UNITS AND MEASUREMENT

Physical quantities :
MODULE 1 (Pg-25)

The quantities which can be measured and are used to describe the laws of physics

A physical quantities can be specified, if its an :

i) Magnitude : Pure ratio.
eg: Strain $= \frac{\Delta l}{l} = \frac{\text{Change in length}}{\text{Initial length}}$

Coefficient of friction $(\mu) = \frac{\text{Friction Force}}{\text{Normal Force}} = \frac{f_s}{N}$

ii Magnitude and units : Scales
eg: Speed, distance and time

iii Magnitude, units and direction : Vector
eg: Displacement = 10 m due east $= 10 m \hat{i}$
Velocity = 100 m/s due north $= 100\ m/s\ \hat{j}$

Classification of physical quantities on the basis of dependency :
MODULE 1 (Pg-25)

i Fundamental or base quantities: Those physical quantities which does not depend on other physical quantities
eg: Mass, Length and Time

ii Derived quantities: Those physical quantities which can be derived from fundamental quantities
eg: Area, volume, speed, Force, momentum etc.

Units of Physical Quantities (classification)
MODULE 1 (Pg-26)

i Fundamental units: The units of fundamental quantities are known as fundamental units.
eg: Mass → Kilogram (Kg)
Length → Metre (m)
Time → Second (s)

ii Derived units: The units of derived quantities are known as derived units.
eg: Area : m^2
Volume : m^3
Speed : m/s or ms^{-1}
Acceleration : m/s^2 or ms^{-2}
Momentum : Kgm/s or $kgms^{-1}$

* Force : N (newton).

$F = ma$

$= kgm/s^2$

$\therefore 1N = 1 kgms^{-2}$

* Frequency $(n, v, f) = \frac{1}{\text{Time period}}$

$$= \frac{1}{T}$$

$$= \frac{1}{S} = S^{-1} = Hz.$$

iii Supplementary units:

- Radian : 1 radian is the angle subtented at the centre of the circle by an arc equal in length to the radius of the circle.

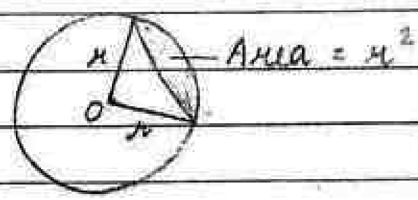
- Steradian : 1 steradian is the solid angle subtended at the centre of the sphere by the surface of the sphere whose area is equal to the square of the radius (i.e r^2).

Steradian (Sr) : Solid angle

Solid angle $(\Omega) = \frac{\text{Area}}{r^2}$

Its area $= r^2$

$\therefore \Omega = \frac{r^2}{r^2} = 1$ Steradian $= 1$ Sr.



Q-1 Determine the angle subtended by a solid sphere at the centre of the circle.

Angle at its centre $= \frac{\text{surface area}}{r^2}$

$w/\Omega = \frac{4\pi r^2}{r^2} = 4\pi$ steradian (Sr)

Q-2 Find the angle subtended by one-forth of a solid sphere at its centre.

Area at its centre = $\frac{\text{surface area}}{r^2}$

$$\omega / \Omega = \frac{4\pi r^2}{r^2} \times \frac{1}{4} = \pi \text{ steradian (sr)}$$

iv) Practical Units :

• ~~Astronomical units~~

a) Distance / length :

• Astronomical units (AU) : The average distance from the centre of the sun to the centre of the earth

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$
$$\simeq 1.5 \times 10^{11} \text{ m}$$

• Light Year (ly) : The distance travelled by the light in vaccum in one year.

$$1 \text{ ly} = \text{speed (m/s)} \times \text{time (s)}$$
$$= (3 \times 10^8) \times (365 \times 24 \times 60 \times 60)$$
$$= 9.46 \times 10^{15} \text{ m}$$

• Parsec (Pc) : It is the largest unit of distance

$$1 \text{ Pc} = 3.1 \times 10^{16} \text{ m}$$
$$\simeq 3.26 \text{ ly (light year)}$$

$\because 1^\circ = 60'$

$1' = 60''$

$\therefore 1^\circ = 3600''$

Smalles units of distance:

- $1\ \mu m = 10^{-6}\ m$ (micron)
- $1\ nm = 10^{-9}\ m$
- $1\ \mathring{A} = 10^{-10}\ m$
- $1\ pm = 1\ \mu\mu m = 10^{-12}\ m$
- 1 fermi $= 1$ femto $= 10^{-15}\ m$

b) Mass:

- Largest unit of mass is 'Chandra Shekhar Limit' (CSL)

$$1\ CSL = 1.4 \times \text{Mass of the sun.}$$
$$= 1.4\ M_{sun}$$

Mass of the sun $= 10^{30}$ kg.

$$\therefore 1\ CSL = 1.4 \times 10^{30}\ kg$$

- Smallest unit of mass is 'atomic mass' (amu)

$$1\ amu = 1.6 \times 10^{-27}\ kg$$ (mass of p^+ = mass of n^0 = 1.672×10^{-27} kg)

c) Time:

- ~~Largest unit of time is 'Lunar month'.~~

Lunar month: It is the time taken by the moon to complete one revolution around the earth.

$\therefore$ 1 Lunar month = 27 days.

- Smallest unit of time is 'Shake'.

$\therefore$ 1 Shake ~~(= 10^{-8} seconds (s))~~

$= 10^{-8}$ s

System of Units

MODULE 1 (Pg- 23)

i) FPS or British Enginnering system:

Fundamental quantities : Units (symbol)

- Length : foot (ft)
- Mass : pound (lb)
- Time : seconds (s)

* 1 m = 3.828 ft

1 lb = 454 gm.

ii) CGS or Gaussian units :

Fundamental quantities : Units (symbol)

- Length : centimetre (cm)
- Mass : gram (g)
- Time : second (s)

iii) MKS system :

Fundamental Quantities : Units (symbol)

- Length : Metre (m)
- Mass : Kilogram (Kg)
- time : Second (s)

iv) SI units or International system of units.:

Fundamental Quantities : Units (symbol)

- length : Metre (m)
- Mass : Kilogram (Kg)
- time : Second. (s)
- temperature : Kelvin (K)

- Electric current: Ampere (A)
- Luminous intensity: Candela ~~(Cd)~~ (cd)
- Amount of substance: Mole (mol)

Properties of Physical Quantities and conversion factor.

- Magnitude of a physical quantities can be represented as, PQ = Numeric value x unit.

  $\therefore PQ = n \times u = nu.$

- When we convert / change the value (magnitude) of PQ from one unit system to another unit system then the magnitude of that PQ must be same (constant)

  i.e. $n_1 u_1 = n_2 u_2$

  $nu = \text{constant}.$

  $\therefore n = \frac{\text{constant}}{u}$

  $n \propto \frac{1}{u}$

It means the numeric value of a physical quantity is inversaly proportional to its base unit.

eg: Physical quantity (SI) (CGS)

Mass: $1\,Kg = 1000\,g$

$n_1 = 1 \quad n_2 = 1000$

$u_1 = Kg \quad u_2 = g$

eg: Physical Quantity (SI) (CGS)

Distance: $1\,km = 1000\,m$

$n_1 = 1 \quad n_2 = 1000$

$u_1 = Km \quad u_2 = m$

Q.3 If acceleration due to gravity $(g) = 9.8\, m/s^2$ then find its value in ft/s^2

$$g = 9.8\ m/s^2$$

$$\because\ 1\ m = 3.28\ ft.$$

$$\therefore\ g = 9.8\ m/s^2 = (9.8 \times 3.28)\ ft/s^2$$

$$= 31.948\ ft/s^2$$

$$\simeq 32\ ft/s^2$$

Q.4 The SI unit of length is m. If in a new unit system, the unit of length is 5m then find the numerical value of area of $12 m^2$ in new unit system.

Let SI unit system be $n_1 u_1$ and new unit system be $n_2 u_2$

$$\because\ n_1 u_1 = n_2 u_2$$

$$12\ m^2 = n_2 (5m)^2$$

$$\therefore\ n_2 = \frac{12\ m^2}{25\ m^2} = \frac{12}{25}$$

Q.5. The SI unit of length is m, If in a new unit system, the unit of length is "ym", then find numerical value of volume of $5m^3$ in new unit system.

Let SI unit system be $n_1 u_1$ and new unit system be $n_2 u_2$

$$\because\ n_1 u_1 = n_2 u_2$$

$$5 m^3 = n_2 (ym)^3$$

$$\therefore\ n_2 = \frac{5\ m^3}{y^3 m^3} = \frac{5}{y^3}$$

Dimensions

MODULE 1 (P7-P9)

- Dimensions of a physical quantities are the power (exponents) to which the base quantities are raised to represent the physical quantities.

