VECTORS

Based on directions, there are two types of physical quantities:
Vector Supplement (VS) Pg-1

i) Scalers : Those physical quantities which require only magnitude but not the direction for their complete definition.

eg: Mass, Time, Temperature, volume, distance, speed, work, power, energy etc.

ii) Vector quantity : Those physical quantities which have magnitude as well as direction and obeys all the laws of vector algebra.

eg: Displacement, velocity, acceleration, Momentum, force etc.

Representation of Vectors:
(VS) Pg-1

- Magnitude ~~is~~ $\propto$ Length of vector.
- Magnitude of a vector is always positive.

Types of vectors:

i) Parallel Vectors:
(VS) Pg-1

- Same direction
- Angle between parallel vectors is always 0°

ii) Equal vectors:
(VS) Pg-2

- Equal magnitude and same direction
- the angle between equal vector is also 0°.

iii) Anti-parallel vector:
(VS) Pg-2

- opposite directions
- the angle between two anti-parallel vector is always $180° = \pi$ radian.

iv) Opposite vectors / negative vectors:
(VS) Pg-2

- Equal magnitude but opposite directions.
- the angle between two opposite / negative vector is also $180° = \pi$ radian

v) coinitial vectors:
(VS) Pg-2

- same starting point.

vi) concurrent vectors:
(VS) Pg 3

- they pass from same point.

vii) coplaner vector:
(VS) Pg-3

- They lie on the same plane.

viii) Unit vector :
(VS) Pg - 3

- Vectors whose magnitude = 1
- Formula of unit vector. $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$
- It is used to specify the direction of a given vector.

Three standard unit vectors.
(VS) Pg - 4

- $\hat{i}$ represents x - axis
- $\hat{j}$ represents y - axis
- $\hat{k}$ represents z - axis.

Q-1 Which of the following cannot be the magnitude of Force?
a. 10 N
b. 0
c. 20 N
d. - 5 N

Ans) d) - 5N
Since magnitude of a vector can't be negative.

ix) zero vector / Null vector :
(VS) Pg - 5

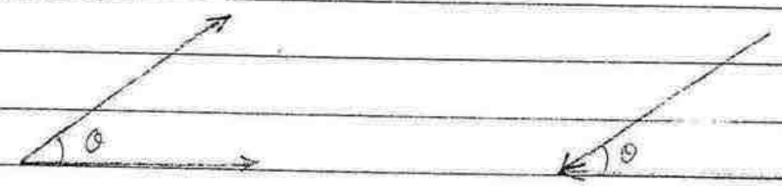
- A vector whose magnitude is zero.

- Its direction is arbitrary (any direction).

To find angle between two vectors when vector diagram is given: (VS) Pg-5

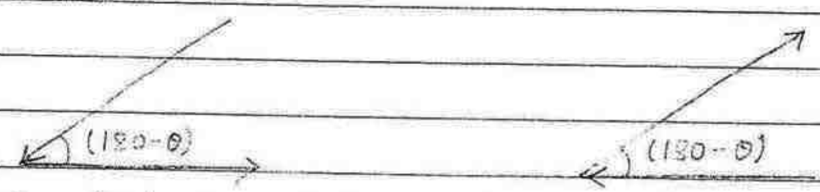
θ = Interior angle

- CASE I :



Angle between two vectors = θ when tails or heads of two vectors are connected to each other.

- Case II :



Angle between two vectors = $180-\theta$ when a tail of one vector is connected to the head of the another vector.

Addition of vectors :

1. Triangle law of vector addition : If two vectors represents two sides of a given triangle in the same order, then their vector sum is represented by the third side of that triangle in reverse order.

(VS) Pg - 6

- For finding magnitude of resultant vector (R) when magnitude of both the vectors are given along with the angle (θ) between them
(VS) Pg - 8

☆ $|\vec{R}|$ or $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$
(when magnitude of both the vectors, A and B are different)

- For finding direction of resultant vector (R) when magnitude of both the vectors are given (different magnitude) along with the angle (θ) between them.
(VS) Pg - 8

~~tan~~ 'α' = angle between $\vec{A}$ and $\vec{R}$ and
'β' = angle between $\vec{B}$ and $\vec{R}$.

☆ $\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$ and

★ $\tan\beta = \dfrac{A\sin\theta}{B + A\cos\theta}$

2. Parallelogram law of vector addition : If two vectors represents two adjacent sides of a parallelogram passing away from your common point, then their resultant vector is represented by the diagonal which also passes away from the same common point
(VS) Pg - 8

Q-10 The resultant of 2N and 3N forces are 4N. Find the angle between 3N and 2N Force.

$$\because \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$4 = \sqrt{2^2 + 3^2 + 2\times2\times3\cos\theta}$$

Squaring both sides,

$$16 = 4 + 9 + 12\cos\theta.$$

$$12\cos\theta = 16 - 13 = 3.$$

$$\therefore \cos\theta = \frac{3}{12} = \frac{1}{4}$$

$$\therefore \theta = \cos^{-1}\left(\frac{1}{4}\right)$$

Q-11 The angle between 4N and 6N forces are 60°. Find :

i) Resultant

ii) Angle of $\vec{R}$ from 4N force.

i)

$$\because \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$R = \sqrt{(4)^2 + (6)^2 + 2\times4\times6\times\cos 60^\circ}$$

$$R = \sqrt{16 + 36 + \cancel{2}\times24\times\frac{1}{\cancel{2}}}$$

$$R = \sqrt{52 + 24} = \sqrt{76}\ N = 2\sqrt{19}\ N$$

ii)

$$\because \quad \tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\tan\alpha = \frac{6\times\sin 60^\circ}{4 + 6\times\cos 60^\circ}$$

$$\tan\alpha = \frac{6\times\frac{\sqrt{3}}{2}}{4 + 6\times\frac{1}{2}} = \frac{3\sqrt{3}}{7}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{3\sqrt{3}}{7}\right)$$

[unit is assumed in radian]

Three special case of $|\vec{R}|$ or R :

i) Case I :
(VS) Pg-9

If $\vec{A}$ and $\vec{B}$ are parallel, then $\theta = 0^\circ$
$\cos\theta = \cos 0 = 1$ (Max. value of $\cos\theta$)

$R_{Max} = A + B$.

ii) Case II :
(VS) Pg-9

If $\vec{A}$ and $\vec{B}$ are perpendicular, then $\theta = 90^\circ$
$\cos\theta = \cos 90^\circ = 0$

$R = \sqrt{A^2+B^2}$ and.

$\tan\alpha = \frac{B}{A}$ and.

$\tan\beta = \frac{A}{B}$

iii) Case III :
(V.S) Pg-10

If $\vec{A}$ and $\vec{B}$ are anti-parallel, then $\theta = 180^\circ$
$\cos\theta = \cos 180^\circ = -1$ (Min value of $\cos\theta$).

$R_{Min} = A \sim B$ (Bigger value - Smaller value)

☆ Range of resultant :

$$R = (A \sim B) \text{ to } (A+B).$$

OR

$$(A \sim B) \leq R \leq (A+B)$$

Q.16 The resultant of $\vec{A}$ and $\vec{B}$ ~~ao~~ is $\perp$ $\vec{A}$. Find the angle between $\vec{A}$ and $\vec{B}$

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\tan 90^\circ = \frac{B \sin \theta}{A + B \cos \theta} = \infty = \frac{1}{0}$$

$$\therefore \quad A + B \cos \theta = 0 \times B \sin \theta$$

$$A + B \cos \theta = 0.$$

$$B \cos \theta = -A$$

$$\cos \theta = \frac{-A}{B}.$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{-A}{B}\right)$$

☆ Conclusions :

i) If $\vec{R}$ is $\perp$ $\vec{A}$

then, $\cos \theta = \frac{-A}{B}$

ii) If $\vec{R}$ is $\perp$ $\vec{B}$

then, $\cos \theta = \frac{-B}{A}$

* Electric current is scaler because it does not obey all the law of vector algebra.

