Ray Optics

Basic concept:

→ Electromagnetic wave.

Light: It is a form of energy which produces the sensation of vision whenever it falls on our eyes.

> The speed of light is different in different medium.

> Its speed is maximum in vacuum

> $C = 299792458$ m/s

$= 2.99792458 \times 10^8$ m/s

$\simeq 3 \times 10^8$ m/s

> white light consist of 7 colours.

V I B G Y O R

λ = (3800 Å) (7600 Å)

> Our eyes is most sensitive to yellow / (yellow-green) colour

> ⊗ When light passes from one medium to another medium, frequency remains same. because frequency depends on source, speed changes, wavelength changes

$V = f\lambda$ → source dependent

> Intensity may or may not change (if medium absorbs some part of light, then intensity will decrease)

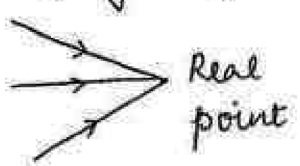
Amplitude may or may not change

($\boxed{\text{Intensity} \propto \text{Amplitude}^2}$)

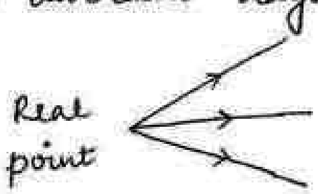
Ray: It is the straight line path followed by light in homogeneous medium.

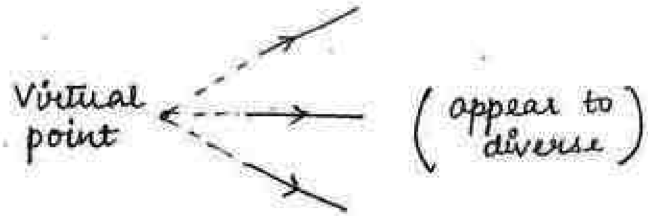
a) Parallel rays : If rays are parallel, then the object or image is located at infinity

b) converging rays :



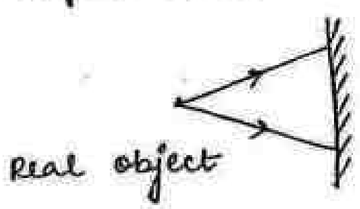
c) diversent rays :





Object: The source of light for an optical element is called object

> It may be luminous or non-luminous.
> Object is formed by the intersection of incident rays
> Object is real if incident rays are diverging
> Object is virtual if incident rays are converging.



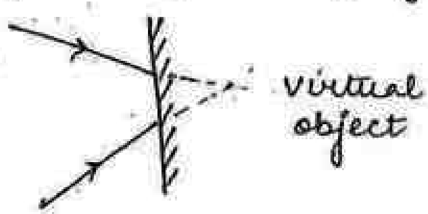
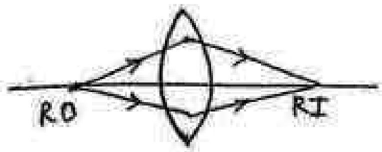
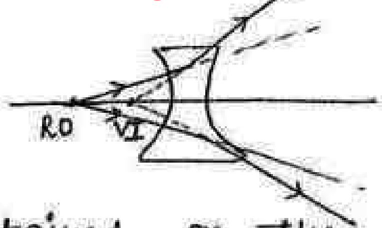


Image: It is formed by the intersection of reflected or refracted rays.

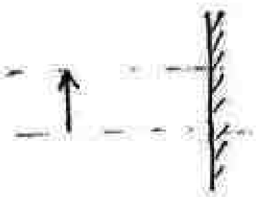
> If their is actual intersection of reflected or refracted rays then the image is real.
> The image is real if reflected or refracted rays are converging.
> The image is virtual if reflected or refracted rays are diverging.



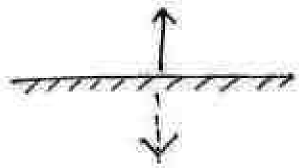


> Real image can be obtained on the screen whereas virtual image cannot be ~~obj~~ obtained on screen

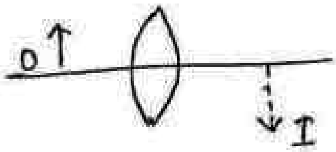
Erect or Inverted :



erect



inverted



inverted

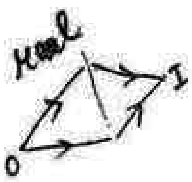
Magnification (m)

$$m = \frac{\text{size of image}}{\text{size of object}}$$

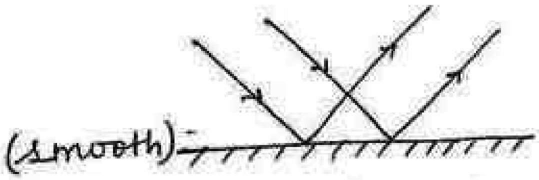
If $|m| = 1$, same size

If $|m| > 1$, enlarged

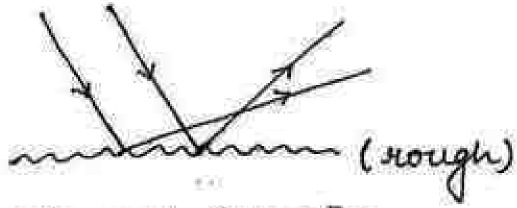
If $|m| < 1$, diminished



Reflection of light: when light returns to the same medium after interaction with an optical element,



Regular Reflection

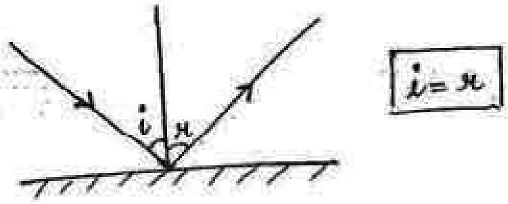


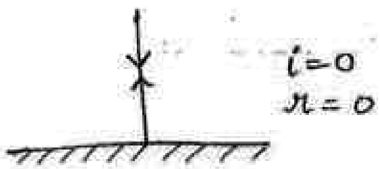
Diffused Reflection

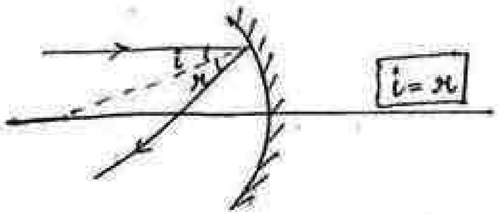
Laws of Reflection:

(i) Incident ray, reflected ray and normal all lie in the same plane and same point.

(ii) Angle of incidence = angle of reflection.



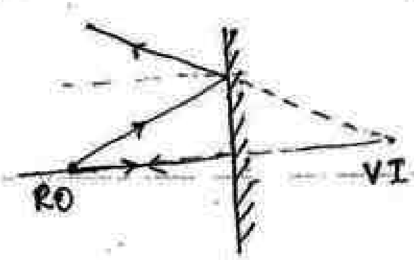


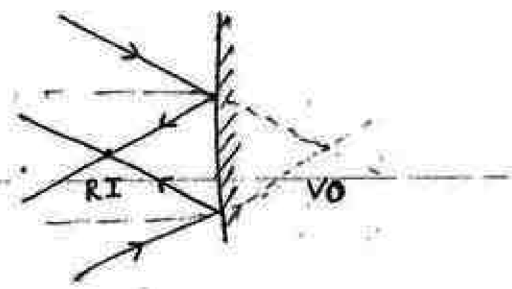


1 Plane Mirror:

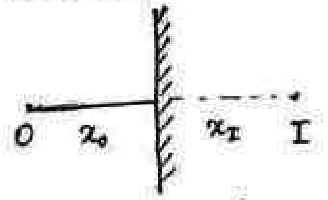
Formation of Image

- Plane mirror forms virtual image of the real object and real image of the virtual object.



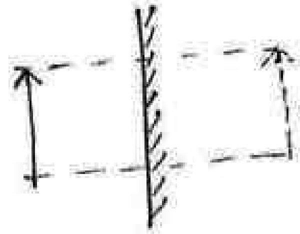


- distance of image from plane mirror = distance of object from plane mirror.



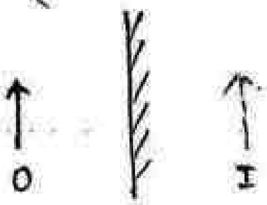
$\therefore \; x_I = -x_0$

- The size of image = the size of object

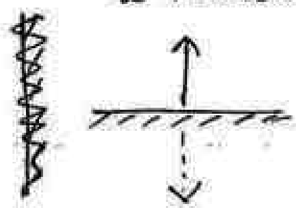


- It can form erect and inverted.

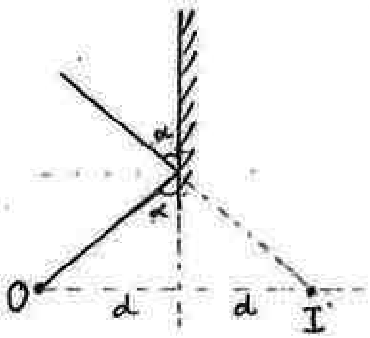
erect : (when O is parallel to mirror)



inverted : (when O is not parallel to mirror)

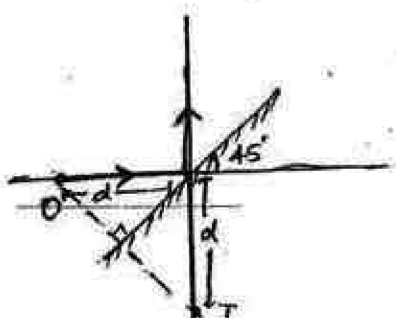


Q.1. Draw the image of the object



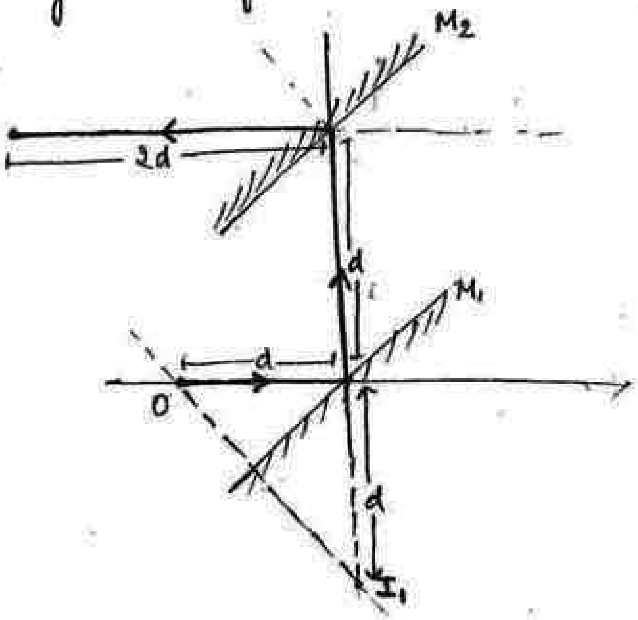
distance of O = distance of I

Q.2 Find the co-ordinate of the image



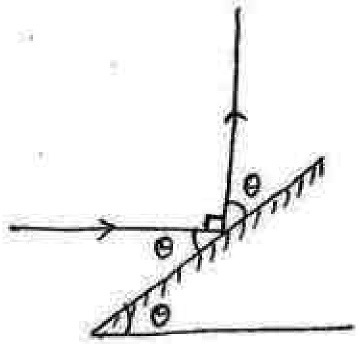
$(0, -d)$

Q.3 Find the co-ordinates of image formed by mirror M_2 Taking 1st reflection with mirror M_1



