ALL INDIA TEST SERIES

TEST - 2

JEE (Advanced)

Time Allotted: 3 Hours **Maximum Marks: 198**

General Instructions:

- The test consists of total **54** questions.
- Each subject (PCM) has **18** questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Three Sections**: **Section-A, Section – B & Section-C.**

Section-A (01 – 06, 19 – 24, 37– 42) *this section contains* ***18 multiple*** *choice questions.*
Each question has FOUR options. ***ONE OR MORE THAN ONE*** *of these four option(s) is (are) correct answer(s).*
For each question, choose the option(s) corresponding to (all) the correct answer(s)
Answer to each question will be evaluated according to the following marking scheme:
Full Marks **: +4** If only (all) the correct option(s) is (are) chosen:
Partial Marks **: +3** If all the four options are correct but ONLY three options are chosen;
Partial Marks **: +2** If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks **: +1** If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks **: 0** If none of the options is chosen (i. e. the question is unanswered);
Negative Marks **: –2** In all other cases

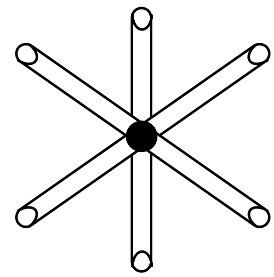
Section-B (07 – 12, 25 – 30, 43– 48) *contains* ***18 Numerical*** *based questions with* ***Single digit integer*** *as answer, ranging from* ***0 to 9*** *and each question carries* ***+3 marks*** *for correct answer and*
–1 mark *for wrong answer.*
Section-C (13 – 18, 31 – 36, 49– 54) *contains* ***18 Numerical*** *answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer and* ***0 marks*** *for wrong answer.*

Physics PART – I

SECTION – A
(One or More than one correct type)

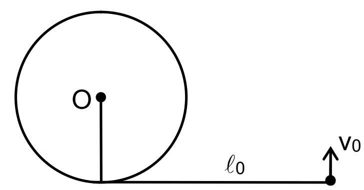
This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

1. An object having all of mass concentrated at centre and having six identical rods of length ℓ each attached to it symmetrically is placed on an inclined rough surface of inclination α. Object looks like a wheel with six equally spaced spokes without rim. Assume that there is no slipping and collision between the spokes and plane is perfectly inelastic and wheel always remains in the vertical plane. Pick correct statement(s) for the motion of wheel.



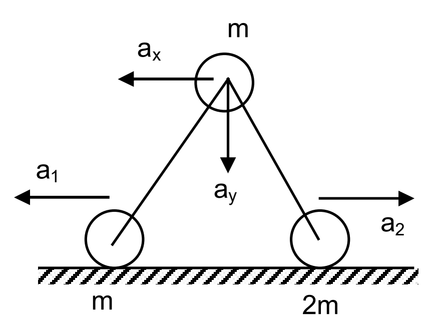
(A) Average velocity of the centre of object increases and attains a constant value (Assuming object remains in contact with the plane and initial kick was given to start motion)
(B) Mechanical energy of object remains conserved
(C) Work done by friction is zero
(D) The value of constant average velocity is $\sqrt{\dfrac{8g\ell\sin\alpha}{3}}$

2. A point mass m is attached to an ideal string, which in turns is wrapped on a vertical pole. Point mass is kept on a smooth horizontal surface and string is horizontal. A horizontal velocity is imparted to the point mass perpendicular to length of string.



(A) Speed of mass will become twice of initial value if length of string between the point mass and the pole becomes half of the original value.
(B) Kinetic energy of mass increases.
(C) Work done by tension on the point mass is zero.
(D) Energy of the mass remains constant.

3. Three masses are joined using pin joints (can move freely). Length of both rods are identical and system starts from rest. Accelerations of the balls are shown in figure. Angle between the rods at this moment is 90°. The horizontal surface is smooth. Choose the correct option(s).
(A) $2a_2 = a_1 + a_x$
(B) $2a_y = a_1 + a_2$
(C) $2a_x = a_1 - a_2$
(D) $3a_1 = 5a_2$



4. A particle of mass m is subjected to two forces

$\vec{F}_1 = f(r)\dfrac{\vec{r}}{|\vec{r}|}$ and $\vec{F}_2 = -\alpha\vec{v}$

Where f(r) is a real valued function depends on the distance of particle from origin, $\vec{r}$ is position vector of particle and α is positive constant. If at t = 0, $\vec{v}$ vector is in x-y plane and angular momentum of the particle about the origin is L_0. Choose the correct option(s).

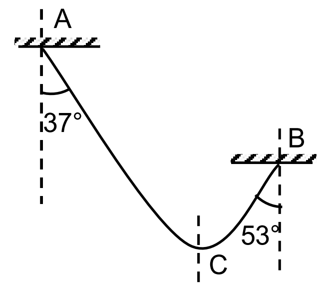
(A) Plane of motion of particle does not change.

(B) Angular momentum of particle about the origin is given by $L_0 e^{-\frac{\alpha t}{m}}$

(C) Angular momentum of particle about the origin is remains constant.

(D) The value of angular momentum about the origin will depend on f(r)

5. A uniform massive string of mass m, length ℓ is fixed between two rigid supports. Angles made by the tangents on the string at points of suspension are shown in the figure. The point C is the lower most point of the string. Then choose the correct option(s).



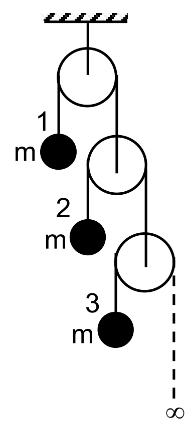
(A) Length of string left of the point C is $\frac{16\ell}{25}$

(B) Tension in the string at point C is $\frac{12mg}{5}$

(C) Radius of curvature of the string at point C is $\frac{12\ell}{25}$

(D) Tension in the string at point B is $\frac{3mg}{5}$

6. An Atwood's machine having infinite number of masses connected with ideal strings as shown in the figure. All the masses are identical and equal to m. All the pulleys are ideal. Then choose the correct option(s).



