ALL INDIA TEST SERIES

TEST - 1

JEE (Advanced)

Time Allotted: 3 Hours **Maximum Marks: 198**

General Instructions:

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-C.**

Section-A (01 – 06, 19 – 24, 37– 42) contains **18 multiple** choice questions which have **ONLY ONE CORRECT ANSWER**. Each question carries **+3 marks** for correct answer and **–1 mark** for wrong answer.

Section-A (07 – 12, 25 – 30, 43 – 48) *this section contains* ***18 multiple*** *choice questions.*
Each question has FOUR options. ***ONE OR MORE THAN ONE*** *of these four option(s) is (are) correct answer(s).*
For each question, choose the option(s) corresponding to (all) the correct answer(s)
Answer to each question will be evaluated according to the following marking scheme:

Full Marks **: +4** If only (all) the correct option(s) is (are) chosen:
Partial Marks **: +3** If all the four options are correct but ONLY three options are chosen;
Partial Marks **: +2** If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks **: +1** If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks **: 0** If none of the options is chosen (i. e. the question is unanswered);
Negative Marks **: –2** In all other cases.

Section-C (13 – 18, 31– 36, 49 – 54) *contains* ***18 Numerical*** *answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer and* ***0 marks*** *for wrong answer.*

Physics PART – I

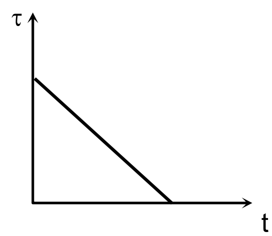
SECTION – A
(One Options Correct Type)

This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. A mass of 3M moving at a speed v collides with a mass of M moving directly towards it, also with a speed v. If the collision is completely elastic, the total kinetic energy after the collision is K_e. If the masses stick together, the total kinetic energy after the collision is K_s. What is the ratio $\frac{K_e}{K_s}$?

(A) $\frac{1}{4}$
(B) 4
(C) $\frac{1}{2}$
(D) 2

2. A steel ball bearing bounces vertically on a steel plate. If the speed of the ball just before a bounce is v_i, the speed of the ball immediately afterwards is $v_f = \alpha v_i$ with α < 1. Which one of the following graphs best shows the time between successive bounces (τ) as a function of time (t)?

(A)


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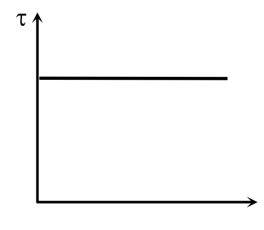

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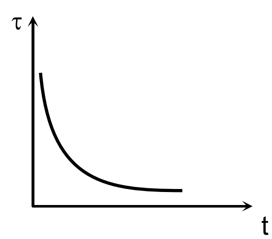

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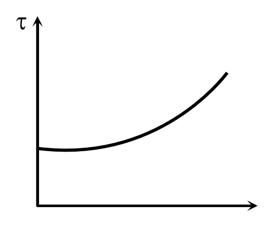


3. The power output from a certain experimental car design to be shaped like a cube is proportional to the mass m of the car. The force of air friction is proportional to Av^2, where v is the speed of the car and A is the cross sectional area. On a level surface the car has a maximum speed v_{max}. Assuming that all versions of this design have the same density, then which of the following is true?

(A) $v_{max} \propto m^{2/3}$
(B) $v_{max} \propto m^{1/7}$
(C) $v_{max} \propto m^{1/9}$
(D) $v_{max} \propto m^{3/4}$

4. Two particles with mass m_1 and m_2 are connected by a mass less rigid rod of length L and placed on a horizontal frictionless table. At time t = 0, the first mass receives an impulse perpendicular to the rod, giving it speed v. At this moment, the second mass is at rest. The minimum time after which the second mass will again come to rest is

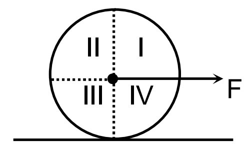
(A) $t = \dfrac{2\pi m_1 L}{(m_1 + m_2)v}$

(B) $t = \dfrac{2\pi m_1 m_2 L}{(m_1 + m_2)^2 v}$

(C) $t = \dfrac{\pi(m_1 + m_2)L}{m_2 v}$

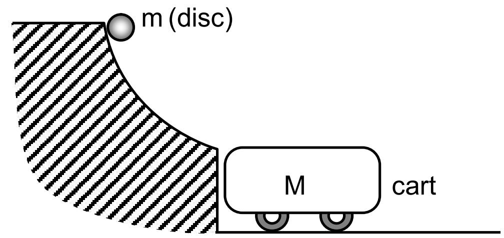
(D) $t = \dfrac{2\pi L}{v}$

5. A uniform circular disc is being pulled by a force F through a string attached to its centre of mass. Assume that the disc is rolling without slipping. At a certain instant of time, in which region of the disc (if any) is there a point with zero acceleration?



(A) Region II
(B) Region III
(C) Region IV
(D) All points on the disc have non zero acceleration.

6. A uniform circular disc of mass m = 12 kg slides down along a smooth frictionless hill, which ends in a horizontal plane without break. The disc is released from rest at a height of h = 1.25 m with zero initial velocity and zero angular velocity and rides on the top of a cart of mass M = 6kg, which can move on a frictionless surface. The coefficient of kinetic friction between the cart and the disc is μ = 0.4. Find the minimum length of the cart so that the disc begins to roll without slipping before loosing contact with the cart.



(A) $\frac{7}{8}$m

(B) $\frac{7}{4}$m

(C) $\frac{21}{4}$m

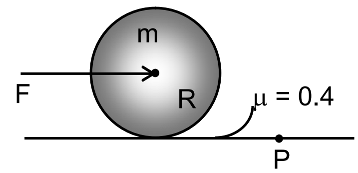
(D) $\frac{3}{8}$m

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

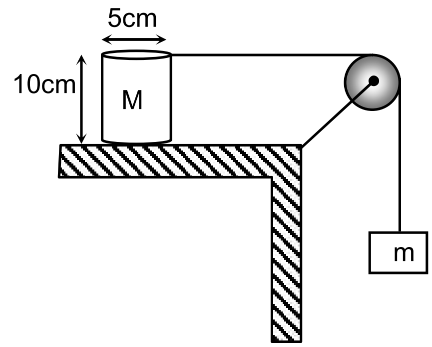
7. A uniform circular disc of mass m = 10 kg and radius R = 10 cm is placed on a rough horizontal surface. A horizontal time dependent force F = 10t N starts acting on the centre of the disc from t = 0 as shown in the figure. The coefficient of friction between the disc and the surface is 0.4. Choose the correct option(s). (Take g = 10 m/s^2)

