

## MODE OF HEAT TRANSFER ...

Differences between conduction convection and radiation :

<u>Conduction</u>	<u>Convection</u>	<u>Radiation</u>
Heat transferred due to temperature difference between two ends of solid and liquid (Hg)	Heat transferred due to density difference between two ends in liquid and gases	Heat transferred due to electro-magnetic waves and propagate by all medium.
Heat transferred by the vibrational motion of medium particle and free electron (no actual motion)	Heat transferred by the actual motion of the medium particle from one place to another place	Medium is not required for the transmission it can transmit in vacuum also.
slowest process	slow process	Fastest Process ( $3 \times 10^8$ m/sec)
Irregular path	Irregular path	straight line path (similar to light radiation)

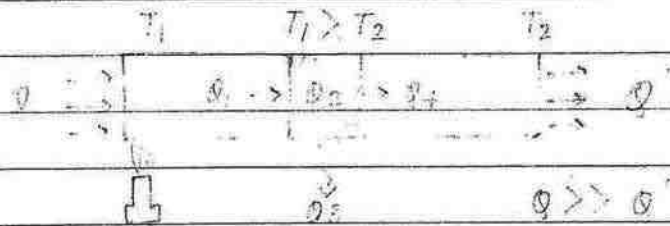
## 1. Conduction:

i) Energy is transferred due to temperature difference between the two ends of solid by the vibrational motion of the medium particle

ii) Heat transfer always from high temperature side to low temperature side from one particle to another particle.

Two states during conduction:

i) Variable state:



$$Q_1 = Q_2 + Q_3 + Q_4$$

and

$$Q_1 = Q_2 + Q_3 \quad (\text{when } Q_4 = 0 \text{ i.e. insulated})$$

where,

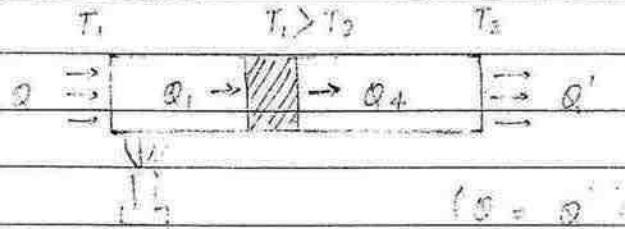
$Q_1$  = heat taken by the previous section

$Q_2$  = heat absorbed by the system

$Q_3$  = heat emitted by cylindrical surface in the form of radiation or convection.

$Q_4$  = heat transferred to the next system

ii) Steady State state:



$$Q_2 = Q_3 = 0$$

$$\therefore Q = Q_1 = Q_4 = Q'$$

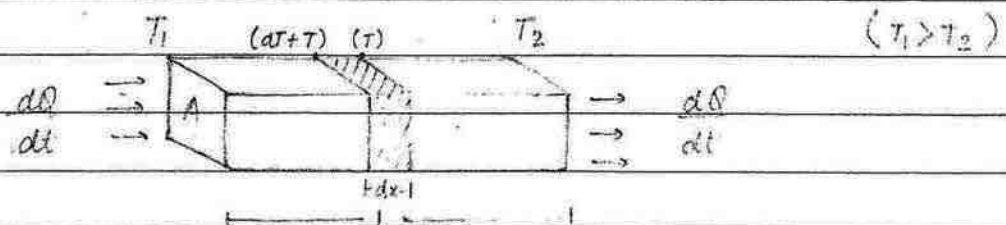
$$\therefore E \propto I \propto \frac{1}{d^2} \text{ (intensity)}$$

\* In steady state heat absorption by any section is zero

\* Temperature of different cross section is different and decreases in the direction of heat flow.

\* Temperature of every one cross section is different remains constant but not same

Rate of heat flow:



$$\therefore \frac{dQ}{dt} \propto A$$

$$\frac{dQ}{dt} \propto K$$

$$\frac{dQ}{dt} \propto \frac{-dT}{dx} \quad \text{rate of } \downarrow \text{ in temp per unit length}$$

$$\therefore \frac{dQ}{dt} = -KA \frac{dT}{dx}$$

For uniform rod,

$$\frac{dQ}{dt} = \frac{-KA(T_2 - T_1)}{L}$$

$$* \quad \therefore \frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{L} = \frac{T_1 - T_2}{R} \quad \left[ \begin{array}{l} \text{M.P.T} \\ \because R = \frac{L}{KA} \end{array} \right]$$

where,  $A$  = cross section area

$K$  = coefficient of thermal conductivity  
(depends on property of material only)

$L$  = length of the rod.

Order of  $K$  in metals:

$$K_{\text{silver}} > K_{\text{copper}} > K_{\text{gold}} > K_{\text{aluminium}} \\ Ag > Cu > Au > Al.$$

Order of  $K$  in phase:

$$K_{\text{solid}} > K_{\text{liquid}} > K_{\text{gas}}$$

$$\text{Unit of } K : \frac{\text{Joule}}{\text{m-sec K}} \quad \text{or} \quad \frac{\text{watt}}{\text{mK}} \quad \left[ \begin{array}{l} \because K = \frac{J \cdot s^{-1}}{J \cdot A \cdot T} \end{array} \right]$$

$$\text{dimensions: } [M^1 L^1 T^{-3} K^{-1}]$$

temperature gradient:  $\frac{dT}{dx}$

Rate of fall in temperature with distance.

Comparison with electric current:



$$V_1 - V_2 = i \times R \quad \text{[ } \because V = IR \text{ ]} \quad T_1 - T_2 = \frac{dQ}{dt} \times \frac{L}{KA}$$

•  $\therefore V_1 - V_2$  (voltage difference)       $\therefore T_1 - T_2$  (Temperature difference)

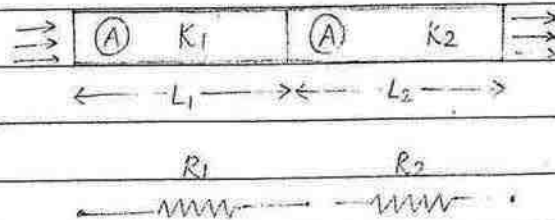
•  $i$        $\frac{dQ}{dt}$  = thermal current

•  $R = \frac{\rho l}{A}$        $R = \frac{L}{KA}$  = thermal resistance

$$\therefore \frac{dQ}{dt} = \frac{T_1 - T_2}{R}$$

To calculate equivalent thermal conductivity.

i) series combination:



$$R_{eq} = R_1 + R_2$$

$$\therefore \frac{L_1 + L_2}{K_{eq} A} = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A}$$

- For same cross section area:

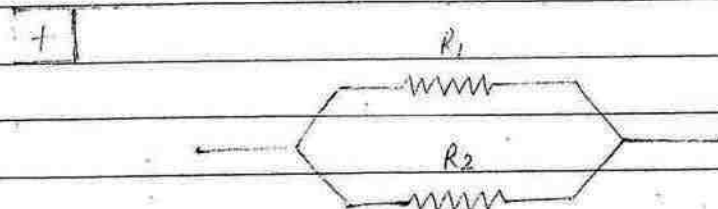
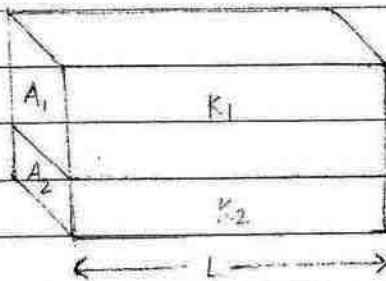
$$K_{eq} = \frac{L_1 + L_2}{\frac{L_1}{K_1} + \frac{L_2}{K_2}}$$

$$\therefore K_{eq} = \frac{\Sigma(L)}{\Sigma\left(\frac{L}{K}\right)}$$

- If  $L_1 = L_2 = L$  (same length):

$$K_{eq} = \frac{2K_1 K_2}{K_1 + K_2} \quad \text{Harmonic Mean (H.M)}$$

- ii) Parallel combination:



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore \frac{K_{eq} (A_1 + A_2)}{L} = \frac{K_1 A_1}{L} + \frac{K_2 A_2}{L}$$

$$\therefore K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

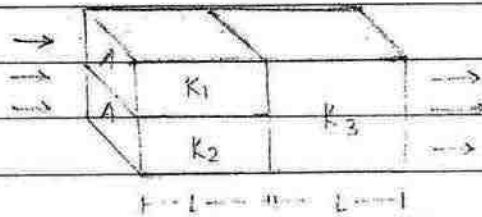
- For same length:

$$K_{eq} = \frac{\sum (KA)}{L} = A$$

- If  $A_1 = A_2 = A$  (same cross section area):

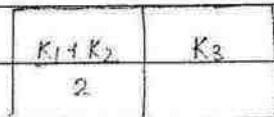
$$\therefore K_{eq} = \frac{K_1 + K_2}{2} \quad \text{Arithmetic Mean (AM)}$$

Q.1 Calculate thermal conductivity in the following combination:



$\therefore K_1$  and  $K_2$  are in parallel combination

$$\therefore K_{eq} = \frac{K_1 + K_2}{2} \quad \text{(A.M)}$$



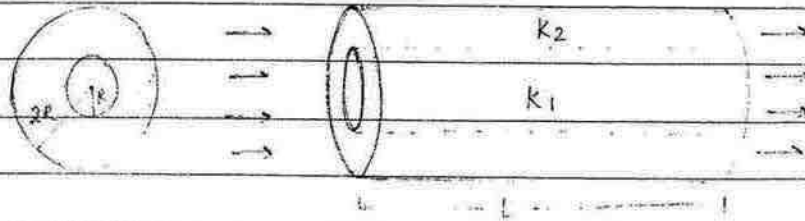
$\therefore \frac{K_1 + K_2}{2}$  and  $K_3$  are in series combination

$$\therefore K_{eq} = \frac{2 \left( \frac{K_1 + K_2}{2} \right) K_3}{2} \quad \text{(H.M)}$$

$$\frac{K_1 + K_2}{2} + K_3$$

$$= \frac{2 K_3 (K_1 + K_2)}{2 K_3 + K_1 + K_2}$$

Q.2 Calculate equivalent thermal conductivity in the following combination:



$$A_1 = \pi R^2$$

$$A_2 = \pi (2R)^2 - \pi R^2 \\ = 4\pi R^2 - \pi R^2 = 3\pi R^2$$

$\therefore K_1$  and  $K_2$  are in parallel combination

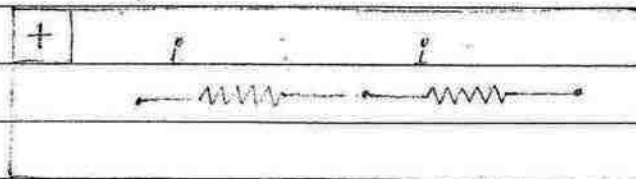
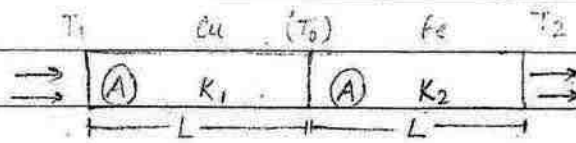
$$\therefore K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

$$= \frac{K_1 \pi R^2 + K_2 (3\pi R^2)}{\pi R^2 + 3\pi R^2}$$

$$= \frac{\pi R^2 (K_1 + 3K_2)}{\pi R^2 (1 + 3)}$$

$$= \frac{K_1 + 3K_2}{4}$$

To calculate junction temperature:



$$\therefore \left( \frac{dQ}{dt} \right)_{Cu} = \left( \frac{dQ}{dt} \right)_{Fe}$$



