Chapter 6

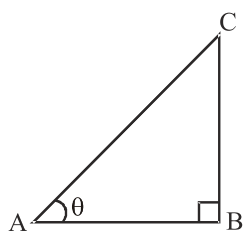
Trigonometry

Introduction to Trigonometry

- **Trigonometrical Ratios : (T - Ratios)**

For rt Δ ABC, $B = 90°$, $A = \theta$

(i) $\frac{BC}{AC} = \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$



(ii) $\frac{AB}{AC} = \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

(iii) $\frac{BC}{AB} = \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

(iv) $\frac{AC}{BC} = \text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

(v) $\frac{AC}{AB} = \sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Base}}$

(vi) $\frac{AB}{BC} = \cot\theta = \frac{1}{\tan\theta} = \frac{\text{Base}}{\text{Perpendicular}}$

These six ratios are called trigonometric-ratios for the angle θ. We can write

$$\tan\theta = \frac{BC}{AB} = \frac{BC/AC}{AB/AC} = \frac{\sin\theta}{\cos\theta} \quad \text{and} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

- **Reciprocal Relations :**

(i) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

(ii) $\sec\theta = \frac{1}{\cos\theta}$

(iii) $\cot\theta = \frac{1}{\tan\theta}$

Note : The side, opposite to concerned angle θ is always taken as perpendicular and adjacent side to the angle θ is base. Therefore, here AB and BC of ΔABC will be treated as base and perpendicular respectively for the angle θ.

Remarks :

(i) T-Ratios defined above are true for all values of angle A.

(ii) The symbol sin A is used as an abbreviation for 'the sine of the angle A'.

(iii) sin A is not the product of 'sin' and A.

(iv) Only 'sin' has no meaning.

(v) $(\sin A)^2 = \sin^2 A$ (Read as sin square A)

(vi) $(\sin A)^{-1} \neq \sin^{-1} A \neq \frac{1}{\sin A}$

(vii) $(\sin A)^{-1} = \frac{1}{\sin A} = \text{cosec}\,A$

(viii) The values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

- **Trigonometric Ratios of some specific angles :**

The table for all T-ratios of some angles is given below :

θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

- **Trigonometric Ratios of Complementary Angles :**

  We have following relationship for complementary angles :

  $\sin(90° - \theta) = \cos\theta$, $\cos(90° - \theta) = \sin\theta$

  $\tan(90° - \theta) = \cot\theta$, $\cot(90° - \theta) = \tan\theta$

  $\sec(90° - \theta) = \mathrm{cosec}\,\theta$, $\mathrm{cosec}(90° - \theta) = \sec\theta$

  for all values of θ lying between 0° and 90°.

  Note : $\tan 0° = 0 = \cot 90°$

  $\sec 0° = 1 = \mathrm{cosec}\,90°$

  and $\sec 90°$, $\mathrm{cosec}\,0°$, $\tan 90°$ $\cot 0°$ are not defined.

  So, $\sin 15° = \sin(90° - 75°) = \cos 75°$

  Similarly, $\tan 42° = \cot 48°$ etc.

- **Trigonometric Identities**

Identity I : $\sin^2\theta + \cos^2\theta = 1$
Identity II : $\sec^2\theta = 1 + \tan^2\theta$
Identity III : $\mathrm{cosec}^2\theta = 1 + \cot^2\theta$

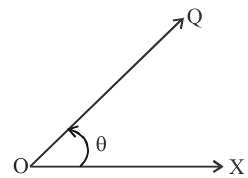
} These are concerned with any right angled triangle with any acute angle θ.

Some Application of Trigonometry

- **Line of sight :**

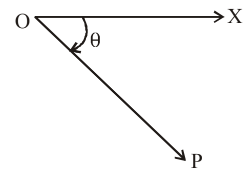
  Suppose we are viewing an object standing on the ground. Clearly, the line of sight (or line of vision) to the object is the line from our eyes to the object, we are viewing.

- **Angle of elevation :**



Let 'O' be the eye of observer and Q be an object and OX is a horizontal line then ∠XOQ is called the angle of elevation and generally it is denoted by Theeta or Phie i.e 'θ' or ϕ.

