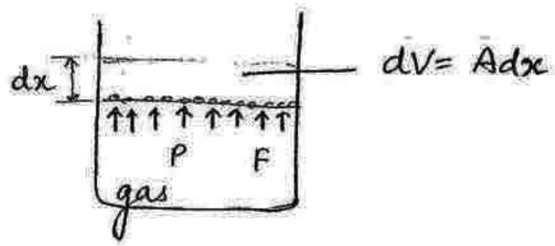
Thermodynamics

Work done in Thermodynamics:

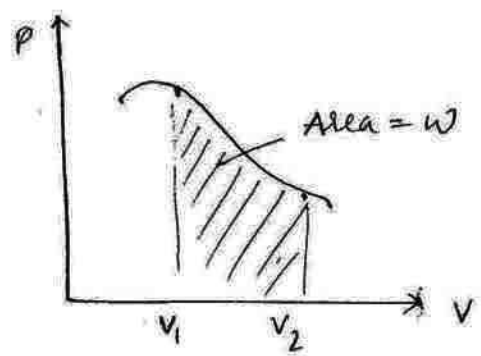
i) work done by system (gas) for expansion:



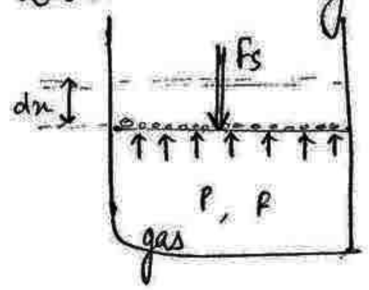
$\therefore \; dW = F dx \cos\theta$

$= PA dx$

$\boxed{dW = PdV} \qquad \therefore \boxed{W = \int_{V_1}^{V_2} P dV}$



ii) work done by surrounding:



$dW_s = F_s \, dx \cos 180^\circ$

$= -Fdx \quad = -PAdp$

$\boxed{dW_s = -PdV}$

$\therefore \boxed{W_s = -\int_{V_1}^{V_1} PdV}$

iii) Work done in diff. Processes:

a) Isochoric process

$V = K \quad \therefore dV = 0 \quad \therefore dW = PdV = 0 \quad \therefore \boxed{W = 0}$

b) Isobaric process

$P = K \quad \therefore W = \int_{V_1}^{V_2} PdV \quad \therefore W = P(V_2 - V_1)$

$\therefore \boxed{W = P\Delta V = nR\Delta T}$

c) ~~Isoba~~ Isothermal process:

$PV = (nRT) \quad \therefore P_1V_1 = P_2V_2$

$W = \int_{V_1}^{V_2} PdV \Rightarrow W = \int_{V_1}^{V_2} \frac{nRT}{V} dV$

$\therefore \boxed{W = nRT \ln\left(\frac{V_2}{V_1}\right)} \quad \boxed{W = nRT \ln\left(\frac{P_1}{P_2}\right)}$

d) Adiabatic process

$PV^{\gamma} = K$

$\therefore P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\therefore W = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{K}{V^{\gamma}} dV = K\left[\int V^{1-\gamma} dV\right]$

$\therefore \boxed{W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}}$

$\boxed{W = \frac{nR(T_1 - T_2)}{\gamma - 1}}$

e) Polytropic process :

$$PV^{x} = K$$

$$\therefore \boxed{W = \frac{P_1 V_1 - P_2 V_2}{x-1}}$$

$$\boxed{W = \frac{nR(T_1 - T_2)}{x-1}}$$

Q) A gas follows the process $\left(P = aV + \frac{b}{V}\right)$ Find the work done by the gas when the vol^m changes from V_0 to $2V_0$

$$\because P = \left(aV + \frac{b}{V}\right)$$

$$\therefore W = \int_{V_0}^{2V_0} P\,dV$$

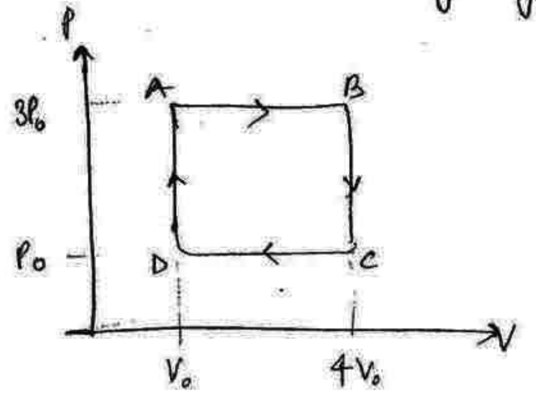
$$= \int_{V_0}^{2V_0} \left(aV + \frac{b}{V}\right) dV$$

$$W = \left[\frac{aV^2}{2} + \ln V\right]_{V_0}^{2V_0} \qquad \therefore W = \frac{3aV_0^2}{2} + b\ln 2$$

