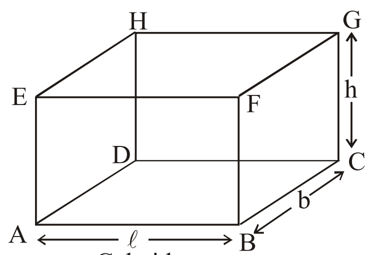
Chapter 5 Mensuration

Volume and Surface Area

- **Volume (Capacity) of a solid:** The measure of space occupied by a solid body is called its volume.
  The units of volume are cubic centimeters (written as cm^3) or cubic meters (written as m^3).
- **Cuboid:** A solid bounded by six rectangular faces is called a cuboid.
  Surface area of a cuboid:
  Let us consider a cuboid of length = l units
  Breadth = b units and height = h units
  Then we have :
  (i) Total surface area of the cuboid
  $= 2(l \times b + b \times h + h \times l)$ sq. units
  (ii) Lateral surface area of the cuboid
  $= [2\,(l + b) \times h]$ sq. units
  (iii) Area of four walls of a room = $[2\,(l + b) \times h]$ sq. units.
  = (Perimeter of the base × height) sq. units
  (iv) Surface area of four walls and ceiling of a room
  = lateral surface area of the room + surface area of ceiling
  $= 2\,(l + b) \times h + l \times b$
  (v) Diagonal of the cuboid $= \sqrt{l^2 + b^2 + h^2}$



Cuboid

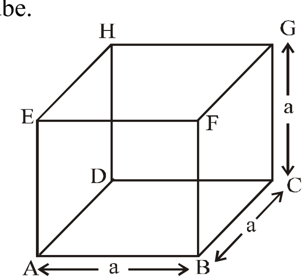
Volume of a cuboid:

Volume of a cuboid = Area of the base × height $V = l \times b \times h$

So, volume of a cuboid = base area × height = length × breadth × height

A rectangular room is in the form of a cuboid and its 4 walls are its lateral surfaces.

- **Cube:** A cuboid whose length, breadth and height are all equal, is called a cube.



Surface area of a cube : Consider a cube of edge a unit.

(i) The Total surface area of the cube = $6a^2$ sq. units

(ii) Lateral surface area of the cube = $4a^2$ sq. units.

(iii) The diagonal of the cube $= \sqrt{3}\,a$ units.

Volume of a cube :

Volume of a cube = edge × edge × edge $= a^3$

- **Cylinder:** Solids like circular pillars, circular pipes, circular pencils, road rollers and gas cylinders etc. are said to be in cylindrical shapes.

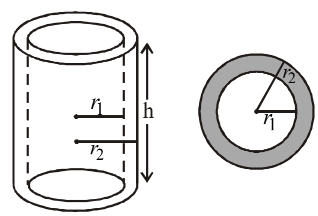
Surface area of cylinder :

(i) Curved surface area of a cylinder $= 2\pi rh$

(ii) Total surface area of the cylinder $= 2\pi r(r + h)$.

(iii) If a cylinder is a hollow cylinder whose inner radius is r_1 and outer radius r_2 and height h then

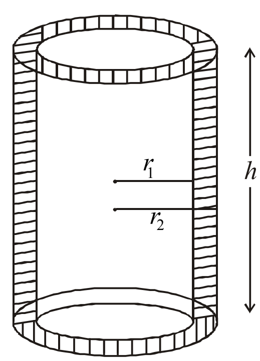
Total surface area of the cylinder $= 2\pi(r_1 + r_2)[h + r_2 - r_1]$



Volume of cylinder :

Volume of a cylinder = $\pi r^2 h$

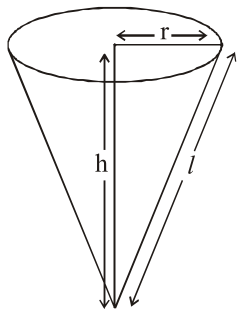
Volume of used material in a hollow cylinder. If height is h and inner and outer radius r_1 and r_2 respectively.



Then the volume of the hollow cylinder

$$= \pi r_2^2 h - \pi r_1^2 h \ = \pi(r_2^2 - r_1^2)h$$

- **Cone :**



Surface area of a cone :

(i) Curved surface area of a cone $= \frac{1}{2} \times l \times 2\pi r = \pi r l$

where r is base radius and l its slant height.

(ii) Total surface area of the right circular cone

= curved surface area + Area of the base

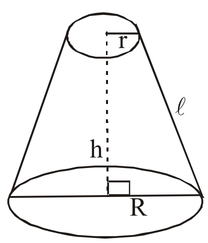
$= \pi r l + \pi r^2 = \pi r(l + r)$

Note : $l^2 = r^2 + h^2$

Volume of cone :

Volume of a cone $= \frac{1}{3}\pi r^2 h$, where r is the base radius and h is the height of the cone.

- **Frustum of a Cone :** A cone is cut by a plane parallel to the base of the cone



then the portion between the plane and base is called frustum of the cone.

(i) Volume of frustum of cone $= \frac{\pi h}{3}[R^2 + r^2 + Rr]$ cubic unit

(ii) L . S. A or C. S. A $= \pi\ell(R + r)$ Sq units

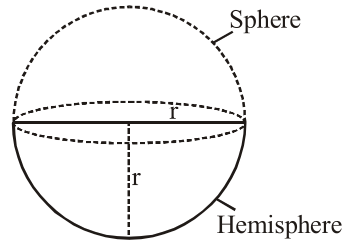
where $\ell^2 = h^2 + (R - r)^2$

(iii) T. S. A $= \pi R^2 + \pi r^2 + \pi\ell(R + r)$ Sq. units.

(Area of base + Area of top + Area of lateral)

(iv) Slant height $(\ell) = \sqrt{h^2 + (R - r)^2}$

- **Sphere and Hemisphere :**



(i) Surface area of sphere $= 4\pi r^2$

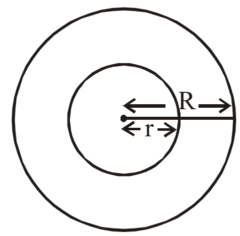
(ii) Volume of a sphere $= \frac{4}{3}\pi r^3$

(iii) Volume of hemisphere $= \frac{2}{3}\pi r^3$

(iv) C.S.A. of hemisphere $= 2\pi r^2$

(v) Total surface area of Hemi-sphere $= 2\pi r^2 + \pi r^2 = 3\pi r^2$

- **Spherical shell and Hemispherical shell :**



(i) Outer surface area of spherical shell $= 4\pi R^2$

(ii) Inner S.A. of spherical shell $= 4\pi r^2$

(iii) Total surface area of spherical shell $= 4\pi(R^2 + r^2)$

(iv) Volume of spherical shell of external radius R and internal radius ‘r’ $= \frac{4}{3}\pi(R^3 - r^3)$

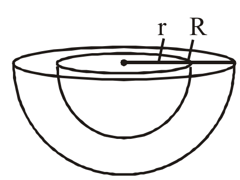
(v) Outer curved surface area of hemispherical shell $= 2\pi R^2$

(vi) Inner curved surface area of hemispherical shell $= 2\pi r^2$

(vii) Thick hemispherical bowl of external and internal radii R and r,

Total S.A. $= \pi(3R^2 + r^2)$nb

(viii) Volume of hemispherical shell of external radius 'R' and internal radius 'r' $= \frac{2}{3}\pi(R^3 - r^3)$.



