ONE DIMENSIONAL MOTION

The study of motion without considering the force is known as kinematics.

1-D supplement (1-DS) Pg-1

Motion: If the position of the particle changes with time, then it is in motion.

Rest: If the position of the particle does not changes with time, then it is in rest.

* Motion and rest are relative terms i.e. they depend on the frame of reference. Hence, they cannot be absolute.

Types of Motion:
(1-DS) Pg-1

i. One dimensional motion (1-D Motion) or ~~Rectin~~ Rectilinear motion:

- If the particle moves along a straight line and its minimum 1 co-ordinate changes with time.

eg: A body dropped from a height
Motion of vehicle along straight line.
Motion of train along track.

ii Two dimensional motion (2-D motion) or motion in a plane:

- If a particle moves in a plane and its minimum two co-ordinate changes with time.

eg: Motion of queen on carrom board.
Projectile motion
Circular motion.

iii Three - dimensional motion (3-D motion) or motion in plane:

- If a particle moves in a space and all its co-ordinates changes with time.

eg: Motion of Aeroplane
Motion of Birds
Motion of kite

One dimensional motion:

a. Distance:
(1-DS) Pg - 2

- It is the actual path covered by a body in a given time interval.
- Its dimension is $[M^0 L^1 S^0]$ and SI unit is meter (m).
- It is a scaler quantity.
- For a moving body distance travelled always increases with time, it can't decrease.
- For a moving body, distance travelled is always positive, it can't be negative or zero.

- If the distance travelled by a particle is zero, or constant, then it must be in rest.

b. Displacement:
(1-DS) Pg-2

- A vector drawn from initial position to final position of a particle.
- The magnitude of displacement is the shortest distance covered between initial and final position.
- Its dimensions are $[M^0 L^1 T^0]$ and SI unit is meter (m).
- It depends upon the initial and final position and not upon the actual path.
- For a moving body, displacement can increase or decrease with time. It increases when body moves away from initial position and decreases when body moves towards initial position.
- For a moving body, it can be positive, negative or zero. It becomes zero when body ~~moves~~ returns back to the initial position.
- For the motion between two points, the displacement can have only one unique value but the distance travelled can have more than one - value depending upon the path.

Comparison in between distance and displacement:

Distance	Displacement
• It is the actual path between initial and final position.	It is the shortest path between initial and final position.
• Its dimensions are $[M^0L^1T^0]$ and SI unit is metre (m).	Its dimensions are $[M^0L^1T^0]$ and SI unit is metre (m)
• It is a scaler quantity	It is a vector quantity
• It can be positive or zero.	It can be positive, negative or zero.
• If zero, body is in rest or constant	If zero, body maybe in rest or its initial and final position concides with each other, when motion
• It always increases with time when in motion.	It may increase or decrease with time when in motion.

Relation between distance and displacement
(1-DS) Pg-3

• Distance travelled $\geq$ |displacement|

$$\therefore \frac{\text{distance travelled}}{|\text{displacement}|} \geq 1.$$

Position vector:
(1-DS) Pg-3

$$\vec{AB} = (\Delta x)\hat{i} + (\Delta y)\hat{j} + (\Delta z)\hat{k}$$

- $\therefore$ displacement can also be defined as the change in position vector $(\Delta \vec{r})$

Q.1. (1-DS) Pg-4

i) Displacement vector $= \Delta \vec{r}$

$$= (\Delta x)\hat{i} + (\Delta y)\hat{j} + (\Delta z)\hat{k}$$

$$= (4-1)\hat{i} + (8-2)\hat{j} + (3-1)\hat{k}$$

$$= (3\hat{i} + 6\hat{j} + 2\hat{k})\ m$$

ii) Minimum distance between initial and final positions $= |\overrightarrow{\text{displacement}}|$

$$|\vec{s}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= \sqrt{3^2 + 6^2 + 2^2}$$

$$= \sqrt{9+36+4} = \sqrt{49} = 7m.$$

iii) Distance travelled = can't be determined, because actual path is not specified.

Q.2 (1-DS) Pg-4
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i) A to B:

distance : $\frac{2\pi r}{4} = \frac{\pi r}{2}$

$|\overrightarrow{\text{displacement}}|$: $\sqrt{r^2 + r^2} = \sqrt{2r^2} = \sqrt{2}\,r$

ii) A to C:

distance : $\frac{2\pi r}{2} = \pi r$

$|\overrightarrow{\text{displacement}}|$: $r + r = 2r$

iii) A to D:

distance : $\frac{2\pi r}{2} + \frac{\pi r}{2} = \frac{3\pi r}{2}$

$|\overrightarrow{\text{displacement}}| = \sqrt{r^2+r^2} = \sqrt{2r^2} = \sqrt{2}\,r$

iv) A to A:

distance : $2\pi r$

$|\overrightarrow{\text{displacement}}| = 0.$

Q.3. (1-DS) Pg-4

$|\text{displacement}|$ = diagonal of the room.

$\therefore$ diagonal of cuboidal room $= \sqrt{l^2+b^2+h^2}$

$= \sqrt{7^2+4^2+4^2}$

$= \sqrt{49+16+16} = \sqrt{81}$

$= 9\text{ m}.$

Velocity and speed.

Average velocity ($\vec{v}_{avg}$)

(1-DS) Pg-4

- It is the ratio of displacement to the total time taken.

$\therefore \overrightarrow{v_{avg}} = \frac{\text{displacement}}{\text{Total time taken}} = \frac{\Delta\vec{r}}{\Delta t \text{ or } t} = \frac{\vec{r}_2-\vec{r}_1}{t_2-t_1}$

- It is a vector quantity whose direction is always along the displacement. (from initial to final position).

- Its dimensions are $[M^0L^1T^{-1}]$ and SI unit is m/s or ms^{-1}

Average speed. (V_{avg})

- It is the ratio of distance travelled to the total time taken.

$$\therefore V_{avg} = \frac{\text{Total distance covered}}{\text{Total time taken.}}$$

- It is a scalar quantity.
- Its dimensions are $[M^0L^1T^{-1}]$ and SI unit is m/s or ms^{-1}

- For a moving body, distance travelled can never decrease but average speed may decrease.

Relation between Average Velocity ($\vec{V}_{avg}$) and average speed (V_{avg}).

(1-DS) Pg - 4

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$$\because \frac{\text{Total distance travelled}}{\text{Time Taken}} \geq \frac{|\text{displacement}|}{\text{time taken}}$$

$$\therefore \text{Average speed} \geq |\text{Average velocity}|$$

$$\therefore \frac{\text{Average speed}}{|\text{Average velocity}|} \geq 1.$$

Instantaneous velocity:

(1-DS) Pg-5

- The velocity of an object at a particular time instant or moment.
- By making the time interval negligibly small ($\Delta t \to 0$), average speed gets converted into instantaneous velocity.

$$\therefore \vec{V}_{inst} \text{ or } \vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$\therefore$ Instantaneous rate of rate of change of position vector w.r.t to time = rate of displacement.

* The direction of average velocity is along the net displacement but the direction of instantaneous velocity is along tangential to the path.

Instantaneous speed:

(1-DS) Pg-5

- It is always equal to magnitude of instantaneous velocity because in a very small intervale of time, the particle will be moving in a straight line in same direction.

$\therefore$ ~~It~~ Instantaneous speed = | Inst. velocity |

OR

$$v = |\vec{v}|$$

Acceleration:
(1D-S) Pg-5

Average acceleration:

- It is the rate of change in velocity to the time taken.

$$\therefore \vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

- It is a vector quantity whose direction is along ~~change~~ the direction of change in velocity ($\Delta \vec{v}$)

- Its dimensions are $[M^0L^1T^{-2}]$ and SI unit is m/s^2 or ms^{-2}

Instantaneous acceleration:

- The acceleration of a particle at a particular time instant or moment.

$$\therefore \vec{a}_{inst} \text{ or } \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

- It can be positive, negative or zero and zero acceleration means that velocity is constant in both magnitude and direction.

* If velocity is constant, then speed must be constant and the path of the particle will be straight line but if speed is constant, velocity may or may not be constant.

Classification of speed, velocity and acceleration as uniform and non-uniform.
(1-DS) Pg-6

i) Uniform speed: constant speed.
ii) Non-uniform speed: variable speed.
iii) Uniform velocity: constant velocity
(both magnitude and direction are constant)
iv) Non-uniform velocity: variable velocity
(either magnitude or direction or both are changed).
v) Uniform acceleration: constant acceleration
(both magnitude and direction are constant)
vi) Non-uniform acceleration: variable acceleration
(either magnitude or direction or both are changed).

Q.4 (1-DS) Pg-6

i) Average speed : $\dfrac{\text{Total distance}}{\text{Total Time Taken.}}$

$$= \frac{\pi r}{5} = \frac{\pi \times 10}{5} = 2\pi$$

$$= 2 \times 3.14 \text{ m/s} = 6.28 \text{ m/s}$$

ii) |Average velocity| : $\dfrac{\text{Total displacement}}{\text{Total Time Taken.}}$

$$= \frac{2r}{5} = \frac{2 \times 10}{5} = 4 \text{ m/s.}$$

Q.5. (1-DS) Pg-6

i) displacement: final position − initial position.

$$-40 - 0 = -40 \cdot m.$$

~~∵ displacement can't be negative.~~

i.e. displacement = 40 m along $(-x)$ axis.

ii) Average velocity : $\dfrac{\overrightarrow{\text{displacement}}}{\text{Time taken.}}$

$$= \frac{-40\,m}{10\,s} = -4\,m/s.$$

i.e. 4 m/s along $(-x)$ axis.

iii) Total distance travelled: 30 m + 30 m + 40 m.

$$= 100\ m$$

iv) Average speed : $\dfrac{\text{Total distance}}{\text{Total Time Taken.}}$

$$= \frac{100\,m}{10\,s} = 10\,m/s.$$

Q.6. (1-DS) Pg-6

$$A_x = A\cos\theta = A\cos 30^\circ = 40 \times \frac{\sqrt{3}}{2} = 20\sqrt{3}\,\hat{i}$$

$$A_{\hat{y}} = A\sin\theta = A\sin 30^\circ = 40 \times \frac{1}{2} = 20\,\hat{j}$$

$$\therefore \overrightarrow{OA} = 20\sqrt{3}\,\hat{i} + 20\,\hat{j}$$

• In real direction:

$\overrightarrow{OA}$ = 40 m due E 30° N

read as 'at 30° from east towards north' OR

$\overrightarrow{OA}$ = 40 m due N 60° E

read as 'at 60° from north towards east'.