Q) A mole of gas follows the process $PV^2 = \text{constant}$ find the work done by it when its Temp. changes from T_0 to $2T_0$

$$W = \frac{nR(T_1 - T_2)}{x-1} \Rightarrow \frac{R(T_0 - 2T_0)}{2-1} = -RT_0$$

Q) Find work done by gas in each process:



i) ~~From~~ W_{AB} (Isobaric):

$$P\Delta V = 3P_0(4V_0 - V_0) = 9P_0V_0$$

ii) $W_{BC} = 0$ (Isochoric)

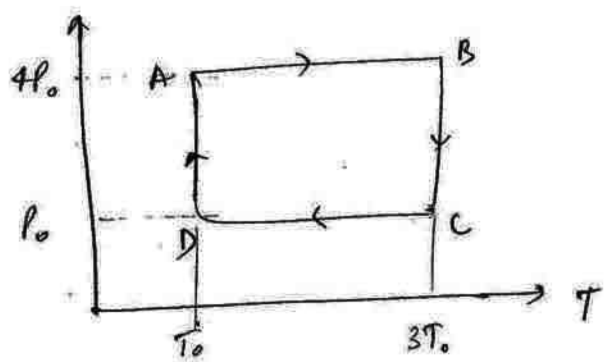
$W_{CD} =$ (Isobaric)

$$P\Delta V = -P_0(3V_0) = -3P_0V_0$$

iv) $W_{DA} = 0$ (Isochoric

$\therefore$ Total work done $= (9-3)P_0V_0 = 6P_0V_0$

Q) Find work done by the gas in each process.



$(n=1)$

W_{AB} = Isobaric

$$nR\Delta T = 1 \times R \times 2T_0 = 2RT_0$$

ii) W_{BC} (Isothermal)

$$nRT \ln \frac{P_1}{P_2} \Rightarrow 3RT_0 \ln \frac{4\cancel{P_0}}{\cancel{P_0}}$$

$$= 3RT_0 \ln 4$$

iii) $W_{CD} = -2RT_0$

iv) $W_{DA} = RT_0 \ln \frac{1}{4} = -RT_0 \ln 4$

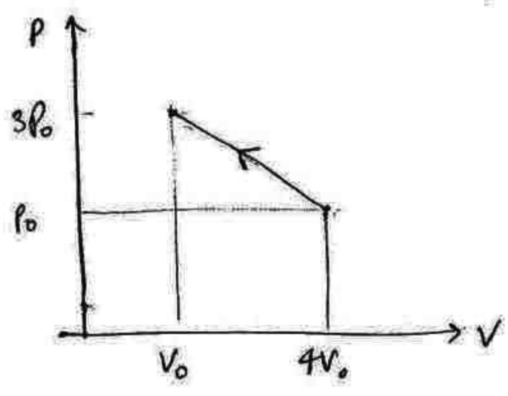
Q) 2 moles of He is heated from 300 K to 400 K in adiabatic process. Find work done by gas.

$$W = \frac{nR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{2R(300-400)}{5/3 - 1} = \frac{2R(-100)}{2/3}$$

$$= -300R$$

Q) Find work done by the gas :

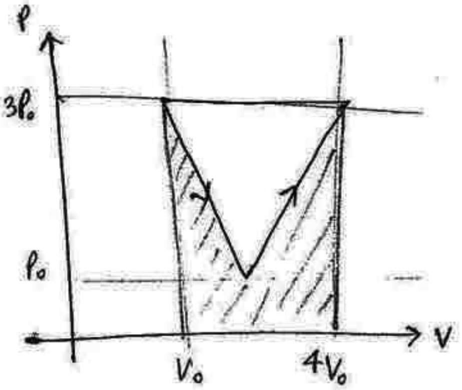


$$W = -\frac{1}{2}(P_0 + 3P_0) \times 3V_0$$

$$= -6P_0V_0$$

∵ Vol^n ↓es

Q) Find work done by the gas.



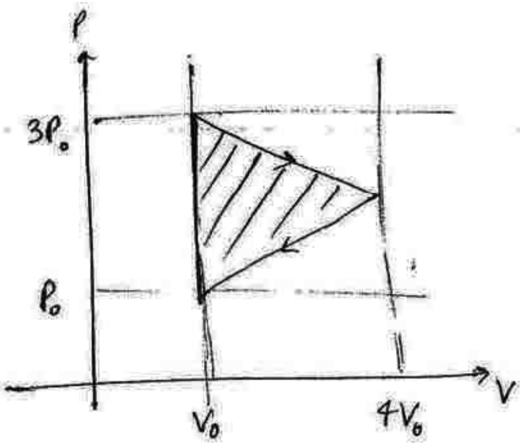
$w =$ area of $\square$ $-$ area of Δ

$w = 3P_0 \times 3V_0 - \frac{1}{2} \times 3V_0 \times 2P_0$

$w = 9P_0V_0 - 3P_0V_0 \Rightarrow 6P_0V_0$

Q)

$V_1 = V_2$

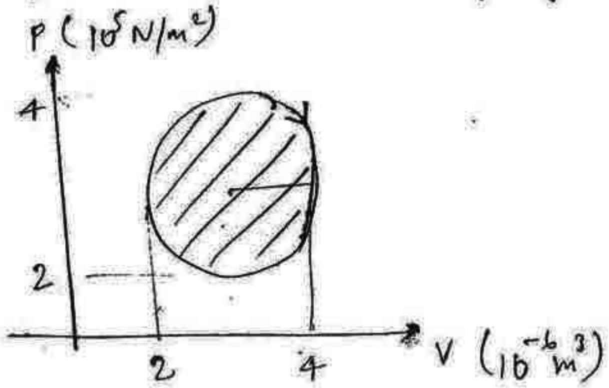


$w =$ ~~area of $\square$~~ area of Δ

$\frac{1}{2} \times 2P_0 \times 3V_0 = 3P_0V_0$

Q) Find work done by gas.