Area Related to Circles

- **Area of the circle** $= \pi r^2$
- **Perimeter of circle** $= 2\pi r$

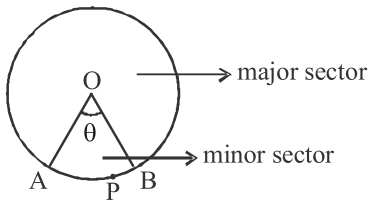
  where r is the radius of the circle.

  Perimeter of a circle is known as circumference of a circle.
- **Area of sector of circle :** The part of the circle inclined between two radii (OA & OB) is called sector of circle.

  (i) Area of the sector OAPB $= \frac{\theta}{360°} \times \pi r^2$

  (ii) Length of an arc of sector OAPB

  $=$ length of arc AB $= \frac{\theta}{360°} \times 2\pi r$ Where θ is the measure of arc AB



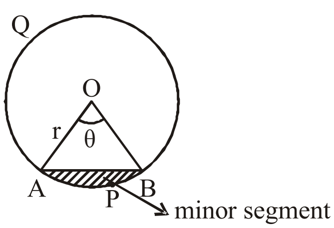
  (iii) Perimeter of the sector (minor sector)

  $= \frac{\theta}{360} \times 2\pi r + 2r.$
- **Area of segment of circle :** Area of Any chord AB divides circle into two parts. The bigger part is known as major segment and smaller one is called minor segment.

  (i) Area of minor segment APB

  $=$ Area of sector OAPB – area of ΔOAB

  $= \frac{\theta}{360°} \times \pi r^2 -$ area ΔOAB



  (ii) Area of major segment OAQB $= \pi r^2$ – area of minor segment APB.

Note : Area of Δ OAB with $\angle AOB = \theta = \frac{1}{2}(OA)(OB)\sin\theta$

- **Area enclosed by the two circles :** If R and r are the radii of two concentric circles such that $R > r$ then area enclosed by the two circles $= \pi R^2 - \pi r^2$

Some Useful Results

(i) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

(ii) If two circles twouch externally, then the distance between their centres is equal to the sum of their radii.

(iii) Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.

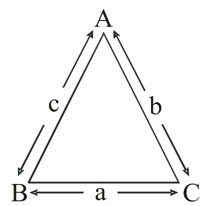
(iv) The number of revolutions completed by a rotating wheel in one minute $= \frac{\text{Distance moved in one minute}}{\text{Circumference}}$

Heron's formula

- **Heron's formula :** If three sides of a triangle are given then the area of triangle is derived by "Heron's formula".

  Area of Δ ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

  where $s = \frac{a+b+c}{2}$ is called the semiperimeter of the triangle and a, b, c are the lengths of opposite sides of vertex angles A, B and C respectively.



- **Area of an isosceles triangle by using Heron's formula :** If one of the equal side of an isosceles triangle is a and the third side is b then the semi perimeter of isosceles triangle is

  $s = \frac{a+a+b}{2} = \frac{2a+b}{2}$

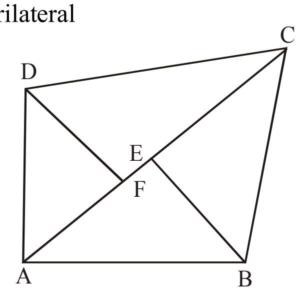
  Then by Heron's formula

  Area of isosceles triangle $= \frac{b}{4}\sqrt{4a^2 - b^2}$
- **Area of equilateral triangle by using Heron's formula :** The length of one side of equilateral triangle is a then its semi perimeter be

  $s = \frac{a+a+a}{2} = \frac{3a}{2}$

  Therefore by Heron's formula

  Area of equilateral triangle $= \frac{a^2\sqrt{3}}{4}$ sq. units
- **Applications of Heron's formula in finding area of Quadrilaterals :** A plane figure bounded by four sides is called a quadrilateral



Area of quadrilateral

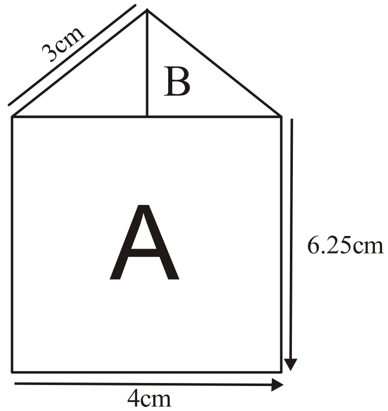
$= \frac{1}{2} \times$ (Diagonal) $\times$ (Sum of the perpendiculars drawn from vertices on the diagonal).

Exercise 1

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) out of which only one is correct.*