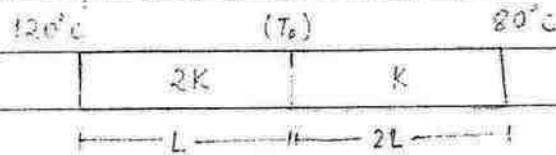
$\therefore$  same rate of heat transfer:

\*

$$\therefore \frac{K_1 A (T_1 - T_0)}{L_1} = \frac{K_2 A (T_0 - T_2)}{L_2}$$

Q.3

Calculate the junction temperature.



$$\therefore \left( \frac{dQ}{dt} \right)_L = \left( \frac{dQ}{dt} \right)_{2L}$$

$$\frac{2KA (120 - T_0)}{L} = \frac{KA (T_0 - 80)}{2L}$$

$$240 - 2T_0 = \frac{1}{2} (T_0 - 80)$$

$$480 - 4T_0 = T_0 - 80$$

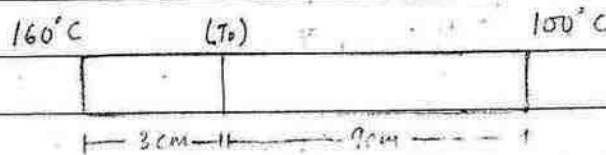
$$480 + 80 = 4T_0 + T_0$$

$$560 = 5T_0$$

$$\therefore T_0 = \frac{560}{5} = 112^\circ\text{C}$$

Q.4

Calculate junction temperature of uniform rod?



$$\therefore \left( \frac{dQ}{dt} \right)_3 = \left( \frac{dQ}{dt} \right)_9$$

$$= \frac{KA (160 - T_0)}{3} = \frac{KA (T_0 - 100)}{9}$$

$$= 3(160 - T_0) = \frac{KA}{9} (T_0 - 100)$$

$$480 - 3T_0 = T_0 - 100$$

$$480 + 100 = 3T_0 + T_0$$

$$580 = 4T_0$$

$$\therefore T_0 = \frac{580}{4} = 145^\circ\text{C}$$

OR

Short trick : Temperature gradient

$$\frac{dT}{dx} = \frac{160 - 100}{12}$$

$$= \frac{60}{12} = 5^\circ\text{C/cm}$$

$$\therefore \text{Decrease in temperature in 3 sec} = 3 \times 5 = 15^\circ\text{C}$$

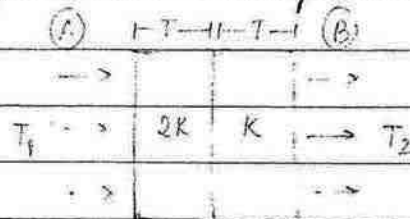
$\therefore$  Temperature after 3 sec =  $160^\circ - 15^\circ = 145^\circ\text{C}$

$$= 3 \times 5 = 15^\circ\text{C}$$

$\therefore$  Temperature after 3 sec =  $160^\circ - 15^\circ = 145^\circ\text{C}$

Q.5

Calculate junction temperature :



given :  $T_1 - T_2 = 36^\circ\text{C}$  . calculate  $T_1 - T_0$  .

$$\therefore \left(\frac{dq}{dt}\right)_A = \left(\frac{dq}{dt}\right)_B$$

$$= \frac{2KA}{T} (T_1 - T_0) = \frac{KA}{T} (T_0 - T_2)$$

$$2(T_1 - T_0) = T_0 - T_2$$

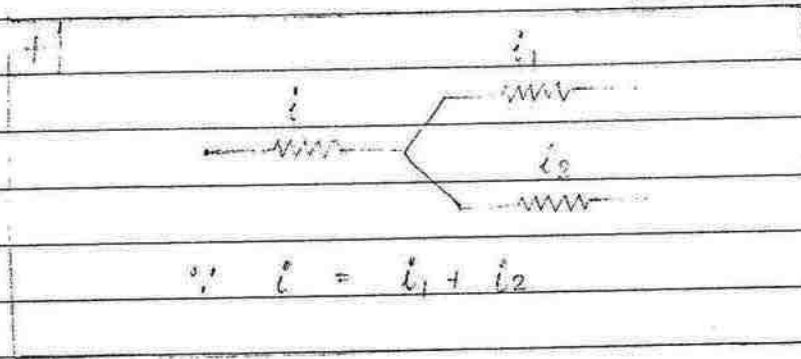
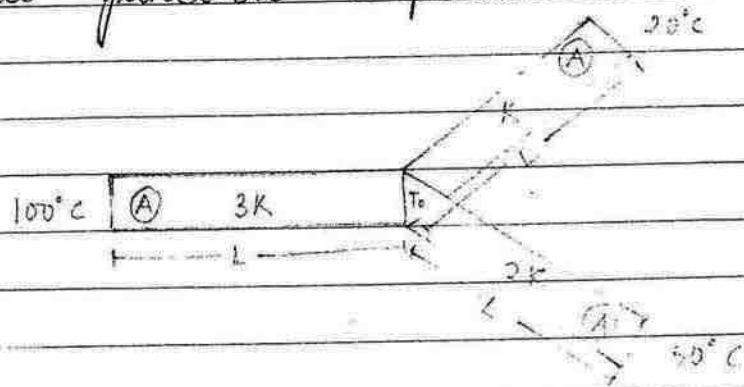
$$2T_1 - 2T_0 = T_0 - T_2 + 36 \quad (\text{given that } T_2 = T_1 - 36)$$

$$\therefore 2T_1 + T_1 = 2T_0 + T_0 + 36$$

$$3T_1 - 3T_0 = 36$$

$$\therefore T_1 - T_0 = 12^\circ\text{C}$$

Q. 6. Calculate junction temperature:



$$\therefore \left( \frac{dQ}{dt} \right)_{3K} = \left( \frac{dQ}{dt} \right)_K + \left( \frac{dQ}{dt} \right)_{2K}$$

$$\frac{3KA(100 - T_0)}{L} = \frac{KA(T_0 - 20)}{L} + \frac{2KA(T_0 - 50)}{L}$$

$$300 - 3T_0 = T_0 - 20 + 2T_0 - 100$$

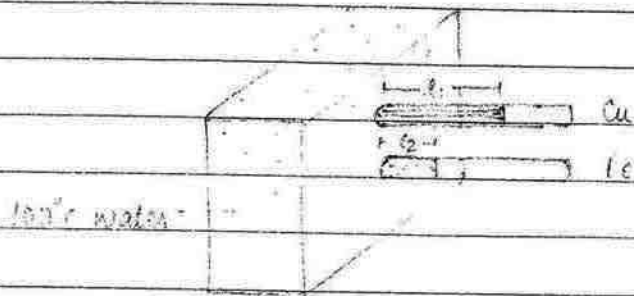
$$20 + 300 + 100 = T_0 - 20 + 2T_0 + 3T_0$$

$$420 = 6T_0$$

$$\therefore T_0 = \frac{420}{6} = 70^\circ\text{C}$$

Application of conduction:

- i) Ingen horse experiment: It is used to compare the conductivity of different material



$$\therefore \frac{dQ}{dt} \propto K$$

$$\therefore K \propto l^2$$

where  $l$  is the length of the melting wax.

$$\left( \frac{K_{Cu}}{K_{Fe}} \right) = \left( \frac{l_1}{l_2} \right)^2$$

\* This experiment has to be used, whenever there is a coating of wax.

ii) Weidemann - Franz law: At a given constant temperature, the ratio of thermal conductivity and electrical conductivity remains constant

At constant temperature,

$$\frac{K}{\sigma} = \text{constant}$$

$$\therefore K \propto \sigma$$

where  $K$  = thermal conductivity

$\sigma$  = electrical conductivity

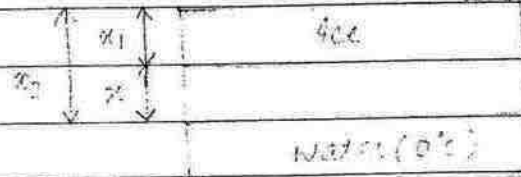
It means that good conductor of heat is also a good conductor of electricity.

iii) Growth of ice on water surface:  
MODULE 2 (Pg-166)

Initially thin layer of ice formed on the water surface is explained by radiation and then growth of ice is explained by conduction.

(low temperature)

Air ( $-7^{\circ}\text{C}$ )



(high temperature)

$$x_1 \xrightarrow{t} x_2$$

$$\therefore t = \frac{1}{2} \frac{\rho L x}{K T} (x_2^2 - x_1^2)$$

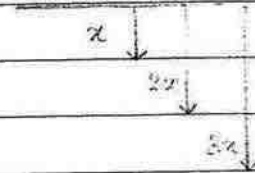
•  $t$  should be in  $^{\circ}\text{C}$

Case I:

$$0^{\circ} \xrightarrow{t_1} x$$

$$0^{\circ} \xrightarrow{t_2} 2x$$

$$0^{\circ} \xrightarrow{t_3} 3x$$



$$\therefore t \propto (x_2^2 - x_1^2)$$

$$t_1 : t_2 : t_3 = (x^2 - 0) : [(2x)^2 - 0] : [(3x)^2 - 0]$$

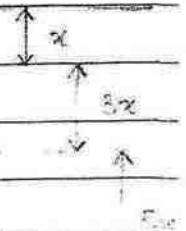
$$1 : 4 : 9$$

Case II:

$$0^{\circ} \longrightarrow x$$

$$x \longrightarrow 2x$$

$$2x \longrightarrow 3x$$

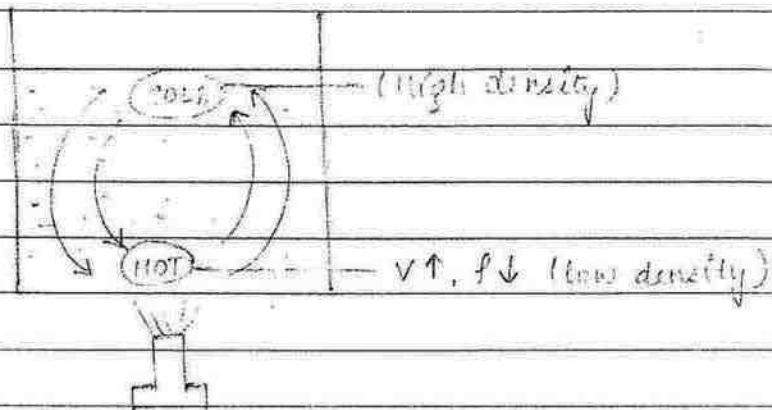


$$t_1 : t_2 : t_3 = (x^2 - 0) : [(2x)^2 - x^2] : [(3x)^2 - (2x)^2]$$

$$1 : 3 : 5$$

## 2. Convection:

- i) Heat is transferred due to density difference in liquid and gas is known as convection.
- ii) Heat transferred by the actual motion of medium particle from one place to another place.