$V_1 = V_2$



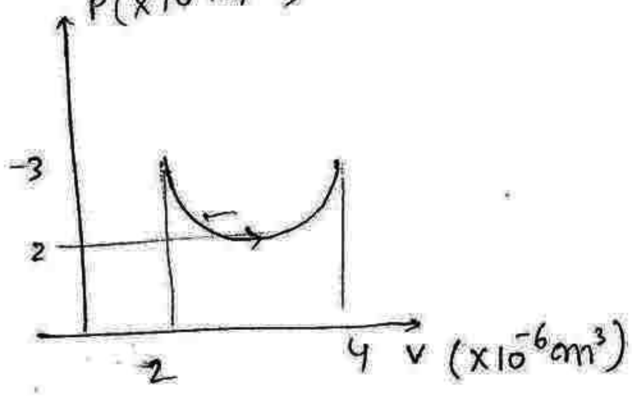
$w_+ = \pi r_1 \times r_2$

$\pi (1 \times 10^{-6}) \times (1 \times 10^{5})$

$= \frac{\pi}{10}$ J

Q)1

$v_1 \neq v_2$



$$w = \text{area of } \square - \text{area of } \triangledown$$

$$= (3\times10^{5})\times(2\times10^{-6}) - \frac{\pi}{2}\,10^{5}\times10^{-6}$$

$$= \left(0.6 - \frac{\pi}{20}\right)\,J$$

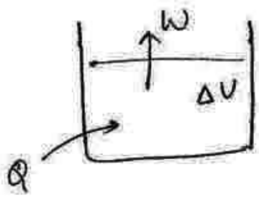
i) zeroth law of thermodynamics : defines temp.

A B C

$\because$ if $T_A = T_B$ and $T_A = T_C$

$\therefore$ $T_B = T_C$

ii) 1st law of thermodynamics : It is based on the law of conservation of energy.
i-e it can neither be created nor be destroyed



$\boxed{Q = W + \Delta U}$ or $\boxed{\Delta U = Q + W_s}$

W: by system

W_s: by surroundy

sign convention.

» for expansion, work done by system is ⊕ve

» for compression, work done on ~~by~~ system is ⊖ve

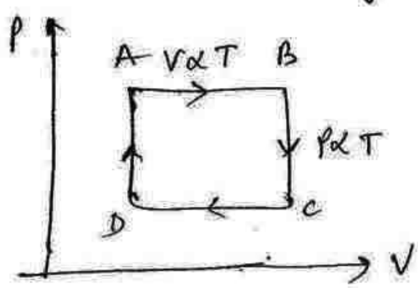
» Q is +ve if heat is added to the system

» Q is −ve if heat is rejected from/by the system

» ΔU is ⊕ve if temp ↑es

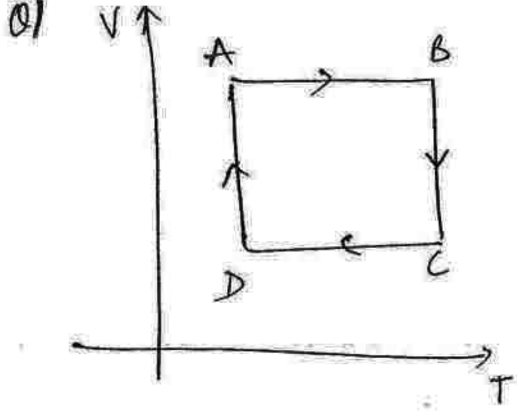
» ΔU is ⊖ve, if temp ↓es

Q) write the sign of ΔU and ($Q = n C_p \Delta T$)



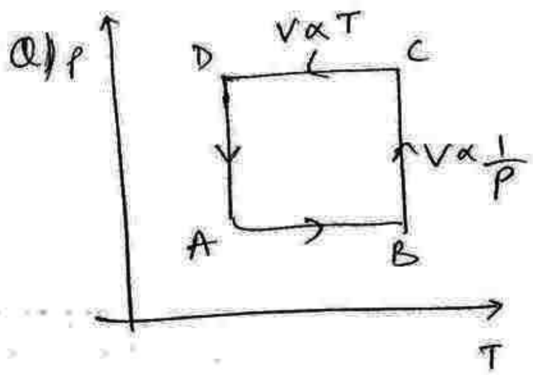
	W	ΔU	Q	
AB	+	+	+	⇒ Isobaric
BC	0	−	−	
CD	−	−	−	⇒ Isobaric
DA	0	+	+	