1. If the radius of a right circular cylinder, open at both the ends, is decreased by 25% and the height of the cylinder is increased by 25% then the surface area of the cylinder, thus formed
   (1) Remains unaltered (2) Is increased by 25%
   (3) Is decreased by 25% (4) Is decreased by 6.25%
2. A vessel contains a mixture of milk and water in the ratio 3 : 1. A certain part, say x, of the mixture is taken away from the vessel and an equal amount of water is now poured into the vessel. It is found that the amount of milk in the vessel is equal to that of water. Then the value of x is
   (1) 1/4 (2) 1/3
   (3) 2/3 (4) 3/4
3. Two volumes of a book are resting side by side in order on a bookshelf with Vol. I left of the Vol. II. Cover of each book is 0.5 cm. thick and each book without its cover is 2.25 cm. thick. A bookworm eats its way directly from page 1 Vol. I to the last page Vol. II. It will travel
   (1) 5.5 cm. (2) 4.75 cm
   (3) 3.25 cm. (4) 1.0 cm.
4. A cylindrical pencil of diameter 1.2 cm. has one of its end sharpened into a conical shape of height 1.4 cm. The volume of the material removed is
   (1) 1.056 cm^3 (2) 4.224 cm^3
   (3) 10.56 cm^3 (4) 42.24 cm^3
5. OA, OB, OC are three mutually perpendicular lines of length a, b, c units. Volume of the tetrahedran OABC is
   (1) $\frac{ab}{c}$ cu. units (2) abc cu. units
   (3) (a + b – c) cu. units (4) 1/6 abc cu. units
6. A conical tent has a floor area of 154 sq. m. Its height is 24m. How much canvas is required for the tent ?
   (1) 500 sq. m. (2) 550 sq. m.
   (3) 700 sq. m. (4) 450 sq. m.
7. The total surface area of rt. circular cylinder whose base area is 346 .5 cm^2 and whose height is 24 cm is
   (1) 2727 cm^2 (2) 2772 cm^2
   (3) 2277 cm^2 (4) 7722 cm^2
8. The volume of a cylinder is 448 π cm^3 and height 7 cm. Its lateral surface area is
   (1) 253 cm^2 (2) 352 cm^2
   (3) 532 cm^2 (4) 325 cm^2
9. The base radii of two rt. Circular cones of the same height are in the ratio 3 : 5, the ratio of their volumes is
   (1) 25 : 9 (2) 9 : 25
   (3) 3 : 25 (4) 9 : 5
10. The diameter of a garden roller is 1.4 m, and 2m Long. How much area will it cover in 5 revolutions?
   (1) 44 m^2 (2) 140 m^2
   (3) 440 m^2 (4) 220 m^2
11. How many spherical ball can be made out of a solid cube of lead whose edge is 44 cm, each ball being 4 cm. in diameter?
   (1) 2451 (2) 2541
   (3) 1254 (4) 1452
12. The larger sphere is carved out of a cube of side 7cm. The volume of the sphere is
   (1) 178 cm^3 (2) 177 cm^3
   (3) 179.66 cm^3 (4) 176
13. A toy is in the form of a cone mounted on a hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm., The total surface area is (use $\pi = 3\frac{1}{7}$)
   (1) 241.5 cm^2 (2) 214.5 cm^2
   (3) 412.5 cm^2 (4) 124.5 cm^2
14. If the perimeter of one face of a cube is 20 cm, then its surface area is
   (1) 120 cm^2 (2) 150 cm^2
   (3) 125 cm^2 (4) 400 cm^2
15. If the diameter of the sphere is doubled, the surface area of the resultant sphere becomes x times that of the original one. Then x would be
   (1) 2 (2) 3
   (3) 4 (4) 8
16. The area of the sector of a circle, whose radius is 6 m, when the angle at the centre is 42° is
   (1) 13.2 m^2 (2) 14.2 m^2
   (3) 13.4 m^2 (4) 14.4 m^2
17. If the radii of two concentric circles are 15 cm and 13 cm, respectively, then the area of the circulating ring in sq. cm will be
   (1) 176 (2) 178
   (3) 180 (4) 200
18. The ratio of the volume and surface area of a sphere of unit radius
   (1) 4 : 3 (2) 3 : 4
   (3) 1 : 3 (4) 3 : 1
19. The slant height of a cone is increased by P%. If radius remains same, the curved surface area is increased by
   (1) P% (2) P^2%
   (3) 2P% (4) None of these
20. A hollow spherical ball whose inner radius is 4 cm is full of water. Half of the water is transferred to a conical cup and it completely filled the cup. If the height of the cup is 2 cm, then the radius of the base of cone, in cm is
   (1) 4 (2) 8 π
   (3) 8 (4) 16

21. A is a right circular cylinder on which a cone B is placed. The entire structure is melted and spheres are formed each having radius 1 cm. How many spheres can be formed?



(1) 18 (2) 20
(3) 21 (4) 23

22. The length of the longest rod that can be fit in a cubical room of 4 cm side is
(1) 8.66 m (2) 5.196 m
(3) 6.928 m (4) 7.264 m

23. If a right circular cone of vertical height 24 cm has a volume of 1232 cm^3, then the area of its curved surface is
(1) 1254 cm^2 (2) 304 cm^2
(3) 550 cm^2 (4) 154 cm^2

24. The radii of two cylinders are in the ratio of 2 : 3 and their heights in ratio of 5 : 3, their volumes will be
(1) 4 : 9 (2) 27 : 20
(3) 20 : 27 (4) 9 : 4

25. If the area of a circle is 154 cm^2, then the perimeter is
(1) 11 cm (2) 22 cm
(3) 44 cm (4) 55 cm

26. Area of the largest triangle that can be inscribed in a semi-circle of radius *r* units is
(1) r^2 (2) $\frac{1}{2}r^2$
(3) $2r^2$ (4) $\sqrt{2}r^2$

27. The sum of the radii of two circles is 84 cm and the difference of their areas is 5544 cm^2. Then the radii of the two circles are
(1) 63 cm, 21 cm (2) 52.5 cm, 31.5 cm
(3) 42 cm, 42 cm (4) none of these

28. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is
(1) 56 cm (2) 42 cm
(3) 28 cm (4) 16 cm

29. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then
(1) $R_1 + R_2 = R$ (2) $R_1^2 + R_2^2 = R^2$
(3) $R_1 + R_2 < R$ (4) $R_1^2 + R_2^2 < R^2$.

30. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then
(1) $R_1 + R_2 = R$
(2) $R_1 + R_2 > R$
(3) $R_1 + R_2 < R$
(4) Nothing definite can be said about the relation among R_1, R_2 and R.

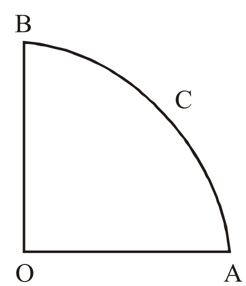
31. If the circumference of a circle and the perimeter of a square are equal, then
(1) Area of the circle = Area of the square
(2) Area of the circle > Area of the square
(3) Area of the circle < Area of the square
(4) Nothing definite can be said about the relation between the area of the circle and square.

32. Area of a sector of angle p° of a circle with radius R is
(1) $\frac{p}{180} \times 2\pi R$ (2) $\frac{p}{180} \times \pi R^2$
(3) $\frac{p}{360} \times 2\pi R$ (4) $\frac{p}{720} \times 2\pi R^2$

33. The area of the circle that can be inscribed in a square of side 6 cm is
(1) 36π cm^2 (2) 18π cm^2
(3) 12π cm^2 (4) 9π cm^2

34. If the area of a circle of radius r is A and its circumference is C, then
(1) $A = rC$ (2) $AC = 4r^2$
(3) $AC = 2r$ (4) $A = \frac{1}{2}rC$

35. In the adjoining figure, OACB is a quadrant of a circle of radius 7 cm. The perimeter of the quadrant is



(1) 11 cm (2) 18 cm
(3) 25 cm (4) 36 cm

36. Two triangles have their sides measuring 5 cm, 6 cm and 9cm, and 9 cm, 7.5 cm and 13.5 cm respectively. Their areas are in the ratio of
(1) 2 : 3 (2) 4 : 9
(3) 9 : 4 (4) 3 : 2

37. Midpoints of the sides of a triangle are joined to form another triangle. The ratio of the area of the new triangle to the rest of the original triangle is
(1) 1 : 2 (2) 1 : 3
(3) 1 : 4 (4) None of these

38. The sides of a triangle are in the ratio of 4 : 6 : 7, then
(1) the triangle is obtuse-angled.
(2) the triangle is acute-angled.
(3) the triangle is right-angled.
(4) the triangle is impossible.