* (VS) Pg-10
Two vectors of unequal magnitude can never produce zero (0) resultant. If the resultant of two vector is zero, then they must be equal in magnitude and opposite in direction.

3 Polygon law of vector addition: If some sides of a polygon represents some vectors in the same order, then their resultant vector is represented by the closing side of that polygon in the reverse order.

(VS) Pg-11

* ~~I~~ (VS) Pg-11
In a closed polygon, if all the vectors are in same order then their resultant is always a zero vector

* The resultant of 3 vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ can be zero, only if.

☆ $$(A \sim B) \leq C \leq (A + B)$$

Conditions for zero vector:
(VS) Pg-12

- Minimum number of vectors of equal magnitude to produce zero resultant is 2

- Minimum number of co-planer vectors having different magnitude to give zero resultant is 3.

- Minimum number of vectors in different planes to give zero resultant is 4.

Important properties of vector addition.
(VS) Pg-12

i) the resultant of two vectors are always co-planer.

ii) Vector addition is commutative

iii) Vector addition is associative.

iv) If $|\vec{A}| = |\vec{B}|$, then their resultant force bisects the angle between them

$\therefore \ \alpha = \beta = \frac{\theta}{2}$

v) If $|\vec{A}| > |\vec{B}|$, then $\alpha < \beta$

- The resultant vector inclines more towards the vector of bigger magnitude

vi) If $|\vec{A}| = |\vec{B}| = a$, then

$R = 2a \cos \frac{\theta}{2}$

vii) If $|\vec{A}| = |\vec{B}| = a$ and $\theta = 120^\circ$, then

$R = a$

At $\theta = 120^\circ$

$|\vec{A}+\vec{B}|$ or $|\vec{R}|$ or $R = |\vec{A}| = |\vec{B}| = a$

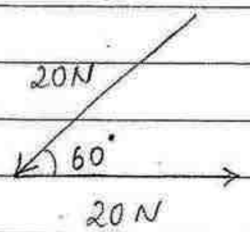
viii) If the angle between two unit vector is 120°, then their resultant is also a unit vector

OR

If the resultant of two unit vector is also a unit vector, then the angle between them is 120°

ix) If n (no of vectors), coplaner vectors of equal magnitude are seperated by equal angle of $\frac{360^\circ}{n}$, then their resultant vector is zero vector.

Q.23 For the given figure, find the value of resultant.



$\theta = 180^\circ - 60^\circ = 120^\circ$

$|\vec{A}| = |\vec{B}| = 20\,N$

Using (vii) property of vector addition, $|\vec{R}|$ or $R = 20\,N$

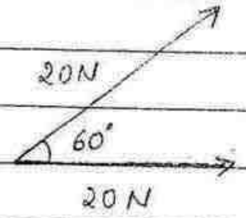
OR

$$R = 2a\cos\frac{\theta}{2}$$

$$= 2 \times 20 \times \cos\frac{120^\circ}{2} = 40^\circ \times \cos 60^\circ$$

$$= 40 \times \frac{1}{2} = 20\,N$$

Q.24 For the given figure, find the value of resultant.



$\theta = 60^\circ$

$|\vec{A}| = |\vec{B}| = 20N.$

Using (vi) property of vector addition.

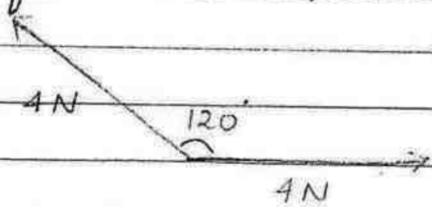
$$R = 2a\cos\frac{\theta}{2}$$

$$= 2 \times 20 \times \cos\frac{60^\circ}{2}$$

$$= 40\cos 30^\circ = 40 \times \frac{\sqrt{3}}{2}$$

$$= 20\sqrt{3}\ N$$

Q.25. For the given figure, find the value of resultant and angle between $\vec{R}$ and forces



$\theta = 120^\circ$

$|\vec{A}| = |\vec{B}| = 4N$

Using (vii) property of vector addition.

$|\vec{A}| = |\vec{B}| = |\vec{R}|$ or $R = 4N$.

$\because \alpha = \beta = \frac{\theta}{2}$ (Using (iv) property of vector addition)

$\therefore \alpha = \beta = \frac{120}{2}$

$= 60^\circ$

$\therefore \alpha = \beta = 60^\circ$

- $|\vec{A}| + |\vec{B}| = A + B$
- $|\vec{A} + \vec{B}| = |\vec{R}|$ or R

★ Subtraction of vectors

(VS) Pg - 14

- $|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

- $\tan\alpha = \dfrac{B\sin\theta}{A - B\cos\theta}$.

- $\tan\beta = \dfrac{A\sin\theta}{B - A\cos\theta}$

Important properties of vector subtraction.

(VS) Pg-14

i) Vector subtraction is not commutative.

ii) Vector subtraction is not associative

iii) $|\vec{A}| = |\vec{B}| = a$, then
$|\vec{A} - \vec{B}| = 2a\sin\dfrac{\theta}{2}$

iv) If $|\vec{A}| = |\vec{B}| = a$ and $\theta = 60^\circ$, then
$|\vec{A} - \vec{B}| = a$

At $\theta = 60^\circ$
$|\vec{A} - \vec{B}| = |\vec{A}| = |\vec{B}| = a$

v) If the angle between two unit vector is 60° then their vector difference is also a unit vector

OR

If the vector difference of two unit vector is also a unit vector, then the angle between them is 60°

Q-26. If $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$, then find angle between $\vec{A}$ and $\vec{B}$

$|\vec{A}+\vec{B}|$ or $|\vec{R}| = |\vec{A}-\vec{B}|$

$\sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{A^2+B^2-2AB\cos\theta}$

Squaring both sides,

$A^2+B^2+2AB\cos\theta = A^2+B^2-2AB\cos\theta$.

$2AB\cos\theta = -2AB\cos\theta$.

$4AB\cos\theta = 0$.

$\cos\theta = 0. = \cos 90°$

$\therefore \theta = 90°$

Means, $\vec{A} \perp \vec{B}$

- If two vectors are perpendicular to each other, then the magnitude of their vector sum and vector difference are equal.

OR

If the magnitude of vector sum or vector difference are equal, then the given two vectors are perpendicular to each other.

Q.27 If $\vec{A}+\vec{B} = \vec{A}-\vec{B}$, then

a. $\theta = 0$

b. $\theta = 90°$

c. $\vec{A} = \vec{0}$

d. $\vec{B} = \vec{0}$

Ans) d) $\vec{B} = \vec{0}$

$\vec{A}+\vec{B} = \vec{A}-\vec{B}$

$\vec{B} = -\vec{B}$

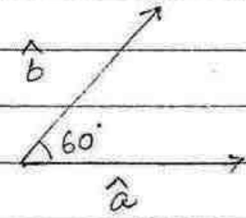
$2\vec{B} = \vec{0}$.

$\therefore \vec{B} = 0/2 = \vec{0}$.