(A) Acceleration of mass-1 is g/2

(B) Ratio of acceleration of mass-1 and mass-2 is 2

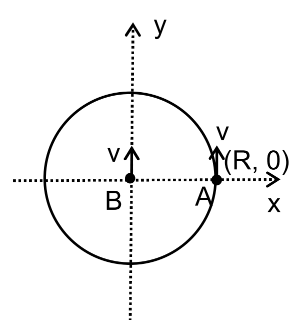
(C) Total work done by tension is zero

(D) Acceleration of mass-1 can't be determined

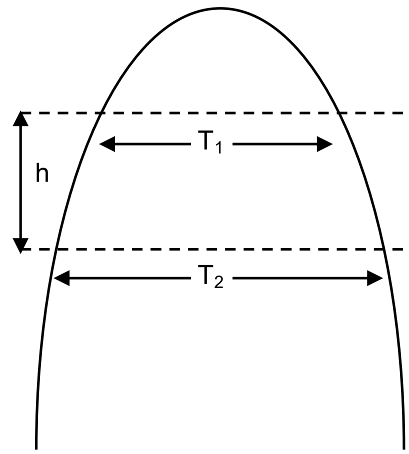
SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

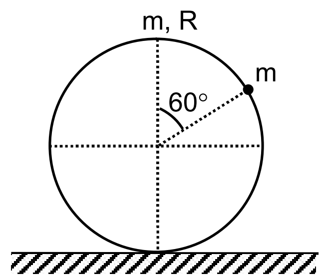
7. Two particles move with same speed v in x-y plane. Particle A moves on the circle of radius R centered at origin while particle B moves along the positive y-axis. At t = 0 particle A is at (R, 0) and B is at the origin. Average angular velocity of B as seen by A in the time interval $\frac{\pi R}{2v}$ is $\frac{\alpha v}{R}$. Find the value of α.



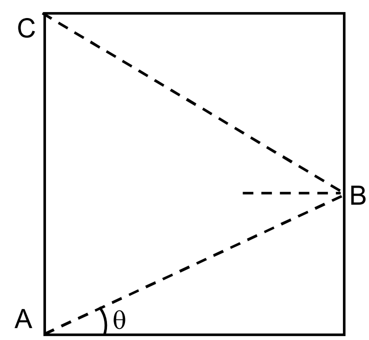
8. A particle is projected from the horizontal surface. Times taken by the particle to pass through two horizontal lines separated by height h are T_1 and T_2. If $T_2 = 3T_1$ then find the value of $\left(\frac{gT_1^2}{h}\right)$.



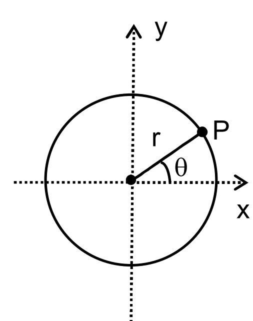
9. A particle of mass m is rigidly attached to the circumference of a uniform circular ring of mass m and radius R and placed on a rough horizontal surface. The ring is released from rest from the initial position as shown in the figure and it starts rolling without slipping on the rough horizontal surface. If the frictional force acting on the ring due to horizontal surface when the particle is getting at the end of the horizontal diameter is $k\left(\frac{mg}{8}\right)$. Find the value of k.



10. Inside a freely falling cubical elevator a point mass is to be projected from the bottom left corner A towards right wall such that it collides with the wall at B and finally reaches the point C, top left corner of the elevator. Points A, B and C lie in the same vertical plane. Coefficient of restitution for the collision between the wall and the ball is $e = \frac{1}{2}$. Then find the value of cot θ. (where θ = angle made by initial velocity of the ball with the horizontal x-axis.



11. Potential energy of a particle moving on a circle in x-y plane is given by $V = V_0 r^3 \cos\theta$. Where r is the radius of circle and θ is the angle made by the radius vector of the particle with the positive x-axis. Magnitude of x-component of force, when the particle is at $\left(\frac{r}{\sqrt{2}}, \frac{r}{\sqrt{2}}\right)$ is given by $\frac{\alpha V_0 r^2}{8}$. Then find the value of α.



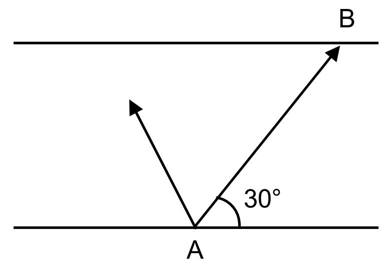
12. A droplet of water in uniform fog is formed. It then falls sweeping up the fog lying in its path. Assume that it retains all the fog in its path, remains spherical and experiences zero drag. Drop attains a constant value of acceleration after a large time interval. Its acceleration is given by $\frac{g}{K}$. Find the value of K.

SECTION – C
(Numerical Answer Type)

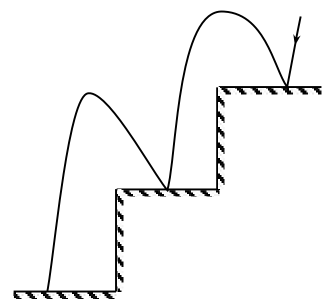
This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

13. Engine of a car can deliver a maximum power of 50 kW. Car starts from rest and experiences a drag force $\vec{F} = -k\vec{V}$, where k = 5 N-s/m. If mass of the car is 500 Kg, find the maximum velocity (in m/s) that the car can attain while it moves on a straight horizontal road is

14. While crossing a river of width 10 m a swimmer wants to follow path AB, as shown in the figure. Velocity of the river current is 5 ms^{-1}. Velocity of the swimmer in still water is also 5 ms^{-1}. Swimmer starts swimming making an angle θ with the direction of the river current. The value of θ (in degrees) is

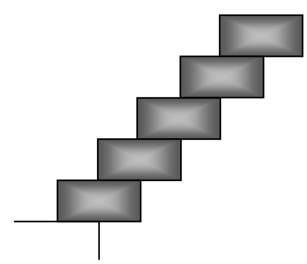


15. Length and height of each step is 10 cm. A ball bounces down the stairs in regular manner, hitting each step at identical points and then the ball bounces upto same height above each step. Coefficient of restitution between the stairs and the ball is e = 1/3. Find the horizontal component of velocity of the ball (in ms^{-1}). (Assume that this process can go on for infinite times).