$(-2d, d)$

Q.4



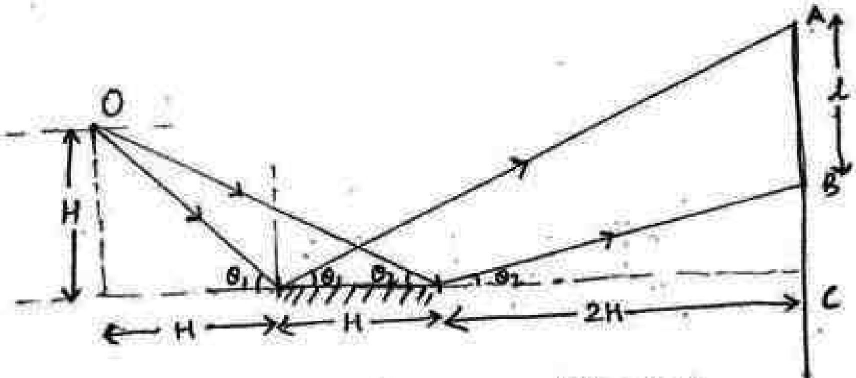
find angle θ for which horizontal incident ray becomes vertical after reflection

$\therefore \theta + 90 + \theta = 180^\circ$

$2\theta = 90^\circ$

$\boxed{\theta = 45^\circ}$

Q.5



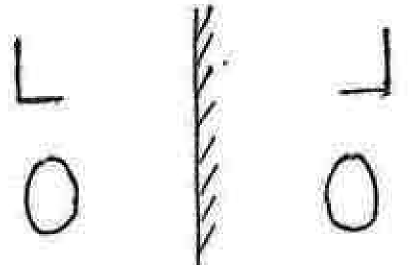
Find the length l on the screen over which reflected rays are present.

$\tan \theta_1 = \frac{H}{H} = \frac{AC}{3H}$ $\therefore AC = 3H$

$\tan \theta_2 = \frac{H}{2H} = \frac{BC}{2H}$ $\therefore BC = H$

$\therefore AB = AC - BC = 3H - H = 2H$

2 Lateral Inversion.

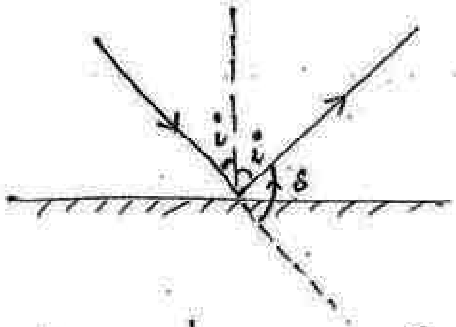


Q) which of the capital letter of each alphabets do not show lateral inversion.

A H I M O T U V W X Y (11)

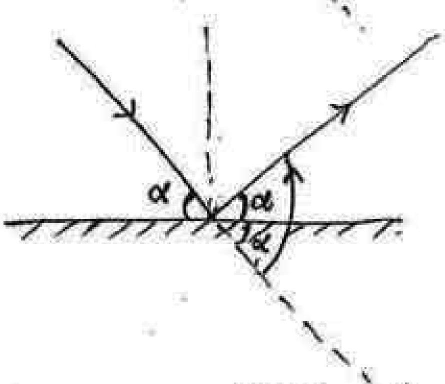
3 Deviation produced by plane mirror:

Angle of deviation by one plane mirror



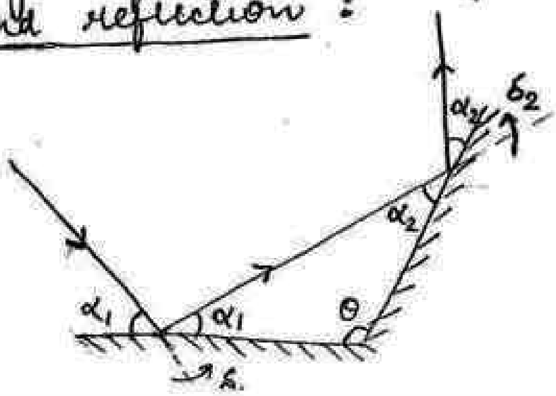
$\delta + i + i = 180°$

$\therefore \boxed{\delta = (180 - 2i)}$



* $\boxed{\delta = 2\alpha}$

Angle of deviation by 2 plane mirror after <u>2nd reflection</u>:



$\therefore \delta = \delta_1 + \delta_2$

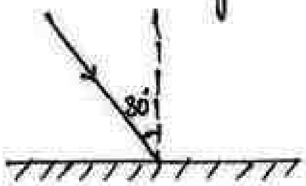
$= 2\alpha_1 + 2\alpha_2 = 2(\alpha_1 + \alpha_2)$

$\because \alpha_1 + \alpha_2 + \theta = 180°$

$\therefore 2(180 - \theta)$

* $\boxed{\delta = 360 - 2\theta}$

Q.7 Find angle of deviation

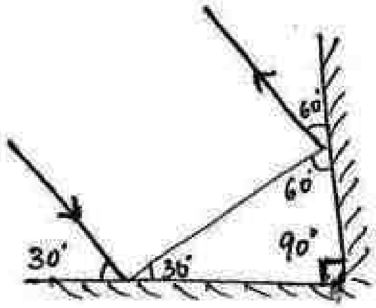


$\alpha = 60°$

$\because \delta = 2\alpha = 60 \times 2$

$\boxed{\delta = 120°}$

*Q.8 Find total deviation after <u>2nd reflection</u>.

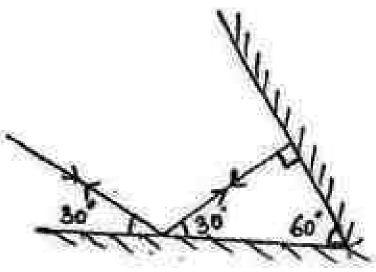


$\theta = 90°$

$\delta = 360° - 2\theta$

$= 360° - 180° = \boxed{180°}$

* When the angle b/w the 2 mirror are 90°, then incident rays are antiparallel to emergent rays.



$\delta_1 = 30 \times 2 = 60°$

$\delta_2 = 90 \times 2 = 180°$

$\therefore \delta = \delta_1 + \delta_2$

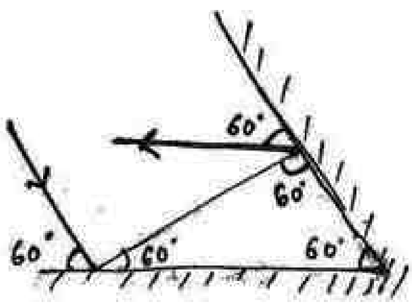
$= 180° + 60° = \boxed{240°}$

OR

$\delta = 360 - 2\theta = 360 - 120$

$= \boxed{240}$

*



$\delta_1 = 60 \times 2 = 120°$

$\delta_2 = 60 \times 2 = 120°$

$\delta = \delta_1 + \delta_2 = \boxed{240}°$

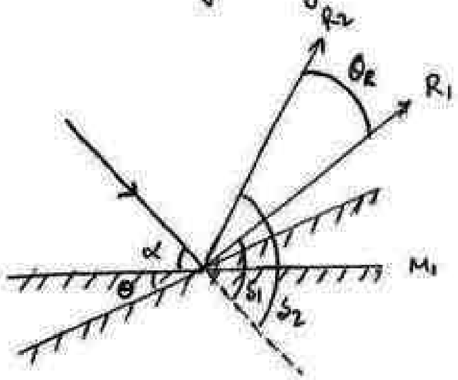
OR

$\delta = 360 - 2\theta = 360 - 120 = \boxed{240}$

* If the angle b/w mirrors is 60° and incident ray is parallel to one of the mirror, then emergent ray is ∥ to another mirror

4 Rotation of Reflected Rays :

Keeping the incident ray fixed, the mirror is rotated by angle θ



$$\delta_1 = 2\alpha$$

$$\delta_2 = 2(\alpha + \theta)$$

$$= 2\alpha + 2\theta$$

$$\therefore \theta_R = \delta_2 - \delta_1$$

$$= (2\alpha + 2\theta) - 2\alpha$$

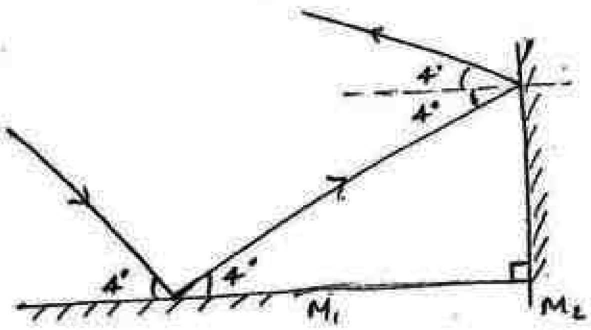
$$\boxed{\theta_R = 2\theta}$$

$\therefore$ when incident ray is kept fixed and mirror is rotated by angle θ. The reflected ray will rotate by 2θ in the same direction.

Q.9 Keeping the incident ray fixed the mirror is rotated by 8°. Find the rotation of reflected ray.

$$\theta_R = 2\theta = 2 \times 8 = 16^\circ$$

Q.10



M_1 is horizontal and M_2 is vertical. Find angle by which M_2 should be rotated its reflected ray becomes Horizontal

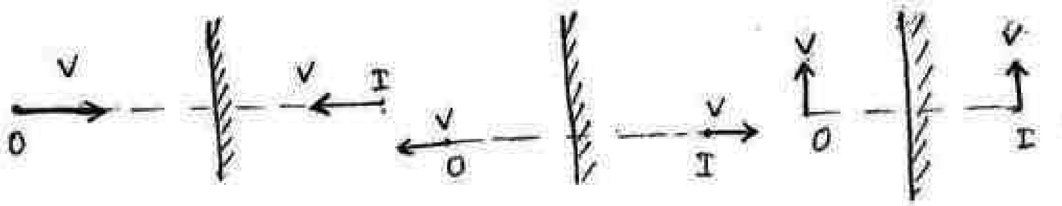
$$\because \theta_R = 2\theta$$

$$4^\circ = 2\theta$$

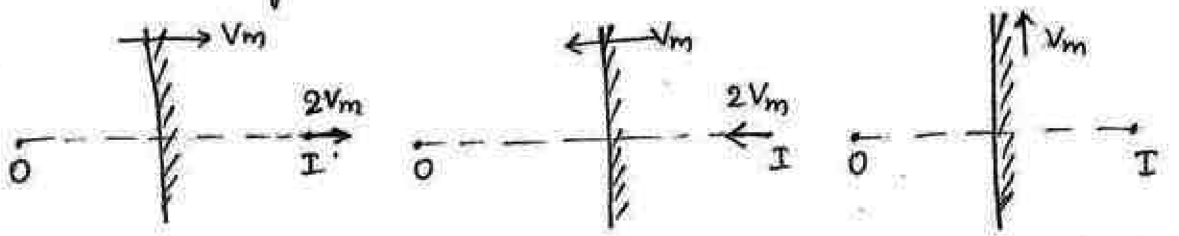
$$\therefore \theta = 2^\circ$$

5 Velocity of image :

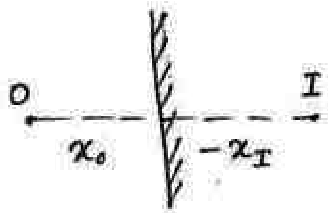
a) when mirror is at Rest.



b) when object is at Rest



* There will be no effect in the velocity of the image



$$x_{I/M} = -x_{O/M}$$

$$\frac{d}{dt} x_{I/M} = -\frac{d}{dt} x_{O/M}$$

$$V_{I/M} = -V_{O/M} \quad \text{(Relative motion)}$$

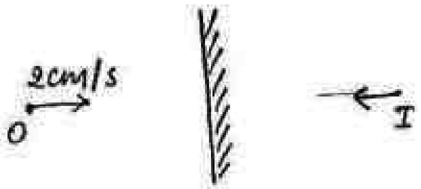
$$V_I - V_M = -(V_O - V_M)$$

$$\boxed{V_I = -V_O + 2V_M}$$

* All these velocities are perpendicular to the plane mirror.

