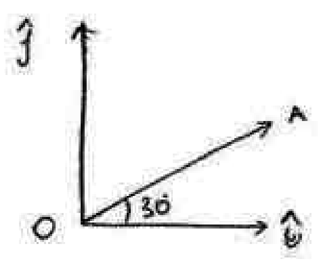
Kinematics

Q) A: If speed is constant then velocity <u>may be</u> const
R: If velocity is constant then speed <u>must be</u> constant
(A)

<u>dirⁿ of:</u>

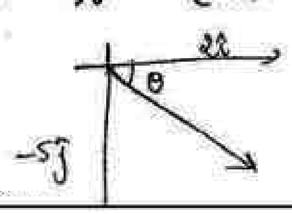
- $\overrightarrow{\text{displacement}}$ = from initial to final posⁿ
- $\overrightarrow{V_{avg}}$ = along the displacement
  (from initial to final posⁿ)
- $\overrightarrow{V_{inst}}$ = Tangential to the path
  dirⁿ of motion and momentum are also same
- $\overrightarrow{a_{avg}}$ = along the change in velocity
- $\overrightarrow{F_{avg}}$ = along the change in momentum
  (dirⁿ of impulse is also same)

Q) If $\overrightarrow{OA}$ = 10m (30° N of E) then express in terms of component (or due E 30° N)



$$\overrightarrow{OA} = (10\cos 30)\hat{i} + (10\sin 30)\hat{j}$$
$$= (5\sqrt{3}\,\hat{i} + 5\hat{j})\ m$$

Q) $\vec{r} = (2\hat{i} - 5\hat{j})$ m , express in real dirⁿ



dirⁿ = E $\tan^{-1}(5/2)$ S

Q) $\vec{V} = (20\hat{i} + 20\hat{j})$ m/s

$\vec{V} = 20\sqrt{2}$ m/s dir^n = E $\tan^{-1}\left(\frac{20}{20}\right)$ N.

E – N
N – E
N 45° E

Q) A particle travel 10 m along east dir^n for 3 sec then it takes a 90° left turn & moves 10 m in 2 sec & then 135° left turn and trave $20\sqrt{2}$ in 5 sec

i) avg speed
ii) displace
iii) avg velocity

i) $\dfrac{20 + 20\sqrt{2}}{3+2+5} = 2 + 2\sqrt{2}$

ii) $\cancel{10\sqrt{2}}$ $10\hat{i} + 10\hat{j} + 20\cancel{\sqrt{2}}\left(\dfrac{-\hat{i} - \hat{j}}{\cancel{\sqrt{2}}}\right)$

$\therefore \left(-10\hat{i} - 10\hat{j}\right) = 10\sqrt{2}$ m, in SW

iii) $\dfrac{10\sqrt{2}}{10} = \sqrt{2}$ due SW

~~Q)~~ A particle travel with speed v for tym t along North dir^n and then it takes a 90° right turn & travel with the same speed for same tym

i) total distance travel
ii) avg speed
iv) displacement
v) avg velocity

i) $2vt$

ii) $\frac{2vt}{2t} = v$

iii) 0

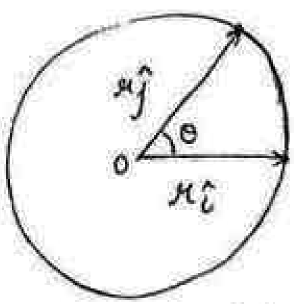
iv) $(vt)\hat{j} + (vt)\hat{i} = vt\sqrt{2}$ NE

v) $\frac{v}{\sqrt{2}}$ NE

vi) $\Delta\vec{v} = v\hat{i} - v\hat{j} = v\sqrt{2}$ due SE

vii) $\vec{a}_{avg} = \frac{v}{\sqrt{2}t}$ due SE

* If a particle travels along a circular track of radius r and subtend θ angle at the centre then its displacement and distance travelled can be obtained as follows.



$|\vec{disp}| = |\Delta\vec{r}| = |\vec{r}_f - \vec{r}_i| = \boxed{2r\sin\left(\frac{\theta}{2}\right)}$

dist travelled = length of arc AB = $\boxed{r\theta}$

(in radian)

here $|\vec{r}_i| = |\vec{r}_f| = r$

Q) A particle travels along a circular path of radius r with constant speed v and subtend 60° angle at centre.

i) magnitude of displacement

ii) distance travelled

iii) Ratio of distance travel to disp.

iv) Time Taken

v) avg speed

vi) change in speed

vii) mag of avg velocity

viii) mag of change in velocity

ix) mag of avg accn

i) $2r\sin\left(\frac{60}{2}\right) = r$

ii) $r\theta = \frac{r\pi}{3}$

iii) $\frac{\pi}{3}$

iv) $\frac{\pi r}{3V}$

v) v

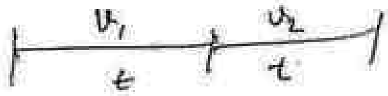
vi) 0

vii) $|\vec{v_{avg}}| = \frac{|\vec{dis}|}{t} = \frac{r}{\left(\frac{\pi r}{3V}\right)} = \frac{3V}{\pi}$

viii) $|\Delta \vec{V}| = |\vec{V_f} - \vec{V_i}| = 2V\sin\left(\frac{60}{2}\right) = V$

Q) A particle travels with speed v_1 for the first half time ~~vert~~ of its motion, then wid speed ~~v~~ v_2 for remaining tym. Calculate avg speed.

$$\frac{v_1+v_2}{2} \qquad \text{AM}$$

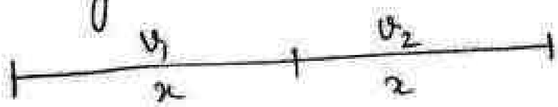


$$v_{avg} = \frac{v_1 t + v_2 t}{2t} = \frac{v_1+v_2}{2}$$

If the travelling tym is distributed in equal parts then Vavg is A.M of all the given speed.

$$\boxed{v_{avg} = \frac{v_1 + v_2 \cdots\cdots v_n}{n}}$$

Q) A particle travels wid speed v_1 for the first half distance and wid v_2 for remaining distance Find its V_{avg}



$$V_{avg} = \frac{2x}{\frac{x}{v_1}+\frac{x}{v_2}} = \frac{2v_1 v_2}{v_1+v_2} \qquad \text{H.M}$$

If the distance travelled is distributed in equal part then avg speed is harmonic mean of all the given speed.

$$\boxed{\frac{n}{V_{avg}} = \frac{1}{v_1} + \frac{1}{v_2} \cdots\cdots \frac{1}{v_n}}$$

Q) A dog leaves its trainer with the speed of 40 km/h and return back wid the speed of 60 km/h. Find V_{avg}

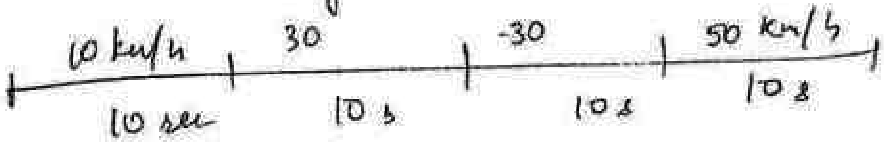
$$\frac{2 \times v_1 v_2}{v_1 + v_2} = \frac{2 \times 40 \times 60}{100} = 48 \text{ km/h}$$

Q) A particle travels wid $v_1 = 10$ m/s for 1st $\frac{1}{3}$ distance $v_2 = 20$ m/s for next $\frac{1}{3}$nd " $v_3 = 30$ m/s for remaining. Calculate V_{avg}

$$\frac{3}{V_{avg}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{30} \qquad \Rightarrow \frac{3}{V_{avg}} = \frac{6+3+2}{60}$$

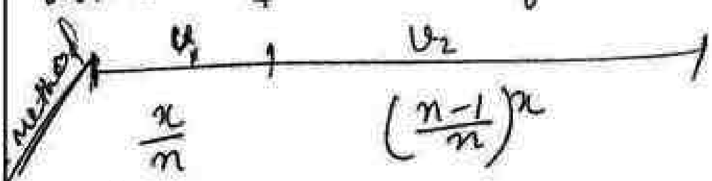
$\therefore V_{avg} = 16.3$ m/s

Q) A particle travels wid speed of 10 km/h for 10 sec then $v_2 = 30$ km/h for next 20 sec finally wid $v_3 = 50$ km/h for 10 sec. Find V_{avg}



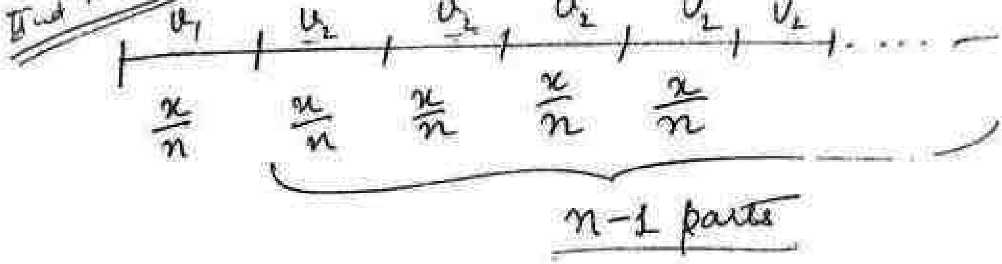
$$AM = \frac{10 + 30 + 30 + 50}{4} = \frac{120}{4} = 30 \text{ km/h}$$

** Q) A particle travel with v_1 for $\frac{1}{n}$ part of its total distance x with v_2 for remaining. Find V_{avg}



$$\therefore V_{avg} = \frac{x}{\frac{x}{\ldots} + \frac{(n-1)x}{\ldots}} = \frac{n v_1 v_2}{v_2 + (n-1) v_1}$$

Find



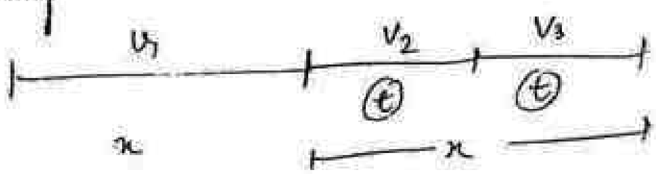
∵ equal distance travelled.

$\therefore$ H.M $\Rightarrow$ $$\frac{n}{V_{avg}} = \frac{1}{v_1} + \frac{1}{v_2}(n-1)$$

$$\therefore V_{avg} = \frac{n \cdot v_1 v_2}{V_2 + (n-1)V_1}$$

Q) A particle travels ~~wid~~ $\frac{1}{2}$ dis wid v_1 and remaining half in 2 equal tym interval v_2 and v_3

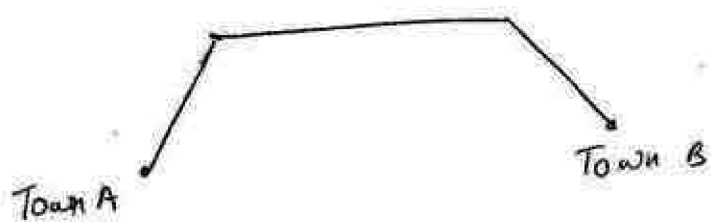
Find v_{avg}



$$\frac{v_2 + V_3}{2}$$

$$\therefore V_{avg} = \frac{2V_1\left(\frac{V_2+V_3}{2}\right)}{V_1 + \left(\frac{V_2+V_3}{2}\right)} = \frac{2V_1(V_2+V_3)}{2V_1+V_2+V_3}$$

*Q)



$V_{up} = 2$ Km/h

$V_{plane} = 3$ km/h

$V_{down} = 6$ km/h

$t_{A\to B} = 4$ hrs

$t_{B\to A} = 6$ hrs

Find total road distance b/w town A and B

$t_{A \to B} = 4\ hr$

$$\frac{a}{2} + \frac{b}{3} + \frac{c}{6} = 4 \qquad \therefore \frac{3a+2b+c}{6} = 4$$

$$\therefore 3a + 2b + c = 24 \quad \text{—(I)}$$

$t_{B \to A} = 6\ hr$

$$\frac{a}{6} + \frac{b}{3} + \frac{c}{2} = 6 \qquad \therefore \frac{a+2b+3c}{6} = 6$$

$$a + 2b + 3c = 36 \quad \text{—(II)}$$

(I) + (II)

$$4a + 4b + 4c = 60$$

$$\therefore a + b + c = \frac{60}{4} = 15$$

Question based on calculus

pos/dis/disp $\xrightarrow{\text{diff}}$ speed/velocity $\left[v = \frac{dx}{dt}\right]$ $\xrightarrow{\text{diff}^n}$ acc^n $\left[a = \frac{dv}{dt} \text{ or } a = \frac{v\,dv}{dx}\right]$

change in pos^n $\xleftarrow{\text{inte}}$; Δv or v $\xleftarrow{\text{inte}}$

$\Downarrow$

$u = 0$

Q) If $v = (3t^2 - 2t)\ m/s$ t (sec) find i) acc^n $(t = 2)$

ii) dis $(t = 1 \text{ to } 3 \text{ sec})$

$6t - 2 \quad \therefore a = 16\ m/s^2$

$$\frac{\cancel{3}t^3}{\cancel{3}} - \frac{\cancel{2}t^2}{\cancel{2}} = \left[t^3 - t^2\right]_1^3 \therefore (27 - 9) - (1^3 - 1^2) = 18$$

Q) If $x = (4t + 3t^2 - t^3)\ m$ t (sec). Find i) $v_{initial}$ (4)

ii) ~~acc^n when $(v=0)$~~ velocity when $(a = 0)$

i) $v = 4 + 6t - 3t^2 \quad \therefore v_{in} = 4\ m/s$

ii) $a = 6 - 6t = 0 \quad \therefore t = 1$

$\therefore v = 4 + 6 - 3 = 7$

Q) If $v = (3t^2 + t^3)$ m/s. Find distance travelled in 4th sec

$$x = \left[t^3 + \frac{t^4}{4}\right]_3^4 = \left[64 + \frac{256}{4}\right] - \left[27 + \frac{81}{4}\right]$$

$$= 128.00 - 47.25 = 80.75$$

Q) If $x \propto \sqrt{t}$, then find i) V - t relation

ii) a - t relation

iii) a - V relation

ie $x \propto t^{1/2}$

i) $V \propto t^{-1/2}$ $\quad \therefore v \propto \frac{1}{\sqrt{t}}$

ii) $a \propto t^{-3/2}$ $\quad \therefore a \propto \frac{1}{\sqrt{t^3}} \propto \frac{1}{t\sqrt{t}}$

iii) $a \propto V^3$

Q) If $v = x^2$, then find i) acc^n in terms of x

ii) acc^n in terms of 'v'

iii) ~~acc in~~ x in terms of t

iv) v in terms of t

i) $a = v\frac{dv}{dt} = x^2 \frac{d(x^2)}{dt} = x^2 2x = 2x^3$

ii) $V = x^2 \quad \therefore x = v^{1/2} \quad \therefore a = 2x^3 = 2(v^{1/2})^3 = 2v^{3/2}$

iii) $V = x^2 \quad \therefore \frac{dx}{dt} = x^2 \quad \therefore \frac{dx}{x^2} = dt$

Integrating,

$\int x^{-2}\,dx = \int dt \quad \therefore -\frac{1}{x} = t \quad \therefore x = -\frac{1}{t}$

iv) $v = x^2 = \left(\frac{-1}{t}\right)^2 = \frac{1}{t^2}$

Q) If $V = \sqrt{x^2+1}$, then find.

i) a in terms of x

ii) a in terms of v

i) $a = \dfrac{v\,dv}{dx} = (x^2+1)^{\frac{1}{2}} \dfrac{d}{dx}(x^2+1)^{\frac{1}{2}}$

$a = (x^2+1)^{\frac{1}{2}} \dfrac{1}{2}(x^2+1)^{-\frac{1}{2}} \times 2x \quad \therefore \; a = x$

ii) $v^2 = x^2+1 \Rightarrow \quad \therefore x^2 = v^2-1$

$\therefore \; a = x = \sqrt{v^2-1}$

Q) If $v = k\sqrt{x}$ i) x in terms of t

ii) v in terms of t

iii) accⁿ

i) $\dfrac{dx}{dt} = k\sqrt{x} \qquad \therefore \; \dfrac{dx}{\sqrt{x}} = k\,dt$

Integrating, $\int x^{-\frac{1}{2}}\,dx = \int k\,dt$

$\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}} = kt \qquad \therefore \; \sqrt{x} = \dfrac{1}{2}kt$

$\therefore \; \boxed{x = \dfrac{k^2t^2}{4}}$

ii) $v = k\sqrt{x} = k\left(\dfrac{kt}{2}\right) = \dfrac{k^2t}{2}$

iii) accⁿ $= \dfrac{dv}{dt} = \dfrac{d}{dt}\left(\dfrac{k^2t}{2}\right) = \dfrac{k^2}{2}$

Q) A particle is constrained to move along x-axis
If it starts from $x = 3m$ and its v-t relationship is given by $v = 12t - 3t^2$. then find its posn when $\underline{v = max}$

for 'v' to be max, $\frac{dv}{dt} = 0$

$$12 - 6t = 0 \quad \therefore \boxed{t = 2s}$$

$$\frac{dx}{dt} = v \quad \& \quad \int_3^x dx = \int_0^2 v\,dt$$

$$x - 3 = \left[\frac{12t^2}{2} - \frac{3t^3}{3}\right]_0^2 = \left[6t^2 - t^3\right]_0^2$$

$$x = 3 + (24 - 8) \quad = 19\,m$$

Non uniform motion (accelerated motion).