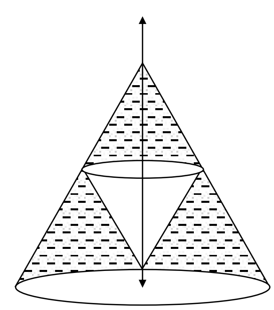


16. A uniform spherical shell of radius 10 cm is divided into eight identical parts. Find the distance (in cm) between the geometric centre of the complete spherical shell and the centre of mass of one part. (Take $\sqrt{3} = 1.732$)

17. Five identical bricks of uniform density and length 10 cm each are placed on the top of each other at the edge of a table such that they remain in equilibrium and corner of top most brick is farthest from the edge of the table. Horizontal projection of the distance between the farthest corner of top most brick and edge of the table is (in cm).



18. An inverted cone of height h is scooped out of a uniform cone of radius 2h and height 2h symmetrically as shown in the figure. If moment of inertia of the removed portion about the axis shown is I and the moment of inertia of the remaining portion is λI. Find the value of λ.



Chemistry PART – II

SECTION – A
(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

19. Select the correct statement with respect to Na_2O_2
    (A) It decolourises acidified $KMnO_4$
    (B) It is obtained along with Na metal when Na_2O is heated to a temperature more than $400^{o}C$
    (C) On heating with oxygen at $450^{o}C$ and 300 atm it becomes paramagnetic
    (D) It gives both H_2O_2 and O_2 with water

20. Which of the following disproportionate on heating with NaOH?
    (A) P_4
    (B) S_8
    (C) Cl_2
    (D) Boron

21. Which of the following are correct regarding s-Block elements
    (A) The reducing character of the alkali metal hydride follows the order: $CsH > RbH > KH > NaH > LiH$
    (B) The bicarbonates of alkaline earth metal are only stable in solution
    (C) LiI is the most ionic in nature among the alkali metal halides
    (D) Magnesium nitrate on heating decomposes to give NO_2 and O_2

22. Which of the following are correct regarding Si compounds?
    (A) Cross link silicones are obtained by hydrolysis of $RSiCl_3$
    (B) Linear silicone are obtained by hydrolysis of R_2SiCl_2
    (C) $Sc_2(Si_2O_7)$ is a example of pyrosilicate
    (D) In three dimensional sheet silicates three oxygen atom of SiO_4^{-4} is shared with adjacent SiO_4^{-4} unit

23. Which of the following statement are correct?
    (A) Ionic carbide are formed by highly electropositive metals
    (B) Carborundum and boron carbide are covalent compound
    (C) Carborundum and boron carbide are used as abrasive
    (D) Al_4C_3 and CaC_2 on hydrolysis gives C_2H_2

24. Which of the following are polar?
    (A) XeO_3
    (B) $XeOF_4$
    (C) XeF_4
    (D) BrF_6^{+}

SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

25. $xU(SO_4)_2 + yKMnO_4 + zH_2O \longrightarrow H_2SO_4 + K_2SO_4 + MnSO_4 + UO_2SO_4$
Find the value of x + y + z? (Minimum integral value of x, y, z)

26. 100 ml of sample of tap water contain 0.192 mg of Mg^{+2}, what is the hardness as parts of $CaCO_3$ per million part of water.

27. Two bulb P and Q are emitting monochromatic light of wave length such that P can just ionize H-atom and Q can just ionize He^{+} ion. If power of P and Q are 60 W and 80 W. The ratio of number of photons emitted per sec by bulb P and Q is:

28. For the following reaction
$A + B \rightleftharpoons C + D \qquad K_c = 16$
if 0.4 mole of A and 0.4 mole of B, 2 moles of C and 2 moles of D are mixed in container of volume 80 ml. Then the equilibrium concentration of B will be.

29. During a decay of 952 gram ${}_{92}U^{238}$ to ${}_{82}Pb^{206}$, total 18.066×10^{24} $\alpha-$particle are emitted, number of half lives completed in this decays are

30. What is the concentration ratio of sodium acetate to acetic acid when pH of solution is 5.35. [pKa of acetic acid = 4.75]

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

31. A diatomic molecule has a dipole moment of 1.2 D. if its bond length is equal to 10^{-10}m then the percentage ionic character is x%. Find x. (1 D = 3.336×10^{-30} C $\times$ metre)

32. Bromophenol is an acid indicator having dissociation constant 10^{-5}, the pH of solution at which indicator is 80% ionized. [log2 = 0.3]

33. When one mole of $K_4[Fe(CN)_6]$ is heated with conc. H_2SO_4, the evolved gas during reaction is reacted with excess of I_2O_5 .find how many moles of I_2 will be liberated during the reaction

34. A flask is filled with $N_2O_3(g)$ at 2 bar pressure, and following equilibrium is established.
Reaction 1: $N_2O_3(g) \rightleftharpoons NO_2(g) + NO(g)$; $K_{p_1} = 2.5$
Reaction 2: $2NO_2(g) \rightleftharpoons N_2O_4(g)$; $K_{p_2} = ?$
If at equilibrium partial pressure of NO (g) is 1.5 bar, then find equilibrium constant for second reaction

35. $A \xrightarrow{K_1(\text{sec}^{-1})} B$

$C \xrightarrow{K_2(M^{-1}\ \text{sec}^{-1})} D$

Starting with 1 molar initial conc. of reactant of each reaction, it takes 10 sec. to reach to 0.5 molar in each reaction. Then find the ratio of k_1/k_2.

36. In the following reaction started only with A

$2A \rightleftharpoons 2B + 3C + D$

mole fraction of C is found to be 0.36 at a total pressure of 200 atm. The mole fraction of A at equilibrium is (Note : Reactant and product are gases).