Q.11 Find velocity of image with respect to ground and with respect to object.

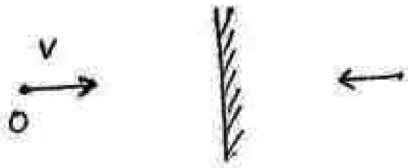
a)



$\therefore V_I = 2\,cm/s\ (\leftarrow)$

$V_{IO} = 2 + 2 = 4\,cm/s$ $\because (V_{rel})$

b)

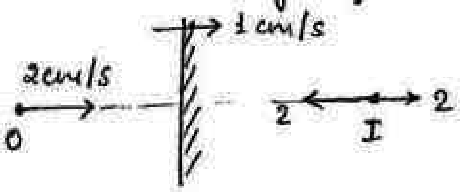


$V_I = V\ (\leftarrow)$

$V_{IO} = 2V\ (\because V_{rel})$

Q.12. Find velocity of image :

a)

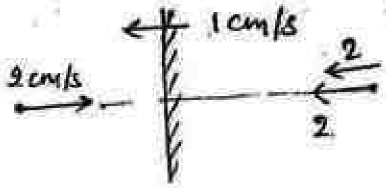


$\therefore V_I = -V_0 + 2V_m$

$= -2 + 2\times 1$

$= -2+2 = 0$

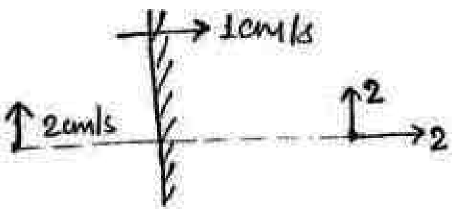
b)



$\therefore V_I = 4\,cm/s$

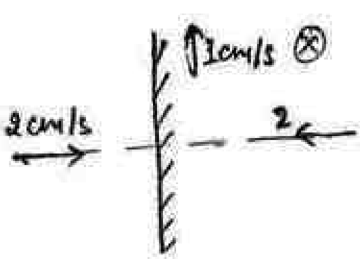
Q.13 Find velocity of image.

a)



$\therefore \vec{V_I} = (2\hat{i} + 2\hat{j})\,cm/s$

b)



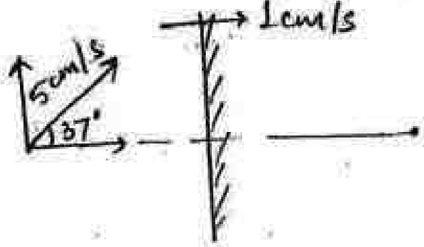
$\vec{V_I} = 2\,cm/s\ (-\hat{i})$

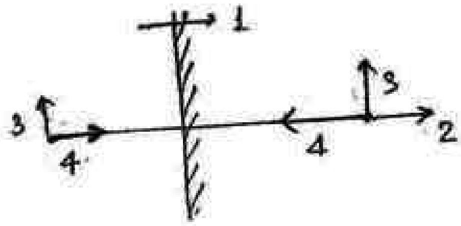
OR

$= -2(\hat{i})$

Q. 14 Find velocity of image:

a)

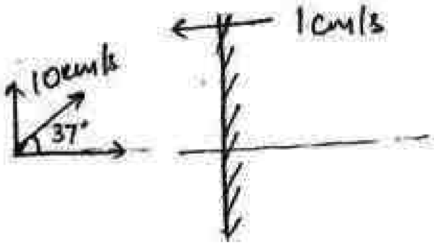


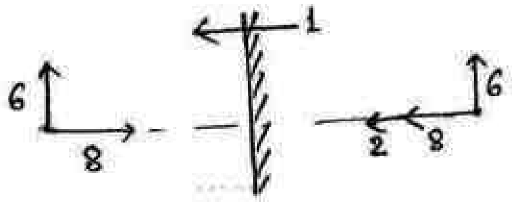


$(4-2)-\hat{i}+3\hat{j}$

$=-2\hat{i}+3\hat{j}$

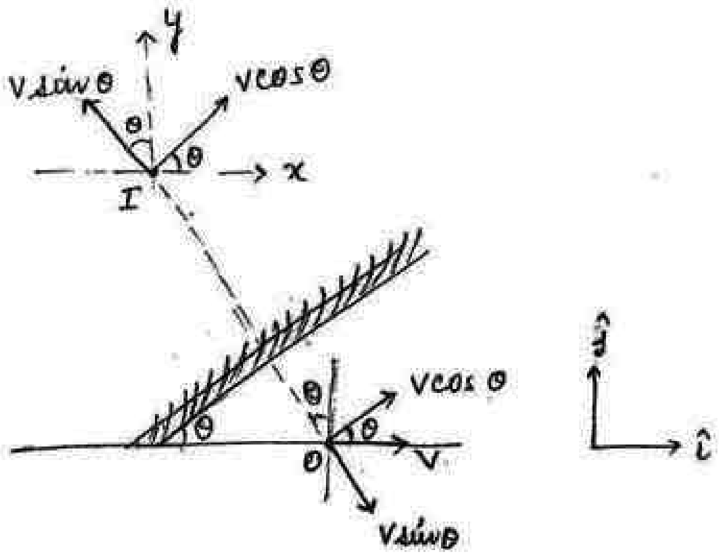
b)





$(8+2)(-\hat{i})+6\hat{j}$

$=-10\hat{i}+6\hat{j}$



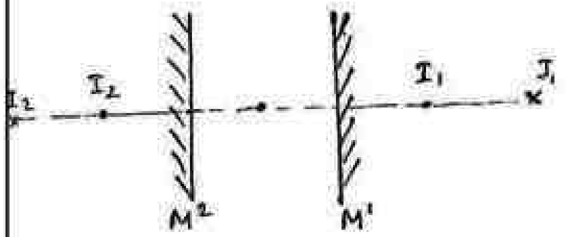
$$\vec{v}_i = [(v\cos\theta)\cos\theta - (v\sin\theta)\sin\theta]\,\hat{i} + [(v\cos\theta)\sin\theta + (v\sin\theta)\cos\theta]\,\hat{j}$$

$$= (v\cos 2\theta)\,\hat{i} + (v\sin 2\theta)\,\hat{j}$$

$\left[\because \cos 2\theta = \cos^2\theta - \sin^2\theta,\quad \sin 2\theta = 2\sin\theta\cos\theta\right.$

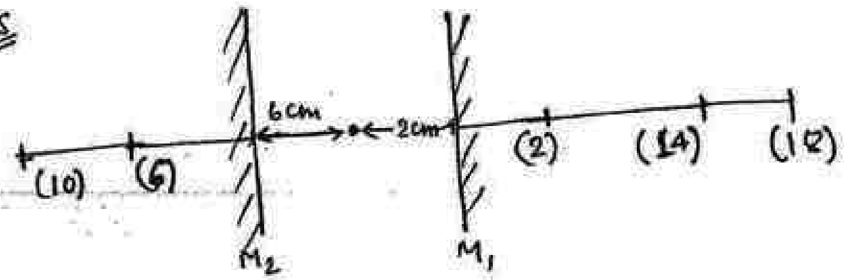
Number of images by plane mirror :

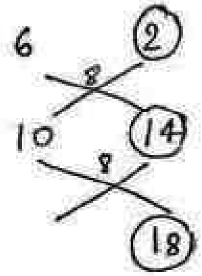
Case I: when the mirrors are parallel.



Infinite images are formed because the image of the 1st reflection behaves as an object for the next reflection.

Q.15

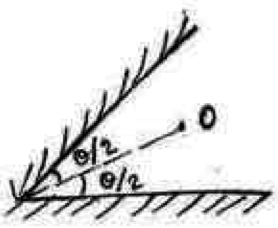




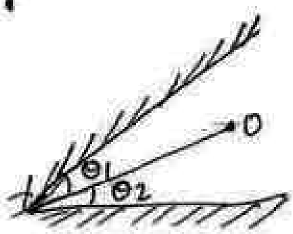
∴ Images formed are at 2, 14 and 18

Case II: when the angle b/w the mirrors is θ

i) object is located symmetrically

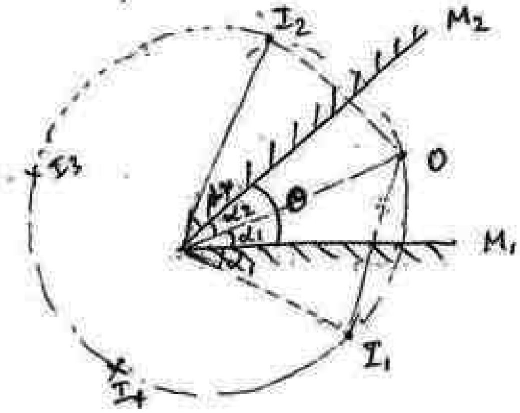


ii) Object is not located symmetrically



$(\theta_1 \neq \theta_2)$

All the images and the object lie on a circle whose radius is equal to distance of object from the intersection of mirror.



The image will act as an object only when its angle with the mirror is less than 180°

Let n = no of images

$$m = \frac{360}{\theta}$$

a) If m is <u>even</u> (2, 4, 6 ....)

$\therefore$ $\boxed{n = (m-1)}$

b) If m is <u>odd</u> (1, 3, 5 .....)