Q.7 (1-DS) Pg-6.

$Ax = A\cos\theta = A\cos 45^\circ = 100 \times \frac{1}{\sqrt{2}} = 50\sqrt{2}\,\hat{i}$

$Ay = A\sin\theta = A\sin 45^\circ = 100 \times \frac{1}{\sqrt{2}} = 50\sqrt{2}\,\hat{j}$

$\therefore \vec{OA} = 50\sqrt{2}\,\hat{i} + 50\sqrt{2}\,\hat{j}$

• In real direction,

$\vec{OA}$ = 100 m due E 45° N. **OR**

= 100 m due N 45° E **OR**

= 100 m due E-N

* If angle is 45°, then components are same. and if the components are same, then the angle must be 45°

Q.8 (1-DS) Pg-7

In real direction,

$|\vec{r}| = \sqrt{Ax^2 + Ay^2} = \sqrt{4^2 + 3^2}$

$\sqrt{16+9} = \sqrt{25} = 5\,m$

for direction,

$\tan\theta = \frac{3}{4} = \tan 37^\circ$ OR $\left[\cos\theta = \frac{Ax}{A} = \frac{4}{5} = \cos 37^\circ\right]$

$\therefore \theta = 37^\circ$ $\therefore \theta = 37^\circ$

$\therefore$ real direction = 5m due E 37° N.

Q.9 (1-DS) Pg-7

In real direction,

$|\vec{r}| = \sqrt{Ax^2 + Ay^2} = \sqrt{3^2 + (-2)^2}$

$\sqrt{9+4} = \sqrt{13}$ m.

for direction,

$\tan\theta = \frac{2}{3}$.

$\therefore \theta = \tan^{-1}\left(\frac{2}{3}\right)$

$\therefore$ real direction $= \sqrt{13}$ m due E $\tan^{-1}\left(\frac{2}{3}\right)$ ~~B~~.S.

Q.10 (J-DS) Pg-7

$d_1 = 30\text{ m } (\hat{j})$

$d_2 = 20\text{ m } (\hat{i})$

$d_3 = 20\sqrt{2}$ m along S-W direction.

$20\sqrt{2}\cos 45^\circ(-\hat{i}) + 20\sqrt{2}\sin 45^\circ(-\hat{j})$

$(-20\hat{i} - 20\hat{j})$ m

OR

$20\sqrt{2}$ m along S-W direction

$20\sqrt{2} \times \left(\frac{-\hat{i}-\hat{j}}{\sqrt{2}}\right) = (-20\hat{i} - 20\hat{j})$ m

$\therefore \vec{d_{net}} = \vec{d_1} + \vec{d_2} + \vec{d_3}$

$= 30\hat{j} + 20\hat{i} - 20\hat{i} - 20\hat{j}$

$= 30\hat{j} - 20\hat{j} = 10\hat{j}$ (m)

10 metre along north.

~~Q.12~~ * Direction convention:

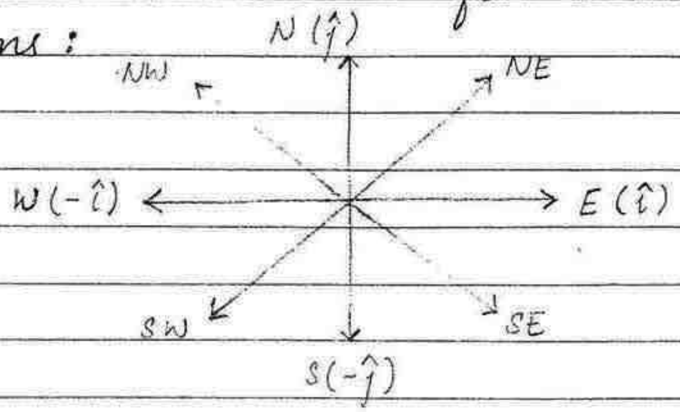
i) $\hat{i} + \hat{j} = \sqrt{2}$ m due N-E

ii) $\hat{i} - \hat{j} = \sqrt{2}$ m due S-E

iii) $-\hat{i} + \hat{j} = \sqrt{2}$ m due N-W

iv) $-\hat{i} - \hat{j} = \sqrt{2}$ m due S-W

Important unit vectors for standard directions :



i) North : $\hat{j}$
ii) South : $-\hat{j}$
iii) East : $\hat{i}$
iv) West : $-\hat{i}$
v) North-east : $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$
vi) North-west : $\frac{-\hat{i} + \hat{j}}{\sqrt{2}}$
vii) South-east : $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$
viii) South-west : $\frac{-\hat{i} - \hat{j}}{\sqrt{2}}$
ix) Upward / Outward : $\hat{k}$
x) Downward / Inward : $-\hat{k}$

Q 11 (1-DS) Pg-7

i) distanced travelled : $|d_1| + |d_2|$

$= 20\,m + 40\,m = 60\,m$

ii) Average speed : $\frac{\text{Total distance covered}}{\text{Total Time Taken}}$

$= \frac{60\,m}{5\,s} = 12\,m/s.$

(iii) Displacement : $\vec{d_{net}} = \vec{d_1} + \vec{d_2}$

$= 20\hat{i} - 40\hat{j}$

in real direction,

$\sqrt{20^2 + 40^2}$ m due E $\tan^{-1}\left(\frac{40}{20}\right)$ S

$= 20\sqrt{5}$ m due E $\tan^{-1}(2)$ S

(iv) Average velocity: $\frac{\text{Total displacement}}{\text{Total time taken}}$

$= \frac{20\sqrt{5}}{5} = 4\sqrt{5}$ m due E $\tan^{-1}(2)$ S.

Q.12 (1-DS) Pg-7

☆☆

~~i) disp~~ $\vec{d_1} = vt\,(\hat{i})$ $[\because v_1 = v\hat{i}]$

$\vec{d_2} = vt\,(\hat{j})$ $[\because v_2 = v\hat{j}]$

i) displacement: $\vec{d_{net}} = \vec{d_1} + \vec{d_2}$

$= vt(\hat{i}) + vt(\hat{j})$

in real direction.

$= \sqrt{(vt)^2 + (vt)^2}$ m due E 45° N.

$= vt\sqrt{2}$ due E-N

ii) Average velocity: $\frac{\text{Total displacement}}{\text{Total time taken}}$

$= \frac{vt\sqrt{2}}{t+t} = \frac{vt\sqrt{2}}{2t} = \frac{v\sqrt{2}}{2}$

$= \frac{v}{\sqrt{2}}$ ~~m~~ due E-N.

iii Total distance travelled : $|d_1| + |d_2|$

$= vt + vt$

$= 2vt$

iv) Average speed : $\dfrac{\text{total distance}}{\text{total time taken}}$

$$= \frac{2vt}{2t} = v \cancel{m/s}.$$

v) Change in velocity $(\Delta \vec{v})$: $\vec{v}_f - \vec{v}_i$

$$= v\hat{j} - v\hat{i}$$

in real directions

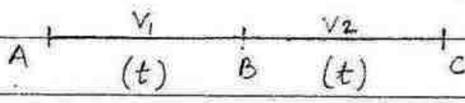
$$= \sqrt{v^2 + v^2} \text{ due } N\ 45^\circ\ W$$

$$= v\sqrt{2} \text{ due } N\text{-}W$$

vi) Average acceleration : $\dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{v\sqrt{2}}{t+t}$

$$= \frac{v\sqrt{2}}{2t} = \frac{v}{t\sqrt{2}} \text{ due } N\text{-}W.$$

Q.13. (1-DS) Pg. 7



$\because$ $d_1 = v_1 t$

$d_2 = v_2 t$

$\therefore$ Average speed = $\dfrac{\text{Total distance covered}}{\text{Total Time Taken}}$

$$= \frac{v_1 t + v_2 t}{t + t} = \frac{v_1 t + v_2 t}{2t}$$

$$= \frac{v_1 + v_2}{2} \quad \text{(Arithmatic Mean).}$$

* If a particle travels with speed $v_1, v_2, v_3 \ldots v_n$ for <u>equal time interval</u> then the average speed for complete motion is 'arithmatic mean' of all the given speed.