A uniform disc



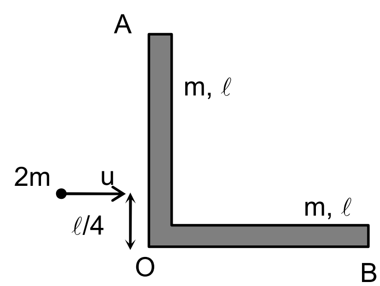
(A) The disc is performing pure rolling at t = 6 sec.
(B) Friction force acting on the disc at t = 6 sec is 40 N
(C) Friction force acting on the disc at t = 20 sec is 40 N
(D) Angular momentum of the disc about point P is $L = t^2$ kg-m²/sec, where t is time

8. A uniform cylinder of height 10 cm, diameter 5 cm and mass M is placed on a horizontal smooth table. One end of a string passing over an ideal pulley is connected to top point of the cylinder and the other end is connected to a block of mass m as shown in the figure. The system is released from rest from the situation shown. Then



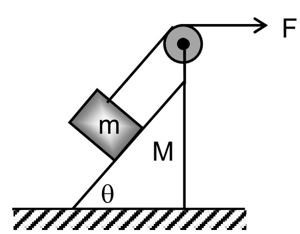
(A) If $m = \frac{M}{3}$ then $|\vec{a}_m| = |\vec{a}_M|$
(B) If m = M then $\vec{a}_m = \vec{a}_M$
(C) If m = 3M then $|\vec{a}_m| = |\vec{a}_M|$
(D) If m = M then the cylinder is at the verge of toppling

9. An L-shaped body is made of two rigid uniform rod each of mass m and length ℓ. The two rods are welded together at 90° at ends. The body is kept horizontally stationary and movable on a smooth horizontal surface. A particle of mass 2m strikes the body normally with horizontal velocity u at a perpendicular distance of $\ell/4$ from point O. The coefficient of restitution for the collision is e = 1/2.



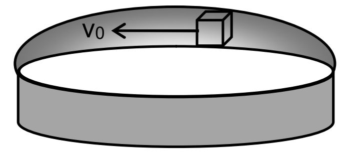
(A) Angular impulse acting on the body (AOB) about its centre of mass is $\frac{3mu\ell}{8}$
(B) Angular velocity of the body (AOB) after collision is $\frac{9u}{16\ell}$
(C) The loss in kinetic energy of the (particle + rod) system during collision is $\frac{3}{8}mu^2$
(D) After the collision, the particle keeps on moving in the direction of velocity before the collision.

10. A wedge of mass M is placed on a horizontal smooth surface. A block of mass m is placed on the smooth wedge and it is pulled by a contact force F with the help of a string passing over an ideal pulley. m is not sliding on the wedge and both m and M are moving with the same acceleration. (Given M > $\sqrt{2}$m)



(A) Pseudo force on the m as seen from M is $\frac{m^2 g \sin\theta}{M + m - m\cos\theta}$
(B) Pseudo force on M as seen from m is $\frac{MF}{M+m}$
(C) Pseudo force on M as seen from m is zero.
(D) The normal force between M and m for $\theta = 45^\circ$ is greater than $\frac{mF}{M+m}$.

11. A block of mass m moves on a horizontal circle against the inner walls of a fixed cylindrical drum of inner radius R. The floor of the drum on which the block moves is smooth but the coefficient of friction between the wall and the block is μ. The block is given an initial velocity v_0 along the tangent of the drum as shown in the figure. Then choose the correct option(s).



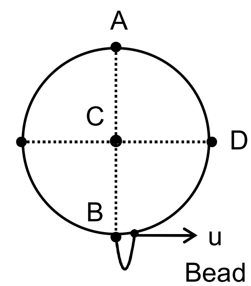
(A) The time at which velocity of the block reduces to $\frac{v_0}{2}$ is $\frac{R}{\mu v_0}$

(B) The magnitude of average power developed by the resultant forces acting on the block from $v = v_0$ to $v = \frac{v_0}{2}$ is $\frac{\mu m v_0^3}{8R}$

(C) The magnitude of average power developed by the resultant forces acting on the block from $v = v_0$ to $v = \frac{v_0}{2}$ is $\frac{3\mu m v_0^3}{8R}$

(D) The instantaneous power developed by the friction force when $v = \frac{v_0}{2}$ is $-\frac{\mu m v_0^3}{8R}$

12. A bead of mass m is threaded by a light elastic string of natural length L. Initially the bead is very close to the bottommost point B of a fixed vertical ring of radius L and the string is slacked initially. The other end of the string is fixed to B and the bead can slide along the frictionless ring. The elastic constant of the string is $\frac{4mg}{L}$. The bead is projected from B with speed u. Mark correct option(s).



(A) The smallest value of $u = u_0$ for which the bead will be able to complete the circular path is $\sqrt{8gL}$

(B) If bead is projected with $2u_0$, then normal force on the bead at topmost point is 19mg and it acts radially inward.

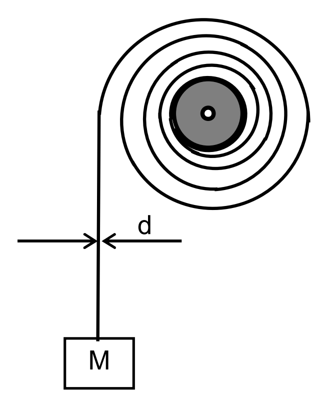
(C) If bead is projected with $2u_0$, then normal force on the bead at topmost point is 19 mg and it acts radially outward.

(D) The tangential acceleration of the bead when it passes through point D is $4g(\sqrt{2}-1)$

SECTION – C
(Numerical Answer Type)

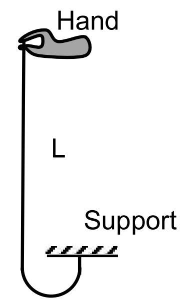
This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

13. A mass M = 1 kg attached to the end of a small flexible rope of diameter d = 1 cm is raised vertically by winding the rope on a reel. If the reel is turned uniformly at the angular velocity of 4π rad/sec, what will be the tension in the rope in newton? Neglect inertia of the rope and slight lateral motion of the suspended mass. (Take g = 10 m/s^2 and π = 3.14)



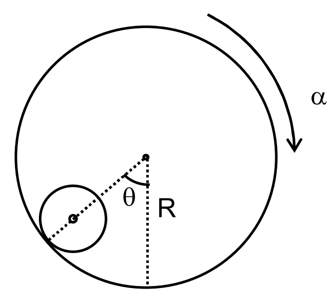
14. Water flows along a horizontal pipe with a curve having a radius of R = $\frac{250}{9}$ m. Find the lateral pressure (in N/m^2) of the water (assuming to be uniform in the diameter of the pipe). The pipe diameter is d = 20 cm and m = $\sqrt{\pi} \times 10^5$ kg of water pass through the pipe in t = 1 hour at a steady rate.