- If object is located <u>symmetrically</u>

$\therefore$ $\boxed{n = (m-1)}$

- If object is <u>not</u> located <u>symmetrically</u>

$\therefore$ $\boxed{n = m}$

c) If m is not an integer, then we take <u>nearest even number.</u>

$m = 3.8 \quad \therefore \quad n = 4$

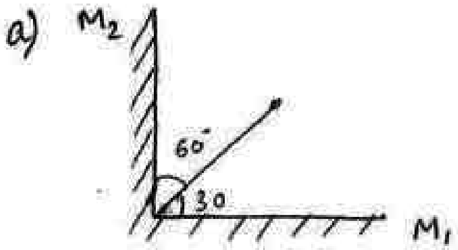
$m = 3.1 \quad \therefore \quad n = 4$

$m = 2.9 \quad \therefore \quad n = 2$

$m = 2.2 \quad \therefore \quad n = 2$

Q.16 Find total no of images

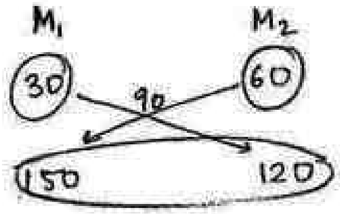
a)



$\theta = 90$

$\therefore m = \frac{360}{\theta} = \frac{360}{90} = 4$ (even)

$\therefore n = m - 1 = 4 - 1 = \boxed{3}$

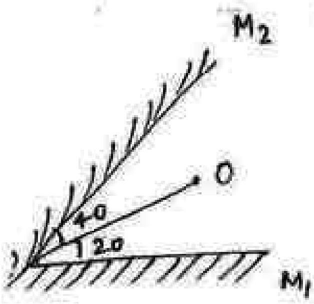


$\therefore$ 150 + 120 + 90 = 360

$\therefore$ last image coincides

$\boxed{n = 3}$

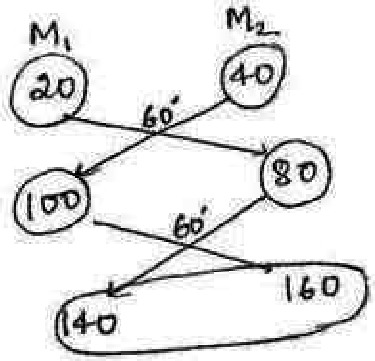
b)



$\theta = 60°$

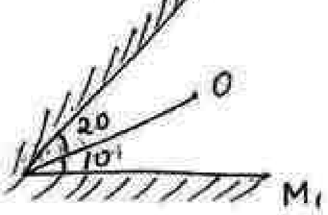
$\therefore m = \frac{360}{\theta} = \frac{360}{60} = 6$ (even)

$\therefore n = m - 1 = 6 - 1 = \boxed{5}$



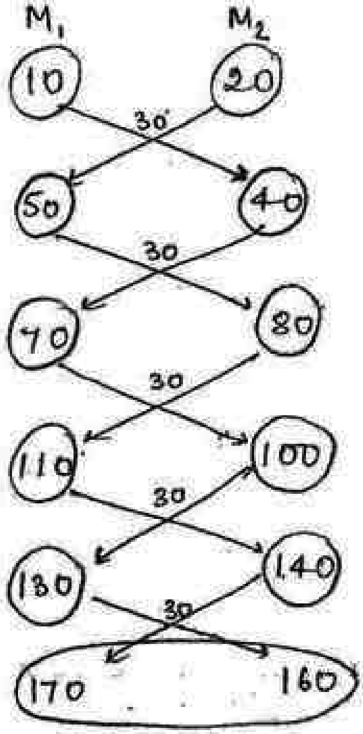
$\therefore$ 140 + 160 + 60 = 360

c)



$$m = \frac{360}{\theta} = \frac{360}{30} = 12$$

$\therefore$ $n = 12 - 1 = \boxed{11}$

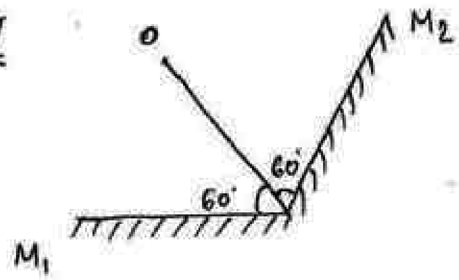


$\therefore$ 170
160
30

360

Q. 17

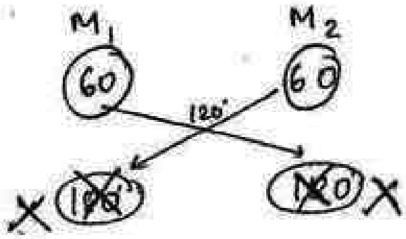
a)



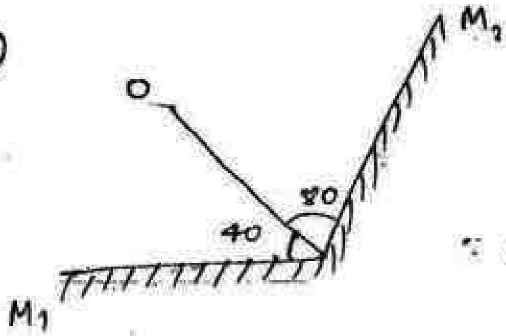
$\theta = 120°$

$\therefore$ $m = \frac{360}{120} = 3$ (odd)

$\therefore$ $n = 3 - 1 = \boxed{2}$ ($\because$ symmetrical)



b)



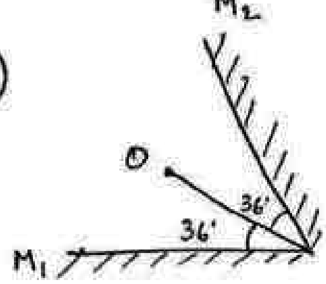
$\theta = 120$

$m = \frac{360}{120} = 3$ (odd)

$\because$ unsymmetrical $\therefore$ $m = n$

$= \boxed{3}$

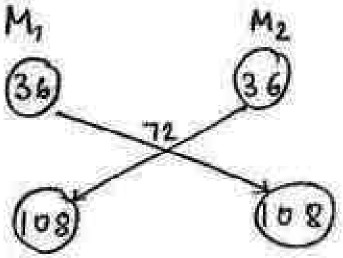
c)



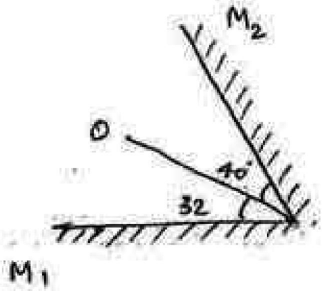
$\because m = \frac{360}{\theta} = \frac{360}{36} = 10$ (even)

$= \frac{360}{72} = \boxed{5}$

$\therefore n = 5 - 1 = \boxed{4}$



d)



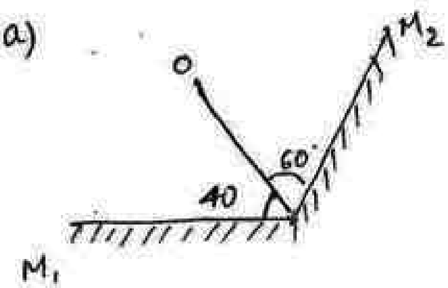
$m = \frac{360}{\theta} = \frac{360}{72} = 5$ (odd)

$\therefore$ unsymmetrical

$\therefore n = m = \boxed{5}$

Q.18 Find no of images ?

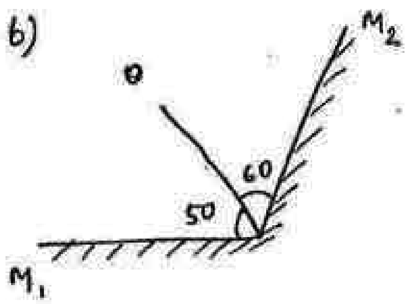
a)



$m = \frac{360}{\theta} = \frac{360}{100} = 3.6$

$\therefore m = 4$ [EVEN] nearest

b)



$m = \frac{360}{110} = 3.2$

$\therefore m = 4$ [EVEN] nearest

Q.19 In a room, two adjacent walls and ceilings are fitted with plane mirrors. A person is standing in the room. Find total no of images.

→ For mirrors on the walls, $\theta = 90$

$\therefore m = \frac{360}{90} = 4$ (even)

$\therefore n = 4 - 1 = 3$

These images will be erect

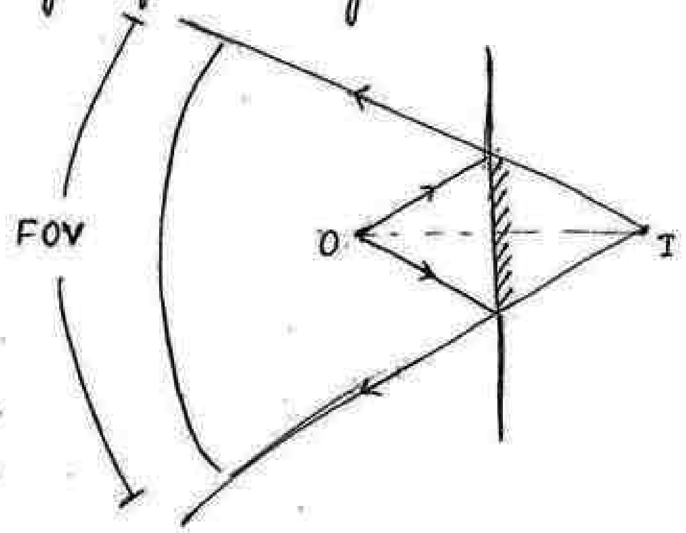
→ For mirror on the ceiling, their are 4 objects.

$\therefore$ it will form 4 images. These images will be inverted $\therefore$

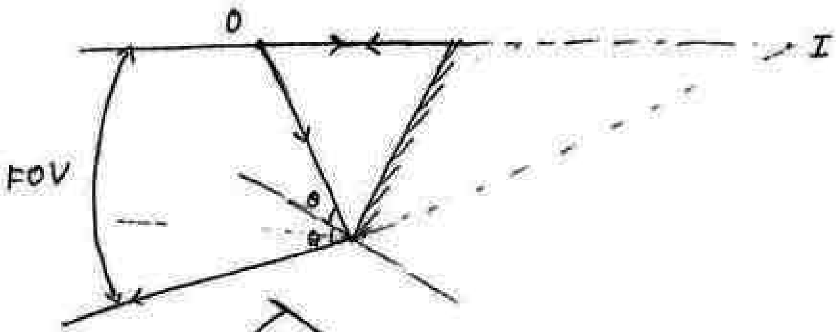
$\therefore$ Total images $= 3 + 4 = 7$ images.

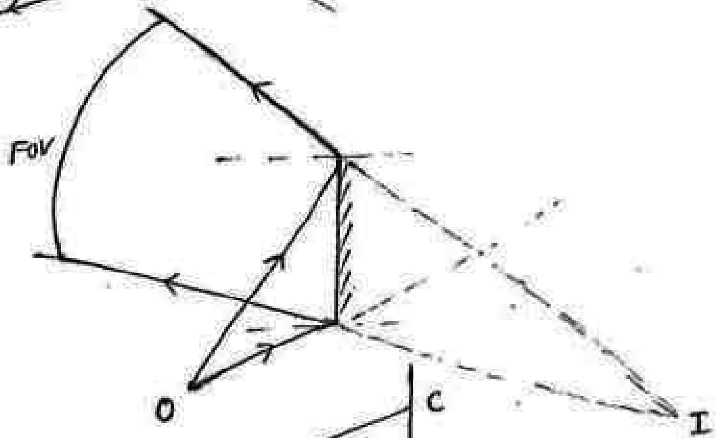
Field of view :

It is the region from where one can see the image of the object.