* $V_{avg} = \frac{v_1 + v_2 + v_3 \dots v_n}{n}$

Q.14 (1-DS) Pg-7

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

$$= \frac{v_1 t + v_2 t}{t + t} = \frac{t(v_1 + v_2)}{2t}.$$

$$\therefore V_{avg} = \frac{v_1 + v_2}{2}$$

$$= \frac{40 + 80}{2} = 60 \text{ km/hr.}$$

Q.15 (1-DS) Pg-7

$$V_{avg} = \frac{v_1 + v_2 + v_3}{3}$$ [∵ the particle covers in equal time interval]

$$= \frac{50 + 60 + 55}{3} = \frac{165}{3} = 55 \text{ km/h}$$

Q.16 (1-DS) Pg-7

$$\text{average speed} = \frac{\text{total distance covered.}}{\text{total time taken}}$$

$$= \frac{d + d}{\frac{d}{v_1} + \frac{d}{v_2}}$$

$$= \frac{2d}{\frac{dv_2 + dv_1}{v_1 v_2}} = \frac{2d(v_1 v_2)}{d(v_1 + v_2)} = \frac{2v_1 v_2}{v_1 + v_2}$$

(Harmonic mean)

* If a particle travels equal distances with different speed $v_1, v_2, v_3 \dots v_n$ then the average speed for complete motion is harmonic mean for the given speed.

$$\frac{n}{V_{avg}} = \frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} \dots \frac{1}{v_n}$$

Q.17 (1-DS) Pg-7

$$\text{Average Speed} = \frac{\text{Total distance covered}}{\text{Total Time Taken}}$$

$$= \frac{d+d}{\frac{d}{20} + \frac{d}{30}}$$

$$= \frac{2d}{\frac{30d+20d}{600}} = \frac{2d \times 600}{50d}$$

$$= \frac{1200}{50} = 24 \text{ km/h.}$$

OR

$$\frac{2}{V_{avg}} = \frac{1}{v_1} + \frac{1}{v_2}$$ [∵ the particle covers equal distance]

$$\frac{2}{V_{avg}} = \frac{1}{20} + \frac{1}{30}$$

$$= \frac{30+20}{600} = \frac{2}{V_{avg}}$$

$$\therefore V_{avg} = \frac{600 \times 2}{50} = 24 \text{ km/h}$$

Q.18. (1-DS) Pg-7

$$\frac{n}{V_{avg}} = \frac{1}{v_1} + \frac{1}{v_2} \quad \left[\because \text{he travels equal distances}\right]$$

$$\frac{2}{V_{avg}} = \frac{1}{40} + \frac{1}{60}$$

$$\frac{2}{V_{avg}} = \frac{60+40}{2400}$$

$$\therefore V_{avg} = \frac{2400 \times 2}{100}$$

$$= 48 \text{ km/h}$$

Q.19
$$\frac{n}{V_{avg}} = \frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3} \quad \left[\because \text{the particle covers equal distances}\right]$$

$$\frac{3}{V_{avg}} = \frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3}.$$

$$\frac{3}{V_{avg}} = \frac{\cancel{V_2 + V} \; V_2V_3 + V_1V_3 + V_1V_2}{V_1V_2V_3}$$

$$\therefore V_{avg} = \frac{3V_1V_2V_3}{V_2V_3 + V_1V_3 + V_1V_2}$$

Kinematic equation of uniformly accelerated motion or equations of motion:

(1-DS) Pg-9

In scaler form:

i) $v = u + at$

ii) $s = ut + \frac{1}{2}at^2$

iii) $v^2 = u^2 + 2as$

In vector form:

i) $\vec{v} = \vec{u} + \vec{a}t$

ii) $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

iii) $\vec{v}\cdot\vec{v} = \vec{u}\cdot\vec{u} + 2\vec{a}\cdot S$

Q.20 (1-DS) Pg-10

$\vec{u} = (\hat{i} + \hat{j})\ m/s.$

$\vec{a} = (0.2\hat{i} + 0.3\hat{j})\ m/s^2$

$t = 10\ s$

i) final velocity:

using (i) eq of motion,

$\vec{v} = \vec{u} + \vec{a}t$

$= (\hat{i}+\hat{j}) + (0.2\hat{i} + 0.3\hat{j})\ 10 \quad m/s$

$= (\hat{i} + \hat{j} + 2\hat{i} + 3\hat{j})\ m/s$

$= (3\hat{i} + 4\hat{j})\ m/s$

ii) final speed.

$\because V_{inst} = |\vec{V}_{inst}|$

$\therefore V_{inst} = \sqrt{3^2 + 4^2} = \sqrt{9+16}$

$= \sqrt{25} = 5 m/s.$

Q.21 (1-DS) Pg-10

$m = 2kg$

$u = 0$

$\vec{F} = (2\hat{i} + 3\hat{j} + 4\hat{k})\ N$

$\because \vec{F} = m\vec{a}$

$\therefore \vec{a} = \dfrac{\vec{F}}{m} = \dfrac{(2\hat{i} + 3\hat{j} + 4\hat{k})\ N}{2}$

i) displacement in 4 sec.

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$= 0 \times 4 + \frac{1}{2}\frac{(2\hat{i} + 3\hat{j} + 4\hat{k})}{2} \times 4^2$

$= 4(2\hat{i} + 3\hat{j} + 4\hat{k})\,m$
$= (8\hat{i} + 12\hat{j} + 16\hat{k})\,m$

ii) Final position vector after 4 sec.

$\vec{r}_f - \vec{r}_i = \vec{s}$

$\therefore \vec{r}_f = \vec{s} + \vec{r}_i$

$= (8\hat{i} + 12\hat{j} + 16\hat{k})\,m + (0\hat{i} + 0\hat{j} + 0\hat{k})\,m$

$= (8\hat{i} + 12\hat{j} + 16\hat{k})\,m$

iii) Final co-ordinates after 4 sec

$(8, 12, 16)$

~~Q.22~~ When a particle starts its motion from origin then its final position vector becomes equal to displacement.

Q.22 (1-DS) Pg-9

$u = 0$
$a = \text{uniform } a$
$s_1 = s$

i) distance travelled by it in first 2 sec:

$\because s_1 = ut + \frac{1}{2}at^2$

$= 0 \times 1 + \frac{1}{2} a \times 1^2 = s$

$\therefore \frac{a}{2} = s$

$\therefore a = 2s.$

when $t = 2s$,

i.e. $s_2 = ut + \frac{1}{2}at^2$

$= 0 \times 2 + \frac{1}{2}(2S)\, 2^2 \text{ m}$

$= (2S)\,2 \text{ m} = 4S \text{ m}$

ii) distance travelled in 2nd sec:

$S_{2nd} = S_2 - S_1$

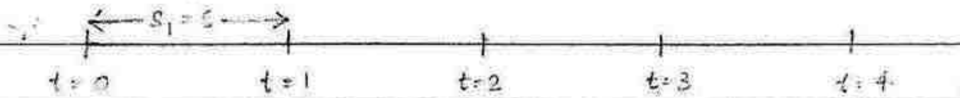
$= 4S - S = 3S$

Q.23. (1-DS) Pg-9

$u = 0$

$a =$ uniform a

$S_1 = S$



$\therefore$ distance travelled in next 3 sec, i.e. $t = 4$

$S_4 = 0 \times 4 + \frac{1}{2} a\, \cancel{2}^2\, 4^2$

$= \cancel{2a}\ 4a\ S$

$\because S_1 = 0 + \frac{a}{2} = S.$

$\therefore a = 2S.$

$S_4 = 2(2a) = \cancel{(2 \times 2S)2} = (4 \times 2)a = 8a = 16S$

$S_1 = S.$

$S_{\text{in next 3 sec}} = S_4 - S_1$

$\cancel{= 4S - S = 3}$

$16S - S = 15S$

Q.24 (1-DS) Pg-9

$u = 0$

$a = \cancel{\text{uniform } a}\ 2 \text{ m/s}^2$

i) $S_4 : ut + \frac{1}{2}at^2$ m

$= 0 \times 4 + \frac{1}{2} \times 2 \times 4^2$ m

$= \frac{4^2}{\cancel{4}} = 16$ m

ii) $S_5 : 0 \times 5 + \frac{1}{2} \times 2 \times 5^2$ m

$= 5^2 = 25$ m

iii) $S_{5th} : S_5 - S_4$

$= 25\text{ m} - 16\text{ m} = 9$ m

To find the distance travelled in nth sec.

(1-DS) Pg-9

$$S_{nth} = u + \frac{a}{2}(2n-1),$$

where n is serial number of second.

Q.25 (1-DS) Pg-10

$u = 10$ m/s

$a = 2$ m/s^2

i) $S_7 : 10 \times 7 + \frac{1}{2} \times 2 \times 7^2$ m

$= (70 + 49)$m $= 119$ m

ii) $S_{7th} : u + \frac{a}{2}(2n-1)$

$= 10 + \frac{2}{2}(2 \times 7 - 1)$ m

$= 10 + (14 - 1)$ m $= 13$ m $+ 10$ m $= 23$ m

iii) S_6 : ~~10 + 6~~ $10 \times 6 + \frac{1}{2} \times 2 \times 6^2$

$= (60 + 36)$ m $= 96$ m

OR

S_6 : $S_7 - S_{7th}$

$= 119$ m $- 23$ m $= 96$ m

Q.26 (1-DS) Pg-10

$u = 0$

$a =$ uniform a

$S_n = ut + \frac{1}{2} at^2$ m

$= 0 \times n + \frac{1}{2} \times a \times n^2$ m

$= \frac{an^2}{2}$ m

$S_{nth} = u + \frac{a}{2}(2n - 1)$

$= 0 + \frac{a}{2}(2n-1) = \frac{a}{2}(2n-1)$ m

$\therefore$ $S_n : S_{nth}$

$= \frac{an^2}{2} : \frac{a}{2}(2n-1)$

$= n^2 : (2n-1)$

Q.27 (1-DS) Pg-10

$u = 0$

$a =$ uniform a

i) In first 1 sec, first 2sec, first 3 sec .... n sec.

$S_1 : S_2 : S_3 : S_4 \; .... \; S_n$

$ut_1 + \frac{1}{2}at_1^2 : ut_2 + \frac{1}{2}at_2^2 : ut_3 + \frac{1}{2}at_3^2 \; ...... \; ut_n + \frac{1}{2}at_n^2$

$\because u = 0.$

$\therefore ut_1 = ut_2 = ut_3 \; .... \; ut_n = 0$

$= t_1^2 : t_2^2 : t_3^2 : t_4^2 \; ...... \; t_n^2$

$1 : 4 : 9 : 16 \; ...... \; n^2$

Hence, it is the ratio of square of natural numbers.

ii) In 1^{st} sec, 2^{nd} sec, 3rd sec ...... nth sec

$S_{1st} : S_{2nd} : S_{3rd} : S_{4th} \; .... \; S_{nth}$

$u + \frac{a}{2}(2n_1 - 1) : u + \frac{a}{2}(2n_2 - 1) : u + \frac{a}{2}(2n_3 - 1) \; .... \; u + \frac{a}{2}(2n_n -$

$\because u = 0$

$\therefore (2n_1 - 1) : (2n_2 - 1) : (2n_3 - 1) : \; ...... (2n_n - 1)$

$1 : 3 : 5 : 7 : 9 \; ...... \; (2n-1)$

Hence, it is the ratio of successive odd no.

