ALL INDIA TEST SERIES

TEST - 3

JEE (Advanced)

Time Allotted: 3 Hours **Maximum Marks: 198**

<u>General Instructions:</u>

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-C.**

Section-A (01 – 06, 19 – 24, 37– 42) contains **18 multiple** choice questions which have **ONLY ONE CORRECT ANSWER**. Each question carries **+3 marks** for correct answer and **–1 mark** for wrong answer.

Section-A (07 – 12, 25 – 30, 43 – 48) *this section contains* ***18 multiple*** *choice questions.*
Each question has FOUR options. ***ONE OR MORE THAN ONE*** *of these four option(s) is (are) correct answer(s).*
For each question, choose the option(s) corresponding to (all) the correct answer(s)
Answer to each question will be evaluated according to the following marking scheme:

Full Marks **: +4** If only (all) the correct option(s) is (are) chosen:
Partial Marks **: +3** If all the four options are correct but ONLY three options are chosen;
Partial Marks **: +2** If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks **: +1** If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks **: 0** If none of the options is chosen (i. e. the question is unanswered);
Negative Marks **:** –**2** In all other cases.

Section-C (13 – 18, 31– 36, 49 – 54) *contains* ***18 Numerical*** *answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer and* ***0 marks*** *for wrong answer.*

Physics PART – I

SECTION – A
(One Options Correct Type)

This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. A point charge q is kept at a point $\left(a, 2a, \frac{a}{2}\right)$ in the space. Consider a triangular plane surface whose co-ordinates of vertices are (a, 2a, 0), $\left(\frac{3a}{2}, 2a, 0\right)$ and $\left(\frac{3a}{2}, \frac{5a}{2}, 0\right)$. The electric flux passing through the considered triangular surface is

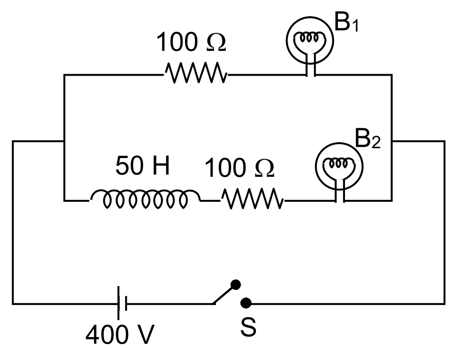
(A) $\frac{q}{12\varepsilon_0}$

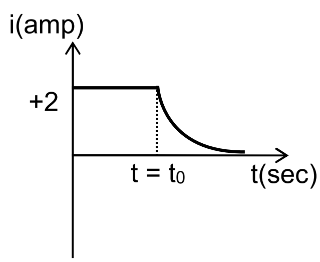
(B) $\frac{q}{24\varepsilon_0}$

(C) $\frac{q}{48\varepsilon_0}$

(D) $\frac{q}{96\varepsilon_0}$

2. An inductor of 50 H, two resistors of 100 Ω each and two identical bulbs of 100 Ω each are connected with a battery of 400 volt through a switch 'S' as shown in the figure. At t = 0, switch is closed for a long time so that steady state is reached. Now at t = t_0, switch is opened, the variation of current through the bulb B_1 with time is best represented by



(A)


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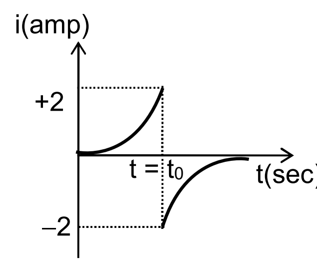
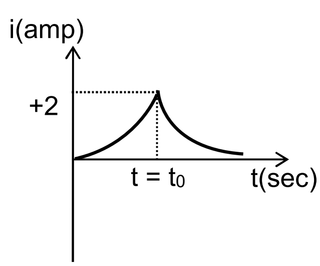


(C)


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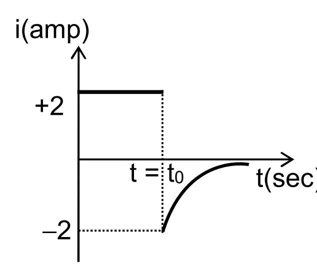
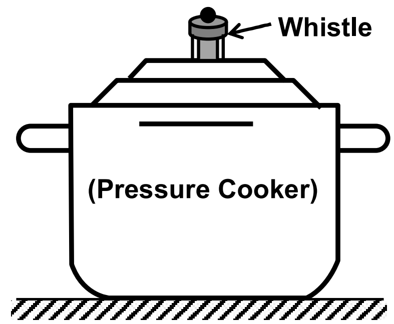
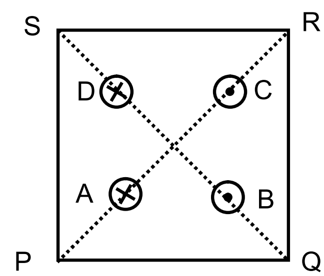