Mathematics PART – III

SECTION – A
(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

37. Let $I_1 = \int_0^{512} \frac{\{x^{1/3}\}}{\sqrt[3]{x^2}} dx$ and $I_2 = \int_0^2 x^2 \{x^3\} dx$ (where {.} denotes fractional part of function), then
(A) $I_1 = I_2$
(B) $I_1 = 9I_2$
(C) $I_1 = 12$
(D) $I_2 = 4$

38. Let the functions f: (–1, 1) $\rightarrow$ R and g: (–1, 1) $\rightarrow$ be defined as f(x) = |3x – 2| + |3x + 2| and g(x) = x – [x] (where [x] denotes greatest integer less than or equal to x). Let fog : (–1, 1) $\rightarrow$ R be the composite function defined by (fog)(x) = f(g(x)). Suppose c is the number of points in the interval (–1, 1) at which fog is NOT continuous and suppose d is the number of points in the interval (–1, 1) at which fog is NOT differentiable, then
(A) c = 1
(B) c = 2
(C) d = 3
(D) d = 2

39. Which of the following inequalities is/are true?
(A) $\int_0^1 x \tan x dx \geq \frac{2}{5}$
(B) $\int_0^1 x^2 \cos x \leq \frac{1}{3}$
(C) $\int_0^1 x^3 \sin x dx \geq \frac{3}{20}$
(D) $\int_{-1}^0 x \sin x \geq \frac{3}{10}$

40. If f(x) is a function defined on the set of non negative integers and having co-domain also as set of non negative integers, given that
(i) $x - f(x) = 20\left[\frac{x}{20}\right] - 80\left[\frac{f(x)}{80}\right]$ and (ii) 1990 < f(2020) < 2050.
Then possible values of f(2020) can take (where [.] denote greatest integer function)
(A) 1920
(B) 3960
(C) 2000
(D) 4000

41. A function f(x) is defined as $f(x)=\begin{cases}6x-5-x^2 & x\le 3\\ 24x-32-4x^2 & x>3\end{cases}$

Tangents are made on f(x) in the first quadrant. Let tangents $T_1 : y = m_1x + b_1$ and $T_2 : y = m_2x + b_2$ have highest and lowest y intercepts of all tangents made in first quadrant then

(A) $b_1 + b_2 = 36$
(B) $b_1 + b_2 = 28$
(C) $m_1 + m_2 = -4$
(D) area made by y axis, $T_1 = 0$ and $T_2 = 0$ is 54 square units

42. Let f: R → R be given by $f(x) = (x - 2)(x - 6)(x^2 - 7x + 10)(x - 10)$. Define $F(x)=\int_0^x f(t)\,dt,\ x>0$.

Then which of the following options is/are correct?

(A) F(x) has a local minimum at x = 5
(B) F(x) has a local maximum at x = 10
(C) F(x) has a local maximum at x = 2
(D) F(x) has, two local minima and one local maxima in $(0, \infty)$

SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

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43. If f(x) is a differentiable function in interval $(-\infty, \infty)$ such that $f(x) = f(16 - x)$, then the value $\left[\int_{-8}^{8} f'(8+x)x^2e^{x^2}\right]$ is equal of (where [.] denote greatest integer function)

44. Let $f(x)=\begin{cases}\ln(x-[x]) & , \ x\notin I\\ 0 & , \ x\in I\end{cases}$. Area enclosed by the curve y = f(x), y = f(|x|), $-2 \le x \le 0$ and y = 0 is $k-\log_e p$, then k + p is equal to

45. For the function $f(x)=\lim_{n\to\infty}\dfrac{\ln([1+9x^2])-9^nx^{2n}\cos(9x^2)}{1+9^nx^{2n}}$ number of points of discontinuity in (–3, 3) is

46. If $f(x) = a_0 + a_1x + a_2x^2 + a_3x^2 + a_3x^3 + \ldots\ldots a_nx^n + \ldots\ldots$ and $\dfrac{f(x)}{1-x} = b_0 + b_1x + b_2x^2 + \ldots\ldots b_nx^n + \ldots\ldots$ If $a_0 = 1$, $b_1 = 3$ and $b_3 = k$, then area bounded by the $y(1 + x^2) = k$, x-axis, y-axis and $x=\sqrt{3}$ is A, then [A] is (where [.] denotes greatest integer function) (Given $a_0, a_1, a_2, \ldots\ldots$ are in G.P.)

47. A curve y = f(x) whose differential equation is given by $x\,dy + 4\ln x\,dx = 2(y + 1)dx$ passes through (1, 3), then $\left[f(\sqrt{2})\right]$ is equal to (where [.] denotes greatest integer function)

48. Let $I_1=\int_0^{\pi/2}\dfrac{\sin^2x+\sin x}{1+\sin x+\cos x}dx$ and $I_2=\int_0^{\pi/2}\dfrac{\cos^2x+\cos x}{1+\cos x+\sin x}dx$, then $\dfrac{I_1}{I_2}$ is equal to

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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49. Let $I_1 = \int\limits_0^{\pi/2} (\cos x)^{\sqrt{2}+1}\,dx$ and $I_2 = \int\limits_0^{\pi/2} (\cos x)^{\sqrt{2}-1}\,dx$, then $\frac{I_1}{I_2}$ (take $\sqrt{2} = 1.41$) is equal to

50. Let f(x) = $a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$ be a polynomial to degree 5 which increases in the interval $(-\infty, 1]$ and $[3, \infty)$ and decreases in the interval (1, 3). Given that f(0) = 2, $f(1) = \frac{88}{15}$ and $f'(2) = 0$, then a_5 is equal to

51. $\lim\limits_{t\to 0} \frac{\sin(t^3) - t^3\cos(t^3) + t^{15}}{t^3\left(e^{3t^3} - 1 - 3t^3\right)}$ is equal to

52. If $\int \frac{x+2}{\sqrt{x^4 + 8x^3 + 20x^2 + 16x + 4}} = \frac{1}{2}\ln f(x) + c$, then f(1) is equal to

53. Let f(x) be a differentiable function such that $f'(x) + f(x) = 4xe^{-x}\sin 2x$ and f(0) = 0.
If $\sum\limits_{n=0}^{7} f\left(\frac{(4n+1)\pi}{4}\right) = e^{-\frac{\pi}{4}}\left(\frac{e^{-k\pi} - 1}{e^{-\pi} - 1}\right)$, then k is equal to

54. Let f : R → R and g : R → R be two bijective function such that they are mirror image of each other about the 4y – 17 = 0. If h(x) = f(x) + g(x), then h(0) is equal to

ALL INDIA TEST SERIES

TEST - 2

JEE (Advanced)