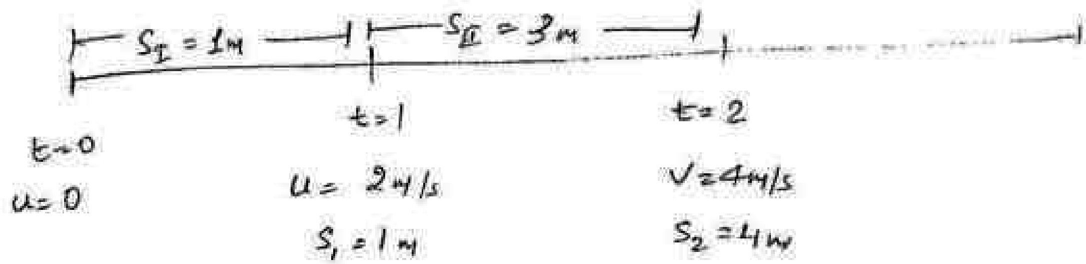
uniform accn
$v \propto t$, $x \propto t^2$, $x \propto v^2$
(use eqn of motion)

non-uniform accn
(Use calculus)

> distance = speed × time [when $a = 0$ or $\vec{v}$ = constant]
> distance = avg speed × time [always]
> $V_{avg} = \frac{u + v}{2}$ [when a = constant]
> $s = \left(\frac{u + v}{2}\right) t$ [when a = constant and not given]

eg $u = 0$ by $v = u + at \Rightarrow v = 0 + 2t \quad \therefore \boxed{v = 2t}$

$a = 2\,m/s^2$ by $s = ut + \frac{1}{2}at^2 \quad \boxed{s \propto t^2}$



<u>Important conclusion</u>

- Initial and final velocity depends upon selected segment of path

  for eg from 1 to 3 sec $u = 2\,m/s$ and $v = 6\,m/s$

  $S_1 = S_I$ but $S_2 \neq S_{II}$ instead $S_2 = S_I + S_{II}$

  similarly $S_3 = S_I + S_{II} + S_{III}$

- The ratio of distances travelled in 1s, 2s, 3s is the ratio of square of natural no

  $S_1 : S_2 : S_3 \ldots = 1 : 4 : 9 \ldots\ldots$

- The ratio of distances travelled in successive 1 sec is the ratio of the successive odd no.

  $S_I : S_{II} : S_{III} \ldots = 1 : 3 : 5 \ldots$

- The ratio of successive odd no is also applicable for successive equal time intervals.
- Both the above ratio are applicable only if $u = 0$ and $a =$ constant

Q) A particle of mass 3 kg starts its motion from rest from the point $(2, 1, 2) = r_i$ under the influence of a force, $\vec{F} = (2\hat{i} + 3\hat{j} + 4\hat{k})\,N$. Find

i) velocity after 3 sec

ii) speed after 3 sec

iii) displacement in 6 sec $6\sqrt{29}$

iv) $\vec{r}_f$ after 6 sec

v) New co-ordinates after 6 sec

i) $\vec{a} = \frac{\vec{F}}{m} = \frac{2\hat{i} + 3\hat{j} + 4\hat{k}}{3}\ m/s^2$

by $\vec{v} = \vec{u} + \vec{a}t$

$= 0 + \frac{2\hat{i} + 3\hat{j} + 4\hat{k}}{3} \times 3 = (2\hat{i} + 3\hat{j} + 4\hat{k})\ m/s$

ii) speed $= |\vec{v}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}\ m/s$

iii) $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 0 + \frac{1}{2} \times \left(\frac{2\hat{i} + 3\hat{j} + 4\hat{k}}{3}\right) \times 6^2 = (12\hat{i} + 18\hat{j} + 24\hat{k})\,m$

iv) $\vec{r}_f - \vec{r}_i = s$

$\vec{r}_f = \vec{s} + \vec{r}_i = (12\hat{i} + 18\hat{j} + 24\hat{k}) + (2\hat{i} + \hat{j} + 2\hat{k})$

$= (14\hat{i} + 19\hat{j} + 26\hat{k})\,m$

v) $(14, 19, 26)$

Q) A car is moving with speed u, when breaks are applied. it ~~starts~~ retards uniformly and stops after travelling a distance S. (Known as stopping distance). Now if by $v^2 - u^2 = 2as$

$0 = u^2 - 2as \quad (\because a \text{ is } -ve)$

$2as = u^2$

$$s' = \frac{(u')^2}{2a} = \frac{(nu)^2}{2a} = n^2\left(\frac{u^2}{2a}\right) = n^2 s$$

$\therefore \boxed{s' = n^2 s}$

Q) A car is moving with speed u when breaks are applied, it retards uniformly and stops after travell for time t (which is known as stopping time) Now if the same car moves with speed nu then find the stopping on application of sm retardation

by : $v = u + at$

$0 = u - at \quad (\because a \text{ is } -ve) \qquad \therefore at = u$

$\therefore t' = \frac{u'}{a} = \frac{nu}{a} = nt \qquad \therefore \boxed{t' = nt}$

speed	stopping dis	stopping time
2u	4s	2t
3u	9s	3t

If a car is moving with speed u stops after travelling a distance s, in time t under uniform retardation then stopping dis & stopping ~~po~~ time are as follows

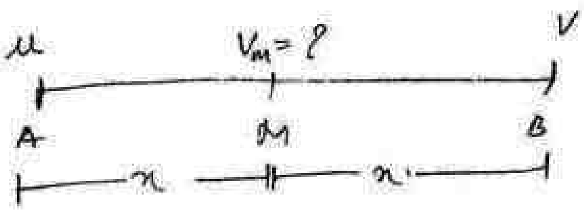
Q) A driver takes 0.25 sec ~~to see~~ to apply the breaks after he sees a red signal for it. If he is driving a car with speed of 72 km/h and breaks cause retardation of $10 m/s^2$. Find distance travelled by car after the driver sees the red signal

during retardation time ⇒ distance $= s \times t = 20 \times 0.25 = 5$ m

during retarding motion, $S_2 = \frac{u^2}{2a} = \frac{20^2}{2 \times 10} = 20$ m

Total distance travelled $= 5 + 20 = 25$

Q) A particle is moving along a st. line with uniform acc^n. Its speed is u while passing through point A and it becomes v when passing through another point B. Find its speed at the midpoint of line AB



using III eq of motion, $v^2 - u^2 = 2as$

$V_M^2 - u^2 = 2ax$ —(1) $v^2 - V_M^2 = 2ax$ —(ii)

$V_M^2 - u^2 = v^2 - V_M^2$ $\therefore 2V_M^2 = u^2 + v^2$

$V_M = \sqrt{\frac{u^2+v^2}{2}}$

Q) A particle starts from rest and ~~moves with~~ moves with constant acc^n. If the distance travelled by it in 1st 10 sec is S.

Find distance travelled in next 10 sec = 3S

20 to 30 sec = 5S

Q) If $u = 10 m/s$, $a = 2 m/s$, find

i) distance travelled in 2 sec

ii) S_{III}

iii) $\frac{S_{IV}}{S_{V}}$

i) $S = ut + \frac{1}{2}at^2 = 10 \times 2 + \frac{1}{2} \times 2 \times 2^2 = 24 m$

ii) $S_{III} = u + \frac{a}{2}(2n-1) = 10 + \frac{2}{2}(2 \times 3 - 1) = 15$

iii) $\frac{10 + \frac{2}{2}(2 \times 4 - 1)}{10 + \frac{2}{2}(2 \times 5 - 1} = \frac{17}{19}$

Q) Find $\frac{S_n}{S_{n^{th}}}$, if $u = 0$, and $a =$ constant

$$\frac{S_n}{S_{n^{th}}} = \frac{un + \frac{1}{2}an^2}{u + \frac{a}{2}(2n-1)} = \frac{n^2}{2n-1}$$

Q) $u = 0$, $a =$ uniform, velocity becomes 36 km/h after it has travelled for 10s. find distance travelled by it during this interval

$$S = \left(\frac{u+v}{2}\right)t = \frac{0 + 10}{2} \times 10 = 50 m$$

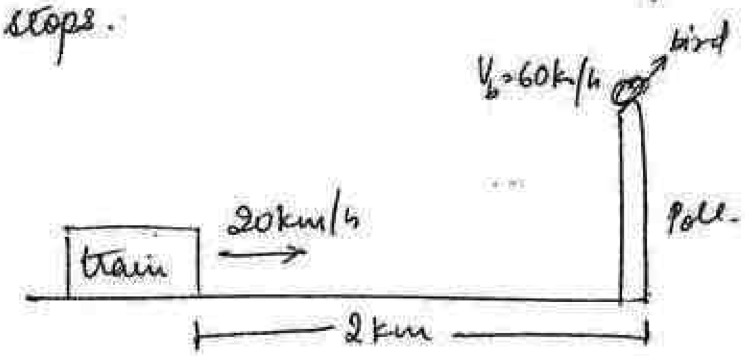
*Q) A train is moving with uniform velocity, suddenly its last compartment gets detached from it and stops (last compartment) after travelling a dis d under uniform retardation. Find the distance travelled by the train during this for compartment. by $s = \left(\frac{u+v}{2}\right) t$

$$d = \left(\frac{V_T + 0}{2}\right) t \qquad \therefore \boxed{t = \frac{2d}{V_T}}$$

for train. distance = speed × time

$$\cancel{V_T} \times \frac{2d}{\cancel{V_T}} = 2d.$$

Q. Train is retarding uniformely such that it stops finally at the pole. A bird sitting at the pole starts flying with uniform speed of 60 km/h towards the train touches its front and return back at the pole and repeats this process again & again. Find total distance travelled by the bird till the train stops.



for train, $s = \left(\frac{u+v}{2}\right) t \qquad \therefore 2 = \left(\frac{20+0}{2}\right) t \qquad \therefore t = \frac{1}{5} hr$

For bird, distance = speed × time

$$60 \times \frac{1}{5} = 12 \text{ km}$$

Q) A particle is dropped by h height. Find time taken and velocity on reaching the ground.

$S = ut + \frac{1}{2}gt^2 \quad \therefore h = 0 + \frac{1}{2}gt^2 \quad \therefore \boxed{t = \sqrt{\frac{2h}{g}}}$

by $v^2 - u^2 = 2as$

$v^2 = 2gh \quad \therefore \quad v = \sqrt{2gh}$

Q) A body is dropped from some height and it reaches the ground in 4 sec. Find its initial height and velocity

$h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 4^2 = 80\,m$

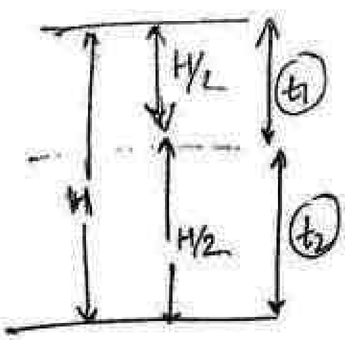
$v = u + at = 0 + 10 \times 4 = 40\,m/s$..

OR

$5 + 15 + 25 + 35 = 80\,M$

1s = 40 due change $\therefore$ in 4 sec = 40 m/s

Q) A body is dropped from some height such that it reaches the ground in given t. Find time taken by it to travel 1st half height and the ratio of time taken to cover upper half and lower half height



$H = \frac{1}{2}gt^2 \quad (1)$

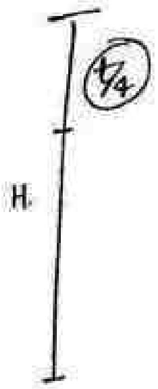
$\frac{H}{2} = \frac{1}{2}gt_1^2$

$\frac{1}{2}gt^2 = gt_1^2 \quad \therefore t_1 = \frac{t}{\sqrt{2}}$

$t - t_1 = t_2 = t - \frac{t}{\sqrt{2}} = t\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$

$$\frac{t_1}{t_2} = \frac{t/\sqrt{2}}{t\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)} = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} \Rightarrow \frac{\sqrt{2}+1}{1}$$

Q) A body is dropped from h height. Find its height above the ground wen it has fallen for the 1st $\frac{1}{4}$ time of its total descending tym

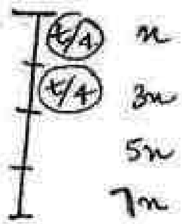


$$x = \frac{1}{2} \times g\left(\frac{t}{4}\right)^2 \qquad \because H = \frac{1}{2}gt^2$$

$$= \frac{1}{2}g\frac{t^2}{16} \Rightarrow \frac{H}{16}$$

$$\therefore \text{height fm ground} = H - \frac{H}{16} = \frac{15H}{16}$$

OR

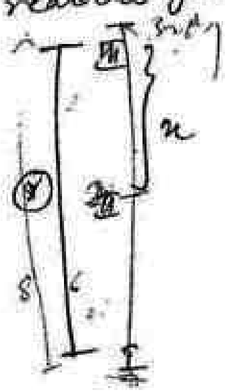


$$x + 3x + 5x + 7x = H$$

$$16x = H \quad \therefore x = \frac{H}{16}$$

$$\therefore \text{distance from ground} = \frac{15H}{16}$$

Q) water drops are falling from a water tap at regular intervals of time such that when the 1st drop reaches ground. 3rd drop is abt to leave the tap. If the height of tap is 8m. Find height of 2nd drop at the instant when 1st reaches ground



1st drop $H = \frac{1}{2}gt^2$ $\quad \therefore 8 = \frac{1}{2} \times 10t^2 \quad \therefore t = \sqrt{\frac{8}{5}}$ sec

$s = \frac{1}{2} \times 10 \times t^2$

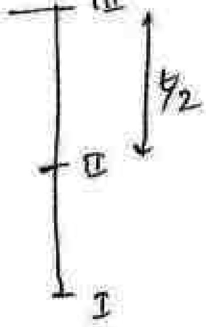
$s = \frac{1}{2} \times 10 \quad 5\left(\frac{8}{5}\right)^2$

for IInd drop $t = \frac{\sqrt{8/5}}{2} = \sqrt{\frac{2}{5}}$ sec

$= 5(t^2 + 1 - 10t)$

$5t^2 + 5 - 50t$

$\frac{1}{2}gt^2 = \frac{1}{2} 10 \times \frac{2}{5} = 2$ height fm ground $= 6$m



$$x + 3x = 8$$

$$4x = 8 \quad \therefore x = 2$$

$$\therefore ht = 3x = 2 \times 3 = 6$$

Q) A body is dropped fm sum height such dt it travel equal distance in 1st 5 sec and last sec of its. Find its total lgm of fall and initial height

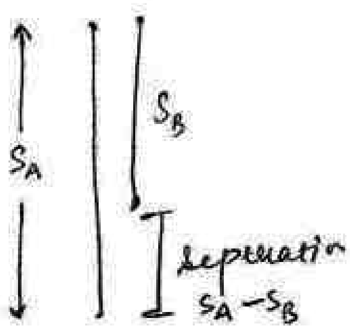
$$S_5 = S_{last}$$

$$0 + \frac{1}{\cancel{2}}\cancel{g}\, 5^2 = 0 + \frac{\cancel{g}}{\cancel{2}}(2n-1)$$

$$25 + 1 = 2n \quad \therefore n = 13$$

Initial height $(h) = \frac{1}{2} \times g\, t^2 = \frac{1}{2} \times 10 \times 169 = 845\,m$

Q) Two particles A and B are released fm sm heigt but at diff instant. Particle B is released 1 sec after 1 find seperation b/w them after 4 sec of releasing A



$$Sep = S_A - S_B$$

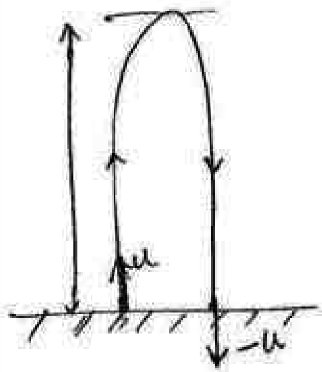
$$\frac{1}{2} g 4^2 - \frac{1}{2} g\, 3^2$$

$$\frac{g}{2}(16 - 9) = 35\,m$$

$$S_{n\,th} \quad S_{n\,4th} = a + \frac{10}{2}(2 \times 4 - 1)^{5}$$

$$= 35\,m$$

Q) A particle is thrown vertically upward with speed u from ground. Find its time of flight & H_{max}



let ↑ be +ve

For complete motion.

by $v = u + at$ $\therefore$ $-u = u - gT$

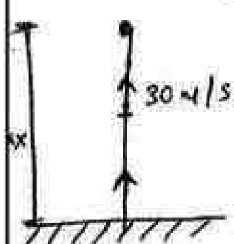
$\therefore gT = 2u$ $\therefore$ $\boxed{T = \frac{2u}{g}}$

for ascending motion

$0 = u^2 + 2as$ $\Rightarrow$ $0 = u^2 + 2(-g)H_{max}$

$\therefore$ $\boxed{H_{max} = \frac{u^2}{2g}}$

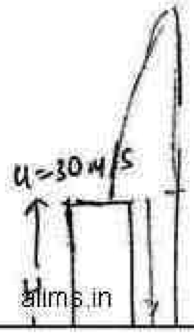
Q) A body is thrown vertically upward frm the ground such that its velocity becomes 30 m/s when it attains $\frac{1}{2}$ of max height. Find max height attained by it



For upper $\left(\frac{H_{max}}{2}\right)$ height :

$\frac{H_{max}}{\cancel{2}} = \frac{u^2}{\cancel{2}g}$ $\therefore$ $H_{max} = \frac{(30)^2}{10} = 90\,m$

Q) A body is projected frm top of the tower with $u = 30$ m/s and it reaches the ground after 8 sec. Find the height of tower and total distance travelled by the body.



