KINETIC THEORY OF GASES (KTG)

MODULE 2 (Pg-77)

Father of KTG : Bernouli

- Ideal gases ~~to~~ do not have intermol~~ecar~~ecular force (IMF), hence, their potential energy is also zero.

States of matter

solid — liquid — gases

$KE \lll |P.E|$ — $KE \approx |PE|$ — $KE \ggg |P.E|$

dense liquid has more P.E as compared to K.E

* We use modulus with the Potential energy because potential energy is always negative, with its negative value, it cannot be compared with the positive Kinetic energy.

Special Points :

i) Total Internal Energy, $U = KE + PE$

- Kinetic energy depends only on the temperature
  $\therefore KE \propto$ Temperature
- Potential energy depends on IMF (inter-molecular force) and IMF depends on PVT (Press, Vol^m, Temp
  $\therefore PE \propto -E$

  $PE \neq mgh$ because there ~~is~~ is no gravitational effect due to its low mass

ii) For real gas; $U = KE + PE$
Total internal energy of real gas depends upon Pressure, Volume and temperature.

iii) For ideal gas; $U = KE + 0$
Total internal energy of ideal gas depends upon temperature only

iv) Order of Inter molecular force (IMF):
solid > liquid > gas > Ideal gas (zero)

* v) Order of Potential energy (always -ve)
solid < liquid < gas < ideal gas (zero)

- when volume increases, the intermolecular force within the particles ~~increases~~ decreases, it means, the potential energy of the particle decreases in negative means it ultimately increases.

∴ V ↑, IMF ↓ and PE ↑

- when volume decreases, the intermolecular force within the particles increases, it means the potential energy of the particle increases in negative means it ultimately decreases.

∴ V ↓, IMF ↑ and PE ↓

Special Notation :

1. Number :

i) no. of gas molecules [N]
ii) Avogadro no. $[N_0]$ or $[N_A]$

$\because$ $N_0 = N_A = 6.023 \times 10^{23}$ molecules/mole

2. Mass :

i) mass of one gas molecule [m] (gm)
ii) mass of gas [M]

$$\boxed{M = mN \text{ gm}}$$

iii) molecular weight of the gas $[M_w]$

$$\boxed{M_w = mN_0 \text{ gram/mole}}$$

(iv) molar amount of gas $[\mu]$ (mu)

$$\boxed{\mu = \frac{M}{M_w} = \frac{mN}{mN_0} = \frac{N}{N_0}}$$

* If $\mu = 1$, then $M = M_w$ and $N = N_0$

3. Density :

i) molecular density (n)

$$\boxed{n = \frac{N}{V}}$$

ii) gas density $[\rho]$ (row)

$$\boxed{\rho = \frac{M}{V}}$$

Two containers can have same molecular density but not gas density and if the gas density is equal then molecular density will be equal.

* Relation between molecular density and gas density :

$$\rho = \frac{M}{V} = \frac{mN}{V} = m\frac{N}{V}$$

$$\therefore \boxed{\rho = mn}$$

$$\therefore \rho \propto n$$

i.e. density $\propto$ Pressure

4. P, V, T (Pressure, volume and temperature)
(Parameter of thermodynamics)

i) Pressure of gas [P]
SI unit : N/m^2 or Pa (Pascal)

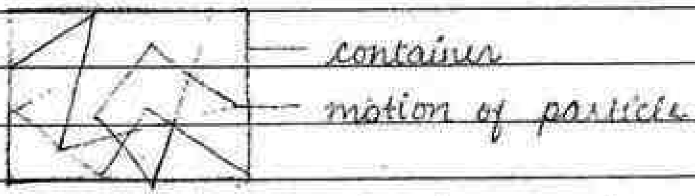
ii) Temperature of gas [T]
SI unit : K (Kelvin)

Absolute temp → K temp
Minimum temp → 0K

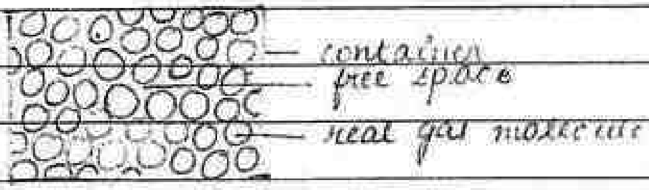
iii) volume of gas [V]
SI unit : m^3

* The volume of the gas will be the volume occupied by the gas molecule by the motion

- Volume of ideal gas = volume of container



- Volume of real gas = (volume of container) − (volume of free space)



Molecular volume : $\frac{4}{3}\pi r^3 \times N$

gas volume : volume of container

i) Pressure : It is exerted when molecules collides with the wall of the container not among themselves.

$1 \text{ atm} = 1.01325 \text{ bar} \simeq 1 \text{ bar}$

(atm = atmospheric pressure)

$\because [1 \text{ bar} = 10^5 \text{ N/m}^2]$

$\therefore$ $1 \text{ atm} = 1.01325 \times 10^5 \text{ N/m}^2$

$1 \text{ atm} = 1.01325 \times 10^5 \text{ Pa (Pascal)}$

$[\because 1 \text{ N/m}^2 = 1 \text{ Pascal}]$

$$\boxed{\begin{array}{l} 1 \text{ atm} = 760 \text{ mm of Hg} \\ 1 \text{ atm} = 760 \text{ torr} \end{array}}$$

Unit Conversion of pressure :

1st method :

$\because 760 \text{ mm of Hg} = 10^5 \text{ N/m}^2$

$\therefore 1 \text{ mm of Hg} = \frac{10^5}{760} \text{ N/m}^2$

$\therefore x \text{ mm of Hg} = \frac{10^5}{760} \times x \text{ N/m}^2$

2nd method :

$P = \rho_{Hg} hg.$

where ρ_{Hg} = density of mercury

h = height of mercury column

g = acceleration due to gravity

$\because \rho_{Hg} = 13600 \text{ kg/m}^3$ and $g = 10 \text{ m/s}^2$

$\therefore P = 13600 \frac{\text{kg}}{\text{m}^3} \times x \times 10^{-3} \text{ m} \times 10 \frac{\text{m}}{\text{s}^2}$

$= \boxed{136x \text{ N/m}^2}$

Unit conversion of volume:-

∵ 1 litre = 10^{-3} m^3 = 10^3 cm^3 = 10^3 cc = 1000 ml

$$\therefore \boxed{1\ m^3 = 1000\ l}$$

$$\boxed{\text{density of water, } \rho_{water} = 1000\ kg/m^3 = 1 g/cm^3}$$

density of ice, ρ_{ice} = 900 kg/m³

density of mercury, $\rho_{mercury}$ = 13600 kg/m³

For only water,

1 litre = 1 kg = 10^3 gm = 10^3 cc = 10^3 ml

∴ 1g = 1cc = 1ml

	N.T.P (Normal temperature and pressure)	S.T.P (Standard temperature and pressure)
i)	Temperature, $T = 0^\circ C = 273.15\ K$	Temperature, $T = 0.01^\circ C = 273.16\ K$
ii)	Pressure, P = 1 atm	Pressure, P = 1 atm
iii)	Volume of 1 mole of ideal gas = 22.4 l at N.T.P	Volume of 1 mole of ideal gas = 22.4 l at S.T.P.

[For numericals, N.T ≃ S.T ≃ 273 K]

5. Velocity:

i) Average velocity / Mean velocity, [$\vec{v}_{avg}$] or [$\langle\vec{v}\rangle$]

$$\boxed{\vec{v}_{avg} = \langle\vec{v}\rangle = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \ldots\ldots \vec{v}_N}{N} = 0}$$

average velocity / mean velocity is always equal to zero.

ii) Average speed/ Mean speed, $[V_m]$ or $[V_{avg}]$

$$\boxed{V_m = V_{avg} = \frac{v_1 + v_2 + v_3 + v_4 \ldots\ldots v_N}{N} \neq 0}$$

average speed or mean speed is always a positive value (+ve)

iii) Mean square velocity, $[\langle v^2 \rangle]$ or $[\overline{v^2}]$

$$\boxed{\langle v^2 \rangle = \overline{v^2} = \frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 \ldots\ldots v_N^2}{N} \neq 0}$$

mean square velocity is always a positive value

iv) root mean square velocity, $[V_{rms}]$ or $[\sqrt{\overline{v^2}}]$

$$\boxed{V_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 \ldots\ldots v_N^2}{N}}}$$

* To make the velocity of all the molecules uniform, we use V_{rms}

Proof:

KE (non-uniform) = KE (uniform)

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 \ldots\ldots \frac{1}{2}mv_N^2 = \frac{1}{2}mv^2 \times N$$

$$\frac{1}{2}m\left(v_1^2 + v_2^2 + v_3^2 \ldots\ldots v_N^2\right) = \frac{1}{2}m\left(v^2 \times N\right)$$

$$\therefore v_1^2 + v_2^2 + v_3^2 \ldots\ldots v_N^2 = v^2 \times N$$

$$\therefore V^2 = \frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 \ldots\ldots v_N^2}{N}$$

$$\therefore V = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 \ldots v_N^2}{N}} = V_{rms}$$

Hence proved

v) most probable velocity, [V_{mp}]
at a given temperature, velocity for which no. of molecules are maximum (Maximum Probability)

vi) Speed of sound in gases medium, [V_s]

Q.1 There are six molecules with 1, 2, 2, 2, −3 and −4 m/s as their velocities respectively, then find :

i)
$$\overrightarrow{V_{avg}} \text{ or } \langle \vec{V} \rangle = \frac{\vec{V_1} + \vec{V_2} + \vec{V_3} \ldots + \vec{V_N}}{N}$$

$$= \frac{1 + 2 + 2 + 2 + (-3) + (-4)}{6}$$

$$= \frac{7 - 3 - 4}{6} = \frac{7 - 7}{6} = \frac{0}{6} = 0$$

ii)
$$V_{avg} \text{ or } V_m = \frac{V_1 + V_2 + V_3 + V_4 \ldots V_N}{N}$$

$$= \frac{1 + 2 + 2 + 2 + (3) + (4)}{6}$$

$$= \frac{7 + 3 + 4}{6} = \frac{14}{6} = 2.3 \text{ m/s}$$

iii)
$$\langle V^2 \rangle \text{ or } \overline{V^2} = \frac{V_1^2 + V_2^2 + V_3^2 \ldots V_N^2}{N}$$

$$= \frac{1^2 + 2^2 + 2^2 + 2^2 + 3^2 + 4^2}{6}$$

$$= \frac{1 + 4 + 4 + 4 + 9 + 16}{6} = \frac{38}{6} = \frac{19}{3} \frac{m^2}{s^2}$$

iv) V_{rms} or $\sqrt{\overline{v^2}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 \dots\dots v_N^2}{N}}$

$$= \sqrt{\frac{1^2 + 2^2 + 2^2 + 2^2 + 3^2 + 4^2}{6}}$$

$$= \sqrt{\frac{1+4+4+4+9+16}{6}} = \sqrt{\frac{19}{3}}\ m/s$$

v) V_{mp} = most probable = 2 m/s

~~6. Some constant :~~

* $\langle \vec{v} \rangle = 0$ (always)
$\langle v^2 \rangle$ or $\langle v^4 \rangle$ or $\langle v^6 \rangle \neq 0$ (i.e. +ve)
$\langle v^3 \rangle$ or $\langle v^5 \rangle$ or $\langle v^7 \rangle$ = may or may not be 0

6. Some constant :

i) Universal gas constant [R]

$$\boxed{R = 8.31 \frac{\text{Joule}}{\text{Mole Kelvin}} = 8.31 \frac{J}{\text{mole K}} \simeq 2 \frac{cal}{\text{mol K}}}$$

ii) Boltzmann constant [K]

$$K = \frac{R}{N_0} = \frac{8.31\ J/\text{mole K}}{6.023 \times 10^{23}\ /\text{mole}}$$

$$\boxed{K = \frac{8.31\ J/K}{6.023 \times 10^{23}} = 1.38 \times 10^{-23}\ J/K}$$

iii) Specific gas constant [r] ~~or [R]~~

$$\boxed{r = \frac{R}{M_w} = \frac{cal/\text{mole K}}{g/\text{mole}} = \frac{cal}{g\,K}}$$

Absolute zero temperature

0 K (Kelvin) or −273 °C [not possible]

- For ideal gas at 0 Kelvin temperature. Velocity, Kinetic energy, V_{rms}, momentum, Force, Pressure, volume of gas and total internal energy ($U = KE$ for ideal gas) are zero.