Q.28. For the given unit vector, find :

i) $|\hat{a} + \hat{b}|$

ii) $|\hat{a} - \hat{b}|$



i) ~~$2a$ $R = 2a \sin \frac{\theta}{2}$~~

$|\hat{a}+\hat{b}| = R = 2a\cos \frac{\theta}{2}$

$\therefore R = 2 \times 1 \times \cos \frac{60°}{2}$

$= 2 \times \cos 30° = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$ unit

ii) $|\hat{a} - \hat{b}| = 2a \sin \frac{\theta}{2}$

$= 2 \times 1 \times \sin \frac{60°}{2} \Rightarrow 2 \times \sin 30°$

$= 2 \times \frac{1}{2} = 1$ unit.

Q-29 If $|\hat{a} + \hat{b}| = \sqrt{2}$, then find :

i) θ, angle between both the vectors

ii) $|\hat{a} - \hat{b}|$

i) $|\hat{a} + \hat{b}| = R = \sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{2}$

* squaring both sides.

$1^2 + 1^2 + 2 \times 1 \times 1 \cos\theta = 2$

$2 + 2\cos\theta = 2$

$\cos\theta = \frac{0}{2} = 0 = \cos 90°$

$\therefore \theta = 90°$

ii) $|\hat{a} - \hat{b}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$= \sqrt{1^2+1^2+2\times1\times1\cos 90^\circ}$

$= \sqrt{1+1+2\times0} = \sqrt{2+0} = \sqrt{2}$ unit.

or when $\theta = 90^\circ$, $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

Q.30. If the resultant of two unit vector is also a unit vector, then find the magnitude of their vector difference

Using (viii) property of vector addition

If $R = a$, then $\theta = \cancel{0}\ 120^\circ$

$|\hat{a}-\hat{b}| = 2a\sin\frac{\theta}{2}$ (since, its unit vector)

$= 2\times1\times\sin\frac{120^\circ}{2}$

$= 2\times\sin 60^\circ = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$ unit.

Use of vector subtraction:

(VS) Pg - 15

- Used for finding change in vector quantity
- Change(Δ) = final value - Initial value

Q-31 If $\vec{A}$ is reversed, then find,

i) $\Delta\vec{A}$ "Change in $\vec{A}$"

ii) $|\Delta\vec{A}|$ "Magnitude of change in $\vec{A}$"

iii) $\Delta|\vec{A}|$ "change in magnitude of $\vec{A}$"

given $\vec{A_i}$ (initial) $= \vec{A}$ and

$\vec{A_f}$ (final) $= -\vec{A}$

i) $\Delta\vec{A} = \vec{A_f} - \vec{A_i}$

$= -\vec{A} - \vec{A} = -2\vec{A}$

ii) $|\Delta\vec{A}| = |\vec{A_f} - \vec{A_i}| = |-\vec{A}-\vec{A}| = |-2\vec{A}| = 2\vec{A}$

iii) $\Delta|\vec{A}| = |\vec{A_f}| - |\vec{A_i}|$

$= |-\vec{A}| - |\vec{A}|$

$= \vec{A} - \vec{A} = \vec{0}$

- On reversing a vector, only its direction changes ~~but~~ and the magnitude remain same

Q.32. A particle moving from point A (3, 1, 1) m to the point B (7, 7, 3) m, then, find the displacement vector

$$\because \vec{S} = (\Delta x)\hat{i} + (\Delta y)\hat{j} + (\Delta z)\hat{k}$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$
$$= (7-3)\hat{i} + (7-1)\hat{j} + (3-1)\hat{k}$$
$$= (4\hat{i} + 6\hat{j} + 2\hat{k})\ m.$$

Important Points :
(VS) Pg- 15

i) In a parallelogram, one diagonal represent vector addition and other diagonal represent vector substration.

ii) If $\vec{A} \perp \vec{B}$ but $|\vec{A}| \neq |\vec{B}|$, then, they will form a rectangle.
Hence, $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ and
$(\vec{A} + \vec{B})$ is not perpendicular to $(\vec{A} - \vec{B})$

iii) If $|\vec{A}| = |\vec{B}|$ but '$\vec{A}$' is not perpendicular to $\vec{B}$. then, they will form a rhombus.
Hence, $|\vec{A} + \vec{B}| \neq |\vec{A} - \vec{B}|$ and
$(\vec{A} + \vec{B}) \perp (\vec{A} - \vec{B})$

iv) If $|\vec{A}| = |\vec{B}|$ and $\vec{A} \perp \vec{B}$, then they will form a square
Hence, $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ and
$(\vec{A} + \vec{B}) \perp (\vec{A} - \vec{B})$

x) Position vector:

(VS) Pg-16

- used to satisfy the position of a particle or point.

eg: $\vec{s} = (\Delta x)\hat{i} + (\Delta y)\hat{j} + (\Delta z)\hat{k}$

Displacement can be defined as change in position vector.

Resolutions of Vectors:

(VS) Pg-17

- The process of finding component of a vectors.
- They are known as perpendicular or orthogonal or rectangular components, if angle between them is 90°.

* Resolution of vector in a plane (in 2-D)

(V.S) Pg-17

- $A_x = A\cos\theta$
- $A_y = A\sin\theta$
- According to parallelogram law of vectors

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

- According to pythogoras theorem.

$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

- For finding direction of $\vec{A}$

$\tan\theta = \dfrac{A_y}{A_x}$

here, θ is the angle of $\vec{A}$ from x-axis.

* **Special points :**
(VS) Pg-17

i) If $\vec{A}$ makes angle θ from a given direction then its component along that direction is always $A \cos\theta$ and the remaining perpendicular component is $A \sin\theta$.

ii) The component of any vector along its perpendicula direction is always zero.

Q-33 If $\vec{A} = 2\hat{i} + 6\hat{j}$, then find :
i) x - component of $\vec{A}$, A_x
ii) Projection of $\vec{A}$ on Y - axis, A_y
iii) $|\vec{A}|$
i) $A_x = 2$
ii) $A_y = 6$
iii) $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$
$= \sqrt{2^2 + 6^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$ unit

Q.34 * If $\vec{A} = 3\hat{i} + 4\hat{j}$, then find :
i) Projection of $\vec{A}$ on x- axis, A_x
ii) y- component of $\vec{A}$, A_y
iii) $|\vec{A}|$
iv) Unit vector parallel to $\vec{A}$ or unit vector along $\vec{A}$, $\hat{A}$
v) Angle of $\vec{A}$ from x- axis.
vi) Angle of $\vec{A}$ from y- axis
i) $A_x = 3$.
ii) $A_y = 4$
iii) $|\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$ unit
iv) $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{3\hat{i} + 4\hat{j}}{5}$

v) $\tan\theta = \frac{A_y}{A_x} = \frac{4}{3}$

$\therefore \theta = \tan^{-1}\left(\frac{4}{3}\right) = 53^\circ$

vi) $\tan\theta = \frac{A_x}{A_y} = \frac{3}{4}$

$\therefore \theta = \tan^{-1}\left(\frac{3}{4}\right) = 37^\circ$

- The component from which we had to find the angle are taken in denominator.

Q. 35 Check whether the following vectors are unit vectors ~~are unit vectors~~ or not?

i) $\hat{i}$

ii) $\hat{j}$

iii) $\hat{i} + \hat{j}$

iv) $\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$

v) $\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}$

i) $A_x = 1$; Yes.

ii) $A_y = 1$; Yes.

iii) $|\hat{i} + \hat{j}| = \sqrt{A_x^2 + A_y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\because$ magnitude $\neq 1$; No

- $\because$ Sum of two unit vector is also a unit vector when angle between them is 120° but angle between $\hat{i}$ and $\hat{j}$ is 90°

iv) $\left|\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right| = \sqrt{A_x^2 + A_y^2}$

$$= \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{\frac{2}{2}} = \sqrt{1}$$

$\sqrt{1} = 1$; Yes

v $\left|\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}\right| = \sqrt{A_x^2 + A_y^2}$

$$= \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{-1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{\frac{2}{2}} = \sqrt{1}$$

$\sqrt{1} = 1$; Yes.