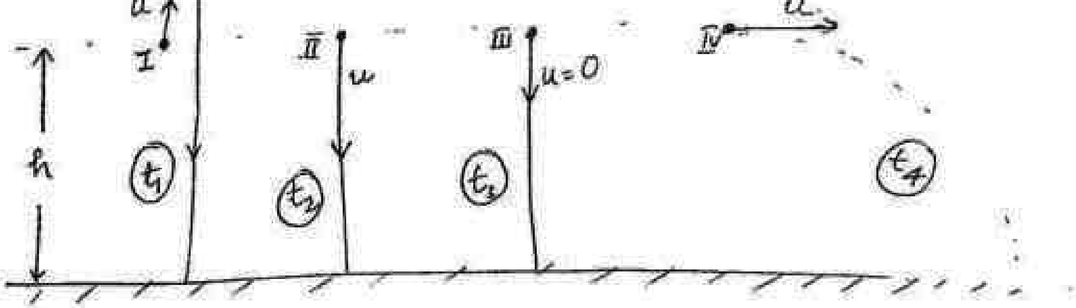
For complete motion (↑ = +ve)

$[u = 30\ ,\ a = -10\ ,\ t = 8,\ H = |\vec{s}| = ?]$

$S = ut + \frac{1}{2}at^2$ $= 30 \times 8 + \frac{1}{2}(-10)\ 8^2$

$\therefore$ $S = -80\,m$ $\therefore$ $H = 80\,m$

Q)



$t_{II} < (t_3 = t_4) < t_1$

$h = \frac{1}{2} g t_3^2$

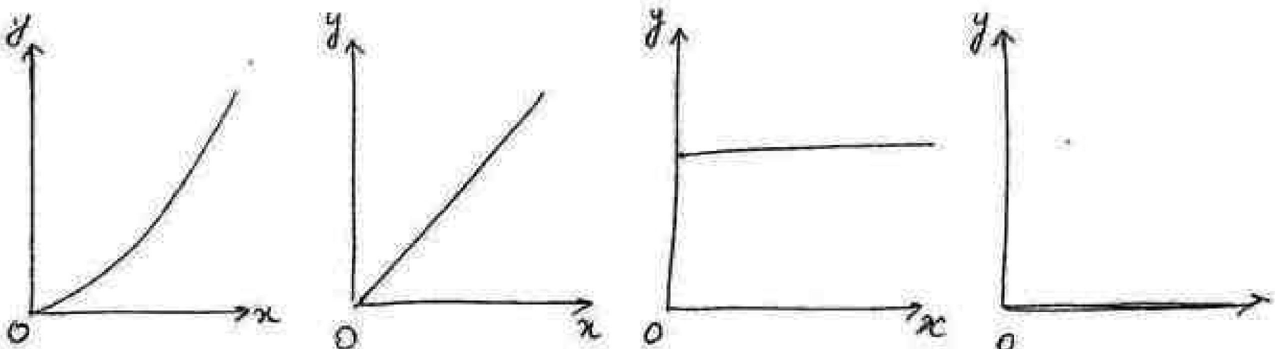
$\therefore t_3 = \sqrt{\frac{2h}{g}}$

$S_y = u_y t + \frac{1}{2} a_y t_4^2$

$h = 0 + \frac{1}{2} g t_4^2$

$\boxed{\therefore t_4 = \sqrt{\frac{2h}{g}} = t_3}$

Differentiation order :

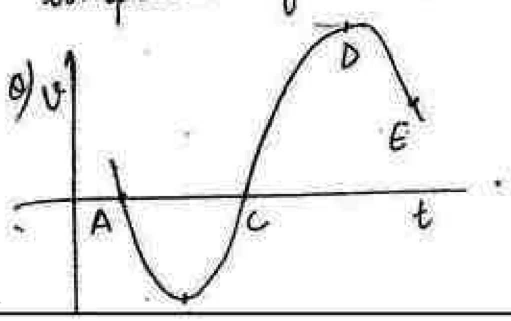


$y \propto x^2$ $\quad y' \propto x$ $\quad y'' =$ constant $\quad y''' = 0$

Integration order

Graphs with time can be used to study only one component of motion at a time.

Q)



check the sign of acc^n

A = $(a < 0)$

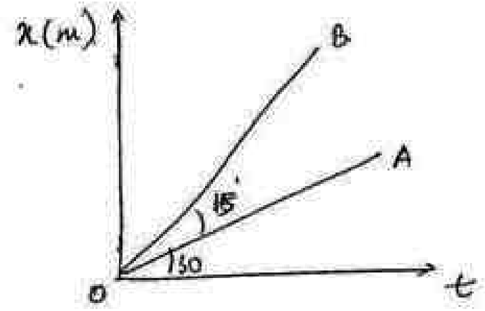
B = $(a = 0)$

C = $(a > 0)$

D = $(a = 0)$

Speed ↑ (A to B) (C to D)

Speed ↓ (B to C) (D to E)

a) 

i) V_A = slope = $\tan 30° = \frac{1}{\sqrt{3}}$ m/s $\qquad a_A = 0$

ii) V_B = " $\tan 45° = 1$ m/s $\qquad a_B = 0$

2) A particle starts from origin and moves along x-axis. Its v-t graph is given below. Find

i) displacement in 4 sec $= \cancel{45}\,15 + 15 + 10 = 40$ m

ii) distance travelled in 6 sec $\cancel{60}\,15 + 15 + 10 + 20 = 60$m

~~iii)~~ max distance from the origin at $t = 4$s (40 m)

iv) V_{avg} over 4 sec $\frac{40}{4} = 10$ m/s

v) $\vec{V_{avg}}$ over 6 sec $\frac{\cancel{45}\,15+15+10-20}{6} = \frac{20}{6}$

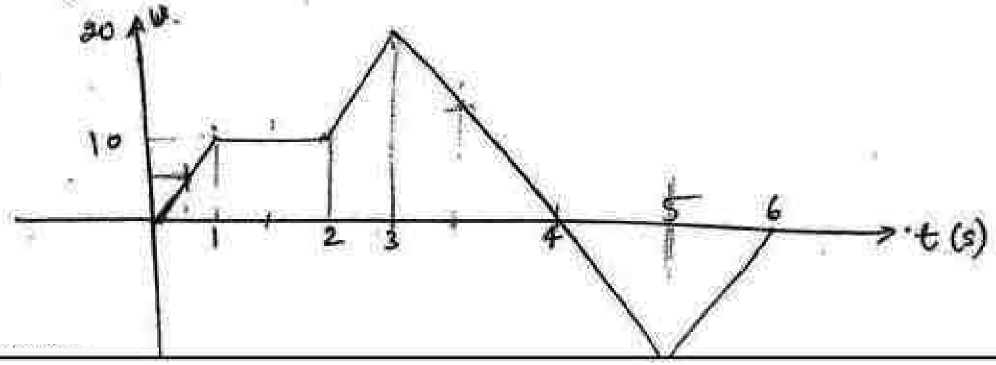
vi) avg. accn over 5 sec $= \frac{-20-0}{5} = -4$ m/s^2

vii) $a_{t=0.5s} = 10$ m/s^2

viii) $a_{t=1.5s} = 0$ ($\because$ v is constant)

ix) $a_{t=3.5s} = -20$ m/s^2

x) $V_{t=0.5s} = 5$ m/s



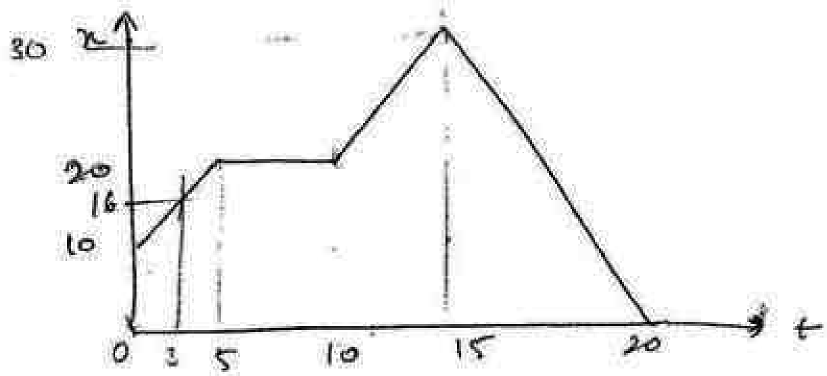
Q) A particle is moving along x-axis. its pos - t graph is given. Find

i) avg. velocity over 15 sec $= \frac{x_{t=15} - x_{t=0}}{15} = \frac{30-10}{15} = 1.3$

ii) avg velocity over 3 sec $= \frac{x_{t=3} - x_{t=0}}{3} = \frac{6}{3} = 2$ m/s

iii) Velocity at $t = 17$ sec $= -\frac{30}{5} = -6$ m/s

iv) acc^n at $t = 12$ sec $= 0$ (constant velocity)



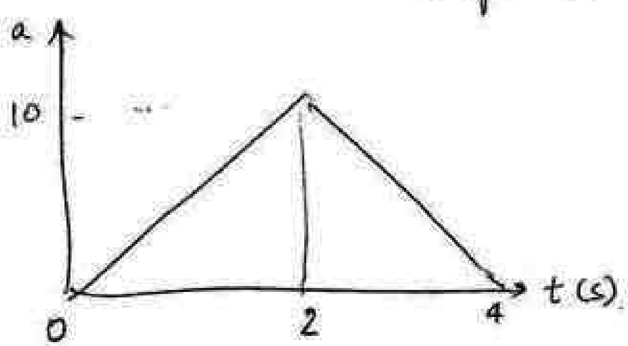
Q) For a particle moving along x-axis. a-t graph is given. If $u = 10$ m/s, Find

i) Velocity at $t = 2$ sec $\quad V_{t=2s} - V_{t=0} = 10 \quad \therefore V_{t=2s} + 10 = 10$

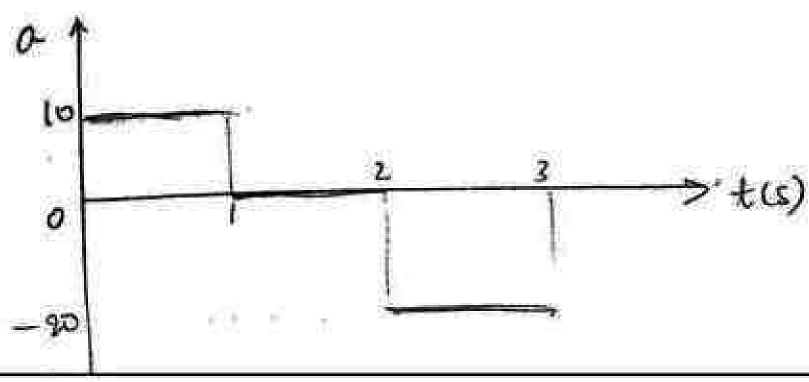
$\therefore V_f = 20$ m/s.

ii) " " $t = 4$ sec $\quad \Delta V = V_f - V_i =$

$\therefore V_f = 20 + 10 = 30$ m/s



Q)



Q) A particle is undergoing 1-D motion, its (a-t) graph is given. If $u = 50$ m/s. Find

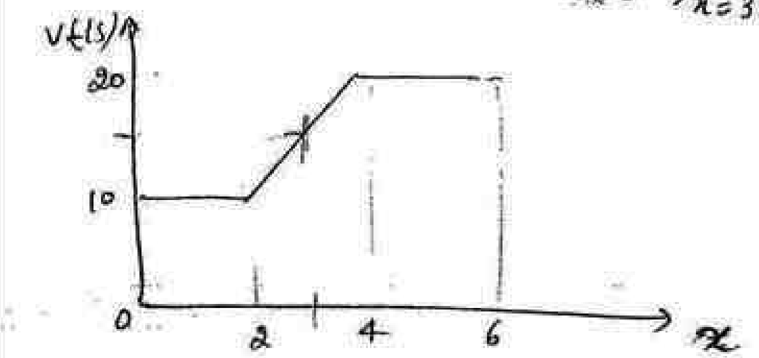
i) $V_{t=1\,sec} = 50 + 10 = 60$ m/s

ii) $V_{t=2\,sec} = V_{t=1s} = 60$ m/s

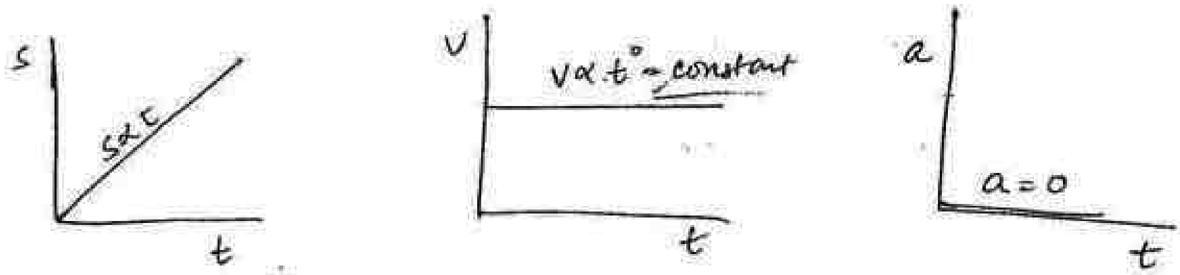
iii) $V_{t=3\,sec} = 60 - 20 = 40$ m/s

Q) For a particle moving along v-x graph is given

Find i) $a_{x=3m} = V_{x=3m}\left(\frac{dv}{dx}\right)_{x=3m} = 15 \times 5 = 75\ m/s^2$

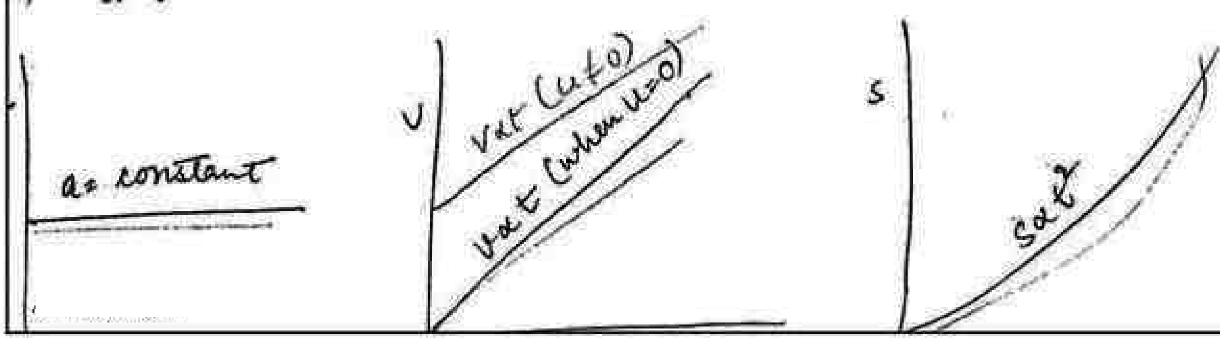


Q) If $s \propto t$, then plot, i) s-t graph ii) v-t iii) a-t



Q) For uniformly accelerated motion. Plot

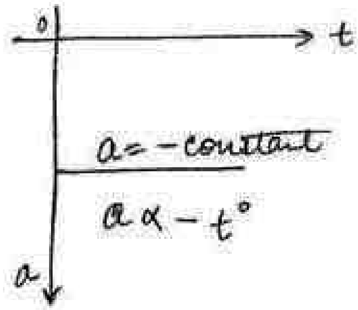
i) a-t $\quad a \propto t^0$ $\quad$ ii) v-t $\quad$ iii) s-t



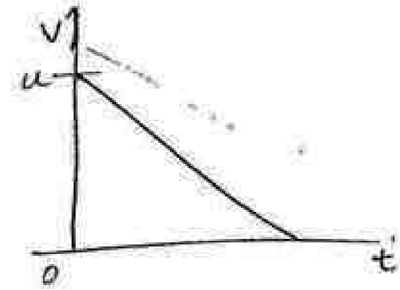
pos may ... displacement always starts from zero.

Q) for uniformly retarded motion.