- For real gas at 0 Kelvin temperature Potential energy and total internal energy ($U = KE + PE$ for real gas) are non-zero.

$$\because U = KE + PE$$

$$\because (KE = 0)$$

$$\therefore U = 0 + PE = PE$$

$$\text{Where } PE \neq 0$$

* At zero Kelvin temperature, density of the gas is non-zero for both ideal and real gas.

$$\because \text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{i.e. } \rho = \frac{(M \neq 0)}{(V_{container} \neq 0)} \neq 0$$

Ideal gas concept:

- There is no intermolecular attraction between ideal gas molecules ($\because IMF = 0$)

- Volume of ideal gas is negligible as compared to volume of container because ideal gas molecule have point mass negligible volume.

Volume of ideal gas = volume of container
It is ~~available~~ applicable for all the temperature except 0 kelvin.

Equation of state of ideal gas OR
Ideal gas equation :
for μ mole of ideal gas,

$$\begin{tikzcd}
 & \star\ \boxed{PV = \mu RT} \arrow[dl] \arrow[dr] & \\
\star\ \boxed{PV = \frac{M}{M_w} RT} \arrow[d] & & PV = \frac{N}{N_0} RT \arrow[d] \\
P = \frac{M}{V} \frac{RT}{M_w} \arrow[d] & & \star\ \boxed{PV = NKT} \quad \left[\because K = \frac{R}{N_0} \right. \arrow[d] \\
\star\ \boxed{P = \frac{\rho RT}{M_w}} & & P = \frac{N}{V} KT \quad \left[n = \frac{N}{V} = \frac{\rho}{m} \right. \arrow[d] \\
 & & P = nKT \arrow[d] \\
 & & P = \frac{\rho}{m} KT \quad [\because \rho = mn] \arrow[d] \\
 & & \star\ \boxed{\frac{P}{\rho} = \frac{KT}{m}}
\end{tikzcd}$$

Q.2 A vessel contains one mole of O_2 gas at a temperature of 27°C, the pressure of the gas is 2atm, an identical vessel containing two mole of He gas at a temperature of 327°c has a pressure of.

$$\because PV = \mu RT$$

$$\therefore \quad V = \frac{\mu_1 R T_1}{P_1} = \frac{1 \times R \times 300}{2} \quad \left[\because 27°C = 27+273 = 300K\right]$$

$$V = 150R \quad — ①$$

$$\therefore \quad V = \frac{\mu_2 R T_2}{P_2}$$

$$V = \frac{2 \times R \times 600}{P_2} \quad — (II) \quad \left[\because 327°C = 327+273\ K = 600\ K\right]$$

Putting ① in (II), we get

$$150R = \frac{1200\ R}{P_2}$$

$$\therefore P_2 = \frac{1200\ R}{150\ R} = 8\ atm$$

OR

$$\because PV = \mu RT$$

P_1 = 2 atm, $V_1 = V$, μ_1 = 1 mole

T_1 = 27°C = 27 + 273 K = 300K

and

$P_2 = P_2$, $V_2 = V$, μ_2 = 2 mole

T_2 = 327°C = 327 + 273K = 600 K

$$\therefore \quad P_1 V_1 = \mu_1 R_1 T_1$$

$$P_2 V_2 = \mu_2 R_2 T_2$$

$\because$ Its an identical vessel

$\therefore$ Its volume is same = V

$$\therefore \frac{2 \times V}{P_2 \times V} = \frac{1 \times R \times 300K}{2 \times R \times 600\ K}$$

$$= \frac{2}{P_2} = \frac{1}{4}$$

$$\therefore P_2 = 4 \times 2 = 8 \text{ atm}$$

Q.3. If the volume of air at $-182°c$ and 2 atm is 8 litres, its volume in litre at N.T.P.

$$\because PV = \mu RT$$

here μ and R are constant

$$\therefore PV \propto T$$

$$\frac{P_1V_1}{P_2V_2} = \frac{T_1}{T_2}$$

$P_1 = 2atm$, $V_1 = 8$ litres

$T_1 = -182°c = -182 + 273 = 91\ K$

and

$P_2 = 1$ atm, $V_2 = V_2$,

$T_2 = 273\ K$ at N.T.P

$$\therefore \frac{2 \times 8}{1 \times V_2} = \frac{91\ K}{273 K}$$

$$\therefore V_2 = 2 \times 8 \times 3 \text{ litres}$$

$$= 48 \text{ litres}$$

Q.4 The approximate no. of molecule in a 250 cc of an ideal gas at a temperature of $27°c$ and pressure at 10^{-3} mm of Hg

$$\because PV = NKT$$

$$\therefore N = \frac{PV}{KT}$$

$V = 250$ cc

$$\because (1m^3 = 10^6\ cm^3 = 10^6\ cc)$$

$$\therefore (1\ cc = 10^{-6}\ m^3)$$

$\therefore (250 \text{ cc} = 250 \times 10^{-6} \text{ m}^3)$

~~V~~ $P = 10^{-3}$ mm of Hg

$\because (1 \text{ mm of Hg} = 136 \text{ N/m}^2)$

$\therefore (10^{-3} \text{ mm of Hg} = 136 \times 10^{-3} \text{ N/m}^2)$

$K = 1.38 \times 10^{-23}$ J/K

$T = 27^\circ C = 27 + 273 = 300 K$

$$\because N = \frac{PV}{KT}$$

$$= \frac{136 \times 10^{-3} \times 250 \times 10^{-6}}{1.38 \times 10^{-23} \times 300}$$

$$= \frac{13600 \times 10^{23-6-3} \times 250}{138 \times 300}$$

$$= \frac{250}{3} \times 10^{14}$$

$$= 83.3 \times 10^{14} = 8.33 \times 10^{15}$$

Q.5 A gas is filled in a cylinder, its temperature increases by 20% on kelvin scale and pressure is reduced by 10%. How much percentage of gas has to leak?

Let $P_1 = P$

$$\therefore P_2 = P \times \frac{10}{100} = \frac{P}{10}$$

$$P - \frac{P}{10} = \frac{10P - P}{10} = \frac{9P}{10} \quad \text{and}$$

Let $T_1 = T$

$$\therefore T_2 = T \times \frac{20}{100} = \frac{2T}{10}$$

$$T + \frac{2T}{10} = \frac{10T + 2T}{10} = \frac{12T}{10}$$

$\because PV = \mu RT$

here V and R are constant

$\therefore P \propto \mu T$

$$\frac{P_1}{P_2} = \frac{\mu_1 T_1}{\mu_2 T_2}$$

$$\frac{10P}{9P} = \frac{\mu_1}{\mu_2} \times \frac{10T}{12T}$$

$$\therefore \frac{\mu_1}{\mu_2} = \frac{10}{9} \times \frac{12}{10} = \frac{4}{3}$$

Hence, percentage of gas to be leaked =

$$\left(\frac{\mu_1 - \mu_2}{\mu_1} \times 100\right)\%$$

$$= \left(\frac{4-3}{4} \times 100\right)\%$$

$$= \left(\frac{1}{4} \times 100\right)\% = 25\%$$

OR

Let $P_1 = 100P \quad \therefore P_2 = 100P - 10P = 90P$ and

$T_1 = 100T \quad \therefore T_2 = 100T + 20T = 120T$

$\mu_1 = 100\mu$

$\because PV = \mu RT$

here V and R are constant

$\therefore P \propto \mu T$

$$\frac{P_1}{P_2} = \frac{\mu_1 T_1}{\mu_2 T_2}$$

$$\frac{100P}{90P} = \frac{100\mu}{\mu_2} \times \frac{100T}{120T}$$

$$\therefore \mu_2 = 100\mu \times \frac{10}{12} \times \frac{9}{10}$$

$$\mu_2 = 75\mu$$

leak mole $= \mu_1 - \mu_2$

$= 100\mu - 75\mu = 25\mu$

∴ Leak percentage $= \frac{25\mu}{100} \times 100$

$= 25\ \%$

Properties of ideal gas:

1. Gas which obeys all the gas laws (charles's law, Gay lussac's law and Boyle's law) and equation of state (ideal gas equation) is known as ideal gas.

2. Ideal gas molecules can do only translation motion so, their kinetic energy is only translation kinetic energy and is directionally proportional to absolute temperature only.

3. Inter molecular attraction force (IMF) between ideal gas molecule is 0, it means total internal energy is ~~also~~ only KE and is also directly proportional to absolute temperature.

$$\underline{U} = KE$$

4. The liquification and solidification of ideal gas is not possible

* Real gas can be liquified but not ideal gas because for liquification, ~~th~~ ~~o~~ the (IMF) has to be increased but IMF of ideal gas is always 0. Hence, the liquification

and solidification of ~~real~~ ideal gas is not possible.

* Real gas converts into ideal gas when Intermolecular spaces between them is brought down to 0.

⑤ Specific heat of ideal gas doesn't depends on temperature

6. All real gas behaves as an ideal gas at high temperature and low pressure and low density (IMF =0) but ideal gas can never behave as a real gas because ideal gas actually don't exist.

7. Molecules of ideal gas is very small and speed is very high so effect of gravity on the motion of the molecule is negligiable

\+ Ideal gas:

- Speed of ideal gas = $x \times 10^5$ m/s
- Molecular density of the ideal gas is considered only if its density is $10^{23}/cm^3$ i.e. $10^{23}/cc$

Types of Motion:

i Translational motion
ii Rotational motion
iii Vibrational motion

Gas law:

All the gas law and equation of state is valied only when molar amount is constant

1. Charles law:

μ and $\textcircled{P}$ are constant

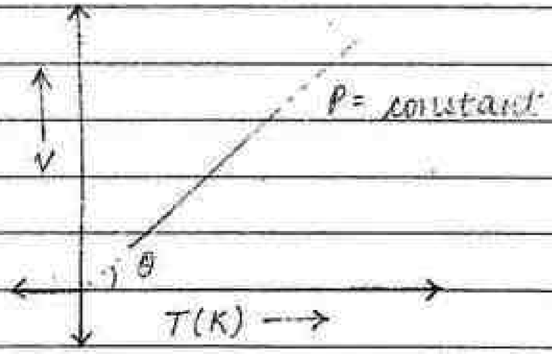
$$\because PV = \mu RT$$
$$\therefore \boxed{V \propto T}$$

hence, comparison eq = $\frac{V_1}{V_2} = \frac{T_1}{T_2}$ and.

$\frac{V}{T}$ = constant

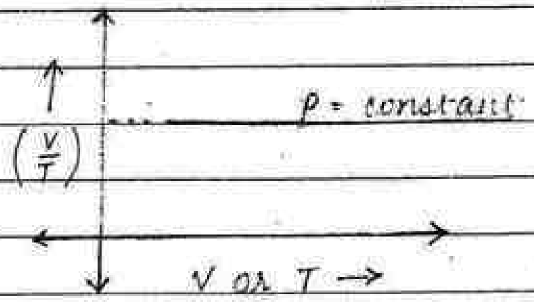
* Terms : In open container and freely moving piston

i)



$$\because PV = \mu RT$$
$$\therefore V = \frac{\mu R}{P} \times T \qquad \left[\because \frac{\mu R}{P} = \text{constant}\right]$$
$$y = mx$$
$$m = \tan\theta = \frac{\mu R}{P}$$
$$\therefore \text{Slope} \propto \frac{1}{P}$$

ii)