39. The sides of a triangle are 5 cm, 12 cm and 12.5 cm. Its area is
(1) 30 cm^2 (2) less than 30 cm^2
(3) more than 30 cm^2 (4) 31.25 cm^2

40. The area and base of a right angled triangle are 36 sq. cm and 9 cm. respectively, the length of the perpendicular is
(1) 8 cm. (2) 4 cm.
(3) 16 cm. (4) 32 cm.

41. The parallel sides of a trapezium are 32m and 20m and the distance between them is 15m, then the area of the trapezium is
(1) 290 sq. m (2) 390 sq. cm
(3) 390 sq. m (4) 400 sq. m

Exercise 2

MCQ Based Questions

DIRECTIONS ***(Qs. 1 to 10)*** **:** *This section contains multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) out of which only one is correct.*

1. A petrol tank is a cylinder of base diameter 21 cm. and length 18 cm fitted with conical ends each of the axis length 9 cm. Determine the capacity of the tank ?
(1) 8136 cm^3 (2) 8163 cm^3
(3) 8316 cm^3 (4) 8631 cm^3

2. If the height and radius of a cone are doubled, then the volume of the cone becomes
(1) 2 times (2) 4 times
(3) 8 times (4) 12 times

3. If two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1, then the ratio of their volumes is
(1) 1 : 3 (2) 3 : 1
(3) 3 : 2 (4) 2 : 3

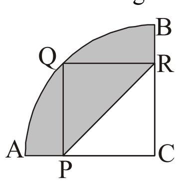
4. If a solid of one shape is converted to another, then the surface area of the new solid
(1) remains same (2) increases
(3) decreases (4) can't say

5. If a solid sphere of radius 6 cm is melted and drawn into a wire of radius 0.2 cm, then the length of the wire is
(1) 72 m (2) 75 m
(3) 72 cm (4) 75 cm

6. If a sphere and a cube have equal surface areas, then the ratio of the diameter of the sphere to the edge of the cube is
(1) 1 : 2 (2) 2 : 1
(3) $\sqrt{\pi} : \sqrt{6}$ (4) $\sqrt{6} : \sqrt{\pi}$

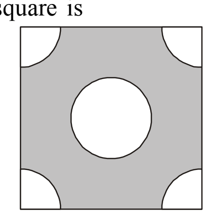
7. If a cone, a hemisphere and a cylinder have equal bases and have same height, then the ratio of their volumes is
(1) 1 : 3: 2 (2) 2 : 3 : 1
(3) 2 : 1: 3 (4) 1 : 2 : 3

8. The quarter-circle shown below has centre C and radius 10 units. If the perimeter of rectangle CPQR is 26 units,



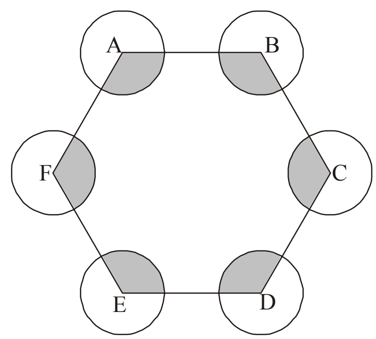
then the perimeter of the shaded region O is
(1) $5\pi + 17$ (2) $5\pi + 20$
(3) $5\pi + 10$ (4) $5\pi + 27$

9. From each corner of a square of side 4 cm; a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the figure. Then the area of the remaining portion of the square is



(1) $\frac{66}{7} cm^2$ (2) $\frac{68}{7} cm^2$
(3) $\frac{64}{7} cm^2$ (4) $\frac{62}{7} cm^2$

10. ABCDEF is any hexagon with different vertices A, B, C, D, E and F as the centres of circles with same radius r are drawn. The area of the shaded portion is



(1) πr^2 (2) $2\pi r^2$
(3) $3\pi r^2$ (4) $4\pi r^2$

Matching Based MCQ

DIRECTIONS ***(Qs. 11 to 13)*** **:** *Match the Column-I with Column-II and select the correct answer given below the columns.*

11.

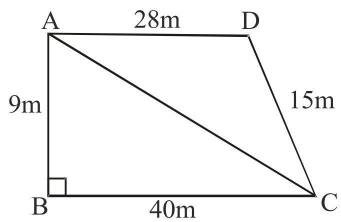
	Column-I		Column-II
(A)	A.................... has three dimensions: length, breadth and height.	(p)	Centre
(B)	.... is measured in cubic units	(q)	Surface Area
(C)	.... is measured in square units.	(r)	radius

(D) In hollow right circular cone, the slant height of the cone is equal to the .... of the circle. (s) Solid

(E) In hollow right circular cone, the vertex of the cone is the .... of the circle. (t) volume

(1) A → (s); B → (t); C → (r); D → (q); E → (p)
(2) A → (t); B → (s); C → (q); D → (r); E → (p)
(3) A → (s); B → (t); C → (q); D → (r); E → (p)
(4) A → (t); B → (s); C → (r); D → (q); E → (p)

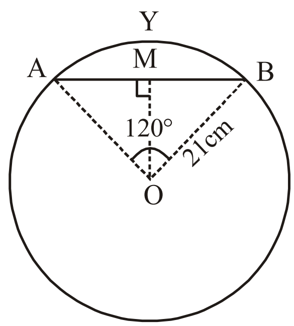
12.



Column I		Column II
(A) Area of Δ ABC	(p)	41
(B) Area of Δ ACD	(q)	126
(C) Total area	(r)	306
(D) AC	(s)	180

(1) A → (s); B → (r); C → (q); D → (p)
(2) A → (s); B → (q); C → (r); D → (p)
(3) A → (q); B → (s); C → (p); D → (r)
(4) A → (q); B → (p); C → (s); D → (r)

13.