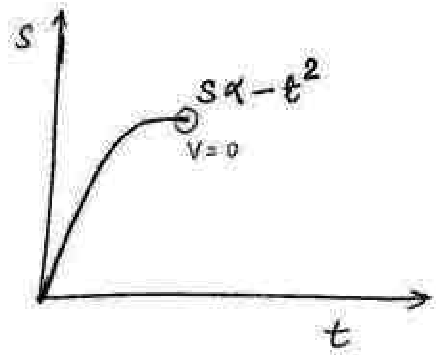
a - t

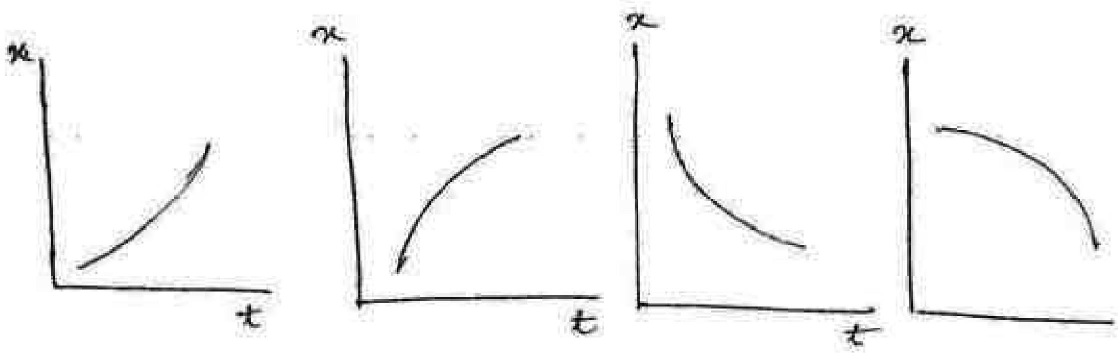


v - t (only for +ve value)



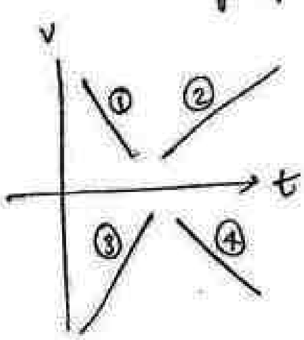
s - t





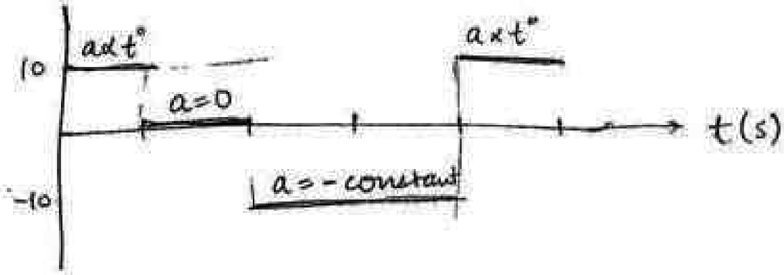
sign of velocity	+ve	+ve	−ve	−ve
sign of a	+ve	−ve	+ve	−ve
speed	↑	↓	↓	↑

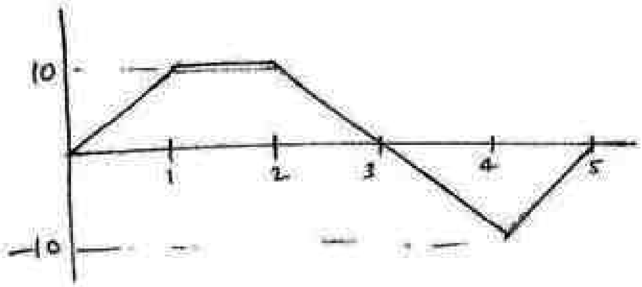
Q) The v-t graph for four particles is given below

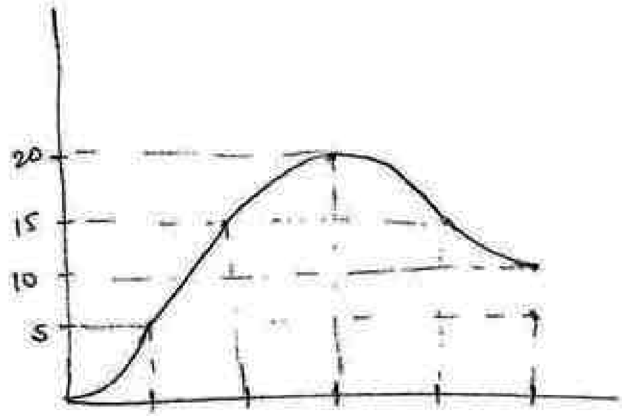


Particle	1	2	3	4
sign of vel.	+ve	+ve	−ve	−ve
sign of a	−ve	+ve	+ve	−ve
speed	↓	↑	↓	↑

Q) For a particle, $u=0$, moving along x axis a-t graph is given below. Draw v-t and x-t for 5 sec





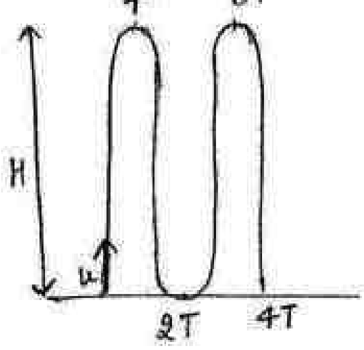


Q) A ball is thrown vertically upward with velocity u from the ground. It reaches the max height and returns back at the ground and collides elastically and rebounds again to the same height. Taking vertically upward dir^n as +ve and the point of projection as the origin. draw the following graph for 2 cycle

i) a-t graph

iv) displacement $-t$

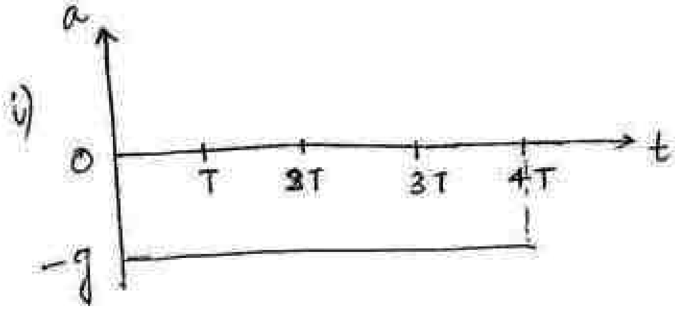
v) v-t

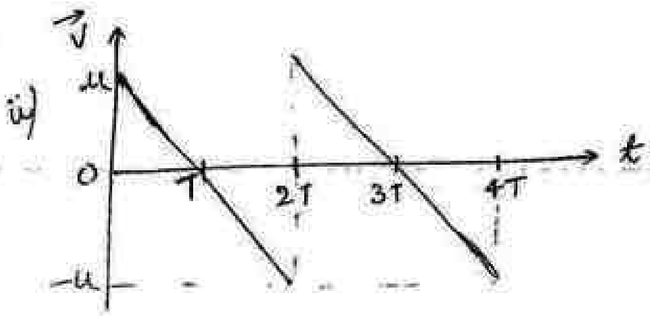


let $\uparrow = +ve$

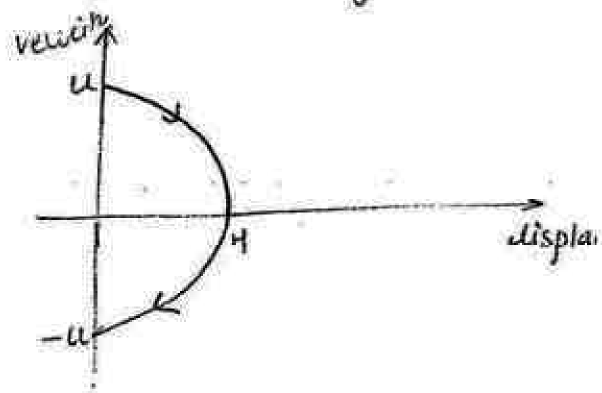
$$H = \frac{u^2}{2g} \quad , \quad T = \frac{u}{g}$$

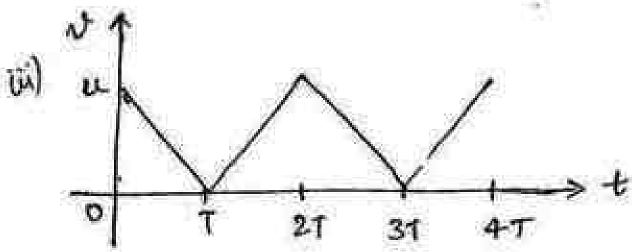
$$a = -g, \quad a \propto -t^0, \quad V \propto -t,$$
$$S \propto -t^2$$

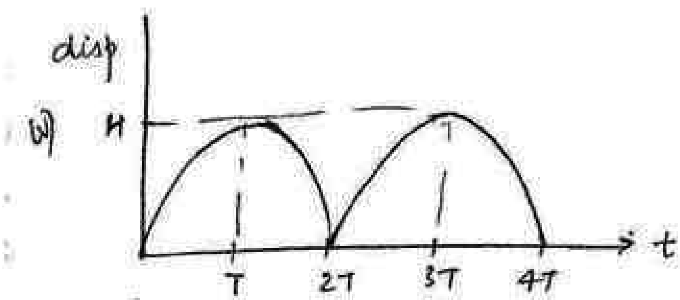


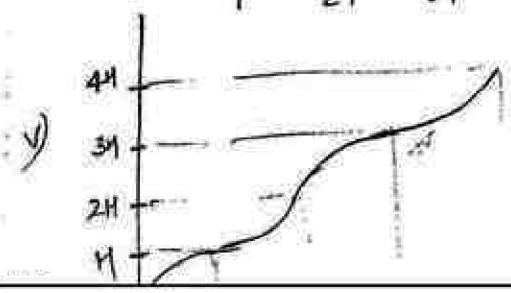


$\because v^2 = u^2 - 2gS$

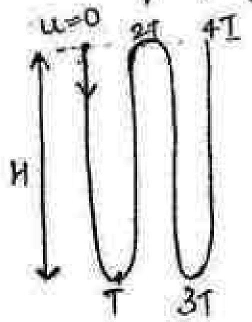








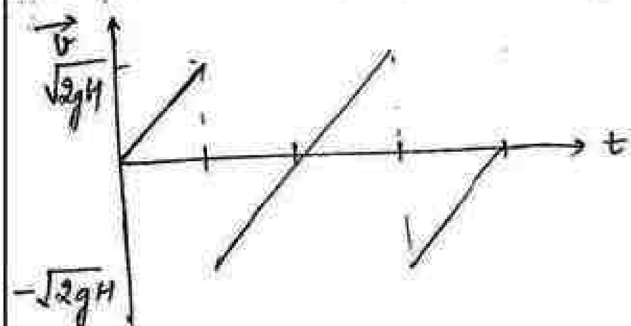
9) A particle is dropped 4m H height, It strikes the ground collides elastically and rebounds upto same initial height. Taking vertically downward dirn as +ve and points of projection at origin. Plot d graph for 2 cycle

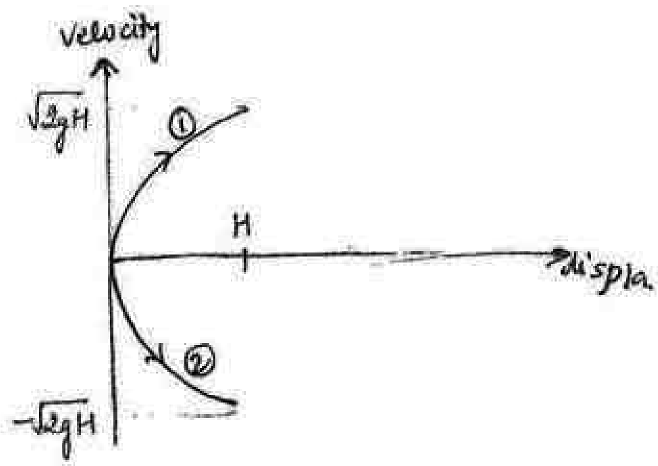


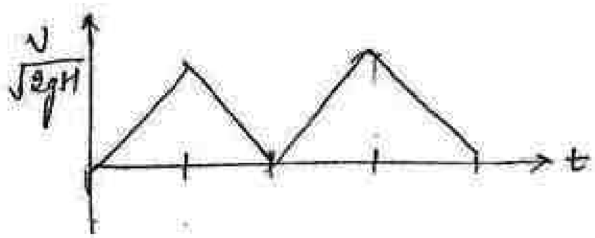
let $\downarrow$ = +ve

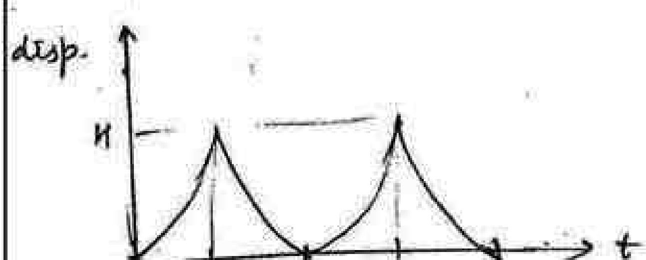
$v=\sqrt{2gH}$, $T=\sqrt{\frac{2H}{g}}$

$a=g$, $a\propto t^0$, $v\propto t$, $s\propto t^2$

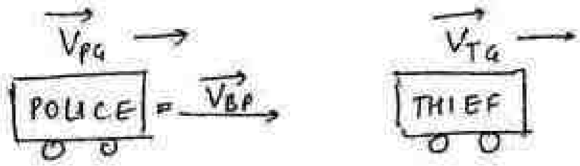








Relative motion



here, $\vec{V_{TP}} = \vec{V_{TG}} - \vec{V_{PG}}$

$$\vec{V_{BT}} = \vec{V_{BP}} - \vec{V_{TP}}$$

$$= \vec{V_{BP}} - (\vec{V_{TG}} - \vec{V_{PG}})$$

OR.

$$\vec{V_{BT}} = \vec{V_{BG}} - \vec{V_{TG}}$$

$$\vec{V_{BT}} = \vec{V_{BP}} + \vec{V_{PG}} - \vec{V_{TG}} \quad \left[\because \vec{V_{BP}} = \vec{V_{BG}} - \vec{V_{PG}}\right]$$

- If $a_x = 0$, then use $(\text{speed})_x = \dfrac{(\text{dis})_x \text{ or seperation covere}}{\text{time}}$
- If a_x = constant then use $V_x = u_x + a_x t$ — (i)

$$S_x = u_x t + \frac{1}{2} a_x t^2 \quad \text{— (ii)}$$

$$V_x^2 = u_x^2 - 2a_x S_x \quad \text{— (iii)}$$

Q) Two trains of length 200 m and 400 m are moving on parallel tracks with constant speeds, when one overtakes the other in same dirn. It takes 60s but when it crosses the other in oppo dirn, it takes 10 sec. Find individual speed of both the trains.

sm dirn $\quad V_1 - V_2 = \dfrac{200 + 400}{60}$

$$V_1 + V_2 = \frac{600}{10}$$

) A car is moving wid $\vec{V} = 10\, m/s\ \hat{i}$ and experinces dt a truck is moving wd the velocity of $20\, m/s\ \hat{j}$. Find the actual velocity of Truck

$$\vec{V_{TC}} = \vec{V_T} - \vec{V_C}$$

$$\vec{V_T} = \vec{V_{TC}} + \vec{V_C}$$

$$= 20\hat{j} + 10\hat{i}$$

$$= \sqrt{20^2+10^2}\ m/s \text{ due } N \tan^{-1}\left(\frac{10}{20}\right) E$$

$$10\sqrt{5}\ m/s \text{ due } N \tan^{-1}\left(\frac{1}{2}\right) E$$

$$\text{due } E \tan^{-1}(2)\, N$$

Q) A car is moving $\vec{V} = 10\, m/s\ \hat{j}$ and a truck is moving in dir^n E 30° N. wd the sm speed find velocity of Car w.r.t. truck

$$\vec{V_{CT}} = \vec{V_C} - \vec{V_T}$$

$$= 10\, m/s\ \hat{j} - \left[10\cos 30^\circ\, \hat{i} + 10 \sin 30^\circ\, \hat{j}\right]$$

$$= 10\hat{j} - 5\sqrt{3}\,\hat{i} - 5\hat{j}$$

$$= \left(5\hat{j} - 5\sqrt{3}\,\hat{i}\right) m/s.$$

$$= 10\, m/s\ N \tan^{-1}\frac{\cancel{5}\sqrt{3}}{\cancel{5}}\ W$$

$$= 10\, m/s \text{ due } N\ 60^\circ\ W$$

Q) A Train is moving wid the velocity of 60 m/s along north dir^n. A monkey is running on its roof wid $\vec{V} = 10\, m/s$ w.r.t train against its motion. Find the velocity of monkey w.r.t ground.

$$V_{MT} = V_{MG} + V_{TG} \qquad \therefore -10\hat{j} + 60\hat{j} = 50\hat{j}$$

velocity of 10 km/h in east dirⁿ. He feels dt wind is blowing from south wd sm speed. Find.

i) $\vec{V_{WG}}$ (wind w.r.t ground)

ii) If the speed of man is doubled, then find $\vec{V_{WM}}$ (wind w.r.t man)

i) $\vec{V_{WG}} = \vec{V_{WM}} + \vec{V_{MG}} = 10\hat{j} + 10\hat{i}$

$= 10\sqrt{2}$ km/h due NE

ii) $= \vec{V_{WM}} = \vec{V_{WG}} - \vec{V_{MG}}$

$= 10\hat{j} + 10\hat{i} - 20\hat{i}$

$= 10\sqrt{2}$ km/h due N-W

Q) A bird is flying wd constant speed u above a train which is moving wid constant speed v, find the avg speed of bird w.r.t train when it flies to and fro along the train (end to end)

$$\text{Avg speed} = \frac{\text{dis. travelled}}{\text{total time}}$$

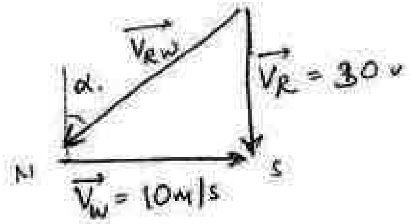
$$V_{avg} = \frac{L + L}{\frac{L}{u-v} + \frac{L}{u+v}}$$

$$= \frac{u^2 - v^2}{u}$$

<u>OR</u>

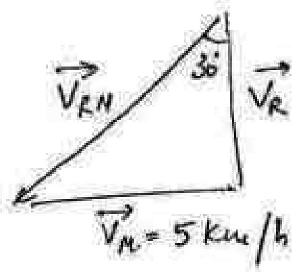
$$V_{avg} = \frac{2(u-v)(u+v)}{(u-v) + (u+v)} \Rightarrow \frac{2(u^2 - v^2)}{2u} = \frac{u^2 - v^2}{u}$$

Q) Rain is falling vertically with the speed of 30m/s. A woman is riding a bicycle is $\vec{v} = 10$ m/s in N → S dir^n In which dir^n she should hold her umbrella to avoid raindrop



$\tan\alpha = \frac{V_W}{V_R} = \frac{10}{30}$ ∴ $\alpha = \tan^{-1}\left(\frac{1}{3}\right)$ from vt. towards south.

Q) A man is walking on a straight road in the horizontal dir^n with the speed of 5 km/hr. He feels at raindrops are hitting him at an angle of 30° from vt. find V_R and V_{RM}



∴ $\sin 30 =$

Q) A boat moves in b/w 2 fix point which are 1 km apart in a river. When it goes upstream, it takes 10 hrs but when it goes downstream, it takes 8 hr. Find speed of boat

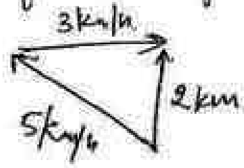
$V_{net} = \frac{dist}{time}$

up ⇒ $V_B - \cancel{V_R} = \frac{1}{10}$ —(I)

$V_B + \cancel{V_R} = \frac{1}{8}$ —(II)

∴ $2V_B = \frac{9}{40}$ ∴ $V_B = \frac{9}{80}$ km/h

Q) A man can swim with the speed of 5 km/h in still water ($\vec{V_{SR}}$). width of a river = 2km. and it is flowing with the speed of 3km/h. If he wanna cross the river with minimum displacement, then find the tym of crossing & dirⁿ of swimming.