ANSWERS, HINTS & SOLUTIONS

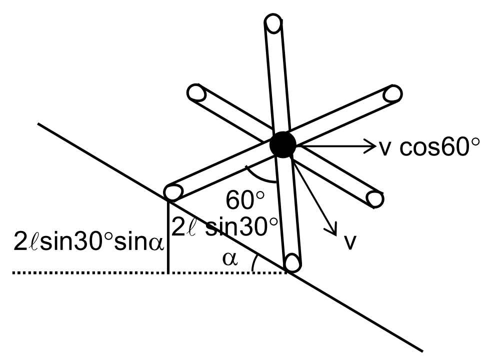
Physics **PART – I**

SECTION – A

1. A, C, D

Sol. When the pivot point is changed the speed of central point changes suddenly and the kinetic energy is lost. Velocity component perpendicular to new spoke survives.
Average velocity becomes constant when the loss
in K.E. due to changing contact point = gain in P.E.

$$\frac{1}{2}mv_0^2\left(1-\cos^2\frac{\pi}{3}\right)=mg\ell.\ 2\sin\frac{\pi}{6}\sin\alpha$$



2. C, D

Sol. The force on the particle is not the central force.

3. A, B, C, D

Sol. $a_x + a_y = a_1$ …(i)

$a_y - a_x = a_2$ …(ii)

$a_{cm(x)} = 0$

$2a_2 = a_1 + a_x$ …(iii)

4. A, B

Sol. $L = mv_{\theta}r$ ...(i)

$\frac{dL}{dt} = -\alpha v_{\theta} r$

$\Rightarrow \frac{dL}{dt} = -\frac{\alpha L}{m}$ from (i)

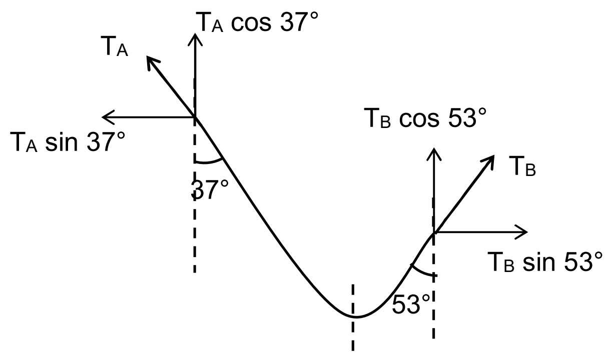
$\Rightarrow \int_{L_0}^{L} \frac{dL}{L} = -\frac{\alpha}{m}\int_0^t dt$

$L = L_0 e^{-\frac{\alpha t}{m}}$

5. A, C, D

Sol. $3T_A = 4T_B$

$\frac{4T_A}{5} + \frac{3T_B}{5} = mg \Rightarrow T_B = \frac{3}{5}mg$



6. A, B, C

Sol. $Ta_1 + \frac{T}{2}a_2 + \frac{T}{4}a_3 + ... = 0$

$\left(\frac{T}{m} - g\right) + \frac{1}{2}\left(\frac{T}{2m} - g\right) + \frac{1}{4}\left(\frac{T}{4m} - g\right) + ... = 0$

$\frac{T}{m}\left(1 + \frac{1}{4} + \frac{1}{16} + ...\right) = g\left(1 + \frac{1}{2} + \frac{1}{4} + ...\right)$

$T = \frac{3mg}{2}$

$\therefore \quad a_1 = \frac{g}{2}$

SECTION – B

7. 1

Sol. $\left[<\omega> = \frac{\Delta\theta}{\Delta t} \right]$

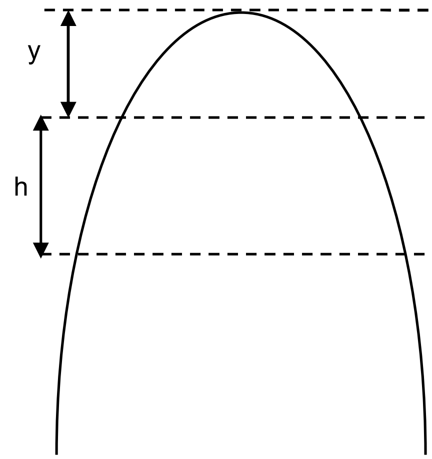
8. 1

Sol. Writing equation of motion in y-direction

$$y = \frac{1}{2}g\left(\frac{T_1}{2}\right)^2$$

$$h + y = \frac{1}{2}g\left(\frac{T_2}{2}\right)^2$$

$$g = \frac{8h}{T_2^2 - T_1^2}$$



9. 3

Sol. For a pure rolling motion

$a = \alpha R$

Using conservation of energy

$$mg\frac{R}{2} = \frac{1}{2}I_P\omega^2$$

$$mg\frac{R}{2} = \frac{1}{2}\times 4mR^2\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{g}{4R}} \quad \ldots(i)$$

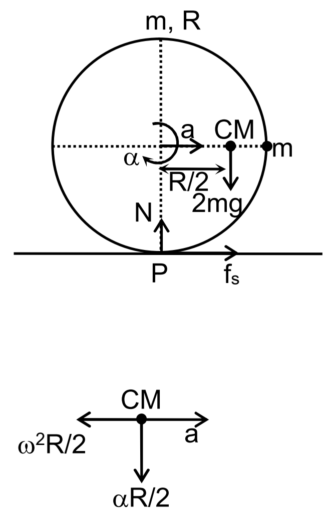
$$\tau_P = I_P\alpha$$

$$2m(g + \omega^2 R)\frac{R}{2} = 4mR^2\alpha$$

$$2m\left(g + \frac{g}{4}\right)\frac{R}{2} = 4mR^2\alpha \Rightarrow \alpha = \frac{5g}{16R} \quad \ldots(ii)$$

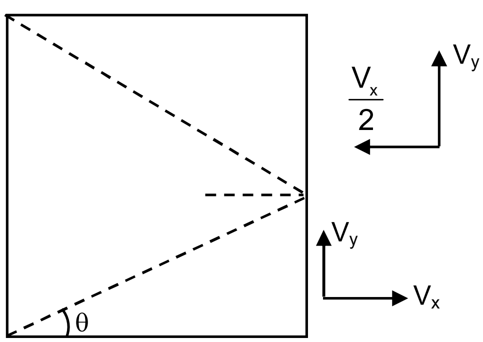
$$f_s = 2m\left(a - \frac{\omega^2 R}{2}\right)$$

$$f_s = 2m\left(\frac{5g}{16} - \frac{g}{8}\right), \quad f_s = \frac{3mg}{8}$$



10. 3

Sol.