Column I		Column II
(A) Area of segment AYB	(p)	$\frac{441}{4}\sqrt{3}$
(B) Area of sector OAYB	(q)	$\frac{21}{4}(88-21\sqrt{3})$
(C) Area of Δ OAB	(r)	462
(D) OM	(s)	21/2

(1) A → (q); B → (p); C → (r); D → (s)
(2) A → (p); B → (q); C → (s); D → (r)
(3) A → (p); B → (s); C → (q); D → (r)
(4) A → (q); B → (r); C → (p); D → (s)

Statement Based MCQ

14. Consider the following statements :
(I) For a hemisphere total surface area = $2\pi r^2$.
(II) The surface areas of a sphere and a cube are equal. Then the ratio of the volumes of the sphere and the cube is 15 : 11.
(III) In hollow right circular cone, curved surface area of the cone is equal to the area of the sector.
(IV) If the volume and the surface area of a solid sphere are numerically equal, then its radius is also equal to π.
Which of these statement(s) is/are correct?
(1) (I) and (II) (2) (I) and (III)
(3) only (II) (4) only (III)

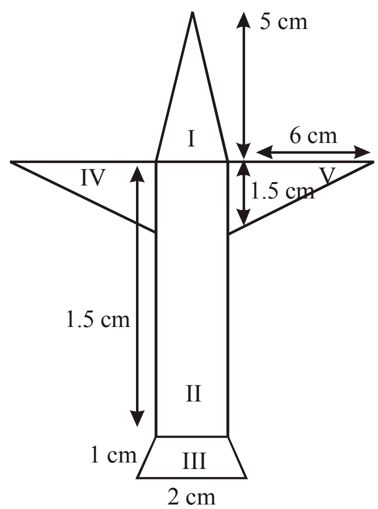
15. Consider the following statements :
(I) Volume of the solid is measured in cubic units.
(II) Area is the length of the boundary of a closed figure.
(III) Area is the total surface covered by a closed figure.
(IV) The volume of sphere of diameter is $\frac{\pi d^3}{6}$.
Which of these statement(s) is/are not correct?
(1) (I) and (II) (2) (I) and (III)
(3) only (II) (4) only (III)

Passage Based MCQ

DIRECTIONS (Qs. 16 to 28) : Read the passage(s) given below and answer the questions that follow.

PASSAGE - 1

Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used.

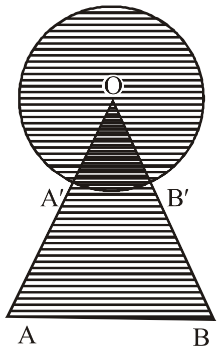


16. Area of region I is
(1) 2.5 cm^2 (2) 2 cm^2
(3) 5 cm^2 (4) 3 cm^2

17. Area of region II is
(1) 6 cm^2 (2) 5 cm^2
(3) 6.5 cm^2 (4) 7 cm^2

18. Area of region III is
(1) 1 cm^2 (2) 3 cm^2
(3) 2 cm^2 (4) 1.3 cm^2

PASSAGE - 2



In the above given figure, *a* circular arc of radius 6 cm has been drawn with vertex *O* of an equilateral triangle *OAB* of side 12 cm as centre.

19. The area of the sector $A'OB'$ is
    (1) 6π cm^2 (2) π cm^2
    (3) 6 cm^2 (4) $(6+\pi)$ cm^2
20. The area of the shaded region is
    (1) 156.522 cm^2 (2) 156.552 cm^2
    (3) 165.552 cm^2 (4) 561.552 cm^2

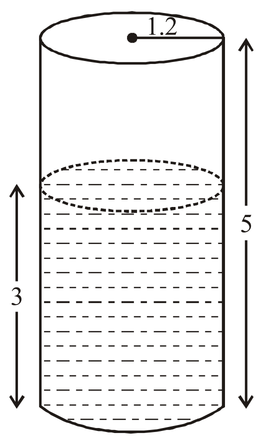
PASSAGE - 3

A test tube is in the form of a right circular cylinder, surmounted by a cone. The diameter of the cylinder is 24 m. The height of the cylindrical portion is 11m, while the vertex of the cone is 16 m above the ground.

21. The curved surface area of the cylindrical portion is
    (1) $(246\pi)m^2$ (2) $(264\pi)m^2$
    (3) $(426\pi)m^2$ (4) $(462\pi)m^2$
22. The slant height of the cone is
    (1) 31 *m* (2) 25 *m*
    (3) 13 *m* (4) 5 *m*
23. The area of the canvas required for the tent is
    (1) 1320 m^2 (2) 3120 m^2
    (3) 2130 m^2 (4) 1230 m^2

PASSAGE - 4

A well is a vertical open cylinder of radius 1.2 m and height 5 m. The well contains water to a depth of 3 m.(use : π = 3.142)



24. The total internal area of the curved surface of the well and the bottom of the well is
    (1) 42.2 to 42.25 m^2 (2) 40.2 to 40.25 m^2
    (3) 24.2 to 24.25 m^2 (4) 42.25 to 42.00 m^2
25. The volume of water in the well, is (1m^3 = 1000 litres)
    (1) 13560 (2) 31560
    (3) 53160 (4) 51360

PASSAGE - 5

Wood is required to make a closed box of external length 1.2 m, breadth 90 cm and height 72 cm. The thickness of the walls of the box being 2 cm throughout.

26. The Capacity of the box is
    (1) 678638 cm^3 (2) 678368 cm^3
    (3) 768368 cm^3 (4) 768386 cm^3
27. The Volume of the wood used, is
    (1) 99232 cm^3 (2) 99322 cm^3
    (3) 99223 cm^3 (4) 92932 cm^3
28. The total cost of the wood at the rate of 20 paise per cm^3 is
    (1) 19864.40 (2) 19684.40
    (3) 19486.40 (4) 19846.40

Assertion Reason Based MCQ

DIRECTIONS (Qs. 29 to 36) : *Following questions consist of two statements, one labelled as the* ***'Assertion (A)'*** *and the other as* ***'Reason' (R)****. You are to examine these two statements carefully and select the answer to these items using the code given below.*

Code :

(1) Both A and R are individually true and R is the correct explanation of A:
(2) Both A and R are individually true but R is not the correct explanation of A.
(3) A is true but R is false
(4) A is false but R is true.

29. **Assertion:** An edge of a cube measures *r* cm. If the largest possible right circular cone is cut out of this cube, then the volume of the cone is $\frac{1}{6}\pi r^3$.
    Reason: Volume of the cone is given by $\frac{1}{3}\pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cone.
30. **Assertion:** The area of an equilateral triangle is $16\sqrt{3}$ cm^2 whose each side is 8 cm.
    Reason: Area of an equilateral triangle is given by $\frac{\sqrt{3}}{4}(\text{side})^2$.
31. **Assertion:** In a cylinder, if radius is halved and height is doubled, the volume will be halved.
    Reason: In a cylinder, radius is doubled and height is halved, curved surface area will be same.
32. **Assertion:** If a ball in the shape of a sphere has a surface area of 221.76 cm^2, then its diameter is 8.4 cm.
    Reason: If the radius of the sphere be r, then surface area , $S = 4\pi r^2$, i. e. $r = \frac{1}{2}\sqrt{\frac{S}{\pi}}$

33. **Assertion:** No. of spherical balls that can be made out of a solid cube of lead whose edge is 44 cm, each ball being 4 cm. in diameter, is 2541

Reason : Number of balls = $\dfrac{Volume\ of\ one\ ball}{Volume\ of\ lead}$

34. **Assertion :** Area of the triangle having three sides 4 m, 6 m and 8 m is 135 sq.m.