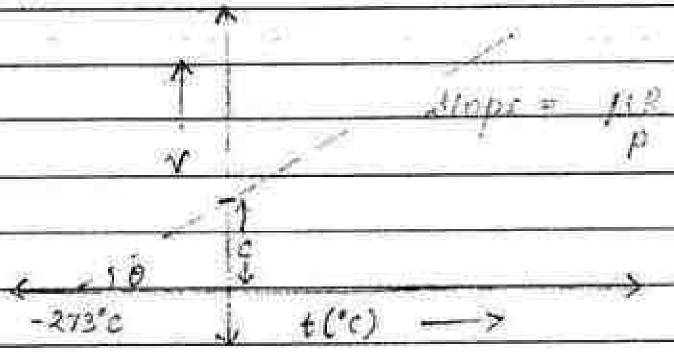
$$\because PV = \mu RT$$

$$\therefore V = \frac{\mu R}{P} \times T$$

$$\frac{V}{T} = \text{constant} \qquad \left[\because \frac{\mu R}{P} = \text{constant}\right]$$

$$\therefore y = \text{constant}$$

iii)



$$\because PV = \mu RT$$

$$\therefore V = \frac{\mu R}{P} \times T = \frac{\mu R}{P}(t + 273°)$$

$$\therefore \underset{\downarrow}{V} = \underset{\downarrow}{\boxed{\frac{\mu R}{P}}} \times \underset{\downarrow}{t} + \underset{\downarrow}{\boxed{\frac{\mu R}{P} \times 273}}$$

$$y = m \quad x \quad + \quad c$$

$$\text{i.e } y = mx + c$$

2. <u>Gay Lussac Law :</u>

μ and ⓥ are <u>constant</u>

$$\because PV = \mu RT$$

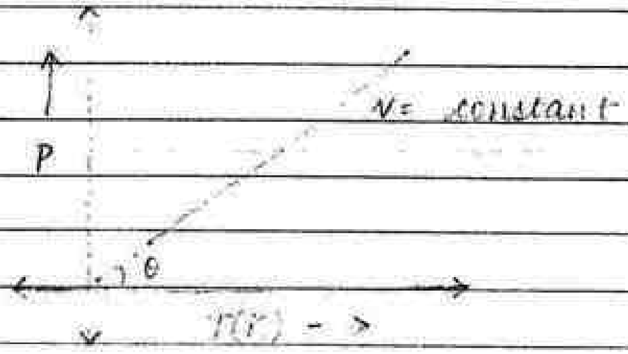
$$\therefore \boxed{P \propto T}$$

hence, comparison eq = $\frac{P_1}{P_2} = \frac{T_1}{T_2}$ and

$\frac{P}{T}$ = constant

* <u>Terms :</u> <u>In closed container</u> and <u>fixed piston.</u>

i)



$$\because PV = \mu RT$$

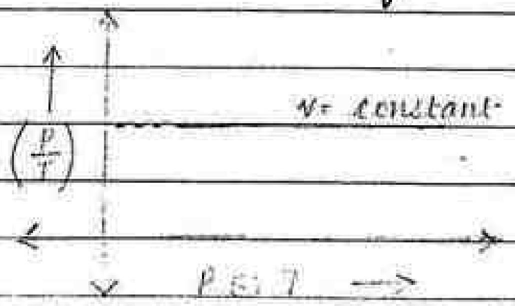
$$\therefore \underset{\downarrow}{P} = \underset{\downarrow}{\left(\frac{\mu R}{V}\right)} \times \underset{\downarrow}{T} \qquad \left[\frac{\mu R}{V} = \text{constant}\right]$$

$$y = mx$$

$$m = \tan\theta = \frac{\mu R}{V}$$

$$\therefore \text{slope} \propto \frac{1}{V}$$

ii)



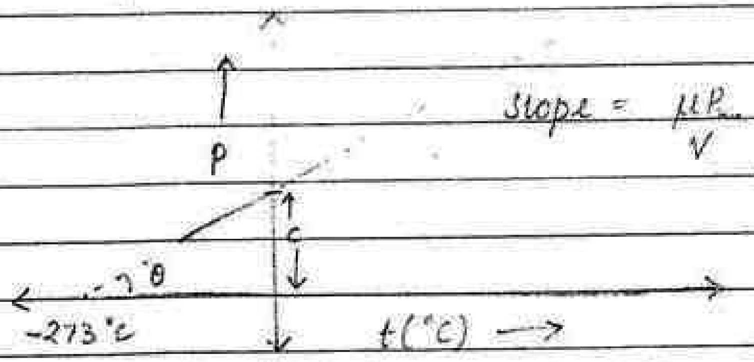
$$\because PV = \mu RT$$

$$\therefore P = \frac{\mu R}{V} \times T$$

$$\frac{P}{T} = \text{constant} \quad \left[\because \frac{\mu R}{V} = \text{constant}\right]$$

$$\therefore y = \text{constant}.$$

iii)



$$\because PV = \mu RT$$

$$\therefore P = \frac{\mu RT}{V} = \frac{\mu R}{T}\left(t + 273^\circ\right)$$

$$\therefore \underset{\downarrow}{P} = \underset{\downarrow}{\left(\frac{\mu R}{T}\right)} \times \underset{\downarrow}{t} + \underset{\downarrow}{\left(\frac{\mu R}{T} \times 273\right)}$$

$$y = m \quad x \quad + \quad c$$

$$\therefore y = mx + c$$

3. __Boyle's law:__

μ and Ⓣ are constant

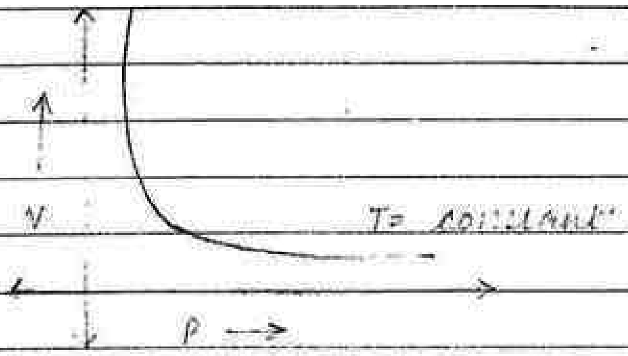
$$\because PV = \mu RT$$

$$\therefore PV = \text{constant}$$

$$P \propto \frac{1}{V}$$

hence, comparing eq $= \frac{P_1}{P_2} = \frac{V_2}{V_1}$

i)

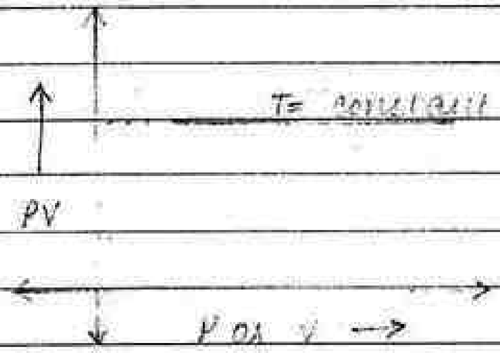


$\because PV = \mu RT$

$\therefore xy = \text{constant}$ [$\because \mu RT = \text{constant}$]

$\therefore \boxed{x \propto \frac{1}{y}}$

ii)



$\because PV = \mu RT$

$\therefore PV = \text{constant}$

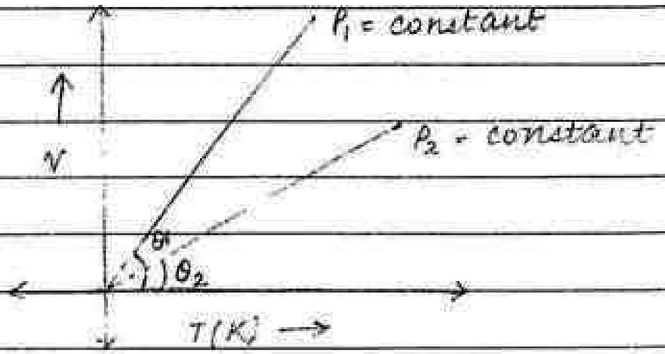
$\therefore y = \text{constant}.$

iii) There will be no graph for Boyle's temperature and boyle's relation because temperature is constant in Boyle's law

Q- 6. Identify the relation between P_1 and P_2

$\because PV = \mu RT$

$\therefore V = \frac{\mu RT}{P}$



$$\because \underset{\downarrow}{V} = \underset{\downarrow}{\boxed{\frac{\mu R}{P}}} \times \underset{\downarrow}{T} \qquad \left[\because \frac{\mu R}{P} = \text{constant}\right]$$

$$y = m\,x$$

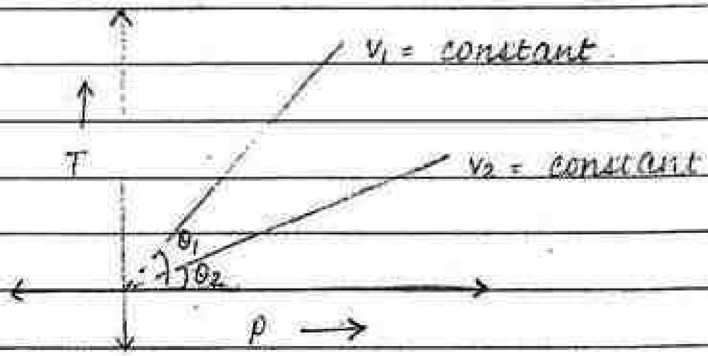
$$\therefore \text{slope} = m = \tan\theta = \frac{\mu R}{P}$$

$$\therefore \text{slope} \propto \frac{1}{P}$$

$$\because \theta_1 > \theta_2 \qquad \therefore \tan\theta_1 > \tan\theta_2$$

$$\therefore P_1 < P_2$$

Q. 7 Identify the relation between V_1 and V_2



$$\because PV = \mu RT$$

$$\therefore \cancel{P = \frac{\mu R}{V} \times T}$$

$$T = \frac{V}{\mu R} \times P$$

$$\underset{\downarrow}{T} = \underset{\downarrow}{\boxed{\frac{V}{\mu R}}} \times \underset{\downarrow}{P}$$

$$y = mx$$

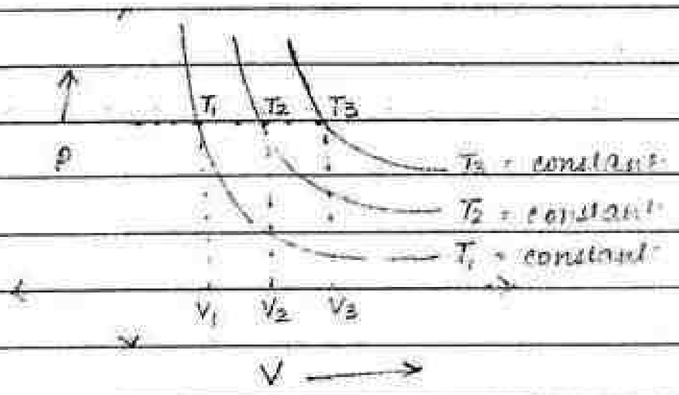
$$\text{Slope} = m = \tan\theta = \frac{V}{\mu R}$$

$$\therefore \text{slope} \propto V$$

$$\because \theta_1 > \theta_2 \qquad \therefore \tan\theta_1 > \tan\theta_2$$

$$\therefore V_1 > V_2$$

Q.8 Identify the relation between T_1, T_2 and T_3



At a constant pressure,

$$\because PV = \mu RT$$

$$\therefore V = \underset{\underset{\text{constant}}{\downarrow}}{\boxed{\frac{\mu R}{P}}} \times T$$

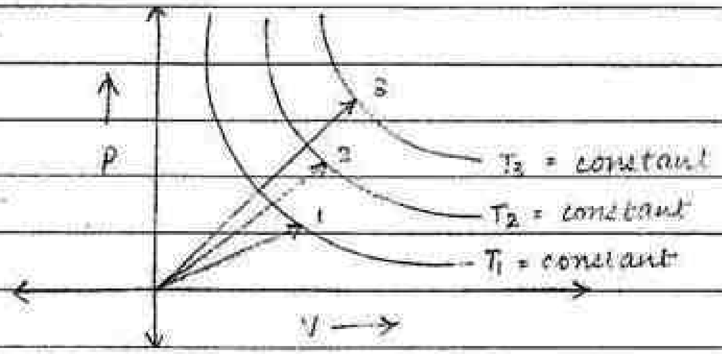
$$\therefore V \propto T$$

$$\because V_1 < V_2 < V_3$$

$$\therefore T_1 < T_2 < T_3$$

OR

Short trick :



Draw a line passing from the origin to the constant temperature.
Hence, in the above construction, the longer line will indicate higher temperature

$\because$ $1 < 2 < 3$ (length)
$\therefore$ $T_1 < T_2 < T_3$ (temperature)

Q-9 A bulb containing air at normal atm pressure and 41°c is sealed, the bulb is heated to 198°c and then the bulb burst, the bursting pressure is

P_1 = Pressure at NTP = 1 atm
T_1 = 41°C = (41 + 273) K = 314 K
T_2 = 198°C = (198 + 273) K = 471 K
V = constant

$\because$ $PV = \mu RT$
where V, μ and R are constant
$\therefore$ $P \propto T$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\frac{1}{P_2} = \frac{314\ K}{471\ K}$$

$$\therefore P_2 = \frac{471}{314} = 1.5 \text{ atm}$$

Q.10 An ideal gas at 17°C and 760 mm of Hg of pressure, the gas is compressed at constant temperature untill its volume is halved, the final pressure of the gas will be.