15. A rope with mass M = 2.3 kg and length L = 10 m is held in the position shown with one end attached to support. Assume that only a negligible length of the rope hangs below the support. The rope is released at t = 0. Find the force (in newton) that the support applies to the rope at t = 1 sec. (Take g = 10 m/s^2)



16. A brick is thrown (from ground level) at an angle θ with respect to horizontal with a speed v = 10 m/s. Assume that the larger face of the brick remains parallel to the ground at all times and there is no deformation in the ground or the brick when the brick hits the ground. After hitting the ground, the brick does not rebound. If the coefficient of friction between the brick and the ground is μ = 0.4. Angle θ is chosen so that the brick travels the maximum total horizontal distance before finally coming to rest. Find the maximum horizontal distance (in meter) covered by the brick before coming to rest. (Take g = 10 m/s^2)

17. A uniform sphere of mass m = 2kg rolls without slipping on the inside of cylinder of radius R = 2m. The cylinder spins around its axis (which points horizontally) with angular acceleration α. What should α (in rad/s^2) be if it is desired that the centre of the sphere to remain motionless at an angle θ = 30° from the vertical (Take g = 10 m/s^2)



18. The motion of a point like mass can be split into two parts. In the first part its average speed is v_1 = 90 km/hr and in the second its average speed is v_2 = 40 km/hr. For the whole motion the average speed is the geometric mean of the speed v_1 and v_2 that is v_3 = 60 km/hr Then the ratio of the covered distance in the first part and the second part is

Chemistry PART – II

SECTION – A
(One Options Correct Type)

This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

19. If an orbital is represented by $\psi = \frac{2}{3}\left(\frac{1}{3a_0}\right)^{3/2}(\sigma-1)\left(\sigma^2-8\sigma+12\right)\sigma e^{-\sigma/2}\cos\theta$ belong to which orbital
(A) $6d_{x^2-y^2}$
(B) $5p_z$
(C) $5p_y$
(D) $6d_{z^2}$

20. For an elementry reaction
$mA \longrightarrow \text{Product}$, the graph is plotted between $\log_{10}\left[-\frac{d[A]}{dt}\right]$ vs $\log_{10}[A]$. It gives straight line with intercept equal to 0.6 and showing an angle 45° with origin, then
(A) rate constant = 3.98 time^{-1} and m = 1
(B) rate constant = 3.98 mole L^{-1} time^{-1} and m = 1
(C) rate constant = 1.99 mole L^{-1} time^{-1} and m = 1
(D) rate constant = 1.99 mole L^{-1} time^{-1} and m = 2

21. In manufacture of Na_2CO_3 from solvay process the raw material used is
(A) NaOH
(B) Na_2SO_4
(C) NaCl
(D) $NaHCO_3$

22. Which of the following is the correct statement?
(A) $T\ell^{+3}$ and Al^{+3} act as an oxidizing agent in aqueous solution
(B) In $T\ell I_3$, $T\ell$ oxidization state is (+3)
(C) Hydrogen bonding in H_3BO_3 gives a layered structure
(D) $B(OC_2H_5)_3$ burns with blue edge flame

23. The correct order second ionization potential of C, N, O and F is
(A) C > N > O > F
(B) O > N > F > C
(C) O > F > N > C
(D) F > O > N > C

24. In which of the following molecule/ions are all the bond angles not equal
(A) SiF_4
(B) ICl_4^-
(C) SF_4
(D) PCl_4^+

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

25. Choose the correct statement(s)
    (A) $BeCO_3$ is kept in the atmosphere of CO_2 since it is thermally unstable.
    (B) Be is dissolved in alkali forming $\left[Be(OH)_4\right]^{-2}$.
    (C) BeF_2 form complex ion with NaF in which Be goes with cation.
    (D) BeF_2 form complex ion with NaF in which Be goes with anion.

26. The orbital angular momentum is $\sqrt{3}\frac{h}{\pi}$, which of the following should not be the permissible value of orbit angular momentum of this electron revolving in Bohr orbit?
    (A) $\frac{h}{2\pi}$
    (B) $\frac{h}{\pi}$
    (C) $3\times\frac{h}{2\pi}$
    (D) $\frac{2h}{\pi}$

27. For a reaction $A + B \rightleftharpoons 2C$, $K_c = 1$. If the initial concentration of A, B and C are 1 M, 1 M and 2 M then at equilibrium
    (A) [A] = [B] = [C]
    (B) $[A] = \frac{4}{3}M$
    (C) $[B] = \frac{2}{3}M$
    (D) $[A] = \frac{1}{2}[C]$

28. Which of the following statements are correct?
    (A) In ClF_3, the axial Cl – F bond length is larger than equilateral Cl – F bond length.
    (B) In ClF_3, all the Cl – F bond angles are 90°.
    (C) In SF_4, F – S – F equitorial bond angle is not 120° but 102° due to lp – bp repulsion.
    (D) In OBr_2, the bond angle is less than OCl_2.

29. Identify the correct statement
    (A) time of 75% completion of reaction is thrice of half-life in second order reaction.
    (B) for a reaction whose equilibrium constant greater than one. Activation energy of forward reaction is greater than backward reaction
    (C) molecularity of slowest step will be zero or fraction.
    (D) In zero order reaction half life is directly proportional to initial concentration

30. Which are correct for radio activity?
(A) Half-life is independent of initial amount of radioactive substance
(B) The increasing the temperature half-life will decrease
(C) Half-life of C-14 in CO_2 is different from that C-14 in $C_6H_{12}O_6$
(D) Half-life of U-235 and U-238 are different.

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

31. How many ml of 0.1 M NaOH is added to 60 ml of 0.15 M H_3PO_4 (pK_{a_1}, pK_{a_2} and pK_{a_3} for H_3PO_4 are 3, 8 and 13). So, that pH of resulting buffer solution would be 8.3. (log2 = 0.3)

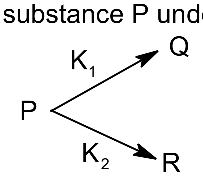
32. In 200 ml solution of 0.1 M NaOH and 0.2 M NH_4OH, 300 ml of 0.1 M HCl is added. Find pH of solution $\left(pK_b \text{ of } NH_4OH = 5\right)$. (and given log3 = 0.48).

33. What is the pH after addition of 40 ml of 0.1 M NaOH in the 50 ml of 0.1 M anilinium chloride $\left(pK_a \text{ of anilinium ion} = 4.6\right)$

34. Hydrogen like species is in some excited state 'P' and on absorbing a photon of wavelength 'a' eV reached to a new state 'Q' on de-excitation back to the ground state a total of 10 different wave lengths were emitted in which seven have energy greater than 'a' eV. The ionization energy in the ground state will be "xa" eV .Find the value of x.

35. In a sample of H atoms, electron are de-excited from a level 'n' to ground state. Total number of Lyman lines emitted are 3. If the electron are ionized from this level 'n' by a photon of 15 eV. Then kinetic energy of emitted photon is x eV. Find x.