Reason : If a, b, c are the lengths of the sides of a triangle then

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{a+b+c}{2}$$

35. **Assertion :** If the circumference of a circle is 176 cm, then its radius is 28 cm.

Reason : Circumference = $2\pi \times$ radius

36. **Assertion :** If the outer and inner diameter of a circular path is $10m$ and $6m$, then area of the path is $16\pi\ m^2$.

Reason : If R and r be the radius of outer and inner circular path respectively then area of path = $\pi\ (R^2 - r^2)$

Correct Definition Based MCQ

37. Cuboid is
(1) a solid bounded by six rectangular faces.
(2) a solid bounded by six square faces.
(3) a solid bounded by four square faces.
(4) a solid bounded by four rectangular faces.

38. A cube is
(1) a cuboid whose length, breadth and height are all equal.
(2) a cuboid whose length, breadth and height are all not equal.
(3) a solid bounded by four rectangular faces.
(4) a solid bounded by four square faces.

39. A perimeter of a circle is
(1) the width of its boundary.
(2) the circumference of the c ircle.
(3) the length of its boundary.
(4) the surface area of its boundary.

Feature Based MCQ

40. On the basis of following features identify the type of solid.
I. Total surface area = $6a^2$ sq. unit
II. Length of diagonal = $\sqrt{3}$ a unit.
III. Volume = a^3
Where 'a' is side of the solid.
(1) Cube (2) Cuboid
(3) Cylinder (4) Cone

41. On the basis of the following identify the type of triangle.
I. Area = $\dfrac{b}{4}\sqrt{4a^2 - b^2}$
II. Two sides are equal.
III. Semi-perimeter = $\dfrac{2a+b}{2}$
where 'a' and 'b' are the sides of the triangle.
(1) Scalene triangle (2) Isosceles triangle
(3) Equilateral triangle (4) Both (1) and (2)

42. On the basis of the following identify the type of solid.
I. Volume = $\dfrac{\pi h}{3}[R^2 + r^2 + Rr]$ cubic unit
II. Curved surface area = $\pi\ell$ (R + r) sq. units where $\ell^2 = h^2 + (R - r)^2$
III. Total surface area = $\pi R^2 + \pi r^2 + \pi\ell$ (R + r) sq. units
(1) Cone (2) Cylinder
(3) Frustum of a cone (4) Hollow cylinder

Hints & SOLUTIONS

Exercise I

1. **(4)** **2.** **(2)** **3.** **(1)**
4. **(1)** **5.** **(4)** **6.** **(2)**

7. **(3)** Let radius of base of cylinder = r cm

$\therefore 346.5 = \pi r^2$ (Base area of cylinder)

$\Rightarrow r^2 = \dfrac{346.5 \times 7}{22} \Rightarrow r = \sqrt{\dfrac{346.5 \times 7}{22}}$

$\Rightarrow r = \sqrt{110.25} \Rightarrow r = 10.5$ cm

$\therefore$ Total surface area of cylinder

$= 2\pi r(r+h) = 2\left(\dfrac{22}{7}\right) \times 10.5\ (10.5 + 24)$

$= 2 \times 22 \times 1.5 \times 34.5 = 2277$ Sq. cm.

8. **(2)** Volume of cylinder = $\pi r^2 h$

$448\pi = \pi r^2 \times 7$

$\Rightarrow r^2 = \dfrac{448}{7} \Rightarrow r = \sqrt{\dfrac{448}{7}} \Rightarrow r = \sqrt{64} \Rightarrow r = 8$ cm.

$\therefore$ L. S. A or C. S. A = $2\pi rh = 2 \times \dfrac{22}{7} \times 8 \times 7 = 352$ cm^2

9. **(2)** Let radii of two cones be 3x and 5x.

$$\frac{\text{Volume of Cone I}}{\text{Volume of Cone II}} = \frac{\frac{1}{3}\pi(3x)^2 h}{\frac{1}{3}\pi(5x)^2 h} = \frac{9}{25}$$

$\therefore$ Ratio of their volume = 9 : 25

10. **(1)** $r = \frac{1.4}{2} = 0.7\text{m. and } h = 2\text{m}$

Area covered = C.S.A × number of revolutions

$= 2\pi rh \times 5 = 10\,\pi rh$

$\Rightarrow 10\left(\frac{22}{7}\right)(0.7)(2) = 44\text{ m}^2$

11. **(2)** Number of balls = $\frac{\text{Volume of lead}}{\text{Volume of a ball}} = \frac{(44)^3}{4/3\pi r^3}$

[∵ volume of a cube = $(\text{edge})^3$]

$= \frac{44\times44\times44}{\frac{4}{3}\times\frac{22}{7}\times(2)^3}$ [∵ Diameter of a ball = 4 cm.

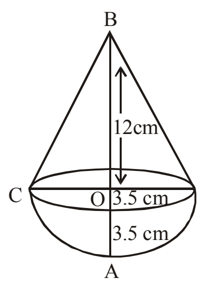
∴ radius of a ball = 2 cm.]

$= \frac{44\times44\times44\times21}{4\times22\times8} = 2541$

12. **(3)** Volume of sphere = $\frac{4}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^3 = 179.66\text{ cm}^3$

13. **(2)** Curved surface area of hemisphere,

$S_1 = 2\pi r^2 = 2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2} = 77\text{ cm}^2,$



For the conical part, r = 3.5 cm, h = 15.5 – 3.5 = 12 cm.

Now, Slant height, $\ell = \sqrt{r^2+h^2}$

$= \sqrt{(3.5)^2+(12)^2} = \sqrt{12.25+144} = \sqrt{156.25} = 12.5\text{ cm}$

Curved surface area of cone ,

$S_2 = \pi r\ell = \frac{22}{7}\times\frac{7}{2}\times12.5 = 137.5\text{ cm}^2.$

∴ Total surface area of toy = $S_1 + S_2 = 77 + 137.5 = 214.5\text{ cm}^2$.

14. **(2)** [Hint. Edge of cube = $\frac{20}{4}$ cm = 5 cm,

Surface area = $6 \times 5^2\text{ cm}^2 = 180\text{ cm}^2$]

15. **(3)**

16. **(1)** Area of sector = $\frac{42}{360}\times\pi r^2 = \frac{42}{360}\times\frac{22}{7}\times6\times6 = 13.2\text{ m}^2.$

17. **(1)** R = 15 cm, r = 13 cm.
where R is radius of larger circle and
r is radius of smaller circle
Area of the circulating ring = π (R + r) (R – r)

$= \frac{22}{7}(15+13)\times(15-13) = \frac{22}{7}\times28\times2 = 176$ sq. cm.