$P_1 = 760$ mm of Hg

$T_1 = 17^\circ C = (17 + 273) K = 290 K$

$T =$ constant (i.e. $T_1 = T_2 = 290 K$)

$V_1 = V$ and $V_2 = \frac{V}{2}$

$\because PV = \mu RT$

where μ, R and T are constant

$\therefore P \propto \frac{1}{\cancel{1}V}$

$$\frac{P_1}{P_2} = \frac{\cancel{T_2}\ V_2}{\cancel{T_1}\ V_1}$$

$$\frac{760 \text{ mm of Hg}}{P_2} = \frac{V}{2 \times V}$$

$\therefore P_2 = (760 \times 2)$ mm of Hg

$= 1520$ mm of Hg

Q.11 A cylinder of 10 l capacity filled with air at 1 atm pressure is connected at constant temperature with an cylinder of 30 l capacity. Find the resulting pressure in cm of Hg.

$V_1 = 10 l.$

$V_2 = 10 l + 30 l = 40 l$

$P_1 = 1$ atm $= 760$ mm of Hg

$= 76$ cm of Hg

$\because PV = \mu RT$

where μ, R, T are constant

$$\therefore \frac{P_1}{P_2} = \frac{V_2}{V_1}$$

$$\frac{76}{P_2} = \frac{40}{10}$$

$$\therefore P_2 = \frac{76 \times 10}{40} = 19 \text{ cm of Hg.}$$

Q.12 By increasing the temperature of an ideal gas in a closed container by 5°c, the pressure increases by 0.8 % from its initial value. calculate initial temperature of the gas.

$$PV = \mu RT$$

here V, μ and R are constant

$$\therefore P \propto T$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\because \Delta T = T_2 - T_1 = 5^\circ C$$

$$= (t_2 + 273) - (t_1 + 273) = 5K$$

$$= t_2 - t_1 = 5K$$

* $[\therefore \Delta T = 5^\circ C = 5K] \therefore \Delta T = \Delta^\circ C = \Delta K$

$$P_1 = 100P \quad \therefore P_2 = 100.8\,P$$

$$\frac{100\,P}{100.8\,P} = \frac{T_i}{T_i + 5}$$

$$100T_i + 500 = 100.8\,T_i$$

$$100.8\,T_i - 100T_i = 500$$

$$0.8\,T_i = 500$$

$$\therefore T_i = \frac{500}{0.8} = \frac{5000}{8}$$

$$= 625 \text{ K}$$

$$625K = (625 - 273)^\circ c = 352^\circ c$$

OR

$$P_i \propto T_i \quad —①$$

$$\therefore \Delta P = \Delta T \quad —⑪$$

dividing eq ⑪ from ①

$$\frac{\Delta P}{P_i} \times 100 = \frac{\Delta T}{T_i} \times 100$$

$$0.8 = \frac{5}{T_i} \times 100 \left[\because \frac{\Delta P}{P_i} \times 100 = 0.8 \right]$$

$$\therefore T_i = \frac{500}{0.8}$$

$$= 625\ K = 352^\circ C$$

* Concept of Mixture :

$$\because PV = \mu RT$$

$$\therefore \boxed{P_{mix} V_{mix} = \mu_{mix} R T_{mix}}$$

- $N_{mix} = N_1 + N_2 + N_3 \dots$ (number)
- $M_{mix} = M_1 + M_2 + M_3 \dots$ (mass)
- $\mu_{mix} = \mu_1 + \mu_2 + \mu_3 \dots$ (mole)
- $\mu_{mix} = \frac{M_1}{M_{w_1}} + \frac{M_2}{M_{w_2}} + \frac{M_3}{M_{w_3}} \dots$
- $M_{w(mix)} = \frac{M_{mix}}{\mu_{mix}} = \frac{M_1 + M_2 + M_3 \dots}{\frac{M_1}{M_{w_1}} + \frac{M_2}{M_{w_2}} + \frac{M_3}{M_{w_3}} \dots}$
- $V_{mix} = \frac{\mu_{mix} R T_{mix}}{P_{mix}}$

i) when <u>both</u> the container are joined together :

$$V_{mix} = \underline{V_1 + V_2} + V_3 \dots$$

ii) when both the gases are put in one container :

$$V_{mix} = V_{putting}$$

$$\rho_{mix} = \frac{M_{mix}}{V_{mix}}$$

Q.13. 16 g of oxygen, 14g of Nitrogen are mixed in a container of volume 5000 cc and 27°c the resulting pressure of mixture is :

$$V_{mix} = 5000\ cc = 5000\ cm^3$$

$$\therefore V_{mix} = 5000 \times 10^{-6}\ m^3 \quad \left[\because 1\ cm^3 = 10^{-6}\ m^3\right]$$

$$T_{mix} = 27°c = (273+27)\ K = 300\ K$$

$$\mu_{mix} = \frac{M_1}{M_{w1}} + \frac{M_2}{M_{w2}}$$

$$= \frac{16}{32} + \frac{14}{28} = \frac{1}{2} + \frac{1}{2}$$

$$= 1 \text{ mole}$$

$$\because P_{mix} V_{mix} = \mu_{mix} R\ T_{mix}$$

$$\therefore P_{mix} = \frac{\mu_{mix} R\ T_{mix}}{V_{mix}}$$

$$= \frac{1 \times 8.31 \times 300}{5000 \times 10^{-6}}$$

* $\boxed{\text{for numericals } 8.31 \times 3 \simeq 25}$ Short trick ✓

$$\therefore P_{mix} = 5 \times 10^5\ N/m^2$$

$$= 5\ atm$$

Q.14 If a mixture of 12 g of Helium, 2g of Hydroge and 32 g of oxygen is contained in a vessel at 400 K and 16.6 atm, the density in g/l of the ~~volume~~ mixture will be.

$$M_{mix} = 12g + 2g + 32g = 46g$$

$$\mu_{mix} = \frac{M_1}{M_{w1}} + \frac{M_2}{M_{w2}} + \frac{M_3}{M_{w3}}$$

$$= \frac{12}{4} + \frac{2}{2} + \frac{32}{32}$$

$= 3 + 1 + 1 = 5$ mole

$T_{mix} = 400\ K$

$P_{mix} = 16.6$ atm

$= 16.6 \times 10^5\ N/m^2$

∵ $P_{mix} V_{mix} = \cancel{\mu R}\ \mu_{mix} R\, T_{mix}$

∴ $$V_{mix} = \frac{\mu_{mix} R\, T_{mix}}{P_{mix}}$$

$$= \frac{5 \times 8.31 \times 400}{16.6 \times 10^5}$$

$= 10 \times 10^{-3}\ m^3$

$= 10\ l$ [∵ $10^{-3}\ m^3 = 1$ litre]

∵ $$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{M_{mix}}{V_{mix}}$$

$$= \frac{46\ g}{10\ l} = 4.6\ g/l.$$

Q.15 A container of volume 20 l contains a mixture of Ne ($M_W = 20$) and Ar ($M_W = 40$) gases at a temperature of 27°c and 1 atm Total mass of the mixture is 28 g. calculate individual mass of the~~ir~~ mixture gases.

$M_1 + M_2 = 28\ g$ —①

∵ $P_{mix} = 1$ atm $= 1 \times 10^5\ N/m^2$

$V_{mix} = 20\ l$ ∴

[∵ $1\ m^3 = 10^3\ l$

∵ $1\ l = 10^{-3}\ m^3$]

∴ $V_{mix} = 20 \times 10^{-3}\ m^3$

$T_{mix} = (27 + 273)\,K = 300\,K$

$\therefore$ ~~Pmi~~ $\because \mu_{mix} = \frac{M_1}{M_{w_1}} + \frac{M_2}{M_{w_2}}$

$= \frac{M_1}{20} + \frac{M_2}{40}$

$\therefore P_{mix} V_{mix} = \mu_{mix} R\, T_{mix}$

$\therefore \cancel{\mu_{mix}} = \cancel{\frac{R \times T_{mix}}{P_{mix} \times V_{mix}}}$

$= \cancel{\frac{8.31 \times 300}{1 \times 10^5 \times 20 \times 10^{-3}}}$

$\therefore \mu_{mix} = \frac{P_{mix}\, V_{mix}}{R\, T_{mix}}$

$\frac{M_1}{20} + \frac{M_2}{40} = \frac{1 \times 10^5 \times 20 \times 10^{-3}}{8.31 \times 300}$

$\frac{2M_1 + M_2}{40} = \frac{4}{5}$

$\therefore 2M_1 + M_2 = \frac{40 \times 4}{5} = 32\,g$ — (ii)

From eq ① and (ii)

~~$2M_1 +$~~ $M_2 = 28 - M_1$

$\therefore 2M_1 + (28 - M_1) = 32$

$\therefore 2M_1 - M_1 = 32 - 28$

$= M_1 = 4\,g$ (Neon)

$\therefore M_2 = 28 - 4 = 24\,g$ (Argon)

A

Q.16* Two identical container containing helium and oxygen gas at 2.25 atm and 7.25 atm respectively. Find the mixture pressure if:

Case I : Both the gases are put in one container

Case II : Both the containers are joined

$P_1 = 2.25$ atm

$P_2 = 7.25$ atm

$V_1 = V_2 = V =$ constant

$\because PV =$ constant

$\therefore P_{mix} V_{mix} = P_1V_1 + P_2V_2$

$$\therefore P_{mix} = \frac{P_1V_1 + P_2V_2}{V_{mix}}$$

<u>Case I:</u> $$\frac{2.25 \times V + 7.25 \times V}{V}$$

$$= 9.5 \text{ atm}$$

<u>Case II:</u> $$\frac{2.25 \times V + 7.25 \times V}{2V}$$

$$= \frac{9.5}{2} = 4.75 \text{ atm}$$

Short Trick:

$$xy = \text{constant} \begin{cases} \rightarrow x = P, T, \rho \\ \rightarrow y = N, M, \mu, V \end{cases}$$

* 'x' represents those quantities which can't be added easily whereas 'y' represents those quantities which can be added easily

$$\because x \propto \frac{1}{y}$$

$$\therefore \boxed{x_{mix} = \frac{x_1y_1 + x_2y_2 + x_3y_3 \ldots}{y_{mix}}}$$

i) $PV =$ constant

$$P_{mix} = \frac{P_1V_1 + P_2V_2 + P_3V_3 \ldots}{V_{mix}}$$

ii) $\mu T =$ constant

$$T_{mix} = \frac{\mu_1T_1 + \mu_2T_2 + \mu_3T_3 \ldots}{\mu_{mix}}$$

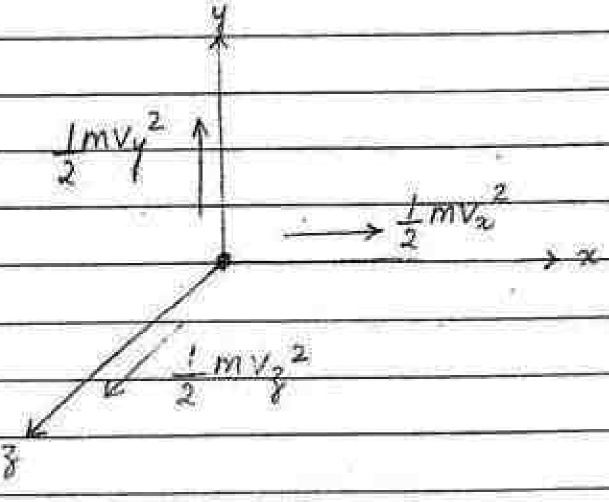
iii) $\rho V =$ constant

$$\rho_{mix} = \frac{\rho_1V_1 + \rho_2V_2 + \rho_3V_3 \ldots}{V_{mix}}$$

Degree of Freedom (f) for real gas:

The number of independent direction in which a gas molecule can do a free motion is called as degree of freedom.