- **Angle of Depression:** 'P' be the object lies below the horizontal line OX, then ∠XOP is called the angle of depression and is denoted by 'θ' or 'ϕ'



Exercise 1

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) out of which only one is correct.*

1. The value of

$$\frac{\cos(90°-\theta)\sec(90°-\theta)\tan\theta}{\text{cosec}(90°-\theta)\sin(90°-\theta)\cot(90°-\theta)}+\frac{\tan(90°-\theta)}{\cot\theta} \text{ is}$$

(1) 1 (2) – 1
(3) 2 (4) – 2

2. If θ and 2θ – 45° are acute angles such that $\sin\theta = \cos(2\theta - 45°)$, then tan θ is equal to

(1) 1 (2) –1
(3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

3. If $\frac{x\,\text{cosec}^2 30°\ \sec^2 45°}{8\cos^2 45°\sin^2 60°} = \tan^2 60° - \tan^2 30°$, then x =

(1) 1 (2) – 1
(3) 2 (4) 0

4. If $x \sin(90° - \theta)\cot(90° - \theta)$, then x =

(1) 0 (2) 1
(3) – 1 (4) 2

5. sin 2A = 2 sin A is true when A =

(1) 0° (2) 30°
(3) 45° (4) 60°

6. If A, B and C are interior angles of a triangle ABC, then $\sin\left(\frac{B+C}{2}\right) =$

(1) $\sin\frac{A}{2}$ (2) $\cos\frac{A}{2}$
(3) $-\sin\frac{A}{2}$ (4) $-\cos\frac{A}{2}$

7. $\tan^2\theta.\cos^2\theta$ is equal to :

(1) $1+\sin^2\theta$ (2) $1+\cos^2\theta$
(3) $1-\sin^2\theta$ (4) $1-\cos^2\theta$

8. $\sin 81° + \tan 81°$ is equal to

(1) $\text{cosec}\,81° + \cot 81°$ (2) $\cos 81° + \cot 81°$
(3) $\cos 9° + \cot 9°$ (4) $\cos 81° + \sec 81°$

9. If sinθ = cos(θ – 6°) where (3θ) and (θ – 6°) are both acute angles, then the value of θ is

(1) 18° (2) 24°
(3) 36° (4) 30°

10. A tree 6 m tall casts a 4 m long shadow. At the same time, a flag pole casts a shadow 50 m long. How long is the flag pole?

(1) 75 m (2) 100 m
(3) 150 m (4) 50 m

11. A 25 m ladder is placed against a vertical wall of a building. The foot of the ladder is 7 m from the base of the building. If the top of the ladder slips 4 m, then the foot of the ladder will slide

(1) 5 m (2) 8 m
(3) 9 m (4) 15 m

12. In ΔABC, ∠A = 30° and ∠B = 90°. If AC = 6 cm, then its area is

(1) $16\sqrt{3}$ cm^2 (2) 16 cm^2
(3) $8\sqrt{3}$ cm^2 (4) $6\sqrt{3}$ cm^2

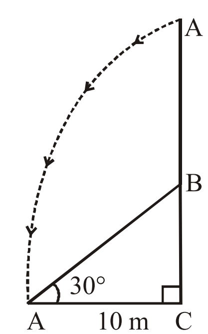
13. If $\sin\theta = \frac{24}{25}$ and θ lies in the second quadrant, then $\sec\theta + \tan\theta =$

(1) –7 (2) 6
(3) 4 (4) –5

14. If the length of the shadow of a tower is $\sqrt{3}$ times that of its height, then the angle of elevation of the sun is

(1) 15° (2) 30°
(3) 45° (4) 60°

15. The top of a broken tree has its top touching the ground (shown in the figure) at a distance of 10 *m* from the bottom. If the angle made by the broken part with ground is 30°, then the length of the broken part is



(1) $10\sqrt{3}$ m (2) $\frac{20}{\sqrt{3}}$ m
(3) 20 m (4) $20\sqrt{3}$ m

16. If $x = a\cos^2\theta + b\sin^2\theta$ then $(x - a)(b - x)$ is equal to

(1) $(a - b)\sin\theta\cos\theta$
(2) $(a - b)^2\sin^2\theta\cos^2\theta$
(3) $(a - b)^2\sin\theta\cos\theta$
(4) $(a - b)\sin^2\theta\cos^2\theta$

