ALL INDIA TEST SERIES

TEST – 10

JEE (Advanced)

Time Allotted: 3 Hours **Maximum Marks: 198**

General Instructions:

- The test consists of total **54** questions.
- Each subject (PCM) has **18** questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Three Sections**: **Section-A, Section – B & Section-C.**

Section-A (01 – 06, 19 – 24, 37– 42) *this section contains* ***18 multiple*** *choice questions.*
Each question has FOUR options. ***ONE OR MORE THAN ONE*** *of these four option(s) is (are) correct answer(s).*
For each question, choose the option(s) corresponding to (all) the correct answer(s)
Answer to each question will be evaluated according to the following marking scheme:

Full Marks **: +4** If only (all) the correct option(s) is (are) chosen:
Partial Marks **: +3** If all the four options are correct but ONLY three options are chosen;
Partial Marks **: +2** If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks **: +1** If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks **: 0** If none of the options is chosen (i. e. the question is unanswered);
Negative Marks **: –2** In all other cases

Section-B (07 – 12, 25 – 30, 43– 48) *contains* ***18 Numerical*** *based questions with* ***Single digit integer*** *as answer, ranging from* ***0 to 9*** *and each question carries* ***+3 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

Section-C (13 – 18, 31 – 36, 49– 54) *contains* ***18 Numerical*** *answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer and* ***0 marks*** *for wrong answer.*

Physics

PART – I

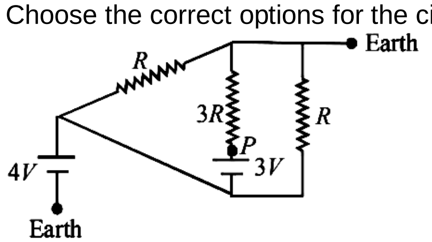
SECTION – A
(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

1. A force $\vec{F} = x^2y^2\hat{i} + x^2y^2\hat{j}(N)$ acts on a particle which moves in XY plane. Choose the correct option(s)
   (A) $\vec{F}$ is a conservative force
   (B) Work done for path ABC is $\frac{a^5}{3}(J)$
   (C) Work done for path ADC is $\frac{a^5}{3}(J)$
   (D) Work done for path AC is $\frac{2a^5}{3}(J)$

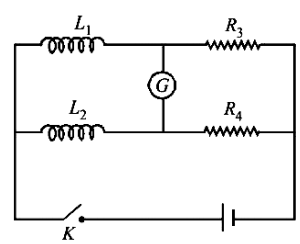
2. A metal sphere of radius R and specific heat C is rotated about an axis passing through its centre at a speed n rotation/second. It is suddenly stopped and 50% of its energy is used in increasing its temperature, then choose the correct statement(s) from the following?
   (A) Kinetic energy used to raise the temperature of the sphere is $\frac{2\pi^2n^2}{5}MR^2$
   (B) Kinetic energy used to raise the temperature of the sphere is $\frac{5\pi^2n}{3}MR$
   (C) The rise in the temperature of the sphere is $\frac{4\pi^2n^2R^2}{7C}$
   (D) The rise in the temperature of the sphere is $\frac{2\pi^2n^2R^2}{5C}$

3. Choose the correct options for the circuit shown.



   (A) Potential of the point P is 7V
   (B) Current in the 3R resistance is 7/R
   (C) Current in the 3R resistance is 7/3 R
   (D) Potential of the point P is 3V

4. Two inductors of self-inductances L_1 and L_2 and of resistances R_1 and R_2 (not shown here) respectively, are connected in the circuit as shown in figure. At the instant t = 0, key K is closed. Choose the correct options for which the galvanometer will show zero deflection at all times after the key is closed.



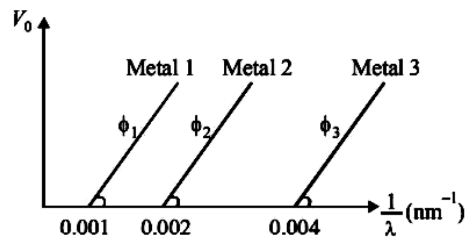
   (A) $\frac{L_1}{L_2} = \frac{R_3}{R_4}$

(B) $\frac{L_1}{L_2} = \frac{R_1}{R_2}$

(C) $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

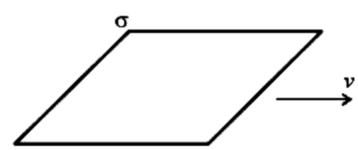
(D) None of these

5. The graph between the stopping potential (V_0) and $\left(\frac{1}{\lambda}\right)$ is shown in the figure. ϕ_1, ϕ_2 and ϕ_3 are work functions. Which of the following is/are correct?



(A) $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$
(B) $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$
(C) $\tan\theta \propto \frac{hc}{e}$
(D) ultraviolet can be used to emit photoelectrons from metal 2 and metal 3 only

6. A large plate with uniform surface charge density σ is moving with constant speed v as shown in the figure. The magnetic field at a small distance from plate is



(A) $\mu_0\sigma v$ in magnitude
(B) $\frac{\mu_0\sigma v}{2}$ in magnitude
(C) perpendicular to plate
(D) parallel to plate

SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

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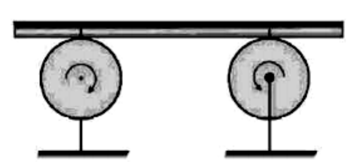
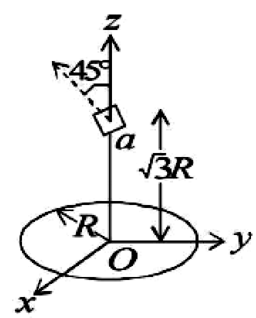
7. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is

8. A uniform rod is placed on two spinning wheels as shown in figure. The axes of the wheels are separated by a distance $\ell = \frac{50}{\pi^2}$ m, the coefficient of friction between the rod and the wheels is $\mu = 0.1$. The rod performs harmonic oscillations. The period of these oscillations is 10x sec. Find the value of x.



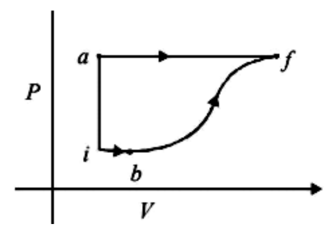
9. A ball of mass of mass 10^{-2} kg and having a charge + 3×10^{-6} C is tied at one end of a 1 m long thread. The other end of the thread is fixed and a charge -3×10^{-6} C is placed at this end. The ball can move in a circular orbit of radius 1 m in the vertical plane. Initially, the ball is at the bottom. Find minimum initial horizontal velocity of the ball so that it will be able to complete the full circle.

10. A circular wire loop of radius R is placed in the x-y plane 1 centered at the origin O. A square loop of side a(a << R) having two turns is placed with its centre at $z = \sqrt{3}R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2}R'}$ then the value of p is



11. A laser beam has intensity $2.5 \times 10^{14} \frac{W}{m^2}$. If the amplitude of magnetic field in the beam is $16k \times 10^{-2}T$. Find the value of k.

12. A thermodynamic system is taken from an initial state I with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are w_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{ifa}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, The ratio $\frac{Q_{bf}}{Q_{ib}}$ is



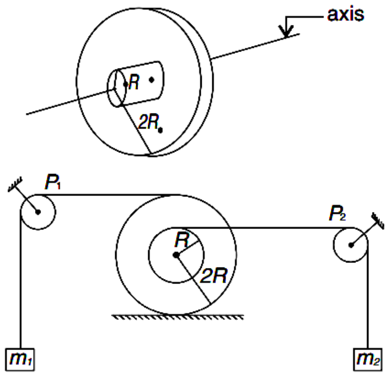
SECTION – C
(Numerical Answer Type)

This section contains **06 questions**. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **SECOND DECIMAL PLACE**; e.g. XXXXX. XX).