Dimentional equation :

$$P.Q. = \boxed{[M^{a} L^{b} T^{c}]}$$

dimensions

dimentional formula

- Physical quantities with their dimensional formula :

i) Mass : $[M^1 L^0 T^0]$ or $[M^1]$
Unit : Kilogram (Kg)

ii) Length : $[M^0 L^1 T^0]$ or $[L^1]$
Unit: Metre (m)
It is also used for width, height, depth, distance, displacement, arc, radius, diametre light year etc.

iii) Time : $[M^0 L^0 T^1]$ or $[T^1]$
Unit : Second (s)

iv) Area : Length x Breadth $= L \times B$

$= \cancel{L} L^1 \times L^1 = L^2$

$[M^0 L^2 T^0]$ or $[L^2]$

Unit : Sq. metre (m^2)

Any pure number ($1, 2, 3, 10, 10^6, 3.14, \frac{10}{3}$ ... etc).
π, e, $\log_e x$, $\sin x$, $\cos x$, $\tan x$ are dimensionless i.e. $[M^0L^0T^0]$

Dimensional formula of any area is always $[M^0L^2T^0]$

v) Volume: length x breadth x height $= L^1 \times L^1 \times L^1$
$= [M^0L^3T^0]$ or $[L^3]$
Unit: Cubic metre (m^3).

Dimensional formula of any volume is always $[M^0L^3T^0]$

vi) Density: $\frac{\text{Mass}}{\text{Volume}} = \frac{M^1}{L^3} = [M^1L^{-3}T^0]$
Unit: Kilogram per cubic metre (kg/m^3)

vii) Speed: $\frac{\text{Distance}}{\text{time}} = \frac{L^1}{T^1} = [M^0L^1T^{-1}]$
Unit: Metre per second (m/s)

It is also used for ~~volume~~ velocity.

viii) Acceleration: $\frac{\text{Velocity}}{\text{Time}} = \frac{L^1T^{-1}}{T^1} = [M^0L^1T^{-2}]$
Unit: Metre per second square (m/s^2)

ix) Force: Mass x acceleration.
$M^1 \times L^1T^{-2} = [M^1L^1T^{-2}]$
unit: ($kg\ m/s^2$) or (N) newton.
~~It is also used for stress.~~

x) Work : Force x displacement

$$M^1L^1T^{-2} \times L^1 = [M^1L^2T^{-2}]$$

Unit : Joule (J)

xi) It is also used for energy (kinetic energy, Potential energy, chemical energy, heat energy, internal energy etc) and calory have same dimensional formula

xi) Power (P) : $\frac{\text{Work}}{\text{time}} = \frac{M^1L^2T^{-2}}{T^1} = [M^1L^2T^{-3}]$

Unit : Watt (W)

xii) Momentum (P): Mass x velocity

$$= M^1 \times L^1T^{-1} = [M^1L^1T^{-1}]$$

Unit: (Kgm/s)

xiii) Impulse (I): Force x time

$$= M^1L^1T^{-2} \times T^1 = [M^1L^1T^{-1}]$$

Unit: Newton-second (NS)

* Momentum and Impulse have same dimensional formula. (Momentum - impulse theorem)

xiv) Pressure (P) : $\frac{\text{Force}}{\text{Area}} = \frac{M^1L^1T^{-2}}{L^2} = [M^1L^{-1}T^{-2}]$

Unit: (N/m^2) or (Pa) Pascal

* Pressure and stress have same dimensional formula.

xv) Strain : $\frac{\Delta l}{l} = \frac{L^1}{L^1} = [M^0L^0T^0]$

Hence, it is dimensionless i.e. Unitless.

xvi) Young's Modulus of elasticity : $\frac{\text{Stress}}{\text{Strain}}$

$$= \frac{M^1L^{-1}T^{-2}}{M^0L^0T^0} = [M^1L^{-1}T^{-2}]$$

Unit : (N/m^2)

* Pressure, stress and Young's Modulus of elasticity have same dimensional formula.

xvii) Angle (θ) : $\frac{\text{Arc}}{\text{Radius}} = \frac{L^1}{L^1} = [M^0L^0T^0]$

Hence, it is also dimensionless but not unitless.

Unit : Radian (rad)

* Angle and Angular displacement have same dimensional unit.