* Both the above ratio are applicable only if $u = 0$ and $a =$ constant

If a particle starts from rest and moves with uniform acceleration, then the ratio of distance travelled in successive equal time intervals is $1 : 3 : 5 : 7 \; ...... \; (2n-1)$

Q.28 (1-DS) Pg-10

$u = 0$

$a =$ uniform a

$S_{4th} : S_{5th}$

Using ratio of $(2n_4 - 1) : (2n_5 - 1)$

$(2 \times 4 - 1) : (2 \times 5 - 1)$

$7 : 9$

Q.29 (1-DS) Pg-10

$$\left.\begin{array}{rl} \text{initial velocity} : & u_1 \\ \text{final velocity} : & 0 \\ s : & s_1 \end{array}\right] \text{case I}$$

$\because v^2 = u^2 + 2as.$

$$\therefore \quad a = \frac{v^2 - u^2}{2s} = \frac{0 - u_1^2}{2s_1} = \frac{-u_1^2}{2s_1}$$

$$\left.\begin{array}{rl} \text{initial velocity} : & u_2 \\ \text{final velocity} : & 0 \\ a : & \dfrac{-u_1^2}{2s_1} \end{array}\right] \text{case II}$$

$$\therefore \quad S_2 = \frac{v_2^2 - u_2^2 \times 2S_1}{2 \times -u_1^2}$$

$$= \frac{0 - u_2^2 \times S_1}{-u_1^2}$$

$$= \frac{-u_2^2 \times S_1}{-u_1^2}$$

$$S_2 = \left(\frac{u_2}{u_1}\right)^2 \times S_1$$

Q.30 (1-DS) Pg-10

$\because \quad S_2 = \left(\frac{u_2}{u_1}\right)^2 \times S_1$

$\therefore \quad S_2 = \left(\frac{60}{30}\right)^2 \times 10$

$= 4 \times 10$

$= 40 \text{ m}$

Q.31 (1-DS) Pg-10

$u = 0$

$v = 18 \text{ km/h} = 18 \times \frac{5}{18} \text{ m/s} = 5 \text{ m/s}$

$t = 5 \text{ sec}$

$\therefore \quad a = \frac{v-u}{t} \Rightarrow \frac{5-0}{5} = \frac{5}{5} = 1 \text{ sec}$

$\because \quad v^2 = u^2 + 2as$

$\therefore \quad s = \frac{v^2 - u^2}{2a} = \frac{25-0}{2 \times 1}$

$= \frac{25}{2} = 12.5 \text{ m}$

OR

Short trick:

$S \neq$ relation between displacement, time, constant acceleration and their velocities

$$S = \left(\frac{u+v}{2}\right) t$$

- It is applicable only when there is unknown constant acceleration.

$\therefore \quad s = \left(\frac{0+5}{2}\right) 5 = \frac{25}{2} = 12.5 \text{ m}$

Stopping distance: When the brakes are applied, distance travelled by a vehicle before stopping.

$$\therefore s \propto u^2$$

$$s_2 = \left(\frac{u_2}{u_1}\right)^2 \times s_1$$

Reaction time: Time taken by a person to think and act is known as 'Reaction time'

$$s = \left(\frac{u+v}{2}\right) t$$

Vertical motion under gravity:

C (1-D3) Pg-10

- Gravitational ~~fro~~ force of attraction is known as gravity.
- Acceleration due to gravity is the acceleration produced in a body due to gravitational attraction of earth.

$$g = 9.8\ m/s^2 \approx 10\ m/s^2 \ (\text{In MKS})$$

$$g = 980\ cm/s^2 \approx 1000\ cm/s^2 \ (\text{In CGS})$$

$$g = 32\ ft/s^2 \ (\text{FPS})$$

- The direction of acceleration due to gravity is always vertically downward irrespective of upward or downward motion of body.

- If a body is dropped or ~~reo~~ released or it starts falling freely, initial velocity is taken as zero. ($u=0$)

- When a hydrogen balloon or a rocket starts its motion, initial velocity is taken as zero. ($u=0$)

- If a body is projected / thrown / fired, then its initial velocity is ~~is~~ non-zero. ($u \neq 0$)

Important conclusions :

- If air resistance and air friction is neglected, then ascending time is equal to descending time

- speed and acceleration remain constant at any point of ascending and descending but the direction of the velocity changes. (reverses)

- The speed of the body one second before and one second after the top most position are same

- At the top most position, velocity is zero but acceleration is non-zero, means it is possible to have zero velocity with non-zero acceleration

- $x_{I} > x_{II} > x_{III}$ It means that the distance travelled by body in successive one second cannot be equal in one particular direction

- The distances travelled by body in the last second of ascending motion and in the first second of descending motion are equal. (nearly 5 m)

- Hence, the distances travelled in last 'n' seconds of ascending motion and first 'n' seconds of descending motion are same

- During the ascending, motion speed decreases and while descending motion speed increases, It is because, when acceleration is in the direction of velocity, it increases speed but when acceleration is in opposite direction of velocity, it decreases.

eg: If v = +ve and a = +ve, then speed ↑
If v = -ve and a = -ve, then speed ↑
If v = +ve and a = -ve, then speed ↓
If v = -ve and a = +ve, then speed ↓

Q.32 (1-DS) Pg - 11

Let downward direction be +ve

i) $u = 0, \quad s = +h, \quad a = g$

$\because s = ut + \frac{1}{2}at^2$

$\therefore h = \frac{1}{2} \times gt^2$

$\therefore t^2 = \frac{h \times 2}{g} \Rightarrow \boxed{t = \sqrt{\frac{2h}{g}}}$

to find time taken by a body to fall 'h' height.

ii) $v^2 = u^2 + 2as$

$= 0 + 2gh$

$\therefore v = \sqrt{2gh}$ downwards

OR

$v = u + at$

$= 0 + g \times \sqrt{\frac{2h}{g}}$

$= \sqrt{2gh}$

Q. 33 (1-DS) Pg-12

let downward direction be +ve

$u = 0, \quad t = 5\ sec, \quad g = 10\ m/s^2, \quad s = h$

i) $h = \frac{1}{2} \times 10 \times 5^2$

$= \frac{1}{2} \times 10 \times 5 \times 5 = 125\ m$

ii) $v = u + gt$

$0 + 10 \times 5 = 50\ m$ downwards.

Q. 34 (1-DS) Pg-12

let downward direction be +ve

$u = 0, \quad h = 80\ m, \quad g = 10\ m/s^2$

i) $s = ut + \frac{1}{2}at^2$

$80 = \frac{1}{2} \times 10 \times t^2$

$\therefore 80 = 5t^2$

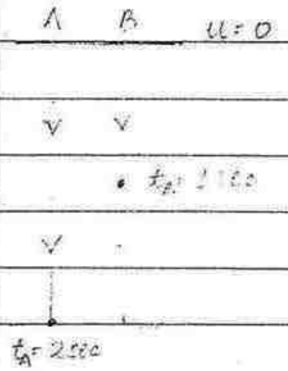
$\therefore t = \sqrt{\frac{80}{5}} = \sqrt{16} = 4\ sec$

ii) $v = u + gt$

$0 + 10 \times 4 = 40$ m downwards.

Q. 35 (1-DS) Pg-12

let downward direction be +ve



$u = 0$; $t_B = 1$ sec and $t_A = 2$ sec

$a = g = 10\ m/s^2$

$\because h_A = ut + \frac{1}{2} g t_A^2$

$= \frac{1}{2} \times 10 \times 2^2$

$= 20$ m and.

$h_B = ut + \frac{1}{2} g t_B^2$

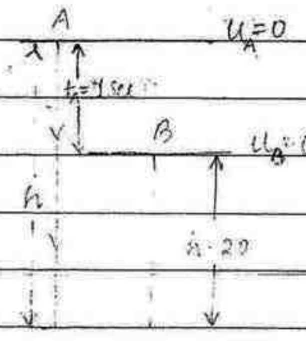
$= \frac{1}{2} \times 10 \times 1^2 = 5m$

$\therefore$ Seperation distance $= 20m - 5m$

$= 15$ m

Q. 36. (1-DS) Pg-12

let downward direction be +ve



$u_A = 0$, $u_B = 0$, $t_A = t$ sec

$S_A = h$, $S_B = h - 20$, $t_B = (t-1)$ sec

$h_A = ut + \frac{1}{2} g t_A^2$

$h = \frac{1}{2} \times 10 \times t^2 = 5t^2$

$h = 5t^2$ — (1)

$$h_B = ut + \frac{1}{2} g t_B^2$$

$$h - 20 = \frac{1}{2} \times 10 \times (t-1)^2$$

$$h - 20 = 5(t^2 + 1 - 2t)$$
$$= 5t^2 + 5 - 10t$$

from eq (1).

$$-20 + 5t^2 = 5t^2 + 5 - 10t$$
$$5t^2 = 5t^2 + 20 + 5 - 10t$$
$$25 = 10t$$
$$\therefore t = \frac{25}{10} = 2.5 \text{ sec}$$

h_A, height of tower $= 5t^2$
$= 5 \times (2.5)^2$
$= 5 \times 6.25 = 31.25$ m.

Q.37 (1-D) Eg-12

i) $S_1 : S_2 : S_3 : S_4 : \ldots\ldots$

$\because u = 0$ and a = constant

$\therefore$ distance travelled is the ratio of square of the natural numbers.

$1 : 4 : 9 : 16 : 25 \ldots\ldots$

ii $S_{1st} : S_{2nd} : S_{3rd} : S_{4th} \ldots\ldots$

$\because u = 0$ and a = constant in equal time interval

$\therefore$ distance travelled is the ratio of consecutive odd no.