17. If $a\cos^3\theta + 3a\cos\theta\sin^2\theta = m$ and $a\sin^3\theta + 3a\cos^2\theta\sin^2\theta = n$ then $(m+n)^{2/3} + (m-n)^{2/3}$ is equal to

(1) $a^{2/3}$ (2) $2a^{1/3}$

(3) $2a^{2/3}$ (4) $a^{1/3}$

18. If $\tan^2\theta = 1 - a^2$ then the value of $\sec\theta + \tan^3\theta \operatorname{cosec}\theta$ is

(1) $(2-a^2)$ (2) $(2-a^2)^{1/2}$

(3) $(2-a^2)^{2/3}$ (4) $(2-a^2)^{3/2}$

19. Which of the following relationship is true ?

(1) $\cos A \sec A = 1$

(2) $\sin A \cot A = 1$

(3) $\sin A + \operatorname{cosec} A = 1$

(4) $\sec A \cot A = 1$

20. If $\frac{x}{a\cos\theta} = \frac{y}{b\sin\theta}$ and $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$ then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ is

(1) 1 (2) – 1

(3) $\sin^2\theta$ (4) $\cos^2\theta$

21. If $\tan\theta = \frac{x\sin\phi}{1 - x\cos\phi}$ and $\tan\phi = \frac{y\sin\theta}{1 - y\cos\theta}$ then the value of $\frac{x}{y}$ is

(1) $\frac{\sin\phi}{\sin\theta}$ (2) $\frac{\sin\theta}{\sin\phi}$

(3) $\frac{\sin\theta}{1-\cos\theta}$ (4) $\frac{\sin\theta}{1-\cos\phi}$

22. If angle θ be divided into two parts such that the tangent of one part is k times the tangent of the other and ϕ is their difference, then sin θ =

(1) $\frac{k+1}{k-1}\sin\phi$ (2) $\frac{k-1}{k+1}\sin\phi$

(3) $\frac{2k-1}{2k+1}\sin\phi$ (4) None of these

23. If θ is an acute angle of a right triangle, then the value of $\sin\theta\cos(90° - \theta) + \cos\theta\sin(90° - \theta)$ is

(1) 0 (2) $2\sin\theta\cos\theta$

(3) 1 (4) $2\sin^2\theta$

24. If $\cos(40° + x) = \sin 30°$ and $40° + x$ is an acute angle, then the value of x is

(1) 20° (2) 30°

(3) 40° (4) 45°

25. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, $0° < \theta_1, \theta_2, \theta_3 \le 90°$, then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 =$

(1) 3 (2) 1

(3) 0 (4) 2

26. If $\sin x + \operatorname{cosec} x = 2$, then $\sin^{19}x + \operatorname{cosec}^{20}x$ is equal to

(1) 2^{19} (2) 2^{20}

(3) 2 (4) 2^{39}

27. A ladder is included to a wall making an angle of 30° with it. A man is ascending the ladder at the rate of 2 metres/ second. How fast is he approaching the wall?

(1) 2 m/s (2) 1.5 m/s

(3) 1 m/s (4) 2.5 m/s

28. A professor standing on one end of a football field observes the elevation of the top of a flood light tower at an angle of α. He then walks a distance equal to twice the height of the tower and finds that the elevation of the top of the top is now at an angle of 2α. What is the value of α?

(1) 30º (2) 60º

(3) 15º (4) 45º

29. AB is a vertical pole. The end A is on the level ground, C is the middle point of AB and P is a point on the level ground. The portion CB subtends an angle β at P. If AP = nAB, then tan β equal

(1) $\frac{n}{2n^2+1}$ (2) $\frac{n}{n^2-1}$

(3) $\frac{n}{n^2+1}$ (4) None of these

30. The angle of elevation of the top of a tower, as seen from two points A and B situated in the same line and at distance p and q respectively from the foot of the tower, are complementary, then the height of the tower is.

