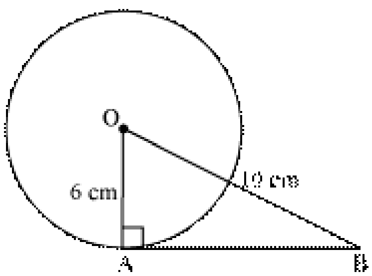
Circles

- **Concept of tangent at any point of the circle**

Theorem: The tangent at any point on a circle is perpendicular to the radius through the point of contact.

Example:
A tangent AB at a point A of a circle of radius 6 cm meets a line through the centre O at the point B, such that OB = 10 cm. Find the length of AB.

Solution:



It is known that the tangent at any point on a circle is perpendicular to the radius through the point of contact.
$OA \perp AB$

By applying Pythagoras theorem in right triangle OAB, we obtain
$OA^2 + AB^2 = OB^2$
$\Rightarrow 6^2 + AB^2 = 10^2$
$\Rightarrow AB^2 = (100 - 36)\ cm^2$
$\Rightarrow AB^2 = 64\ cm^2$
$\Rightarrow AB=64\ cm2=8\ cm$

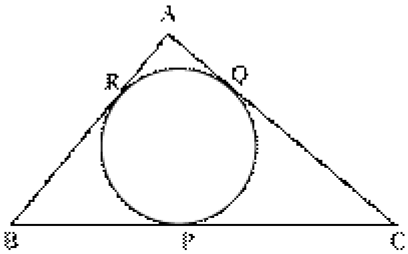
No tangent can be drawn to a circle passing through a point lying inside the circle.
One and only one tangent can be drawn to a circle passing through a point lying on the circle.
Exactly two tangents can be drawn to a circle through a point lying outside the circle.

- **Tangent drawn from an external point to a circle**

Length of the tangent: The length of the segment of the tangent from an external point P to the point of contact with the circle is called the length of the tangent from the point P to the circle.

Theorem: The lengths of tangents drawn from an external point to a circle are equal.

Example:
In the given figure, a circle is inscribed in ΔABC touching the points, P, Q, and R.



If AB = 7 cm, BC = 9 cm, CA = 8 cm, then find the measures of AR, AQ, BR, BP, CP, and CQ.

Solution:
It is known that the lengths of tangents drawn from an external point to a circle are equal.
AR = AQ = a (say)
BR = BP = b (say)
CP = CQ = c (say)

AB + BC + CA = (7 + 9 + 8) cm = 24 cm

$\Rightarrow (a + b) + (b + c) + (c + a) = 24$ cm
$\Rightarrow 2(a + b + c) = 24$ cm
$\Rightarrow a + b + c = 12$ cm

AB = 7 cm
$\Rightarrow a + b = 7$ cm
$\therefore c + 7$ cm = 12 cm
$\Rightarrow c = (12 - 7)$ cm = 5 cm

BC = 9 cm
$\Rightarrow b + c = 9$ cm
$\Rightarrow b = 9 - c = (9 - 5)$ cm = 4 cm

$a + b + c = 12$ cm
$\therefore$ 9 cm + a = 12 cm
$\Rightarrow a = (12 - 9)$ cm = 3 cm

Hence, AR = AQ = 3 cm,
BR = BP = 4 cm,

CP = CQ = 5 cm.

Results: If two tangents are drawn to a circle from an external point, then

1.
1.
2. 1. they subtend equal angles at the centre.
3. 2. they are equally inclined to the segment, joining the centre to that point.