36. For a substance P undergoing parallel reaction



$K_1 = 0.372 \text{min}^{-1}$ and $K_2 = 0.321 \text{ min}^{-1}$.

What will be concentration of P after 30 sec where initial concentration of P is 1 molar

Mathematics PART – III

SECTION – A
(One Options Correct Type)

This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

37. Let R be the region in the first quadrant bounded by the x-axis, the line 2y = x, and the ellipse $\frac{x^2}{9}+y^2=1$. Let R′ be the region in the first quadrant bounded by the y-axis, the line y = mx, and the ellipse, then the value of m such that R and R′ have the same area is

(A) $\frac{2}{9}$

(B) $\frac{3}{11}$

(C) $\frac{2}{5}$

(D) $\frac{2}{7}$

38. If f(x): [1, 3] → [–1, 1] satisfies $\int_1^3 f(x)dx = 0$, then the maximum value of $\int_1^3 \frac{f(x)}{x}dx$ is

(A) $\ln\left(\frac{3}{2}\right)$

(B) $\ln\left(\frac{4}{3}\right)$

(C) $\frac{8}{3}$

(D) $\ln\left(\frac{9}{2}\right)$

39. The value(s) of the real parameter ‘a’ for which the inequality
$x^6 - 6x^5 + 12x^4 + ax^3 + 12x^2 - 6x + 1 \geq 0$ is satisfied for all real x

(A) [–12, 37]
(B) [–12, 38]
(C) [–17, 14]
(D) [–17, 35]

40. Let f(x) : [–1, 1] → R is twice differentiable function such that $|f(x)| \leq 1$ and $|f''(x)| \leq 1$, then choose the correct option

(A) $|f'(x)| \leq 2\ \forall\ x \in [-1, 1]$
(B) $f'(x_1) = 3$ for some $x_1 \in (-1, 1)$
(C) $f'(x_1) = 4$ for some $x_1 \in (-1, 1)$
(D) none of these

41. $\int_0^\infty \frac{\tan^{-1}(\pi x)-\tan^{-1}x}{x}dx$ is equal to

(A) $\frac{\pi}{2}\ln\pi$

(B) $\pi \ln \pi$

(C) $\frac{2}{\pi}\ln\pi$

(D) none of these

42. The domain of $f(x)=\sqrt[4]{x\left(2\cdot 3^x-\frac{4x^2+x+2}{x^2+x+1}\right)}$ is

(A) [–7, 101]
(B) [–9, 103]
(C) [–9, 93 ln 2]
(D) $(-\infty, \infty)$

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

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43. Which of the following statements are INCORRECT? (where {.} denotes the fractional part of function)

(A) $f(x)=\{x\}\cos^2\left(\left(\frac{2x-1}{2}\right)\pi\right)$ is differentiable for all $x \in R$

(B) $f(x) = |\sin x|\cos^{-1}(\cos x)$ is differentiable $\forall\ x \in (0, 2\pi)$

(C) $f(x) = ||x-2|-|x-6|| - 3|x| + 2x + 1$ is not differentiable at 3 points

(D) $f(x) = \{x\}|\sin \pi x|$ is not differentiable at all integers

44. Let $f(x) : R \to [-1, 1]$ be twice differentiable and $f^2(0) + (f'(0))^2 = 4$, then which of the following(s) is/are TRUE?

(A) There exist $\alpha, \beta \in R$ where $\alpha < \beta$, such that f is one-one on the interval (α, β)

(B) There exists $c \in (0, 2)$ such that $|f'(c)| \le 1$

(C) $\lim_{x\to\infty} f(x) = 1$

(D) There exists some x_0 such that $f(x_0) + f''(x_0) = 0$ but $f'(x_0) \ne 0$

45. If $f(x) = 2|x+1| - |x-1| + 3|x-2| + 2x + 1$ and $h(x)=\begin{cases}2x & ;\ x\ge 3\\ 1 & ;\ x<3\end{cases}$, then choose the correct statements

(A) y = h(f(x)) is continuous $\forall\ x \in R$
(B) y = h(f(x)) is discontinuous at 3 points
(C) y = h(f(x)) is not differentiable at 3 points
(D) y = h(f(x)) is not differentiable at 4 points

46. Let $f(x)=\frac{x^2+4x+3}{x^2+7x+14}$ and $g(x)=\frac{x^2-5x+10}{x^2+5x+20}$, then

(A) greatest value of f(x)g(x) is 5
(B) greatest value of $(g(x))^{f(x)}$ is 9
(C) greatest value of f(x) is 2
(D) greatest value of $(g(x))^{f(x)}$ is 8

47. Let f: [0, 1] $\to$ R be a continuous function, g(x) be a non-increasing function on [0, 1] and h(x): [0, ∞) $\to$ [0, ∞) be a continuous strictly increasing function with h(0) = 0, then

(A) $\alpha\int_0^1 g(x)dx \le \int_0^\alpha g(x)dx$ for any $\alpha \in (0, 1)$

(B) $\left(\int_0^1 f(t)dt\right)^2 \le \int_0^1 f^2(t)dt$

(C) $\int_0^a h(x)dx + \int_0^b h^{-1}(x)dx \ge ab$

(D) $\int_0^a h(x)dx + \int_0^b h^{-1}(x)dx \le ab$

48. Let $n \ge 0$ be an integer and $I_n = \int_0^\pi \frac{1-\cos nx}{1-\cos x}dx$, then

(A) $I_0, I_1, I_2, ...$ is an A.P.
(B) $I_0, I_1, I_2 .....$ is a G.P.
(C) $I_n = n\pi$; $n \ge 1$
(D) absolute value of the integrand is bounded

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

---

49. Let f(x), g(x) : R $\to$ R be periodic function with period $\frac{3}{2}$ and $\frac{1}{2}$ respectively such that $\lim_{x\to 0}\frac{f(x)}{x} = 1$, $\lim_{x\to 0}\frac{g(x)}{x} = 2$, then $\lim_{n\to\infty}\frac{f\left(\frac{3}{2}\left(3+\sqrt{7}\right)^n\right)}{g\left(\frac{1}{2}\left(2+\sqrt{2}\right)^n\right)}$ is equal to

50. Let f: (0, ∞) $\to$ (0, ∞) be a differentiable function such that $f'\left(\frac{a}{x}\right) = \frac{x}{f(x)}$ where a is positive constant and $f'(1) = 1$, $f'(2) = 2$, then f(5) is equal to

51. Let f(x) be a differentiable function with $f'(0) = 2$ and $f(0) = 3$ such that $f'(x) = \frac{f(x+n) - f(x)}{n}$ $\forall$ x $\in$ R and n $\in$ N, then $\frac{f(5)}{f(1)}$ is