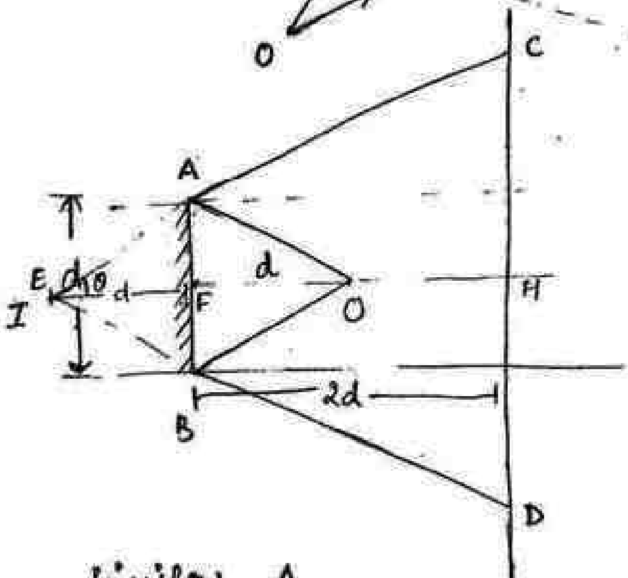


Q. Draw the field of view of the image





Q.



Find the distance on the line over which image of the object can be seen

By using similar Δ,
(In Δ EAB and ECD)

$$\frac{d}{AB} = \frac{3d}{CD} \qquad \therefore \ CD = \frac{3d}{d} \times AB = \frac{3d}{d} \times d = \boxed{3d}$$

OR

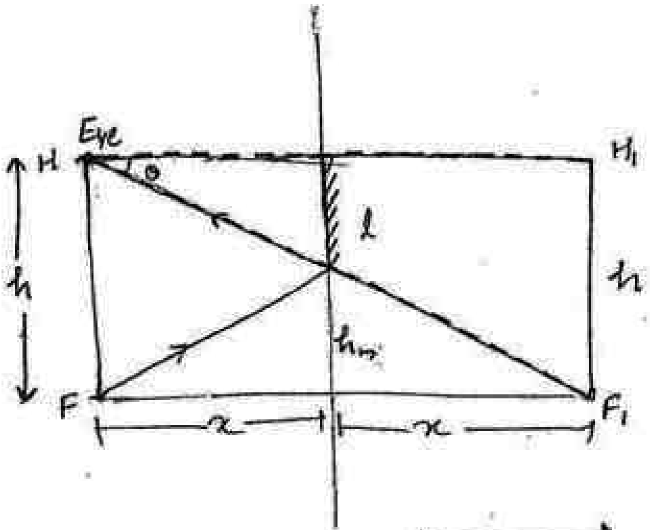
By trignometry,
(In Δ EAF and Δ ECH)

$$\tan\theta = \frac{AF}{EF} = \frac{CH}{EH} \qquad \text{i.e.} \quad \tan\theta = \frac{d}{2d} = \left(\frac{1}{2} = \frac{CH}{EH} = \frac{CH}{3d}\right)$$

$$\therefore \ CH = \frac{3d}{2} \qquad \because \ CD = 2\,CH \ \text{ i.e. } \ 2 \times \frac{3d}{2} = \boxed{3d}$$

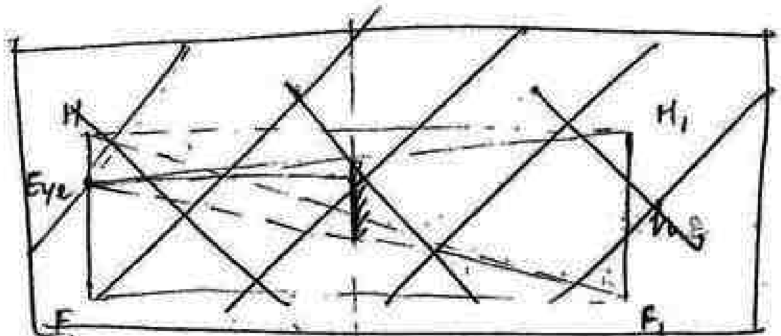
8 Minimum legth of Plane mirror :

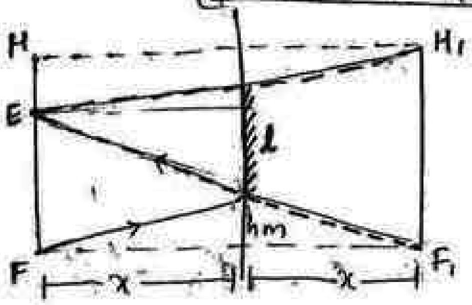
A person wants to see his complete image.



(In Δ HH'F') $\frac{l}{x} = \frac{h}{2x}$ ∴ $\boxed{l = \frac{h}{2}}$ (by similar Δ)

(In Δ HFF₁) ∴ $\frac{h_m}{x} = \frac{h}{2x}$ ∴ $\boxed{h_m = \frac{h}{2}}$ (by trignometry)



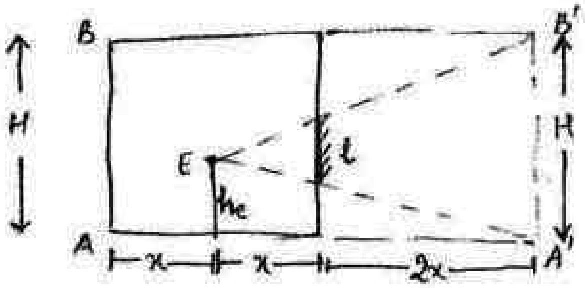


$\frac{l}{x} = \frac{h}{2x}$ ∴ $\boxed{l = \frac{h}{2}}$

∴ $\frac{h_m}{x} = \frac{EF}{2x}$ ∴ $\boxed{h_m = \frac{EF}{2}}$

* Therefore, minimum length of the plane mirror is half of the height of person. and the lower portion of the mirror should be at the height of <u>half of the eye level</u> of the person.

Q. A person is standing at the centre of a room on the front wall. a plane mirror needs to be fixed, find minimum length of the plane mirror required so that the person can see the complete image of the wall behind him.



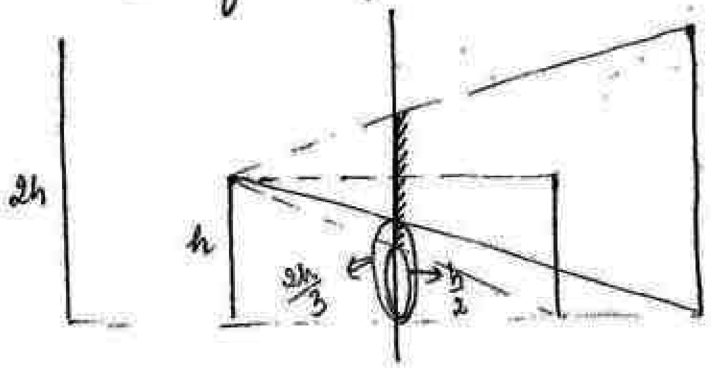
In Δ EA'B'

by similar Δ, we get, $\frac{l}{x} = \frac{H}{3x}$

$$\therefore \boxed{l = \frac{H}{3}}$$

$$\frac{h_m}{2x} = \frac{h_e}{3x} \qquad \therefore \boxed{h_m = \frac{2h_e}{3}}$$

** Q. A person of height h is standing in a room of height $2h$ at the centre. Find the minimum height of the plane mirror required so that he can see complete image and of the wall behind him.



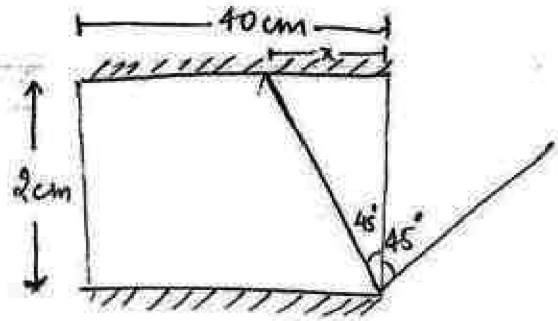
To see wall : $l = \frac{2h}{3}$

$$\frac{2h}{3} - \frac{h}{2} = \frac{4h-3h}{6} = \frac{h}{6}$$

∴ length of the mirror

$$= \frac{2h}{3} + \frac{h}{6} = \boxed{\frac{5h}{6}}$$

Q.



Find total no of Reflections.

$$\tan 45^\circ = \frac{x}{2} = 1$$

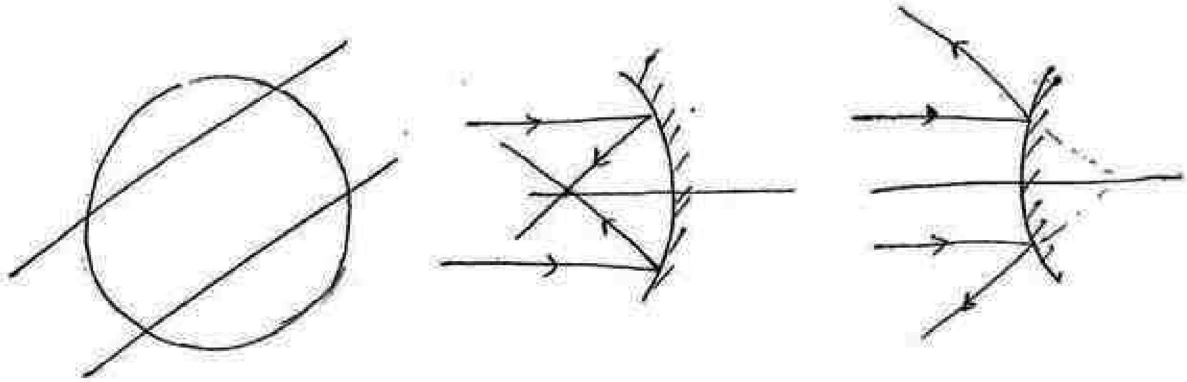
$$\therefore x = 2$$

$$Nx = 40$$

$$\therefore N = \frac{40}{x} = \frac{40}{2} = 20$$

∴ No of reflection $= 20 + ① = 21$

Spherical mirror :

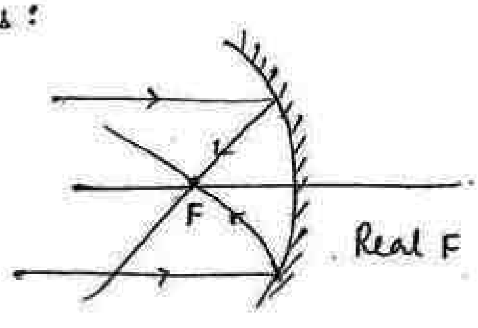


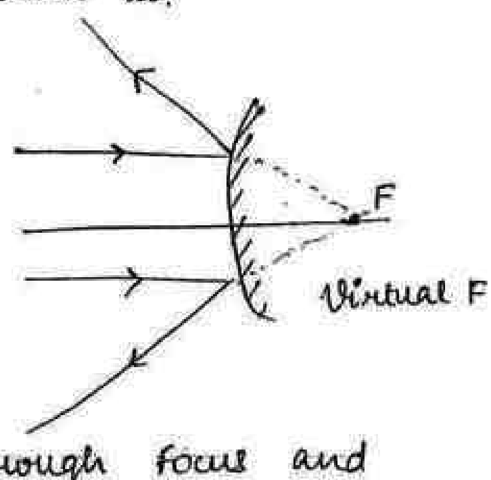
- Pole: Midpoint of the mirror is called is pole
  All the distances are measured from the pole

- Centre of curvature: Centre of the sphere of which spherical mirror is a part is called centre of curvature
  Any line passing through the centre of curvature is <u>normal</u> to the spherical mirror.