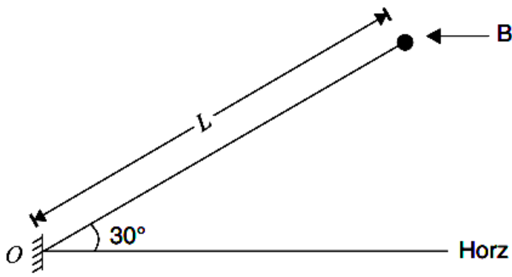
13. In a sample of radioactive material, the probability that a particular nucleus will decay in next 2 hour is 8×10^{-4}. Find the half-life of the radioactive sample
[Take ln2 = 0.693, ln(0.9992) = -8×10^{-4}]

14. Sunrays pass through a pinhole in the roof of a hut and produce an elliptical spot on the floor. The minor and major axes of the spot are 6 cm and 12 cm respectively. The angle subtended by the diameter of the sun at our eye is 0.5°. Calculate the height of the roof.

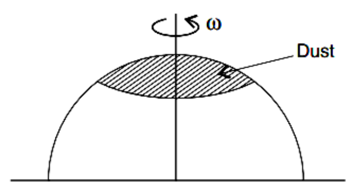
15. A spool has the shape shown in figure. Radii of inner and outer cylinders are R and 2 R respectively. Mass of the spool is 3 m and its moment of inertia about the shown axis is $2mR^2$. Light threads are tightly wrapped on both the cylindrical parts. The spool is placed on a rough surface with two masses $m_1 = m$ and $m_2 = 2m$ connected to the strings as shown. The string segment between spool and the pulleys P_1 and P_2 are horizontal. The centre of mass of the spool is at its geometrical centre. System is released from rest. What is minimum value of coefficient of friction between the spool and the table so that it does not slip?



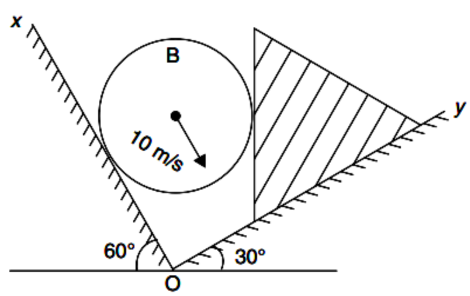
16. A pendulum has length L = 1.8 m. The bob is released from position shown in the figure. Find the tension in the string when the bob reaches the lowest position. Mass of the bob is 1 kg.



17. A metallic hemisphere is having dust on its surface. The sphere is rotated about a vertical axis passing through its centre at angular speed $\omega = 10$ rad s^{-1}. Now the dust is visible only on top 20% area of the curved hemispherical surface. Radius of the hemisphere is R = 0.1 m. Find the coefficient of friction between the dust particle and the hemisphere [$g = 10\ ms^{-2}$].



18. In the arrangement shown in the figure A is an equilateral wedge and the ball B is rolling down the incline XO. Find the velocity of the wedge (of course, along OY) at the moment velocity of the ball is 10 m/s parallel to the incline XO.



Chemistry

PART – II

SECTION – A
(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

19. Which is/are correct statement?
    (A) Packing fraction in 2D-hcp is 0.785
    (B) Packing fraction in AAA ..... is 0.52
    (C) Packing fraction in ABAB .... is 0.74
    (D) Void fraction in ABCABC ..... is 0.26

20. Indicate the correct statement for equal volumes of $N_2(g)$ and $CO_2(g)$ at 25°C and 1 atm.
    (A) The average translational K.E. per molecule is the same for N_2 and CO_2
    (B) The rms speed is same for both N_2 and CO_2
    (C) The density of N_2 is less than that of CO_2
    (D) The total translational K.E. of both N_2 and CO_2 is the same

21. Select correctly matched.
    (A) $Cr_2O_7^{2-} \rightarrow$ two tetrahedral units are joined by their common corner
    (B) $S_2O_6^{2-} \rightarrow$ centre of one tetrahedral is the corner of other tetrahedral
    (C) $S_2F_{10} \rightarrow$ two octahedral joined together
    (D) $S_2O_8^{2-} \rightarrow$ two tetrahedral unit joined by their corners

22. Which of the following represent correct similarity between sulphur and chromium?
    (A) Both exhibit hexavalency
    (B) Sulphate and chromate of Ba^{2+} are water insoluble
    (C) Trioxide (MO_3) both are acidic
    (D) Sulphate (SO_4^{2-}) and chromate (CrO_4^{2-}) have same colouration

23. H_2A is a weak diprotic acid. If the pH of 0.1M H_2A solution is 3 and concentration of A^{2-} is 10^{-12} at 25°C.
    Select correct statement(s)
    (A) $[H^+]_{total} \approx [H^+]$ from first step of ionization of acid H_2A
    (B) Concentration of OH^- in solution is 10^{-3} M
    (C) The value of K_{a_1} is nearly 10^{-5}
    (D) $pK_{a_2} - pK_1 = 9$

24. Determine which of the following reactions taking place at constant pressure represents system that do work on the surrounding environment
    I. $Ag^+(aq) + Cl^-(aq) \rightarrow AgCl(s)$
    II. $NH_4Cl(s) \rightarrow NH_3(g) + HCl(g)$
    III. $2NH_3(g) \rightarrow N_2(g) + 3H_2(g)$
    (A) I
    (B) III
    (C) II and III
    (D) I and II

SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

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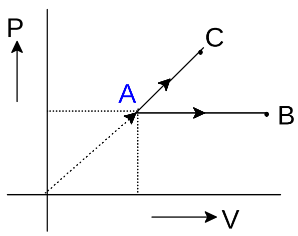
25. If edge fraction unoccupied in ideal anti-fluorite structure is x. Calculate the value of Z. Where $Z = \frac{x}{0.097}$

26. Consider the following list of reagents:
Acidified $K_2Cr_2O_7$, alkaline $KMnO_4$, $CuSO_4$, H_2O_2, Cl_2, O_3, $FeCl_3$, HNO_3 and $Na_2S_2O_3$. The total number of reagents that can oxidise aqueous iodide to iodine is:

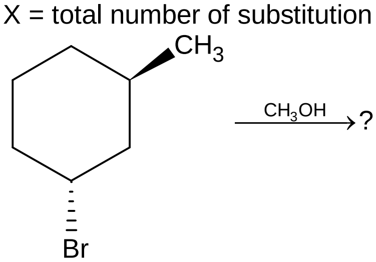
27. One mole ideal monoatomic gas is heated according to path AB and AC. If temperature of state B and state C are equal. Calculate $\frac{q_{AC}}{q_{AB}} \times 10$.



28. Find number of substance which produce H_2 gas when react with cold water.
$Ca, CaH_2, Mg, MgCl_2, Ba, BaO_2, NaHCO_3, Ca_3N_2$.

29. A molten mixture contain following substance,
Cu, FeO, PbO, CaO, MgO, Ag
Find number of substances which are removed when molten mixture is treated with SiO_2.

30. X = total number of substitution and elimination product(s). Find the value of X.



SECTION – C
(Numerical Answer Type)

This section contains **06 questions**. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **SECOND DECIMAL PLACE**; e.g. XXXXX. XX).