Q.36. given $\vec{A} = 0.6\hat{i} + \alpha\hat{j}$. If $\vec{A}$ is a unit vector. then find α

$|\vec{A}| = \sqrt{A_x^2 + A_y^2} = 1.$ ($\because$ Its a unit vector)

$\therefore \sqrt{(0.6)^2 + \alpha^2} = 1.$

Squaring both sides.

$0.36 + \alpha^2 = 1.$

$\therefore \alpha^2 = 1 - 0.36 = 0.64$

$\therefore \alpha = \sqrt{0.64} = \pm 0.8$

Q.37 A force of 10 N is applied on a body at an angle of 45° from horizontal, find:

i) horizontal component

ii) vertical component

i) $A_x = A\cos\theta = F\cos\theta$

$= 10 \times \cos 45° = 10 \times \frac{1}{\sqrt{2}}$

$= 5\sqrt{2}$ N

ii) $A_y = A\sin\theta = F\sin\theta$

$= 10 \times \sin 45° = 10 \times \frac{1}{\sqrt{2}}$

$= 5\sqrt{2}$ N

Q.38 A force of 20N is acting on a body at an angle of 30° from vertical, find:

i) Horizontal component

ii) vertical component

i) $A_x = A\sin\theta = F\sin\theta$

$= 20 \times \sin 30^\circ$

$= 20 \times \frac{1}{2} = 10\,N$

ii) $A_y = A\cos\theta = F\cos\theta$

$= 20 \times \cos 30^\circ$

$= 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}\,N.$

Q.39. A force of 50 N is acting in a vertical direction, find its horizontal component?

$A_x = A\cos\theta = F\cos 90^\circ$

$= 50 \times 0 = 0$

$\because$ F $\perp$ horizontal direction.

$\therefore$ its horizontal component = 0.

Similarly, the vertical component of horizontal vector is also 0.

☆ Resolution of a vector in space (3-D)

(VS) Pg-15

- $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
- $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
- Directions cosines of $\vec{A}$

$\cos\alpha = \frac{A_x}{A}$

$\cos\beta = \frac{A_y}{A}$

$\cos\gamma = \frac{A_z}{A}$

- Relation between directional cosines:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Q-40 Find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$

$\sin^2\theta = 1 - \cos^2\theta.$

$\therefore \sin^2\alpha = 1 - \cos^2\alpha. \quad - (I)$

$\sin^2\beta = 1 - \cos^2\beta \quad - (II)$

$\sin^2\gamma = 1 - \cos^2\gamma \quad - (III)$

On adding (I), (II) and (III)

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1 - \cos^2\alpha + 1 - \cos^2\beta$
$+ 1 - \cos^2\gamma$
$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$
$= 3 - 1 = 2$

Q.41 If $\vec{A} = 6\hat{i} + 3\hat{j} + 2\hat{k}$, then find:

☆ i) x component of $\vec{A}$

ii) Projection of $\vec{A}$ on y-axis

iii) A_z

iv) Magnitude of $\vec{A}$

v) Unit vector parallel to $\vec{A}$

vi) Directional cosines of $\vec{A}$

vii) Angle of $\vec{A}$ from x-axis

viii) Angle of $\vec{A}$ from y-axis.

ix) Angle of $\vec{A}$ from z-axis.

i) $A_x = 6.$

ii) $A_y = 3.$

iii) $A_z = 2$

iv) $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{6^2 + 3^2 + 2^2}$
$= \sqrt{36 + 9 + 4}$
$= \sqrt{49} = 7$ unit

v) $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{6\hat{i} + 3\hat{j} + 2\hat{k}}{7}$

vi) $\cos\alpha = \dfrac{A_x}{A} = \dfrac{6}{7}$

$\cos \beta = \frac{A_y}{A} = \frac{3}{7}$

$\cos \gamma = \frac{A_z}{A} = \frac{2}{7}$

vii) $\cos \alpha = \frac{6}{7}$

$\therefore \alpha = \cos^{-1}\left(\frac{6}{7}\right)$

viii) $\cos \beta = \frac{3}{7}$

$\therefore \beta = \cos^{-1}\left(\frac{3}{7}\right)$

ix) $\cos \gamma = \frac{2}{7}$

$\therefore \gamma = \cos^{-1}\left(\frac{2}{7}\right)$

Q.42 Find the projection of $\vec{A} = 2\hat{i} + 3\hat{j}$ on z axis.

Projection of $\vec{A}$ on z-axis, $A_z = 0$.

$\therefore A_z$; coefficient of $\hat{k} = 0$.

Q.43. Given $\vec{A} = 0.2\hat{i} + 0.3\hat{j} + p\hat{k}$, if $\vec{A}$ is a unit vector, find the value of p.

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} = 1$ ($\because$ Its a unit vector)

$= \sqrt{(0.2)^2 + (0.3)^2 + p^2} = 1$

Squaring both sides.

$0.04 + 0.09 + p^2 = 1$

$p^2 = 1 - 0.13 = 0.87$

$\therefore p = \pm\sqrt{0.87}$

Q-44 Given $\vec{A} = \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$, check whether $\vec{A}$ is a unit vector or not

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$
$$= \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2}$$
$$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = \sqrt{\frac{3}{3}} = \sqrt{1} = 1$$
$$\sqrt{1} = 1$$

$\because$ magnitude $= 1$

$\therefore$ Its a unit vector.

Q.45 If $\vec{A} = 5\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} + \hat{k}$,
★★ then find:

i) $\vec{A} - \vec{B}$

ii) $|\vec{A} - \vec{B}|$

iii) $\vec{R}$

iv) $|\vec{R}|$

v) unit vector parallel to resultant of $\vec{A}$ and $\vec{B}$

vi) Angle of resultant of $\vec{A}$ and $\vec{B}$ from x-axis.

* vii) Find a vector $\vec{c}$ which on adding with the resultant of $\vec{A}$ and $\vec{B}$ gives a unit vector along x-axis.

i) $\vec{A} - \vec{B} = 5\hat{i} + 3\hat{j} + 3\hat{k} - (2\hat{i} + \hat{j} + \hat{k})$
$$= 5\hat{i} - 2\hat{i} + 3\hat{j} - \hat{j} + 3\hat{k} - \hat{k}$$
$$= 3\hat{i} + 2\hat{j} + 2\hat{k}$$

ii) $|\vec{A} - \vec{B}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
$$= \sqrt{3^2 + 2^2 + 2^2} = \sqrt{9+4+4} = \sqrt{17} \text{ unit}$$

iii) $\vec{R} = \vec{A} + \vec{B}$
$$= 5\hat{i} + 3\hat{j} + 3\hat{k} + 2\hat{i} + \hat{j} + \hat{k}$$
$$= 7\hat{i} + 4\hat{j} + 4\hat{k}$$

iv) $|\vec{R}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$= \sqrt{7^2 + 4^2 + 4^2}$$

$$= \sqrt{49 + 16 + 16} = \sqrt{81} = 9 \text{ unit}$$

v) $\hat{R} = \dfrac{\vec{R}}{|\vec{R}|} = \dfrac{7\hat{i} + 4\hat{j} + 4\hat{k}}{9}$

vi) $\cos\alpha = \dfrac{A_x}{A} = \dfrac{R_x}{R} = \dfrac{7}{9}$

$\therefore \alpha = \cos^{-1}\left(\dfrac{7}{9}\right)$

vii) $\vec{C} + \vec{R} = \hat{i}$

$\therefore \vec{C} = \hat{i} - \vec{R}$

$= \hat{i} - (7\hat{i} + 4\hat{j} + 4\hat{k})$

$= \hat{i} - 7\hat{i} - 4\hat{j} - 4\hat{k}$

$= -6\hat{i} - 4\hat{j} - 4\hat{k}$

Important point about resolution of vectors:

(VS) Pg - 18

- Maximum no. of ~~regular~~ rectangular components of any vector is space (3-D) is 3.
- Maximum no. of ~~regular~~ rectangular components of any vector in plane (2-D) is 2
- Maximum no. of component of any vector is infinity (∞).
- A vector does not depends upon orientation of axis but the components of a vector depends upon orientiation of axis.