18. **(3)** Volume of sphere = $\frac{4}{3}\pi r^3$ and it's surface area = $4\pi r^2$.

Ratio = $\frac{4}{3}\pi r^3 : 4\pi r^2 = \frac{r}{3}:1$

But r = 1 unit (unit radius)

∴ ratio = $\frac{1}{3}$: 1 or 1 : 3.

19. **(1)** Curved surface area = πrl.

New curved surface area = $\pi r\left(l+\frac{Pl}{100}\right)$

Percentage increase = $\frac{\pi r\left(l+\frac{Pl}{100}\right)-\pi rl}{\pi rl}\times100 = P\%$

20. **(3)** Volume of water in hollow spherical ball = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\frac{22}{7}\times64\text{ cm}^3$ [r = 4 cm]

Half the water = $\frac{44\times64}{21}\text{cm}^3$.

Volume of cone = $\frac{1}{3}\pi r^2h = \frac{1}{3}\times\frac{22}{7}\times r^2\times2 = \frac{44r^2}{21}$

Hence, $\frac{44\times64}{21} = \frac{44r^2}{21}$ or $r^2 = 64$ or r = 8 cm.

21. **(3)** Clearly, radius of base of cone B = 2 cm

∴ Height of cone = $\sqrt{3^2-2^2} = \sqrt{5}$ cm

Let the number of spheres = n.
n × volume of 1 sphere = volume of cylinder A
+ volume of cone B

$n \times \frac{4}{3}\pi(1)^3 = \pi(2)^2\times6.25+\frac{1}{3}\pi(2)^2\times(\sqrt{5})$

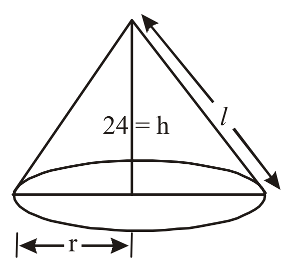
$\frac{4}{3}n = 25+\frac{4\sqrt{5}}{3}$

$\Rightarrow n = \frac{3}{4}\times\left(\frac{75+4\sqrt{5}}{3}\right) = 20.99 \approx 21.$

22. **(3)** The length of longest rod = diagonal of the cube

$= a\sqrt{3} = 4\times1.732$ m [a = 4 m] = 6.928 m.

23. **(3)**



Let radius of base and slant height be r and *l* respectively.
Vertical height, h = 24 cm

Volume = $\frac{1}{3}\pi r^2h = 1232$

$\Rightarrow \quad r^2 = \frac{1232 \times 3}{24} \times \frac{7}{22}$

$\Rightarrow \quad r^2 = 49 \quad \Rightarrow r = 7$ cm

Slant height ,

$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{625} = 25\,\text{cm}$

Area of curved surface = $\pi r l = \frac{22}{7} \times 7 \times 25 = 550\,\text{cm}^2$.

24. **(3)** Let the radii of first and second cylinder be 2x and 3x; and let their heights be 5y and 3y, respectively.

$$\frac{\text{Volume of first cylinder}}{\text{Volume of second cylinder}} = \frac{\pi \times (2x)^2 \times 5y}{\pi \times (3x)^2 \times 3y} \quad [\because V = \pi r^2 h]$$

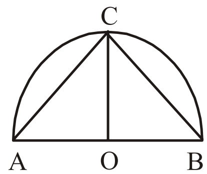
$$= \frac{20}{27}.$$

25. **(3)** $\pi r^2 = 154 \Rightarrow r = 7$

Perimeter = $2\pi r = 44$ cm

26. **(1)** Largest triangle will be a right angled isosceles triangle ABC with OA = OB = OC and OC $\perp$ AB.

Area = $\frac{1}{2}(2r)(r) = r^2$.



27. **(2)** **28.** **(2)** **29.** **(1)**
30. **(1)** **31.** **(2)**

32. **(4)** holds. $\left[\because \text{Area} = \frac{p}{360} \times \pi R^2 = \frac{p}{720} \times (2\pi R^2)\right]$

33. **(4)** holds

Let ABCD be a square with side = 6 cm. Then the radius of the circle touches the square = 3 cm.

Its area $= \pi r^2 = \pi(3)^2 = 9\pi\ \text{cm}^2$

34. **(4)** [**Hint :** $C = 2\pi r \Rightarrow \pi r = \frac{C}{2}$, $A = \pi r^2 = \pi r . r = \frac{C}{2} r$.]

35. **(3)** [**Hint :** Perimeter $= \frac{1}{4} \times 2\pi r + 2r$

$= \left(\frac{1}{2} \times \frac{22}{7} \times 7 + 2 \times 7\right)$ cm = 25 cm.

36. **(3)** **37.** **(2)** **38.** **(2)**
39. **(4)** **40.** **(1)** **41.** **(3)**

Exercise 2

1. **(3)** $V = \pi\left(\frac{21}{2}\right)^2(18) + 2\left[\frac{1}{3}\pi\left(\frac{21}{2}\right)^2(9)\right] = 8316\,\text{cm}^3$

2. **(3)** [**Hint.** New volume = $\frac{1}{3}\pi(2r)^2$ (2h) = $8 \times \frac{1}{3}\pi r^2 h$.]

3. **(2)** Let the heights of two cones be h and 3h, and their radii be 3r and r respectively.

$$\frac{\text{Volume of first cone}}{\text{Volume of second cone}} = \frac{\frac{1}{3}\pi(3r)^2 h}{\frac{1}{3}\pi r^2 \times 3h} = \frac{3}{1}\ ]$$

4. **(4)**

5. **(3)** [**Hint.** Let l cm be the length of wire, then]

$\pi \times \left(\frac{2}{10}\right)^2 \times l = \frac{4}{3}\pi \times 6^3 \Rightarrow l = \frac{4}{3} \times 216 \times 25$ cm = 72 × 100 cm

6. **(4)** [**Hint.** Let the diameter of the sphere be d units and the edge of the cube be a units, then $4\pi\left(\frac{d}{2}\right)^2 = 6a^2 \Rightarrow \frac{d^2}{a^2} = \frac{6}{\pi}$

$\Rightarrow \frac{d}{a} = \frac{\sqrt{6}}{\sqrt{\pi}}$.]

7. **(4)** [**Hint.** Let radius of each be r units, then the height of hemisphere = r units.