(1) pq (2) $\frac{p}{q}$

(3) $\sqrt{pq}$ (4) none of these

Exercise 2

Matching Based Questions

DIRECTIONS (Qs. 1 to 2) : *Match the Column-I with Column-II and select the correct answer given below the columns.*

1.

	Column-I		Column-II
(A)	If $2\sin 2\theta = \sqrt{3}$ then the value of θ is	(p)	20°
(B)	If $2\cos 3\theta = 1$ then the value of θ is	(q)	$\frac{7}{25}$
(C)	If $\tan 3x = \sin 45° \cos 45° + \sin 30°$ then the value of x is	(r)	0
(D)	If $\tan\theta = \frac{3}{4}$ then $\cos^2\theta - \sin^2\theta$ is	(s)	15°
(E)	The value of $\cos^2 17° - \sin^2 73°$ is	(t)	30°

(1) A → t; B → p; C → s; D → q; E → r
(2) A → t; B → p; C → q; D → s; E → r
(3) A → p; B → t; C → s; D → q; E → r
(4) A → r; B → q; C → t; D → s; E → p

2.

	Column-I		Column-II
(A)	The angle of elevation of the sun if the length of the shadow of a tower is $\sqrt{3}$ times the height of the tower, is	(p)	45°
(B)	The angle of elevation of the top of a tower from a point 20 metres away from its base is 45°. The height of the tower is	(q)	70 m
(C)	At a point $15\sqrt{3}$ metres away from the base of a $15\sqrt{3}$ metres high house. The angle of elevation of the top is	(r)	20 m
(D)	The angle of depression of the top of a tower at a point 70 m from the house is 45°. Then the height of the tower is	(s)	30°

(1) A → p; B → r; C → s; D → q
(2) A → s; B → r; C → p; D → q
(3) A → p; B → r; C → q; D → s
(4) A → r; B → s; C → q; D → p

Statement Based Questions

3. Consider the following statements :

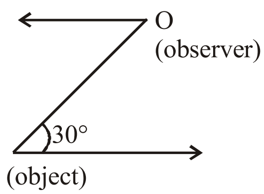
I. The value of sinA or cos A never exceeds 1.
II. The value of secA or cosecA is always equal to 1.
III. The value of $\sin\theta + \cos\theta$ for $\theta = 0°$ is 1.
IV. The value of $\sin\theta$ is $\left(a+\frac{1}{a}\right)$, where 'a' is a positive number.

Which of the statements given above is/are correct?

(1) 'I' and 'III' (2) 'II' and 'IV'
(3) Neither 'I' nor 'III' (4) Neither 'II' nor 'IV'

4. Consider the following statements :

I. The angle of elevation or depression is always measured from horizontal line through the point of observation.
II. If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45°.
III. In the adjoining figure, the positions of observer and object are marked. The angle of depression is 20°.



IV. The height of an object or the distance between distant objects can be determined with the help of trigonometric ratios.

Which of the statements given above are correct?

(1) I, II and III (2) I, II and IV
(3) I, III and IV (4) II, III and IV

5. Consider the following statements:

I. Sin A is not the product of 'sin' and 'A'.
II. Only 'sin' has no meaning.

Which of the statement (s) given above is/are correct?

(1) only I (2) only II
(3) Both I and II (4) Neither I nor II

6. Consider the following identities:

I. $\sin^2\theta + \cos^2\theta = 1$
II. $\sec^2\theta = 1 + \tan^2\theta$
III. $\text{cosec}^2\theta = 1 + \cot^2\theta$

Which of the identities given above is/are correct?

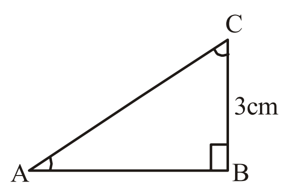
(1) Both I and II (2) Both I and III
(3) Neither I, II nor III (4) All the above

Passage Based Questions

DIRECTIONS (Qs. 7 to 12) : *Read the passage(s) given below and answer the questions that follow.*

PASSAGE - 1

In ΔABC, right angled at B



$AB + AC = 9$ cm and $BC = 3$cm.