$|\vec{A} + \vec{B}| = 2a \cos\left(\dfrac{\theta}{2}\right)$ and $|\vec{A} - \vec{B}| = 2a \sin\left(\dfrac{\theta}{2}\right)$

* (VS) Pg-19

A vector $\vec{A}$ multiplied by a real no λ, is also a vector.

Magnitude = λ time magnitude of $\vec{A}$, $\lambda|\vec{A}|$

direction = it depends upon positive and negative sign of λ.

- If λ is positive, $\vec{A}$ and $\vec{B}$ are in same dir^n
- If λ is negative, $\vec{A}$ and $\vec{B}$ are in opposite dir^n

Multiplication of vectors:

(VS) Pg.-19

Products of vectors

Scaler product	Vector Product
$\vec{A} \cdot \vec{B}$	$\vec{A} \times \vec{B}$
(dot product)	(cross product)

1 Dot Product or Scaler Product.

(VS) Pg-19

☆ • $\vec{A} \cdot \vec{B} = AB\cos\theta$

eg: Work = $\overrightarrow{\text{Force}} \cdot \overrightarrow{\text{Displacement}}$

$$W = \vec{F} \cdot \vec{S}$$

eg: Power = $\overrightarrow{\text{Force}} \cdot \overrightarrow{\text{Velocity}}$

$$P = \vec{F} \cdot \vec{V}$$

Important points of dot products.

(VS) Pg-20

- Dot product of two vector is always a scaler.

ii) $\vec{A} \cdot \vec{B} = AB \cos \theta$.

- If $\cos \theta = (+ve)$, $\theta < 90^\circ$, then dot product $= +ve$
- If $\cos \theta = (-ve)$, $90^\circ < \theta < 180^\circ$, then dot product $= (-ve)$

$\therefore$ Scalers can be negative.

iii) If $\vec{A}$ and $\vec{B}$ are parallel, then $\theta = 0^\circ$

$\cos \theta = \cos 0^\circ = 1$ (Max value of $\cos \theta$).

$$(\vec{A} \cdot \vec{B})_{Max} = AB$$

iv) If $\vec{A}$ and $\vec{B}$ are antiparallel, then $\theta = 180^\circ$

$\cos \theta = \cos 180^\circ = -1$ (Min value of $\cos \theta$).

$$(\vec{A} \cdot \vec{B})_{Min} = -AB.$$

** v) If $\vec{A} \perp \vec{B}$, then $\theta = 90^\circ$

$\cos \theta = \cos 90^\circ = 0$

Dot product of two perpendicular vectors are always 0.

OR

If the dot product of two non-zero vectors are 0, then, they are perpendicular to each other

vi) Dot product of two orthogonal unit vector is always 0

$\therefore \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0.$

vii) The value of self dot product is square of magnitude of that vector

$\therefore \vec{A} \cdot \vec{A} = A^2$

viii) The self dot product of any unit vector is always 1.

$\therefore \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = \hat{n}.\hat{n} = 1$

ix) Dot Product is commutative

x) Dot Product is distributive.

xi) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Q-46 Find the value of $\hat{i}\cdot(\hat{i}+\hat{j}+\hat{k})$

$\hat{i}.\hat{i} + \hat{i}.\hat{j} + \hat{i}.\hat{k}$ ($\because$ dot product is ~~commutative~~, distributive)

$1 + 0 + 0 = 1$

Q-47 Find the value of $(\hat{i}+\hat{j})\cdot(\hat{i}-\hat{j})$

$\hat{i}.\hat{i} - \hat{i}.\hat{j} + \hat{i}.\hat{j} - \hat{j}.\hat{j}$ [$\because$ dot product is distributive]

$1 - 0 + 0 - 1$

$1 - 1 = 0$

Q.48 Find angle between $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$

$\hat{i}.\hat{i} - \hat{i}.\hat{j} + \hat{i}.\hat{j} - \hat{j}.\hat{j}$ [$\because$ dot product is distributive]

$1 - 0 + 0 - 1$

$1 - 1 = 0$

$\cos\theta = 0 = \cos 90^\circ$

$\therefore \theta = 90^\circ$

$\because (\hat{i}+\hat{j}) \perp (\hat{i}-\hat{j})$

If two vectors are perpendicular to each other, their dot product is 0.

$\therefore$ angle between $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$ is 0.

Q.49 If $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$ then find the ~~angle~~ value of $\vec{A}\cdot\vec{B}$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$
$$= (3 \times 1) + (1 \times 2) + (2 \times 3)$$
$$= 3 + 2 + 6 = 11$$

Q. 50 If $\vec{F} = (2\hat{i} - \hat{j} + \hat{k})\,N$ and $\vec{s} = (3\hat{i} + \hat{j} - 2\hat{k})\,m$ then find the work done?

$$W = \vec{F} \cdot \vec{s}$$
$$\vec{F} \cdot \vec{s} = F_x S_x + F_y S_y + F_z S_z$$
$$= (2 \times 3) + (-1 \times 1) + [1 \times (-2)] \; J$$
$$= 6 + (-1) + (-2) \; J$$
$$= 6 - 1 - 2 = 6 - 3 = 3 \; J$$

Q. 51 If $\vec{F} = (2\hat{i} + 4\hat{j})\,N$ and $\vec{s} = (\hat{i} + 3\hat{j} + 5\hat{k})\,m$ then, find the work done?

$$W = \vec{F} \cdot \vec{s}$$
$$= (2 \times 1) + (4 \times 3) + (0 \times 5) \; J$$
$$= 2 + 12 + 0 = 14 \; J$$

Q. 52 If $\vec{F} = (3\hat{i} + 4\hat{k})\,N$ and $\vec{V} = (3\hat{j} + 2\hat{k})\,m/s$. then, find the power?

$$P = \vec{F} \cdot \vec{V}$$
$$= (3 \times 0) + (0 \times 3) + (4 \times 2) \text{ watt}$$
$$= 0 + 0 + 8 = 8 \text{ watt}$$

Q. 53. If $\vec{F} = (3\hat{i} - 4\hat{k})\,N$ and $\vec{s} = (4\hat{j})\,m$, then find the work done?

$$W = \vec{F} \cdot \vec{s}$$
$$= (3 \times 0) + (0 \times 4) + (-4 \times 0) \; J$$
$$= 0 + 0 - 0 = 0$$

Since, their dot product is 0.

$\therefore \vec{F} \perp \vec{s}$

Q.54 If $\vec{A} = \hat{i} + 4\hat{j} + c\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} + \hat{k}$, and $A \perp B$, then find value of c.