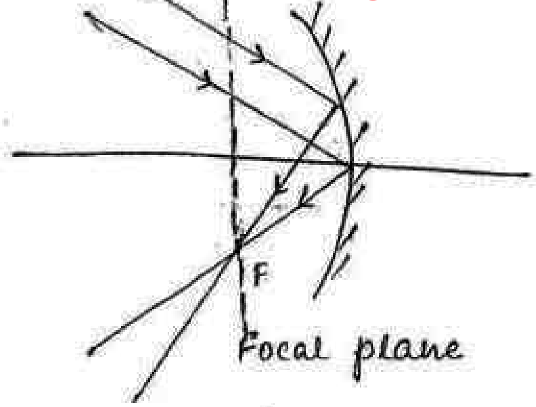
- Principal axis: line passing through pole and centre of curvature.
  All the distances are measured along the principal axis are perpendicular to ~~each other~~ it.

- Focus:



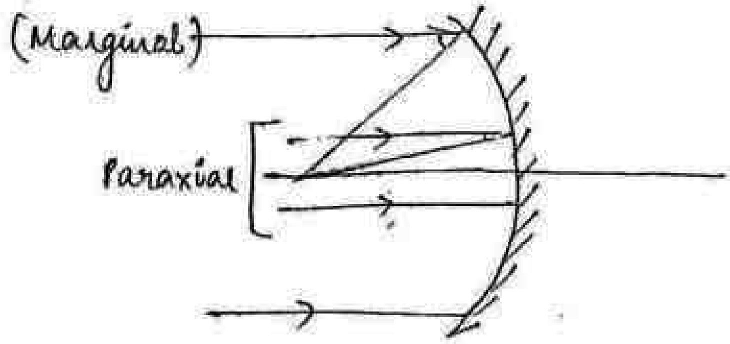


- Focal plane: A plane passing through focus and perpendicular to principal axis.



> Paraxial rays : Rays parallel and close to the principal axis are called paraxial rays.
The angle of incidence for these rays is very small

> Marginal rays : These rays are away from the principal axis. The angle of incidence for these rays is not very small.
These rays do not pass through the single point.

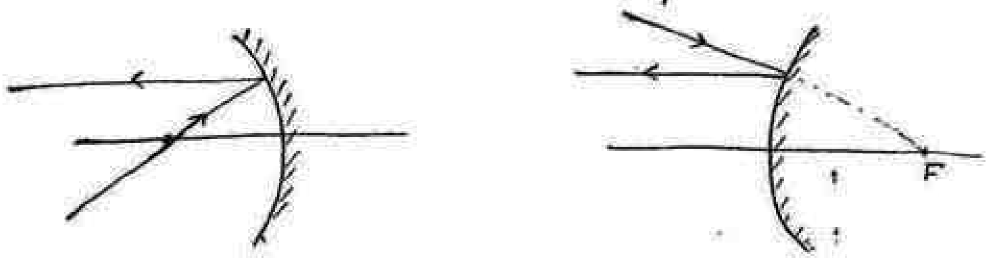


> Aperture : The portion of the spherical mirror where actual reflection takes place.
For paraxial ray, aperture of the mirror is small.

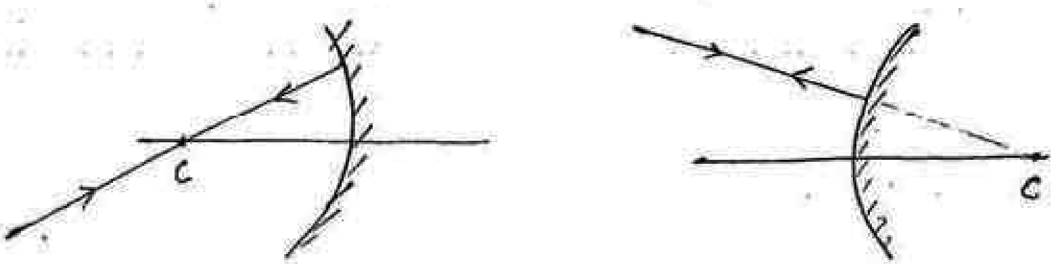
Rules of Image tracing :

i) A ray parallel to the principal axis passes through the focus after reflection

ii) A ray passing through the focus becomes parallel to the principal axis after reflection.



iii) A ray passing through the centre of curvature, retraces its path after reflection.



iv)

<u>Image formation in concave mirror.</u>

i) when object is at infinity.

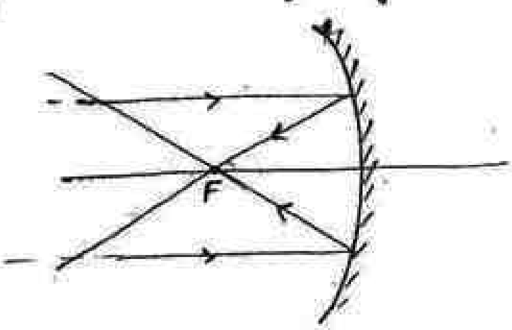


Image is at focus
It is real
highly diminished

ii) when object is beyond C

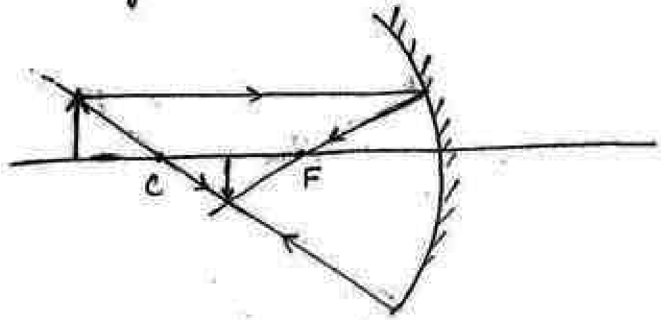


Image is b/w F and C
It is real and inverted
diminished.

iii) when object is at C

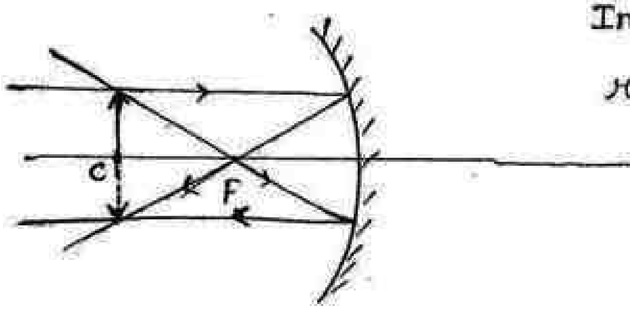


Image is at C
real and inverted
same size

iv) when object is b/w F and C

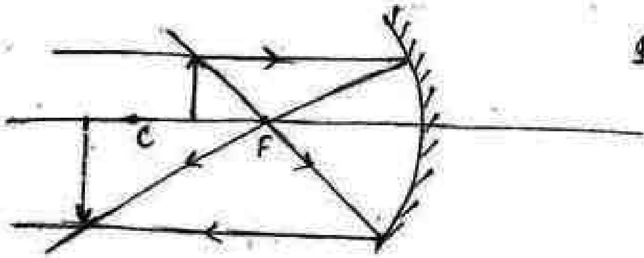


Image at beyond C
real and inverted
enlarged

v) when object is at F

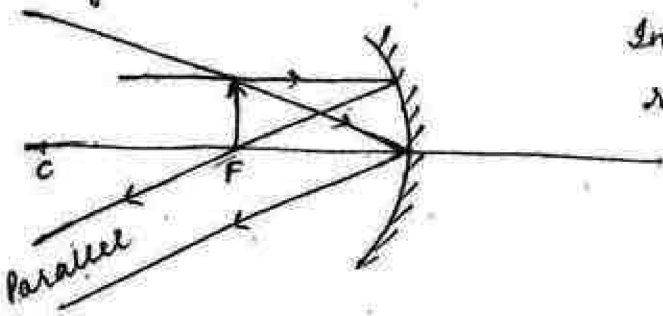
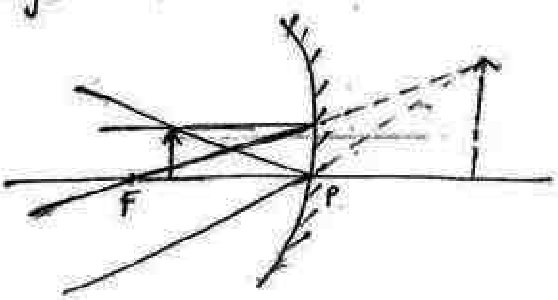


Image at infinity
real and inverted
highly enlarged

vi) when object is b/w F and P



behind the mirror
virtual and erect
enlarged

vii) when object is virtual

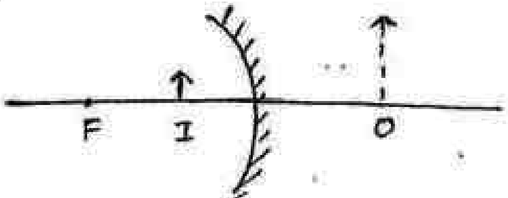


Image is real and erect b/w F and P diminished

∵ light follow principal of reversibility.