Angular displacement : dimensionless and its unit is radian.

xviii) Angular velocity (ω) : $\frac{\text{Angle}}{\text{Time}} = \frac{M^0L^0T^0}{T^1}$

$$[M^0L^0T^{-1}]$$

Unit : Radian/second (rad/s)

xix) Angular acceleration (α) : $\frac{\text{Angular velocity}}{\text{Time}}$

$$= \frac{M^0 L^0 T^{-1}}{T^1} = [M^0 L^0 T^{-2}]$$

Unit : Radian per second square (rad/s^2)

xx) Torque(τ) : Force x ($\perp$ distance)

$$M^1 L^1 T^{-2} \times L^1 = [M^1 L^2 T^{-2}]$$

Unit : (N-m)

* Torque and work have same dimensional formula

xxi) Momentum of Inertia (I) : Mass x ($\perp$ distance)^2

$$M^1 \times L^1 L^1 = [M^1 L^2 T^0]$$

Unit : (Kgm^2)

xxii) Frequency (n, f, ν) : $\frac{1}{\text{Time taken}} = \frac{1}{T^1} = [M^0 L^0 T^{-1}]$

Unit : Hertz (Hz)

* Frequency and angular velocity have same dimensional formula

xxiii) Angular Momentum (J, L) : ~~Impulse x~~ Momentum of Inertia x Angular velocity

$$= M^1 L^2 \times T^{-1} = [M^1 L^2 T^{-1}]$$

Unit : (Kgm^2/s)

xxiv) Plank's constant (h) : $\frac{\text{Energy}}{\text{Frequency}} = \frac{M^1 L^2 T^{-2}}{T^{-1}}$

$$[M^1 L^2 T^{-1}]$$

Unit : Joule second (Js)

* Angular momentum and Planks constant have same dimensional formula.

xxv) Surface Tension : $\frac{\text{Force}}{\text{length}} = \frac{M^1L^1T^{-2}}{L^1}$

$[M^1L^0T^{-2}]$

Unit : newton/ metre (N/m)

xxvi) Force constant (spring constant) (K) :

$\because$ Force, F = Kx (displacement)

$\therefore K = \frac{\text{Force}}{\text{displacement}} = \frac{M^1L^1T^{-2}}{L^1}$

$[M^1L^0T^{-2}]$

Unit: newton / metre (N/m)

* Force constant and surface tension have same dimensional formula

xxvii) Electric current (I) : $[M^0L^0T^0A^1]$ or $[A^1]$

Unit : Ampere (A)

xxviii) Charge (Q) : Electric current x time.

$A^1 \times T^1 = [M^0L^0T^1A^1]$

Unit: coulomb (c)

xxix) Electric potential or Potential difference (EMF):

$= \frac{\text{work}}{\text{charge}} = \frac{M^1L^2T^{-2}}{T^1A^1} = [M^1L^2T^{-3}A^{-1}]$

Unit: volt (V)

xxx) Resistance : $\frac{\text{Potential difference}}{\text{electric current}} = \frac{M^1L^2T^{-3}A^{-1}}{A^1}$

$[M^1 L^2 T^{-3} A^{-2}]$

Unit : Ohm (Ω)

xxxi) coefficient of viscosity (η) : (read as Eta)

$\because F$ (Force) $= 6\pi\eta r v$

$$\therefore \eta = \frac{F}{6\pi r v} = \frac{M^1 L^1 T^{-2}}{L^1 M^0 \times L^1 T^{-1}}$$

$$= [M^1 L^{-1} T^{-1}]$$

Unit : (N-s/m^2)

xxxii) Universal gravitational constant (G):

$$\because F = \frac{G m_1 m_2}{r^2}$$

$$\therefore G = \frac{F r^2}{M_1 M_2} = \frac{M^1 L^1 T^{-2} \times L^2}{M^1 M^1}$$

$$= [M^{-1} L^3 T^{-2}]$$

Unit : $\left(\frac{Nm^2}{kg^2}\right)$

xxxiii) Temperature : $[K^1]$ or $[\theta^1]$

Unit : Kelvin (K)

xxxiv) Specific heat (s or c) :

$\because$ (heat) $Q = ms\Delta\theta$, where θ = temp^n

$$\therefore s = \frac{Q}{m\Delta\theta}$$

$$= \frac{M^1 L^2 T^{-2}}{M^1 \theta^1} = [M^0 L^2 T^{-2} \theta^{-1}]$$

Unit: $\left(\frac{J}{kg \times K}\right)$

xxxv) Energy density : $\frac{\text{Energy}}{\text{volume}}$

$$= \frac{M^1L^2T^{-2}}{L^3} = [M^1L^{-1}T^{-2}]$$

Unit : $\left(\frac{J}{m^3}\right)$

OR

energy density $= \frac{1}{2}\epsilon_0 E^2$

where ϵ_0 = permittivity of free space (read as epsilon)

E = Electric field

energy density $= [M^1L^{-1}T^{-2}]$

xxxvi) Light velocity (c) : $[L^1T^{-1}]$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = (\mu_0\epsilon_0)^{-1/2}$$

where μ_0 = permiability of free space.