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31. The rate of a first order reaction is 0.04 mol L^{-1} s^{-1} at 10 minutes and 0.03 molt's ' at 20 minutes after initiation. Find the half-life of the reaction

32. A solution containing 1 M XSO_4(aq) and 1M YSO_4(aq) is electrolysed. If conc. Of X^{2+} is 10^{-2M} when deposition of Y^{2+} and X^{2+} starts simultaneously, calculate the value of Z.

Given:

$$\frac{2.303RT}{F} = 0.06$$

$$E^o_{X^{2+}/X} = -0.12V; E^o_{Y^{2+}/Y} = -0.24V$$

33. Given the two concentration of HCN $(K_a = 10^{-9})$ are 0.1 M and 0.001 M respectively. What will be the ratio of degree of dissociation?

34. The degree of hydrolysis of 0.1 M RNH_3Cl solution is 1.0%. If the concentration of RNH_3Cl is made 0.4M, what is the new degree of hydrolysis (in percentage)?

35. Calculate wave number (in cm^{-1}) corresponding to shortest wavelength transition in balmer series of atomic hydrogen.

36. The equilibrium constant for the following reaction in aqueous solution is 0.90.

$$H_3BO_3 + \text{glycerin} \rightleftharpoons (H_3BO_3 - \text{glycerin})$$

How many moles of glycerin should be added per litre of 0.10 M H_3BO_3 so that 80% of the H_3BO_3 is converted to the boric-acid-glycerin complex?

Mathematics PART – III

SECTION – A
(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

37. Let $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 .... + a_{2n}x^{2n}$, where *n* is odd integer, if

$S_1 = a_0 + a_4 + a_8 + ...$

$S_2 = a_1 + a_5 + a_9 + ...$

$S_3 = a_2 + a_6 + a_{10} + ...$

$S_4 = a_3 + a_7 + a_{11} + ...$ then

(A) $S_1 = S_3$

(B) $S_2 = S_4$

(C) $S_2 + S_4 = 0$

(D) either $S_1 = S_2 = S_3$ or $S_1 = S_3 = S_4$

38. Let $f_1(x) = \sin^{-1}\left(\cos(\sin^2 x)\right)$, $f_2(x) = \cos^{-1}(\sin(\cos^2 x))$

$f_3(x) = \sin^{-1}\left(\cos(\cos^2 x)\right)$, $f_4(x) = \cos^{-1}(\sin(\sin^2 x))$, then choose the correct option(s)

(A) $\sum_{i=1}^{4} f_i(x) = 2\pi - 2$

(B) $\sum_{i=1}^{4} (-1)^i f_i(x) = 0$

(C) $\sum_{i=1}^{4} (-1)^{\left[\frac{i}{3}\right]} f_i(x) = 0$ (where [.] is greater integer function)

(D) $f_1(x) > f_2(x) \quad \Rightarrow \cos 2x > 0$

39. If the area of a triangle is given Δ and angle C is given and if the value of the side C opposite to angle C is minimum then

(A) a = $\sqrt{\frac{2\Delta}{\sin C}}$

(B) b = $\sqrt{\frac{2\Delta}{\sin C}}$

(C) a = $\frac{4\Delta}{\sin C}$

(D) b = $\frac{4\Delta}{\sin C}$

40. If the equation $x^5 - 10a^3x^2 + b^4x + c^5 = 0$ has three equal roots, then

(A) $2b^2 - 10a^3b^2 + c^5 = 0$

(B) $6a^5 + c^5 = 0$

(C) $2c^5 - 10a^3b^2 + b^4c^5 = 0$

(D) $b^4 = 15a^4$

41. Which of the following is /are true ?

(A) $\int_0^1 \sin(x^2 + 2x + 1)dx - \int_1^2 \sin x^2 dx = 0$

(B) $\int_{-1}^1 e^{\sin x}dx - \int_0^1 e^{\sin(2x+1)}dx = 0$

(C) $\int_{-5}^{-4} \sin(x^2 - 3)dx + \int_{-2}^{-1} \sin(x^2 + 12x + 33)dx = 0$

(D) $\int_{-4}^{4} \cos x^2\, dx - 8\int_0^1 \cos 16(2x - 1)^2 dx = 0$

42. Let $f(x) = \sec^{-1}([1+\sin^2 x])$ ([.] denotes the greatest integer function). Then set of points, where f(x) is not continuous is

(A) $\left\{\frac{n\pi}{2}, n \in I\right\}$

(B) $\left\{(2n-1)\frac{\pi}{2}, n \in I\right\}$

(C) $\left\{(2n+1)\frac{\pi}{2}, n \in I\right\}$

(D) None of these

SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

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43. A pair of tangents is drawn from the point A (23, 3) to the circle whose equation is $x^2 + y^2 - 2x - 4y + 4 = 0$ to touch the circle in B and C. A third tangent intersects the segment AB in P and AC in R and touch the circle in Q. If P is the perimeter of the triangle APR, than |p – 50| = ...

44. If z is a complex number and the minimum value of |z| + | z – 1| + |2z –3| is λ and if y = 2 [x] + 3 = 3 [x – λ], then find the value of $\frac{[x+y]}{10}$ (where [.] denotes the greatest integer function}

45. John has x children by his first wife. Mary has x + 1 children by her first husband. They marry and have children their own. In the whole family there are 24 children, Assuming that two children of the same parents do no fight. If the maximum no. of fights that can take place is λ then 382/λ is.

46. Value of $\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{1}{3^k}\left({}^kC_r\right)$ is

47. If equation $4x^4 - ax^3 + bx^2 - cx + 6 = 0$ has four distinct real roots say $x_1 < x_2 < x_3 < x_4$ such that $\frac{1}{x_1} + \frac{2}{x_2} + \frac{3}{x_3} + \frac{4}{x_4} = 8$, $a, b, c \in R^+$ then $\frac{x_4}{x_1}$ is equal to

48. If α, β are the roots of the equation x2 – 2x + 3 = 0, then find the constant term of the equation whose roots are α3 – 3α2 + 5α – 2 & β3 – β2 + β + 5.

SECTION – C
(Numerical Answer Type)

This section contains **06 questions**. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **SECOND DECIMAL PLACE**; e.g. XXXXX. XX).

49. 10 IIT & 2 PET students sit in a row. The total number of ways in which exactly 3 IIT students sit between 2 PET students is K then $\frac{K}{1000}$ is

50. The number of integral solution of this equation x + y + z + w < 25 such that x > – 2, y > 1, z $\geq$ 0, w > 3 is

51. The value of $\frac{y_1.y_2.y_3}{(y_1 - x_1)(y_2 - x_2)(y_3 - x_3)}$ when $(x_i, y_i), i = 1, 2, 3$ satisfy both $x^3 - 3xy^2 = 2005$ & $y^3 - 3x^2y = 2004$ is

52. If $z = \frac{1}{3} + \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + ...\infty$ then find value of $\frac{z^2 + 2z + 100}{10}$ is equal to

53. Let f(x) is a quadratic expression with positive integral coefficients such that for every. α $\beta \in$ R, $\beta > \alpha$.
$\int_{\alpha}^{\beta} f(x)dx > 0$ Let g(t) = f"(t) . f(t) and g(0) = 12. If no of quadratics are ab where a and b are natural number than $\frac{a+b}{2}$ is equal to. (ab is 2 digit number)

54. Find the integral value of a, for which the quadratic expression $ax^2 + (a – 2) x – 2$ is negative for exactly two integral values of x.