52. $\lim_{n\to\infty} n^2\int_0^{\frac{1}{n}} x^{x+1}dx$ is equal to

53. If $\lim_{n\to\infty}\left(\frac{2^{\frac{1}{n}}}{n+1}+\frac{2^{\frac{2}{n}}}{n+\frac{1}{2}}+.....+\frac{2^{\frac{n}{n}}}{n+\frac{1}{n}}\right)=\frac{1}{\ln(k)}$, then k is

54. Number of solution(s) of the equation sin(sin(sin(sin(sin(x))))) = $\frac{x}{3}$ is/are

ALL INDIA TEST SERIES

TEST - 1

JEE (Advanced)

ANSWERS, HINTS & SOLUTIONS

Physics **PART – I**

SECTION – A

1. B

Sol. $K_e = \frac{1}{2}3Mv^2 + \frac{1}{2}Mv^2 = 2Mv^2$

$K_s = \frac{1}{2}(3M + M)\left(\frac{2Mv}{4M}\right)^2 = \frac{1}{2}Mv^2$

So, $\frac{K_e}{K_s} = 4$

2. A

Sol. $\tau_n = \frac{2v_n}{g}$, where $v_n = \alpha^n v_i$

So, $\tau_n = \tau_0\alpha^n$, where $\tau_0 = \frac{2v_i}{g}$

$t_n = \tau_0\Sigma\alpha^n = \tau_0\frac{1-\alpha^n}{1-\alpha} = \frac{\tau_0 - \tau_n}{1-\alpha}$

So, $\tau_n = \tau_0 - (1-\alpha)t_n$

3. C

Sol. The side of the cube is $L \propto m^{1/3}$ (density is constant)

So, $A \propto m^{2/3}$

For constant speed,

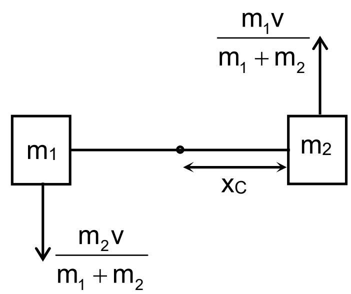
$m \propto P(=Fv)$

So, $m \propto Av^3 \propto m^{2/3}v^3$

So, $v \propto m^{1/9}$

4. D

Sol. Relative to COM frame, the rod is undergoing pure rotatory motion with a constant angular velocity, $\omega = \frac{v}{L}$

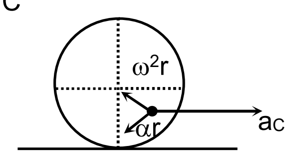


$$x_C = \frac{m_1 L}{m_1 + m_2}$$

So, $t = \frac{2\pi L}{v}$

5. C

Sol.



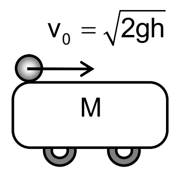
6. A

Sol. $(v_0 - \mu g t) - \alpha t R = \frac{\mu m g}{M} t$

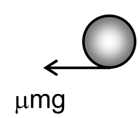
So, $t = \frac{v_0}{\mu g\left(3 + \frac{m}{M}\right)}$

So, in the frame of the cart

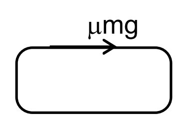
$$L = v_0 t - \frac{1}{2}\mu g\left(1 + \frac{m}{M}\right)t^2 = \frac{7}{8}m$$



At t = 0



F.B.D. of disc



F.B.D. of cart

7. A, C

Sol. $a_C = \frac{2FR^2}{3mR^2} = \frac{2F}{3m}$

So, $F - f_s = m\frac{2F}{3m}$

So, $f_S = \frac{F}{3}$

$f_S \le \mu mg$

So, $\frac{F}{3} \le 40$ N

So, $F \le 120$ N

So, the disc undergoes pure rolling motion upto t = 12 sec

Torque about point P = (10t) (0.1) N-m

So, L = $\frac{t^2}{2}$ kg-m^2/sec

8. A, D

Sol. For moving together, $a = \frac{mg}{M+m}$

So, $T = \frac{Mmg}{M+m}$

For toppling the cylinder $(T)\left(\frac{10}{2}\right) > (Mg)\left(\frac{5}{2}\right)$

So, $\frac{Mmg}{M+m}\left(\frac{10}{2}\right) > Mg\left(\frac{5}{2}\right)$

So, M < m

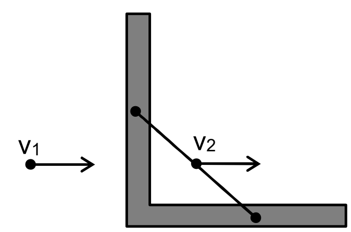
9. C, D

Sol. From conservation of linear momentum

$2mu = 2mv_1 + 2mv_2$

And for e, $\frac{1}{2}u = v_2 - v_1$

So, $v_1 = \frac{u}{4}$ and $v_2 = \frac{3u}{4}$

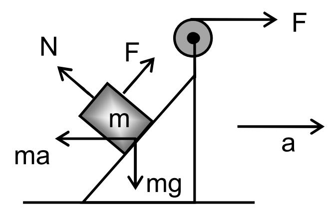


Impulse is passing through centre of mass of the body. So there is no rotation after the collision

Loss in kinetic energy = $\frac{1}{2}2mu^2 - \frac{1}{2}2m(v_1^2 + v_2^2) = \frac{3}{8}mu^2$

10. A, B, D

Sol. $a = \frac{F}{M+m}$



m is in equilibrium w.r.t to M

So, $F = \frac{(M+m)mg\sin\theta}{M+m-m\cos\theta}$

So, pseudo force on m w.r.t to M = $\frac{mF}{M+m}$

= $\frac{m^2 g\sin\theta}{M+m-m\cos\theta}$

Pseudo force on M w.r.t. to m = $\frac{MF}{M+m}$

Also, F cos θ = N sin θ + ma

$\Rightarrow$ N sin 45° = $\frac{F}{\sqrt{2}} - \frac{mF}{M+m}$

So, $N = \frac{mF}{M+m}\left[\frac{M+m-\sqrt{2}m}{m}\right]$

So, N > $\frac{mF}{M+m}$

11. A, C, D

Sol. $\frac{\mu mv^2}{R} = -m\frac{dv}{dt}$

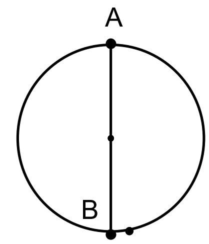
So, $\int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v^2} = -\frac{\mu}{R}\int_0^t dt \Rightarrow \; t = \frac{R}{\mu v_0}$

$|P_{avg}| = \frac{|\Delta K.E.|}{t} = \frac{3}{8}\frac{\mu m v_0^3}{R}$

At v = $v_0/2$, instantaneous power due to kinetic friction force = $-\frac{\mu m}{R}\frac{v_0^2}{4}\frac{v_0}{2} = -\frac{\mu m v_0^3}{8R}$

12. A, B

Sol. Extension in the spring at highest point A is L.