1 Monoatomic or Ideal gas:
(One molecule is made up of 1 atom)
eg: He, Ne, Ar, Kr, Xe etc



$$f = \underbrace{1 + 1 + 1}_{\text{translational}} + \underbrace{0}_{\text{rotational}} + \underbrace{0}_{\text{vibrational}}$$

$$\therefore \boxed{f = \underset{(T)}{3} + \underset{(R)}{0} + \underset{(V)}{0}}$$

$$\therefore \boxed{f = 3}$$

* It has no rotational and vibrational motion of movement because the atoms are very small (negligiable radius)

+ Rotational Kinetic energy:

Rotational $KE = \frac{1}{2} I w^2$

where, I (momentum of Inertia)

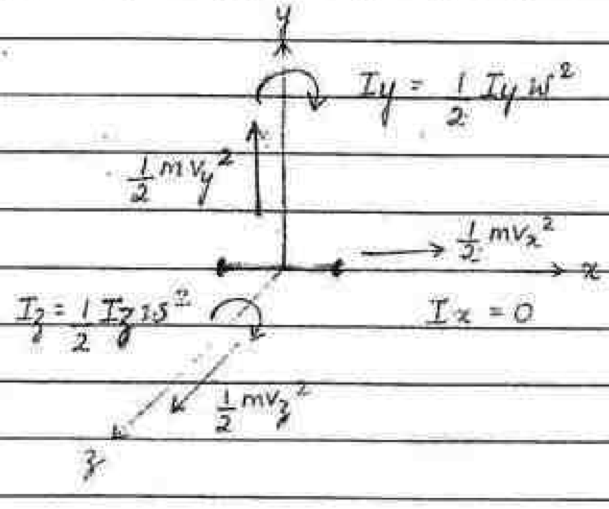
$I = mR^2$

$\therefore$ Rotational $KE = \frac{1}{2} mR^2 w^2$

2. Diatomic gas:

(one molecule is made up of 2 atoms)

eg: H_2, O_2, Cl_2, N_2, F_2 etc



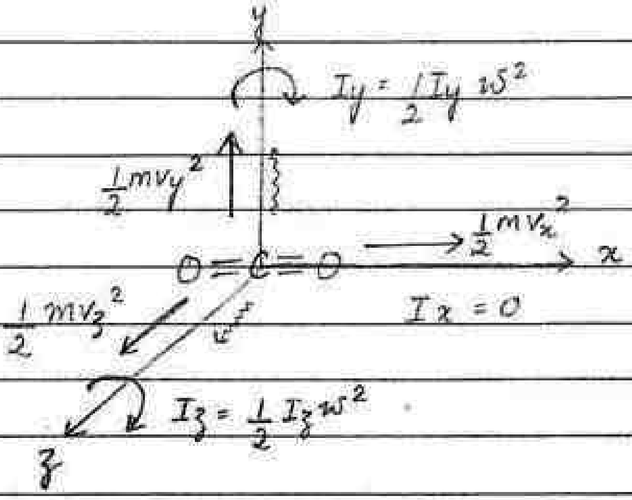
$$f = \underbrace{1+1+1}_{\text{translational}} + \underbrace{0+1+1}_{\text{rotational}} + \underbrace{0}_{\text{vibrational}}$$

$$\therefore \boxed{f = \underset{(T)}{3} + \underset{(R)}{2} + \underset{(V)}{0}}$$

$$\therefore \boxed{f = 5}$$

3. Triatomic or Polyatomic gas:
(one molecule is made up of 3 or more than 3 atoms)
~~(They occur in layer)~~

i) Linear gas: when atoms are arranged in one straight line or configuration is in the same line.
eg: CO_2, C_2H_2 (only two)
$O = C = O$ and $H - C \equiv C - H$



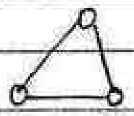
$$f = \underbrace{1+1+1}_{\text{translational}} + \underbrace{0+1+1}_{\text{rotational}} + \underbrace{0+1+1}_{\text{vibrational}}$$

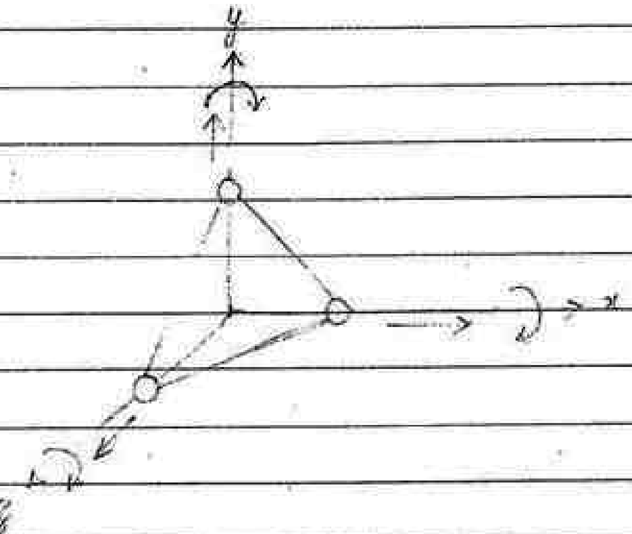
$$\therefore \boxed{f = \underset{(T)}{3} + \underset{(R)}{2} + \underset{(V)}{2}}$$

$$\therefore \boxed{f = 7}$$

ii) Non-linear gas: when the configuration is not in straight line

eg: O_3





$$f = \underbrace{1+1+1}_{\text{translational}} + \underbrace{1+1+1}_{\text{rotational}} + \underbrace{\quad 0 \quad}_{\text{vibrational}}$$

$$\therefore f = \underset{(T)}{3} + \underset{(R)}{3} + \underset{(V)}{0}$$

$$\therefore \boxed{f = 6}$$

Pressure expression:

$$\overline{V^2} = \overline{V_x^2} + \overline{V_y^2} + \overline{V_z^2}$$

$$\because \overline{V_x^2} = \overline{V_y^2} = \overline{V_z^2}$$

because $P_x = P_y = P_z = P$ (Pressure) — ①

Particles (atoms) are exerting equal pressure on the walls of the container.

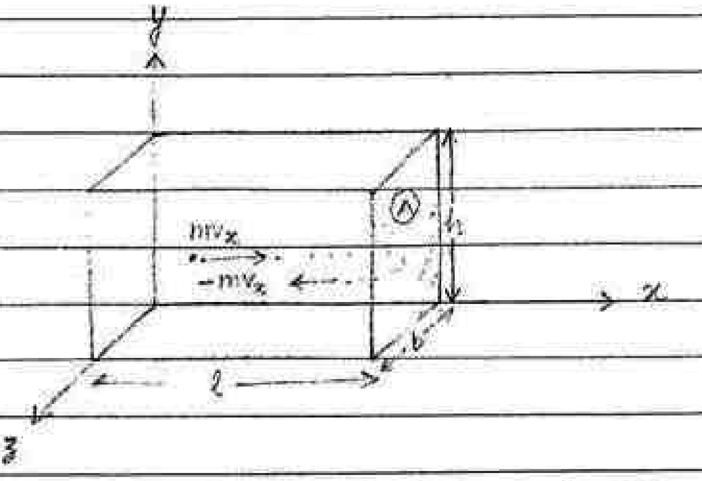
$$\therefore \overline{V^2} = 3\overline{V_x^2}$$

$$V_{rms} = \sqrt{\overline{V^2}} = \sqrt{3\overline{V_x^2}}$$

$$\therefore V_{rms}^2 = \overline{V^2} = 3\overline{V_x^2}$$

$$\therefore V_{rms}^2 = 3\overline{V_x^2} \quad \text{— (11)}$$

$$\therefore \quad \boxed{\overline{V_x^2} = \frac{V_{rms}^2}{3}}$$



Area (A) = Breadth (b) × height (h)

$\therefore A = b \times h$

Volume = length × breadth × height

$\therefore V = l \times b \times h = l \times A$

- Step 1: Change in momentum of the gas molecules

$$\overrightarrow{\Delta P_{molecule}} = \overrightarrow{P_f} - \overrightarrow{P_i}$$
$$= (-mV_x) - (mV_x)$$
$$= -mV_x - mV_x = -2mV_x$$

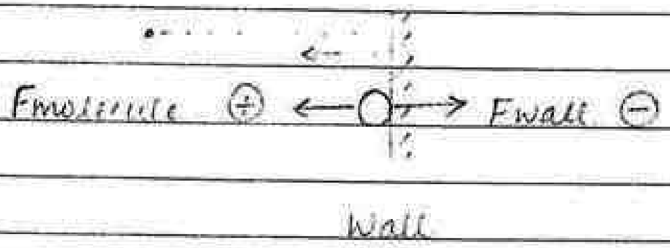
- Step 2: Successive collision time

$$t_c = \frac{distance}{speed} = \frac{2l}{V_x}$$

- Step 3: Force exerted on the wall

$$F_{molecule} = \frac{\overrightarrow{\Delta P_{molecule}}}{t_c}$$
$$= \frac{-2mV_x}{\frac{2l}{V_x}} = -\frac{mV_x^2}{l}$$

$$\therefore F_{wall} = \frac{mV_x^2}{l}$$ (+ve after rebounding)



Step 4: Pressure exerted ~~by~~ on the wall by 1 molecule,

$$\because \text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$P = \frac{F}{A} = \frac{mv_x^2}{l \times A} = \frac{mv_x^2}{Vol^m}$$

∴ Pressure exerted on the wall by n molecule

$$\therefore P_x = P_1 + P_2 + P_3 + \dots\dots\dots P_N$$

$$= \frac{mv_{x_1}^2}{vol^m} + \frac{mv_{x_2}^2}{Vol^m} + \frac{mv_{x_3}^2}{Vol^m} \dots\dots \frac{mv_N^2}{Vol^m}$$

$$= \frac{mN}{Vol^m}\left(\frac{v_{x_1}^2 + v_{x_2}^2 + v_{x_3}^2 \dots\dots v_{x_N}^2}{N}\right)$$

$$= \frac{mN}{V} \times \overline{\cancel{m}v_x^2}$$

From eq ① and ⑪

$$P = m\frac{N}{V}\frac{v_{rms}^2}{3} \quad \left[\because P_x = P \text{ and } \overline{V_x^2} = \frac{v_{rms}^2}{3}\right]$$

$$\therefore P = \frac{1}{3}\frac{M}{V}v_{rms}^2 \quad [\because mN = M]$$

★ $$\therefore \boxed{P = \frac{1}{3}\rho v_{rms}^2}$$

Different Kinetic energy for ideal gases

Translational Kinetic energy (E_T) :

$$E_T = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 \ldots\ldots \frac{1}{2}mv_N^2$$

$$= \frac{1}{2}mN\left(\frac{v_1^2 + v_2^2 + v_3^2 + \ldots\ldots v_N^2}{N}\right)$$

by multiplying numerator and denominator by N.

$$\therefore E_T = \frac{1}{2}mN\overline{v^2} \quad \left[\because \frac{v_1^2 + v_2^2 + \ldots\ldots v_N^2}{N} = \overline{v^2}\right]$$

$$= \frac{1}{2}M\overline{v^2} \quad \left[\because mN = M\right]$$

$$= \frac{1}{2}M v_{rms}^2 \quad \left[\because \overline{v^2} = v_{rms}^2\right]$$

$$= \frac{3}{2}PV \quad \left[\begin{array}{l}\because P = \frac{1}{3}\frac{M}{V}v_{rms}^2 \\ \therefore 3PV = Mv_{rms}^2\end{array}\right]$$

$$☆ \therefore \boxed{E_T = \frac{3}{2}PV}$$

$$\because PV = \mu RT$$

- when $V =$ constant, $P \propto T$
  $$\therefore E_T \propto T$$
- when $P =$ constant, $V \propto T$
  $$\therefore E_T \propto T$$
- when $T =$ constant, $P \propto \frac{1}{V}$
  $$\therefore E_T = \text{constant}$$

Hence, Kinetic energy depends only on temperature. It doesn't depends on pressure & volume

✯ $\therefore \boxed{E_T = \frac{3}{2}PV = \frac{3}{2}\mu RT = \frac{3}{2}NKT = \frac{3}{2}\frac{M}{M_w}RT}$

Energy density (E_V)

E_V : (read as 'energy per unit volume')

$$\therefore E_V = \frac{E_T}{V}$$

$$\therefore E_V = \frac{\frac{3}{2}PV}{V} = \frac{3}{2}P$$

$$\therefore \boxed{E_V = \frac{3}{2}P \quad \text{and} \quad P = \frac{2}{3}E_V}$$

$$\therefore E_V = \frac{3}{2} \times \frac{1}{3} \rho v_{rms}^2$$

$$\boxed{E_V = \frac{1}{2}\rho v_{rms}^2}$$

For $\mu = 1$ mole,

- Molar KE
- Mean Molar KE
- KE of 1 mole gas
- KE of M_w gm gas
- KE of N_0 molecules
- Gram - molar KE
- Molecular molar KE
- Gram - molecular KE

$$\therefore E = \frac{3}{2}RT$$

For $N = 1$ molecule

- Molecular KE
- Mean Molecular KE
- KE of 1 gas molecule
- KE of n(name of the gas) gas molecule

$$\therefore \bar{E} = \frac{3}{2} KT$$

$$= \frac{3}{2} \frac{R}{N_0} T \quad \left[\because K = \frac{R}{N_0} \right]$$

$$\therefore \bar{E} = \frac{E}{N_0} \quad \left[\because \frac{3}{2} RT = E \right]$$