7. The value of cot C is

(1) $\frac{3}{4}$ (2) $\frac{1}{4}$

(3) $\frac{5}{4}$ (4) none

8. The value of sec C is

(1) $\frac{4}{3}$ (2) $\frac{5}{3}$

(3) $\frac{1}{3}$ (4) none

9. $\sin^2 C + \cos^2 C =$

(1) 0 (2) 1

(3) −1 (4) none

PASSAGE - 2

From the top of a tower, the angles of depression of two objects on the same side of the tower are found to be α and β where $\alpha > \beta$.

10. If the distance between the objects is 'p' metres, then the height 'h' of the tower is

(1) $\frac{p\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$ (2) $\frac{\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$

(3) $\frac{p(\tan\alpha-\tan\beta)}{\tan\alpha\tan\beta}$ (4) none of these

11. The height of the tower if $p = 50$m, $\alpha = 60°$ and $\beta = 30°$, is

(1) 120 m (2) 130 m

(3) 140 m (4) none of these

12. The distance of the extreme object from the top of the tower is

(1) 65 m (2) 130 m

(3) 260 m (4) none of these

Assertion Reason Based Questions

DIRECTIONS (Qs. 13 to 17) : *Following questions consist of two statements, one labelled as the **'Assertion' (A)** and the other as **'Reason' (R)**. You are to examine these two statements carefully and select the answer to these items using the code given below.*

Code :

(1) Both A and R are individually true and R is the correct explanation of A:

(2) Both A and R are individually true but R is not the correct explanation of A.

(3) A is true but R is false

(4) A is false but R is true.

13. Assertion : In a right angled triangle, if $\tan\theta = \frac{3}{4}$, the greatest side of the triangle is 5 units.

Reason : $(\text{greatest side})^2 = (\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$.

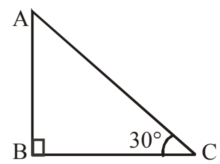
14. Assertion : In a right angled triangle, if $\cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$, then $\tan\theta = \sqrt{3}$

Reason : $\tan\theta = \frac{\sin\theta}{\cos\theta}$

15. Assertion : In a right angled triangle, sin 47° = cos 43°.

Reason : $\sin\theta = \cos(90+\theta)$, where θ is an angle in the right angled triangle.

16. Assertion : If the above figure, if BC = 20 m, then height AB is 11.56 m.



Reason : $\tan\theta = \frac{AB}{BC} = \frac{\text{perpendicular}}{\text{base}}$ where θ is the angle $\angle ACB$.

17. Assertion : If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45°.

Reason : According to pythagoras theorem, $h^2 = l^2 + b^2$, where h = hypotenuse, l = length and b = base

Correct Definition Based Questions

18. Which of the following is correct definition of sin θ ?

(1) $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$ (2) $\sin\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

(3) $\sin\theta = \frac{\text{Perpendicular}}{\text{Base}}$ (4) $\sin\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

19. Which of the following is correct definition of cot θ ?

(1) $\cot\theta = \frac{\text{Perpendicular}}{\text{Base}}$ (2) $\cot\theta = \frac{\text{Base}}{\text{Perpendicular}}$

(3) $\cot\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$ (4) $\cot\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

Hints SOLUTIONS

Exercise 1

1. (3) **2.** (1) **3.** (1)
4. (2) **5.** (1) **6.** (2)

7. (4) We have,

$$\tan^2\theta\cos^2\theta = \frac{\sin^2\theta}{\cos^2\theta}.\cos^2\theta$$

$$= \sin^2\theta = 1 - \cos^2\theta$$

8. (3) We have,

$\sin 81° + \tan 81°$

$= \sin(90° - 9°) + \tan(90° - 9°)$

$= \cos 9° + \cot 9°$

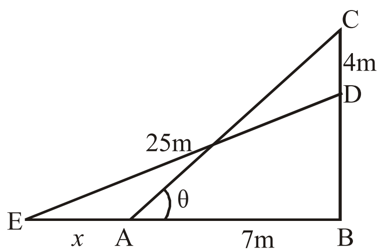
9. (2) $\sin 3\theta = \cos[90 - (96 - \theta)]$
$= \sin(96 - \theta)$
$\Rightarrow 3\theta = 96 - \theta$
$\Rightarrow \theta = 24$

10. (1) Let h be the length of the pole.