11. 4

Sol. Tangential component of force = $V_0 r^2 \sin\theta$

Magnitude of the x-component of force = $V_0 r^2 \sin^2\theta$.

12. 7

Sol. $\frac{4}{3}\pi R^3 \rho_1, \frac{dv}{dt} + v\frac{d}{dt}\left(\frac{4}{3}\pi R^3 \rho_1\right) = \frac{4}{3}\pi R^3 \rho_1 g.$

$R\frac{dv}{dt} + 3v\frac{dR}{dt} = Rg$ …(i)

Also, $\pi R^2 v \rho_2 = \frac{dm}{dt}$

$v = \frac{4\rho_1}{\rho_2} \cdot \frac{dR}{dt}$ …(ii)

After a long time when acceleration becomes constant v = at will satisfy our differential equation.

$v = at \qquad v = \frac{4\rho_1}{\rho_2}\frac{dR}{dt}$

$R = \frac{at^2 \rho_2}{8\rho_1}$

From equation (i) and (ii)

$\frac{at^2 \rho_2 a}{8\rho_1} + \frac{3\rho_2}{4\rho_1}(at)^2 = \frac{at^2 \rho_2 g}{8\rho_1}$

$\frac{a}{8} + \frac{3a}{4} = \frac{g}{8}$

$a = \frac{g}{7}$

SECTION – C

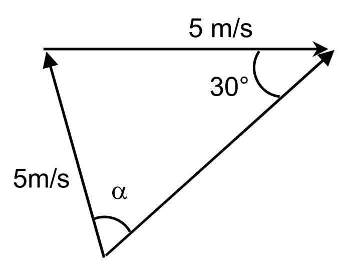
13. 00100.00

Sol. $5V^2 = 50 \times 10^3$

$V = 10^2\ ms^{-1}$

14. 00060.00

Sol. $\frac{5}{\sin\alpha} = \frac{5}{\sin 30^\circ} \Rightarrow \alpha = 30^\circ$



15. 00000.50

Sol. $\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + mg\ell$

$v_{2y}^2 = v_{1y}^2 + 2g\ell$ …(i)

$ev_{2y} = v_{1y}$

$$v_{1x} = v_{2x} = \sqrt{\frac{g\ell}{2}\left(\frac{1-e}{1+e}\right)} = 0.50 \quad m/s$$

16. 00008.66

Sol. $\frac{R}{2}\sqrt{3} = 10 \times \frac{1.732}{2} = 8.66$ cm

17. 00011.42

Sol. Centre of mass should not cross the corner

$$d = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10}\right) \times 10\,\text{cm} = 11.42\ \text{cm}$$

18. 00031.00

Sol. $I \propto h^5$

PART – II

SECTION – A

19. A, B, C

Sol. $Na_2O \xrightarrow{400^\circ C} Na_2O_2 + Na$

$Na_2O_2 + O_2 \xrightarrow{450^\circ C} NaO_2$

$Na_2O_2 + H_2O \longrightarrow 2NaOH + H_2O_2$

20. A, B, C

Sol. $NaOH + P_4 \longrightarrow PH_3 + NaH_2PO_2$

$S_8 + NaOH \longrightarrow Na_2S + Na_2SO_3$

$Cl_2 + NaOH \longrightarrow NaCl + NaClO_3$

$B + NaOH \longrightarrow Na_3BO_3 + H_2$

21. A, B, D

22. A, B, C

Sol. (D) four oxygen atoms are shared.

23. A, B, C

Sol. $Al_4C_3 + H_2O \longrightarrow Al(OH)_3 + CH_4$

$CaC_2 + H_2O \longrightarrow Ca(OH)_2 + C_2H_2$

24. A, B

SECTION – B

25. 9

Sol. $5U(SO_4)_2 + 2H_2O + 2KMnO_4 \longrightarrow 2MnSO_4 + 5UO_2SO_4 + 2H_2SO_4 + K_2SO_4$

26. 8

Sol. 100 ml water contain = 0.192 mg

1000 ml water contain = 1.92 mg

Milli mole of $Mg^{+2} = \frac{1.92}{24} = 0.08$

Hence milli mole of $CaCO_3$ = 0.08

Hence mass of $CaCO_3$ with 1L = 0.08 × 100 = 8 mg

Hardness will be = 8 ppm.

27. 3

Sol. $60 = \frac{hc}{\lambda_1}\left(\frac{n_1}{t_1}\right)$

$80 = \frac{hc}{\lambda_2}\left(\frac{n_2}{t_2}\right)$

$$\lambda_1 = \frac{1}{R_H}$$

$$\lambda_2 = \frac{1}{4R_H}$$

$$\frac{(n_1 / t_1)}{n_2 / t_2} = 3$$

28. 6

Sol.

$$\begin{array}{cccc} A + B & \rightleftharpoons & C & + D \\ 0.4+x \quad 0.4+x & & 2-x & 2-x \quad K_c = 16 \end{array}$$

x = 0.08

$$[B]_{eq} = \frac{(0.4+0.08)}{0.08} = 6$$

29. 4

Sol. $_{92}U^{238} \longrightarrow {}_{82}Pb^{206} + 8\alpha + 6\beta$

$$\left[4 - 4\left(\frac{1}{2}\right)^n\right] \times 8 \times 6.02 \times 10^{23} = 18.066 \times 10^{24}$$

n = 4

30. 4

Sol. $5.35 = 4.75 + \log\frac{[CH_3COO^{-}]}{[CH_3COOH]}$

$$\frac{[CH_3COO^{-}]}{[CH_3COOH]} = 4$$

SECTION – C

31. 00025.02

Sol. $\% = \frac{1.2 \times 3.336 \times 10^{-30}}{1.6 \times 10^{-19} \times 10^{-10}} \times 100\%$

$$= \frac{3 \times 3.336 \times 10}{4}$$

= 25.02%

Hence, x = 25.02

32. 00005.60

Sol. $pH = 5 + \log\frac{80}{20}$

$$= 5 + \log 4 = 5.6$$

33. 00001.20

Sol. $K_4[Fe(CN)_6] + 6H_2SO_4 + 6H_2O \xrightarrow{\Delta} 2K_2SO_4 + FeSO_4 + 3(NH_4)_2SO_4 + 6CO\uparrow$

$$5CO + I_2O_5 \longrightarrow 5CO_2 + I_2$$

34. 00000.48

Sol.