Q)



	W	ΔU	Q	
AB	0	+	+	
BC	−	0	−	
CD	0	−	−	
DA	+	0	+	

Q)



	W	ΔU	Q
AB	+	+	+
BC	−	0	−
CD	−	−	−
DA	+	0	+

diff thermodynamic processes.

i) Isochoric : a) $V = K$, $\therefore dV = 0$

b) $PV = nRT$ $\therefore$ $(\Delta P)V = nR(\Delta T)$

c) $W = 0$

d) $C_v = \frac{fR}{2}$

e) $Q = nC_v dT$

f) $Q = W + \Delta U$

$\therefore$ $\boxed{Q = \Delta U}$

Isobaric

a) $P = K$, $dP = 0$

b) $PV = nRT$ $\therefore$ $P\Delta V = nR\Delta T$

c) $C_p = C_v + R$

d) $W = P\Delta V = nR\Delta T$

$\therefore$ $\Delta U = \frac{f}{2} nR\Delta T = nC_v dT$ $\therefore$ $\Delta U = \frac{f}{2} W$

e) $\frac{\Delta U}{Q} = \frac{nC_v\Delta T}{nC_p\Delta T} = \frac{C_v}{C_p} = \frac{1}{\gamma}$

$\frac{W}{Q} = \left(1 - \frac{1}{\gamma}\right)$

Gas	γ	$\Delta U/Q$	W/Q
Monoato	$5/3$	60%	40%
diatomic	$7/5$	71%	29%
Polyatomic	$4/3$	75%	25%

$\gamma = \left(1 + \frac{2}{f}\right)$

Q) 400 J is given to He in isobaric process Find work done by it and ↑ in its internal energy

$\therefore$ $\frac{\Delta U}{Q} = \frac{1}{\gamma} =$ $\therefore$ 240 J

$W = Q - \Delta U \Rightarrow 160\text{ J}$

or $400 \times \frac{60}{100} = 240$ (240)

$400 \times \frac{40}{100} = 160$

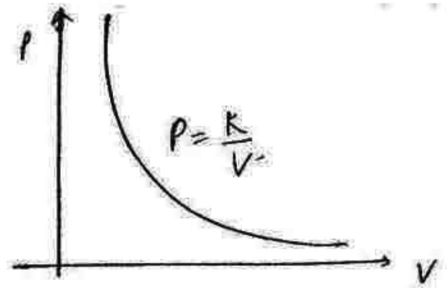
Isothermal

a) $T = K$, $dT = 0$

b) $PV = nRT$

$P_1 V_1 = P_2 V_2$ i.e $\boxed{P \propto \frac{1}{V}}$

c)



$$\text{Slope} = \frac{dP}{dV}$$

$PV = \text{constant}$

$$\frac{d.(PV)}{dV} = \frac{d}{dV} K$$

$$P + V\frac{dP}{dV} = 0$$

$$\therefore \frac{dP}{dV} = \left(\frac{-P}{V}\right)$$

d) $W = nRT \ln\left(\frac{V_2}{V_1}\right)$

or

$nRT \ln\left(\frac{P_1}{P_2}\right)$

e) $C = \frac{Q}{n\Delta T} = \infty$

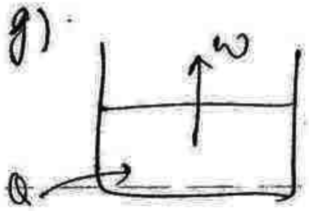
$Q \cancel{= nC\Delta T}$ $\therefore Q = W + \Delta U$

$\therefore Q = W$

f)

Isothermal process is an ideal process cz it can't be ~~sall~~ realised practically

g)



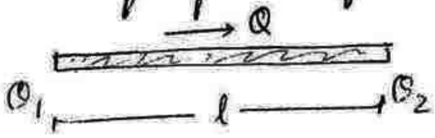
$\left.\begin{array}{l} T = K \\ U = K \end{array}\right\}$ def. req.

$\left.\begin{array}{l} \Delta T = 0 \\ \Delta U = 0 \end{array}\right\}$ achieved

The wall of the container should be conducting and the process should be carried out very slowly.

MODE OF HEAT TRANSFER

i) Rate of flow of heat

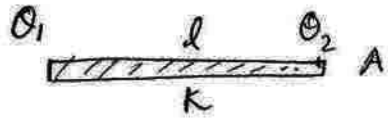


$$\boxed{\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}}$$

$$\frac{dQ}{dt} = -KA\left(\frac{d\theta}{dl}\right)$$

ii) Temp. gradient $\Rightarrow$ $\frac{-d\theta}{dx} = \left(\frac{\theta_1 - \theta_2}{l}\right)$

iii) Thermal resistance $\Rightarrow$



$$\boxed{R = \frac{l}{KA}}$$

$\because R = \frac{\rho l}{A}$

$$\boxed{R = \frac{l}{\sigma A}}$$

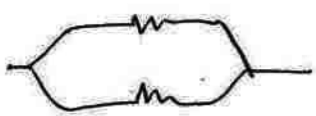
heat current $\Leftarrow$ $\boxed{H = \frac{\theta_1 - \theta_2}{R}}$