ALL INDIA TEST SERIES

TEST – 10

JEE (Advanced)

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. B, C, D

Sol. If $\vec{F}$ is conservative, then

$$F_x = -\frac{\partial U}{\partial x}, F_y = -\frac{\partial U}{\partial y}$$

And so $\frac{\partial F_x}{\partial y} = -\frac{\partial^2 U}{\partial y \partial x} = -\frac{\partial^2 U}{\partial x \partial y} = \frac{\partial F_y}{\partial x}$

$$W = \int \vec{F}.\overline{ds} = (x^2y^2\hat{i} + x^2y^2\hat{j}).(dx\hat{i} + dy\hat{j})$$

$$= \int x^2y^2dx + \int x^2y^2dy$$

$$W_{BC} = \int_0^a a^2y^2dy = \frac{a^5}{3}$$

Thus, $W_{ABC} = W_{AB} + W_{BC} = \frac{a^5}{3}(J)$

$$W_{AC} = 2\int_0^a x^4dx = \frac{2a^5}{3}(J)$$

2. A, D

Sol. Kinetic energy of the sphere

$$K = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2} \times \frac{2}{5} MR^2 (2\pi n)^2$$

$$= \frac{4}{5} \pi^2 n^2 MR^2$$

Kinetic energy used to raise, the temperature

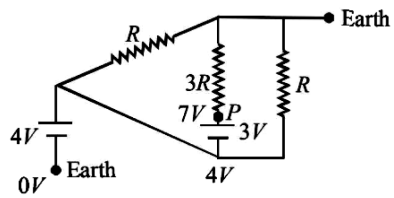
$$= 0.5\left(\frac{4}{5}\pi^2 n^2 MR^2\right)$$

$$= \frac{2}{5}\pi^2 n^2 mR^2$$

$\therefore \quad \Delta T = \frac{2\pi^2 n^2 R^2}{5C}$

3. A, C

Sol.



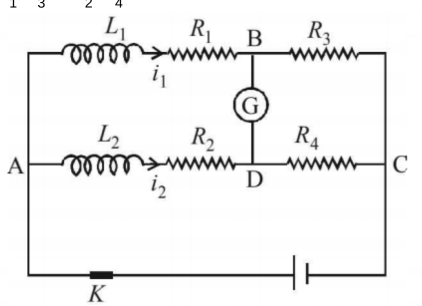
$\therefore \quad V_P = 7V; i = \frac{7-0}{3R} = \frac{7}{3R}$

4. A, B, C, D

Sol. $V_B = V_D$

$L_1 \frac{di_1}{dt} + i_1 R_1 = L_2 \frac{di_2}{dt} + i_2 R_2$ ...(1)

$i_1 R_3 = i_2 R_4$ ...(2)



From Equation (1) and (2)

$$\frac{di_1}{dt} R_3 = \frac{di_2}{dt} R_4$$

or $\frac{di_1}{dt} = \frac{R_4}{R_3}\frac{di_2}{dt}$ and $i_1 = i_2 = \frac{R_4}{R_3}$

$\left(L_1 \frac{R_4}{R_3} - L_2\right)\frac{di_2}{dt} = i_2\left[R_2 - \frac{R_1 R_4}{R_3}\right]$ ....(3)

At t = 0, $i_2 = 0$

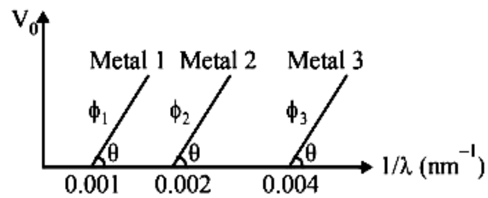
$\therefore \quad \frac{L_1}{L_2} = \frac{R_3}{R_4}$ ...(4)

$$R_2 - \frac{R_4 R_1}{R_3} = 0 \therefore \frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \ldots(5)$$

From Equation (4), (5), $\frac{L_1}{L_2} = \frac{R_1}{R_2} = \frac{R_3}{R_4}$

5. A, C

Sol. $\phi_1 : \phi_2 : \phi_3 = eV_{0_1} : eV_{0_2} : eV_{0_3}$



$= V = V_{0_1} : V_{02} : V_{03} = 0.001 : 0.002 : 0.004 = 1 : 2 : 4$

Therefore (a) is correct

$$\Rightarrow \quad V = \frac{hc}{e\lambda} - \frac{\phi}{e} \quad \ldots(1)$$

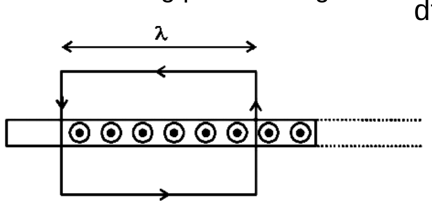
$$m = \frac{hc}{e} = \tan\theta$$

$$\frac{1}{\lambda_{0_1}} = 0.001\,nm^{-1} \Rightarrow \lambda_{0_1} = \frac{1}{0.001} = 1000mn$$

Also, $\frac{1}{\lambda_{0_2}} = 0.002\,nm^{-1} \Rightarrow \lambda_{02} = 500mn$ and $\lambda_{0_2} = 250nm$

6. B, D

Sol. Current flowing per unit length $= \sigma \frac{dx}{dt} = \sigma v$



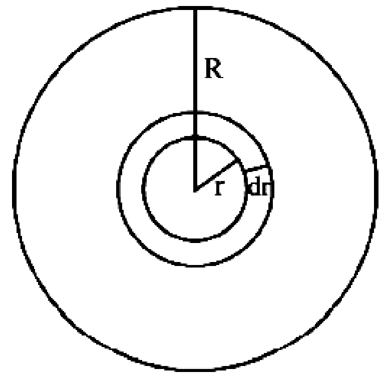
By Ampere's law

$$B\ell + B\ell = \mu_0 \sigma v \ell \Rightarrow B = \frac{\mu_0 \sigma v}{2}$$

SECTION – B

7. 6

Sol. $I=\int_0^R (dm)r^2$



$\therefore \quad I=\int_0^R \rho\times 4\pi r^2 dr\times r^2$

$\therefore \quad I=\int_0^R \rho\times 4\pi r^2 dr\times r^2$

$\therefore \quad I_A=4\pi\int_0^R k\frac{r}{R}\times r^4 dr=\frac{4\pi K}{R}\int_0^R r^5 dr$

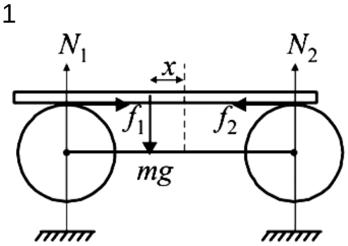
$=\frac{4\pi K}{R}\left(\frac{R^6}{6}\right)=4\pi K\frac{R^5}{6}$

$I_B=4\pi\int_0^R K\left(\frac{r}{R}\right)^5 r^4 dr=\frac{4\pi K}{R^5}\times\frac{R^{10}}{10}=4\pi K\frac{R^5}{10}$

$\therefore \quad \frac{I_B}{I_A}=\frac{6}{10}\Rightarrow n=6$

8. 1

Sol.