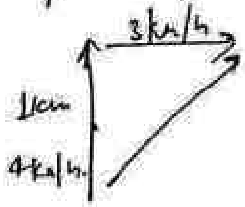


$\therefore\ t = \dfrac{d}{V_{SG}} = \dfrac{d}{\sqrt{V_{SR}^2 - V_R^2}} = \dfrac{2}{\sqrt{5^2-3^2}} = \dfrac{1}{2}$ hr

$\sin\theta = \dfrac{V_R}{V_{SR}} = \dfrac{3}{5} \quad \therefore\ \theta = 37^\circ$

at an angle of 87° $\perp$ to flow of river in upstream dirⁿ

Q) width of a river is 1 km and it is flowing with the speed of 3 km/h. a man can swim with the speed of 4 km/h in still water. If he makes his stroke normal to the river current then find tym of crossing and drift along the river



$t = \dfrac{d}{V_{SR}} = \dfrac{1}{4}$ hr = 15 min

drift = $V_R \times t = 3 \times \dfrac{1}{4} = \dfrac{3}{4}$ km = 750 M

Projectile Motion

Q) A particle is projected, $\vec{V} = u$ from ground such dt its min speed in its trajectory is $\frac{u}{2}$. Find.

i) θ ii) T iii) R iv) H

i) $u\cos\theta = \frac{u}{2} \quad \therefore \quad \theta = 60^\circ$

ii) $\frac{2u_y}{g} = \frac{\sqrt{3}u}{g}$

iii) $\frac{u^2\sin 2\theta}{g} = \frac{u^2\sqrt{3}}{g\,3}$

iv) $\frac{u^2\sin^2\theta}{2g} = \frac{3}{8}\frac{u^2}{g}$

Q) A particle is projected fm the ground wd velocity $\vec{u} = (\overset{u_x}{8\hat{i}} + \overset{u_y}{6\hat{j}})$ m/s taking $g = 10\,m/s^2$ find θ, T, R, H

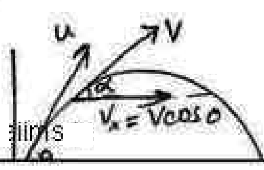
i) $\tan\theta = \frac{u_y}{u_x} = \frac{6}{8} = \frac{3}{4} \;\therefore\; \theta = 37^\circ$

ii) $\frac{2u_y}{g} = \frac{12}{10} = 1.2$ sec

iii) $\frac{2u_x u_y}{g} = \frac{2\times8\times6}{10} = \frac{96}{10} = 9.6\,m$

iv) $\frac{u_y^2}{2g} = \frac{6^2}{2\times10} = 1.8\,m$

Q) A particle is projected fm the ground wd u velocity at θ angle fm horizontal. Find its speed when it is moving at α and fm horizontal during its ascending motion

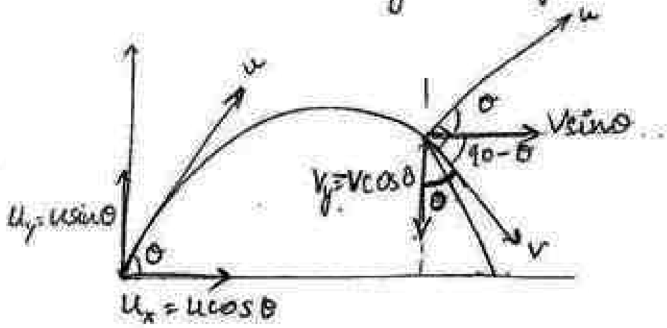


$\because$ Hz ~~cc~~ component of velocity remain constant.

$V\cos\alpha = u\cos\theta$

$\therefore V = \frac{u\cos\theta}{\cos\alpha} = u\cos\theta\sec\alpha$

6) A body is projected from the ground with velocity u at θ angle from Hz. Find its speed when its final velocity becomes to initial velocity also find time after which it occurs.



let ↑ = +ve

$(a_y = -g)$

Hz velocity remains same $\therefore$ $u\cos\theta = V\sin\theta$

$\therefore$ $\boxed{V = u\cot\theta}$

fm 1st eq^n of motion,

$V_y = u_y + a_y t$ $\quad\therefore\quad t = \dfrac{V_y - u_y}{a_y}$

$t = \dfrac{-V\cos\theta - u\cos\theta}{-g} \Rightarrow \therefore \dfrac{u\cot\theta\cos\theta - u\cos\theta}{-g}$

$\therefore$ $\dfrac{u\cos^2\theta - u\cos\theta\sin\theta}{\sin\theta\,(-g)}$

IInd method

$\vec{u} \perp \vec{V}$ $\quad\therefore\quad$ $\vec{u}\cdot\vec{V} = 0.$

$(u_x\hat{i} + u_y\hat{j})\cdot(v_x\hat{i} + v_y\hat{j}) = 0.$

$\therefore$ $u_x v_x + u_y v_y = 0.$ $\quad\because v_y = u_y + a_y t$

$(u\cos\theta)(u\cos\theta) + u\sin\theta(u\sin\theta - gt) = 0$

$u^2\cos^2\theta + u^2\sin^2\theta = gt\,u\sin\theta.$

$\therefore$ $t = \dfrac{u}{g\sin\theta}$

Q) A player throws a ball such that it attains its max Hz range of 100 m. Find. i) max height attained by ball.
ii) Max height upto which a player can throw the ball

$$R_{max} = \frac{u^2}{g} = 100 \quad [\because \theta = 45^\circ]$$

i) $$H = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2/g}{4} \therefore \frac{R_{max}}{4} = \frac{100}{4} = 25\text{ m}$$

$$\star \boxed{H = \frac{R_{max}}{4}} \quad \text{At } \theta = 45^\circ$$

ii) At $\theta = 90^\circ$ then, $$H_{max} = \frac{u^2}{2g} = \frac{100}{2} = 50\text{ m}$$

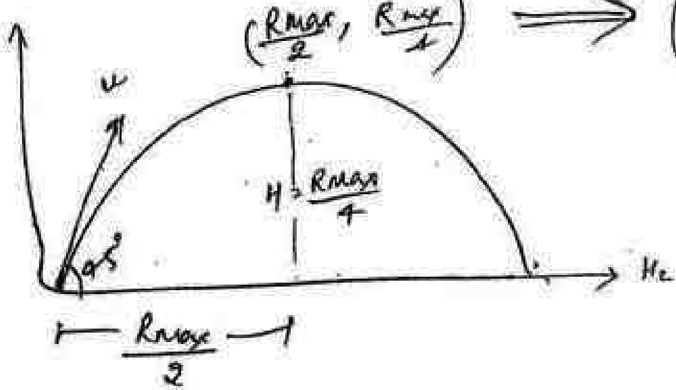
Q) A player can throw a ball upto the max height of 100 m. Find max Hz distance to which he can throw the same ball

$$H_{max} = \frac{u^2}{2g} = 100 \qquad \therefore R = \frac{u^2}{g} = 100 \times 2 = 200\text{ m}$$

Q) A particle is projected from the ground at 45° from Hz. R = 500 m. Taking point of projection as origin, find co-ordinates of a point where its KE becomes min.

$$\left(\frac{R_{max}}{2}, \frac{R_{max}}{4}\right) \Longrightarrow \left(\frac{500}{2}, \frac{500}{4}\right)$$

$$= (250, 125)\text{ m}$$



Q) A ~~part~~ particle is projected from ground at $\theta = 30^\circ$ from Hz. R = 100 m. Find co-ordinate at top assuming point of projection at origin

$\theta = 30^\circ$, R = 100 m

$$\boxed{\frac{u^2}{...} = 100} \text{ — (1)}$$

$$\therefore \; H = \frac{u^2 \sin^2\theta}{2g} = \frac{100}{\sqrt{3}} \times \frac{1}{4} = \frac{25}{\sqrt{3}} \text{ m}$$

$$(x_{top}, y_{Top}) = \left(\frac{R}{2}, H\right) = \left(\frac{100}{2}, \frac{25}{\sqrt{3}}\right) \text{m}$$

Q) If $R = nH$, find θ

$$\frac{u^2 \, 2\sin\theta\cos\theta}{g} = n \, \frac{u^2 \sin^2\theta}{2g}$$

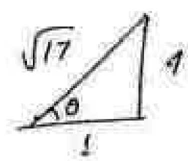
$$\therefore \; \frac{4}{n} = \frac{\sin\theta}{\cos\theta} \qquad \therefore \; \boxed{\tan\theta = \frac{4}{n}}$$

$$\therefore \; \theta = \tan^{-1}\left(\frac{4}{n}\right)$$

Q) If $R = H$, Find, θ, R and H

$$\therefore \; \tan\theta = \frac{4}{n} \qquad \therefore \; \tan\theta = \frac{4 \rightarrow P}{1 \rightarrow B} \quad (\because n = 1)$$

$$\theta = \tan^{-1}(4) \simeq 76^\circ$$



$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin\theta\cos\theta \times 2}{g}$$

$$\sin\theta = \frac{4}{\sqrt{17}}$$

$$\cos\theta = \frac{1}{\sqrt{17}}$$

$$= \frac{u^2 \frac{4}{\sqrt{17}} \times \frac{1}{\sqrt{17}} \times 2}{g} = \frac{8u^2}{17g}$$

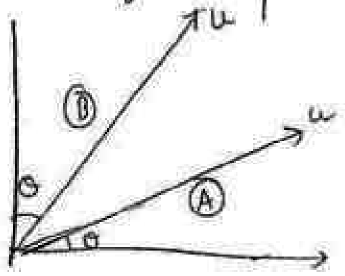
$$H = R = \frac{8u^2}{17g}$$

* If $\theta = 76^\circ$, then $R = H$

If $\theta > 76^\circ$, then $R < H$

If $\theta < 76^\circ$, then $R > H$

Q) Find following Ratio



i) $\dfrac{T_A}{T_B} = \dfrac{2u_y}{g} \propto \dfrac{(u_y)_A}{(u_y)_B} = \dfrac{u\sin\theta}{u\cos\theta} = \tan\theta$

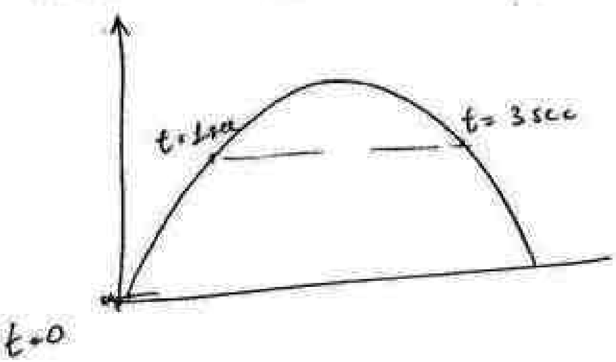
ii) $\dfrac{H_A}{H_B} = \dfrac{\cancel{u^2}\sin^2\theta}{\sin^2(90-\theta)} = \dfrac{u^2\sin^2\theta}{u^2\cos^2\theta} = \tan^2\theta$

iii) $\dfrac{R_A}{R_B} = \dfrac{2u_{xA}\,u_{yA}}{2u_{xB}\,u_{yB}} = 1.$

If $u_1 = u_2$ and θ_1 and θ_2 are complimentary angles (sum=90°) then $R_1 = R_2$.

θ	15°	30°	45+α
90−θ	75°	60	45−α

Q)



If V_4 Height is same at t=1sec and 3 sec find T and Hmax attained.

$$(S_y)_{t=1\,sec} = (S_y)_{t=3s}$$

$$u_y(1) - \frac{1}{2}g(1)^2 = u_y(3) - \frac{1}{2}g(3)^2$$

$$40 = 2u_y \qquad \therefore u_y = 20\ m/s$$

$$\therefore T = \frac{2u_y}{g} = \frac{2\times 20}{10} = 4\,sec$$

$$\therefore H = \frac{u_y^2}{2g} = \frac{20^2}{20} = 20\ m$$

OR

$$H = \frac{T^2 g}{8} = \frac{4^2\times 10}{8} = 20\ m$$

Q) The eq^n of trajectory of a projectile is given by

$y = \sqrt{3}x - 5x^2$

$\sqrt{3}x\left(1 - \frac{5x}{\sqrt{3}}\right)$ $\therefore$ $\theta = 60^\circ$

$R = \frac{\sqrt{3}}{5}$

& If $x = \frac{R}{2}$, then $y = H$

Q) $y = (4x - 3x^2)m$. Find θ and H

Q) If $x = a\cos t$ and $y = b\sin t$

If $x = a\cos t$ $\quad$ $y = b\sin t$

$\cos t = \frac{x}{a}$ $\quad$ $\sin t = \frac{y}{b}$

squaring 'n' adding ① & ②

$\cos^2 t + \sin^2 t = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $\quad$ $\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore$ Path = elliptical

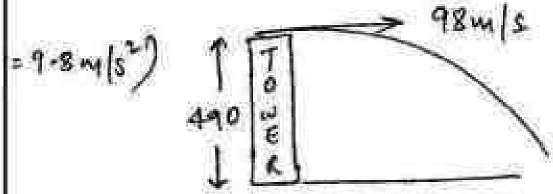
Q) If $x = 2t$ and $y = 6t + 4$

$t = \frac{x}{2}$ $\quad$ $\therefore y = \frac{6x}{2} + 4$ $\quad$ $\therefore y = 3x + 4$

$\therefore$ path = st. line

Q) $x = 3t$, $y = 9t^2 + 9$

Q) For the given projectile, find i) T ii) R iii) speed at ground iv) angle of velocity from vertical when body hits ground



i) $T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 490}{9.8}} = 10$

ii) $R = 98 \times 10 = 980$

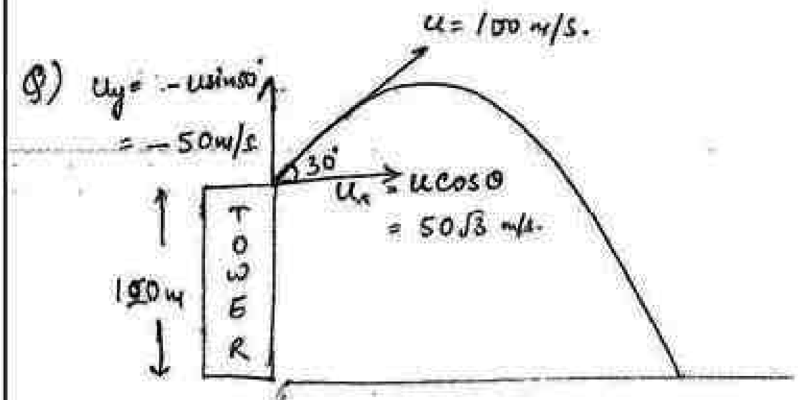
iii) $v = \sqrt{u^2 + (gt)^2}$

$= \sqrt{98^2 + 98^2} = \sqrt{2} \times 98$

iv) $\tan\theta = \frac{V_x}{V_y} = \frac{u}{gT} = \frac{98}{98 \times 10} = 1$

$\therefore \theta = 45°$

Q)



Find T and R

$S_y = 120$ m

$u_y = -50$ m/s

$a_y = 10$ m/s²

$T = ?$

$S_y = u_y T + \frac{1}{2} a_y T^2$

$120 = -50T + 5T^2$

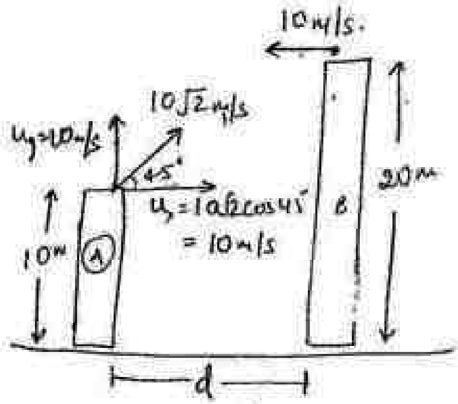
$\therefore T^2 - 10T - 24 = 0$

$\therefore T^2 - 12T + 2T - 24 = 0$ $\therefore T = 12$ sec

$\therefore R_x = uT = 50\sqrt{3} \times 12$

Q)

Find the Hz separation b/w the building so dt both may collide



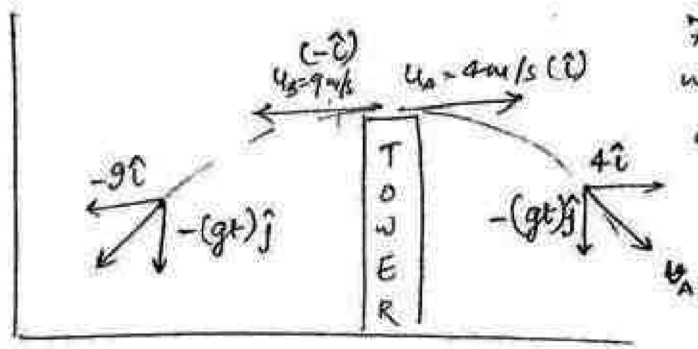
In vt. dirn: $t = \frac{\text{sep}^n \text{ covrd}}{(\text{speed})_{rel}}$ $\therefore t = \frac{10}{10+0} = 1 \text{ sec}$

In Hz dirn: $\text{sep}^n \text{ covrd} = \cancel{\text{sep}} (\text{speed})_{rel} \times t$

$d = (10+10) \times 1 = 20 \text{ m}.$

Q)

Find the tym after which the velocities of both the projectile will become ⊥ to each other.