$$N_2O_3(g) \rightleftharpoons NO_2(g) + NO$$

$2-x$ $\quad$ $x-2y$ $\quad$ x

$$2NO_2(g) \rightleftharpoons N_2O_4(g)$$

$x-2y$ $\quad$ y

Given x = 1.5

$$2.5 = \frac{(1.5-2y)(1.5)}{0.5}$$

35. 00000.69

36. 00000.28

Sol.

$$2A \rightleftharpoons 2B + 3C + D$$

$\quad$ 0.24 $\quad$ 0.36 $\quad$ 0.12

$$X_A = (1-0.72) = 0.28$$

Mathematics PART – III

SECTION – A

37. B, C

Sol. $I_1 = \int_0^{512} \frac{\{x^{1/3}\}}{\sqrt[3]{x^2}} dx$. Let $x^{1/3} = t \Rightarrow \frac{dt}{dx} = \frac{1}{3}x^{-2/3}$

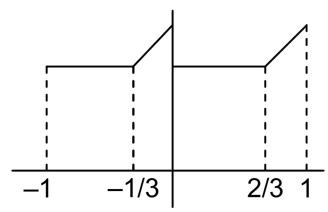
$\Rightarrow I_1 = 3\int_0^8 \{t\} dt = 3 \times 8 \times \int_0^1 \{t\} dt = 24 \times \frac{1}{2} = 12$

$I_2 = \int_0^2 x^2 \{x^3\} dx$. Let $x^3 = t \Rightarrow 3x^2\,dx = dt$

$= \frac{1}{3}\int_0^8 \{t\} dt = \frac{1}{3} \times 8 \times \int_0^1 \{t\} dt = \frac{8}{3} \times \frac{1}{2} = \frac{4}{3}$

38. A, C

Sol. $f(x) = \begin{cases} -6x & -1 < x < -\frac{2}{3} \\ 4 & -\frac{2}{3} \le x \le \frac{2}{3} \\ 6x & \frac{2}{3} < x < 1 \end{cases}$



$g(x) = \{x\}$

$\therefore f(g(x)) = \begin{cases} 4 & 0 \le \{x\} \le \frac{2}{3} \\ 6\{x\} & \frac{2}{3} < \{x\} < 1 \end{cases}$ $\therefore$ c = 1 and d = 3

39. A, B, C, D

Sol. (A) $\int_0^1 x \tan x dx \ge \int_0^1 x\left(x + \frac{x^3}{3}\right) dx \ge \frac{2}{5}$

(B) $\int_0^1 x^2 \cos x \le \int_0^1 x^2 dx \le \frac{1}{3}$

(C) $\int_0^1 x^3 \sin x \ge \int_0^1 x^3\left(x - \frac{x^3}{3!}\right) \ge \frac{37}{210}$

(D) x sin x is an even function

$\therefore \int_{-1}^0 x \sin x = \int_0^1 x \sin x \ge \int_0^1 x\left(x - \frac{x^3}{3!}\right) dx \ge \frac{3}{10}$

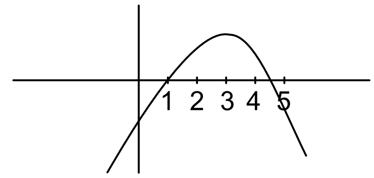
40. A, C

Sol. $\frac{1990}{80} < \frac{f(2020)}{80} < \frac{2050}{80} \Rightarrow \left[\frac{f(2020)}{80}\right] = 24$ or 25

Now put x = 2020 we get f(2020) = 1920 or 2000

41. B, C, D

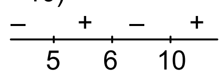
Sol. $f(x) = \begin{cases} -(x-1)(x-5) & x \le 3 \\ -4(x-2)(x-4) & x > 3 \end{cases}$



Lowest y intercept will be of tangent made on (1, 0) and highest y intercept will be of tangent made on (4, 0) tangent at (1, 0) is y = 4x – 4 $\therefore$ $m_2 = 4$, $b_2 = -4$ and tangent at (4, 0) is y = –8x + 32 $\therefore$ $m_1 = -8$, $b_1 = 32$

42. A, D

Sol. $F'(x) = f(x) = (x-2)^2(x-5)(x-6)(x-10)$



F′(x) changes sign from negative to positive at 5 and 10

$\therefore$ F(x) has local minima at x > 5, 10, F(x) has local maxima at x = 6

SECTION – B

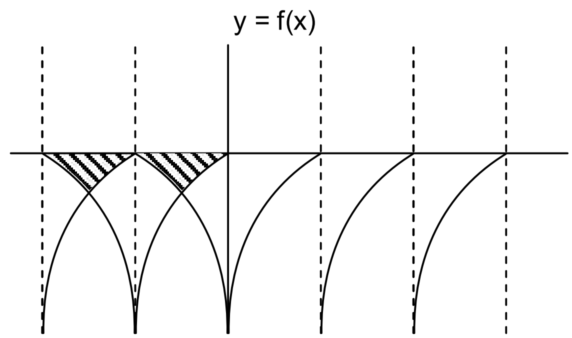
43. 0

Sol. $f(8-x) = f(8+x) \Rightarrow f'(8-x) = -f'(8+x)$

$$I = \int_{-8}^{8} f'(8+x)x^2e^{x^2}dx = -\int_{-8}^{8} f'(8-x)x^2e^{x^2}dx = -I \Rightarrow I = 0$$