$F_{string} = \frac{4mg}{L} \times L = 4mg$

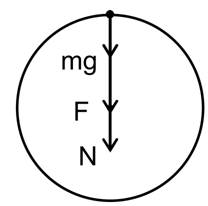
From conservation of energy at B and A

$\frac{1}{2}mu_0^2 = mg(2L) + \frac{1}{2}\left(\frac{4mg}{L}\right)L^2$

So, $u_0 = \sqrt{8gL}$

For u = $2u_0$ at B.

$\frac{1}{2}m\left[2\sqrt{8gL}\right]^2 = \frac{1}{2}mv^2 + mg(2L) + \frac{1}{2}\left(\frac{4mg}{L}\right)L^2$



So, $v = \sqrt{24gL}$

So, F + mg + N = $\frac{m\left[\sqrt{24gL}\right]^2}{L}$

So, N = 19mg (radially inward)

At point D, string is stretched and gravity is also acting on the bead.

SECTION – C

13. 00010.25

Sol. $\frac{dv}{dt} = \omega\frac{dr}{dt} = \omega\frac{\omega}{2\pi}d$

Where $\frac{dv}{dt}$ = acceleration of the mass and r = radius of the reel at any instant

So, T – Mg = $M\frac{\omega^2 d}{2\pi}$

So, T = $M\left(g + \frac{16\pi^2 \times 10^{-2}}{2\pi}\right) = 10.25\text{ N}$

14. 00013.89

Sol. Centripetal force F_C= $Pd\ell.d$

Where $d\ell$ is a small section of pipe

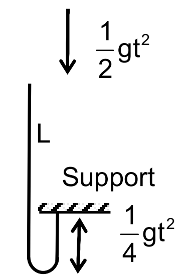
So, $Pd\ell.d$ = $\frac{\pi d^2}{4}\rho(d\ell)\frac{v^2}{R}$

and $v = \frac{4m}{\pi d^2 \rho t}$

So, $P = \frac{4m^2}{\pi\rho R d^3 t^2} = 13.89\text{ N/m}^2$

15. 00017.25

Sol. $\rho Lg - T = \frac{d}{dt}\left(\rho Lgt - \frac{\rho g^2 t^3}{4}\right)$



So, $T = \frac{3}{4}\rho g^2 t^2$

So, at t = 1 sec

$T = \frac{3}{4}\left(\frac{2.3}{10}\right)(10)^2(1)^2 = 17.25\text{ N}$

16. 00014.50

Sol. $d_{air} = \frac{v^2 \sin 2\theta}{g}$

Impulse of normal reaction from ground = mV sin θ

So, after collision, v_x = v cos θ – μv sin θ

So, $d_{ground} = \frac{v^2(\cos\theta - \mu\sin\theta)^2}{2\mu g}$

So, $d_{total} = \frac{v^2}{2\mu g}\left[2\mu\sin 2\theta + (\cos\theta - \mu\sin\theta)^2\right]$

So, $\frac{d}{dt}\left(d_{total}\right) = 0$ So tan θ = μ

So, $d_{total}^{maximum} = \frac{v^2}{2\mu g}(1+\mu^2) = 14.50\text{ m}$

17. 00006.25

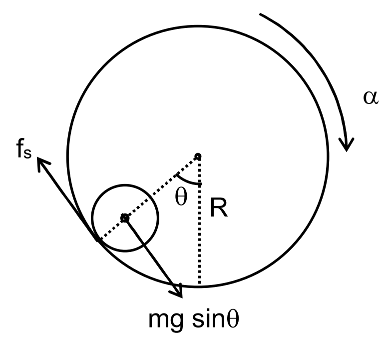
Sol. $f_s = mg\sin\theta$

$\alpha_{sphere} = \frac{R\alpha}{r}$

So, $\tau = I\alpha_{sphere}$

So, mgr sin θ = $\frac{2}{5}mr^2\frac{R}{r}\alpha$

So, $\alpha = \frac{5g\sin\theta}{2R} = \frac{(5)(10)\left(\frac{1}{2}\right)}{(2)(2)} = 6.25\text{ rad/s}^2$



18. 00001.50

Sol. $v_3 = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}} = \sqrt{v_1 v_2}$

After solving $\frac{s_1}{s_2} = \sqrt{\frac{v_1}{v_2}} = 1.50$

SECTION – A

19. B

20. A

Sol. $-\frac{1}{m}\frac{dA}{dt} = k[A]^m$

$-\frac{dA}{dt} = mk[A]^m$

$\log_{10}\left[-\frac{dA}{dt}\right] = \log_{10} mk + m\log[A]$

m = 1, $\log_{10} mk = 0.6 = \log 3.98$

k = 3.98, first order reaction.

21. C

Sol. $NH_3 + H_2O + CO_2 \longrightarrow NH_4HCO_3$

$NH_4HCO_3 + NaCl \longrightarrow NaHCO_3 + NH_4Cl$

22. C

Sol. $T\ell I_3 \longrightarrow T\ell^{+1} + I_3^-$

$B(OC_2H_5)_3$ Burns with green edge flame.

23. C

24. C

25. A, B, D

26. A, B, C

Sol. $\ell = 3$

n = 4

orbital angular momentum $= 4 \times \frac{h}{2\pi} = \frac{2h}{\pi}$

27. A, B

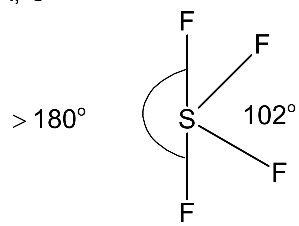
Sol.

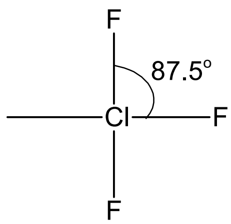
	A	+	B	$\rightleftharpoons$	2C
At equilibrium	$1+x$		$1+x$		$2-2x$

$1 = \frac{(2-2x)^2}{(1+x)(1+x)}$

28. A, C

Sol.