- iii) Convection based on gravity: convection is not possible in zero gravity.

- Regions of zero gravity: orbiting satellite, centre of the earth, free falling lift.

Q. 7 In weightlessness condition, liquid can be heated by:

- i) Conduction (Bulk)
- ii) convection
- iii) Radiation (affects the surface - hot)
- iv) None of these

Ans) i) Conduction

Q-8 In weightlessness condition, liquid can be heated by:

- i) conduction
- ii) convection
- iii) radiation
- iv) liquid can't be heated

Ans) iv) liquid can't be heated

iv) In convection, heat propagate by temperature gradient exist in vertically upward direction not horizontal.

• A (convection + radiation)

(radiation) B

(radiation) C



$$\therefore T_A > T_B = T_C$$

\* Most of the heat transferred on the earth is by convection and the contribution of conduction and radiation is very small.  
(we feel hot due to convection)

Types of convection:

Natural convection

Forced convection

- It is also known as free convection
- It is always in vertically upward direction

It is also known as artificial convection  
It's direction is based on force

•  $\frac{d\theta}{dt} \propto (\theta - \theta_0)^{5/4}$

$\frac{d\theta}{dt} \propto (\theta - \theta_0)$

•  $\frac{5}{4} = 1\frac{1}{2}$  power law

equal power law.

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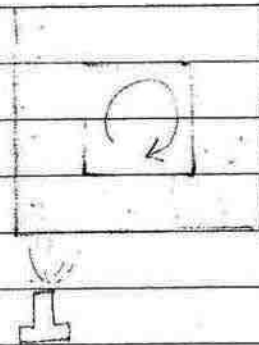
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It is equivalent to 'Newton's laws of ~~putting~~ cooling'

eg: cold air flows from ocean to ground side in the day time and vice-versa

eg: circulation of blood in our body.

eg: circulation of water around boiler.



### 3. Thermal Radiation:

MODULE 2 (Pg-151)

i) Spectrum of radiation is observed by KCl prism, Rock salt prism and flourospar prism



ii) Intensity of radiation is measured by Bolometer and Thermopile

iii) Pyrometer is used to measure high temperature only.

Range: Minimum:  $800^{\circ}\text{C}$

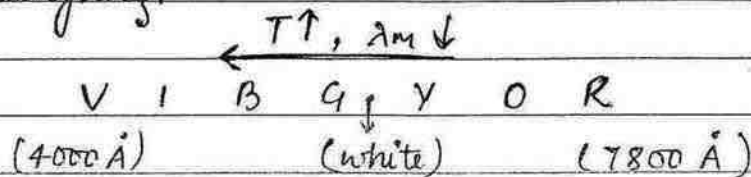
Maximum: No limit

- Temperature of 'sun' and stars are measured by pyrometer.

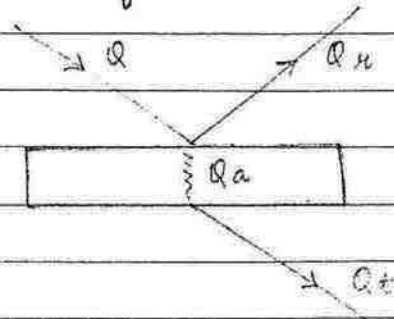
Electromagnetic wave spectrum:  
(By Maxwell's concept)

cosmic rays	$\gamma$ -ray	x-ray	U.V rays	Visible light	Infra-red Thermal radiation	Micro waves	Radio and TV waves
				VIBGYOR			
0	$1\text{\AA}$	$100\text{\AA}$	$4000\text{\AA}$	$7800\text{\AA}$	$4 \times 10^6\text{\AA}$	$\lambda \rightarrow$	

- wavelength range of radiations:  $0$  to  $\infty$
- wavelength of heat radiations:  $7800\text{\AA}$  to  $4 \times 10^6\text{\AA}$   
(infra-red)
- width of visible region:  $4000\text{\AA}$
- The wavelength corresponding to maximum emission of radiation  $\lambda_m$  shifts from longer wavelength to shorter wavelength as the temperature decreases increases. Due to this the colour of the body appears to be changing.



Nature of surface :



$$\therefore Q = Q_r + Q_a + Q_t$$

$$\frac{Q}{Q} = \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q}$$

$$\therefore 1 = r + a + t$$

- For general body / grey body:  
 $0 < r/t/a < 1$
- $r$ ,  $a$  and  $t$  are unitless.

where  $Q$  = amount of incident radiation  
 $Q_r$  = amount of reflected radiation  
 $Q_a$  = amount of absorbed radiation  
 $Q_t$  = amount of transmit radiation

$$\therefore r = \frac{Q_r}{Q} = \text{Reflective power / reflective co-efficient}$$

$$a = \frac{Q_a}{Q} = \text{absorptive power / absorptive co-efficient}$$

$$t = \frac{Q_t}{Q} = \text{transmitive power / transmitive co-efficient}$$

Case I: when  $r = \frac{Q_r}{Q} = 1 \quad \therefore a = t = 0 \quad \therefore Q = Q_r$

• perfectly reflective body  
eg: Ideal Mirror.

Case II: when  $a = \frac{Q_a}{Q} = 1 \quad \therefore r = t = 0 \quad \therefore Q = Q_a$

• perfectly absorptive body  
eg: Ideal black bodies.

Case III: when  $t = \frac{Q_t}{Q} = 1 \quad \therefore a = r = 0 \quad \therefore Q = Q_t$

• perfectly transmissive body.  
eg: Vacuum.

\* These three cases are practically not possible.

Q-9 Total 700 J is incident on the surface  
& 25 cal is transmitted. 45% is absorbed.  
Calculate amount of reflected radiations.

Transmitted radiations = 25 cal

$$= 25 \times 4.2 = 105 \text{ J } (Q_t)$$

$$\text{absorbed radiations} = \frac{45}{100} \times 700 = 315 \text{ J } (Q_a)$$

$$\text{Incident radiations} = 700 \text{ J } (Q)$$

$$\therefore Q = Q_a + Q_t + Q_r$$

$$\therefore Q_r = Q - Q_a - Q_t$$

$$= 700 - 315 - 105$$

$$= 700 - 420$$

$$= 280 \text{ J}$$

Q-10 Total 100 cal is incident on the surface, 120 J reflects and 20% is absorbed. Then, calculate Transmittive power.

$$Q = 100 \text{ cal} = 100 \times 4.2 = 420 \text{ J}$$

$$Q_r = 120 \text{ J}$$

$$Q_a = \frac{20}{100} \times 420 = 84 \text{ J}$$

$$\therefore Q = Q_a + Q_r + Q_t$$

$$\therefore Q_t = Q - Q_a - Q_r$$

$$= 420 - 84 - 120$$

$$= 420 - 204 = 216 \text{ J}$$

$$\therefore \text{transmittive power, } t = \frac{Q_t}{Q}$$

$$= \frac{216 \text{ J}}{420 \text{ J}} = 0.51$$

Some definitions:

i) Absorptive power ( $a$ ): It is the ratio of amount of absorbed radiations and amount of incident radiations.

$$a = \frac{Q_a}{Q} = \frac{\text{Amount of absorbed radiation (0 to } \infty)}{\text{Amount of incident radiation (0 to } \infty)}$$

- It is unitless
- For general bodies (GB):  $0 < a < 1$
- For ideal black body (IBB):  $a = 1$
- It depends on nature of surface only

ii) Spectral Absorptive power ( $a_\lambda$ ):

$a_\lambda = \frac{Q_{a\lambda}}{Q_\lambda}$  = Amount of absorbed radiations from  $\lambda \text{ \AA}$   
 $Q_\lambda$  Amount of incident radiations from  $\lambda \text{ \AA}$

- It is unitless
- For general bodies (GB):  $0 < a_\lambda < 1$
- For ideal black bodies (IBB):  $a_\lambda = 1$
- It depends on the nature of the surface only

iii) Emissive Power ( $e$ ):

At a given temperature, the amount of emitted radiations per unit area in per unit time by a body is called as emissive power of the body.

$$GB \Rightarrow e = \frac{Q_{GB}}{At}$$

$$\therefore Q_{GB} = eAt$$

$$IBB \Rightarrow E = \frac{Q_{IBB}}{At}$$

$$\therefore Q_{IBB} = EAt$$

- Unit:  $\frac{\text{Joule}}{\text{m}^2 \text{ sec}}$  or  $\frac{\text{Watt}}{\text{m}^2}$
- Dimensions:  $[M^1 L^0 T^{-3}]$
- Emissive power of ideal gas body depends on temperature only  
 $[E \propto t^4] \therefore [e \propto e_{RT}^4]$
- But, emissive power of general body depends on temperature and nature of surface.  
 $a \propto e \quad (a \neq e)$

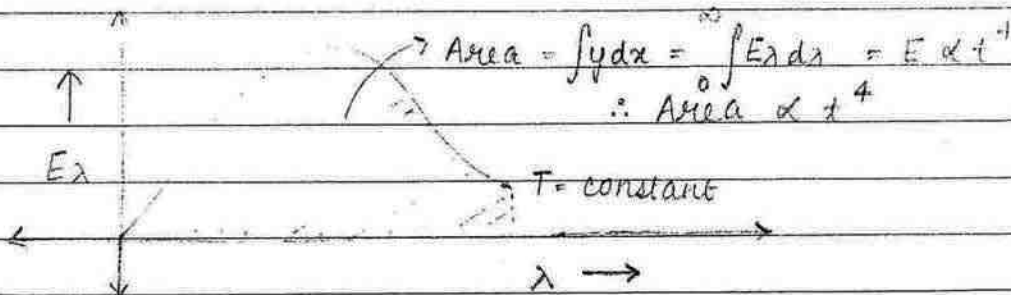
$a \neq e$ , It is because  $a$  is a ratio but  $e$  is the quantity not a ratio.

iv) Spectral Emission power ( $e_\lambda$ ):

At a given temperature, the amount of emitted radiation per unit area per unit time by a body for  $\lambda$  wavelength is called a spectral emissive power.

$$Q_B \Rightarrow e_\lambda = \frac{(Q_\lambda)_{Q_B}}{At}$$

$$I_{BB} \Rightarrow E_\lambda = \frac{(Q_\lambda)_{I_{BB}}}{At}$$



+

$$e_{\lambda_1} + e_{\lambda_2} + e_{\lambda_3} \dots \dots \dots e_{\lambda_\infty}$$

$$= \frac{Q_{\lambda_1}}{At} + \frac{Q_{\lambda_2}}{At} + \frac{Q_{\lambda_3}}{At} \dots \dots \dots \frac{Q_{\lambda_\infty}}{At}$$

$$= \frac{Q_{\lambda_1} + Q_{\lambda_2} + Q_{\lambda_3} \dots \dots \dots Q_{\lambda_\infty}}{At} = Q = E$$

- v) Emissivity / Relative emissivity ( $e_r$ ) :  
 At a given temperature, it is the ratio of emissive power of general body to emissive power of ideal black body at identical condition.

$$e_r = \frac{e}{E}$$

$$\therefore e_r = \frac{e \times A_t}{E \times A_t} = \frac{Q_{GB}}{Q_{TBB}}$$

- It is unitless
- For GB  $\Rightarrow 0 < e_r < 1$
- For TBB  $\Rightarrow e_r = 1$
- It depends upon the nature of the gas

### Ideal Black Bodies :

- Surface which absorbs all the incident radiation ( $0$  to  $\infty \text{ \AA}$ ) at low temperature and emit all these radiations at high temperature.
- For TBB,  $r = t = 0$ ,  $a = 1$   
 $a_\lambda = 1$ ,  $e_r = 1$
- An ideal black body need not be black in colour, eg: sun.
- spectrum of an ideal black body is continuous. so, it is called full or white spectrum.