$\because I = \frac{V_1 - V_2}{R}$

$$\boxed{H = \frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{\theta_1 - \theta_2}{R}}$$

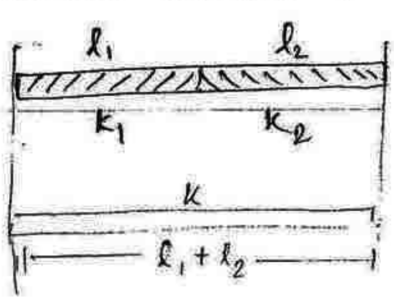
iv)

$R = R_1 + R_2$



$R_{net} = \frac{R_1 R_2}{R_1 + R_2}$ $\left[\because \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}\right]$

5) a) for series combination:



$R = R_1 + R_2$

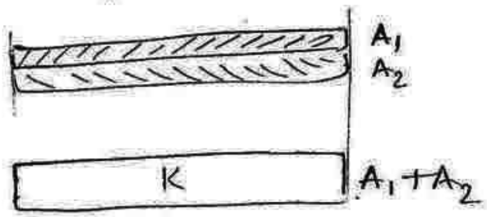
$$\left(\frac{l_1 + l_2}{KA}\right) = \frac{l_1}{K_1 A} + \frac{l_2}{K_2 A}$$

$$K = \frac{l_1 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}}$$

If $l_1 = l_2$ then,

$$K = \left(\frac{2K_1K_2}{K_1 + K_2}\right)$$

b) for parallel combination



$$\because \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

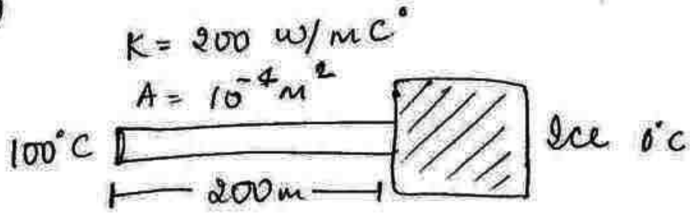
$$\frac{K(A_1 + A_2)}{l} = \frac{K_1 A_1}{l} + \frac{K_2 A_2}{l}$$

$$\therefore K = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

If $A_1 = A_2$, then

$$K = \left(\frac{K_1 + K_2}{2}\right)$$

Q)



Find the amount of ice that melts in 15 min

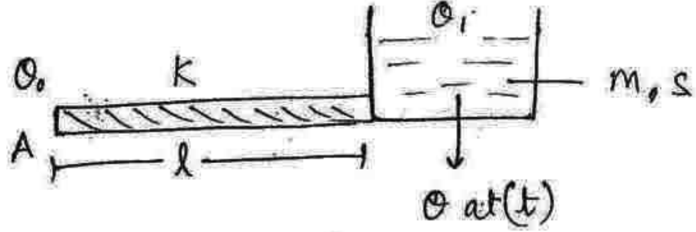
$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}$$

$$\frac{mL}{t} = \frac{KA(100 - 0)}{l}$$

$$\frac{m \times 3.36 \times 10^5}{15 \times 60} = \frac{200 \times 10^{-4} \times 100}{2}.$$

$$\boxed{m \simeq 27\ gm}$$

Q11)



Find the time in which temp. of water increases from θ_1 to θ_2?

$$\frac{d\theta}{dt} = \frac{KA(\theta_0 - \theta)}{l}$$

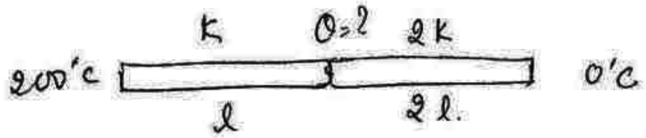
$$\Rightarrow \frac{ms(d\theta)}{dt} = \frac{KA(\theta_0 - \theta)}{l}$$

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta_0 - \theta} = \left(\frac{KA}{msl}\right) \int_0^t dt$$

$$= \ln\left(\frac{\theta_0 - \theta_2}{\theta_0 - \theta_1}\right) = \left(\frac{KA}{msl}\right) t$$

$$\boxed{t = \left(\frac{msl}{KA}\right) \ln\left(\frac{\theta_0 - \theta_1}{\theta_0 - \theta_2}\right)}$$

Q)



Find temp at the junction.