Bond angle of $OBr_2 = 112^\circ$

$OCl_2 = 110^\circ$

29. A, D

30. A, D

SECTION – C

31. 00150.00

Sol. $pH = pK_{a_2} + \log\frac{[HPO_4^{2-}]}{[H_2PO_4^-]}$

$8.3 = 8 + \log\frac{[HPO_4^{2-}]}{[H_2PO_4^-]}$

$\frac{[HPO_4^{2-}]}{[H_2PO_4^-]} = 2$

$H_3PO_4 + NaOH \longrightarrow NaH_2PO_4$

9	x	0
0	x − 9	9

$NaH_2PO_4 + NaOH \longrightarrow Na_2HPO_4$

9	x − 9	0
18 − x	0	x − 9

$\frac{x-9}{18-x} = 2$

$x = 15$

$15 = V \times 0.1,\ V = 150$ ml

32. 00009.48

Sol. $NH_4OH + HCl \longrightarrow NH_4Cl$

40	10	0
30	0	10

$pOH = 5 + \log\frac{1}{3};\ pH = 9.48$

33. 00004.20

Sol. $$C_6H_5NH_3^+ + NaOH \longrightarrow C_6H_5NH_2 + NaCl$$

5	4	0	0
1	0	4	

$$pOH = pK_b + \log\frac{1}{4}$$

= 10.4 – log4 = 10.4 – 0.6 = 9.8

pH = 4.2

34. 00014.06

Sol. $$a\ eV = E_1\left[\frac{1}{3^2} - \frac{1}{5^2}\right]$$

$$E_1 = \frac{225}{16}a\ eV$$

x = 14.06

35. 00014.50

Sol. $$15eV = \frac{13.6}{4^2}eV + KE$$

$$KE = 14.15eV$$

36. 00000.71

Sol. $$K_1 + K_2 = \frac{2.303}{0.5}\log\frac{1}{1-x}$$

Hence, $(1-x) = 0.71$ molar

Mathematics PART – III

SECTION – A

37. A

Sol. Stretch the figure by a factor 3 along the y-axis. So that the point (x, y) goes to (x, 3y), then new line is 2y – 3x

So, for R = R′, $m = \frac{2}{9}$

38. B

Sol. $\frac{1-f(x)}{x}, \frac{1+f(x)}{x} \geq 0 \ \ \forall \ x \in [1, 3]$

$$\int_1^2 \frac{1-f(x)}{x}dx \geq \int_1^2 \frac{1-f(x)}{2}dx \qquad ..... (1)$$

$$\int_2^3 \frac{1+f(x)}{x}dx \leq \int_2^3 \frac{1+f(x)}{2}dx$$

$$\Rightarrow -\int_2^3 \frac{1+f(x)}{x}dx \geq -\int_2^3 \frac{1+f(x)}{2}dx \qquad ..... (2)$$

Adding equation (1) and (2), we get $\ln 2 - \ln\left(\frac{3}{2}\right) \geq \int_2^3 \frac{f(x)}{x}dx$

Equality holds for $f(x) = \begin{cases} 1 & ; \ \ 1 \leq x < 2 \\ -1 & ; \ \ 2 \leq x \leq 3 \end{cases}$

39. B

Sol. $f(x) = \left(x^3 + \frac{1}{x^3}\right) - 6\left(x^2 + \frac{1}{x^2}\right) + 12\left(x + \frac{1}{x}\right) + a \geq 0 \ \ \forall \ x > 0$ and

$f(x) = \left(x^3 + \frac{1}{x^3}\right) - 6\left(x^2 + \frac{1}{x^2}\right) + 12\left(x + \frac{1}{x}\right) + a \leq 0 \ \ \forall \ x > 0$

Let $x + \frac{1}{x} = t$

So, g(t) = $t^3 - 6t^2 + 9t + 12t + a \geq 0; \ t \geq 2$

g′(t) = 3(t – 1)(t – 3) ⇒ g(3) ≥ 0 and g(t) ≤ 0 ∀ t ≤ – 2 ⇒ –12 ≤ a ≤ 38

40. A

Sol. $2|f'(x)| \leq |f(1)| + |f(-1)| + \left|(1-x)^2 \frac{f^2(h)}{2}\right| + \left|\frac{(1+x^2)f''(k)}{2}\right| \ \Rightarrow \ |f'(x)| \leq \frac{3+x^2}{2} \leq 2$

41. A

Sol. Let $I(a) = \int_0^\infty \frac{\tan^{-1}ax - \tan^{-1}x}{x}$

$$I'(a) = \frac{1}{a^2}\int_0^\infty \frac{dx}{x^2 + \frac{1}{a^2}} = \frac{\pi}{2a}$$

$$\therefore \ I = \frac{\pi}{2}\ln a + c$$

So, $I(a) = \frac{\pi}{2}\ln a$

42. D

Sol. **Case-I:** Let $x \le 0$ then $2.3^x \le 2$ and $\frac{4x^2+x+2}{x^2+x+1} \ge 2 \Rightarrow x \le 0$ or $x \ge \frac{1}{2}$. So, $x \le 0$

Case-II: Let $x > 0$, we prove that $\frac{4x^2+x+2}{x^2+x+1} < 2.3^x$

Assume the opposite i.e. $\frac{4x^2+x+2}{x^2+x+1} \ge 2.3^x$

$\because \frac{4x^2+x+2}{x^2+x+1} > 2.3^0 = 2 \Rightarrow x < 0$ or $x > \frac{1}{2}$

Since, $x > 0$. So, $x > \frac{1}{2}$. Hence, $\frac{4x^2+x+2}{x^2+x+1} \ge 2.3^x > 2\sqrt{3} > 3$

$\Rightarrow x^2 - 2x - 1 > 0 \Rightarrow x \in \left(1+\sqrt{2}, \infty\right)$

Thus, $\frac{4x^2+x+2}{x^2+x+1} \ge 2.3^x > 23^{1+\sqrt{2}} > 2 \cdot 3^2 = 18$

But $\frac{4x^2+x+2}{x^2+x+1} < 4$ for any $x > 0$, we get a contradiction

So, domain is R

43. B, C

Sol. (A) $f(x) = \{x\}\sin^2 \pi x$ differentiable $\forall\, x \in R$

(B) not differentiable at $x = (2n + 1)\pi$

(C) not differentiable at $x = 0, 2, 6, 4$

(D) not differentiable at all integers

44. A, B, D

Sol. Let $k(x) = f(x)^2 + f'(x)^2$ using MVT, $|f'(a)| = \left|\frac{f(2)-f(0)}{2}\right| \le 1 : a \in (0, 2)$

So, $k(a) \le 2$

Similarly; $k(a) \le 2$, $b \in (-2, 0)$

$\because$ $k(0) = 4$. So $k(x)$ has a maximum at some interior point of $(-2, 2)$.