$1 : 3 : 5 : 7 : 9 \ldots\ldots$

* It is also known as Galileo's law of odd numbers

Q.38 (1-BS) Pg-12

let ↓ direction be +ve

$h = 5m$, $g = 10\ m/s^2$ $u = 0$.

<u>for 1st drop.</u>

$$S_1 = ut + \frac{1}{2} \times g t^2$$

$$5 = \frac{1}{2} \times 10\ t^2$$

$$\therefore t^2 = 1$$

$$\therefore t = 1 sec$$

∵ Two drops are falling in 1 sec

∴ one drop will fall in half sec.

<u>for 2nd drop,</u>

$t = \frac{1}{2} sec$, $u = 0$, $g = 10 m/s^2$

$$\therefore S_2 = \frac{1}{2} \times 10 \times \frac{1}{2} \times \frac{1}{2} = \frac{5}{4} m = 1.25m$$

∴ height of 2nd drop from the ground :

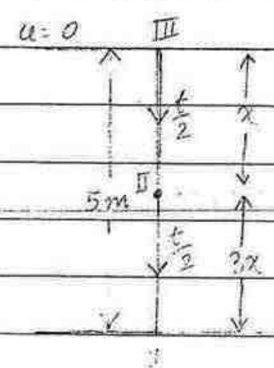
$$5 - \frac{5}{4} = \frac{20-5}{4} = \frac{15}{4} = 3.75m$$

OR

∵ $u = 0$, $a = g$ = constant and time intervals are equal

∴ Ratio of their distances travelled = 1:3

acc to Que.



$$x + 3x = 5$$

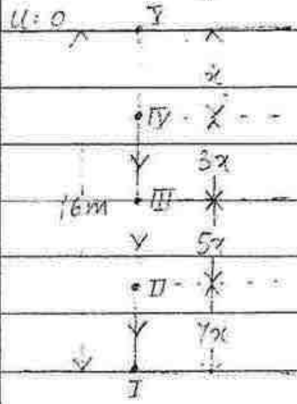
$$4x = 5$$

$$\therefore x = \frac{5}{4} m = 1.25 m$$

∴ height of IInd drop from ground = 3x

$3 \times 1.25\ m = 3.75\ m$

Q.39 (I-DS) Pg - 12



let downward direction be +ve

$\because$ $u = 0$, $a = g$ = constant,

~~$\therefore$ ratio~~ at equal time interval

$\therefore$ Ratio of their distances travelled = 1:3:5:7

acc to que

$$x + 3x + 5x + 7x = 16$$

$$\therefore 16x = 16$$

$$\therefore x = 1 \text{ m}$$

height of IInd drop from ground = $7x$ = 7m

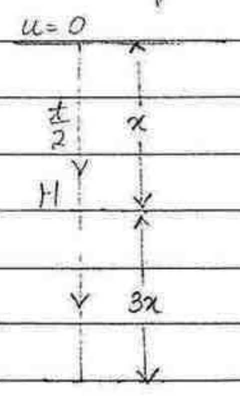
height of IIIrd drop from ground = $7x + 5x$

= 7+5 = 12m

height of IVth drop from the ground = $7x + 5x + 3x$

= 7 + 5 + 3 = 15 m ~~= 7 + 12 + 15 =~~

Q.40 (I-DS) Pg - 12



let downward direction be +ve

$S = H$, $u = 0$, $t = t$, $g = 10 \text{ m/s}^2$

$$\therefore S = ut + \frac{1}{2}gt^2$$

$$H = \frac{1}{2} \times 10 \times t^2$$

$$\therefore H = 5t^2 \quad \text{—(I)}$$

for 1st $\frac{t}{2}$ time,

$$\therefore x = \frac{1}{2} \times 10 \times \frac{t}{2} \times \frac{t}{2}$$

$$x = \frac{5t^2}{4} \quad \text{—(II)}$$

from eq (i) and (ii), we get

$$x = \frac{5 \times H}{4 \times 5}$$

$$\therefore \; x = \frac{H}{4}$$

height of the body from the ground =

$$H - \frac{H}{4} = \frac{4H - H}{4} = \frac{3H}{4}$$

OR

$u = 0$, $a = g$ = constant at equal time interval

$\therefore$ Ratio of their distances travelled = 1 : 3

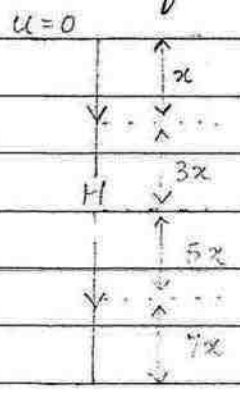
$$x + 3x = H$$

$$4x = H$$

$$\therefore \; x = \frac{H}{4} \quad \text{and}$$

$$3x = \text{height from ground} = 3 \times \frac{H}{4} = \frac{3H}{4}$$

Q. 40 + A body is dropped from height 'H'. It reaches the ground in 't' time. Find the height from the ground after $\frac{t}{4}$ time of releasing it.



let downward direction be +ve

$u = 0$, a = constant at equal time interval

$\therefore$ Ratio of their displacement = 1 : 3 : 5 : 7

$$x + 3x + 5x + 7x = H$$

$$16x = H$$

$$x = \frac{H}{16}$$

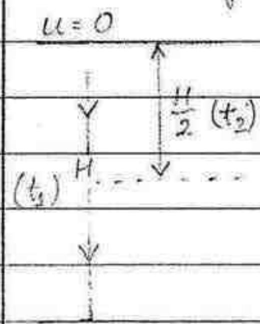
$\therefore$ height from the ground after $\frac{t}{4}$ time =

$$3x + 5x + 7x = 15x$$

$$\because x = \frac{H}{16}$$

$$\therefore 15x = 15 \times \frac{H}{16} = \frac{15H}{16}$$

Q-41 (J-DS) Pg-12



Let downward direction be +ve

$u=0$, $s = H$, time $= t_1$, $g = 10\ m/s^2$

$$s = ut + \frac{1}{2}at^2$$

$$H = \frac{1}{2} \times 10 \times t_1^2$$

$$\therefore H = 5t_1^2 \quad \text{—— (I)}$$

distance travelled in first half time $= \frac{H}{2}$

$$\therefore \frac{H}{2} = \frac{1}{2} \times 10 \times t_2^2$$

$$\therefore H = 10t_2^2 \quad \text{—— (II)}$$

from equation (I) and (II)

$$5t_1^2 = 10t_2^2$$

$$\therefore t_2^2 = \frac{5t_1^2}{10} = \frac{t_1^2}{2}$$

$$\therefore t_2 = \sqrt{\frac{t_1^2}{2}} = \frac{t_1}{\sqrt{2}}$$

OR

$u = 0$, $s = H$, time $= t_1$, $g = 10\ m/s^2$

$\therefore \quad H = \frac{1}{2} \times 10 \; t_1^2$

$H = 5t_1^2$ — (1)

Using $t = \sqrt{\frac{2h}{g}}$ for first $\frac{H}{2}$ ~~time~~ distance

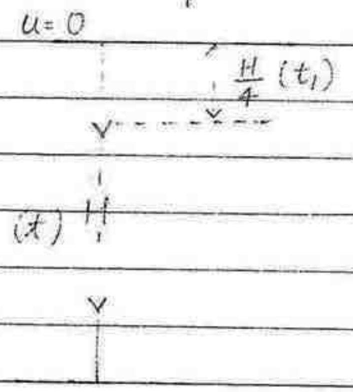
$\therefore \; h = \frac{H}{2}$, $t = t_2$ and $g = 10 m/s^2$

$t_2 = \sqrt{\frac{2 \times H}{10 \times 2}} = \sqrt{\frac{H}{10}}$

$= \sqrt{\frac{5t_1^2}{10}}$ from eq (1)

$\therefore \; t_2 = \frac{t_1}{\sqrt{2}}$

Q.42 (1-DS) Pg-12



Let downward direction be +ve

$u = 0, \; S = H, \; g = 10 \; m/s^2, \; t = t$

$\therefore \; S = ut + \frac{1}{2} g t^2$

$H = \frac{1}{2} \times 10 \times t^2$

$\therefore \; H = 5t^2$ — (1)

distance travelled in first one-forth time $= \frac{H}{4}$

$\therefore \; \frac{H}{4} = \frac{1}{2} \times 10 \times t_1^2$

$\therefore \; H = 20 \, t_1^2$ — (II)

from eq (1) and (II)

$5t^2 = 20 \, t_1^2$

$t^2 = 4 \, t_1^2$

$\therefore \; t_1 = \sqrt{\frac{t^2}{4}} = \frac{t}{\sqrt{4}} = \frac{t}{2}$ OR

$$H = \frac{1}{2} \times 10\, t^2$$

$$H = 5t^2 \quad — (1)$$

Using $t = \sqrt{\frac{2h}{g}}$ for first $\frac{H}{4}$ distance.

$$h = \frac{H}{4}, \quad t = t_1, \quad g = 10\, m/s^2$$

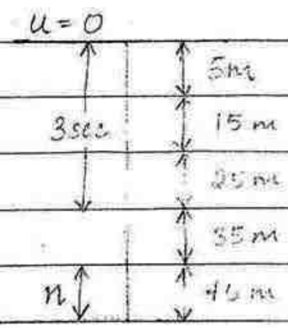
$$t_1 = \sqrt{\frac{2 \times H}{10 \times 4}}$$

$$= \sqrt{\frac{5t^2}{20}} \quad \text{from eq (1)}$$

$$= \sqrt{\frac{t^2}{4}} = \frac{t}{2}$$

Q. 43. (J-DS) Pg - 12

★



i) $S_{nth} = S_3$

$$u + \frac{a}{2}(2n-1) = ut + \frac{1}{2}at^2$$

$\because u = 0$

$$\therefore \frac{a}{2}(2n-1) = \frac{a}{2}(t)^2$$

$$= 2n - 1 = 3^2$$

$$= 2n = 9 + 1 = 10$$

$$\therefore n = 5$$

$\because$ 5th second is the last second.