If $\vec{V_A} \perp \vec{V_B}$, $\therefore \vec{V_A} \cdot \vec{V_B} = 0$

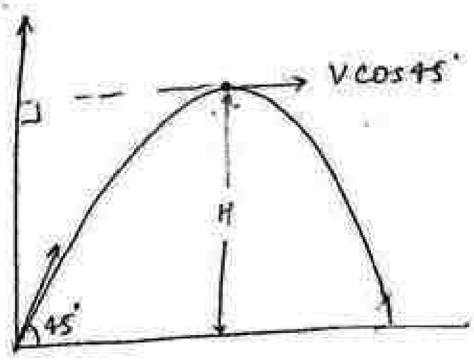
$\therefore \vec{V_A} = 4\hat{i} - gt\,\hat{j}$ $\therefore \vec{V_B} = -9\hat{i} - gt\,\hat{j}$

$\therefore (4\hat{i} - gt\,\hat{j}) \cdot (-9\hat{i} - gt)\,\hat{j} = 0$ $\therefore 36 + gt^2 = 0$

$gt^2 = 6$

$\therefore t = \frac{6}{g} = \frac{6}{10} = 0.6 \text{ sec}$

Q.2
Ex-II
Pg-82 (module 1)



angular momentum = $mvr \sin\theta$

$= mvH$

$= m(v\cos 45°)\left(\frac{u^2 \sin^2 45}{2g}\right)$

$= \boxed{\frac{mv^3}{4\sqrt{2}\,g}}$

Q) The x and Y components of momentum are given by

$p_x = 2\cos t$ kg m/s $\quad p_y = 2\sin t$ kg m/s

i) Find magnitude of momentum

ii) Angle b/w force and momentum

i) $\sqrt{2^2\cos^2 t + 2^2 \sin^2 t} = 2$

ii) $\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(2\cos t\,\hat{i} + 2\sin t\,\hat{j})$

$$\vec{F} = -2\sin t\,\hat{i} + 2\cos t\,\hat{j}$$

$\therefore \vec{F}\cdot\vec{p} = 0 \quad \therefore \quad \vec{F} \perp \vec{p} \quad \therefore$ angle $= 90^\circ$

OR

$\because \quad |\vec{p}| =$ constant

$\therefore \quad \vec{F}$ must be $\perp$ to $\vec{p}$

Q.) A particle of mass 5 kg starts its motion with the velocity of $\vec{u} = (30\hat{i} + 40\hat{j})$ m/s. A force of $\vec{F} = (-4\hat{i} - 5\hat{j})$ N acts on it. Find tym after which the Y-component of velocity becomes zero

$V_y = u_y + a_y t \quad \therefore \quad V_y = u_y + \left(\frac{F_y}{m}\right)t$

$0 = 40 + \left(\frac{-5}{5}\right)t \quad \therefore \quad t = 40$ sec

Parabolic path

Q) When sm force is applied on 2 diff^n object, the acc^n produced in them are $6\,m/s^2$ and $4\,m/s^2$. If the same F is applied on the combinations of these 2 object den find the acc^n of the combination

$F = m_1 a_1 \quad \therefore m_1 = \frac{F}{a_1} \quad \therefore F = m_2 a_2 \quad \therefore m_2 = \frac{F}{a_2}$

by comb^n : $F = (m_1 + m_2) a_c$

$$\therefore \cancel{F} = \left(\frac{\cancel{F}}{a_1} + \frac{\cancel{F}}{a_2}\right) a_c \qquad \therefore \quad 1 = \left(\frac{a_1 + a_2}{a_1 a_2}\right) a_c$$

$$\therefore \; a_c = \frac{a_1 a_2}{a_1 + a_2} = \frac{6 \times 4}{6 + 4} = 2.4 \; m/s^2$$

Q) A particle of mass 2 kg starts its motion wid $\vec{V}_{in} = 4 m/s$ on a horizontl surface. It stops after travelling for 2 sec. Find F required to move it wd uniform velocity of 4 m/s

$$F_{app} = \frac{\Delta P}{\Delta t} = \frac{m(V-u)}{t} = \frac{2(0-4)}{2} = -4 N$$

$\therefore \; F_{req} = 4 N$

Q) A gun fires bullets of mass m wid velocity v If the F_{avg} exerted on the gun is F then find the no of bullets fired per unit tym

Δp of 1 bullet $= mv - 0 = mv$

Δp of n bullet $= n mv$.

by $F_{avg} = \frac{\Delta p}{\Delta t}$ $\quad \therefore F = \left(\frac{n}{t}\right) mv \quad \therefore$

no of bullets fired per unit tym $\frac{n}{t} = \frac{F}{mv}$

Q) Sand is dropped on a conveyor belt at the rate of 20 kg/s. Find additional force to keep the belt moving wd uniform velocity of 5 m/s.

$$F = V \frac{dm}{dt} = 20 \times 5 = 100 N$$

Concept of F_{avg}

In some event, if the tym span of at event is very small and the force is changing rapidly den sumtyms it becums impossible to calculate the instantaneous value of force. In such cases we find the F_{avg} by ~~dividing~~ $\frac{\Delta p}{\Delta t}$

eg: i) To throw a ball on a wall / floor

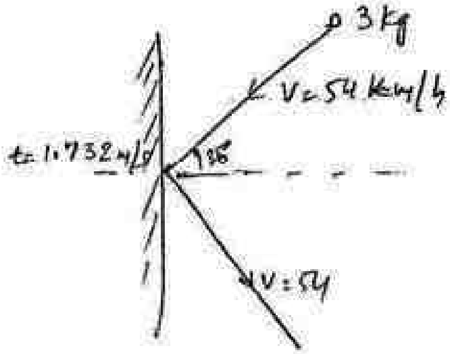
ii) To hit a ball with a bat or hockey stick

iii) To kick a football

iv) To hit a coin on carromboard.

v) Hammering a nail.

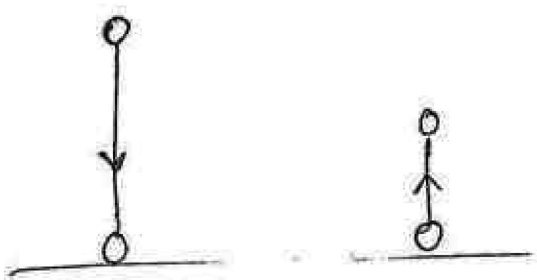
Q) For the given fig. Find $|\overrightarrow{F_{avg}}|$ exerted on ball by wall



$$F = \frac{2mu\cos\theta}{t}$$

$$= \frac{2\times 3\times 15\times \sqrt{3}}{1.732\times 2} = 45\ N$$

Q) A ball of mass 400 g is dropped from a height of 5m on a horizontal floor. It remains in contact with the floor for 1 millisec and rebounds upto the height of 1.8 m Find the ~~st~~ $|\overrightarrow{F_{avg}}|$ exerted on the wall by the floor



$V_b = -\sqrt{2gH}$

$= -\sqrt{2\times 10\times 5}$

$= -10\ m/s$

$V_a = \sqrt{2gh}$

$= \sqrt{2\times 10\times 1.8}$

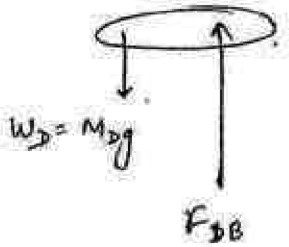
$= 6\ m/s$

$$\therefore F_{avg} = \frac{m(V_a - V_b)}{t}$$

$$= \frac{0.4\,(6+10)}{10^{-3}}$$

$$= 6400\ N$$

Q) A disc of mass 10g is ~~kepp~~ kept floating Hzly in air by firing bullets on it. vertically at the rate of 10 bullets/sec. (mass$_{bullet}$) = 5 gm. If the bullets are returning wd the sm speed after striking wd disk dow find speed of bullets.



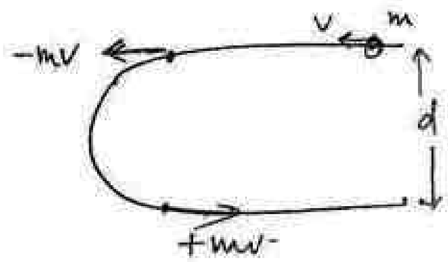
$(\downarrow) W_D = (\uparrow) F_{DB}$

$$m_D g = \frac{\Delta p}{\Delta t} = \frac{(2mv)n}{t}$$

$$\cancel{10} \times 980 = 2 \times 5 \times V_2 \times \cancel{10}$$

$\therefore V = 98$ cm/s

Q). A bead of mass m is sliding on a frictionless path with uniform speed V. The wire is bend in a semicircular form AB from the middle as shown in figure. Find $|\vec{F_{avg}}|$ by the bead on semicircular path AB



$$f_{avg} = \frac{\Delta p}{\Delta t} = \frac{2mV}{t}$$

$$= \frac{2mv}{\pi \frac{d}{2} / V} = \frac{4mV^2}{\pi d}$$

Q) A hammer of mass 1 kg strikes on a nail from a height of 19.6 m to fix it on a Hz surface. If the hammer stops within 10 millisec after striking wd the nail. Then find the $|\vec{F_{avg}}|$ exerted on nail by hammer

$$V = \sqrt{2gH} = \sqrt{2 \times 9.8 \times 19.6} = 19.6$$

$$\therefore |\vec{F_{avg}}| = \frac{\Delta p}{\Delta t} = \frac{0 - 19.6}{10 \times 10^{-3}} = 1960$$

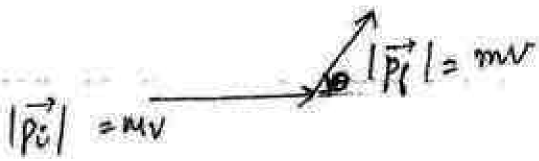
Q) A hosepipe is held [illegible] by man which is [illegible]
water with constant velocity v through nozel.
If Area of cross section is A and the density of H_2O is d
then find the F_{avg} exerted on hosepipe to keep it in its pos^n

$$\therefore \vec{F} = \frac{v\,dm}{dt} = v\frac{d(V\rho)}{dt} = v\frac{d}{dt}(A x d)$$

$$= v A d \frac{dx}{dt} = Av^2d$$

Q) A particle of mass m is moving along the perimeter of a regular polygon of n side with uniform speed v. Find the magnitude of impulse imparted at each corner

exterior angle $= \frac{360^\circ}{n}$ or $\frac{2\pi}{n}$ rad



$$|\overrightarrow{\text{impulse}}| = |\Delta \vec{p}| = |\vec{p_f} - \vec{p_i}|$$

$$= 2mv\sin\left(\frac{\theta}{2}\right) = 2mv\sin\frac{\pi}{n}$$

Q) Mass of a rocket is 1 kg and is launched in gravity free space. $\left(\frac{dm}{dt}\right) = 50$ kg/s and the $V_{exhaust} = 40$ m/s then find i) $\vec{a}_{in}$ of rocket
ii) $\vec{a}$ of rocket after 10 sec

Q) A rocket is set for vt. firing.

$m = 50$ kg and contains 450 kg of fuel. $V_{exhaust} = 0.2$ km/s

Find $\left(\frac{dm}{dt}\right)$ ie rate of fuel consumption.

i) Rocket just leaves launching pad

ii) Rises up wid. accⁿ of 20 m/s

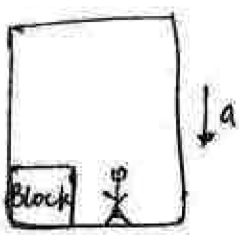
Q) The ratio of apparent wt of a man in a lift moving wid upward and downward accⁿ is 3 : 1. If the accⁿ is sm find accⁿ

$$\frac{\cancel{m}(g+a)}{\cancel{m}(g-a)} = \frac{3}{1}$$

$\therefore \quad g+a = 3g-3a$

$\therefore \quad a = \frac{2\times10}{4} = 5 \text{ m/s}^2$

Q)



Observer is <u>attached</u> to floor of lift with $15 \text{ m/s}^2 \downarrow a$

i) ~~acc~~ $\vec{a}_{bg}$ (block w.r.t ground).

ii) $\vec{a}_{bo}$ (block w.r.t observer)

i) $\vec{a}_{bg} = -10 \text{ m/s}^2$

ii) $\vec{a}_{bo} = \vec{a}_b - \vec{a}_o = -10\hat{j} - (-15\hat{j}) = 5 \text{ m/s}^2 \hat{j}$

or logically v cn say

Q) A block is kept on floor of lift at rest. The lift starts descending with $12\ m/s^2$. Find displacement of lift and block in first 0.2 sec of their motion.

<u>lift</u> $S_L = u_L t + \frac{1}{2} a_L t^2$

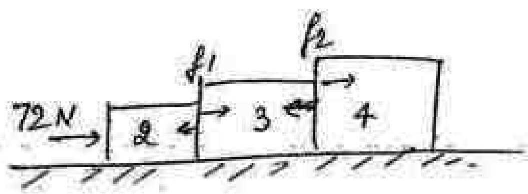
$= 0 + \frac{1}{2} 12 (0.2)^2 = .24\ m = 24\ cm$

<u>block</u> $S_B = u_B t + \frac{1}{2} a_B t^2$

$= 0 + \frac{1}{2} \times 10 (0.2)^2 = .2\ m = 20\ cm$

$\vec{S}_{BL} = 4\ cm$ upwards.

Q) Find contact forces f_1 and f_2

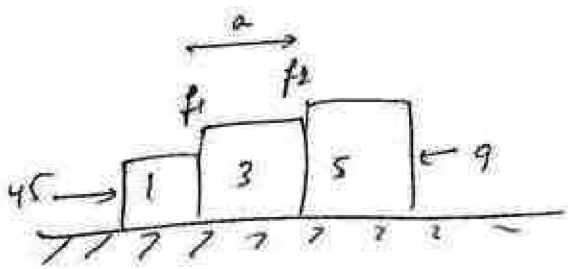


$a = \frac{F}{M_1 + M_2 + m} = \frac{72}{9} = 8$

$f_2 = 4 \times 8 = 32$

$f_1 = 7 \times 8 = 56$

Q).



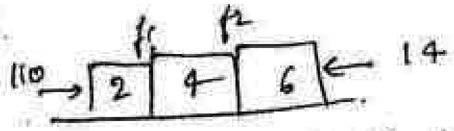
$a = \frac{45 - 9}{1 + 3 + 5} = \frac{36}{9} = 4$

for <u>1 kg</u> $45 - f_1 = a \quad \therefore f_1 = 45 - a = 45 - 4 = 41\ N$

for <u>5 kg</u> $f_2 - 9 = 5 \times a \quad \therefore f_2 = 9 + 5 \times 4 = 29\ N$

$$\boxed{f = (m_{ahead} \times a) + \text{opposing force}}$$

Q).



$a = \frac{110 - 14}{12} = 96$

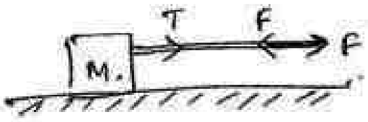
$\therefore f_2 = 6 \times 8 + 14 = 62$

$f_1 = 10 \times 8 + 14 = 94$

Tension in string :

- Tension is produced due to intermolecular forces.
  Its dirⁿ is always away from the point of contact and tied end of the string.

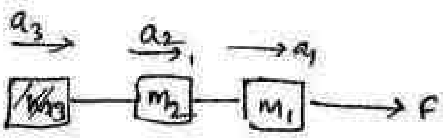
$\therefore \boxed{T = F}$



$\therefore \boxed{T = F}$

- strings are assumed to be - inextensible

eg:

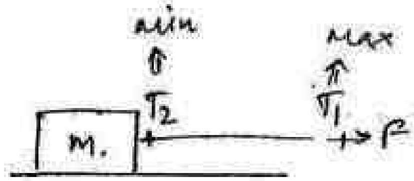


$\therefore a_1 = a_2 = a_3$

- Strings are assumed to be massless

  if massless, then, $T_1 = T_2$

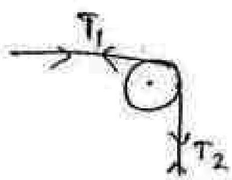
  if not massless then $T_1 > T_2$



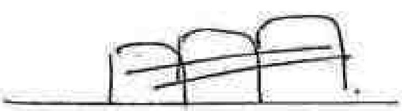
- assumed to be frictionless

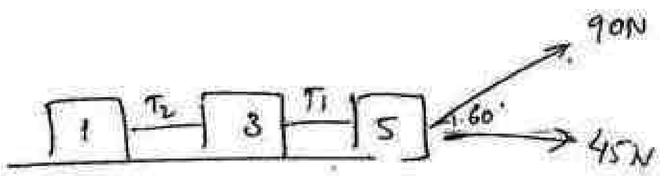
  if frictionless, then $T_1 = T_2$

  if not $\therefore$ then $T_1 \neq T_2$



Q) find T_1 & T_2

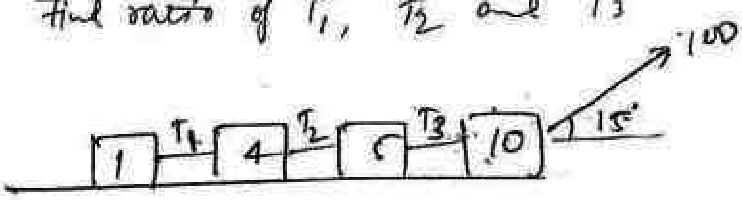




$\therefore \quad a = \frac{45}{9} = 5 \qquad \therefore T_1 = 4 \times 5 = 20\ N$

$T_2 = 1 \times 5 = 5\ N$

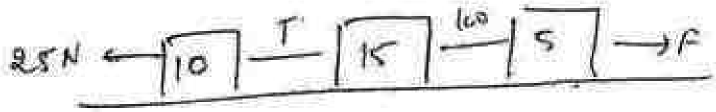
Q) Find ratio of T_1, T_2 and T_3



$T_1 : T_2 : T_3$

$1 \times a : 5 \times a : 10 \times a \qquad \therefore = 1 : 5 : 10$

Q.



$T - 25 = 10a$ $\qquad$ $100 - T = 15a$

$\therefore T = 10a + 25$ $\qquad$ $\therefore T = 100 - 15$

Q).

