respectively.

Reason : The sum of the angles of a quadrilateral is 360º.

40. **Assertion :** If the diagonals of a parallelogram *ABCD* are equal, then $\angle ABC = 90°$.

Reason : If the diagonals of a parallelogram are equal, it becomes a rectangle.

41. **Assertion :** If $ABC \cong PQR$ and area (ΔABC) = 10 sq. units, then area (ΔPQR) = 20 sq. units.

Reason : Two congruent figures have equal areas.

42. **Assertion:** If in a circle, the radius of the circle is 3 cm and distance of a point from the centre of a circle is 5 cm, then length of the tangent will be 4 cm.

Reason: $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$

43. **Assertion:** If in a cyclic quadrilateral, one angle is 40°, then the opposite angle is 140°

Reason: Sum of opposite angles in a cyclic quadrilateral is equal to 360°

44. **Assertion:** If length of a tangent from an external point to a circle is 8cm, then length of the other tangent from the same point is 8 cm.

Reason: length of the tangents drawn from an external point to a circle are equal.

45. **Assertion :** Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is 1 : 1.

Reason : Two parallelograms on the same base (or equal bases) and between the same parallel lines are equal in area.

Feature Based Questions

46. On the basis of following features identify the correct shape

I. The sum of the angles of this is 360°

II. If the sides of this are produced, in order, the sum of the four exterior angles so formed is 360°.

III. It is a convex.

(1) Quadrilateral (2) Triangle

(3) Circle (4) None

47. On the basis of following features identify the correct shape.

I. It's opposite side are equal.

II. It's opposite angles are equal.

III. It's diagonals bisect each other.

IV. It's diagonals are not equal in length

(1) Circle (2) Triangle

(3) Quadrilateral (4) Parallelogram

Correct Definition Based Questions

48. Which of the following is the correct definition Arc of a circle ?

(1) A line segment joining any two points on a circle

(2) A line that passes through the centre of a circle.

(3) A part of a circumference is called a arc

(4) Perimeter of the circles

49. If the three altitudes of a Δ are equal then triangle is

(1) isosceles (2) equilateral

(3) right angled (4) none

50. Each angle of an equilateral triangle is

(1) 60° (2) 45°

(3) 90° (4) 30°

Hints SOLUTIONS

Exercise I

1. (4) **2.** (2) **3.** (3) **4.** (4)
5. (1) **6.** (3) **7.** (1) **8.** (1)
9. (2) **10.** (4) **11.** (2) **12.** (1)
13. (4) **14.** (4) **15.** (2) **16.** (1)
17. (2) **18.** (1)

19. (2) **Hint.** $\frac{AE}{EC} = \frac{AD}{BD} \Rightarrow \frac{AE}{3\text{ cm}} = \frac{3}{4}$

$\Rightarrow AE = \frac{3}{4} \times 3\text{cm}$

20. (3) **Hint.** AB = (2.4 + 4) cm = 6.4 cm;

$\frac{\text{BQ}}{\text{BC}} = \frac{BP}{AB} \Rightarrow \frac{5}{BC} = \frac{4}{6.4}$

21. (1) [**Hint.** As $\Delta ABC \sim \Delta QPR$,
$\angle R = \angle C$ but $\angle C = 180° - \angle A - \angle B$]

22. (1) [**Hint.** $\Delta ABC \sim \Delta DEC \Rightarrow \frac{\text{CE}}{\text{CB}} = \frac{ED}{AB}$

$\Rightarrow \frac{CE}{3.6} = \frac{1.8}{2.7} \Rightarrow \text{CE} = 2.4$ cm]

23. (1) [**Hint.** AB || ED $\Rightarrow \angle A = \angle D$ and
$\angle B = \angle E \Rightarrow \Delta ABC \sim \Delta DEC$]

24. (4) **25.** (1) **26.** (4) **27.** (2)
28. (2) **29.** (4) **30.** (3)

31. (3) $\angle$ OPQ = $\angle$ OQP = 30°, i.e., $\angle$ POQ = 120°.

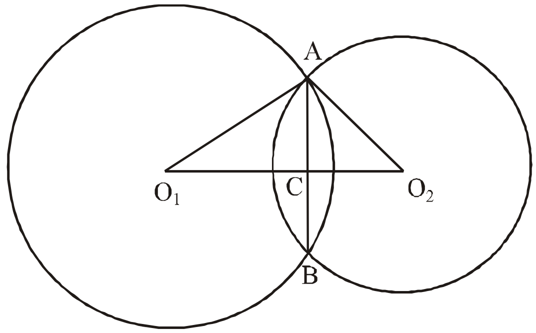
Also, $\angle \text{PRQ} = \frac{1}{2}$ reflex $\angle$ POQ

32. (1) **33.** (3) **34.** (2) **35.** (4)
36. (4)

37. (2) Side of the regular hexagon inscribed in a circle of radius *r* is also *r*, the perimeter is 6r.

38. (3)

39. (2)



C is the mid-point of AB so that AC = 12

$AO_1 = 37$ and $AO_2 = 20$

$\therefore \quad CO_1 = \sqrt{37^2 - 12^2} = 35,$

$CO_2 = \sqrt{20^2 - 12^2} = 16$

$\therefore \quad O_1O_2 = 35 + 16 = 51$

40. (2)
41. (3)
42. (2)
43. (3)
44. (4)
45. (2)
46. (1)

47. (2) ar $(\Delta AFB) = \frac{1}{2}$ ar(|| gm $ABCD$) $= \frac{1}{2} \times 4 \times 4$

$= 8 \text{ cm}^2$

Exercise 2

1. (1)
2. (2)
3. (4)
4. (1) **5.** (2) **6.** (1) **7.** (3)
8. (1) **9.** (4) **10.** (4) **11.** (2)
12. (1) **13.** (2) **14.** (3) **15.** (3)
16. (4) **17.** (4) **18.** (4) **19.** (4)
20. (4)
21. (4) All the given statements are true
22. (4)
23. (4) All the given statements are true.
24. (3)
25. (1)
26. (3)
27. (4)
28. (2) $\angle a = 36°$
29. (3) $\angle b = 54°$
30. (2) $\angle c = 126°$
31. (2) **32.** (1) **33.** (3) **34.** (1)
35. (4) **36.** (1) **37.** (1) **38.** (4)
39. (1) **40.** (1) **41.** (4) **42.** (1)
43. (3) **44.** (1) **45.** (1) **46.** (1)
47. (4) **48.** (3) **49.** (2) **50.** (1)