For $M = 1$ gram

- gram KE
- Mean gram KE
- KE of 1 gram ~~molecule~~ gas.

$$\therefore E_m = \frac{3}{2} \frac{\cancel{N}}{M_w} RT$$

$$E_m = \frac{E}{M_w} \quad \left[\because \frac{3}{2} RT = E \right]$$

Note :

* $$\because E_m = \frac{3 RT}{2 M_w}$$

$$\therefore E_m \propto \frac{1}{M_w} \text{ at constant } T$$

Hence E_m is maximum for lowest M_w.

i.e. $(E_m)_{max}$ = Hydrogen gas

* At constant T

$$\boxed{E > E_m > \bar{E}}$$

$$\begin{tikzcd}E \arrow[d] & E_m \arrow[d] & \bar{E} \arrow[d] \\ (1\text{ mole}) & (1\text{gram}) & (1\text{ molecule})\end{tikzcd}$$

$$\left[\because E = \frac{E}{M_w} = \frac{E}{N_0}\right]$$

$$\left[\begin{array}{l}\because M_w < N_0 \\ \therefore \frac{E}{M_w} > \frac{E}{N_0}\end{array}\right]$$

* Ideal gas do not have atomicity because it has point mass negligiable volume. Hence, it may consist of maximum 1 atom. (monoatomic)

Special Points :

i $E = \frac{\bigcirc}{2} RT$ [1 mole]

$E = \frac{\bigcirc}{2} KT$ [1 molecule]

$E = \frac{\bigcirc}{2} \frac{RT}{M_w}$ [1 gram]

where $\bigcirc$ = either 3 or f.

ii When atomicity is given :

KE → f

Total KE → f

iii When atomicity is not given (only name of ga
is given)
KE → 3
Total KE → f

Total KE = Translation KE + rotational KE
+ vibrational KE

Q.17 Calculate the KE of the following:

i KE of 8 g H_2 gas
ii Total KE of O_2 gas molecules
iii Gram - molecular KE of triatomic linear gas.
iv Gram KE of diatomic ideal gas.
v. Gram molar rotational KE of ammonia gas molecule (NH_3)

i $\frac{\textcircled{3}}{2} \frac{M}{M_w} RT = \frac{3}{2} \times \frac{8}{2} \times RT = 6RT$

ii $\frac{f}{2} NKT = \frac{5}{2} NKT$

iii $\frac{f}{2} RT = \frac{7}{2} RT$ (μ)? 7?

iv $\frac{f}{2} \frac{RT}{M_w} = \frac{5}{2} \frac{RT}{M_w}$ (M)?

v. $\frac{3}{2} RT.$

Root mean square velocity (Vrms)

$$\because P = \frac{1}{3} \rho V_{rms}^2$$

$$\therefore V_{rms} = \sqrt{\frac{3P}{\rho}}$$

$$\left[\because \frac{P}{\rho} = \frac{RT}{M_w} = \frac{KT}{m} \right]$$

★ $$\boxed{\therefore V_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3KT}{m}} = \sqrt{\frac{3RT}{M_w}}}$$

$$\boxed{\therefore V_{rms} = \sqrt{\frac{3RT}{M_w}}}$$ (for numericals)

Q.18 At 27°c Vrms velocity of a gas is 1930 m/s, then the gas is.

$$\because V_{rms} = \sqrt{\frac{3RT}{M_w}}$$

$$1930 = \sqrt{\frac{3 \times 8.31 \times 300}{M_w}}$$

$$\because 27^\circ c = 27 + 273 = 300\ K$$

$$\therefore 1930 \times 1930 = \frac{3 \times 8.31 \times 300}{M_w \times 10^{-3}}$$

$$\therefore M_w = \frac{25 \times 300}{1930 \times 1930 \times 10^{-3}}$$

[(Mw) Molecular weight should be in kg/mole, since, all the other parameters are in SI unit]

$$= \frac{75 \times 10^5}{1930 \times 1930}$$

$$= 2.01 \simeq 2$$

$\because$ Mw = 2 g/mole

Hence, the gas is ~~Helium~~. Hydrogen (H_2)

<u>Case I</u> : Only one gas is given

$\therefore$ Mw = constant

$\because$ Mw = constant $\therefore$ $V_{rms} \propto \sqrt{T}$

$$\therefore \boxed{\frac{V_{rms1}}{V_{rms2}} = \sqrt{\frac{T_1}{T_2}}}$$

<u>Case II</u> : a) When two or more than two gases are given at variable temperature

$$\therefore V_{rms} \propto \sqrt{\frac{T}{Mw}}$$

$$\therefore \boxed{\frac{V_{rms1}}{V_{rms2}} = \sqrt{\frac{T_1}{Mw_1} \times \frac{Mw_2}{T_2}}}$$

b) When two or more than two gases are given at constant temperature

$$\therefore V_{rms} \propto \sqrt{\frac{1}{Mw}}$$

$$\boxed{\therefore \frac{V_{rms1}}{V_{rms2}} = \sqrt{\frac{Mw_2}{Mw_1}}}$$

c) When two or more than two gases of same Vrms are given.

$$\because \frac{T}{Mw} = \text{constant}$$

$$\therefore T \propto Mw$$

$$\boxed{\frac{T_1}{T_2} = \frac{Mw_1}{Mw_2}}$$

✓ Case III : When pressure and temperature both are changed. then, only temperature effect will be considered.

* Vrms velocity doesn't depends on pressure or density, It depends only on temperature

Case IV : a) Escape velocity : Minimum velocity required to escape the particles from the earth's gravitational field.

$$V_{es} = \sqrt{\frac{2GM}{R}}$$

$\therefore$ $(V_{es})_{earth}$ = 11.2 km/sec

$(V_{es})_{moon}$ = 2.36 km/sec

b) ~~Atm~~ Atmosphere is present on earth.

$\therefore$ $(V_{rms})_{gas} <<< (V_{es})_{earth}$ i.e 11.2 km/s.

c) Atmosphere is not present on moon

$\therefore$ $(V_{rms})_{gas} \geq (V_{es})_{moon}$

d) Minimum temperature required to escape gas molecule from earth's atmosphere.

$$(V_{rms})_{gas} \geq (V_{es})_{earth}$$

$$\sqrt{\frac{3RT}{M_w}} \geq V_{es}.$$

$$\therefore \frac{3RT}{M_w} \geqslant V_{es}^2$$

$$\therefore T \geqslant \frac{V_{es}^2 M_w}{3R}$$

$$T_{min} = \frac{V_{es}^2 M_w}{3R}$$

$$= \frac{(11.2 \times 10^3)^2 \times (M_w \times 10^{-3} \text{ kg/mole})}{3 \times 8.31}$$

$$= \frac{125 \times 10^3 \times M_w}{25}$$

$$= 5 \times 10^3 \, M_w$$

$$= \frac{10}{2} \times 10^3 \, M_w$$

$$\therefore \boxed{T_{min} = \frac{M_w}{2} \times 10^4 \text{ K}}$$

$$\therefore T_{H_2} = 10^4 \text{ K}$$

$$T_{N_2} = \frac{28}{2} \times 10^4 \text{ K} = 14 \times 10^4 \text{ K}$$

$$T_{O_2} = \frac{32}{2} \times 10^4 \text{ K} = 16 \times 10^4 \text{ K}$$

$$T_{CO_2} = \frac{44}{2} \times 10^4 \text{ K} = 22 \times 10^4 \text{ K}$$

* when temperature is constant following terms are used:

- Isotherm
- Isothermic
- Thermal equilibrium

Different speed of gas molecules:

1. RMS speed : $\sqrt{\frac{3RT}{M_w}} = 1.73 \times \sqrt{\frac{RT}{M_w}}$

2. Mean speed, V_m : $\sqrt{\frac{8RT}{\pi M_w}} = 1.59 \times \sqrt{\frac{RT}{M_w}}$

3. Most Probable speed, V_{mp} : $\sqrt{\frac{2RT}{M_w}} = 1.41 \times \sqrt{\frac{RT}{M_w}}$

4. Speed of sound in gases medium, V_s :

$$V_s = \sqrt{\frac{\gamma RT}{M_w}}$$

$$\therefore \boxed{1 < \gamma = \frac{C_p}{C_v} < 2} \quad \text{Always.}$$

* $V_{rms} > V_m > V_{mp} > V_s$.

* $\frac{V_{rms}}{V_{mp}} = \sqrt{\frac{3}{2}}$ and

* $\frac{V_{rms}}{V_s} = \sqrt{\frac{3}{\gamma}}$

Q.19 At what temperature, average speed of oxygen is equal to rms speed of hydrogen at 7°C

∴ ~~ACQ,~~

$$\cancel{[(V_{rms})_{O_2}]_T = X}$$

$$\therefore \left[(V_m)_{O_2}\right]_T = \left[(\text{RMS speed})_{H_2}\right]_{7^\circ C}$$

$$\sqrt{\frac{8}{\pi}\frac{RT}{M_w}} = \sqrt{\frac{3RT}{M_w}}$$

$\therefore$ T of H_2 = $7^\circ C$ = $7+273$ = 280 K.

$$= \frac{8}{\pi}\frac{RT}{32} = \frac{3R\cancel{T}280}{2}$$

$$= \frac{T\times 7}{22\times 4} = 3\times 140.$$

$$\therefore T = \frac{3\times 140\times 22\times 4}{7}$$

$$= 5280\ K$$

$$= 5007^\circ C$$

Q.20. Temperature of O_2 gas becomes double then the gas is converted from molecular form to atomic form. what will be effect on Vrms?

$T_1 = T$ and $T_2 = 2T$

Molecular form (O_2), $\therefore Mw_1 = 32\ g$

Atomic form (O), $\therefore Mw_2 = 16\ g$

$$\because V_{rms} = \sqrt{\frac{3RT}{M_w}}$$

$$\therefore V_{rms} \propto \sqrt{\frac{T}{M_w}}$$

$$\frac{V_{rms_1}}{V_{rms_2}} = \sqrt{\frac{T_1}{Mw_1}\times\frac{Mw_2}{T_2}}$$

$$\frac{v}{V_{rms_2}} = \sqrt{\frac{T}{32}\times\frac{16}{2T}}$$

$$\frac{v}{V_{rms_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$\therefore \ V_{rms_2} = 2v$

Hence, on increasing the temperature twice, the velocity becomes twice from ~~at~~ molecular to atomic form

Q. 21 The temperature at which Vrms velocity of O_2 molecule is equal to that of nitrogen molecules at 127°c is.

$$[(V_{rms})_{O_2}]_T = [(V_{rms})_{N_2}]_{127°c}$$

$127°C = 273 + 127 = 400\ K$

$$= \sqrt{\frac{3RT_1}{Mw_1}} = \sqrt{\frac{3RT_2}{Mw_2}}$$

$$\frac{T_1}{Mw_1} = \frac{T_2}{Mw_2}$$

$$\frac{T}{32} = \frac{400}{28}$$

$$\therefore T = \frac{400 \times 32}{28}$$

$$= 457 \cdot 14\ K \simeq 457\ K$$

$457\ K = 457 - 273 = 184°c.$

Q. 22 At what temperature Vrms velocity of O_2 molecule will be one-third of H_2 molecule at $-3°c$

$$\because [(V_{rms})_{O_2}]_T = \frac{1}{3}[(V_{rms})_{H_2}]_{-3°c}$$

$$\sqrt{\frac{3RT_1}{Mw_1}} = \frac{1}{3}\sqrt{\frac{3RT_2}{Mw_2}}$$

Squaring both sides,

$$\frac{3RT_1}{Mw_1} = \frac{1}{9} \times \frac{3RT_2}{Mw_2}$$

$$\frac{T}{32} = \frac{1}{9} \times \frac{270}{2}$$

$T_2 = -3^\circ C = -3 + 273 = 270\ K$

$$\therefore T = \frac{32 \times 270}{9 \times 2}$$

$$= 480\ K$$

$$= 207^\circ C$$

Maxwell law of equal partition of Energy: Total KE of a gas molecule is equally distributed among its all degree of freedom and that energy associated each degree of freedom at absolute temperature 'T' is $\frac{1}{2}KT$