$F=-(f_1-f_2)$

$=-(\mu N_1-\mu N_2)$

$=-\mu(N_1-N_2)$

$N_1 + N_2 = mg$ …(1)

$mg(\ell/2-x)-N_2\ell=0$ …(2)

$\therefore \quad N_2=\frac{mg}{\ell}(\ell/2-x)$

$=\frac{mg}{\ell}\left[\frac{\ell}{2}+x\right]$

$F=\mu\left[\frac{mg}{\ell}\left(\frac{\ell}{2}+x\right)-\frac{mg}{\ell}\left(\frac{\ell}{2}-x\right)\right]$

$=-\mu\frac{mg}{\ell}(2x)$

$=\pi\sqrt{\frac{2\ell}{\mu g}}=10\,\text{sec}$

9. 7

Sol. $(T_A + F_e) - mg = \frac{mv_A^2}{\ell}$ ...(i)

$(T_A + F_e) - mg = \frac{mv_A^2}{\ell}$

$mg - F_e = \frac{mv_B^2}{\ell}$ ...(ii)

$F_e = \frac{1}{4\pi \epsilon_0} \frac{q^2}{\ell^2}$

$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mg(2\ell)$ ...(iii)

$v_A = 7 \text{ m/s}$

10. 7

Sol. $B = \frac{\mu_0}{4\pi} \frac{2\pi i R^2}{(R^2 + 3R^2)^{3/2}} = \frac{\mu_0 i}{16R}$

$M = \frac{N\phi}{i} = \frac{2}{i}\left[\frac{\mu_0 i}{16R} \times a^2 \cos 45^\circ\right]$

$\therefore \quad M = \frac{\mu_0 a^2}{\frac{7}{2^2 R}}$

11. 9

Sol. The intensity is given by

$I = \frac{1}{2} \epsilon_0 E^2 C$

or $2.5 \times 10^{14} = \frac{1}{2} \times (8.86 \times 10^{-12}) \times E_0^2 \times (3 \times 10^8)$

$\therefore \quad E_0 = 4.3 \times 10^8$ V/m

and $B_0 = \frac{E_0}{C} = 1.44T$

12. 2

Sol. Applying first law of thermodynamics to path iaf

$Q_{iaf} = \Delta U_{iaf} + W_{iaf}$

$500 = \Delta U_{iaf} + 200$

$\therefore \quad \Delta U_{iaf} = 300$ J

Now,

$Q_{iaf} = \Delta U_{ibf} + W_{ib} + W_{bf}$

$= 300 + 50 + 100$

$Q_{ib} + Q_{bf} = 450$ J ...(1)

Also, $Q_{ib} = \Delta_{ib} + W_{ib}$

$\therefore \quad Q_{ib} = 100 + 50 = 150$ J ...(2)

From (1) & (2) $\frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$

SECTION – C

13. 01732.50

Sol. Probability that a particular nucleus will decay in next 2 hr is

$\frac{N_0 - N}{N_0} = 8 \times 10^{-4}$ [N = surviving population after 2 hr]

$N_0[1 - 8 \times 10^{-4}] = N$

$\Rightarrow \quad 1 - 8 \times 10^{-4} = e^{-\lambda t} \qquad \left[\because \frac{N}{N_0} = e^{-\lambda t}\right]$

$\Rightarrow \quad e^{-\lambda t} = 0.9992 \Rightarrow -\lambda t = \ell n(0.9992)$

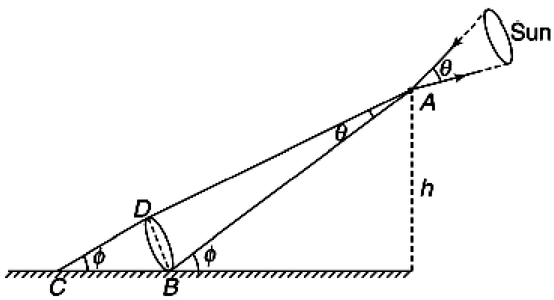
$-\frac{\ell n 2}{t_{1/2}} \times 2 = \ell n(0.9992) \quad [\because t = 2hr]$

$t_{1/2} = \frac{2\ell n 2}{\ell n(0.9992)} hr$

$= \frac{2 \times 0.693}{8 \times 10^{-4}} = 1732.5 \text{ hr}$

14. 00003.44

Sol. The cone of rays passing through the hole at a produce an elliptical spot on the floor.
The circular base having diameter BD will get projected on the floor as an ellipse.



CB = major axis = 12 cm

DB = Minor axis = 6 cm

$DB \simeq AB(\theta) = \frac{h\theta}{\sin\phi} \Rightarrow h = (6cm)\frac{\sin\phi}{\phi}$

But $\quad \sin\phi = \frac{DB}{CB} = \frac{6}{12} = \frac{1}{2}$

$\therefore \quad h = \frac{(6\text{ cm})}{2 \times \theta}$

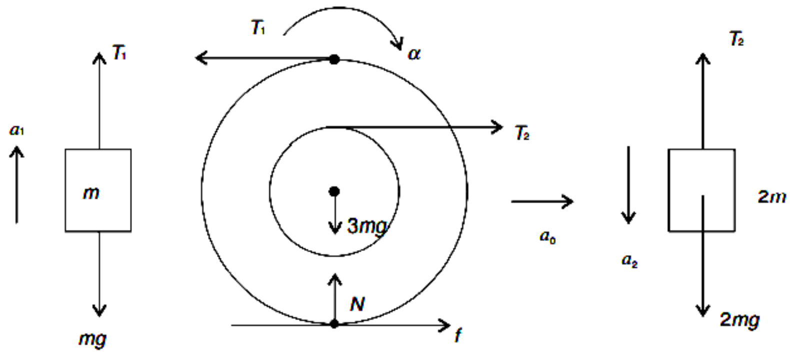
$= \frac{3cm \times 180}{0.5 \times \pi} \quad \left[0.5° = \frac{0.5 \times \pi}{180} \text{radian}\right]$

= 344 cm

= 3.44 m

15. 00000.11

Sol. Let acceleration of COM of the spool be a_0 towards right, and acceleration of m_2 be a_2 downward and that of m_1 be a_1 upward



f = friction, α = acceleration of the spool

$2R\alpha = a_0$ ...(1)

$R\alpha + a_0 = a_2$ ...(2)

$2R\alpha + a_0 = a_1$ ...(3)

$T_1 - mg = ma_1$

$T_1 - mg = m(a_0 + 2R\alpha) = 2ma_0$ ....(4) $\left[\because R\alpha = \frac{a_0}{2}\right]$

$2mg - T_2 = 2ma_2$

$2mg - T_2 = 2m[a_2 + R\alpha] = 3ma_a$ ...(5)

$T_2 + f - T_1 = 3ma_0$ ...(6)

$T_2R - T_12R - f2R = I\alpha$

$T_2 - 2T_1 - 2f = 2mR\alpha \quad [\because I = 2mR^2]$

$T_2 - 2T_1 - 2f = ma_0$ ...(7)

Solving, (4), (5), (6) & (7) we get $f = -\frac{mg}{3}$

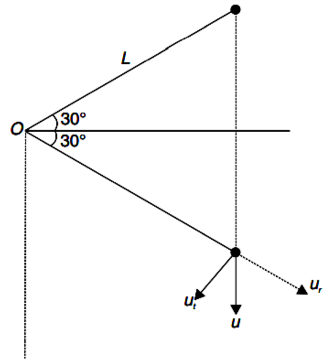
$\because f \le \mu N \quad \therefore \frac{mg}{3} \le \mu 3mg \quad \therefore \frac{1}{9} \le \mu$

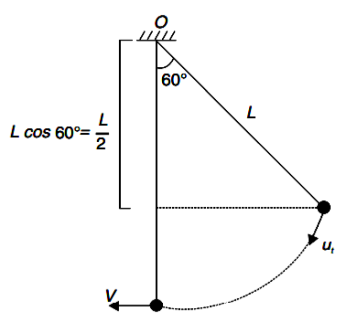
16. 00035.00

Sol. The bob will experience free fall for a distance of L = 1.8 m.