Image formation in convex mirror :

i) for real object

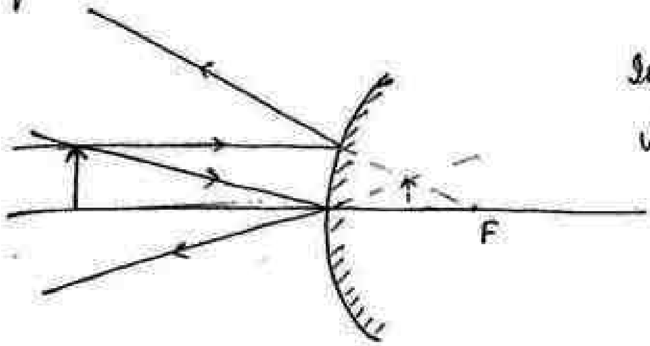


Image is b/w F and P virtual and erect diminished

ii) when the virtual object is located b/w F and P

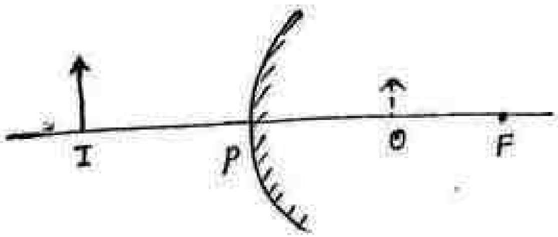
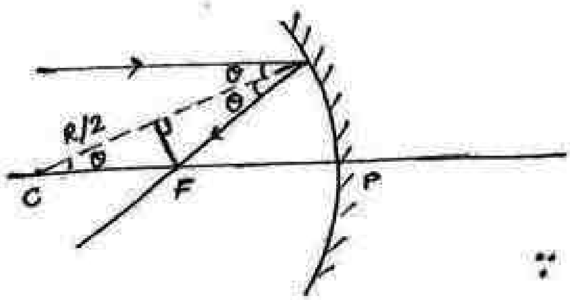


Image is real and erect infornt of mirror enlarged

~~iii) Image is~~

Relation b/w focal length and Radius of curvature



$$\cos\theta = \frac{R/2}{CF}$$

$$\therefore CF = \frac{R/2}{\cos\theta} = \left(\frac{R}{2}\right)\sec\theta$$

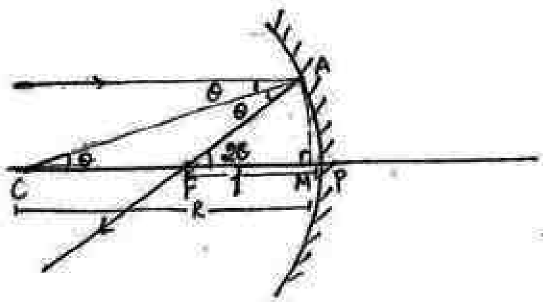
$$\because PF = PC - CF$$

$$f = \left(R - \frac{R}{2}\sec\theta\right)$$

If θ is very small,

$\sec\theta \simeq 1$

$$\therefore f = R - \frac{R}{2} \qquad \therefore \boxed{f = \frac{R}{2}}$$



$\tan\theta = \frac{AM}{CM}$

$\therefore \theta = \frac{AM}{CP}$ — (I)

$\tan 2\theta = \frac{AM}{FM}$

$\therefore 2\theta = \frac{AM}{FP}$ — (II)

Putting (I) in (II), we get.

$$2\left(\frac{AM}{CP}\right) = \frac{AM}{FP}$$

$$\therefore FP = \frac{CP}{2} \quad \text{i.e} \quad \boxed{f = \frac{R}{2}}$$

* If θ is very small, then

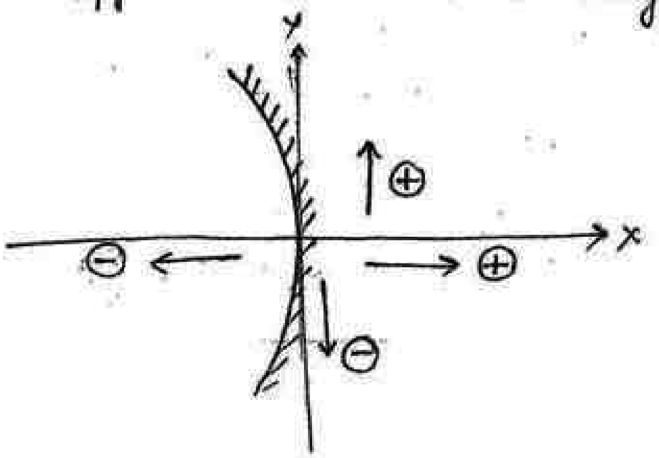
$\sin\theta = \theta$

$\cos\theta = 1$

$\tan\theta = \theta$

Sign convention :

> All the distances are measured from pole
> Distance in the dirⁿ of incident ray are ⊕
> Distances in opp. dirⁿ to incident ray are ⊖



u → (−) Real, (+) virtual (object distance)

v → (−) Real, (+) virtual (Image distance)

f → (−) concave, (+) convex (focal length)

m → (−) Inverted, (+) Erect (magnification)

*. If V.O forms same size image, i.e Plane mirror

If V.O forms diminished image, i.e concave mirror

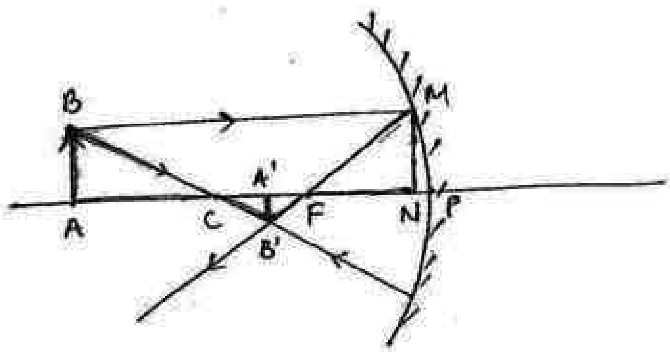
If V.O forms magnified image ie convex mirror

Relation b/w u, v and f.

i) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

ii) $m = \frac{h(I)}{h(O)} = \frac{-v}{u}$

iii) $f = \frac{R}{2}$



$u = -PA$

$v = -PA'$

$f = -PF$

$2f = -PC$

In Δ ABC and $\Delta A'B'C$

$$\frac{AB}{A'B'} = \frac{AC}{A'C} \quad \text{--- (I)}$$

In Δ MNF and $\Delta A'B'F$

$$\frac{MN}{A'B'} = \frac{FN}{FA'}$$

$$\frac{AB}{A'B'} = \frac{FN}{FA'} \quad \text{--- (II)}$$

from (I) and (II)

$$\frac{AC}{A'C} = \frac{FN}{FA'} \qquad \therefore \frac{PA - PC}{PC - PA'} = \frac{FP}{PA' - PF}$$

$$\frac{-u + 2f}{-2f + v} = \frac{-f}{-v + f}$$

$$uv - uf - 2vf + \cancel{2f^2} = \cancel{2f^2} - vf$$

$$\therefore \; uv = uf + vf$$

$$\therefore \; \frac{uv}{uvf} = \frac{uf}{uvf} + \frac{vf}{uvf}$$

$$\Rightarrow \boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}} \; *$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{u-f}{uf}$$

***

$$\therefore \boxed{v = \left(\frac{uf}{u-f}\right)}$$

$$\because m = \frac{-v}{u} = -\left(\frac{f}{u-f}\right) = \boxed{\left(\frac{+f}{f-u}\right) = m}$$ ***

Q. Focal length of concave mirror is 10 cm, find position nature and magnification of image when the real object is at a distance of i) 30 cm ii) 20 cm iii) 5 cm

i) $\because v = \left(\frac{fu}{u-f}\right) \Rightarrow \frac{-30(-10)}{-30+10} = \frac{300}{-20} = -15\text{ cm}$

nature : real and inverted

$$\therefore m = \frac{-v}{u} = -\frac{(-15)}{-30} = \left(-\frac{1}{2}\right)$$

ii) $\because v = \frac{uf}{u-f} = \frac{-20(-10)}{-20+10} = \frac{200}{-10} = -20\text{ cm}$

nature : real and inverted

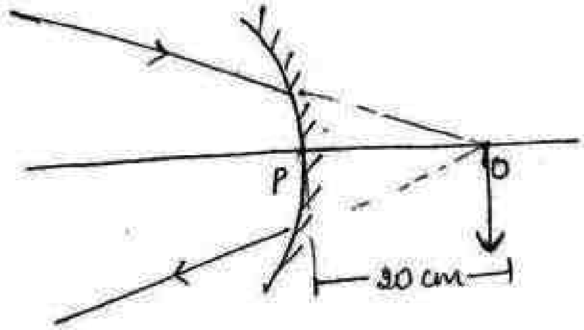
$$m = -\left(\frac{-20}{-20}\right) = -1$$

iii) $v = \frac{uf}{u-f} = \frac{-5(-10)}{-5+10} = \frac{-150}{5} = +10\text{ cm}$

nature : virtual and Erect

$$m = \frac{-10}{-5} = 2$$

Q.2



$f = 10$ cm

Find position nature and magnification of image

$f = -10$ cm

$u = 20$ cm

$$\therefore V = \frac{20(-10)}{20+10} = \frac{-20\not{0}}{3\not{0}} = \frac{-20}{3} \quad \text{(Real)}$$

$$\because m = -\frac{V}{u} = -\left(\frac{-20}{3 \times 20}\right) = +\frac{1}{3} \quad \text{(Erect)}$$

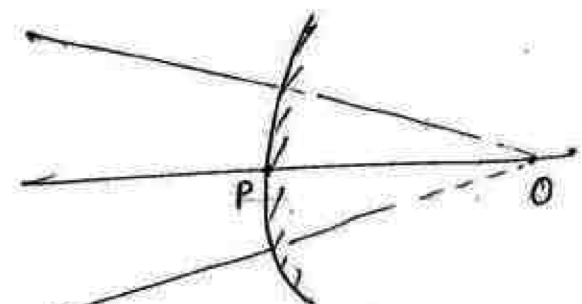
Q. Focal length of convex mirror is 10 cm, Find position, nature and magnification of the image when real object is at the distance of 20 cm

$$\because \quad V = \left(\frac{uf}{u-f}\right) \quad \frac{-20(10)}{-20-10} = \frac{-200}{-30} = \frac{20}{3} \text{ cm}$$

$$m = -\frac{20}{3} \times \frac{1}{-20} = \frac{1}{3}$$

$\therefore$ Virtual and erect

Q.



$f = 10$ cm

Find position, nature and magnification of image if
i) PO = 5 cm
ii) PO = 15 cm
iii) PO = 20 cm

i) $\because \quad v = \frac{uf}{u-f} \Rightarrow \frac{5(10)}{5-10} = \frac{50}{-5} = \boxed{-10 \text{ cm}}$

$\therefore \quad m = -\frac{v}{u} = -\left(\frac{-10}{5}\right) = \boxed{+2}$

∴ Real and Erect

ii) $\because \quad v = \frac{uf}{u-f} = \frac{15(10)}{15-10} = \frac{150}{5} = \boxed{+30 \text{ cm}}$

$\therefore \quad m = -\frac{30}{15} = \boxed{-2}$

∴ Virtual and inverted

iii) $v = \frac{uf}{u-f} = \frac{20(10)}{20-10} = \frac{200}{10} = \boxed{20 \text{ cm}}$

$\therefore \quad m = -\left(\frac{20}{20}\right) = \boxed{-1}$

∴ Virtual and inverted

Note : For curved surfaces.

i) RO ⟶ VI ⟶ Erect

RO ⟶ RI ⟶ Inverted

VO ⟶ RI ⟶ Erect

VO ⟶ VI ⟶ Inverted

Q) Focal length of concave mirror is 10 cm. Find pos^n of real object for which

i) real image is three times the size of object

ii) virtual image is 3 times the size of object

i) $\because$ $m = \dfrac{f}{f-u} = \dfrac{-10}{-10-u} = -3$

$\therefore 30 + 3u = -10$

$\therefore u = \dfrac{-10-30}{3} = \boxed{\dfrac{-40}{3}}$

ii) $\because$ $m = \dfrac{f}{f-u} = \dfrac{-10}{-10-u} = +3$

$\therefore -30 - 3u = -10$

$\therefore u = \dfrac{-10+30}{-3} = \boxed{\dfrac{-20}{3}}$

Q) Focal length of convex mirror is 10 cm. Find position of the real object for which image is $\frac{1}{3}$ the size of object.