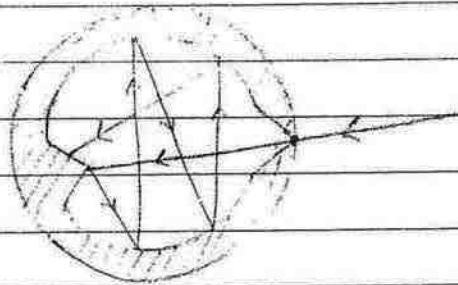
v) Emissive power of ideal black body depends on temperature only but emission power of general body depends on temperature and nature of surface.

vi) In nature, 100% absorption of incident radiation is not possible by any surface.

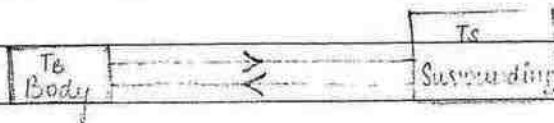
- The maximum absorptive power is of Platinum Black and Carbon black.

vii) Practically made ideal body:

- Ferry's Ideal Black Body:



- Prevost law of heat exchange:  
According to this law emission and absorption of radiation is a continuous process, it means every body emits and absorb radiation to the surrounding at all temperature (except 0 K)

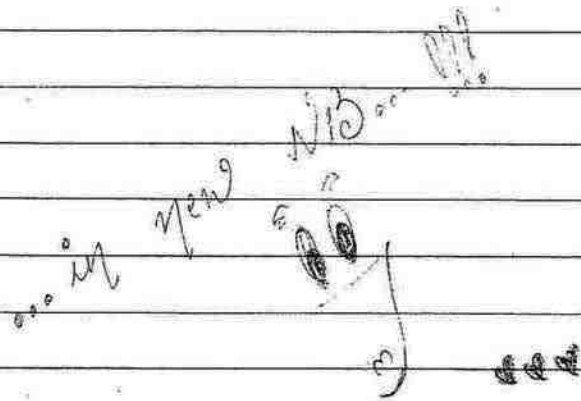




- $T_B > T_S \Rightarrow Q_{emit} > Q_{absorb}$   
 $T_B \downarrow = \text{cooling effect}$   
 eg: ~~311K~~ ~~311K~~ AC room

- $T_B < T_S \Rightarrow Q_{emit} < Q_{absorb}$   
 $T_B \uparrow = \text{heating effect}$   
 eg: ~~311K~~ ~~311K~~ ~~311K~~ ~~311K~~

- $T_B = T_S \Rightarrow Q_{emit} = Q_{absorb}$   
 $T_B = \text{constant}, \text{ No effect}$   
 eg: ~~311K~~ ~~311K~~ ~~311K~~



## ... MODE OF HEAT TRANSFER

Kirchoff's law: At a given constant temperature, ratio of spectral emissive power to spectral absorptive power and all bodies will remain constant and is equal to spectral emissive power of Ideal Black Body at same temperature.

$$\left(\frac{e_\lambda}{a_\lambda}\right)_1 = \left(\frac{e_\lambda}{a_\lambda}\right)_2 = \left(\frac{e_\lambda}{a_\lambda}\right)_3 = \dots = \left(\frac{E_\lambda}{A_\lambda}\right) = \text{constant}$$

G.B
IBB

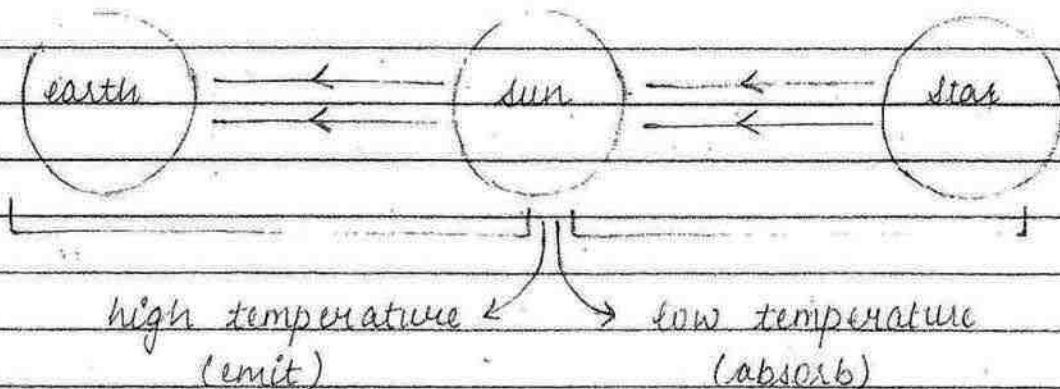
$\therefore$  for IBB,  $A_\lambda = 1$

$$\therefore \left(\frac{e_\lambda}{a_\lambda}\right) = E_\lambda = \text{constant} \quad (\text{at constant temp.})$$

G.B
IBB

$$a_\lambda \propto e_\lambda$$

(at low temp) (at high temp) w.r.t. body



Result:

- Good absorber  $\rightleftharpoons$  good emitter
- Bad absorber  $\rightleftharpoons$  bad emitter  
(at low temp of the body) (at high temp of the body)

Nature of surface (surface finish)

- Dark (black) and rough surface: Good
- Bright (white) and smooth surface: Bad

eg: In desert area, days are hot and nights are cool.

Reason:  $\because$  sand is rough,  
 $\therefore$  It is good absorber as well as good emitter of heat.

eg: Mudhouses are cooler in summer and warmer in winter.

Reason:  $\because$  Mud is smooth  
 $\therefore$  It is a bad absorber as well as bad emitter of heat.

eg: A black skin person experience more heat in summer and more cold in winter as compared to white skin.

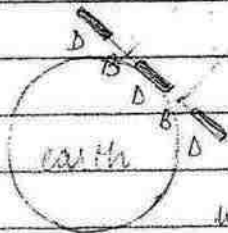
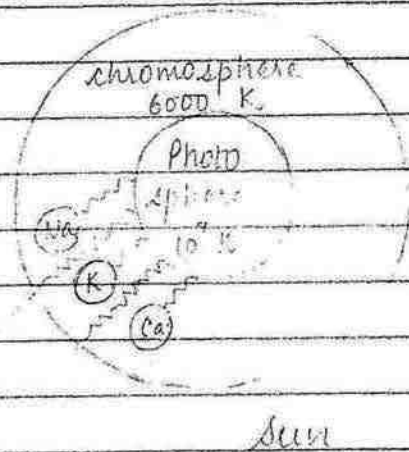
Reason: Black is a good absorber as well as good emitter whereas white is a bad absorber as well as emitter.

## Applications:

### Fraunhofer's line:

- In normal condition (dark lines): It is due to absorption of chromosphere (which is a good absorber)
- At solar eclipse (Bright light): It is due to emission by chromosphere (which is a good emitter)

\* Chromosphere is a good absorber as well as good emitter.



where, D = dark  
and B = Bright

Normal  
condition

Radiation colour: green body appears green because it reflects green colour and absorbs all other colour.

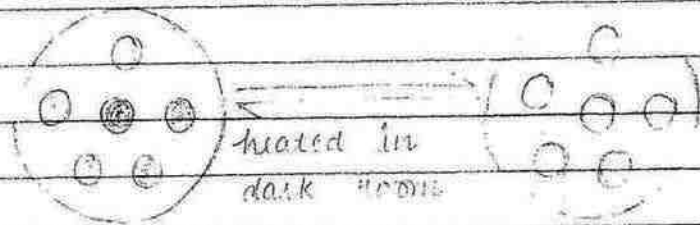
Complementary colours:

Red  $\rightleftharpoons$  Green

Blue  $\rightleftharpoons$  Yellow

Black  $\rightleftharpoons$  White

When a green body is heated in a dark room then it appears red because green is a good absorber of red radiation at low temperature. so it is also a good emitter of red radiation at high temperature.

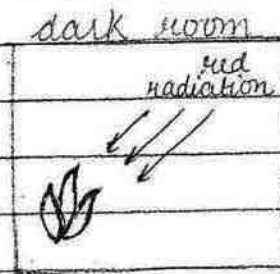


white ball  
Black spot

Black ball  
white spot



Appears Black



Appears Black

Stefan's law:

The amount of emitted radiation per unit area per unit time by a black body is directly proportional to fourth power of its absolute temperature

$$E = \frac{Q_{IBB}}{At}$$

$$\therefore E \propto T^4$$

$$E = \sigma T^4$$

where,  $\sigma$  = Stefan's constant

$$\sigma = 5.67 \times 10^{-8} \text{ Joule } \text{ or } \text{ Watt} \\ \text{m}^2 \text{ sec K}^{-4} \quad \text{m}^2 \text{ sec K}^{-4}$$

dimensions of  $\sigma$ :  $[M^1 L^0 T^{-3} K^{-4}]$

Amount of emitted radiation (Q):

$$Q_{IBB} = EAt$$

$$Q_{IBB} = \sigma T^4 At \quad \text{joule}$$

$$\therefore e_n = \frac{Q_{GB}}{Q_{IBB}}$$

$$Q_{IBB}$$

$$\therefore Q_{GB} = e_n Q_{IBB}$$

$$\therefore Q_{GB} = e_n \sigma A T^4 t \quad \text{joule}$$

Rate of emitted radiation by IBB:  $E_1 = \sigma T^4$   
where,  $T$  = temperature of body

Rate of absorbed radiation by IBB = rate of emitted radiation by surrounding:  $E_2 = \sigma T_0^4$   
where,  $T_0$  = temperature of the surrounding

Net rate of loss of radiation by IBB :

$$E = E_1 - E_2$$

$$\therefore E = \sigma (T^4 - T_0^4)$$

Net amount of loss of emitted radiation:

$$Q_{IBB} = EAt$$

$$\therefore Q_{IBB} = \sigma (T^4 - T_0^4) At \quad \text{joule}$$

$$e_{\pi} = \frac{Q_{GB}}{Q_{IBB}} \quad \therefore Q_{GB} = e_{\pi} Q_{IBB}$$

$$\star\star \quad Q_{GB} = e_{\pi} \sigma A (T^4 - T_0^4) t \quad \text{joule}$$