ϵ_0 = permittivity of free space

Unit : (m/s)

Some special Points:

- All the physical quantities which are unitless, will also be dimensionless but all the dimensionless physical quantities are not unitless.
  eg: Angle : dimensionless with radian as its unit.

- Two physical quantities can be added or subtracted only when, they have

same property. In this condition, the dimension formula are also same.
It is known as 'Law of Homogeneity'

- For multiplication and division, two physical quantities ~~having~~ may have different properties and dimension formula, ~~it may be possible~~

eg: $w = F \times d$

$P = \frac{F}{A}$

eg: Law of Homogeneity

$F = \frac{Mv^2}{r}$

$[M^1L^1T^{-2}] = \frac{M^1 \times L^2T^{-2}}{L^1} = [M^1L^1T^{-2}]$

Q.6 If displacement $x = At^2 + Bt + c$ where $t =$ time then find the dimensional formula of A, B and C along with its unit.

Case I

Using Law of Homogeneity

$x = At^2$

$\therefore A = \frac{x}{t^2} = \frac{L^1}{T^2} = [L^1T^{-2}]$

Unit : (m/s^2)

Case II

$x = Bt$

$\therefore B = \frac{x}{t} = \frac{L^1}{T^1} = [L^1T^{-1}]$

Unit : (m/s)

Case III

$x = c$

$\therefore c = x = [L^1]$

Unit : (m)

Q. 7 Velocity, $v = at + \frac{b}{t+c}$, where t = time

then find dimensional formula of A, B and C along with its unit.

Using law of homogeneity

Case I

$v = at$

$\therefore a = \frac{v}{t} = \frac{L^1T^{-1}}{T^1} = [L^1T^{-2}]$

Unit : (m/s^2)

Case II

$\cancel{v} = \frac{\cancel{b}}{\cancel{t+d}}$, $c = t = [T^1]$

Unit : (s)

Case III

$v = \frac{b}{t+c}$

$\therefore b = vt + vc$

- $b = vt$

$= L^1T^{-1} \times T^1$

$= [L^1]$

Unit : (m) OR

- $b = vc$

$= L^1T^{-1} \times T^1$

$= [L^1]$

Unit : (m)

OR CASE III : $v = \frac{b}{t}$

by short trick

$$\therefore b = vt$$
$$= L^1T^{-1} \times T^1$$
$$= [L^1]$$

Unit : (m)

Q. 8 Power, $p = \frac{a - x^2}{bt}$ where x = distance and t = time, then find the dimensional formula and units of A and B.

Using law of homogeneity

Case I :

$$a = x^2 = [L^2]$$

Unit : (m^2)

Case II :

$$\because p = \frac{x^2}{bt}$$
$$\therefore b = \frac{x^2}{pt} = \frac{L^2}{M^1L^2T^{-3} \times T^1}$$
$$= [M^{-1}T^2]$$

Unit : (s^2/kg)

Q.9 In vanderwaal's equation $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

where P = pressure, V = volume, R = gas constant and T = temperature then find the dimensional formula and units of a and b.

Using law of homogeneity

Case I

$$V = b = [L^3]$$

Unit : (m^3)

Case II

$P = \frac{a}{V^2}$ $\therefore a = PV^2$

$= [M^1L^{-1}T^{-2}] \times [L^3]^2$

$= [M^1L^5T^{-2}]$

Unit : $(kg\,m^5 s^{-2})$

Q.10 If $y = a \sin(Ax - Bt + c)$ where x = distance, y = displacement and t = time

Let $y = a \sin \theta$ where θ is an angle

$\therefore Ax - Bt + C = \theta$

Case I

$Ax = \theta$

$\therefore A = \frac{\theta}{x} = \frac{M^0L^0T^0}{L^1} = [L^{-1}]$ unit : (m^{-1})

Case II

$Bt = \theta$

$\therefore B = \frac{\theta}{t} = \frac{M^0L^0T^0}{T^1} = [T^{-1}]$

Unit : (s^{-1})

Case III

$C = \theta = [M^0L^0T^0]$

Unit : (rad) radian.