So, the height of cone = r units = height of cylinder

$= \frac{1}{3}\pi r^2 \times r : \frac{2}{3}\pi r^3 : \pi r^2 \times r = 1 : 2 : 3$]

8. **(1)** **9.** **(2)** **10.** **(2)**
11. **(3)** **12.** **(2)** **13.** **(4)**

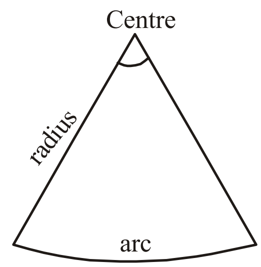
14. **(4)** (I) **False.** (Total surface Area = $3\pi r^2$)

(II) **False.** Required ratio $= \frac{\frac{4}{3}\pi r^3}{a^3}$

But $4\pi r^2 = 6a^2 \Rightarrow \frac{r}{a} = \sqrt{\frac{3}{2\pi}}$

Hence, ratio is $\sqrt{231}$: 11

(III) **True**



(IV) **False.** According to the question,

We have $\frac{4}{3}\pi r^3 = 4\pi r^2 \Rightarrow r = 3$ units.

15. **(3)** (I) True; (II) False; (III) True; (IV) True

16. **(1)** **17.** **(3)** **18.** **(4)**

16-18. For Triangular Area I, we have

$a = 5$ cm, $b = 5$ cm, $c = 1$ cm

as the sides of the triangle

$\therefore \quad s = \frac{a+b+c}{2}, = \frac{5+5+1}{2} = \frac{11}{2} = 5.5$ cm ,

$\therefore \quad$ Area of shaded region I = $\sqrt{s(s-a)(s-b)(s-c)}$,

$= \sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$

$= \sqrt{5.5(.5)(.5)(4.5)} = (.5)\sqrt{(5.5)(4.5)}$

$= (.5)\sqrt{(.5)(11)(.5)(9)} = (.5)(.5)(3)\sqrt{11}$

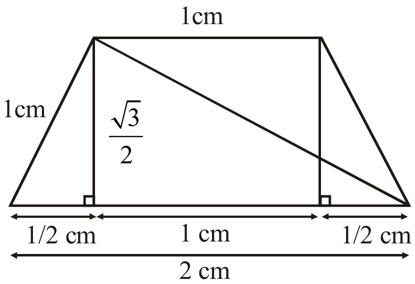
$= 0.75\sqrt{11} = 0.75(3.3)$ (approx.)

$= 2.5\ \text{cm}^2$ (approx.),

Area of shaded region II = Area of rectangle

$= 6.5 \times 1 = 6.5\ cm^2$

For Area of shaded region III draw the figure



Area of region III

$$= \frac{1}{2}\times 2\times\frac{\sqrt{3}}{2}+\frac{1}{2}\times 1\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{4}$$

$$= \frac{3\sqrt{3}}{4}=\frac{3\times 1.732}{4}\text{ (Approx.)}$$

$$= \frac{5.196}{4} = 1.3\ cm^2\text{ (approx.)}$$

Thus, area of region IV = Area of right angled Δ

$$= \frac{6\times 1.5}{2} = 4.5\ cm^2$$

Similarly, Area $V = \dfrac{6\times 1.5}{2} = 4.5\ cm^2$

∴ Total area of the paper used

= Area I + Area II + Area III + Area IV + Area V

$= 2.5\ cm^2 + 6.5\ cm^2 + 1.3\ cm^2 + 4.5\ cm^2 + 4.5\ cm^2$

$= 19.3\ cm^2$. (approx)

19. **(1)** $\angle AOB = 60°$

Area of the sector $A'OB' = \dfrac{60}{360}\pi(6)^2 = 6\pi\ cm^2$

20. **(2)** Area of shaded region

= area of circle + area of ΔOAB – area of sector $A'OB'$

$$= \pi(6)^2+\frac{\sqrt{3}}{4}(12)^2-6\pi = 36\pi+\frac{\sqrt{3}}{4}(144)-6\pi$$

$= 94.2 + 62.352 = 156.552\ cm^2$.

21. **(2)** $R = \text{Radius} = \dfrac{24}{2} = 12$ cm.

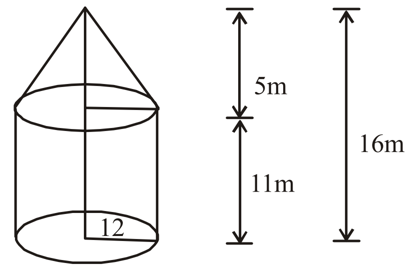
H = Height = 11 m

Curved surface area of the cylindrical portion = $2\pi RH$

$= 2\pi\,(12)(11) = (264\,\pi)\ m^2$ ·

22. **(3)** h = Height of the cylindrical portion = $16 - 11 = 5\ m$

Slant height, $l = \sqrt{h^2+R^2} = \sqrt{25+144} = 13\ m$.



23. **(1)** Area of canvas required for the tent = curved surface area of the cylindrical portion + curved surface area of the cone curved surface area of the cone = $\pi Rl = \pi\,(12)\,(13) = (156\pi)\ m^2$

Hence, Area of canvas = $(264\pi + 156\pi)\ m^2$

$= (420\pi -)\ m^2 = 420\times\dfrac{22}{7} = 1320\ m^2$.

24. **(1)** Required area = $2\pi \times 1.2 \times 5 + \pi \times (1.2)^2 = 42.2$ to $42.25\ m^2$

25. **(1)** Required volume = $\pi \times (1.2)^2 \times 3$

26. **(2)** Required Capacity = Internal Volume = $116 \times 86 \times 68$

27. **(2)** Volume of the wood used

= external volume – internal volume

$= (120 \times 90 \times 72 - 678368)\ cm^2$

28. **(4)** Cost of wood = 20 paise per cm^3

∴ Total cost $= \dfrac{20}{100}\times 99232 = 19846.40$

29. **(4)** Assertion is false but reason is correct.

30. **(1)** Area of triangle $= \dfrac{\sqrt{3}}{4}(8)^2\ cm^2 = \dfrac{\sqrt{3}}{4}\times 8\times 8 = 16\sqrt{3}\ cm^2$.

31. **(2)** Both Assertion and Reason is correct.

32. **(1)** Both assertion and reason are correct and reason is the correct explanation of the assertion.

33. **(3)** Assertion is correct but reason is not correct.

34. **(3)** Assertion

$$s = \frac{4+6+8}{2}=\frac{18}{2}=9$$

∴ Area $= \sqrt{9(5)(3)(1)} = \sqrt{135}$

So, Assertion is incorrect. Reason is correct.

35. **(1)** Both assertion and reason are correct. Also Reason is the correct explanation of the assertion.

$$C = 2\times\frac{22}{7}\times r = 176$$

$$r = \frac{176\times 7}{2\times 22} = 28\text{ cm.}$$

36. **(1)** Both assertion and reason are correct. Also, Reason is the correct explanation of the assertion.

Area of the path $= \pi\left[\left(\dfrac{10}{2}\right)^2-\left(\dfrac{6}{2}\right)^2\right] = \pi(25-9) = 16\pi$

37. **(1)** **38.** **(1)** **39.** **(2)**

40. **(1)** **41.** **(2)** **42.** **(3)**