Q. 11 Two spheres A and B of same colour and having radius 4 cm and 1 cm maintains temperature  $127^\circ\text{C}$  and  $327^\circ\text{C}$  respectively. The surrounding temperature is 200 K. Calculate ratio of energy radiated by A and B.

$$\therefore Q = e_{\pi} \sigma A (T^4 - T_0^4) t$$

$$\therefore Q \propto A (T^4 - T_0^4)$$

$$\therefore Q \propto r^2 (T^4 - T_0^4)$$

$$\frac{Q_1}{Q_2} = \left( \frac{r_1}{r_2} \right)^2 \left[ \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4} \right]$$

$$\therefore \frac{Q_1}{Q_2} = \frac{16}{1} \left[ \frac{400^4 - 200^4}{600^4 - 200^4} \right]$$

$$= \frac{16}{1} \left[ \frac{(2^4 \times 200^4) - 200^4}{(3^4 \times 200^4) - 200^4} \right]$$

$$= \frac{16}{1} \times \frac{200^4 (2^4 - 1)}{200^4 (3^4 - 1)}$$

$$= \frac{16}{1} \times \frac{16-1}{81-1}$$

$$= \frac{16 \times 15}{80} = \frac{3}{1}$$

$$\therefore \frac{Q_1}{Q_2} = \frac{3}{1}$$

Q.12 If the temperature of ideal black body is increased by 25%, then what will be % of rate of emitted radiation emitted from its surface

$$\therefore E = \sigma (T^4 - T_0^4)$$

$$\therefore E \propto T^4$$

$$\frac{E_1}{E_2} \propto \frac{T_1^4}{T_2^4}$$

$$\frac{E_1}{E_2} = \frac{T_1^4}{T_2^4}$$

$$\text{Let } T_1 = 100T \quad \therefore T_2 = 100T + 25T = 125T$$

$$E_1 = 100E \quad \therefore E_2 = ?$$

$$\frac{100E}{E_2} = \left( \frac{100T}{125T} \right)^4$$

$$\frac{100E}{E_2} = \left( \frac{4}{5} \right)^4$$

$$\frac{100E}{E_2} = \frac{256}{625}$$

$$\therefore E_2 = \frac{625 \times 100E}{256}$$

$$= 244E$$

$$\therefore \% \text{ increase} = \frac{E_2 - E_1}{E_1} \times 100 = \frac{244E - 100E}{100E} \times 100$$

$$= \frac{144E}{100E} \times 100 = 144\%$$



$$Q_{RB} = \epsilon \chi \sigma A (T^4 - T_0^4) t$$

$$\therefore dQ = \epsilon \chi \sigma A (T^4 - T_0^4) dt$$

- $R_H = \frac{dQ}{dt}$  = Rate of loss of heat

$$R_H = \frac{dQ}{dt} = \epsilon \chi \sigma A (T^4 - T_0^4) \text{ Joule} = \text{watt}$$

sec

∴  $R_H = \frac{dQ}{dt} = \frac{\epsilon \chi \sigma A (T^4 - T_0^4)}{J} \text{ cal}$

sec

- $R_F = \frac{d\theta}{dt}$  = Rate of fall in temperature

$$dQ = MS d\theta$$

$$\therefore \frac{dQ}{dt} = MS \frac{d\theta}{dt}$$

$$\therefore R_H = Hc R_F$$

$$\left( \because \frac{dQ}{dt} = R_H, MS = Hc \text{ and } \frac{d\theta}{dt} = R_F \right)$$

$$\therefore \frac{d\theta}{dt} = \frac{1}{MS} \frac{dQ}{dt}$$

$$\therefore R_F = \frac{d\theta}{dt} = \frac{\epsilon \chi \sigma A (T^4 - T_0^4)}{MSJ} \text{ } ^\circ\text{C or K}$$

sec sec

- $R_H$  = Rate of loss of heat  
 = rate of emitted radiation per second  
 = emitted power (Joule/sec)

- $R_F$  = Rate of fall in Temperature  
 = Rate of cooling  
 = which is cooling faster?

Case I to V : If  $T, T_0, e_H = \text{same}$   
 $R_H \propto A$  and  $R_F \propto A$   
 MS

Case I : Two solid sphere ( $r_1 > r_2$ ) of different material

- $$\therefore R_H \propto A \propto r^2 \quad (\because A = \pi r^2)$$

$$\therefore \frac{R_{H1}}{R_{H2}} = \frac{r_1^2}{r_2^2}$$

$$R_{H1} > R_{H2} \quad (\because r_1 > r_2)$$

- $$\therefore R_F \propto A \propto A \quad \left( \because f = \frac{M}{V}, \therefore M = fV \right)$$

$$\frac{A}{V} \propto \frac{r^2}{r^3 f} \quad \left( \because A = \pi r^2 \text{ and } V = \frac{4}{3} \pi r^3 \right)$$

$$\therefore R_F \propto \frac{1}{r f}$$

$$\frac{R_{F1}}{R_{F2}} = \frac{r_2 f_2 S_2}{r_1 f_1 S_1}$$

Case II : Two solid sphere ( $r_1 > r_2$ ) of same material.

- $$\therefore R_H \propto A \propto r^2 \quad (\because A = \pi r^2)$$

$$\therefore \frac{R_{H1}}{R_{H2}} = \frac{r_1^2}{r_2^2}$$

$$R_{H1} > R_{H2} \quad (\because r_1 > r_2)$$

- $$\therefore R_F \propto A \propto A \quad \left( \because f = \frac{M}{V}, \therefore M = fV \right)$$

$$\frac{A}{VPS} \propto \frac{r^2}{r^3PS} \quad \left( \because A = \pi r^2 \text{ and } V = \frac{4}{3}\pi r^3 \right)$$

$$\frac{r^2}{r^3PS} \propto \frac{1}{r(P)S} \quad \left( P \text{ and } S = \text{constant} \right)$$

$$\because \text{same material}$$

$$\therefore R_F \propto \frac{1}{r}$$

$$\frac{RF_1}{RF_2} = \frac{r_2}{r_1}$$

$$\therefore RF_1 < RF_2 \quad (\because r_1 > r_2)$$

Time taken for cooling

$$t_{r_1} > t_{r_2} \quad \left( \because RF \propto \frac{1}{dt} \right)$$

Case III: different solid shaped, cube ( $l$ ), cylinder ( $r, l$ ) and sphere ( $r$ )

- $\because R_F \propto A$  (where  $A$  is total surface area)

- cube =  $6l^2$

- cylinder =  ~~$2\pi r$~~   $2\pi rl + 2\pi r^2 = 2\pi r(l+r)$

- sphere =  $4\pi r^2$

- $\therefore R_F \propto \frac{A}{MS} \propto \frac{A}{VPS}$  ( $P, S = \text{constant}$ )

$$\therefore R_F \propto \frac{A}{V}$$

- cube =  $\frac{6l^2}{l^3} = \frac{6}{l}$

- cylinder =  $\frac{2\pi rl + 2\pi r^2}{\pi r^2 l} = \frac{2\pi r(l+r)}{\pi r^2 l}$

- sphere =  $\frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}$

Case IV: different solid shaped. cube ( $l$ ), cylinder ( $r, l$ ), sphere ( $r$ ), disc ( $r$ ) of same material and same volume.

- $\therefore R_H \propto A$

- $\therefore R_F \propto \frac{A}{MS} \propto \frac{A}{VPS}$  ( $\rho, S, V = \text{constant}$ )  
( $\therefore$  same material and volume)

- $\therefore R_F \propto \frac{A}{V}$

Maximum surface area is of disc because maximum mass is on the surface so  $R_H, R_F$  is maximum for disc.

Maximum Minimum surface area is of sphere because maximum mass is inside the sphere, so  $R_H, R_F$  is minimum for sphere.

Case V: A solid and a hollow sphere of same material and same radius.

- $\therefore R_H \propto A \propto r^2$

- $\therefore R_{H1} = R_{H2}$  ( $\therefore r_1 = r_2$ )

- $\therefore R_F \propto \frac{A}{MS} \propto \frac{r^2}{MS}$  (same material and same radius)

- $R_F \propto \frac{1}{M}$

- $\therefore M_{\text{solid}} > M_{\text{hollow}}$

- $R_F \text{ solid} < R_F \text{ hollow}$

- Time taken for cooling,  $T_{\text{solid}} > T_{\text{hollow}}$

Q.13 Case VI : A sphere at temperature 900 K is placed in an environment of temperature 300 K. It's cooling rate is 8°C/min. If its temperature is reduced to 600 K, then its cooling rate in same surrounding will be.

$$\therefore R_F = \frac{e \times \sigma \times A}{(M \cdot S \cdot J)} (T^4 - T_0^4)$$

$$\therefore R_{F1} = (T_1^4 - T_0^4)$$

$$R_{F2} = (T_2^4 - T_0^4)$$

$$8 = 900^4 - 300^4$$

$$R_{F2} = 600^4 - 300^4$$

$$\therefore \frac{8}{R_{F2}} = \frac{3^4 \times 300^4 - 300^4}{2^4 \times 300^4 - 300^4}$$

$$\frac{8}{R_{F2}} = \frac{3^4 \times 300^4 - 300^4}{2^4 \times 300^4 - 300^4}$$

$$\frac{8}{R_{F2}} = \frac{300^4 (3^4 - 1)}{300^4 (2^4 - 1)}$$

$$\therefore \frac{8}{R_{F2}} = \frac{80}{15}$$

$$R_{F2} = 15 \times \frac{8}{80} = \frac{3}{2}$$

$$R_{F2} = \frac{15 \times 8}{80} = \frac{3}{2}$$

$$\therefore R_{F2} = 1.5^\circ \text{C} / \text{min}$$

Newton's law of cooling :

According to this law, the rate of cooling of a body is directly proportional to temperature difference between body and surrounding.

$$R_F = \frac{d\theta}{dt} \propto (\theta - \theta_0)$$

$$\therefore \frac{d\theta}{dt} = -k(\theta - \theta_0)$$

-ve indicates decrease in rate of cooling with  $\theta$ .

$k$  = cooling constant

$$k = \frac{4\epsilon\sigma A\theta_0^3}{MSJ}$$

where,  $\theta$  = temperature of body  
 $\theta_0$  = surrounding temperature  
 $\theta - \theta_0$  = excess temperature

condition :  $\theta - \theta_0 < 35^\circ\text{C}$

eg: A cold milk  $\xrightarrow{5 \text{ min}} 90^\circ\text{C}$   $\xrightarrow{E \propto T^4}$   $80^\circ\text{C}$   
( $100^\circ\text{C}$ )

B  $\xrightarrow{5 \text{ min}} 88^\circ\text{C}$  cold milk  $\xrightarrow{E \propto T^4}$   $78^\circ\text{C}$   
( $100^\circ\text{C}$ )

(Hence more loss of heat at high temperature.)