By the given condition $\frac{6}{4} = \frac{h}{50}$

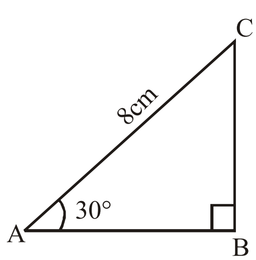
$\Rightarrow h = \frac{6 \times 50}{4} = 75m$

11. (2)



DE = 25 = AC = length of ladder.
Let BD = y m
$\therefore (y + 4)^2 + 7^2 = 25^2$
$\Rightarrow (y + 4)^2 = 625 - 49 = 576$
$\Rightarrow y + 4 = \pm 24 \Rightarrow y = 20$ $[\because y > 0]$
From ΔDBE, $(20)^2 + (x + 7)^2 = (25)^2$
$\Rightarrow (x + 7)^2 = (25)^2 - (20)^2 = (15)^2$
$\Rightarrow x + 7 = 15 \Rightarrow x = 8$
∴ Foot of the ladder slides 8 m.

12. (3) [Hint. $\cos 30° = \frac{AB}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AB}{8cm}$]



$\Rightarrow AB = 4\sqrt{3}$ cm,

$\sin 30° = \frac{BC}{AC} \Rightarrow \frac{1}{2} = \frac{BC}{8cm} \Rightarrow BC = 4cm$

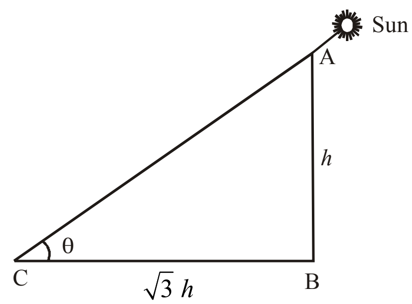
$\therefore$ Area of $\Delta ABC = \frac{1}{2} AB \times BC$

$= \frac{1}{2} \times 4\sqrt{3} \times 4cm^2$

$= 8\sqrt{3}$ cm^2

13. (1) $\sec\theta + \tan\theta = \frac{-25}{7} + \frac{-24}{7} = -7$

14. (2) **Hint :** Let height of tower (AB) be h metres, then length of its shadow (BC) = $\sqrt{3}\,h$ metres.



Let angle of elevation be θ,

then $\tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

15. (2) **Hint :** $\cos 30° = \frac{AC}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{10\text{ m}}{AB} \Rightarrow AB = \frac{20}{\sqrt{3}}$ m.

16. (2) $x - a = b\sin^2\theta - a\sin^2\theta = (b - a)\sin^2\theta$
$b - x = b\cos^2\theta - a\cos^2\theta = (b - a)\cos^2\theta$
$\therefore (x - a)(b - x) = (b - a)^2\sin^2\theta\cos^2\theta$
$= (a - b)^2\sin^2\theta\cos^2\theta$

17. (3) $m + n = a(\cos\theta + \sin\theta)^3$
$(m + n)^{2/3} = a^{2/3}(\cos\theta + \sin\theta)^2$
$(m - n)^{2/3} = a^{2/3}(\cos\theta - \sin\theta)^2$
$\therefore (m + n)^{2/3} + (m - n)^{2/3} = 2a^{2/3}$

18. (4) $\sec\theta + \tan^3\theta\,\text{cosec}\,\theta$

$= \sec\theta + \frac{\sin\theta}{\cos\theta}\tan^2\theta\,\text{cosec}\,\theta$

$= \sec\theta(1 + \tan^2\theta)$
$= (1 + \tan^2\theta)^{3/2} = [1 + (1 - a^2)]^{3/2}$

19. (1) By defination. $\sec A = \frac{1}{\cos A}$

$\therefore \cos A \sec A = 1$

20. (1) On solving both the equations, we get
$x = a\cos\theta, y = b\sin\theta$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2\theta + \sin^2\theta = 1$