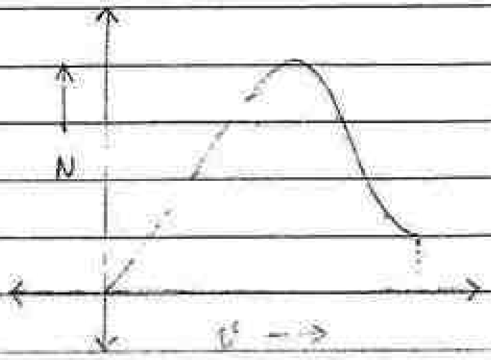
+ distribution of KE

$$\frac{f}{2}KT = \left[\frac{7}{2}KT\right]$$

$$\underbrace{\frac{1}{2}mv_x^2 \quad \frac{1}{2}mv_y^2 \quad \frac{1}{2}mv_z^2} \quad \underbrace{\frac{1}{2}I_y w^2 \quad \frac{1}{2}I_z w^2} \quad \underbrace{\frac{1}{2}KT \quad \frac{1}{2}KT}$$

$$\frac{1}{2}KT \quad \frac{1}{2}KT \quad \frac{1}{2}KT \quad \frac{1}{2}KT \quad \frac{1}{2}KT \quad \frac{1}{2}KT \quad \frac{1}{2}KT$$

(Translational) (Rotational) (vibrational)



$$\text{Area} = \int y\,dx = \int_0^\infty N\,dv$$

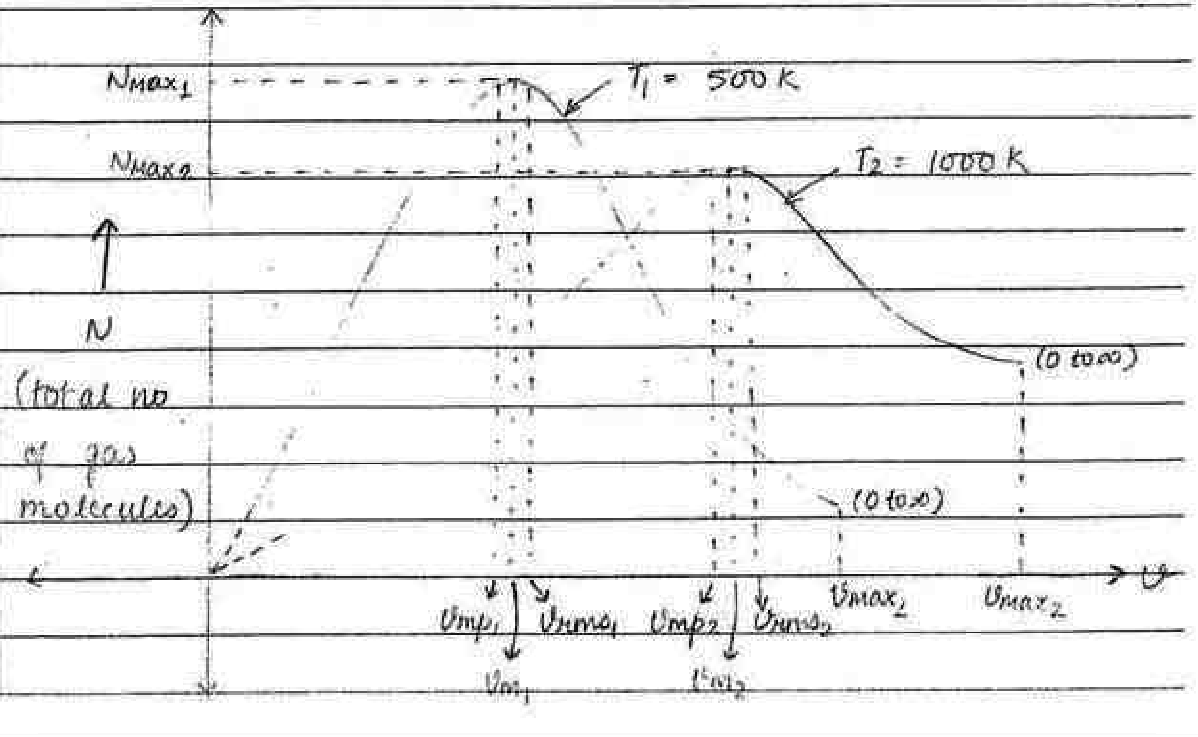
where,

$\int_0^\infty N\,dv$ = Total no of gas molecule in a container.

Special Points :

- Velocity of a gas molecule are ~~from~~ in between 0 to ∞
- At a given temperature, no of gas molecule is minimum corresponding to Vmin and Vmax and no. of gas molecule is maximum corresponding to Vmp. It means as the velocity increases corresponding, to number of molecules, first increases and then decreases.
- At a given temperature, area enclosed between curve and velocity axis represent total number of gas molecules present in a container.

Maxwell velocity distribution curve :



Conclusion :

i) ∵ any speed ∝ $\sqrt{T}$

∴ If temperature increases then,

- v_{rms} ↑
- v_m ↑
- v_{mp} ↑
- v_{max} ↑
- v_s ↑

ii) If temperature increases then

- no of gas molecules decreases corresponding to v_{mp}.
- Number of gas molecules increases corresponding to v_{max}.

Mean free path (λ_m):

- The distance travelled between two successive collison is called mean free path.

OR

- The average distance travelled between two successive collision is called mean free path.

$$\therefore \lambda_m = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \ldots\ldots\ldots \lambda_x}{x}$$

6 ~~$\lambda_m = 1$~~

as the value of x increases, λ_x also increases hence the total value i.e λ_m remains constant.

MFP ★ ∴ $\boxed{\lambda_m = \dfrac{1}{\sqrt{2}\pi d^2 n}}$

where, $n = \dfrac{N}{V}$ $\left[\text{molecular density}\right]$

d = diameter of molecule, and

$\dfrac{1}{\sqrt{2}}$ = correction factor.

$\because n = \dfrac{N}{V}$ $\therefore \lambda_m \propto V$ and.

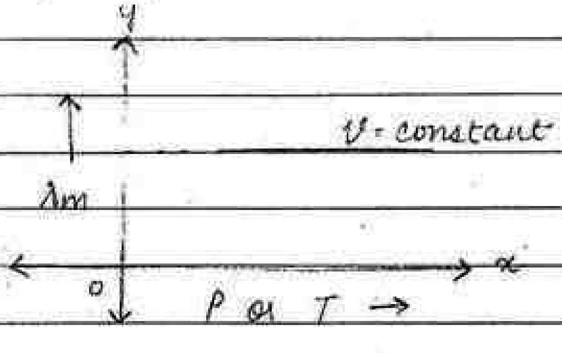
$\lambda_m \propto \dfrac{1}{d^2}$

<u>Case I</u> : $\left[\begin{array}{l}\text{If } N = \text{constant},\ V = \text{constant} \\ P\uparrow \text{ or } T\uparrow,\ \lambda_m = ?(\text{constant})\end{array}\right]$

$\because n = \dfrac{N}{V} = \text{constant}$

$\therefore P\uparrow$ or $T\uparrow$,

hence, λ_m = constant



<u>Case II</u> : $\left[\begin{array}{l}\text{If } N = \text{constant},\ T = \text{constant} \\ P\uparrow,\ \lambda_m = ?\ (\downarrow)\end{array}\right]$

$\because N$ = constant and T = constant and

$\because PV = \textcircled{N}kT$

$\therefore \; PV = \text{constant}$

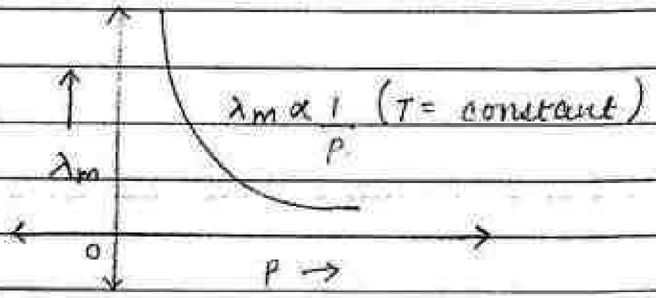
$P \propto \frac{1}{V}$

$\therefore \; P \uparrow , \; V \downarrow$

$\because \; n = \frac{N}{V} , \; V \downarrow$ then $n \uparrow$

$\because \; \lambda_m \propto \frac{1}{n} , \; \therefore \; n \uparrow$ then $\lambda_m \downarrow$

Hence, $\boxed{P \uparrow , \; V \downarrow , \; n \uparrow \text{ and } \lambda_m \downarrow}$



<u>Case III</u> : $\left[\begin{array}{l} \text{If } N = \text{constant}, \; P = \text{constant} \\ T \uparrow , \; \lambda_m = \, ? \; (\uparrow) \end{array} \right]$

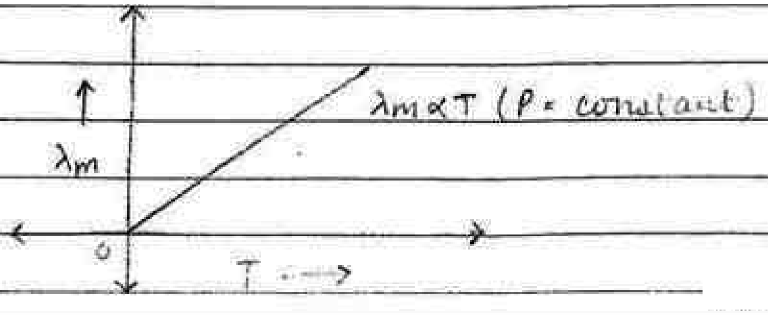
$\because \; PV = NKT$

$\therefore \; V \propto T , \; T \uparrow$ then $V \uparrow$

$\because \; n = \frac{N}{V} , \; V \uparrow$ then $n \downarrow$

$\because \; \lambda_m \propto \frac{1}{n} , \; \therefore \; n \downarrow$ then $\lambda_m \uparrow$

Hence, $\boxed{T \uparrow , \; V \uparrow , \; n \downarrow \text{ and } \lambda_m \uparrow}$



$$\therefore \quad \lambda_m = \frac{1}{\sqrt{2}\pi d^2 n}$$

$$\lambda_m = \frac{V}{\sqrt{2}\pi d^2 N} \qquad \left[\because n = \frac{N}{V}\right]$$

$$\lambda_m = \frac{KT}{\sqrt{2}\pi d^2 P} \qquad \left[\begin{array}{l} \because PV = NKT \\ \therefore \dfrac{V}{N} = \dfrac{KT}{P} \end{array}\right]$$

* If 2 gases at same temperature and pressure, then λ_m is different (due to change in the diameter (d) of the molecule)

$$\because \lambda_m \propto \frac{1}{d^2}$$

$$\therefore (\lambda_m)_{max} = H_2 \text{ gas}$$

* Some Scientist

i) Avagadro no.: If two and more than two gases at same pressure same temperature and ~~are~~ at same volume then no. of gas molecule is also same

$$\because PV = NKT$$

where P, V, K and T are constant

$\therefore$ N remains same.

$$\therefore \boxed{N_1 = N_2} \text{ at constant } P, V, T$$

* ii) Graham law of diffusion (expansion): If the temperature and pressure is same then the rate of diffusion (r_d) is inversaly proportional to square root of gas density.

Rate of diffusion, $\boxed{r_d \propto \frac{1}{\sqrt{\rho}}}$ at constant P, T

$$\because\ r_d \propto \frac{1}{\sqrt{\rho}} \propto \frac{1}{\sqrt{M_w}}$$

$$\therefore\ V_{rms} \propto \frac{1}{\sqrt{\rho}} \times \frac{1}{\sqrt{M_w}}$$

Hence, rate of diffusion (r_d) and V_{rms} will have same order

Q.23. Find out the order of rate of diffusion (r_d) of H_2S, NH_3, N_2O, CH_4 and SO_2

$$\because\ r_d \propto \frac{1}{\sqrt{M_w}}$$

$$\therefore\ H_2S \Rightarrow r_d \propto \frac{1}{\sqrt{34}}$$

$$NH_3 \Rightarrow r_d \propto \frac{1}{\sqrt{17}}$$

$$N_2O \Rightarrow r_d \propto \frac{1}{\sqrt{44}}$$

$$CH_4 \Rightarrow r_d \propto \frac{1}{\sqrt{16}}$$

$$SO_2 \Rightarrow r_d \propto \frac{1}{\sqrt{64}}$$

$\therefore$ Higher $\sqrt{M_w}$ will have low rate of diffusion.

$$\therefore\ \frac{1}{\sqrt{64}} < \frac{1}{\sqrt{44}} < \frac{1}{\sqrt{34}} < \frac{1}{\sqrt{17}} < \frac{1}{\sqrt{16}}$$

$$\therefore\ SO_2 < N_2O < H_2S < NH_3 < CH_4$$

Relation with time,

$$t \propto \sqrt{M_w}$$

iii) Dalton's Partical Pressure law :
(for monoatomic gases)
According to this law, the pressure exerted by mixture of several gases is equal to sum of the pressure exerted by each component present in the mixture.