Q.11 If $P = P_0 e^{-\alpha t^2}$, then find dimensional formula of α

$\because$ the exponential power of base is always dimensionless.

$\therefore \alpha t^2 = M^0L^0T^0$

$$\alpha = \frac{M^0L^0T^0}{t^2} = \frac{M^0L^0T^0}{T^2} = [T^{-2}]$$

Unit : (sec^{-2})

Q.12 If Force, $F = \frac{a}{t} + bt^2$, then find the dimensional formula and units of a and b where t = time

Using law of homogeneity,

Case I

$$F = \frac{a}{t} \quad \therefore a = Ft = M^1L^1T^{-2} \times T^1 = [M^1L^1T^{-1}]$$

Unit : (Kgm/s) or (Ns)

Case II

$F = bt^2$

$$\therefore b = \frac{F}{t^2} = \frac{M^1L^1T^{-2}}{T^2} = [M^1L^1T^{-4}]$$

Unit : (N/s^2)

Application of dimensional analysis:

MODULE 1 (Pg-28)

1. To convert magnitude of a physical quantity from one unit system to another.

The basic concept is 'the magnitude of a physical quantity remains same (constant) whatever unit system is used for measurment'

i.e $n_1u_1 = n_2u_2$

$u_1 = [M_1^aL_1^bT_1^c]$

(read as mass, length and time of first unit system)

$u_2 = [M_2^a L_2^b T_2^c]$

(read as mass, length and time of second unit system)

$\because n_1 u_1 = n_2 u_2$

$\therefore n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

i.e. numerical value of Physical quantity in second unit system.

Q.13 Convert 1 J (SI unit of work) into erg (CGS unit of work)

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

$n_1 = 1$

work $= [M^1 L^2 T^{-2}]$

by comparing,

$a = 1, \quad b = 2$ and $c = -2$

$$n_2 = 1 \left(\frac{kg}{g}\right)^1 \left(\frac{m}{cm}\right)^2 \left(\frac{s}{s}\right)^{-1}$$

$$= 1 \times \frac{1000 g}{g} \times \left(\frac{100\ cm}{cm}\right)^2 \times (1)^{-1}$$

$$= 1000 \times 10000 \times 1 = 10^7$$

Hence,

$1\ J = 10^7$ erg

Q.14 The value of acceleration due to gravity g in CGS system is $980 \frac{cm}{s^2}$, then find its value in SI system.

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

$n_1 = 980$

$g = [M^0 L^1 T^{-2}]$

by comparing.

$a=0$, $b=1$ and $c=-2$

+4/4 $n_2 = 980 \left(\frac{g}{kg}\right)^0 \left(\frac{cm}{m}\right)^1 \left(\frac{s}{s}\right)^{-2}$

$= 980 \times 1 \times \frac{1}{100} \times 1$

$= \frac{980}{100} = 9.8$

Hence.

$980\ cm/s^2 = 9.8\ m/s^2$

Q.15 If density of a material is $10\ kg/m^3$. then find its value in CGS system

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

$n_1 = 10.$

density $= [M^1 L^{-3} T^0]$

by comparing.

$n_2 = 10 \times \left(\frac{kg}{g}\right)^1 \times \left(\frac{m}{cm}\right)^{-3} \times \left(\frac{s}{s}\right)^0$

$= 10 \times \left(\frac{1000\ g}{g}\right)^1 \times \left(\frac{100\ cm}{cm}\right)^{-3} \times 1$

$= 10 \times 1000 \times (100)^{-3}$

$= 10 \times 1000 \times \frac{1}{(100)^3}$

$= 10 \times 1000 \times \frac{1}{1000000} = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$

Hence.

$10\ kg/m^3 = 10^{-2}\ g/cm^3$

Q.16 Convert 100 N, in a unit system in which unit of length is m, unit of mass is kg and unit of time is minute.

$$n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

$n_1 = 100$

Force = $[M^1 L^1 T^{-2}]$

by compairing,

$a = 1$, $b = 1$ and $c = -2$

$$n_2 = 100 \left(\frac{m}{m}\right)^1 \left(\frac{Kg}{Kg}\right)^1 \left(\frac{s}{m}\right)^{-2}$$

$$= 100 \times 1 \times 1 \times \left(\frac{s}{60s}\right)^{-2}$$

$$= 100 \times (60^{-1})^{-2}$$

$$= 100 \times (60)^{+2}$$

$$= 100 \times 3600$$

$$= 3.6 \times 10^5$$

Hence, $100\ N = 3.6 \times 10^5\ kg\,m/min^2$.