$\therefore$ the body has fallen for 5 sec

OR

distance travelled in first 3 sec = ~~1+3+5~~ =

$= 5m + 15m + 25m = 45m$

Hence, for last second, it should cover 45 m and 45 m is covered in 5th sec.

ii) initial height, $s = ut + \frac{1}{2}gt^2$

$= \frac{1}{2} \times 10 \times 5 \times 5$

$= 5 \times 5 \times 5 = 125\ m$

iii) distance travelled = 125 m

iv) $\overrightarrow{\text{displacement}}$ = 125 m, vertically downwards

v) Speed, $v = \cancel{\frac{s}{t} = \frac{125\ m}{5}}$

$v = u + at$

$= 0 + 10 \times 5 = 50\ m/s$

vi) velocity, $\vec{v} = u + at$

$= 10 \times 4 = 40\ m/s$ vertically ↓

Q.43 + A body is dropped from some height. The distance travelled by it in first ~~8~~5 sec of its motion is equal to the distance travelled in the last second of its motion, then find.

i) Time for which body has fallen:

$S_{nth} = S_5$

$u + \frac{g}{2}(2n-1) = ut + \frac{1}{2}g\,t^2$

$\because u = 0$

$\therefore 2n - 1 = 5^2$

$2n = 25 + 1 = 26$

$\therefore n = 13$ sec

$\because$ 13 th sec is the last second.

$\therefore$ The body has fallen for 13 seconds

ii) Initial height of the body

$S = \frac{1}{2} \times 10 \times 13 \times 13$

$= 845\ m$

iii) distance travelled by the body till it reaches the ground.

$S = 845\ m$

iv) displacement of the body till it reaches the ground

$\vec{S} = 845\ m$, vertically downwards.

v) speed of the body when it reaches on ground

$v = 10 \times 13 = 130\ m/s$

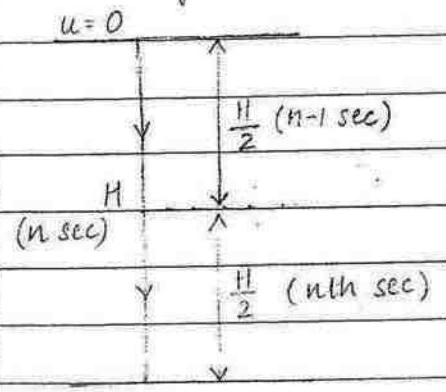
vi) velocity of the body 4 sec after releasing it

$v = u + at$

$= 10 \times 4 = 40\ m/s$, vertically downwards.

Q.44 (I-DS) Pg-12

☆☆



let downward direction be +ve

height = H, $u = 0$, $g = 10\ m/s^2$

let nth sec be n sec

for H height,

$S = ut + \frac{1}{2} at^2$

$\therefore H = \frac{1}{2} \times 10 \times n^2 = 5n^2$

$$H = 5n^2 \quad — (1)$$

for first $\frac{H}{2}$ height, when $t = (n-1 \text{ sec})$

$$\frac{H}{2} = \frac{1}{2} \times 10 \times (n-1)^2$$

$$\therefore H = 10(n-1)^2 \quad — (11)$$

dividing eq (11) from eq (1), we get

$$\frac{H}{H} = \frac{10}{5}\frac{(n-1)^2}{n^2}$$

$$1 = 2\left(\frac{n-1}{n}\right)^2$$

$$\frac{1}{2} = \left(1 - \frac{1}{n}\right)^2$$

~~Squaring~~ root of both sides,

$$1 - \frac{1}{n} = \frac{1}{\sqrt{2}} \qquad \frac{n-1}{n} = \frac{1}{\sqrt{2}}$$

$$\therefore -\frac{1}{n} = \frac{1}{\sqrt{2}} - 1 \qquad \sqrt{2}n - \sqrt{2} = n$$

$$\sqrt{2}n - n = \sqrt{2}$$

$$\frac{1}{n} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}} \qquad n(\sqrt{2}-1) = \sqrt{2}$$

$$n = \frac{1.41}{0.41}$$

$$\therefore n = \frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} \qquad = 3.414$$

$$= \frac{2+\sqrt{2}}{2-1} = 2+\sqrt{2}$$

$$= 2 + 1.414 \text{ sec}$$

$$= 3.414 \text{ sec}$$

hence, the body has fallen for 3.414 sec

OR

For H height,

$$s = \frac{1}{2} \times 10 \times n^2$$

$$H = 5n^2 \quad — (1)$$

for nth sec, i.e. $\frac{H}{2}$ height

$$\cancel{s = \frac{1}{2} \times 10 \times} \quad s = \frac{10}{2}(2n-1)$$

$$\frac{H}{2} = \frac{10}{2}(2n-1)$$

$$\therefore H = 10(2n-1) \quad \text{--- (II)}$$

from eq (I) and (II), we get

$$5n^2 = 20n - 10$$

$$5n^2 - 20n + 10 = 0$$

$$n^2 - 4n + 2 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm \sqrt{8}}{2}$$

$$\frac{4 + 2\sqrt{2}}{2} \text{ or } \frac{4 - 2\sqrt{2}}{2}$$

$$= 2 + \sqrt{2} \text{ or } 2 - \sqrt{2}$$

$$n = 2 + 1.414 \text{ or } 2 - 1.414$$

$$= 3.414 \text{ sec or } 0.586 \text{ sec}$$

$n = 0.586$ (not possible)

because $0.586 < 1$, where 1 sec is the time taken for last nth sec

$\therefore n = 3.414$ sec

hence, the body has fallen for 3.414 sec

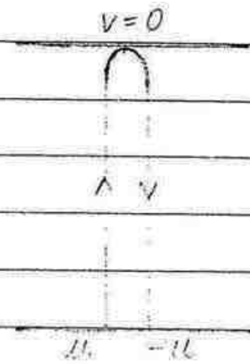
~~Q.45 (1-DS) Pg-13~~

☆

Let upward direction be +ve

$u = u, \; v = 0, \; a = -g$

i) ascending time, t_a



$$\because v = u + at_a$$
$$\therefore t_a = \frac{v-u}{a}$$
$$= \frac{0-u}{-g} = \frac{u}{g}$$
$$\therefore \boxed{t_a = \frac{u}{g}}$$

ii) descending time, t_d

$$\boxed{t_a = t_d = \frac{u}{g}}$$

iii) Time of flight, T

$$T = t_a + t_d = \frac{u}{g} + \frac{u}{g} = \frac{2u}{g}$$
$$\therefore \boxed{T = \frac{2u}{g}}$$

iv) Maximum height attained

$u = u$, $v = 0$, $s = H_{Max}$, $a = -g$

$$\because v^2 = u^2 + 2as$$
$$\therefore s = \frac{v^2 - u^2}{2a}$$
$$H_{Max} = \frac{0-u^2}{2 \times (-g)}$$
$$\therefore \boxed{H_{Max} = \frac{u^2}{2g}}$$

v) Total distance travelled till it returns back to the ground

$$S = \frac{u^2}{2g} + \frac{u^2}{2g} = \frac{2u^2}{2g} = \frac{u^2}{g}$$

$$\therefore \boxed{S = \frac{u^2}{g}}$$

vi) displacement till it returns back to the ground.

displacement = 0.

Q. 46 (I-DS) Pg-13.

For only ascending time, ta = 4 sec

let downwards direction be +ve

$\therefore v = 0,\ t_a = 4\,sec,\ g = 10\ m/s^2$

$\therefore v = u + at$

$u = 0 - 10 \times 4 = -40\ m/s$ vertically ↓

$\therefore u = 40\ m/s$ vertically upwards.

$$H_{Max} = ut + \frac{1}{2}at^2$$

$$= -40 \times 4 + \frac{1}{2} \times 10 \times 4 \times 4$$

$$= -160 + 80$$

$= -80\ m$ downwards.

$\therefore H_{Max} = 80\ m$ vertically upwards.

OR

$$\because T = \frac{2u}{g} = 8\,sec$$

$\therefore u = \frac{8 \times 10}{2} = 40\ m/s$ vertically ↑

$$H_{Max} = \frac{u^2}{2g} = \frac{40 \times 40}{2 \times 10} = 80\,m$$

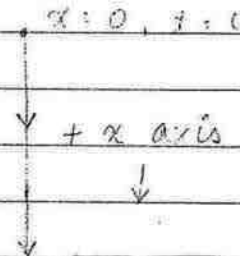
Q.47 (I-DS) Pg-13

$u = 29.4 \ m/s$, $g = 9.8 \ m/s^2$

i) vertically downwards.

ii) At the highest point, $v = 0$ and ~~$a = 9.8 m/s^2$~~
$a = 9.8 \ m/s^2$ vertically downwards

* iii)



Sign of	upward motion	downward motion
Position (x)	+ve ($x > 0$)	+ve ($x > 0$)
velocity (v)	−ve ($v < 0$)	+ve ($v > 0$)
acceleration (a)	+ve ($a > 0$)	+ve ($a > 0$)

iv) $H_{Max} = \frac{u^2}{2g} = \frac{29.4 \times 29.4}{2 \times 9.8}$

$= 14.7 \times 3 = 44.1 \ m$

$T = \frac{2u}{g} = \frac{2 \times 29.4}{9.8}$

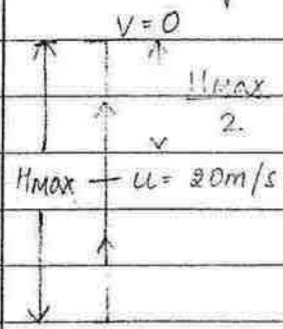
$= 2 \times 3 = 6 \ sec$

v) Total distance travelled : $\frac{u^2}{g}$

$= \frac{29.4 \times 29.4}{9.8}$

$= 29.4 \times 3 = 88.2 \ m$

Q.48 (1-DS) Pg-13



Let upward direction be +ve

For upper $\frac{H_{Max}}{2}$,

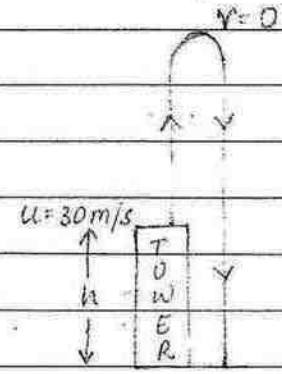
$u = 20 m/s$, $V = 0$, $g = 10 m/s^2$

$\because H_{Max} = \frac{u^2}{2g}$

$\therefore \frac{H_{Max}}{2} = \frac{20 \times 20}{2 \times 10}$

$\therefore H_{Max} = 20 \times 2 = 40 m$

Q.49 (1-DS) Pg-13.