21. (2) $x = \frac{\tan\theta}{\sin\phi + \tan\theta\cos\phi}$

$y = \frac{\tan\phi}{\sin\theta + \tan\phi\cos\theta}$

$\therefore \frac{x}{y} = \frac{\tan\theta}{\tan\phi}\left[\frac{\sin\phi\cos\theta + \sin\theta\cos\phi}{\sin\phi\cos\theta + \sin\theta\cos\phi}\right] \times \frac{\cos\theta}{\cos\phi}$

$= \frac{\sin\theta}{\cos\theta} \times \frac{\cos\phi}{\sin\phi} \times \frac{\cos\theta}{\cos\phi} = \frac{\sin\theta}{\sin\phi}$

22. (1) Let $A + B = \theta$ and $A - B = \phi$
Then $\tan A = k\tan B$

or $\frac{k}{1} = \frac{\tan A}{\tan B} = \frac{\sin A\cos B}{\cos A\sin B}$
Applying componendo and divdendo

$\Rightarrow \frac{k+1}{k-1} = \frac{\sin A\cos B + \cos A\sin B}{\sin A\cos B - \cos A\sin B}$

$= \frac{\sin(A+B)}{\sin(A-B)} = \frac{\sin\theta}{\sin\phi}$

$\Rightarrow \sin\theta = \frac{k+1}{k-1}\sin\phi$

23. (3)

24. (1) $\cos(40° + x) = \sin 30° = \sin(90° - 60°) = \cos 60°$
$\Rightarrow 40° + x = 60° \Rightarrow x = 20°$

25. (3) **26.** (3)

27. (3) Let AB be the ladder inclined at an angle of 30° with the wall BC.
After one second, the man will be at D such that AD = 2 m

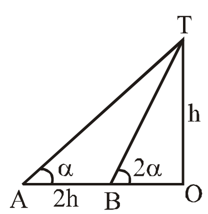
$\therefore AE = AD\cos 60° = 2 \times \frac{1}{2} = 1\,m$

Similarly after 2 seconds, the man will be at D such that AD = 2 m

$\therefore AE = AD\cos 60° = 2 \times \frac{1}{2} = 1\,m$

Similarly after 2 seconds, the man will be at F such that AF = 4 m
$\therefore$ AG = 2 m.
Thus after every second the man is approaching the wall at the rate of 1 m/sec.

28. (3) In the following figure,



A is the point where the professor was standing initially.
OT = h = highest of the tower.
B is the point where the professor is standing after walking the distance 2h.
$h\cot\alpha = OA$, $h\cot 2\alpha = OB$
Therefore, $AB = OA - OB = 2h$
$= h\cot\alpha - h\cot 2\alpha$.

$\Rightarrow 2 = \cot\alpha - \cot 2\alpha$

$\Rightarrow 2 = \frac{\cos\alpha}{\sin\alpha} = \frac{\cos 2\alpha}{\sin 2\alpha}$

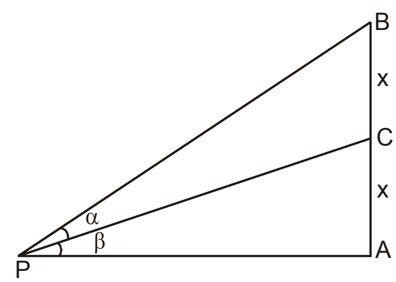
$\Rightarrow \cos\alpha \times \sin 2\alpha - \sin\alpha \times \cos 2\alpha$
$= 2\sin\alpha \times \sin 2\alpha$

$\Rightarrow \sin(2\alpha - \alpha) = 2\sin\alpha \times \sin 2\alpha$

$\Rightarrow 1 = 2\sin 2\alpha \Rightarrow \sin 2\alpha = \frac{1}{2}$

$\Rightarrow 2\alpha = 30°$ or $\alpha = 15°$

29. (1) Let $\angle APC = \alpha$, AC = CB = x.