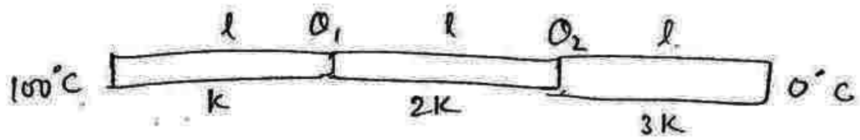
$$H_1 = H_2$$

$$\frac{KA\,(100-\theta)}{l} = \frac{2KA\,(\theta-0)}{2l}$$

$$\therefore\ 100-\theta = \theta-0$$

$$\therefore\ 2\theta = 100 \quad \therefore\ \theta = 50$$

Q)

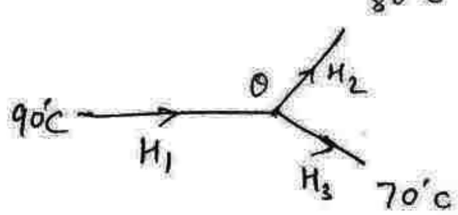


$$H_1 = H_2 = H_3$$

$$\therefore\ \frac{KA\,(100-\theta_1)}{l} = \frac{2KA\,(\theta_1-\theta_2)}{l} = \frac{3KA\,(\theta_2-0)}{l}$$

$$100-\theta_1 = 2\,(\theta_1-\theta_2) = 3\,(\theta_2-0)$$

Q)

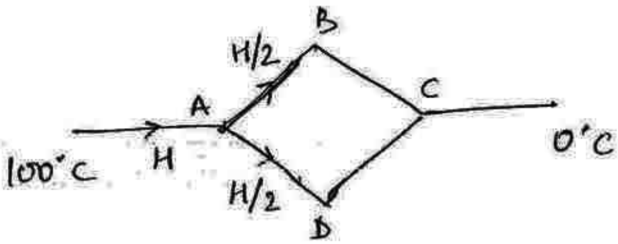


Find θ

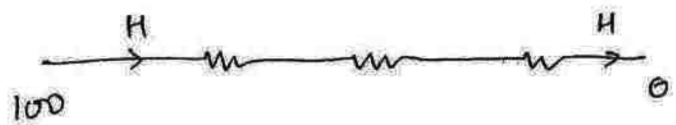
$$H_1 = H_2 + H_3.$$

$$\frac{\cancel{KA}(90-\theta)}{\cancel{l}} = \frac{\cancel{KA}(\theta-80)}{\cancel{l}} = \frac{\cancel{KA}(\theta-70)}{\cancel{l}}$$

Q)



Find temp at A, B, C, D



$$H = \frac{100-0}{3R} = \frac{100}{3R}$$

for <u>1st rod</u>

$$H = \frac{100-\theta_A}{R}$$

$$\frac{100}{3R} = \frac{100-\theta_A}{R}$$

$$\therefore \theta_A = \frac{200}{3}$$

for AB

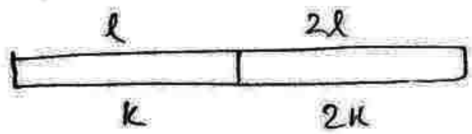
$$\frac{H}{2} = \frac{\theta_A - \theta_B}{R}$$

$$\frac{100}{3R \times 2} = \frac{200}{3} - \theta_B. \qquad \therefore \theta_B = 50C$$

for last rod:

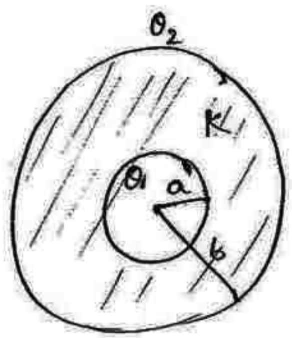
$$H = \frac{\theta_C - 0}{R} \qquad \therefore \frac{100}{3R} = \frac{\theta_C}{R} \qquad \therefore \theta_C = \frac{100}{3}$$

Q) Find equivalent thermal conductivity:



$$K_{eq} = \frac{l_1 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}}$$

Q)



$(\theta_1 > \theta_2)$

Find rate of flow of heat

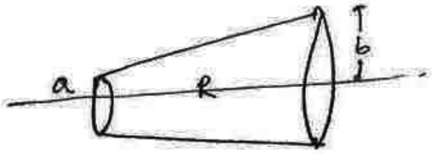
$$r_{eff} = \sqrt{ab}$$

$$A_{eff} = 4\pi r_{eff}^2 = 4\pi ab.$$

$$R = \frac{l}{KA} = \frac{(b-a)}{K(4\pi ab)}$$

$$\therefore H = \frac{\theta_1 - \theta_2}{R}$$

Q)

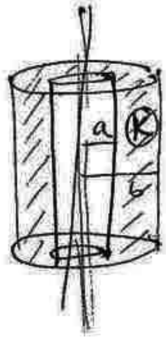


$$r_{eff} = \sqrt{ab}$$

$$A_{eff} = \pi r_{eff}^2 = \pi(ab)$$

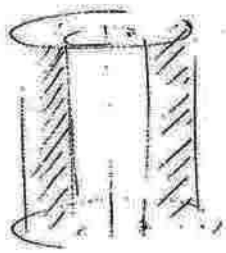
$$R = \frac{l}{KA_{eff}} = \boxed{\frac{l}{K\pi ab}}$$

Q)