Speed of the bob just before the string gets taut is

$u = \sqrt{2gL} = \sqrt{2 \times 10 \times 1.8} = 6 \text{ m/s}$





$$u_t = u\cos 30^\circ = \frac{\sqrt{3}u}{2} = 3\sqrt{3}\ \text{m/s}$$

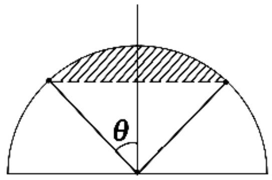
$$\frac{1}{2}mV^2 = \frac{1}{2}mu_t^2 + mg\frac{L}{2}$$

$$V = 3\sqrt{5}\ \text{m/s}$$

$$T = 10 + \frac{45}{1.8} = 35\text{N}$$

17. 00002.45

Sol.

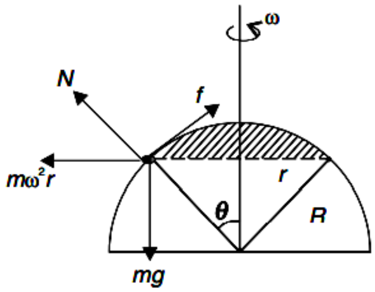


$$s = \Omega R^2 = 2\pi R^2(1-\cos\theta)$$

If $s = (0.2)\,(2\pi R^2)$ then

$0.2 = 1 - \cos\theta$

$\Rightarrow \quad \cos\theta = 0.8 \Rightarrow \theta = 37^\circ$



$$N = mg\cos\theta - m\omega^2 r\sin\theta$$

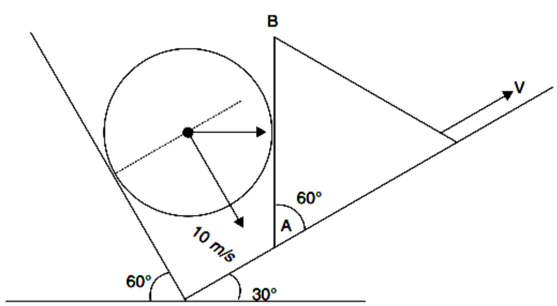
$$\mu(mg\cos\theta - m\omega^2 r\sin\theta) = mg\sin\theta + m\omega^2 r\cos\theta$$

$$\therefore \frac{\mu g\sin\theta + \omega^2 r\cos\theta}{g\cos\theta - \omega^2 r\sin\theta}$$

$$= \frac{10\times 0.6 + 10^2\times 0.06\times 0.8}{10\times 0.8 - 10^2\times 0.06\times 0.6} = \frac{0.6+0.48}{0.8-0.36} = 2.45$$

18. 00005.77

Sol. Wall AB of the wedge is vertical. For ball to remain in contact with the wedge the velocity component of the ball perpendicular to the wall AB must be equal to velocity component of the wedge in horizontal direction (i.e., perpendicular to wall AB)



$$\therefore 10\cos 60^\circ = V\cos 30^\circ$$

$$5 = V\frac{\sqrt{3}}{2}$$

$$V = \frac{10}{\sqrt{3}}\ \text{m/s}$$

Chemistry PART – II

SECTION – A

19. A, B, C, D

Sol. (A) 2-D-hexagonal packing

$$\text{Packing efficiency} = \frac{\pi R^2 + 6 \times \frac{\pi R^2}{3}}{6 \times \frac{\sqrt{3}(2R)^3}{4}}$$

(B) simple cubic $= \dfrac{1 \times \frac{4}{3}\pi R^3}{a^3}$

$a = 2R$

$$\text{H.C.P.} = (3D) = \frac{6 \times \frac{4}{3}\pi R^3}{24\sqrt{2}R^3}$$

$$\text{F.C.C.} = (3D) = \frac{4 \times \frac{4}{3}\pi R^3}{(2\sqrt{2}R)^3}$$

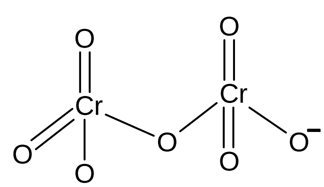
20. A, C, D

Sol. $KE = \frac{3}{2}RT(1\,\text{mole})\,U_{rms} = \sqrt{\frac{3RT}{M_w}}$

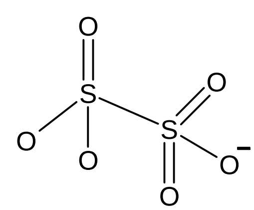
$d = \dfrac{PM_w}{RT}$

21. A, B, C, D

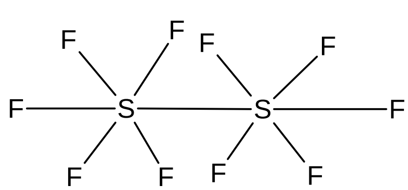
Sol. (A)



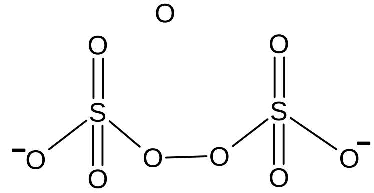
(B)



(C)



(D)



22. A, B, C

Sol. Factual

23. A, C

Sol. $H_2A \rightleftharpoons H^+ + HA^-$; $\quad \because$ y is very less so $\underset{x-y=x}{x+y} \approx x \;\therefore\; K_{a_1} = \dfrac{x^2}{c-x}$

at equilibrium C-X x+y x-y $K_{a_2} = y = 10^{-12}; pK_{a_2} = 12$

$HA^{-1} \rightleftharpoons H^{+} + A^{2-}$
at equilibrium x-y x+y y
$pK_{a_2} - pK_{a_1} = 12 - 5 = 7$

$pH = \frac{1}{2}[pK_{a_1} - \log c]$

$3 = \frac{1}{2}[pK_{a_1} + 1] \quad pK_{a_1} = 5$

24. C
Sol. As ΔV is positive w is negative
$W = -P\Delta V$
So work is done on the system.

SECTION – B

25. 3
Sol. Fraction of edge unoccupied $= \frac{a-2R}{a}$

$a = 2\sqrt{2}R \qquad X = \frac{2(\sqrt{2}-1)}{2\sqrt{2}}$

$X = \frac{0.414}{1.414} = 0.293$

$Z = \frac{X}{0.097} = \frac{0.293}{0.097} = 3$

26. 00007.00
Sol. alk. $KMnO_4$ & $Na_2S_2O_3$ will not
React with kI to give I_2

27. 8
Sol. Process AC = polytropic process (P = KV)
Molar Heat capacity $c_m = c_V + R/2 = 2R$
Process AB = Isobaric
$c_m = c_p = 5R/2$

$$\frac{q_{AC}}{q_{AB}} = \frac{\int_{T_A}^{T_C} nC_m.dT}{\int_{T_A}^{T_B} n.C_{p,m*}dT} = \frac{2R}{\frac{5}{2}R} = 0.8$$