2\. To check the correctness (dimensionally) of given equation / formula.

To check the correctness (dimensionally), the dimensions of left hand side physical quantity is equal to dimensions of right hand side physical quantity

• To check the correctness of the given relation,

i) Time ~~(T)~~ period $(T) = 2\pi \sqrt{\frac{l}{g}}$

where, l = length of pendulum and

g = acceleration due to gravity

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$[T^1] = \sqrt{\frac{L^1}{L^1T^{-2}}}$$

$$= \sqrt{\frac{1}{T^{-2}}} = \sqrt{T^2} = [T^1]$$

Hence, the above formula is dimensionally correct.

ii) Escape velocity $V_e = \sqrt{\frac{2GM}{R}}$,

where G = Universal acceleration constant
M = Mass of the earth
R = Radius of the earth.

$$V_e = \sqrt{\frac{2GM}{R}}$$

$$[L^1T^{-1}] = \sqrt{\frac{M^{-1}L^3T^{-2} \times M^1}{L^1}}$$

$$= \sqrt{L^2T^{-2}}$$

$$= [L^1T^{-1}]$$

Hence, the above formula is dimensionally correct.

iii) Frequency of a compound pendulum (f) =

$$f = \frac{1}{2\pi}\sqrt{\frac{mgl}{I}}$$

where, g = acceleration due to gravity
l = length of the pendulum
I = ~~Electric current~~ moment of Inertia

$$f = \frac{1}{2\pi}\sqrt{\frac{mgl}{I}}$$

$$[T^{-1}] = \sqrt{\frac{M^1L^1T^{-2} \times L^1}{M^1L^2}}$$
$$= \sqrt{\frac{L^2T^{-2}}{L^2}}$$
$$= \sqrt{T^{-2}} = [T^{-1}]$$

Hence, the above formula is dimensionally correct

iv) $S = ut + \frac{1}{3}at^2$

where, u = initial velocity
t = time taken
a = uniform acceleration.

$$S = ut + \frac{1}{3}at^2$$
$$[L^1] = [L^1T^{-1} \times T^1] + [L^1T^{-2} \times T^2]$$
$$= [L^1] + [L^1]$$
$$= 2[L^1]$$
$$= [L^1]$$

Hence, the above formula is dimensionally correct. but incorrect from physics point of view.

* All dimensionally correct relation/formula may or may not be correct from physics point of view but dimensionally incorrect relation/formula is always incorrect from physics point of view.

v) $\tan\theta = \frac{v^2}{rg}$

where, v = velocity and
r = radius.

$\therefore \tan\theta = \frac{v^2}{rg}$

$[M^0L^0T^0] = \frac{(L^1T^{-1})^2}{L^1 \times L^1T^{-2}}$

$= \frac{L^2T^{-2}}{L^2T^{-2}} = [M^0L^0T^0]$

Hence, the above equation is dimentionally correct

vi) $v^2 = u^2 + 2as.$

where, u = initial velocity

a = acceleration (constant).

s = displacement.

$v^2 = u^2 + 2as.$

$[L^1T^{-1}]^2 = [L^1T^{-1}]^2 + [L^1T^{-2}] \times L^1$

$[L^2T^{-2}] = L^2T^{-2} + L^2T^{-2}$

$= 2[L^2T^{-2}]$

$= [L^2T^{-2}]$

Hence, the above equation is dimentionally correct

3. To find the relation between given physical quantity (to derive the formula for given Physical Quantity by dimentional analysis method):

i) Relative density : $\frac{\text{Density of solid}}{\text{Density of water}}$

$= \frac{[M^1L^{-3}]}{[M^1L^{-3}]} = [M^0L^0T^0]$

Hence, its dimensionless ~~or~~ and unitless

ii) Di- electric current (K or ϵ_r)
where ϵ_r = relative permittivity of medium.
(read as epsilon r).

$$\epsilon_r = \frac{\text{Permittivity of medium}}{\text{Permittivity of free space}}$$
$$= [M^0L^0T^0]$$

Hence, its unitless and dimensionless

iii) Magnetic moment = IA
current × Area.
$$= [A^1L^2]$$

iv) Permeability of free space (μ_0) : $\frac{\text{Force (F)}}{\text{length } (l)}$

$$= \frac{F}{l} = \frac{\mu_0}{4\pi} \times \frac{I_1 I_2}{2\pi d}.$$

where, I_1 and I_2 are current and.
d = distance.