$\because A \perp B$

$\therefore \vec{A} \cdot \vec{B} = 0$

$(1 \times 2) + (4 \times 1) + (c \times 1) = 0.$

$2 + 4 + c = 0.$

$\therefore c = -6$

Q.55. If $\vec{A} = 2\hat{i} + 3\hat{j} + \alpha\hat{k}$ and $\vec{B} = \hat{j} - \hat{i} + \hat{k}$ and $A \perp B$, then find the value of α.

$\because A \perp B.$

$\therefore \vec{A} \cdot \vec{B} = 0.$

$[2 \times (-1)] + (3 \times 1) + (\alpha \times 1) = 0.$

$\because -2 + 3 + \alpha = 0.$

$\therefore \alpha = 2 - 3 = -1$

Q.56. The vector $\hat{i} + \hat{j}$ is $\perp$ to :

a. $2\hat{i} - 2\hat{k}$

b. $3\hat{i} + 3\hat{j}$

c. $3\hat{i} - 3\hat{j}$

d. None of the above.

Ans: c) $(\hat{i} + \hat{j}) \perp (3\hat{i} - 3\hat{j})$

$= (3 \times 1) + (-3 \times 1)$

$= 3 - 3 = 0.$

$\therefore$ They are perpendicular to each othe

Q.57 which of the following is $\perp$ to $2\hat{i} + \hat{j}$:

a $\hat{i} - 2\hat{j}$

b. $2\hat{i} - 4\hat{j}$

c. $5\hat{k}$

d. All of these

Ans: d) All of these

* Application of dot product:
(VS) Pg - 21

i) To find the angle between $\vec{A}$ and $\vec{B}$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

ii) To find the projection of $\vec{A}$ on $\vec{B}$

a) In scaler form:

$$A_x = A\cos\theta = A\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \frac{\vec{A}\cdot\vec{B}}{B} = \vec{A}\hat{B}$$

b) In vector form:

$$A_x = (A\cos\theta)\,\hat{B} = A\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)\hat{B} = \left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\hat{B}$$

$$= \left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\hat{B} = (\vec{A}\hat{B})\,\hat{B}$$

Q. 58 If $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{j} + \hat{k}$, then find angle between $\vec{A}$ and $\vec{B}$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

$$= \frac{\hat{i}\cdot\hat{j} + \hat{i}\cdot\hat{k} + \hat{j}\cdot\hat{j} + \hat{j}\cdot\hat{k}}{\sqrt{A_x^2 + A_y^2} \times \sqrt{B_x^2 + B_y^2}}$$

$$= \frac{0 + 0 + 1 + 0}{\sqrt{1+1} \times \sqrt{1+1}}$$

$$= \frac{1}{\sqrt{2}\times\sqrt{2}} = \frac{1}{2} = \cos 60^\circ$$

$$\therefore \theta = 60^\circ$$

Angle between $\vec{A}$ and $\vec{B}$ = 60°

Q.59 If $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i}$, then find :

i) Angle between $\vec{A}$ and $\vec{B}$

ii) Angle of $\vec{A}$ from x-axis

iii) Angle of $\vec{B}$ from y-axis.

i) $$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

$$= \frac{1}{\sqrt{3}\times 1} = \frac{1}{\sqrt{3}} = \cos\theta.$$

$$\therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

ii) $$\cos\alpha = \frac{A_x}{A} = \frac{1}{\sqrt{3}}.$$

$$\therefore \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

iii) $$\cos\beta = \frac{B_y}{B} = \frac{0}{1} = 0 = \cos 90^\circ$$

$$\therefore \beta = 90^\circ$$

Q.60. ☆☆ Find the angle between $\vec{A}$ and resultant of $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$.

The angle between $\vec{A}$ and $[(\vec{A}+\vec{B})+(\vec{A}-\vec{B})]$

= angle between $\vec{A}$ and $(2\vec{A}) = 0$.

$\because$ 2 is positive (+ve).

$\therefore$ both the vectors are in same direction.

i.e. $\vec{A}$ and $2\vec{A}$ are in same direction.

$\therefore \vec{A} \parallel 2\vec{A}$

$\therefore$ angle between $\vec{A}$ and $2\vec{A} = 0^\circ$.

2. Cross product or Vector Product.

(VS) Pg-21

- $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

- cross product of any two vector is always a vector quantity.
  magnitude : AB $\sin\theta$ and
  direction: perpendicular to the plane of both the vectors.(determined by using right-hand thumb rule)

Right hand thumb rule:
(VS) Pg-22

- $\because$ the direction of $(\vec{A}\times\vec{B})$ and $(\vec{B}\times\vec{A})$ are opposite.
  $\therefore$ angle between $(\vec{A}\times\vec{B})$ and $(\vec{B}\times\vec{A})$ is 180°

- $\because$ the cross product is perpendicular to plane of both the vectors.
  $\therefore$ cross product is individually perpendicular to both the given vector. i.e.
  $$(\vec{A}\times\vec{B}) \perp \vec{A} = (\vec{A}\times\vec{B})\cdot\vec{A} = 0$$
  $$(\vec{A}\times\vec{B}) \perp \vec{B} = (\vec{A}\times\vec{B})\cdot\vec{B} = 0$$

Examples of cross product:
(VS) Pg-22

- $\vec{\tau} = \vec{r}\times\vec{F}$
  $\overrightarrow{\text{torque}} = \overrightarrow{\text{radius / position}} \times \overrightarrow{\text{Force}}$

- $\vec{J} = \vec{r}\times\vec{p}$
  $\overrightarrow{\text{Angular Momentum}} = \overrightarrow{\text{radius/position}} \times \overrightarrow{\text{Linear Momentum}}$

- $\vec{v} = \vec{\omega} \times \vec{r}$

$\overrightarrow{\text{Linear velocity}} = \overrightarrow{\text{Angular velocity}} \times \overrightarrow{\text{radius}}$

- $\vec{a} = \vec{\alpha} \times \vec{r}$

$\overrightarrow{\text{Linear acceleration}} = \overrightarrow{\text{Angular acceleration}} \times \overrightarrow{\text{radius}} / \overrightarrow{\text{position}}$

Q.61. If $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = \hat{i} + \hat{j}$, then find

i) Projection of $\vec{A}$ on x-axis.

ii) Projection of $\vec{B}$ on z-axis.

iii) Projection of $\vec{A}$ on $\vec{B}$ in scaler form.

iv) Projection of $\vec{A}$ on $\vec{B}$ in vector form

v) Projection of $\vec{B}$ on $\vec{A}$ in scaler form.

vi) Projection of $\vec{B}$ on $\vec{A}$ in vector form.

i) $A_x = 4$

ii) $B_z = 0$

iii) Projection of $\vec{A}$ on $\vec{B}$ in scaler form:

$$\frac{\vec{A}\cdot\vec{B}}{B} = \frac{(4\hat{i}-3\hat{j}).(\hat{i}+\hat{j})}{\sqrt{B_x^2+B_y^2}}$$

$$= \frac{4-3}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

iv) Projection of $\vec{A}$ on $\vec{B}$ in vector in vector form:

$$\left(\frac{\vec{A}\times\vec{B}}{B}\right)\hat{B} = \frac{4-3}{\sqrt{1+1}}\hat{B} = \frac{1}{\sqrt{2}} \times \frac{\vec{B}}{B}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\hat{i}+\hat{j}}{\sqrt{2}} = \frac{\hat{i}+\hat{j}}{2}$$

v) Projection of $\vec{B}$ on $\vec{A}$ in scaler form:

$$\frac{\vec{B}\cdot\vec{A}}{A} = \frac{4-3}{\sqrt{4^2+(-3)^2}} = \frac{1}{\sqrt{16+9}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$$

vi) Projection of $\vec{B}$ on $\vec{A}$ in vector form:

$$\left(\frac{\vec{B}\times\vec{A}}{A}\right)\hat{A} = \frac{4-3}{\sqrt{25}}\hat{A} = \frac{1}{5} \times \frac{\vec{A}}{A} = \frac{4\hat{i}-3\hat{j}}{5\times5} = \frac{4\hat{i}-3\hat{j}}{25}$$