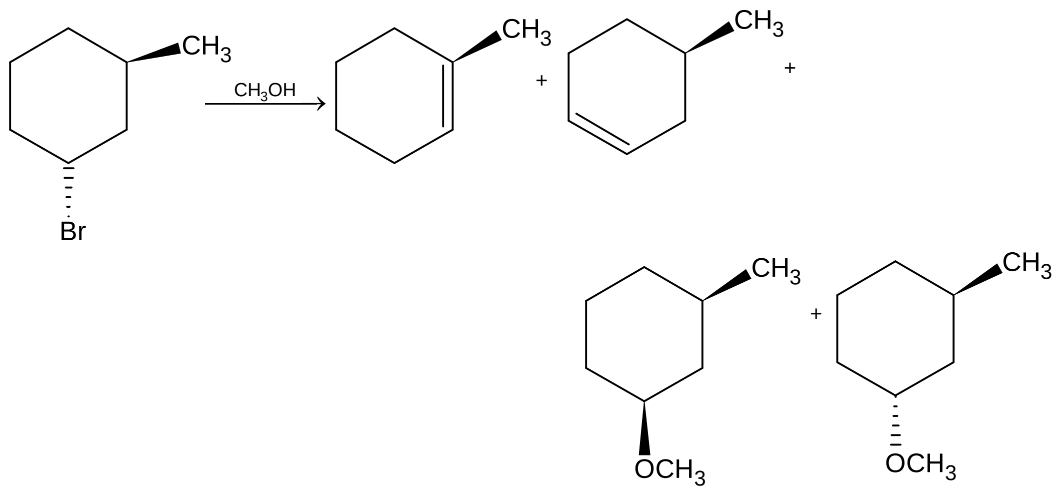
$$\frac{q_{AC}}{q_{AB}} = \times 10 = 0.8 \times 10 = 8$$

28. 3
Sol. Ca, CaH_2 and produce H_2 gas with cold water.

29. 4
Sol. $\underset{\text{(Metal oxide basic)}}{MO} + \underset{\text{(Acedic)}}{SiO_2} \rightarrow \underset{\text{(slag)}}{MSiO_3}$

30. 4

Sol.



SECTION – C

31. 00024.06

Sol. $k = \frac{2.303}{10}\log\frac{0.04}{0.03}$

$t_{1/2} = \frac{0.693}{K}$

32. –0.24

Sol. $-0.12 - \frac{0.0591}{2}\log\left(\frac{1}{x}\right) = -0.24$

$\log\frac{1}{x} = \frac{0.12\times 2}{0.06} = 4$

$x = 10^{-4}$

33. 00000.10

Sol. $\alpha = \sqrt{\frac{Ka}{C}}$

34. 00000.50

Sol. $\frac{\alpha_1}{\alpha_2} = \sqrt{\frac{C_2}{C_1}} \Rightarrow \frac{0.01}{\alpha_2} = \sqrt{\frac{0.4}{0.1}}$

$\alpha_2 = 0.005$ or $\%\,\alpha_2 = 0.5$

35. 27419.25

Sol. $\bar{V} = R \times \left[\frac{1}{2^2} - \frac{1}{\alpha^2}\right]$

36. 00004.52

Sol. $K_c = \frac{[\text{complex}]}{[H_3BO_3][\text{glycerine}]}$.

Mathematics PART – III

SECTION – A

37. A, D

Sol. $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$ …(1)

$\Rightarrow \quad a_0 + a_2 + a_4 \ldots = \dfrac{3^n+1}{2}$

$a_1 + a_3 + a_5 \ldots = \dfrac{3^n-1}{2}$

Put $x = i$ in equation (1) where $i = \sqrt{-1}$

If $n = 4m+1$

$\Rightarrow \quad a_1 - a_3 + a_5 \ldots = 1$

$a_0 - a_2 + a_4 \ldots = 0$

So, $a_1 + a_5 + a_9 \ldots = \dfrac{3^n+1}{4} = S_2$

$a_3 + a_7 + a_{11} \ldots = \dfrac{3^n-3}{4} = S_4$

Similarly if $n = 4m+3$

Then, $S_2 = \dfrac{3^n-3}{4}$ and $S_4 = \dfrac{3^n+1}{4}$

38. A, B, C, D

Sol. $f_1(x) = \dfrac{\pi}{2} - \sin^2 x,\; f_2(x) = \dfrac{\pi}{2} - \cos^2 x$

$f_3(x) = \dfrac{\pi}{2} - \cos^2 x,\; f_4(x) = \dfrac{\pi}{2} - \sin^2 x$

39. A, B

Sol. $c^2 = a^2 + b^2 - 2ab\cos C$

$= (a-b)^2 + 2ab(1-\cos C)$

$\Delta = \dfrac{1}{2}ab\sin C$

or $2ab = \dfrac{4\Delta}{\sin C}$

here $c^2 = (a-b)^2 + 4\Delta\left(\dfrac{1-\cos C}{\sin C}\right)$

$= (a-b)^2 + 4\Delta\tan\dfrac{C}{2}$

$4\Delta\tan\dfrac{C}{2}$ is constant so for c^2 to be minimum a = b thus $2ab = 2a^2 = \dfrac{4\Delta}{\sin C}$

$\Rightarrow a = b = \sqrt{\dfrac{2\Delta}{\sin C}}$.

40. B, D

Sol. f(x) = $x^5 - 10a^3 x^2 + b^4 x + c^5$

$f'(x) = 5x^4 - 20a^3 x + b^4$

$f''(x) = 20 x^3 - 20a^3$

If x = α be a root that is repeated three times,

$\Rightarrow f''(\alpha) = 0, f'(\alpha) = 0, f(\alpha) = 0$

$\Rightarrow \alpha = a, 5a^4 - 20 a^4 + b^4 = 0, a^5 - 10a^5 + ab^4 + c^5 = 0$

$\Rightarrow \alpha = a, b^4 = 15a^4, c^5 + ab^4 - 9a^5 = 0 \Rightarrow c^5 + 15a^5 - 9a^5 \Rightarrow 6a^5 + c^5 = 0.$

41. A, D

Sol. Since $\int_a^b f(x)dx = (b-a)\int_0^1 f\{(b-a)x + a\}dx$,

$$\int_1^2 \sin x^2 dx = \int_0^1 \sin(x+1)^2 dx = \int_0^1 \sin(x^2 + 2x + 1)dx$$

$$\int_{-4}^4 \cos x^2\, dx = 8\int_0^1 \cos(8x-4)^2\, dx$$

$$= 8\int_1^0 \cos 16(2x-1)^2\, dx .$$

42. B, C

Sol. f(x) = 0

If $\sin x \neq \pm 1$ $(1 \leq 1 + \sin^2 x < 2, \ \sin^2 = 2)$

$= \pi/3$ If $\sin x = \pm 1$

So f(x) is not continuous at the points, where $\sin x = \pm 1$ i.e. x is an odd multiple of $\pi/2$

SECTION – B

43. 6

Sol. AB = 22 = AC

Now AP + PR + AR = (AP + PQ) + (QR + AR)

= AB + AC = 44.

44. 3

Sol. $|z| + |z-1| + |2z-3| = |z| + |z-1| + |3-2z| \geq |z + z - 1 + 3 - 2z| = 2$

$\therefore$ $|z| + |z-1| + |2z-3| \geq 2$

$\therefore$ $\lambda = 2$

then $2[x] + 3 = 3[x - \lambda]$

$= 3[x - 2]$

$\Rightarrow$ $2[x] + 3 = 3([x] - 2)$

or $[x] = 9$, then $y = 2.9 + 3 = 21$

$\therefore$ $[x + y] = [x + 21] = [x] + 21 = 9 + 21 = 30$

45. 2

Sol. No. of fights = ${}^{24}C_2 - {}^{x}C_2 - {}^{x+1}C_2 - {}^{23-2x}C_2$

f(x) = $-3x^2 + 45x + 23$.