$f = +10$

$\therefore m = \dfrac{f}{f-u}$

$\dfrac{1}{3} = \dfrac{10}{10-u}$ $\therefore$ $\boxed{u = -20\text{ cm}}$

Q.7. Focal length of concave mirror is f. Find position of the real object for which
i) real image is n times the size of object.
ii) virtual image is n times the size of object

$$f = -f$$

i) $\because \quad m = \frac{f}{f-u}$

$$-n = \frac{-f}{-f-u}$$

$$+nf + nu = -f$$

$$nf + f = -nu$$

$$\therefore f = -\frac{nu}{(n+1)} \qquad \therefore u = -\frac{(n+1)}{n} f$$

ii) $$n = \frac{-f}{-f-u}$$

$$-nf - nu = -f$$

$$-nf + f = +nu$$

$$f(-n+1) = nu$$

$$u = \frac{(1-n)f}{n}$$

Q.8 Focal length of concave mirror is 10 cm and object of height 2 cm is kept at a distance of 15 cm in front of mirror, find height of image?

$$\because \quad m = \frac{h(I)}{h(0)} = \frac{f}{f-u}$$

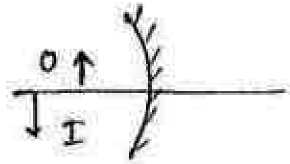
$$= \frac{h(I)}{2} = \frac{-10}{-10+15}$$

$$= \frac{-10 \times 2}{5} = \boxed{-4 \text{ cm}}$$

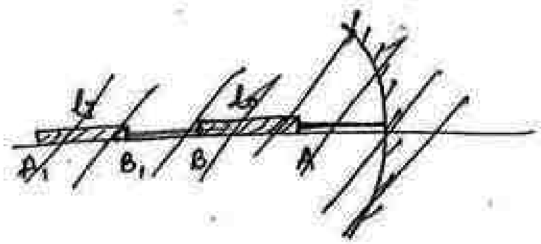
Magnification

i) Lateral Magnification (m): In this case object is ⊥ to principal axis

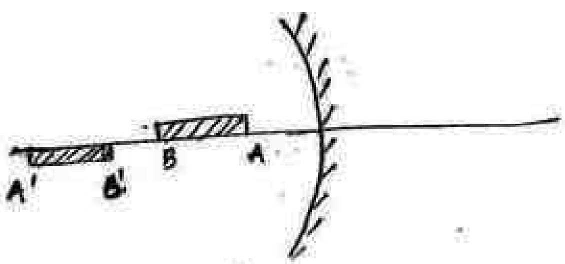
$$** \quad \boxed{m = \frac{h(I)}{h(0)} = \frac{-v}{u} = \frac{f}{f-u}}$$



ii) longitudinal magnification : In this case, object is placed ~~at or kept~~ along the principle axis.



$$m_L = \frac{l_I}{l_o} = \frac{|v_2 - v_1|}{|u_2 - u_1|} = \left|\frac{\Delta v}{\Delta u}\right|$$



$$\boxed{m_L = \frac{l_I}{l_o} = m^2}$$

Proof: $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{d}{dv}\left(\frac{1}{u} + \frac{1}{v}\right) = \frac{d}{dv}\left(\frac{1}{f}\right)$$

$$-\frac{1}{u^2}\left(\frac{du}{dv}\right) - \frac{1}{v^2} = 0 \quad ?$$

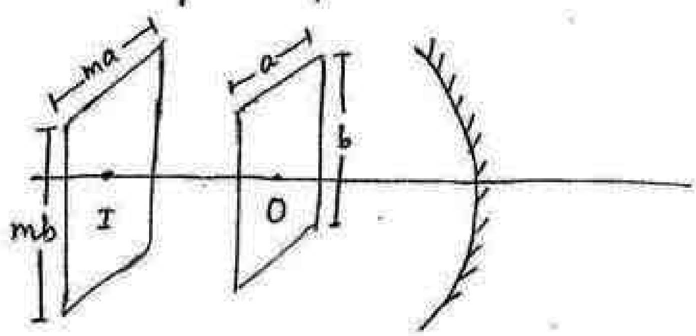
$$\frac{dv}{du} = \frac{-v^2}{u^2} = -m^2$$

$$m_L = \left|\frac{dv}{du}\right| = m^2$$

iii) Aerial magnification :

$$m_A = \frac{\text{Area of image}}{\text{Area of object}} = \frac{A_I}{A_O} = m^2$$

In this case, a two-dimensional object is kept ⊥ to the principal axis.

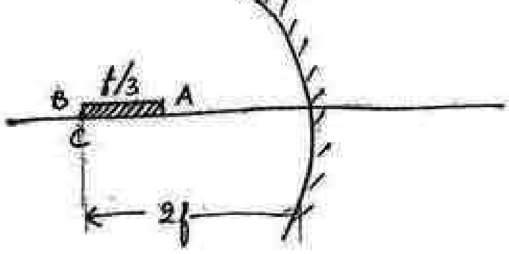


$A_O = a \times b$

$A_I = (ma)(mb) = m^2 ab$

$$\therefore \boxed{A_I = m^2 A_O}$$

Q)



length of the object is $\frac{f}{3}$ and its one end is at C find length of the image

for end A:

$$u_A = -\left(2f - \frac{f}{3}\right) = \frac{-5f}{3}$$

$$\therefore V_A = \frac{uf}{u-f} = \frac{\left(\frac{-5f}{3}\right)(-f)}{-\frac{5f}{3} + \frac{f}{1}}$$

$$\Rightarrow \frac{-5f}{2} = -2.5f$$

for end B:

$$u_A = -2f$$

$$V_B = \frac{-2f(-f)}{-2f+f} = \frac{2f^2}{-f} = -2f$$

$\therefore$ length of the image $= -2f - (-2.5f)$

$$\therefore 2.5f - 2f = \boxed{0.5f}$$

Q) Focal length of concave mirror is f. A very small object of length 'B' is kept along the principle axis at a distance u from the pole. find length of image.

$m_L = m^2$ ($\because$ object is very small)

$$\frac{l_I}{l_o} = \left(\frac{-f}{-f+u}\right)^2 = \left(\frac{f}{f-u}\right)^2$$

$$\frac{l_I}{B} = \left(\frac{f}{f-u}\right)^2$$

$$\therefore\ l_I = B\left(\frac{f}{f-u}\right)^2$$

Q.) Focal length of concave mirror is 10 cm. An object 3 cm x 4 cm is kept ⊥ to the principle axis at a distance of 15 cm from pole find Area of image

$$A_I = A_0\, m^2$$

$$= (3\times4)\left(\frac{f}{f-u}\right)^2$$

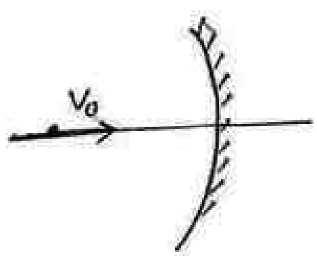
$$= (3\times4)\left(\frac{-10}{-10+15}\right)^2$$

$$= 12\times(-2)^2 = 48\ cm^2$$

Velocity of image :

Case I : when the object is moving along the principal axis.

$$\boxed{V_I = -m^2 V_0}$$



$$\because\ \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

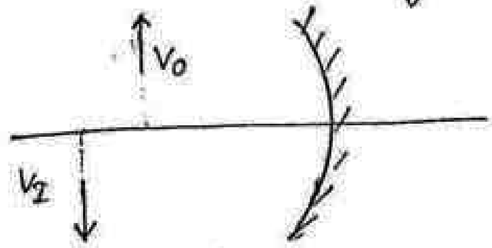
$$\frac{d}{dt}\left(\frac{1}{u}\right) + \frac{d}{dt}\left(\frac{1}{v}\right) = \frac{d}{dt}\left(\frac{1}{f}\right)$$

$$-\frac{1}{u^2}\left(\frac{du}{dt}\right) - \frac{1}{v^2}\left(\frac{dv}{dt}\right) = 0$$

$$-\frac{1}{u^2} V_0 \quad -\frac{1}{v^2} V_I = 0$$

$$\therefore \quad V_I = -\left(\frac{v^2}{u^2}\right) V_0$$

Case II when object is moving ⊥ to principle Axis



$$\boxed{V_I = m V_0}$$

∵ u and v are constant

$$m = \frac{h_I}{h_0}$$

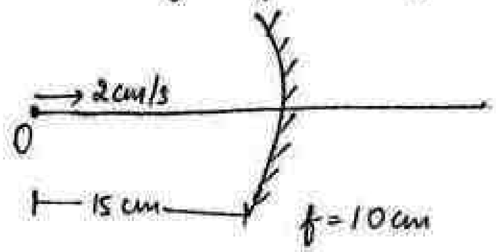
$$h_I = m h_0$$

$$\Rightarrow \frac{d}{dt} h_I = m \frac{d}{dt} h_0$$

$$\therefore \boxed{V_I = m V_0}$$

Q Find velocity of image

i)



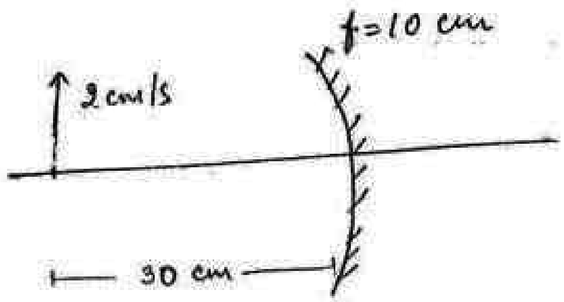
$$m = \frac{f}{f-u} = \frac{-10}{-10+15} = \frac{-10}{5} = (-2)$$

$$\because \quad V_I = -m^2 V_o$$

$$= -(-2)^2 \times 2$$

$$= \boxed{-8 \text{ cm/s}}$$

ii)

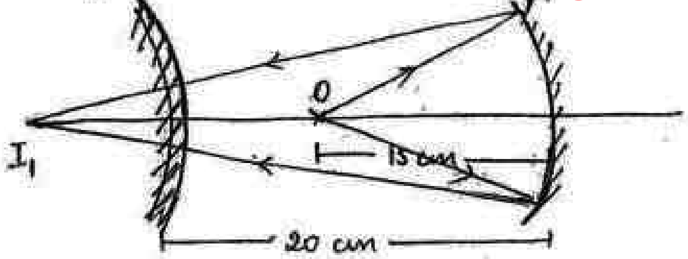


$$m = \left(\frac{f}{f-u}\right) = \frac{-10}{-10+30} = -\frac{1}{2}$$

$$\because \quad V_I = m V_o$$

$$= -\frac{1}{2} \times 2 = \boxed{-1 \text{ cm/s}}$$

Q.)



Taking 1st reflection with concave mirror, find position of image formed by convex mirror

for concave

$u = -15 \qquad f = -10$

$\therefore \; v = \dfrac{uf}{u-f} = \dfrac{-15(-10)}{-15+10} = \dfrac{150}{-5} = -30 \text{ cm}$

for convex

$u = +10 \qquad f = +15$

$v = \dfrac{uf}{u-f} = \dfrac{10 \times 15}{10-15} = \dfrac{150}{-5} = \boxed{-30 \text{ cm}}$

ie 10 cm behind concave mirror