Q-62 Find the value of $(\vec{A}+\vec{B})\cdot(\vec{A}\times\vec{B})$

$$\vec{A}\cdot(\vec{A}\times\vec{B}) + \vec{B}\cdot(\vec{A}\times\vec{B})$$

$$0 + 0 = 0$$

$$\left(\because (\vec{A}\times\vec{B}) \perp \vec{A} \text{ and } (\vec{A}\times\vec{B}) \perp \vec{B} \quad \therefore \vec{A}\cdot(\vec{A}\times\vec{B}) = 0 \text{ and } \vec{B}\cdot(\vec{A}\times\vec{B}) = 0 \right)$$

OR

$$\vec{A} + \vec{B} = \vec{R}$$

$\because$ $\vec{R}$ is located in the same plane but $(\vec{A}\times\vec{B})$ is perpendicular to their plane

$\therefore$ $(\vec{A}+\vec{B}) \perp (\vec{A}\times\vec{B})$

Hence, their dot product is 0 (by (v) properties of dot product)

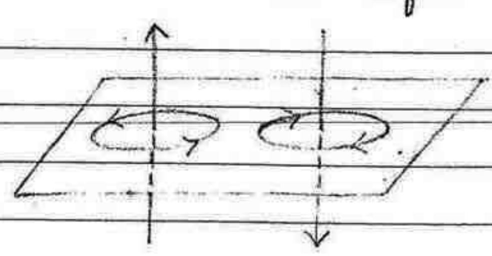
x) Axial Vector:

(VS) Pg - 22

- direction is along the axis of rotation
- Used in rotational motion
- Direction is determined by right hand thumb rule

Right hand thumb rule:

Curl the fingers of right hand along the direction of motion of particle. Now, the right hand thumb rule will indicate the direction of Axial vector



eg: Small Angular displacement ($\vec{d\theta}$)
Angular velocity ($\vec{\omega}$)
Angular acceleration ($\vec{\alpha}$)
Angular momentum ($\vec{J}$ or $\vec{L}$)
Torque ($\vec{\tau}$)

Properties of cross Product :
(VS) Pg-22

- If $\vec{A}$ and $\vec{B}$ are parallel, then $\theta = 0°$
$\vec{A} \times \vec{B} = AB \sin 0° \hat{n} = \vec{0}$, and.
If $\vec{A}$ and $\vec{B}$ are antiparallel, then $\theta = 180°$
$\vec{A} \times \vec{B} = AB \sin 180° \hat{n} = \vec{0}$

* Vectors lying in the same line are known as collinear vectors.
* The cross product of colinear vector is always $\vec{0}$.

- If $|\vec{A} \times \vec{B}| = AB \sin\theta$
$\therefore$ If $\sin\theta$ is maximum, $|\vec{A} \times \vec{B}|$ is also maximum
$\therefore$ $|\vec{A} \times \vec{B}|_{max} = AB \cancel{\sin\theta}$ (at $\theta = 90°$)

- The value of self cross product is always $\vec{0}$.
$\therefore \vec{A} \times \vec{A} = \vec{0}$

- The value of self cross product of ~~zero~~ unit vector is always $\vec{0}$
$\therefore \hat{n} \times \hat{n} = \vec{0}$
$\hat{i} \times \hat{i} = \vec{0}$

- ~~to~~ Cross product of two orthogonal unit vectors:

$$\hat{i} \times \hat{j} = \hat{k} \quad \text{and} \quad \hat{j} \times \hat{i} = -\hat{k}$$

- $\because \vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

  $\therefore$ Cross product is not commutative.

* $|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = AB\sin\theta$

and.

$$(\vec{A} \times \vec{B}) = -(\vec{B} \times \vec{A})$$

- cross product is distributive

Q. 63 Find the value of $\hat{i} \times (-\hat{j})$.

$$= \hat{i} \times (-\hat{j}) = -\hat{k}$$

Q. 64 Find the value of $(-\hat{j}) \times (-\hat{k})$

$$= -\hat{j} \times (-\hat{k}) = \hat{i}$$

Q. 65 Find the value of $\hat{i} \times (\hat{i} + 2\hat{j} + \hat{k})$

$$= \hat{i} \times \hat{i} + \hat{i} \times 2\hat{j} - \hat{i} \times \hat{k}$$

$$= \vec{0} + 2\hat{k} + \hat{j}$$

$$= -\hat{j} + 2\hat{k}$$

Q. 66 Find the value of $\hat{i} \cdot (\hat{j} \times \hat{k})$

$$= \hat{i} \cdot \hat{i} = 1$$

Q. 67 Find the value of $\hat{j} \cdot (\hat{i} \times \hat{k})$

$$= \hat{j} \cdot (-\hat{j}) = -1$$

Q. 68 Find the value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$

$= \hat{i}\cdot\hat{i} + \hat{j}\cdot\hat{j} + \hat{k}\cdot\hat{k}$

$= 1+1+1 = 3$

- The cross product of $\vec{A}$ and $\vec{B}$ is determined by using determiant method.

Determinant Methode:

(VS) Pg-24

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\therefore \vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j}$$
$$(A_xB_y - A_yB_x)\hat{k}$$

Q.69 If $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} + 2\hat{k}$

using determinant methode:

$\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A}) = \vec{A}\times\vec{B}$

$$\begin{array}{cccc} & \hat{i} & \hat{j} & \hat{k} \\ \vec{A} & 2 & 1 & 1 \\ \vec{B} & 1 & 2 & 2 \end{array}$$

$\because \vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$

$= (2-2)\hat{i} - (4-1)\hat{j} + (4-1)\hat{k}$

$= 0\hat{i} - 3\hat{j} + 3\hat{k}$

$\therefore \vec{A}\times\vec{B} = -3\hat{j} + 3\hat{k}$

Q.70. If $\vec{F} = (2\hat{i} + 2\hat{j})$ N and $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k})$ m.

then find $\vec{\tau}$

$\because \vec{\tau} = \vec{r}\times\vec{F}$

Using determinant methode:

$$\begin{array}{cccc} & \hat{i} & \hat{j} & \hat{k} \\ \vec{r} & 1 & 2 & 1 \\ \vec{F} & 2 & 2 & 0 \end{array}$$

$\because \vec{r} \times \vec{F} = [(2\times0)-(2\times1)]\ \hat{i} - [(0\times1) - (2\times1)]\hat{j} + [(2\times1) - (2\times2)]\ \hat{k}$

$= (0-2)\ \hat{i} - (0-2)\ \hat{j} + (2-4)\ \hat{k}$

$= (-2\hat{i} + 2\hat{j} - 2\hat{k})$ units.

Q. 71 If $\vec{\omega} = (\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{r}\ (\hat{i} + \hat{j} + \hat{k})$ then find $\vec{v}$.