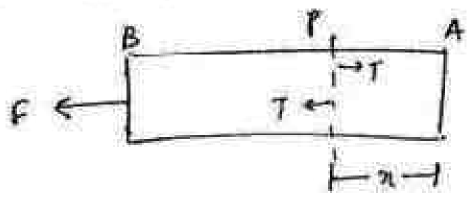
$T = 15 \times a$ $\qquad$ $200 - T = 10 \times a_{10kg}$

$\therefore 15 \times 4$ $\qquad$ $\dfrac{200 - 60}{10} = a$ $\qquad$ $\therefore a = 14\ m/s^2$

$= 60$

Q)

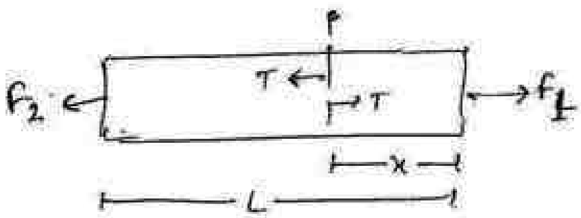
i)



Find T at cross section P in follow

$$T = \frac{\cancel{M}x}{L} \times \frac{F}{\cancel{M}} = \frac{Fx}{L}$$

ii)



$$a = \frac{F_1 - F_2}{M}$$

$$\therefore F_1 - T = \frac{\cancel{M}x}{L}\left(\frac{F_1 - F_2}{\cancel{M}}\right)$$

$$\therefore F_1 - F_1\frac{x}{L} + F_2\frac{x}{L} = T$$

$$\therefore \boxed{T = F_1\left(1 - \frac{x}{L}\right) + F_2\frac{x}{L}}$$

Q) a body 150 kg mass is hanging wd a rope for which breaking strength is 100 kg-wt. Find the min accⁿ of the object so dt the rope doesn't break.

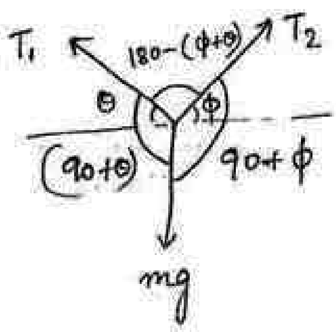
$T \leq$ Breaking strength

$m(g-a) \leq 100$ kg-wt

$150(g-a) \leq 100g$

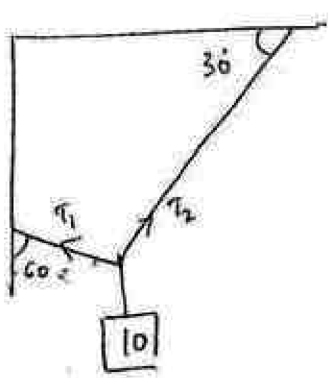
$\frac{g}{3} \leq a \quad \therefore \; a_{min} = \frac{g}{3}$

<u>Lami's Theorem</u>

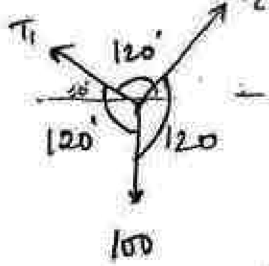


$$\frac{T_1}{\sin(90+\phi)} = \frac{T_2}{\sin(90+\theta)} = \frac{mg}{\sin[180-(\theta+\phi)]}$$

Q)

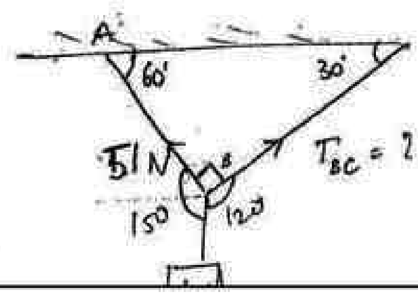


By Lami's theorem,



$\therefore T_1 = T_2 = 100N$

Q. Find T ~~Be~~ in string BC in weight W

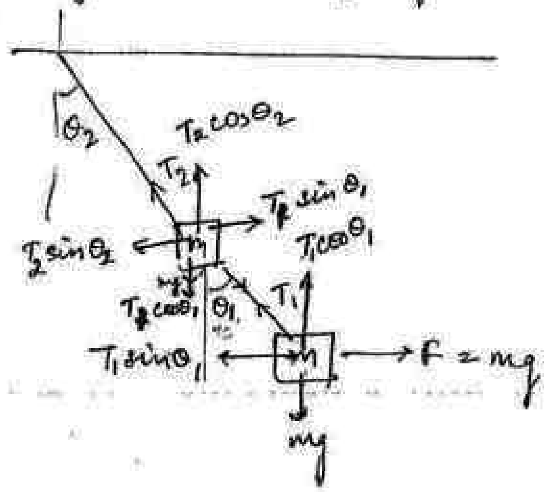


By Lami's $\dfrac{T_{BC}}{\sin 150^\circ} = \dfrac{51}{\sin 120} = \dfrac{W}{\sin 90}$

$$\frac{T_{BC}}{1/2} = \frac{51}{\sqrt{3}/2} = W \quad \therefore \; T_{BC} = \frac{51}{\sqrt{3}} \simeq 30N$$

$$W = \frac{51}{\sqrt{3}} \times 2 \simeq 60N$$

Q) System is in eqm, find T_1, T_2, θ_1 and θ_2



$\therefore \; T_1 \sin\theta_1 = mg$

$T_1 \cos\theta_1 = mg$

$\therefore \; \theta_1 = 45^\circ$

$T_1 = mg\sqrt{2}$

$\therefore$

$T_2 \cos\theta_2 = T_1 \cos\theta_1 + mg$ — (i)

$T_2 \sin\theta_2 = T_1 \sin\theta_1$ — (ii)

$= mg\cancel{\sqrt{2}} \times \dfrac{1}{\cancel{\sqrt{2}}}$

$\therefore \; T_2 \cos\theta_2 = 2mg$

$T_2 \sin\theta_2 = mg$

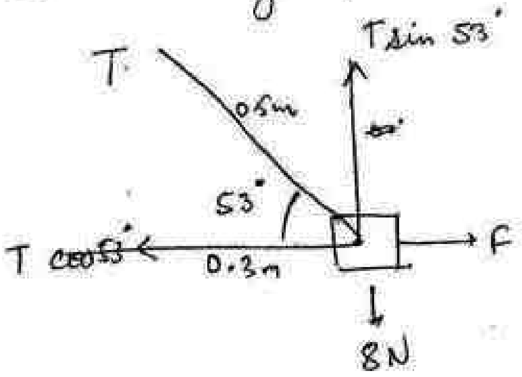
$$\therefore \; T_2^2 = (mg)^2 + (2mg)^2 = 5m^2g^2$$

$$\therefore \; T_2 = \sqrt{5m^2g^2} = mg\sqrt{5}$$

$\cos\theta_2 = \dfrac{2mg}{\cancel{mg}\sqrt{2}} \times \cancel{\cos\theta}$

of length 0.5 m

8) one end of ~~light~~ string is attached with the ceiling at point A. other end of this string is attached with an object of weight 8 N. This object is now pulled with Hz force F so that it moves away by 0.3 m from the vertical through point A. find tension T in string and F force

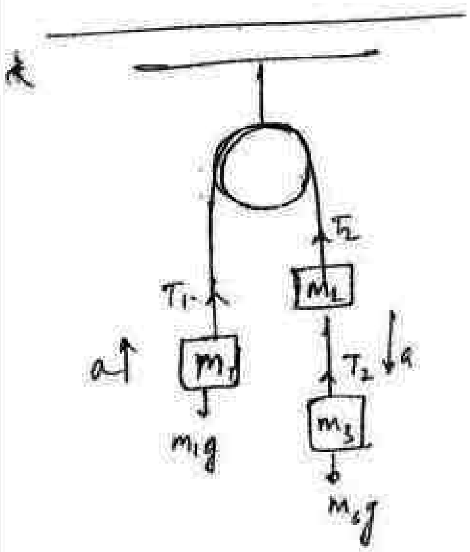


$\therefore T \sin 53 = 8$

$T \times \frac{4}{5} = 8^{2} \quad \therefore T = 10\,N$

$F = T \cos 53 = 10 \times \frac{3}{5} = 6\ N$

*



$T_1 - m_1 g = m_1 a$

$T_2 + m_2 g - T_1 = m_2 a$

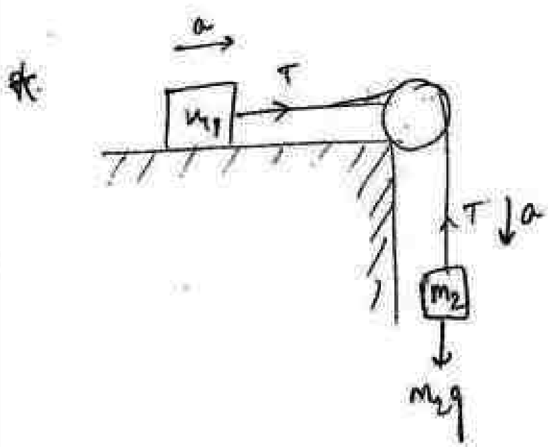
$m_3 g - T_2 = m_3 a$

$\therefore \boxed{a = \frac{(m_2 + m_3)g - m_1 g}{m_1 + m_2 + m_3}}$

$T_1 = m_1 (g + a)$

$T_2 = m_3 (g - a)$

*



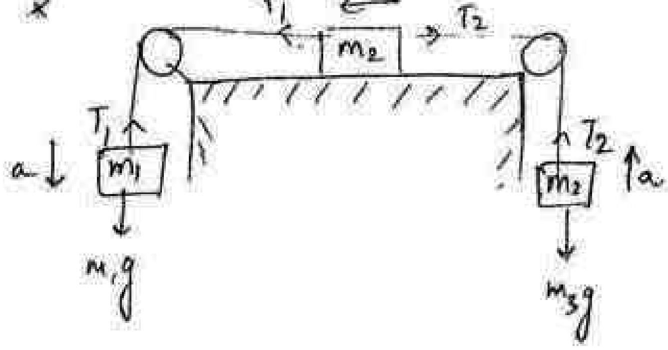
$T = m_1 a \quad$ — (I)

$m_2 g - T = m_2 a \quad$ — (II)

$\therefore m_2 g = (m_1 + m_2) a$

$\therefore \boxed{a = \frac{m_2 g}{m_1 + m_2} = \frac{F_{net}}{mass_{net}}}$

$\therefore T = m\, m_2 g$



$m_1 g - \cancel{T_1} = m_1 a$ —(1)

$\cancel{T_2} - m_3 g = m_3 a$ —(2)

$\cancel{T_1} - \cancel{T_2} = m_2 a$ —(3)

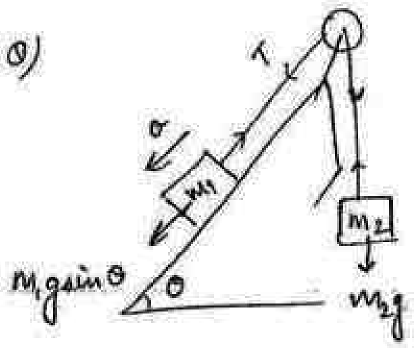
$$\boxed{\frac{g(m_1 - m_3)}{m_1 + m_2 + m_3} = \cancel{m_1+m}\ a}$$

$\therefore T_1 = m_1 (g - a)$

$T_2 = m_3 (g + a)$

*

Q)



$m_1 g \sin\theta - \cancel{T} = m_1 a$ —(1)

$\cancel{T} - m_2 g = m_2 a$ —(2)

$\therefore m_1 g \sin\theta - m_2 g = (m_1 + m_2)\, a$

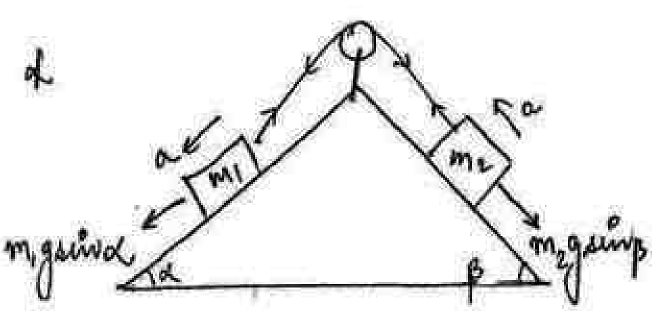
$$\therefore \boxed{a = \frac{m_1 g \sin\theta - m_2 g}{(m_1 + m_2)}} = \frac{F_{net}}{M_{net}}$$

by eqn (2) $\quad T = m_2 (g + a) = m_2 \left[g + \frac{m_1 g \sin\theta - m_2 g}{m_1 + m_2} \right]$

$$T = m_2 g \left[\frac{m_1 m_2 + m_1 \sin\theta - m_2}{m_1 + m_2} \right]$$

$$\therefore \boxed{T = \frac{m_1 m_2 g (1 + \sin\theta)}{m_1 + m_2}}$$

*

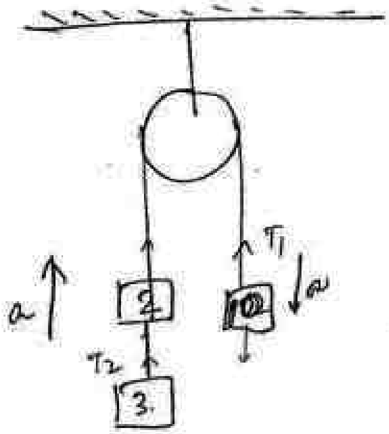


$m_1 g \sin\alpha - \cancel{T} = m_1 a$

$\cancel{T} - m_2 g \sin\beta = m_2 a$

$$\Rightarrow \frac{m_1 g \sin\alpha - m_2 g \sin\beta}{m_1 + m_2} = a$$

Q) In the given arrangement find T_1, T_2 and acc^n of block

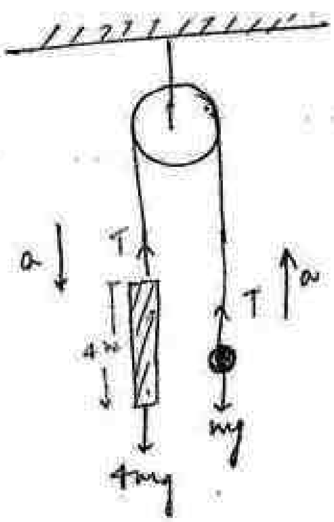


$$a = \frac{10g - (3+2)g}{15} = \frac{g}{3}$$

$$T_1 = 10(g-a) = 10\left(g - \frac{g}{3}\right) = \frac{20g}{3}$$

$$T_2 = 3(g+a) = 3\left(g + \frac{g}{3}\right) = 4g$$

Q) Find Time taken by the particle of mass 'm' to travel the distance AB of a rod of mass 4m and length 4m. If both starts their motion from rest.



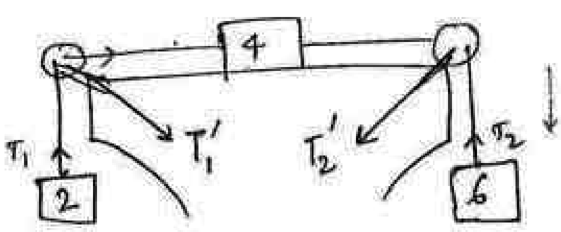
$$a = \frac{4mg - mg}{4m + m} = \frac{3g}{5}$$

$$a_x = a + a = 2a = \frac{6g}{5}$$

$$s_x = u_x t + \frac{1}{2} a_x t^2$$

$$4 = 0 + \frac{1}{2} \times \frac{6g}{5} \times t^2 \qquad \therefore t = \sqrt{\frac{2}{3}} \text{ sec}$$

Q) ~~For a~~ Find tension T_1, T_2 and thrust on pulley T_1' and T_2'



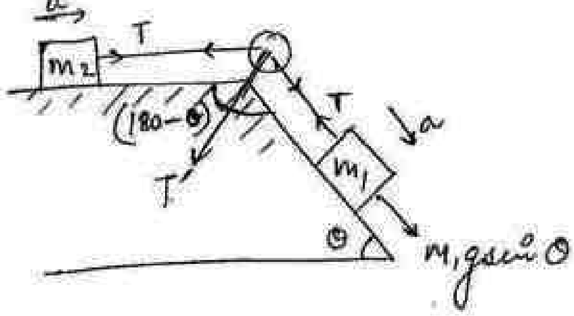
$$a = \frac{6g - 2g}{12} = \frac{g}{3}$$

$$T_1 = m_1(g+a) = 2\left(g + \frac{g}{3}\right) = \frac{8g}{3}$$

$$T_2 = m_2(g-a) = 6\left(g - \frac{g}{3}\right) \quad 4g$$

$$\therefore T_1' = \frac{8\sqrt{2}}{3} g$$

$$T_2' = 4\sqrt{2}\, g$$



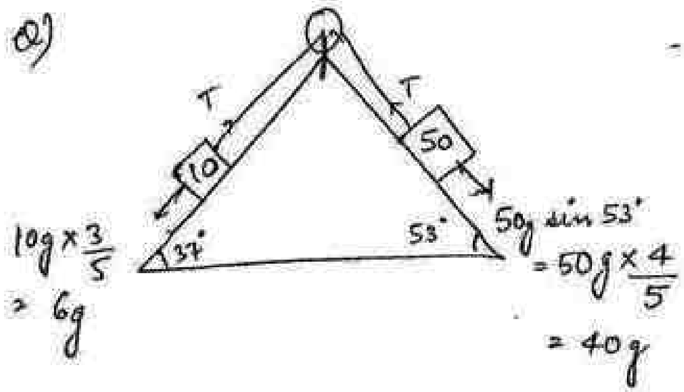
$$a = \frac{m_1 g \sin\theta}{m_1 + m_2}$$

$$\therefore\ T = m_2 a = \frac{m_1 m_2 g \sin\theta}{m_1 + m_2}$$

$$T' = 2T\cos\left(\frac{180-\theta}{2}\right) = 2T\cos\left(90-\right.$$

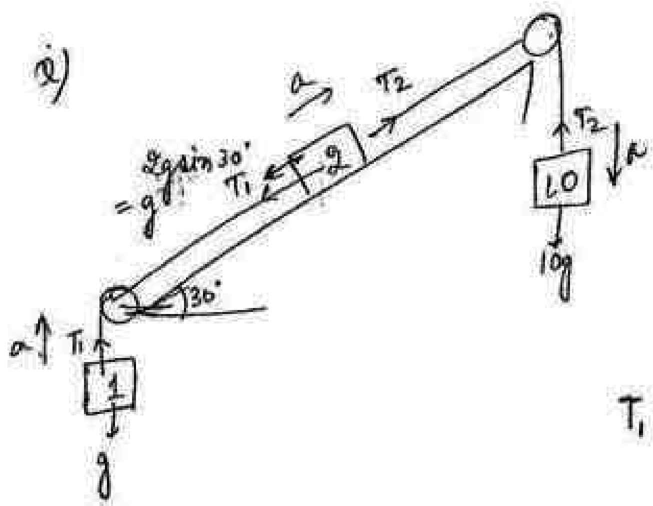
$$= 2T\sin\left(\frac{\theta}{2}\right)$$