Find thermal resistance

$$dR = \frac{dr}{K(2\pi rl)}$$



$$\therefore R = \left(\frac{1}{2\pi Kl}\right) \int_a^b \frac{dr}{r}$$

$$\therefore R = \left(\frac{1}{2\pi Kl}\right) \ln \frac{b}{a}$$

Thermal radiation:

i) Emmissive power: Energy emitted by body per unit time per unit surface area

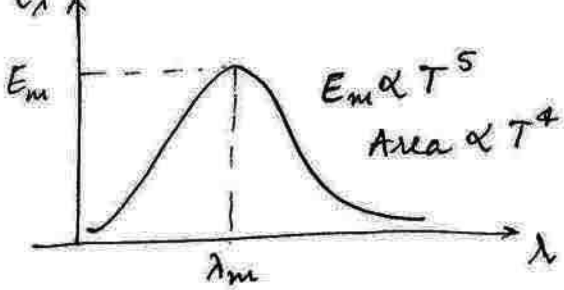
$$E = \frac{Q}{tA} = \frac{P}{A}$$

ii) Stefan Boltzmann law:

$$E \propto t^4 \quad \therefore E = \sigma t^4$$

$$\therefore \frac{P}{A} = \sigma t^4 \quad \therefore \boxed{P = \sigma A t^4} \text{ for Black body}$$

for ordinary body, $P = e\sigma A t^4$ (e = emmissivity) iii)



$E_m \propto T^5$

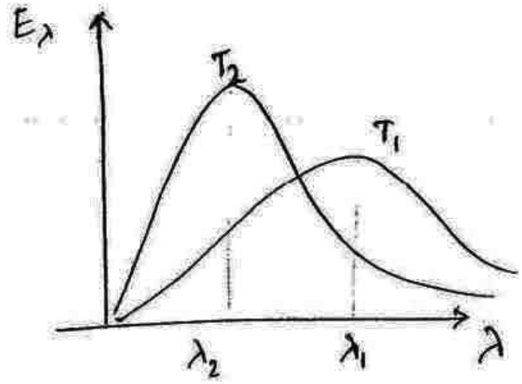
Area $\propto T^4$

(Peak wavelength)

iv) Wein's law:

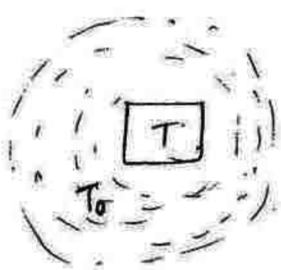
$$\boxed{\lambda_m T = b} \rightarrow \text{wein's constant} \Rightarrow \boxed{2.89 \times 10^{-3} \, (mK)}$$

$$\therefore \quad \lambda_m T = 2.89 \times 10^{-3} \, mK$$



$\boxed{T_2 > T_1}$

v) Rate of ~~fall~~ loss of heat:



$T > T_0$

Power emitted, $P_1 = \sigma e A T^4$

Power absorbed, $P_2 = e\sigma A T_0^4$

$\therefore$ Rate of loss of heat,

$$P = P_1 - P_2$$

$$\frac{dQ}{dt} = e\sigma A\left(T^4 - T_0^4\right)$$

vi) $\frac{-ms\,dT}{dt} = e\sigma A(T^4 - T_0^4)$

∴ Rate of cooling:

$$-\frac{dT}{dt} = \left(\frac{e\sigma A}{ms}\right)(T^4 - T_0^4)$$

vii) Newton's law of cooling:

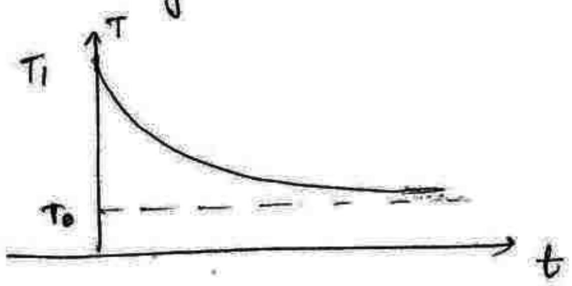
If temp. difference b/w body and surrounding is very small then rate of cooling is directly proportional to the temp difference.

If $\Delta T << T_0$

$$-\frac{dT}{dt} = k\Delta T \quad \text{or} \quad \frac{-dT}{dt} = K(T - T_0)$$

$$\boxed{\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_0\right)}$$

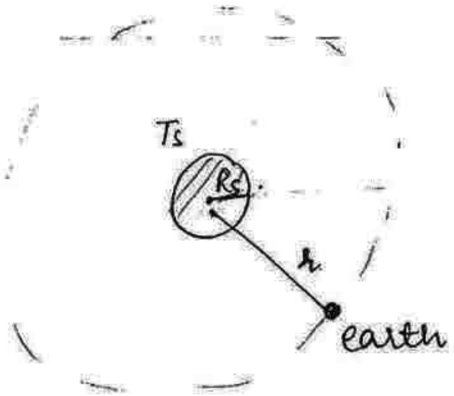
viii) cooling curve:



$$T = T_0 + (T_1 - T_0)e^{-kt}$$

Q) Find solar constant on earth surface
(assume that their is no reflection from the atm.