Let upward direction be +ve

for complete motion,

$u = 30 m/s$, $g = -10 m/s^2$. $t = 8 sec$.

$|\vec{S}| = H$

$\because S = ut + \frac{1}{2}at^2$

$= 30 \times 8 + \frac{1}{2} \times (-10) \times 8 \times 8$

$= 240 - 320$

$= -80 m$ vertically downward

$\therefore$ height of tower, $H = |\vec{S}|$

$= 80 m$.

Formula association:

Acceleration means the change in velocity per unit time.

i) when acceleration is zero $(a=0)$.
we use,
- distance = speed $\times$ time
- displacement = velocity $\times$ time

ii) when acceleration is not equal to zero $(a \neq 0)$ but is constant (a = constant).
we use, equations of motion.
- $v = u + at$
- $s = ut + \frac{1}{2}at^2$
- $v^2 = u^2 + 2as$

iii) when acceleration is variable
we use, calculus
- Integration and
- Differentiation.

Graphs in Kinematics
(1-DS) Pg-13

- $x \xrightarrow[\text{or slope}]{\text{differentiation}} v \xrightarrow[\text{or slope}]{\text{differentiation}} a \xrightarrow[\text{or slope}]{\text{diff.}}$ Nothing

- $a \xrightarrow[\text{or area}]{\text{Integration}} \Delta v$ or $v \xrightarrow[\text{or area}]{\text{integration}} x \xrightarrow[\text{or area}]{\text{integra.}}$ Nothing

-50 to Q-73 (1-DS) Pg [illegible]

Relative Motion:

(1-DS) Pg-26

- Relative displacement:

$$\vec{S}_{AB} = \vec{S}_A - \vec{S}_B$$

- Relative velocity:

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

- Relative acceleration:

$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$$

* Velocity of an object with respect to itself is always 0 because $\vec{v}_{AA} = \vec{v}_A - \vec{v}_A = \vec{0}$
Hence, a moving body cannot experience its own motion.

Q-74 (1-DS) Pg-27

~~Q-75~~ Relative speed:

$$v_{relative} = |\vec{v}_{relative}|$$

OR

$$v_{rel} = |\vec{v}_{rel}|$$

* If the bodies are moving in same ~~direction~~ straight line, then relative speed can be calculated without calculating relative velocity

<u>Case I</u>: When the bodies are moving in same directions

$$v_{relative} = v_A \sim v_B$$

Case II : If the bodies are moving in opposite directions

$$v_{relative} = v_A + v_B$$

~~Q.75~~ $\vec{v_{AB}} = -\vec{v_{BA}}$, and,

$$v_{AB} = v_{BA}$$

Q-75 (I-DS) Pg-28

$$v_C = 70 \text{ km/h}$$

$$v_M = \frac{50}{3} \text{ m/s}$$

$$= \frac{50}{3} \times \frac{18}{5} \text{ km/h} = 60 \text{ km/h}$$

i) same directions :

$$v_{CM} = v_C \sim v_M$$

$$= 70 - 60 = 10 \text{ km/h}$$

ii) different directions :

$$v_{MC} = v_M + v_C$$

$$= 60 + 70 = 130 \text{ km/h}$$

Q-76 (I-DS) Pg-28

$v_t = 15\cancel{10} \text{ m/s } \hat{i}$

$v_b = -10 \text{ m/s } \hat{i}$

$S_t = 500 \text{ m.}$ [= distance travelled
= seperation covered
= relative distance]

Using relation,

relative speed = $\frac{\text{relative distance(seperation covered)}}{\text{time taken}}$

$$(\text{speed})_{rel} = \frac{(\text{distance})_{rel}}{t}$$

$$15 + 10 = \frac{500}{t}$$

$$\therefore t = \frac{500}{25} = 20 \text{ sec}$$

Q-77 (1-DS) Pg-28

$V_m = 10$ km/h

(distance) rel = 200 m [= relative distance = seperation covered. = distance travelled]

$$\therefore (\text{Speed})_{rel} = \frac{(\text{distance})_{rel}}{\text{time}}$$

$$= \frac{200}{9} \times \frac{18}{5}$$

$$= 80 \text{ km/h}$$

But, $V_{relative} = V_T + V_M$

$$80 = V_T + 10$$

$$\therefore V_T = 80 - 10$$

$$= 70 \text{ km/h}$$

Q-78 (1-DS) Pg-28

$V_j = 300$ km/h

$V_{relative} = 1200$ km/h, $V_p = ?$

$\because V_{relative} = V_j + V_p$ [opposite directions]

$$1200 = 300 + V_p$$

$$\therefore V_p = 1200 - 300$$

$$= 900 \text{ km/h}$$

Q.79 (1-DS) Pg - 28

$v_{T1} = 12 \text{ m/s}$

$v_{T2} = 13 \text{ m/s}$

Seperation covered = 75 + 75 = 150 m

i) Opposite directions :

$$\text{time} = \frac{\text{seperation covered}}{v_{relative} / \text{relative speed}}$$

$$= \frac{150}{13+12} = \frac{150}{25} = 6 \text{ sec}$$

ii) Same directions :

$$\text{time} = \frac{\text{seperation covered}}{\text{relative speed}}$$

$$= \frac{150}{13-12} = 150 \text{ sec}$$

* Absolute or Actual speed is always taken as w. r. t. ground

* During overtaking or crossing of train, the seperation covered is equal to sum of length of both the trains.

Q.80 (1-DS) Pg - 29

$v_A = 10 \text{ m/s } \hat{i}$

$v_B = 10 \text{ m/s } \hat{j}$

i) $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$10\hat{i} - 10\hat{j}$

In real directions, $10\sqrt{2}$ m/s due S-E

ii) $\overrightarrow{v_{BA}} = \overrightarrow{v_B} - \overrightarrow{v_A}$

$= 10\hat{j} - 10\hat{i}$

In real direction, $10\sqrt{2}$ due N-W

iii) $v_{AB} = |\overrightarrow{v_{AB}}|$

$= 10\sqrt{2}$ m/s

iv) $v_{BA} = |\overrightarrow{v_{BA}}|$

$= 10\sqrt{2}$ m/s

Q-81 (1-DS) Pg-29

$\overrightarrow{v_M} = 10$ km/h $\hat{i}$

$\overrightarrow{v_{TM}} = 10\sqrt{3}$ km/h $\hat{j}$

$\because \overrightarrow{v_{TM}} = \overrightarrow{v_T} - \overrightarrow{v_M}$

$\therefore \overrightarrow{v_T} = \overrightarrow{v_{TM}} + \overrightarrow{v_M}$

$= 10\sqrt{3}\,\hat{j} + 10\,\hat{i}$

In real direction,

$\overrightarrow{v_T} = \sqrt{(10\sqrt{3})^2 + 10^2}$ due N $\tan^{-1}\left(\frac{10}{10\sqrt{3}}\right)$ E

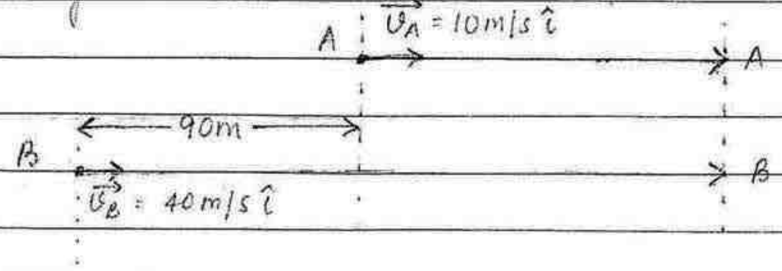
$= \sqrt{400}$ due N $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ E

$\therefore \overrightarrow{v_T} = 20$ km/h due N 30° E or 30° E of N

OR

20 km/h due E 60° N or 60° N of E

Q.82 (1-DS) Pg-29



$V_A = 10 \ m/s$

$V_B = 40 \ m/s$

i) $V_{rel} = B \sim A = B - A = 40 - 10 = 30 \ m/s$

Seperation covered $= 90 \ m$

Using time $= \dfrac{\text{Seperation covered}}{V_{relative}}$

$= \dfrac{90}{30} = 3 \ sec$

ii) $S_B = V_B \times t$ [$\because$ distance = speed $\times$ time]

$= 40 \times 3 \ m$

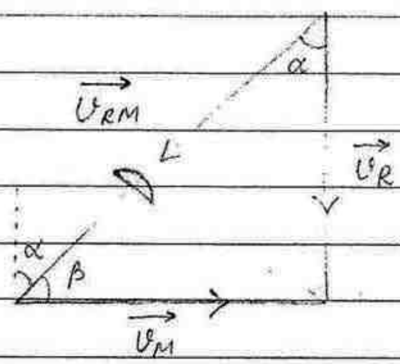
$= 120 \ m$

Rain and Man

(J. BS) Pg-29

$\because \vec{V_{RM}} = \vec{V_R} - \vec{V_M}$

$\therefore \vec{V_R} = \vec{V_{RM}} + \vec{V_M}$



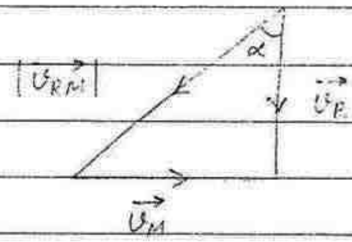
$\because \tan \alpha = \left(\dfrac{V_M}{V_R}\right)$

$\therefore \alpha = \tan^{-1}\left(\dfrac{V_M}{V_R}\right)$ from vertical in forward direction

$$\tan\beta = \left(\frac{V_R}{v_M}\right)$$

$$\therefore \beta = \tan^{-1}\left(\frac{v_R}{v_M}\right)$$ from horizontal in ~~farw~~ forward direction.