Graph between  $\theta$  (temperature of body) and  $t$  (time):

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\int \frac{1}{\theta - \theta_0} d\theta = -k \int dt$$

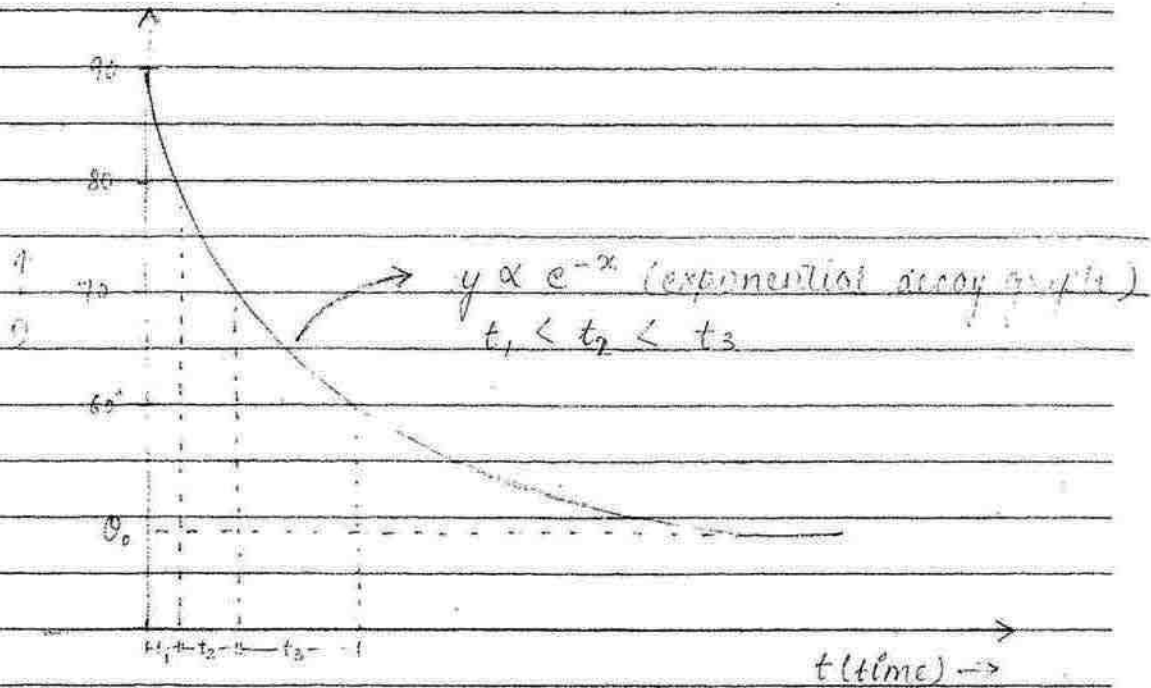
$$\log_e(\theta - \theta_0) = -kt + c$$

$$e^{-kt+c} = \theta - \theta_0$$

$$\theta = e^{-kt+c} + \theta_0$$

$$y = e^{-x} + c$$

$$\left[ \begin{array}{l} \log_{10} 100 = 2 \quad \therefore 10^2 = 100 \\ \log_x y = z \quad \therefore x^z = y \end{array} \right]$$



Graph between  $\log_e (\theta - \theta_0)$  and time ( $t$ ):

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\int \frac{1}{\theta - \theta_0} d\theta = -k \int dt$$

$$\log_e (\theta - \theta_0) = -kt + c$$

$$y = -mx + c$$

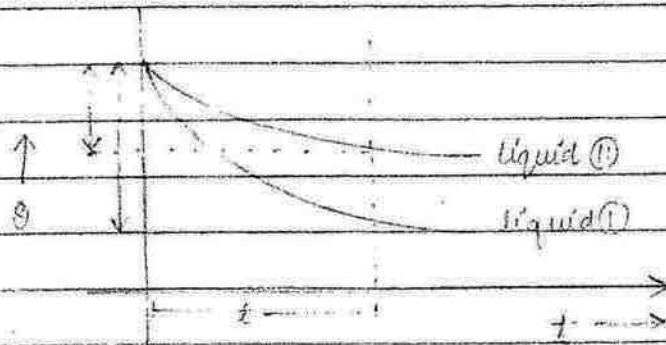
$$y = -\log(\theta - \theta_0)$$

+c

$-\theta_0$

t →

Q.14 Find out the slope of rate of fall in temp



$$\therefore R_F = \frac{d\theta}{dt} = \text{slope}$$

$$\therefore \text{slope} \propto d\theta$$

$$(\text{slope})_{\text{liq (I)}} > (\text{slope})_{\text{liq (II)}}$$

$$R_{F1} > R_{F2}$$

OR

$$R_F \propto d\theta \quad (dt = \text{same})$$

Rate of fall in temperature:

$$\theta_1 \xrightarrow[t]{\text{temperature fall to}} \theta_2$$

$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

$$\frac{\theta_2 - \theta_1}{t} = -K \left[ \frac{\theta_1 + \theta_2}{2} - \theta_0 \right]$$

$$\left( \because \theta_{\text{mid}} = \frac{\theta_1 + \theta_2}{2} = \text{radiation correction} \right)$$

$$-\frac{(\theta_1 - \theta_2)}{t} = -K \left( \frac{\theta_1 + \theta_2}{2} - \theta_0 \right)$$

$$\therefore \frac{\theta_1 - \theta_2}{t} = K \left[ \frac{\theta_1 + \theta_2}{2} - \theta_0 \right]$$



$$Q.15 \quad 60^{\circ}\text{C} \longrightarrow 40^{\circ}\text{C} \quad (\text{time} = 5 \text{ min})$$

$$40^{\circ}\text{C} \longrightarrow 30^{\circ}\text{C} \quad (\text{time} = ?)$$

$$\text{given } \theta_0 = 20^{\circ}\text{C}$$

$$\therefore \frac{\theta_1 - \theta_2}{t} = K \left[ \frac{\theta_1 + \theta_2 - \theta_0}{2} \right]$$

$$\therefore \frac{60 - 40}{5} = K \left[ \frac{60 + 40 - 20}{2} \right]$$

$$4 = K (50 - 20)$$

$$\therefore K = \frac{4}{50 - 20}$$

$$K = \frac{4}{30} = \frac{2}{15}$$

$$K = \frac{4}{30} = \frac{2}{15}$$

$$\therefore \frac{40 - 30}{t} = K \left[ \frac{40 + 30 - 20}{2} \right]$$

$$= \frac{10}{t} = \frac{2}{15} (35 - 20) \quad \left[ \because K = \frac{2}{15} \right]$$

$$= \frac{10}{t} = \frac{2}{15} \times 15$$

$$\therefore t = \frac{10}{2} = 5 \text{ min}$$

$$Q.15 \quad 70^{\circ}\text{C} \longrightarrow 60^{\circ}\text{C} \quad (t_1 = 5 \text{ min})$$

$$60^{\circ}\text{C} \longrightarrow 50^{\circ}\text{C} \quad (t_2 = 8 \text{ min})$$

$$50^{\circ}\text{C} \longrightarrow 40^{\circ}\text{C} \quad (\text{find } t_3 \text{ and } \theta_0)$$

$$\therefore \frac{\theta_1 - \theta_2}{t} = K \left[ \frac{\theta_1 + \theta_2 - \theta_0}{2} \right]$$

$$\therefore \frac{70 - 60}{5} = K \left[ \frac{70 + 60 - \theta_0}{2} \right] \quad \text{--- (I) and}$$

$$\frac{60 - 50}{8} = K \left[ \frac{60 + 50 - \theta_0}{2} \right] \quad \text{--- (II)}$$

dividing (ii) by (i),

$$\frac{10}{5} = K (65 - \theta_0)$$

$$\frac{10}{8} = K (55 - \theta_0)$$

$$\Rightarrow \frac{8}{5} = \frac{65 - \theta_0}{55 - \theta_0}$$

$$= \frac{440 - 8\theta_0}{325 - 5\theta_0}$$

$$= \frac{440 - 325}{8\theta_0 - 5\theta_0}$$

$$= \frac{115}{3\theta_0} = 300$$

$$\therefore \theta_0 = 38.33$$

Now, putting  $\theta_0$  in (i), we get

$$\frac{10}{5} = K (65 - 38.33)$$

$$\therefore K = \frac{2}{26.67} = \frac{1}{13.33}$$

$$\therefore \frac{50 - 40}{t_3} = \frac{1}{13.33} \left[ \frac{50 + 40 - 38.33}{2} \right]$$

$$\frac{10}{t_3} = \frac{1}{13.33} [45 - 38.33]$$

$$\frac{10}{t_3} = \frac{1}{13.33} \times 6.67$$

$$\therefore \frac{10}{t_3} = \frac{1}{2}$$

$$\therefore t_3 = 10 \times 2$$

$$= 20 \text{ min}$$

Q.17 If initial temperature of the body is  $40^\circ\text{C}$ , room temperature is  $20^\circ\text{C}$ . After 5 minutes excess temperature is reduced to its half value, then calculate temp of the body after

next 3 minutes.

$$\theta_1 \xrightarrow{5 \text{ min}} \theta_2 \xrightarrow{3 \text{ min}} \theta_3$$

$40^\circ\text{C}$

$$\therefore \theta_1 - \theta_0 = 40 - 20 = 20^\circ\text{C}$$

$$\theta_2 - \theta_0 = \frac{1}{2} \times 20 = 10^\circ\text{C}$$

$$\therefore \theta_2 = 10 + \theta_0 = 10 + 20 = 30^\circ\text{C}$$

$$\therefore \frac{\theta_1 - \theta_2}{t} = K \left[ \frac{\theta_1 + \theta_2 - \theta_0}{2} \right]$$

$$\frac{40 - 30}{5} = K \left[ \frac{40 + 30 - 20}{2} \right]$$

$$\frac{10}{5} = K [35 - 20]$$

$$2 = K \times 15$$

$$\therefore K = \frac{2}{15}$$

$$\therefore \frac{\theta_2 - \theta_3}{t} = K \left[ \frac{\theta_2 + \theta_3 - \theta_0}{2} \right]$$

$$\frac{30 - \theta_3}{3} = \frac{2}{15} \left[ \frac{30 + \theta_3 - 20}{2} \right]$$

$$30 - \theta_3 = \frac{2}{5} \left[ \frac{30 + \theta_3 - 20}{2} \right]$$

$$\therefore 5(30 - \theta_3) = \theta_3 - 10$$

$$150 - 5\theta_3 = \theta_3 - 10$$

$$150 + 10 = 6\theta_3$$

$$160 = 6\theta_3$$

$$\therefore \theta_3 = \frac{160}{6}$$

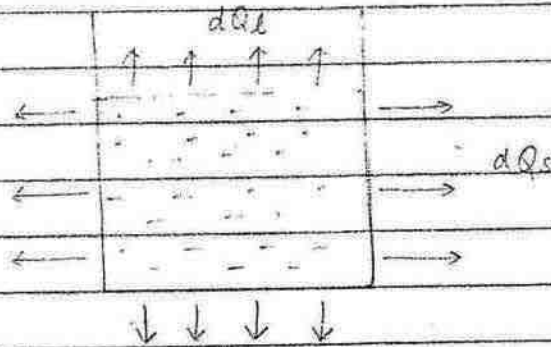
$$= 26.66^\circ\text{C}$$

## Limitations :

- i) Newton's laws of cooling (NLC) can be used only when temperature difference is small  $(\theta - \theta_0) < 35^\circ\text{C}$  but Stefan's law can be used for all temperature difference.
- ii) NLC can be determined by Stefan's law
- iii) NLC can be used only to calculate specific heat of liquid.