Q)



$$T - 6g = 10a$$

i)



$$\cancel{T_1} - g = a$$

$$\cancel{T_2} - \cancel{T_1} - g = 2a$$

$$10g - \cancel{T_2} = 10a$$

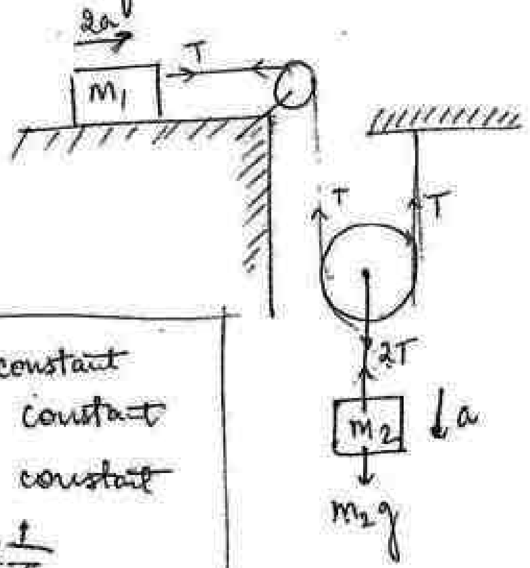
$$\therefore\ 8g = 13a \ \therefore\ \boxed{a = \frac{8}{13}g}$$

$$T_1 = 1\left(g + \frac{8}{13}g\right) = \frac{21g}{13}$$

$$T_2 = 10\left(g - \frac{8g}{13}\right) = \frac{50g}{13}$$

Movable pulley:

The axis of rotation displaces with time



xT = constant

xT = constant

xT = constant

$a \propto \frac{1}{T}$

$T = 2m_1 a$

$m_2 g - 2T = m_2 a$

~~$m_2 g - T = a(2m_1 + m_2)$~~

~~$\therefore a = \frac{m_2 g - T}{2m_1 + m_2}$~~

~~$= \frac{2m_2 g}{}$~~

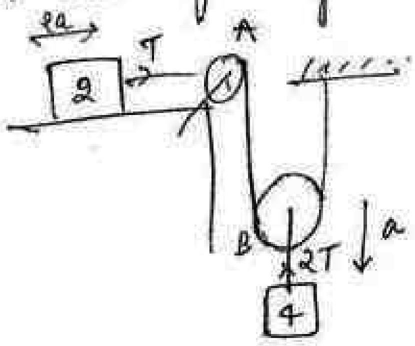
$m_2 g - 2(2m_1 a) = m_2 a$.

$m_2 g = (4m_1 + m_2)a$

$$\therefore \boxed{a = \frac{m_2 g}{4m_1 + m_2}}$$

$$\therefore T = \frac{2m_1 m_2 g}{4m_1 + m_2}$$

Q) Find acc^n of 4kg block and T in string b/w A and B



$T = 2(2a) = 4a$

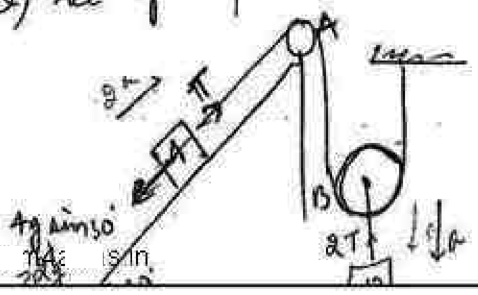
$4g - 2T = 4a$

$4g - 2(4a) = 4a$

$\boxed{a = \frac{g}{3}}$

$\boxed{T = \frac{4g}{3}}$

Q) Acc^n of 4kg block & T in string b/w AB



$T - 2g = 4(2a)$ $\therefore T = 2g + 8a$ — ①

$12g - 2T = 12a$ — ②

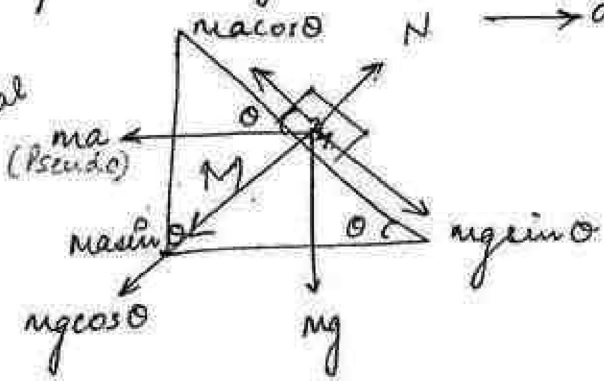
$\therefore 12g - 4g - 16a = 12a$

$\therefore 8g = 28a$ $\therefore \boxed{a = \frac{2g}{7}}$

$T = 2g + \frac{8 \times 2g}{}$ = $\boxed{30g}$

Q) Find accⁿ of wedge ~~...~~ so that a block of mass m placed on it remains at rest. Also find the net force exerted on the block by the wedge.

From non-inertial frame



$$\cancel{m}g\sin\theta = \cancel{m}a\cos\theta$$

$$\therefore g = \frac{a\cos\theta}{\sin\theta} = \frac{a}{\tan\theta}$$

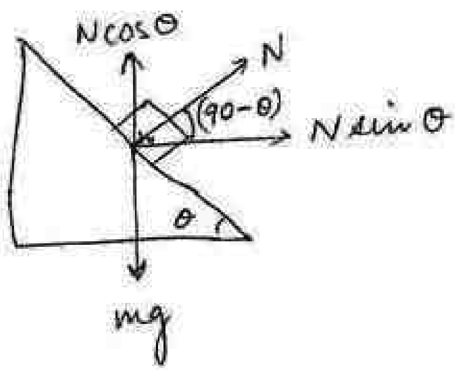
F_{BW} (block by wedge) $\Rightarrow mg\cos\theta + ma\sin\theta$

$$= mg\cos\theta + mg\tan\theta\sin\theta$$

$$= mg\left[\frac{\sin^2\theta}{\cos\theta} + \cos\theta\right]$$

$$= mg\left[\frac{\sin^2\theta + \cos^2\theta}{\cos\theta}\right] = \frac{mg}{\cos\theta} = mg\sec\theta$$

From Inertial frame



$\therefore$ ~~N sin θ = ma~~

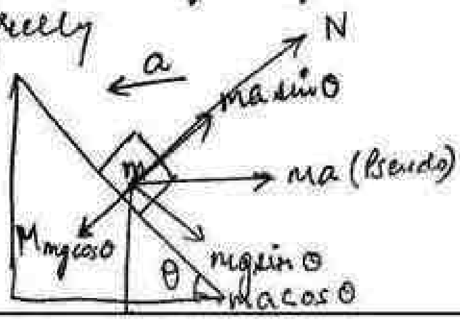
$\because N\cos\theta = mg$

$$\therefore N = \frac{mg}{\cos\theta} = mg\sec\theta$$

$$\therefore N\sin\theta = ma$$

$$\therefore a = g\tan\theta \Leftarrow \frac{mg}{\cos\theta}\sin\theta = ma$$

Q) Find accⁿ of wedge so that the block placed on it falls freely



<u>for eqᵐ ⊥ to incline</u>

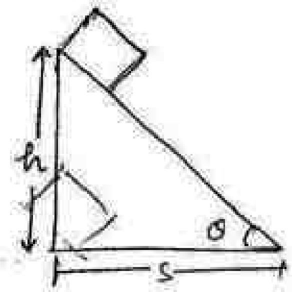
$N + ma\sin\theta = mg\cos\theta$

For block to fall freely, $N = 0$

$$\therefore \cancel{m}a\sin\theta = \cancel{m}g\cos\theta$$

$$\therefore a = g\cot\theta$$

From inertial frame :



$\tan\theta = \frac{h}{s} = \frac{\frac{1}{2}gt^2}{\frac{1}{2}at^2}$

$\therefore a = \frac{g}{\tan\theta}$

$\therefore \boxed{a = g\cot\theta}$

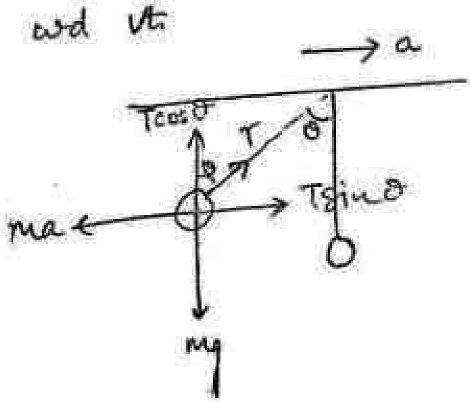
For block: by $S = ut + \frac{1}{2}at^2$

$h = 0 + \frac{1}{2}gt^2$ — (1)

For wedge by $S = ut + \frac{1}{2}at^2$

$s = 0 + \frac{1}{2}at^2$ — (2)

Q) A simple pendulum having a ball of mass m is suspended through a string from ceiling of car which is moving wd uniform accⁿ a on a straight h/z road. Find tension T in string and angle of inclination θ wd vt.



$T\sin\theta = ma$ — (I)

$T\cos\theta = mg$ — (II)

$\therefore \frac{(I)}{(II)} \cdot \tan\theta = \frac{a}{g}$

$\therefore \theta = \tan^{-1}\left(\frac{a}{g}\right)$

Non-inertial frame

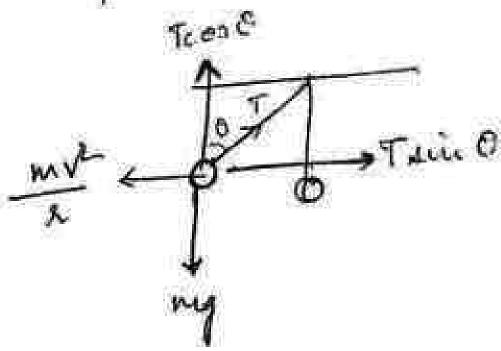
$(I)^2 + (II)^2$

$T^2\sin^2\theta + T^2\cos^2\theta = m^2(a^2+g^2)$

$\therefore T = m\sqrt{a^2+g^2}$

* for inertial frame: $T\cos\theta \uparrow$, $\rightarrow a$, $\rightarrow T\sin\theta$ $\therefore$ same as above

Q) A simple pendulum having a bob of mass m is suspended from ceiling of car taking a circular turn and uniform speed v having radius r. calculate the tension T in string of pendulum and angle of inclination of θ from its initial posn



$$T \sin\theta = \frac{mv^2}{r} \quad \text{— (I)}$$

$$\therefore \; T \cos\theta = mg \quad \text{— (II)}$$

(I) ÷ (II)

$$\tan\theta = \frac{v^2}{rg}$$

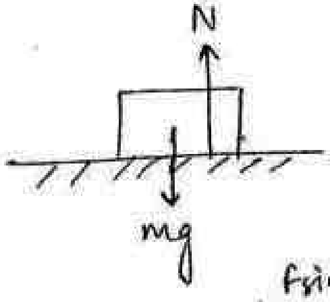
$(I)^2 + (II)^2$

$$T^2 \sin^2\theta + T^2\cos^2\theta = m^2\left[\left(\frac{v^2}{r}\right)^2 + g^2\right]$$

$$T = m\sqrt{\frac{v^4}{r^2} + g^2}$$

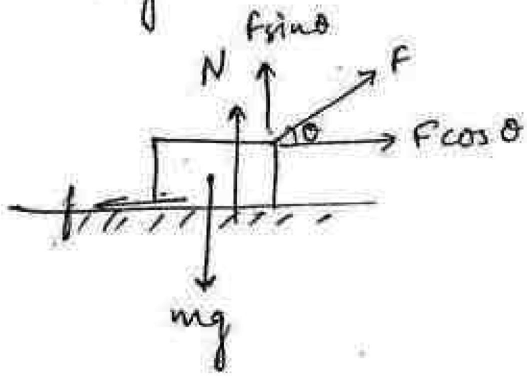
Friction

Case I



$\therefore N = mg$

Case II

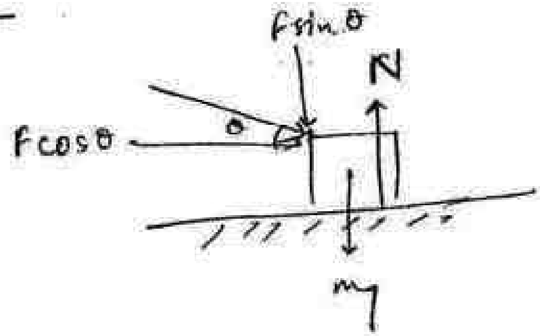


$\therefore N + F\sin\theta = mg$

$\therefore \boxed{N = mg - F\sin\theta}$

Pulling

Case III

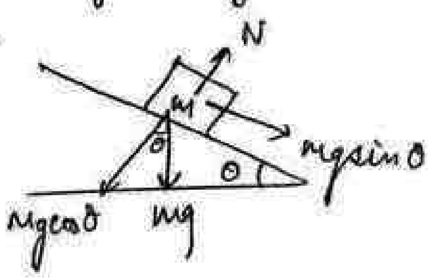


$\boxed{N = mg + F\sin\theta}$

Pushing.

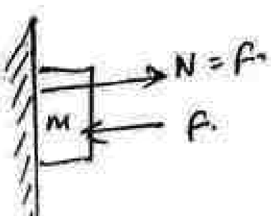
$\because$ N or W_{app} and hence friction are lesser ~~in~~ in case of pulling as compared to pushing $\therefore$ pulling an object is easier than pushing on Hz surface by an oblique force

Case IV



$\therefore \boxed{N = mg\cos\theta}$

Case V



$\therefore \boxed{N = F}$

surface having coefficient of friction = 0.4 calculate the time taken by it to stop and the distance travelled in this duration.

$u = 10\ m/s \qquad v = 0 \qquad \mu = 0.4$

$\therefore \quad F_{net} = m\,a_{net}$

$\mu N = ma$

$\mu \cancel{m} g = \cancel{m} a \qquad \therefore \boxed{a = \mu g}$ retardation

* when only friction exerts on lower surface of body during Hz motion

$v = u + at$

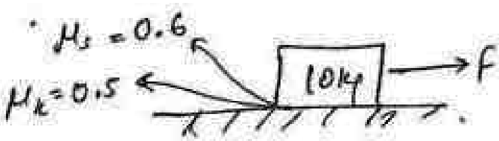
$= u - \mu g t \qquad \cancel{= 10 - 0.4 \times 10}$

$0 = 10 - 0.4 \times 10\,t \qquad \therefore \ t = 2.5\ sec.$

$s = ut - \frac{1}{2}(\mu g)t^2 \quad \Rightarrow \quad 10(2.5) - \frac{1}{2}(0.4 \times 10)(2.5)^2$

$25 - 12.5 = 12.5\ m.$

Q). For the given arrangement find force of friction (f) and acc^n of block.



$f_L = \mu_s N = \mu_s mg = 0.6 \times 10 \times 10$
$= 60\ N$

$f_k = \mu_k N = \mu_k mg = 0.5 \times 10 \times 10$
$= 50\ N$

i) $F = 50\ N$

$\because F_{ext} < f_L$
$\therefore$ Rest

$f = f_s = F_{ext}$
$= 50\ N$

$a = 0$

ii) $F = 60\ N$

$\because F_{ext} = f_L$
$\therefore F_{net} = 0$

$f = 60\ N$

$a = 0$

iii) $F = 70\ N$

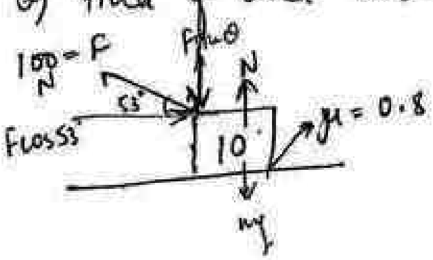
$\because F_{ext} > f_L$

Relative motion

$f_k = 50\ N$

$a = \frac{F_{ext} - f_k}{m} = \frac{70 - 50}{10}$
$= 2\ m/s^2$

Q) Find f and acc^n

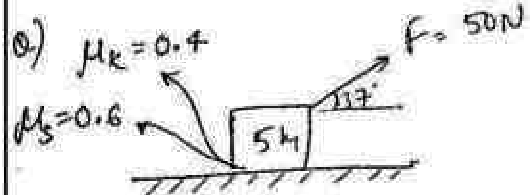


for vt eq^m: $\quad N = F\sin\theta + mg$
$= 100 \times \frac{4}{5} + 100 = 180$

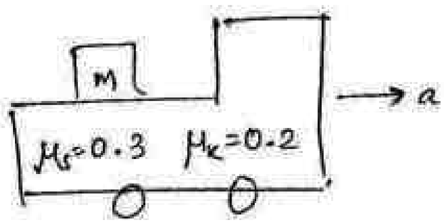
$f_L = \mu N = 0.8 \times 180 = 144\ N$

$F_{ext} = F\cos 53° = 100 \times \frac{3}{5} = 60\ N$

$F_{ext} < f_L \quad \therefore$ Rest

Q) 

Find velocity of block after 4 sec. if it starts from rest

Q) 

To find the max accⁿ of truck so that the block remains at rest w.r.t truck when its accⁿ just exceeds the max value. then find the displacement of block w.r.t truck during 1st sec of its motion