$f(x) = 0 \Rightarrow x = \frac{15}{2} = 7.5$

but x can't be a fraction.

So x = 7 (nearest integer)

$\therefore$ f(x) = 191 Ans.

46. 2

Sol. $k=1 \quad \frac{1}{3}\left({}^1C_0 + {}^1C_0\right) = \frac{1}{3}\times 2 = \frac{2}{3}$

$k=2 \quad \frac{1}{3^2}\left({}^2C_0 + {}^2C_1 + {}^2C_2\right) = \frac{2^2}{3^2} = \left(\frac{2}{3}\right)^2$

$k=3 \quad \frac{1}{3^3}\left({}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3\right) = \frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3$

– –
– –

$\therefore$ Reg. Sum = $\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + ...\infty$

$= \frac{\frac{2}{3}}{1-\frac{2}{3}} = 2$

47. 4

Sol. equation has four positive real root.

$\Rightarrow \quad \frac{\frac{1}{x_1}+\frac{2}{x_2}+\frac{3}{x_3}+\frac{4}{x_4}}{4} \geq \left(\frac{24}{x_1, x_2, x_3, x_4}\right)^{1/4} = 2.$

$\Rightarrow \quad \frac{1}{x_1} + \frac{2}{x_2} = \frac{2}{x_3} = \frac{4}{x_4} = k$

$\Rightarrow \quad k = 2 \quad \therefore \textit{roots are } \frac{1}{2}, 1, \frac{3}{2} \textit{ and } 2$

$\Rightarrow \quad \frac{x_4}{x_1} = 4$

48. 2

Sol. Since $\alpha 2 - 2\alpha + 3 = 0$ & $\beta 2 - 2\beta + 3 = 0$

$\therefore \quad \alpha 3 - 3\alpha 2 + 5\alpha - 2 = \alpha 3 - 2\alpha 2 + 3\alpha - \alpha 2 + 2\alpha - 3 + 1$

$= \alpha(\alpha 2 - 2\alpha + 3) - (\alpha 2 - 2\alpha + 3) + 1 = 1$

And $\beta 3 - \beta 2 + \beta + 5 = \beta 3 - 2\beta 2 + 3\beta + \beta 2 - 2\beta + 5 = \beta(\beta 2 - 2\beta + 3) + (\beta 2 - 2\beta + 3) + 2 = 2$

So the ref. equation is

x2 – (2 + 1) x + 2.1 = 0

x2 – 3x + 2 = 0

SECTION – C

49. 58060.80

Sol. 10 IIT students $T_1, T_2, ...\ T_{10}$ can be arranged in 10 ! ways. Now the number of ways in which two PET student can be placed will be equal to the number of ways in which 3 consecutive IIT students can be taken i.e. in 8 ways and can be arranged in two ways Þ (10 !) (8) (2 !).

Alternatively 3 IIT students can be selected in ${}^{10}C_3$ ways. Now each selection of 3 IIT and 2 PET students in $P_1\ T_1\ T_2\ T_3\ P_2$ can be arranged in (2 !) (3 !) ways. Call this box X. Now this X and the remaining IIT students can be arranged in 8 ! ways

$\Rightarrow$ Total ways ${}^{10}C_3$ (2 !) (3 !) (8 !)]

50. 08855.00

Sol. x + y + z + w < 25

let x + y + z + w + a = 25 such that a > 0 ....(i)

$x = -1 + t_1 \quad \therefore \quad t_1 \geq 0$

$y = 2 + t_2 \quad \therefore \quad t_2 \geq 0$

$z = t_3 \quad \therefore \quad t_3 \geq 0$

$w = 4 + t_4 \quad \therefore \quad t_4 \geq 0$

$a = 1 + t_5 \quad \therefore \quad t_5 \geq 0$

put in equation (i)

$(-1 + t_1) + (2 + t_2) + t_3 + (4 + t_4) + (1 + t_5) = 25$

$t_1 + t_2 + t_3 + t_4 + t_5 = 19$

by fictitious partition solution ${}^{19+\overline{5-1}}C_{19} = {}^{23}C_{19}$

51. 01002.00

Sol. $x^3 - 3xy^2 = 2005 \Rightarrow \left[\left(\frac{x}{y}\right)^3 - 3\left(\frac{x}{y}\right) = \frac{2005}{y^3}\right] \times 2004$ ....(1)

$y^3 - 3x^2y = 2004 \Rightarrow \left[1 - 3\left(\frac{x}{y}\right)^2 = \frac{2004}{y^3}\right] \times 2004$ ....(2)

Subtract (1) & (2) & put $\frac{x}{y} = t$

$2004t^3 + 6015t^2 - 6012t - 2005 = 0$ → t_1, t_2, t_3

$$\frac{y_1.y_2.y_2}{(y_1 - x_1)(y_2 - x_2)(y_3 - y_3)} = \frac{1}{(1-t_1)(1-t_2)(1-t_3)}$$

$$= \frac{1}{1 + (t_1t_2 + t_2t_3 + t_3t_1) - (t_1 + t_2 + t_3) - t_1t_2t_3}$$

put values

= 1002

52. 00010.20

Sol. $\left(1 - \frac{2}{3}\right)^{-\frac{1}{2}} = 1 + \frac{1}{2}.\frac{2}{3} + \frac{\frac{1}{2}.\frac{3}{2}}{\lfloor 2}.\left(\frac{2}{3}\right)^2 + \frac{\frac{1}{2}.\frac{3}{2}.\frac{5}{2}\left(\frac{2}{3}\right)^3}{\lfloor 3}$

$\Rightarrow \left(\frac{1}{3}\right)^{-\frac{1}{2}} - = \frac{1}{3} + \frac{1.3}{(3)^2\lfloor 2} + \frac{1.3.5}{(3)^3\lfloor 3} + ...\infty$

$\sqrt{3} - 1 = z \qquad \Rightarrow z^2 + 2z - 2 = 0$

$\because \qquad z = \frac{-2 \pm \sqrt{4+8}}{2}$

$\because \qquad z = -2 \pm \sqrt{3},\ but\ z \neq -1 - \sqrt{3}$

53. 00003.50

Sol. Let f(x) = ax^2 + bx + c, $\because \int_\alpha^\beta f(x)dx > 0 \quad \forall \alpha, \beta \in R$

$a, b, c \in I^+ \qquad f(x) > 0 \Rightarrow D < 0$

$f''(x) = 2a \qquad \Rightarrow g(t) = 2a(at^2 + bt + c) = 2a^2t^2 + 2abt + 2ac.$

$g(0) = 2ac = 12 \Rightarrow \qquad ac = 6.$

$$\left.\begin{array}{ll} a = 6, & c = 1 \\ a = 1 & c = 6 \\ a = 3 & c = 2 \\ a = 2 & c = 3 \end{array}\right] \Rightarrow b^2 < 24; \qquad b = 1, 2, 3, 4$$

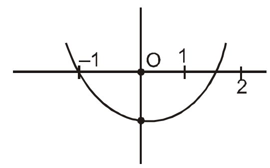
$\therefore$ 16 quadratic are possible

54. 00001.00

Sol. Let f(x) = ax^2 + (a – 2) x – 2

f(0) = –2

and f(–1) = 0



Since the quadratic expression is negative for exactly two integral values

$\Rightarrow$ f(1) < 0 and f(2) $\geq$ 0

$\Rightarrow$ a + a – 2 – 2 < 0 and 4a + 2a – 4 – 2 $\geq$ 0

$\Rightarrow$ a < 2 and a $\geq$ 1

$\therefore$ a $\in$ [1, 2)