To calculate specific heat of a liquid:

- Step 1: Rate of loss of heat by one liquid.



$\theta_1$  temp fall to  $t \rightarrow \theta_2$

$$\therefore \frac{dQ}{dt} = \frac{dQ_l}{dt} + \frac{dQ_s}{dt}$$

$$\frac{dQ}{dt} = M_s s_l (\theta_1 - \theta_2) + M_s s_s (\theta_1 - \theta_2)$$

$$\frac{dQ}{dt} = (M_s s_l + M_s s_s) (\theta_1 - \theta_2)$$

- Step 2: Water equivalent of calorimeter ( $w$ ):  
Water equivalent of any substance is quantity of water in gram whose heat capacity is equal to heat capacity of the substance

$$\therefore (Hc)_{\text{substance}} = (Hc)_{\text{water}}$$

$$M_s S_s = M_w S_w$$

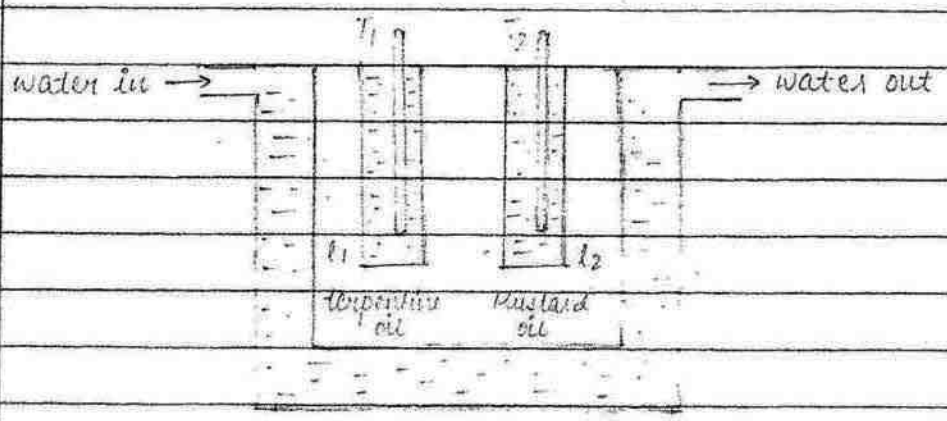
$$M_s S_s = M_w \times 1 \quad [\because S_{\text{water}} = 1 \text{ cal/g}^\circ\text{C}]$$

$$\therefore M_s S_s = w \times 1 \quad (\text{where } w \text{ is in gm})$$

$$\therefore \frac{dQ}{dt} = \frac{(M_s S_s + w)(\theta_1 - \theta_2)}{t} \quad \text{in CGS}$$

- Step 3: Compare two liquids:  
Two liquids are cooled at identical condition (initial temperature, surrounding temperature, surface area, nature of gas, surface, and volume are same) then, rate of loss of heat by both liquid is same

$\theta_1$  temp. fall to  $t \rightarrow \theta_2$



$$R_H = \frac{dQ}{dt} = \epsilon \sigma A (T_1^4 - T_0^4)$$

$$\therefore R_F = \frac{d\theta}{dt} \propto \frac{1}{MS}$$

$$\left( \frac{d\theta}{dt} \right)_{\text{liquid I}} = \left( \frac{d\theta}{dt} \right)_{\text{liquid II}}$$

$$M_1 S_1 + W_1 (\theta_1 - \theta_2) = M_2 S_2 + W_2 (\theta_1 - \theta_2)$$

$t_1 \qquad \qquad \qquad t_2$

$$\therefore \boxed{M_1 S_1 + W_1 = M_2 S_2 + W_2} \quad \text{in CGS}$$

$t_1 \qquad \qquad \qquad t_2$

$$\boxed{V \rho_1 S_1 + W_1 = V \rho_2 S_2 + W_2} \quad \text{in CGS}$$

$t_1 \qquad \qquad \qquad t_2$

$$\left( \because \rho = \frac{M}{V}, \therefore M = \rho V \right)$$

$$dQ_1 \gg \gg \gg dQ_2$$

$$M_1 S_1 \gg \gg \gg M_2 S_2$$

$$\boxed{M_1 S_1 = M_2 S_2}$$

$t_1 \qquad \qquad \qquad t_2$

(when w is not given)

$$\boxed{\rho_1 S_1 = \rho_2 S_2}$$

$t_1 \qquad \qquad \qquad t_2$

Solar constant (S):

The amount of received radiation per unit area per unit time on the earth from the sun is called as solar constant

• The amount of emitted radiation per unit area per unit time from the sun =  $E$

• The amount of emitted radiation by  $4\pi R^2$  area in per unit time from the sun =  $E \times 4\pi R^2$

• The amount of received radiation by  $4\pi R^2$  area in per unit time on the orbit sphere =  $E \times 4\pi R^2$

• The amount of received radiation per unit area per unit time on the orbit sphere. (on the earth),  $S = \frac{E \times 4\pi R^2}{4\pi d^2}$

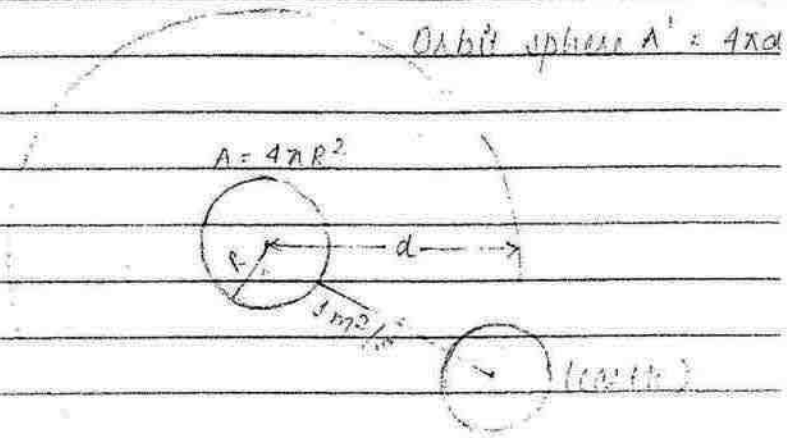
$$\therefore S = \frac{ER^2}{d^2}$$

$$* \quad S = \frac{\sigma T^4 \times R^2}{d^2}$$

$$S \propto \frac{1}{d^2} \quad \therefore \sqrt{S} \propto \frac{1}{d}$$

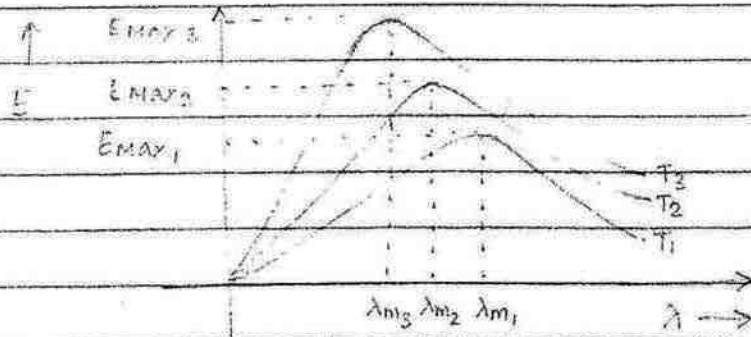
$$\begin{aligned} S_{\text{earth}} &= 1340 \text{ watt/m}^2 \approx 1400 \text{ watt/m}^2 \\ &= 1.4 \text{ K.watt/m}^2 \end{aligned}$$

$$\begin{aligned} S_{\text{earth}} &= 1.94 \text{ cal/cm}^2 \text{mm} \approx 2 \text{ cal/cm}^2 \text{mm} \end{aligned}$$



Solar constant ( $S$ ) is measured by:  
Pyroheliometer

Spectral energy distribution curve:



$$T_1 < T_2 < T_3$$

$$(A\mu\epsilon)_I < (A\mu\epsilon)_{II} < (A\mu\epsilon)_{III}$$

$$E_1 < E_2 < E_3$$

$$E_{max1} < E_{max2} < E_{max3}$$

$$\lambda_{m1} > \lambda_{m2} > \lambda_{m3}$$

$$\nu_{m1} < \nu_{m2} < \nu_{m3}$$

$$\propto T^4$$

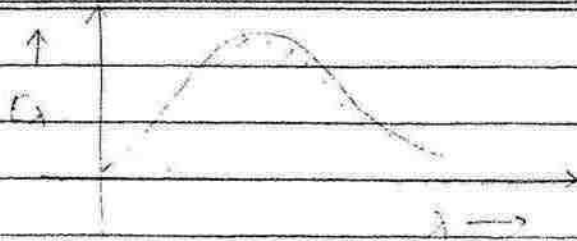
$$\propto T^4$$

$$\propto T^5$$

$$\propto \frac{1}{T}$$

$$\propto T$$





$$\text{Area} = \int_0^{\infty} E_{\lambda} d\lambda = E$$

$$\therefore E \propto T^4$$

$$\text{Area} \propto T^4$$

Results:

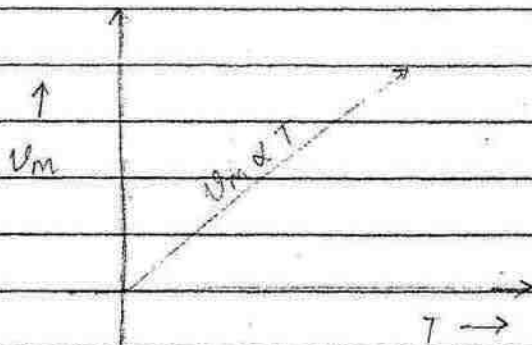
i)  $\text{Area} \propto E \propto T^4$

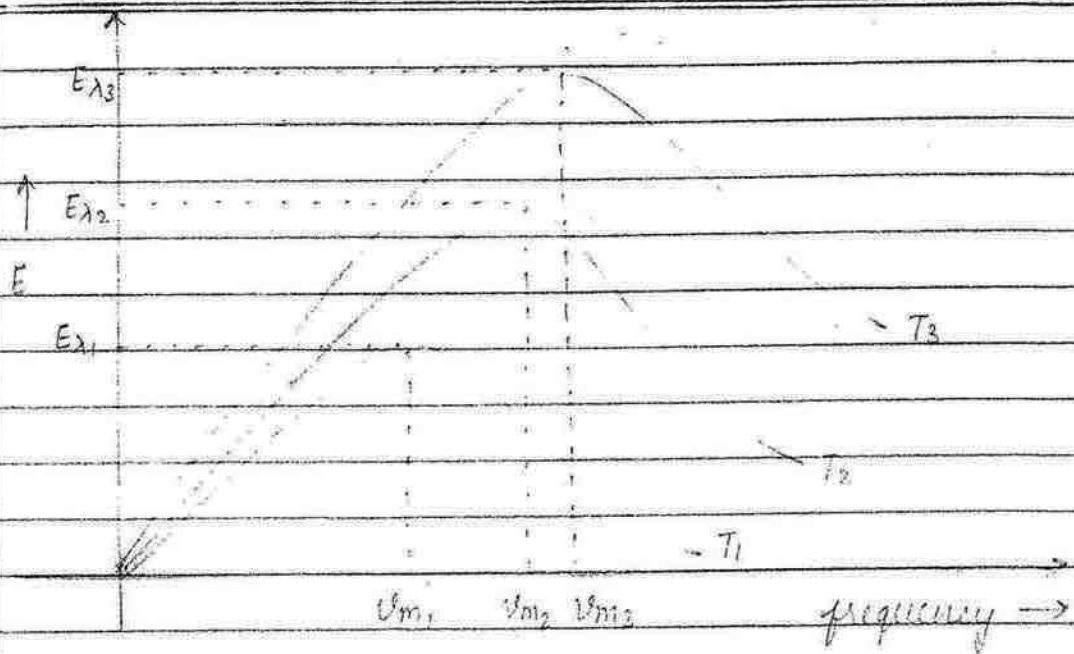
$$\therefore \frac{\text{Area I}}{\text{Area II}} = \left( \frac{T_1}{T_2} \right)^4$$

ii)  $E_{\text{max}} \propto T^5$

iii)  $\lambda_m \propto \frac{1}{T} \quad \therefore \frac{\lambda_{m1}}{\lambda_{m2}} = \frac{T_2}{T_1}$

iv)  $v_m \propto \frac{1}{\lambda_m} \propto T \quad \therefore v_m \propto T$



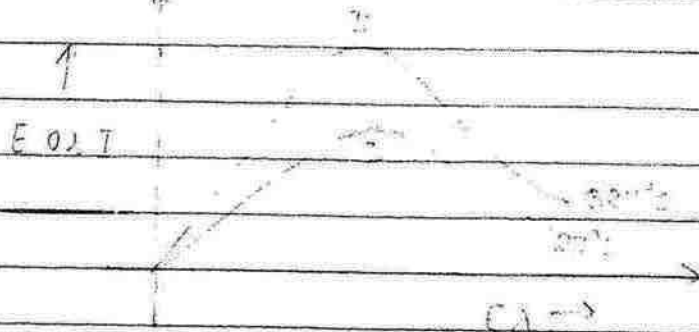


### Special points :

- i) At a given temperature, as the wavelength increases, the amount of emitted radiation becomes maximum and then decreases.
- ii) Emission spectrum of Black Bodies is continuous but amount of emitted radiation is different for different wavelength.
- iii) At a constant temperature, area enclosed between curve and wavelength axis represent total amount of emitted radiation ( $E$ ).
- iv) Wein's 5th power law:  
Maximum amount of emitted radiation from an ideal black body is proportional to 5th power of absolute temperature of IBB.  
 $\therefore E_{\text{max}} \propto T^5$

\* If the temperature of black body becomes double then total amount of emitted radiation is 16 times but maximum amount of emitted radiation becomes 32 times

Q. 18 Calculate the ratio of area, when temperature becomes double.



$$\therefore \text{Area} \propto T^4$$

$$\therefore \frac{\text{Area}_1}{\text{Area}_2} = \left( \frac{T_1}{T_2} \right)^4$$
$$= \left( \frac{300}{600} \right)^4$$

$$\therefore \frac{\text{Area}_1}{\text{Area}_2} = \frac{1}{16}$$

v) Wien's displacement law:

On increasing the temperature of ideal black body, the wavelength corresponding to maximum emission decreases

$$\therefore \lambda_m \propto \frac{1}{T}$$

$$\lambda_m = \frac{b}{T}$$

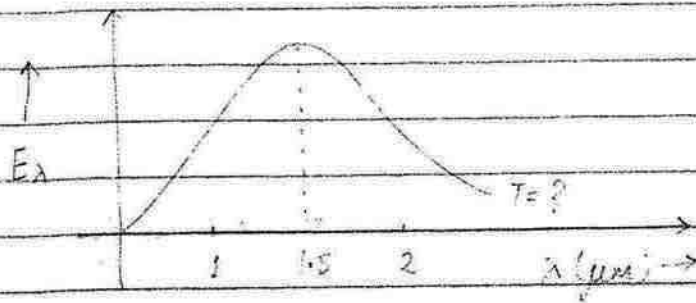
where,  $b = \text{wein's constant}$

$$b = 2.93 \times 10^{-3} \text{ m K}$$

$$\text{dimensions of } b = [M^0 L^1 T^0 K^1]$$

$$\therefore \lambda_{m1} T_1 = \lambda_{m2} T_2 \quad (T \text{ in kelvin})$$

Q.19 calculate temperature corresponding to given wavelength ( $\mu\text{m}$ )



$$\therefore T = \frac{b}{\lambda_m}$$

$$= \frac{2.93 \times 10^{-3} \text{ m K}}{1.5 \times 10^{-6} \text{ m}}$$

$$= 2.93 \times 10^{-3} \text{ m K}$$

$$= 2 \times 10^{6-3} \text{ K}$$

$$= 2 \times 10^3 \text{ K}$$

$$= 2000 \text{ K}$$

$$= 2000 \text{ K}$$

graph between  $\log \lambda_m$  and  $\log T$

$$\lambda_m = \frac{b}{T} \quad \left( \because T = \frac{b}{\lambda_m} \right)$$

$$\therefore \log \lambda_m = \log \left( \frac{b}{T} \right)$$

$$\log \lambda_m = \log b - \log T$$

$$\therefore \log \lambda m = -\log T + \log b$$

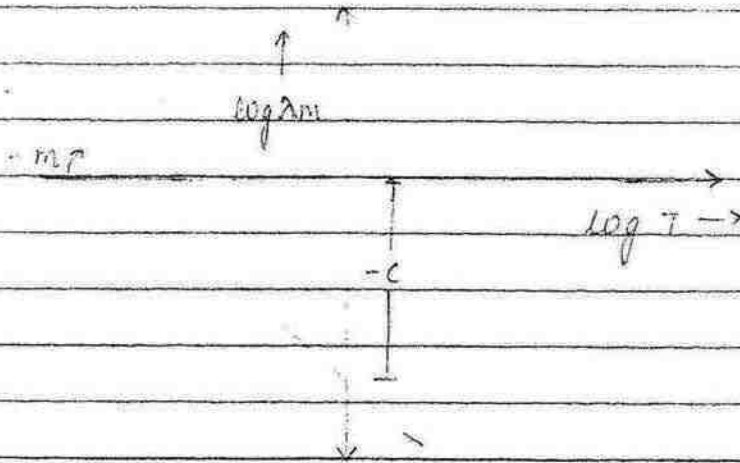
\*

$$y = -mx - c \quad \text{--- Memorise this!}$$

$$\begin{aligned} \therefore \log b &= \log (2.93 \times 10^{-3}) \\ &= \log 2.93 + \log 10^{-3} \\ &= \log 2.93 - 3 \log 10 \\ &= 0.47 - 3 \times 1 \\ \log b &= -2.53 \end{aligned}$$

\* Hence, the value of  $\log$  becomes negative when the power  $b$  is negative.

$$\begin{aligned} \log xy &= \log x + \log y \\ \log \left(\frac{x}{y}\right) &= \log x - \log y \end{aligned}$$



Graph between  $\log E$  and  $\log T$

$$E = \sigma T^4$$

$$\log E = \log (\sigma T)^4$$

$$= \log \sigma + \log T^4$$

$$\log \sigma + 4 \log T$$

$$\therefore \log E = 4 \log T + \log \sigma$$

★

$$y = mx - c \rightarrow \text{memory tips}$$

$$\therefore \log \sigma = \log (5.67 \times 10^{-8})$$

$$\log 5.67 + \log 10^{-8}$$

$$\log 5.67 - 8 \log 10$$

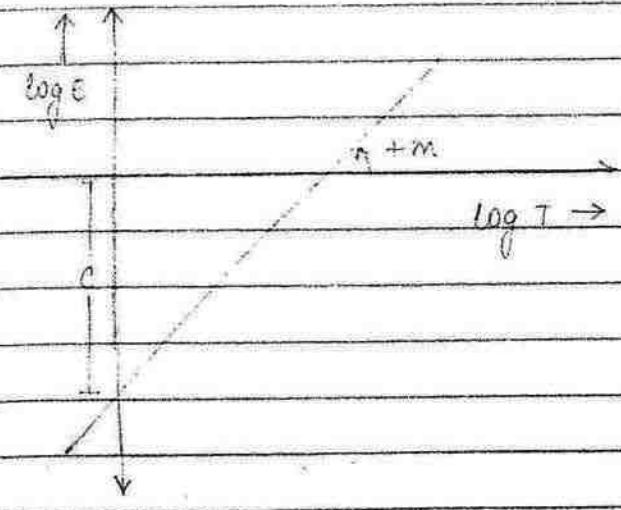
$$0.7 - 8 \times 1$$

$$\log \sigma = -7.3$$

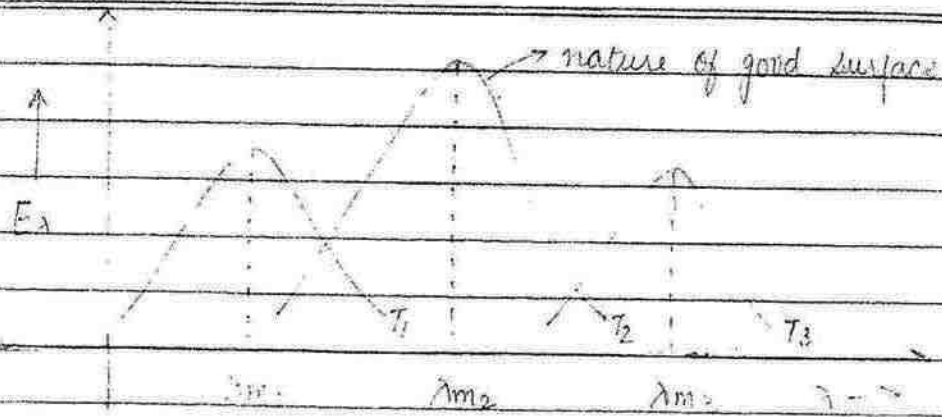
and

$$\log xy = \log x + \log y$$

$$\log \left(\frac{x}{y}\right) = \log x - \log y$$



Q.20. Calculate the order of  $\lambda_m$  and  $T$  from the following graph.



\* Curve of the same body can never intersects each other.

$$\therefore \lambda_{m1} < \lambda_{m2} < \lambda_{m3}$$

$$\therefore T_1 > T_2 > T_3 \quad \left[ \because \lambda_m \propto \frac{1}{T} \right]$$