It is given that AP = nAB or AP = 2nx

$\Rightarrow$ In Δ APC, we have $\tan\alpha = \frac{AC}{AP}$

$\Rightarrow \tan\alpha = \frac{x}{2nx} = \frac{1}{2n}$ ...(1)

From Δ APB, we have

$\tan(\alpha + \beta) = \frac{AB}{AP} \Rightarrow \tan(\alpha + \beta) = \frac{2x}{2nx}$

$\Rightarrow \tan(\alpha + \beta) = \frac{1}{n}$ ...(2)

Now $\tan\beta = \tan[(\alpha + \beta) - \alpha]$

$= \frac{\tan(\alpha+\beta) - \tan\alpha}{1 + \tan(\alpha+\beta)\tan\alpha}$

$\Rightarrow \tan\beta = \frac{1/n - 1/2n}{1 + 1/n.(1/2n)} = \frac{n}{2n^2 + 1}$

[By (1) and (2)]

30. (3)

Exercise 2

1. (1) (A) $\to$t; (B) $\to$ p; (C) $\to$s; (D) $\to$ q; E $\to$ r

2. (2) (A) $\to$ s, (B) $\to$ r, (C) $\to$ p, (D) $\to$ q

3. (1)

4. (2) Statement 'I', 'II' and 'IV' are correct.

5. (3)

6. (4)

7. (1) In ΔABC,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2 \Rightarrow AB = 4$ cm.

$AC = 5$ cm.

$\cot C = \frac{BC}{AB} = \frac{3}{4}$

8. (2) $\sec C = \frac{AC}{BC} = \frac{5}{3}$

9. (2) $\sin C = \frac{4}{5}$

$\cos C = \frac{3}{5}$

L.H.S $= \sin^2 C + \cos^2 C = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{16+9}{25} = 1$

= R.H.S

10. (1) Height of the tower (AB) = h m

distance (CD) = p m

Let distance (BC) = x m

$\angle ACB = \alpha$ and $\angle ADB = \beta$

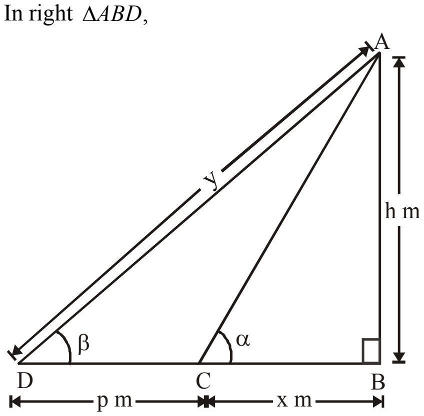
In right ΔABD,

$\frac{AB}{BC} = \tan\alpha$

$\Rightarrow \frac{h}{x} = \tan\alpha$

$\Rightarrow h = x\tan\alpha$ ..... (i)

In right ΔABD,



$\frac{AB}{BD} = \tan\beta$

$\Rightarrow \frac{h}{BC+CD} = \tan\beta$

$\Rightarrow h = (x + p)\tan\beta$ ..... (ii)

From (i), we get

$x = \frac{h}{\tan\alpha}$

Hence, $h = \frac{p \cdot \tan\alpha \cdot \tan\beta}{\tan\alpha - \tan\beta}$.

11. (2) Putting $p = 150$ m, $\alpha = 60°$ and $\beta = 30°$, we get

$$h = \frac{150 \times \tan 30° \times \tan 60°}{\tan 60° - \tan 30°} \text{ m} = 129.9 \text{ m}$$

Hence, the height of the tower = 129.9 m $\simeq 130\, m$.

12. (3) $\sin\beta = \frac{h}{y}, \Rightarrow y = \frac{h}{\sin\beta} = \frac{130}{\sin 30^o} = 260m.$

13. (1) Both Assertion and Reason are correct and Reason is the correct explanation of the assertion.

greatest side $= \sqrt{(3)^2 + (4)^2} = 5$ units.

14. (1) Both assertion and reason are correct and reason is the correct explanation of the assertion.

$\tan\theta = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$.

15. (3) Assertion is true, but reason is not correct.

$\sin\theta = \cos(90 - \theta)$

$\sin 47° = \cos(90 - 47) = \cos 43°$

16. (1) Both the assertion and reason are correct, reason is the correct explanation of the assertion.

$\tan 30° = \frac{AB}{BC} = \frac{AB}{20}$

$AB = \frac{1}{\sqrt{3}} \times 20 = \frac{20}{1.73} = 11.56$ m.

17. (2) Both assertion and reason are correct, but reason is not the correct explanation of the assertion.

18. (1) $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

19. (2)