Let that point be x_0 then certainly $k(x_0) \ge k(0) = 4 \Rightarrow k'(x_0) = 0$

$\Rightarrow f'(x_0)(f'' + f(x_0)) = 0$: clearly, $f'(x_0) \ne 0$

45. A, C

Sol. $h(f(x))$ is continuous $\forall\, x \in R$ and not differentiable at $x = -1, 1, 2$

46. B, C

Sol. $f(x)$ has maximum value 2 when $x = -5$ and $g(x)$ has maximum value 3 when $x = -5$

47. A, B, C

Sol. (A) $\because \frac{1}{1-\alpha}\int_{\alpha}^{1} g(x)\,dx \le \frac{1}{\alpha}\int_{0}^{\alpha} g(x)\,dx$

$$\alpha\int_{\alpha}^{1} g(x)\,dx \le (1-\alpha)\int_{0}^{\alpha} g(x)\,dx$$

Add $\int_0^{\alpha} f(x)dx$ both sides

$$\alpha\int_0^1 f(x)dx \le \int_0^{\alpha} f(x)dx$$

(B) Cauchy-schwarz inequality to the functions f(t) and g(t) = 1 $\forall$ t $\in$ [0, 1]

48. A, D

Sol. $\lim\limits_{x\to 0}\dfrac{1-\cos nx}{1-\cos x} = n^2$

So, absolute value of the integrand is bounded as $x \to 0$ and hence the integral converges

$$\frac{I_{n+1}+I_{n-1}}{2} = \int_0^{\pi}\frac{1-\cos nx\cdot\cos x}{1-\cos x} = \int_0^{\pi}\frac{(1-\cos nx)+\cos nx(1-\cos x)}{1-\cos x} = I_n$$

So, $I_n = \dfrac{1}{2}(I_{n+1}+I_{n-1})$; $n \ge 1$

$\because$ $I_0 = 0$, $I_1 = \pi$, ..., $I_n = n\pi$

SECTION – C

49. 00000.00

Sol. $$\frac{f\left((3+\sqrt{7})^n T_1\right)}{g\left((2+\sqrt{2})^n T_2\right)} = \frac{f\left((3+\sqrt{7})^2 T_1+(3-\sqrt{7})^n T_2-(3-\sqrt{7})^n T_1\right)}{g\left((2+\sqrt{2})^n T_2+(2-\sqrt{2})^n T_2-(2-\sqrt{2})^n T_2\right)} = \frac{f\left(-(3-\sqrt{7})^n T_1\right)}{g\left(-(2-\sqrt{2})^n T_2\right)}\ \forall\ n \ge 1$$

So, $$\lim_{n\to\infty}\frac{f\left(T_1(3+\sqrt{7})^n\right)}{f\left(T_2(2+\sqrt{2})^n\right)} = \frac{T_1}{T_2}\lim_{n\to\infty}\left\{\frac{f\left(-(3-\sqrt{7})^n T_1\right)}{-(3-\sqrt{7})^n T_1}\cdot\frac{-(2-\sqrt{2})^n T_2}{g\left(-(2-\sqrt{2})^n T_2\right)}\cdot\frac{(3-\sqrt{7})^n}{(2-\sqrt{2})^n}\right\} = 0$$

50. 00012.50

Sol. $f'\left(\dfrac{a}{x}\right) = \dfrac{x}{f(x)}$ ..... (1)

$x \to \dfrac{a}{x}$

$f'(x) = \dfrac{a}{xf\left(\dfrac{a}{x}\right)}$ ..... (2)

$f''(x) = -\dfrac{a}{x^2 f\left(\dfrac{a}{x}\right)} + \dfrac{a^2 f'\left(\dfrac{a}{x}\right)}{x^3\left(f\left(\dfrac{a}{x}\right)\right)^2}$ ..... (3)

Using equation (1), (2) and (3), we get $f''(x) = -\dfrac{f'(x)}{x} + \dfrac{(f'(x))^2}{f(x)}$

$\Rightarrow xf(x)\, f''(x) + f(x)\, f'(x) = x(f'(x))^2 \Rightarrow \dfrac{f'(x)}{f(x)} + \dfrac{xf''(x)}{f(x)} - \dfrac{x(f(x))^2}{(f(x))^2} = 0$

Observe $\left(\dfrac{xf'(x)}{f(x)}\right)' = 0$. So, $\dfrac{xf'(x)}{f(x)} = c \Rightarrow \dfrac{f'(x)}{f(x)} = \dfrac{c}{x}$

51. 00002.60

Sol. Put n = 1 ; $f'(x) = f(x+1) - f(x)$, n = 2 ; $f'(x) = \frac{f(x+2)-f(x)}{2}$

So, $f'(x) = \frac{f(x+2)-f(x+1)+f(x+1)-f(x)}{2}$, $f'(x) = \frac{1}{2}f'(x+1) + \frac{1}{2}f'(x)$

$\Rightarrow f'(x) = f'(x+1)\ \forall\ x \in R \Rightarrow (f(x+1) - f(x))' = 0\ \forall\ x \in R$

$\Rightarrow f(x+1) - f(x) = c$ for a constant $c \in R \Rightarrow f'(x) = c \Rightarrow f(x) = cx + d$

52. 00000.50

Sol. $\because \lim\limits_{x\to 0} x^x = 1$

Fix $\in > 0$ and choose $\delta > 0$ such that for $0 < x < \delta$, $|x^x - 1| < \in$ then for $n \geq \frac{1}{\delta}$, we have

$$\left|n^2\int_0^{\frac{1}{n}}(x^{x+1} - x)dx\right| \leq n^2\int_0^{\frac{1}{n}}|x^{x+1} - x|dx = n^2\int_0^{\frac{1}{n}}x|x^x - 1|dx < \in n^2\int_0^{\frac{1}{n}}xdx = \frac{\in}{2}$$

So, $n^2\int_0^{\frac{1}{n}}x^{x+1}dx = \frac{\in}{2} + n^2\int_0^{\frac{1}{n}}xdx = \frac{\in}{2} + n^2 \cdot \frac{1}{2n^2} = \frac{1}{2}$

53. 00002.00

Sol. $S_n = \lim\limits_{n\to\infty}\sum_{r=1}^{n}\frac{2^{\frac{r}{n}}}{1+\frac{1}{m}}\ \because\ \frac{2^{\frac{r}{n}}}{1+\frac{1}{m}} = \frac{2^{\left(\frac{r-1}{n}\right)}2^{\frac{1}{n}}}{1+\frac{1}{m}} = \frac{2^{\left(\frac{r-1}{n}\right)}e^{\frac{\ln 2}{n}}}{1+\frac{1}{rn}} = \alpha$ and

for; $r \geq 2$; $2^{\frac{r}{n}} > \alpha > 2^{\frac{r-1}{n}}\left(\frac{1+\frac{\ln 2}{n}}{1+\frac{1}{rn}}\right) > 2^{\frac{r-1}{n}}$ ($\because$ $e^x > 1 + x$)

So, $\frac{2^{\frac{1}{n}} + 2^{\frac{2}{n}} + ... + 2^{\frac{n-1}{n}}}{n} < S_n < \frac{2^{\frac{2}{n}} + 2^{\frac{3}{n}} + ... + 2^{\frac{n}{n}}}{n}$. Hence, $S_n = \int_0^1 2^x dx = \frac{1}{\ln 2}$

54. 00003.00

Sol. Let f(x) = sin(sin(sin(sin(sin x))))

$f'(0) = 1 > \frac{1}{3}$. Therefore, $f(x) > \frac{x}{3}$ in some neighbourhood of 0

So, there are 3 solutions