44. 6

Sol. Area $= 2 \times 2 \times \int_{-1/2}^{0} \ln\{x\}dx$



$= \left|4 \times \int_{1/2}^{1} (\ln x dx)\right|$

$= \left|4(x\ln x - x)\right|_{1/2}^{1}$

$= \left|4\left(-1-\left(\frac{1}{2}\ln\frac{1}{2}-\frac{1}{2}\right)\right)\right|$

$= 4\left(\frac{1}{2}+\frac{1}{2}\ln\frac{1}{2}\right) = 2(1-\ln 2)$

= 2 – 2 ln 2 = 2 – ln 4 $\therefore$ k = 2 and p = 4

45. 2

Sol. $f(x) = \begin{cases} 0 & 0 \le 9x^2 < 1 \\ \frac{\ln 2 - \cos 1}{2} & 9x^2 = 1 \\ -\cos 9x^2 & 9x^2 > 1 \end{cases}$ discontinuous at $x = \pm\frac{1}{3}$

46. 7

Sol. Comparing the coefficient of x^n both sides we get $b_n - b_{n-1} = a_n$

Also, $b_0 = a_0$ and $b_1 = a_0 + a_1 \Rightarrow b_0 = 1, a_1 = 2$

$\therefore b_3 = a_0 + a_1 + a_2 = 1 + 2 + 4 = 7$

Area bounded by $y = \frac{7}{1+x^2}$

x-axis, y-axis and $x = \frac{\pi}{3}$ will be $\int_0^{\sqrt{3}} \frac{7}{1+x^2}dx = \frac{7\pi}{3}$

47. 6

Sol. $x\frac{dy}{dx}-2y=2-4\ln x \Rightarrow \frac{dy}{dx}-\frac{2}{x}y=\frac{2-4\ln x}{x}$

$\text{I.F}=e^{-\int\frac{2}{x}dx}=\frac{1}{x^2} \;\therefore\; y\frac{1}{x^2}=\int\frac{2-4\ln x}{x^3} \Rightarrow \frac{y}{x^2}=-\frac{1}{x^2}-4\left(\frac{\ln x\, x^{-2}}{-2}+\frac{x^{-2}}{-4}\right)+c$

$\Rightarrow y = cx^2 + 2\ln x$ $\because$ it passes through (1, 3) $\therefore$ c = 3 $\therefore$ $y = 3x^2 + 2\ln x$

48. 1

Sol. $I_1+I_2=\int_0^{\pi/2}1\cdot dx=\frac{\pi}{2}$

$I_1-I_2=\int_0^{\pi/2}\frac{\sin^2 x-\cos^2 x+\sin x-\cos x}{1+\sin x+\cos x}dx=\int_0^{\pi/2}(\sin x-\cos x)dx=0$

$\therefore I_1 = I_2 = \frac{\pi}{4}$

SECTION – C

49. 00000.59

Sol. $I_1=\int_0^{\pi/2}(\cos x)^{\sqrt{2}}\cos x\,dx=(\cos x)^{\sqrt{2}}\sin x\Big|_0^{\frac{\pi}{2}}+\sqrt{2}\int_0^{\pi/2}(\cos x)^{\sqrt{2}-1}\sin^2 x\,dx$

$\Rightarrow I_1=\sqrt{2}I_2-\sqrt{2}I_1 \Rightarrow \frac{I_1}{I_2}=\frac{\sqrt{2}}{1+\sqrt{2}}=2-\sqrt{2}=2-1.41=0.59$

50. 00000.20

Sol. $f'(x) = k(x-1)(x-3)(x-2)^2$ and $k > 0 \Rightarrow f(x) = k\int(x^2-4x+3)(x^2-4x+4)dx$

$= k\int x^4+16x^2-8x^3+7(x^2-4x)+12\,dx = k\left(\frac{x^5}{5}-2x^4+\frac{23x^3}{3}-14x^2+12x\right)+c$

$\because f(0) = 2 \Rightarrow c = 2 \;\therefore\; f(1)=\frac{88}{15} \Rightarrow \frac{88}{15}=k\left(\frac{1}{5}-2+\frac{23}{3}-14+12\right)+c$

$\Rightarrow \frac{58}{15}=k\times\frac{58}{15} \Rightarrow k = 1 \;\therefore\; a_5=\frac{1}{5}$

51. 00000.07

Sol. Let $t^3 = x \Rightarrow \lim_{x\to 0}\frac{\sin x-x\cos x+x^5}{x(e^{3x}-1-3x)}$

Now use expansions of sin x, cos x and e^{3x}

52. 00007.00

Sol. $I=\int\frac{x+2}{\sqrt{(x+2)^4-4(x+2)^2+4}}$. Let $(x + 2)^2 = t \Rightarrow \frac{dt}{dx}=2(x+2) \Rightarrow \frac{dt}{2}=(x+2)dx$

$\Rightarrow I=\frac{1}{2}\int\frac{1}{\sqrt{t^2-4t+4}}dt \Rightarrow I=\frac{1}{2}\int\frac{1}{t-2}dt=\frac{1}{2}\ln|t-2|+c$

$= \frac{1}{2}\ln\left|(x+2)^2-2\right|+c \;\therefore\; f(x) = x^2 + 4x + 2$

53. 00008.00

Sol. I.F = e^x $\therefore$ solution is $f(x)e^x = \int 4x\sin 2x dx \Rightarrow f(x)e^x = 4\left(\frac{-x\cos 2x}{2}+\frac{\sin 2x}{4}\right)$

$\Rightarrow f(x) = (-2x\cos 2x + \sin 2x)e^{-x},\; f\left(\frac{\pi}{4}\right) = e^{-\frac{\pi}{4}}$

$\therefore\; f\left(\frac{5\pi}{4}\right) = e^{-\frac{5\pi}{4}}$

$f\left(\frac{9\pi}{4}\right) = e^{-\frac{9\pi}{4}}$

$e^{-\frac{\pi}{4}}\left(\frac{e^{-8\pi}-1}{e^{-\pi}-1}\right) = \sum_{n=0}^{7} f\left(\frac{(4n+1)\pi}{4}\right)$

54. 00008.50

Sol. $\frac{17}{4} - f(x) = -\frac{17}{4} + g(x) \Rightarrow f(x)+g(x) = \frac{17}{2}\;\forall\; x \in R$