$$\therefore \mu_0 = \frac{F \times 4\pi \times 2\pi d}{l \times I_1 \times I_2}$$
$$= \frac{M^1L^1T^{-2} \times L^1}{L^1 \times A^1 \times A^1}$$
$$= [M^1L^1T^{-2}A^{-2}]$$

Q.17 The time period of a simple pendulum (T) depends on the mass of the bob (m), length of a simple pendulum (l) and acceleration due to gravity is (g), then find the relation between time period of simple pendulum ?

$$\because T \propto m^a l^b g^c$$
$$\therefore T = K m^a l^b g^c \text{ --- } (1)$$

where, K is dimensionless constant

$$\therefore [M^0 L^0 T^1] = [M^1]^a [L^1]^b [L^1 T^{-2}]^c$$

$$= M^a L^b L^c T^{-2c}$$

by comparing, we get.

$a = 0$

$-2c = 1$

$\therefore c = \frac{-1}{2}$

$\because b + c = 0.$

$\therefore b = -\left(\frac{-1}{2}\right) = \frac{1}{2}$

Putting value of a, b and c in equation (1), we get;

$$T = K m^0 l^{1/2} g^{-1/2}$$

$$\therefore T = \frac{K l^{1/2}}{g^{1/2}} = K\sqrt{\frac{l}{g}}$$

$$\therefore T = K\sqrt{\frac{l}{g}},$$

where $K = 2\pi$ (experimentally obtained)

$$\therefore T = 2\pi\sqrt{\frac{l}{g}}$$

Q.18 The centripetal force depends on mass of the body (m), velocity of the body (V) and radius (r), then derive the formula of centripetal force (F) by dimension analysis method.

$$\because F \propto m^a V^b r^c$$

$$\therefore F = K m^a V^b r^c \quad — (1)$$

where, k is dimensional constant

$$[M^1 L^1 T^{-2}] = K[M^1]^a [L^1 T^{-1}]^b [L^1]^c$$

$$= K[M^a L^b T^{-b} L^c]$$

$$[M^1L^1T^{-2}] = K[M^a L^{b+c} T^{-b}]$$

by comparing the powers, we get.

$a = 1$

$-2 = -b$.

$\therefore b = 2$

$b + c = 1$

$\therefore c = 1 - 2 = -1$

Putting the values in eq ①, we get

$$F = K m^1 V^2 r^{-1}$$

$$= \frac{K m V^2}{r}$$

where $K = 1$ (experimentally obtained)

$$\therefore F = \frac{mV^2}{r}$$

Q.19 If Force (F), velocity (V) and time (T) are assumed as a fundamental quantities then find the dimensional formula of mass (m)

$$m = F^a V^b T^c \quad \text{—①}$$

$$[M^1] = [M^1L^1T^{-2}]^a [L^1T^{-1}]^b [T^1]^c ..$$

$$[M^1] = [M^a L^a T^{-2a} L^b T^{-b} T^c]$$

$$[M^1] = [M^a L^{a+b} T^{-2a-b+c}]$$

On comparing quantities, we get

$a = 1$

$a + b = 0$

$\therefore b = -a = -1$

$-2a - b + c = 0$.

$\therefore c = 2a + b = 2 \times 1 + (-1) = 2 - 1 = 1$

Putting these values in eq ①, we get

$$m = F^1 V^{-1} T^1$$

$$m = \frac{FT}{V} \quad ; \text{OR}$$

Short trick ;

$$\because F = ma$$

$$\therefore \cancel{a = \frac{F}{m}} \quad \therefore \quad m = \frac{F}{a}$$

$$m = \frac{FT}{V} = [F^1 T^1 V^{-1}] \qquad \left[\because a = \frac{V}{T}\right]$$

Q.30 In the above question, find the dimensional formula of work

$$\because \quad W = F \times \text{displacement}$$

$$= F \times VT \quad [\because \text{displacement} = V \times T]$$

$$\therefore \quad W = [F^1 V^1 T^1] = FVT$$

Limitations of dimensional analysis method:

i) The value of proportionality constant (K) is not obtained by dimensional analysis method.

ii) We cannot derive the formula or equation which has two or more than two terms in forms of addition or substraction

eg: $s = ut + \frac{1}{2} at^2$

$v^2 = u^2 + 2as$

iii) We cannot derive the equation or formula which has trignometrical function, logarithmic function and exponential function.

eg: $N = N_0 e^{-\lambda t}$

$y = A \sin(wt)$