$\because \vec{v} = \vec{\omega} \times \vec{r}$

Using determinant method,

$$\begin{array}{cccc} & \hat{i} & \hat{j} & \hat{k} \\ \vec{\omega} & 1 & -1 & 2 \\ \vec{r} & 1 & 1 & 1 \end{array}$$

$\because \vec{\omega} \times \vec{r} = (-1\times1) - (2\times1)\ \hat{i} - (1\times1) - (2\times1)\ \hat{j} + (1\times1) - (1\times(-1))\ \hat{k}$

$= (-1-2)\ \hat{i} - (1-2)\ \hat{j} + (1+1)\ \hat{k}$

$= (-3\hat{i} + \hat{j} + 2\hat{k})$ units.

Application of cross product:

(VS) Pg - 24

i) To find the area using cross product

a) Area of parallelogram : $|\vec{A} \times \vec{B}|$

b) Area of triangle : $\frac{|\vec{A} \times \vec{B}|}{2}$

c) Area of a parallelogram, when $\vec{A}$ and $\vec{B}$ represents diagonals : $\frac{|\vec{A} \times \vec{B}|}{2}$

ii) For finding a unit vector which is perpendicular to $\vec{A}$ as well as $\vec{B}$.

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

Q.72 If $\vec{A} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 3\hat{i} + \hat{j} + \hat{k}$, represent two diagonals of parallelogram, then find its area.

Area of llgm $= \frac{1}{2} |\vec{A} \times \vec{B}|$

$$\therefore \vec{A} \times \vec{B} = \begin{array}{cccc} & \hat{i} & \hat{j} & \hat{k} \\ \vec{A} & 1 & 1 & 2 \\ \vec{B} & 3 & 1 & 1 \end{array}$$

$$= (1-2)\hat{i} - (1-6)\hat{j} + (1-3)\hat{k}$$

$$= -\hat{i} + 5\hat{j} - 2\hat{k}$$

$$|\vec{A} \times \vec{B}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= \sqrt{1^2 + 5^2 + (-2)^2}$$

$$= \sqrt{1 + 25 + 4} = \sqrt{30}.$$

$\therefore$ Area of llgm $= \frac{1}{2} \times \sqrt{30} = \frac{\sqrt{30}}{2}$ sq units.

Q.73 If $\vec{A} = \hat{i} + 2\hat{j}$ and $\vec{B} = 3\hat{i} + \hat{j}$, If $\vec{A}$ and $\vec{B}$ are two adjacent sides of the parallelogram then find its area.

$$\therefore \vec{A} \times \vec{B} = \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 3 & 1 & 0. \end{array}$$

$$\therefore \vec{A} \times \vec{B} = (0-0)\hat{i} - (0-0)\hat{j} + (1-6)\hat{k}$$

$$= -5\hat{k}$$

$$|\vec{A} \times \vec{B}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= \sqrt{0 + 0 + (-5)^2}$$

$$= \sqrt{25} = 5.$$

$\therefore$ Area of $\parallel$gm $= |\vec{A} \times \vec{B}| = 5$ sq. units.

Q.74 In the above que, If $\vec{A}$ and $\vec{B}$ are two sides of a triangle, then find its area?

$\because$ Area of triangle $= \dfrac{|\vec{A} \times \vec{B}|}{2}$

$$\therefore \frac{|\vec{A} \times \vec{B}|}{2} = \frac{5}{2} \text{ sq. units.}$$

Q.75 If $\vec{A} = 3\hat{i} + \hat{j}$ and $\vec{B} = \hat{i} + \hat{k}$, then find a unit vector $\perp$ $\vec{A}$ and $\vec{B}$.

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$\because \vec{A} \times \vec{B} = \begin{array}{cccc} & \hat{i} & \hat{j} & \hat{k} \\ \vec{A} & 3 & 1 & 0 \\ \vec{B} & 1 & 0 & 1 \end{array}$$

$$\therefore \vec{A} \times \vec{B} = (1-0)\hat{i} - (3-0)\hat{j} + (0-1)\hat{k}$$

$$= \hat{i} - 3\hat{j} - \hat{k}$$

$$|\vec{A} \times \vec{B}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= \sqrt{1^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{1+9+1} = \sqrt{11}$$

$$\therefore \hat{n} = \frac{\hat{i} - 3\hat{j} - \hat{k}}{\sqrt{11}}$$

Q.76 If $\vec{A}$ is along west and $\vec{B}$ is along north then the direction of $\vec{A} \times \vec{B}$ is

$$\vec{A} = -\hat{i}$$

$$\vec{B} = \hat{j}$$

$$\therefore \vec{A} \times \vec{B} = -\hat{i} \times \hat{j} = -\hat{k}$$

Hence, the direction is downwards/Inwards

Q.77. If $\vec{P}$ = south and $\vec{Q}$ = upwards then, find the direction of $\vec{P} \times \vec{Q}$

$\vec{P} = -\hat{j}$ and $\vec{Q} = \hat{k}$

$\therefore \vec{P} \times \vec{Q} = -\hat{j} \times \hat{k} = -\hat{i}$

Hence, the direction is along ~~east~~ west

Q.78. If $\vec{A}$ = inwards and $\vec{B}$ = west, then find $\vec{B} \times \vec{A}$.

$\vec{A} = -\hat{k}$ and $\vec{B} = -\hat{i}$

$\therefore \vec{B} \times \vec{A} = -\hat{i} \times (-\hat{k}) = -\hat{j}$

Hence, the direction is along south.

Q.79 If $\vec{A}$ is along west and $\vec{B}$ is along east then find the direction of $\vec{A} \times \vec{B}$.

$\vec{A} = -\hat{i}$ and $\vec{B} = \hat{i}$

$\therefore \vec{A} \times \vec{B} = -\hat{i} \times \hat{i} = \vec{0}$

Hence, its direction is arbitary.

Q.80. If $\vec{A} \times \vec{B} = \vec{0}$ and $\vec{A} \times \vec{C} = \vec{0}$ then find the value of ~~$\vec{A}$~~ $\vec{B} \times \vec{C}$.

$\because \vec{A} \times \vec{B} = \vec{0} \quad \therefore \vec{A} \parallel \vec{B}$ —(i)

$\because \vec{A} \times \vec{C} = \vec{0} \quad \therefore \vec{A} \parallel \vec{C}$ —(ii)

From eq (i) and (ii)

$\vec{A} \parallel \vec{B} \parallel \vec{C}$

$\therefore \vec{B} \parallel \vec{C}$

$\because \vec{B} \parallel \vec{C} \quad \therefore \vec{B} \times \vec{C} = \vec{0}$

We can prove this by either parallel or anti-parallel.

Q.81. If $|\vec{A} \times \vec{B}| = \sqrt{3}\, \vec{A} \cdot \vec{B}$, then find the angle between $\vec{A}$ and $\vec{B}$ i.e θ and $|\vec{A} + \vec{B}|$ or R or $|\vec{R}|$

$$|\vec{A} \times \vec{B}| = \sqrt{3}\ \vec{A} \cdot \vec{B}$$

$$= AB \sin\theta = \sqrt{3}\ AB \cos\theta.$$

$$\therefore \sin\theta = \sqrt{3} \cos\theta.$$

$$\frac{\sin\theta}{\cos\theta} = \sqrt{3}.$$

$$\therefore \tan\theta = \sqrt{3} = \tan 60^\circ$$

$$\therefore \theta = 60^\circ$$

and

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB\cos 60^\circ}$$

$$= \sqrt{A^2 + B^2 + 2AB \times \frac{1}{2}}$$

$$= \sqrt{A^2 + B^2 + AB}.$$