Q.83 (1-DS) Pg-30



$\because \vec{v}_M = 10$ km/h

$\vec{v}_R = 10\sqrt{3}$ km/h

$\therefore \vec{v}_{RM} = \vec{v}_R - \vec{v}_M$

Its speed $= |\vec{v}_{RM}|$

Using pythagoras,

$$v_{RM} = \sqrt{v_M^2 + v_R^2}$$
$$= \sqrt{10^2 + (10\sqrt{3})^2}$$
$$= \sqrt{100 + 300} = \sqrt{400} = 20 \text{ km/h}$$

$\therefore$ speed of rain w.r.t man, $v_{RM} = 20$ km/h

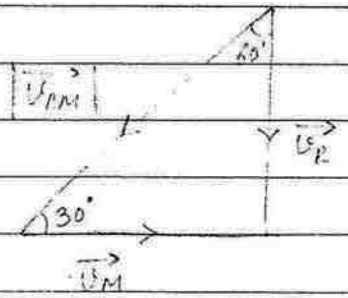
Q.84 (1-DS) Pg-30

$$\because \tan\alpha = \frac{v_M}{v_R} = \frac{10}{10\sqrt{3}} = \frac{1}{\sqrt{3}} = 30^\circ$$

$$\therefore \alpha = 30^\circ$$

Hence, in order to avoid raindrops, he should hold his umbrella at an angle of 30° angle from vertical in the forward direction

Q.85 (1-DS) Pg-30



$\vec{v}_M = 20$ km/h.

i) $\alpha = 60°$

$\therefore \beta = 90° - 60° = 30°$

$\tan\beta = \left(\dfrac{v_R}{v_M}\right) \neq \cancel{\dfrac{\vec{v}_R}{\vec{v}_M}} \neq$

$\tan 30° = \dfrac{v_R}{20} = \dfrac{1}{\sqrt{3}}$

$\therefore v_R = \dfrac{20}{\sqrt{3}}$ km/h

ii) Speed of $\vec{v}_{RM} = |\vec{v}_{RM}|$

$\because \sin 60° = \dfrac{v_M}{v_{RM}} = \dfrac{20}{v_{RM}} = \dfrac{\sqrt{3}}{2}$

$\therefore v_{RM} = \dfrac{20 \times 2}{\sqrt{3}} = \dfrac{40}{\sqrt{3}}$ km/h

Swimming in river:

(1-DS) Pg-30

- $\vec{v}_{SR}$ in still water = $\vec{v}_{SR}$ in flowing water because it is the capability of the swimmer which is constant
- Upstream: Opposite direction of river flow
  $v_{net} = (v_b \text{ or } v_M) \sim v_R$

Downstream: same direction of river flow
  $v_{net} = (v_b \text{ or } v_M) + v_R$

Q-86 (1-DS) Pg-30

$$v_b = \frac{\text{distance}}{\text{time}} = \frac{8+8}{2} = \frac{16}{2} = 8 \text{ km/h.}$$

$$v_w = 4 \text{ km/h}$$

$\because$ time = (t) upstream + (t) downstream

$$= \frac{8}{v_b - v_w} + \frac{8}{v_b + v_w} \quad \left[\because \text{time} = \frac{\text{distance}}{(v_{net})\text{ speed}}\right]$$

$$= \frac{8}{8-4} + \frac{8}{8+4}$$

$$= \frac{8}{4} + \frac{8}{12}$$

$$= \frac{24+8}{12} = \frac{32}{12} = \frac{8}{3} \text{ hr.}$$

$\because$ 1 hr = 60 min

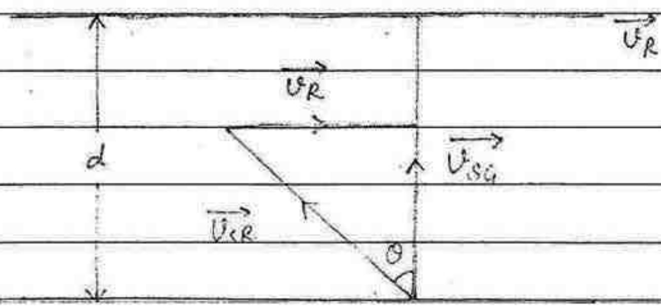
$$\therefore \frac{8}{3} \text{ hr} = \frac{8}{3} \times 60 \text{ min}$$

$$= 160 \text{ min}$$

Crossing of river:

(1-DS) Pg-30

i) Minimum distance or shortest path approach



$$\because \vec{v_{SR}} = \vec{v_{SG}} - \vec{v_R}$$

$$\therefore \cancel{\vec{v_R} = v} \quad \vec{v_{SG}} = \vec{v_{SR}} + \vec{v_R}$$

- $|\vec{v_{SG}}| = \sqrt{v_{SR}^2 - v_R^2}$ (If v_{SR} and v_R are given)

- $|\vec{v_{SG}}| = v_{SR} \cos\theta$ (If v_{SR} and θ are given)

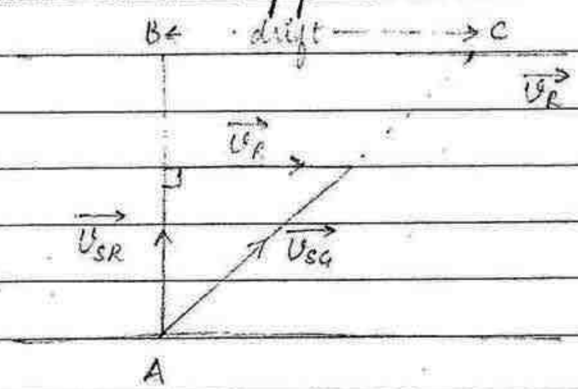
Hence, to cross the river with minimum displacement, swimmer should swim at an angle of θ from perpendicular of river flow in the upstream direction or at an angle of $(90^\circ+\theta)$ from the river flow direction.

- Time of crossing $= \frac{\text{distance}}{\text{speed}} = \frac{d}{v_{SG}}$

$$= \frac{d}{\sqrt{v_{SR}^2 - v_R^2}} = \frac{d}{v_{SR}\cos\theta}$$

- In this condition, displacement is minimum but time is not minimum.

ii) Minimum time approach:



$\because \vec{v_{SR}} = \vec{v_{SG}} - \vec{v_R}$

$\therefore \vec{v_{SG}} = \vec{v_{SR}} + \vec{v_R}$

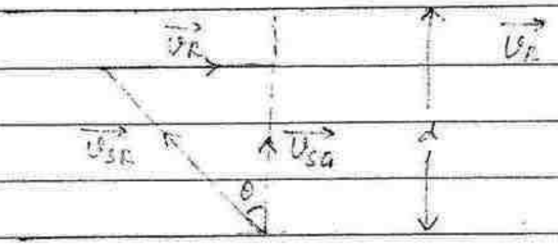
- $|\vec{v_{SG}}| = \sqrt{v_{SR}^2 + v_R^2}$

- Time of crossing $= \dfrac{\text{distance}}{\text{speed}} = \dfrac{AC}{v_{SG}}$

$$\frac{AC}{v_{SG}} = \frac{BC \text{ or drift}}{v_R} = \frac{AB \text{ or } d}{v_{SR}}$$

- Drift $= v_R \times t$

Q.87 (1-DS) Pg-31



$d = 0.5$ km

$\vec{v_{SR}} = 2$ km/h

$\vec{v_R} = 1$ km/h

$\because \vec{v_{SR}} = \vec{v_{SG}} - \vec{v_R}$

$\therefore \vec{v_{SG}} = \vec{v_{SR}} + \vec{v_R}$

$v_{SG} = \sqrt{v_{SR}^2 - v_R^2}$

$= \sqrt{2^2 - 1^2} = \sqrt{4-1} = \sqrt{3}$

$\tan\theta = \dfrac{v_R}{v_{SG}} = \dfrac{1}{\sqrt{3}} = \tan 30^\circ$

$\therefore \theta = 30^\circ$

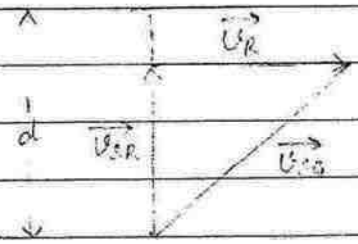
Hence, he should swim at an angle of 30° from the perpendicular of the river in the upstream direction or $90^\circ + 30^\circ = 120^\circ$ from the river flow direction.

Q.88 (1-DS) Pg-31

i) $|\vec{v_{MG}}| = \cancel{v_{SR}}\ |\vec{v_{SG}}| = \sqrt{3}$ km/h

ii) time of crossing $= \dfrac{\text{distance}}{v_{net}} = \dfrac{0.5}{\sqrt{3}}$ sec.

Q.89 (1-DS) Pg-31



$d = 400$ m
$\vec{v_R} = 10$ km/h
$\vec{v_{SR}} = 2$ km/h

i) direction of swimming: It is perpendicular to the river flow

ii) Time of crossing: $\dfrac{\text{distance}}{v_{net}} = \dfrac{\text{distance}}{v_{SR}}$

$$= \frac{400}{2} = 200 \text{ sec}$$

iii) speed of man w.r.t ground: $|\vec{v_{SG}}|$

$$|\vec{v_{SG}}| = \sqrt{v_R^2 + v_{SR}^2}$$
$$= \sqrt{10^2 + 2^2} = \sqrt{100 + 4}$$
$$= \sqrt{104} = 2\sqrt{26} \text{ m/s}$$

iv) Drift along the river: $v_R \times t$

$$= 10 \times 200$$
$$= 2000 \text{ m or } 2 \text{ km}$$