Energy received from sun per unit time per unit surface area



$$S = \frac{P_s}{4\pi r^2} = \frac{\sigma A_s T_s^4}{4\pi r^2}$$

$$= \frac{\sigma (\cancel{4\pi} R_s^2) T_s^4}{\cancel{4\pi} r^2}$$

$$\therefore \boxed{S = \frac{\sigma R_s^2 T_s^2}{r^2}}$$

$\sigma = 5.67 \times 10^{-8}\ w/m^2 k^4$

$R_s = 7 \times 10^8\ m$

$T_s = 5500\ k$

$r = 1.5 \times 10^{11}\ m.$

$\therefore \boxed{S \simeq 1400\ w/m^2}$

Q) Find Total energy received on earth per second from sun

$P_E = S \times \text{Area}$

$$\boxed{P_E = \left(\frac{\sigma R_s^2 T_s^4}{r^2}\right)(\pi R_E^2)}$$ → area of disc

Q) Taking the temp of the sun to be 6000 k find its peak wavelength

$\lambda_m T =$

Q) Temp of surrounding is 40°C. In 10 min the temp of a body ↓es from 52°C to 50°C find the time to ↓ the temp from 50°C to 48°C

$$\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_0\right)$$

$$\left(\frac{52-50}{10}\right) = K\left(\frac{52+50}{2} - 40\right)$$

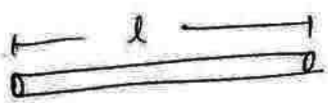
$$\therefore \quad K = \frac{1}{5 \times 11}$$

$$\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_0\right)$$

$$\frac{50-48}{t} = \frac{1}{5 \times 11}\left(\frac{50+48}{2} - 40\right)$$

$$\therefore \quad t = 12 \text{ min}$$

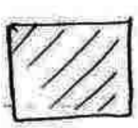
Thermal expansion :

i) 

$$\boxed{\Delta l = \alpha l \Delta t}$$

$$l_2 - l_1 = \alpha l_1 \Delta t$$

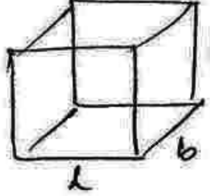
$$\boxed{l_2 = l_1(1 + \alpha \Delta t)}$$

ii) 

$$\boxed{\Delta A = \beta A \Delta t}$$

$$\boxed{A_2 = A_1(1 + \beta \Delta t)}$$

iii)



$$\boxed{\Delta V = \gamma V \Delta t}$$

$$\boxed{V_2 = V_1 (1 + \gamma \Delta t)}$$

$$V = lbh$$

$$\frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}$$

$$\gamma \Delta t = \alpha \Delta t + \alpha \Delta t + \alpha \Delta t$$

$$\boxed{\gamma = 3\alpha}$$

$A = l \times b$.

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

$$\therefore \beta \Delta t = \alpha \Delta t + \alpha \Delta t$$

$$\beta = \alpha + \alpha$$

$$\boxed{\beta = 2\alpha}$$

Application :

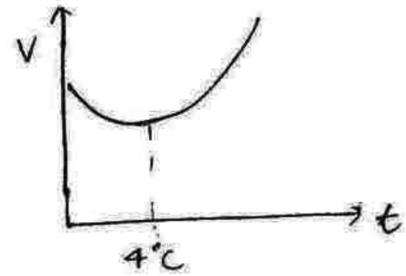
① Density

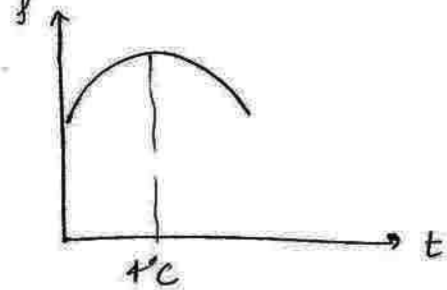
$$\rho = \frac{m}{V} \qquad \therefore \frac{\rho_1}{\rho_2} = \frac{V_1}{V_2}$$

$$\frac{\rho_1}{\rho_2} = \frac{V_1}{V_1 (1 + \gamma \Delta t)}$$

$$\boxed{\rho_2 = \left(\frac{\rho_1}{1 + \gamma \Delta t}\right)} \qquad \boxed{\rho_2 \simeq \rho_1 (1 - \gamma \Delta t)}$$

ii)





$$\boxed{\rho = 1000\ kg/m^3 \text{ at } 4^\circ c}$$

iii) $T = 2\pi\sqrt{\frac{l}{g}}$

$$\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta l}{l}\right)$$

$$\frac{\Delta T}{T} = \frac{1}{2}(\alpha \Delta t) \qquad \therefore \boxed{\Delta T = \frac{1}{2}(\alpha \Delta t)\, T}$$

iv)

apparant expansion of liquid :

$$\Delta V_{contain} = \gamma_c V \Delta t$$

$$\Delta V_L = \gamma_l V \Delta t$$

$\therefore$ Overflow Vol^m

$$\Delta V = \Delta V_l - \Delta V_c$$

$$\Delta V = (\gamma_l - \gamma_c) V \Delta t$$

$$\Delta V = \gamma_A V \Delta t$$

$$\boxed{\gamma_A = \gamma_L - \gamma_C} \quad \therefore$$

real apparant