$\therefore \boxed{P_{mix} = P_1 + P_2 + P_3 + P_4 \ldots\ldots}$ at constant T, V

Real Gas :

Some scientist :

★ 1 Vander-waal gas equation :

$\boxed{PV = RT}$ (for 1 mole of ideal gas)

Volume correction : It is due to finite size of real gas molecules

Reason : volume correction

$V_{real} = (V - b) \rightarrow$ volume of container

where b = vander-wall gas constant

$$b = N_0 \times \frac{4}{3} \pi r^3 \times ④$$

b = free surface occupied by N_0 molecule

$$\because P_{IG} (V - b) = RT$$

(for 1 mole of ideal gas for $(V-b)$ volume)

$$\therefore \boxed{P_{IG} = \frac{RT}{V-b}}$$ (for 1 mole of real gas)

* Unit of b = m^3
* dimension = $[M^0 L^3 T^0]$ or $[L^3]$

* b ~~a~~ depends on : size of gas molecules
Mono > Dia > Triatomic nature of gas.
no. of gas molecules

Pressure correction : It is due to inter molecular force between real gas molecules

$\therefore \quad P_{real} = P_{IG} - \beta$

Reason : Pressure correction

$P_{real} = P_{IG} - \beta$

where β := reducing pressure due to IMF

$\beta \propto \rho^2 \left[\text{for } \mu = \frac{M}{M_w} = 1 \text{ and } M = M_w \right]$

$\therefore \beta = \frac{Km^2}{V^2} = \frac{KM_w^2}{V^2} = \frac{a}{V^2}$

and a = vanderwall gas constant.

$\therefore \quad P_{real} = P_{IG} - \frac{a}{V^2}$

$\therefore \quad \boxed{P_{real} = \frac{RT}{(V-b)} - \frac{a}{V^2}} \quad \left[\because P_{IG} = \frac{RT}{(V-b)} \right]$

$P_{real} + \frac{a}{V^2} = \frac{RT}{(V-b)}$

(for 1 mole of real gas) :

* $$\boxed{\left(P_{real} + \frac{a}{V^2}\right)(V-b) = RT}$$

* Unit of a (Pressure) $= \frac{a}{V^2} = \frac{N}{m^2}$

$$\therefore a = \frac{N}{m^2} \times m^6$$

$$\therefore a = Nm^4$$

* dimensions of a $= [M^1 L^5 T^{-2}]$

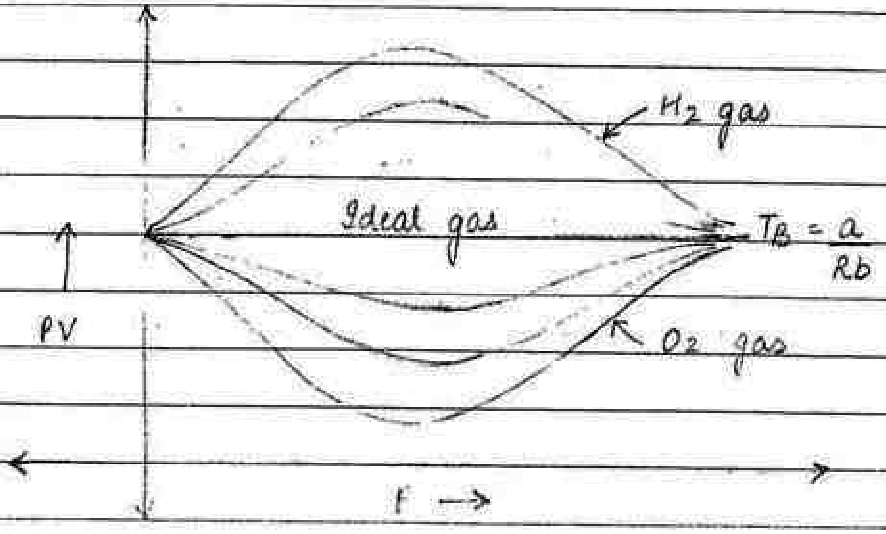
* a depends on : density of gas.
  attraction force between gas molecule.
  no. of collision per second.

2. Amaghat experiment:

- Real gas doesnot follows all the gas law and equation of state.
- The specific temperature at which a real gas starts obeys Boyle's law is called as Boyle's temperature.

Boyle's temperature $(T_B) = \boxed{\frac{a}{Rb}}$

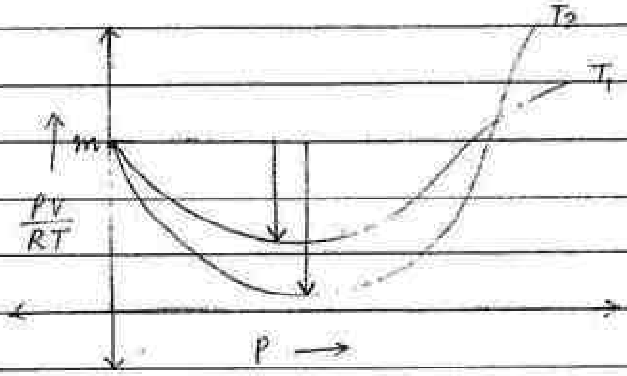
where a and b are vanderwall constant



Q.24 For O_2 gas, $M = 1 \times 10^{-3}$ kg, then find out the following.

i) m

ii) relation between T_1 and T_2



i) $$\because PV = \mu RT$$

$$\therefore \frac{PV}{RT} = \mu = \frac{M}{M_w} = m.$$

$$\therefore m = \frac{M}{M_w} = \frac{1 \times 10^{-3}\ kg}{32 \times 10^{-3}\ kg/mole}$$

$$m = \frac{1}{32} \text{ mole}$$

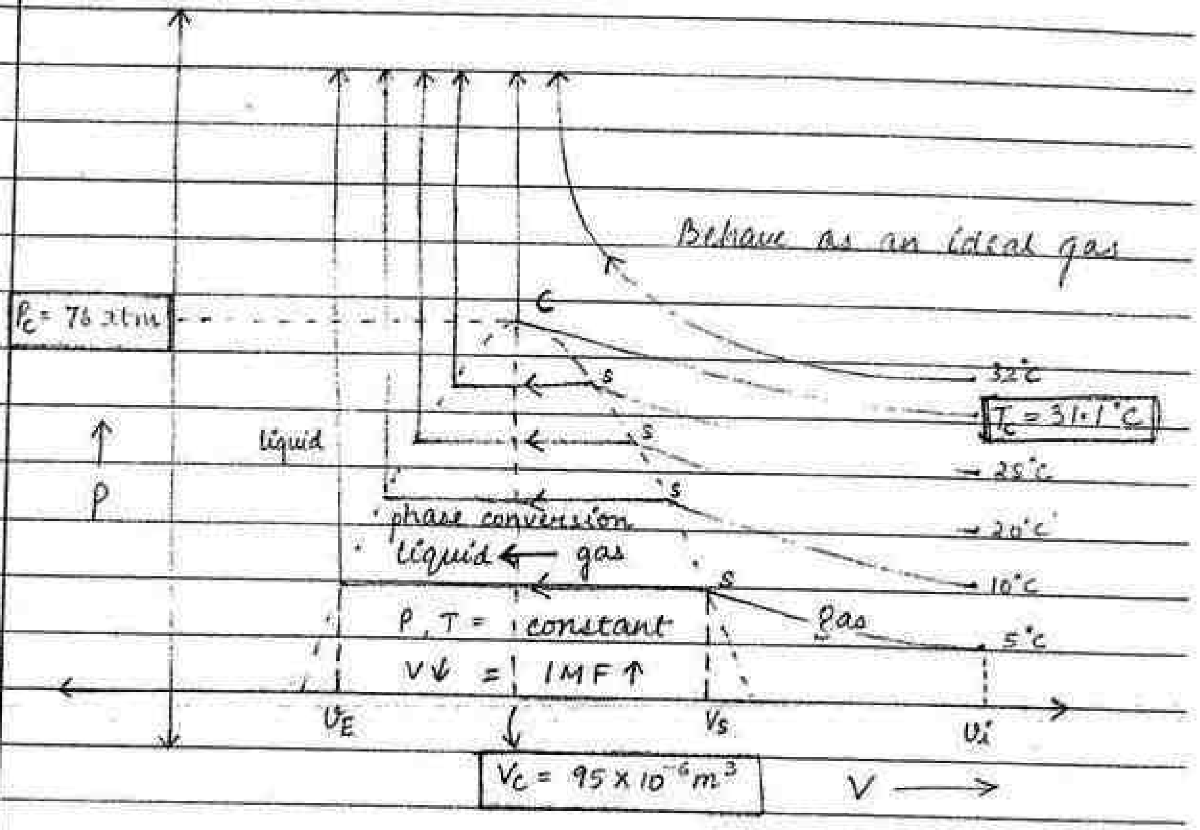
ii) ∵ T_1 is closer to the straight line

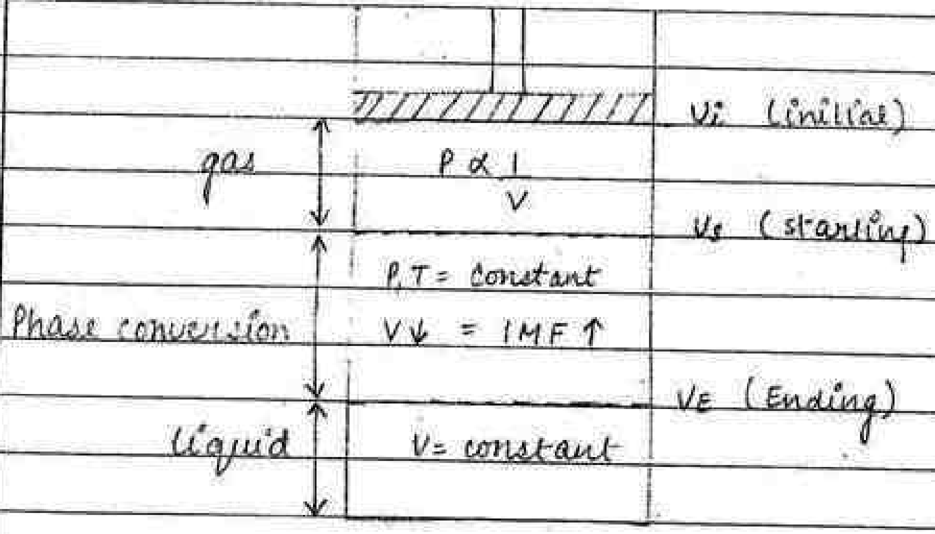
∴ It has gained more amount of heat energy as compared to T_2

Hence $T_1 > T_2$

~~Dr. Andrewz experiment:~~

Dr Andrewz Experiment :





- The peak point of the parabola is known as critical point (c)
- Thermodynamic parameter corresponding to critical point is known as critical parameter (P_c, V_c, T_c)

★ Critical Temperature (T_c) : The temperature below which a gas can be liquified by increasing pressure and above this temperature a gas cannot be liquified whatever the pressure may be increased, it means it behaves as an ideal gas.

$$\boxed{T_c = \frac{8}{27}\frac{a}{Rb}}$$

$$= \frac{8}{27} T_B \quad \left[\because \frac{a}{Rb} = T_B\right]$$

$$\therefore \boxed{T_B > T_c}$$

For $CO_2 \Rightarrow T_c = 31.1^\circ C$

★ Critical Pressure (P_c) : The minimum pressure require to liquify a real gas corresponding to T_c is called ~~#~~ P_c

$$P_c = \frac{a}{27 b^2}$$

For $CO_2 \Rightarrow P_c = 76$ atm

Critical Volume (V_c) : Volume of 1 mole of real gas corresponding to T_c, P_c is called as V_c

$$V_c = 3b$$

For $CO_2 \Rightarrow V_c = 95 \times 10^{-6}\ m^3$

$\boxed{\dfrac{P_cV_c}{RT_c} = \dfrac{3}{8}}$ (for 1 mole of real gas)

$\boxed{\dfrac{PV}{RT} = 1}$ (for 1 mole of ideal gas)