3. A 15 litres pressure cooker has air inside it at room temperature 27° at atmosphere pressure 10^5 Pa. Pressure cooker has a whistle of area 0.15 cm^2 as shown in figure. The mass of whistle is 200 gm and the coefficient of friction between the whistle and the pressure cooker is negligible. Now, air inside the pressure cooker is heated, the temperature of air inside it at which the whistle is just lifted up is



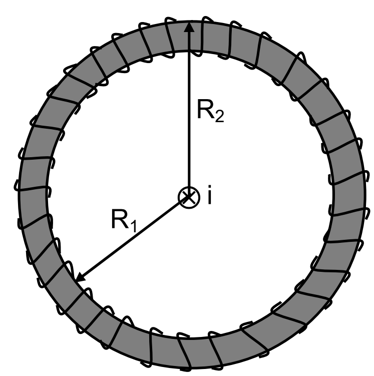
(A) 527°C
(B) 427°C
(C) 127°C
(D) 327°C

4. PQRS is a square of side ℓ_0. A, B, C and D are four long current carrying wires kept perpendicular to the plane of paper as shown in the figure. PA = QB = RC = SD = $\frac{\ell_0}{\sqrt{5}}$. The magnitude of current in the wires A, B, C and D are i_0, $2i_0$, $8i_0$ and $4i_0$ respectively, the direction of current are shown. It is given that $\int_P^Q \vec{B}_A \cdot d\vec{\ell} = -9\mu_0$ Tesla meter and $\int_R^S \vec{B}_B \cdot d\vec{\ell} = 5\mu_0$ Tesla meter, where $\vec{B}_A$ and $\vec{B}_B$ are the magnetic fields due to the wires A and B respectively. The magnitude of i_0 is



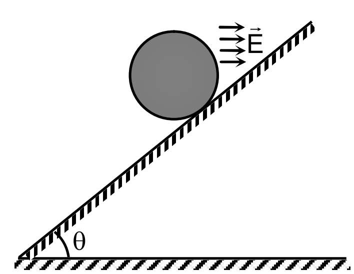
(A) $\frac{23}{2}$ amp
(B) 46 amp
(C) 23 amp
(D) $\frac{23}{4}$ amp

5. A long wire carrying alternating current $i = i_0 \cos \pi t$ is kept along the axis of a toroid. The inner and outer radii of the toroid are R_1 and R_2 respectively. The toroidal loop has rectangular cross-section with 'N' turns and height 'h'. If $R_2 = 4R_1$, i_0 = 3 amp, N = 1000 and h = 2 mm. Then the maximum emf induced in the loop is



(A) $(3\pi \times 10^{-7})\ln 2$ volt
(B) $(6\pi \times 10^{-7})\ln 2$ volt
(C) $(12\pi \times 10^{-7})\ln 2$ volt
(D) $(24\pi \times 10^{-7})\ln 2$ volt

6. A non-conducting uniformly charged sphere is kept on a rough inclined plane. A uniform electric field is present in the horizontal direction as shown in the figure. The angle of inclination is 45° and coefficient of friction between the sphere and the inclined plane is $\frac{1}{7}$. As the sphere is released, it is in the state of impending motion and starts pure rolling. For this the ratio of gravitational force to electrostatic force on the sphere is



(A) 1/3
(B) 3
(C) 1/5
(D) 5

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

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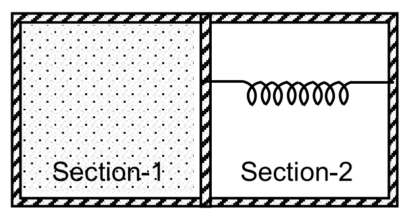
7. A fuse made of a lead wire with a cross-section of 0.2 mm^2 is incorporated into a circuit of copper wire with a cross-section of 2 mm^2. On short circuiting the current reaches 30A. The temperature of wires before short circuiting is $T_0 = 20°C$. Neglect the loss of heat due to thermal conductivity and radiation. Take the specific heat of lead and copper as constant and equal to $C_1 = 0.032$ cal/gm°C and $C_2 = 0.091$ cal/gm°C respectively. Given that resistivity and density of lead are $\rho_1 = 22 \times 10^{-6}$ ohm-cm and $d_1 = 11.34$ gm/cm^3 and for copper $\rho_2 = 1.7 \times 10^{-6}$ ohm-cm and $d_2 = 8.9$ gm/cm^3. The melting point of the lead is $T_1 = 327°C$. Let "t_0" is the time when the lead fuse begins to melt after the short circuit occurs and Δt is the rise in temperature of copper wire during this time. Then
(A) $t_0 = 0.095$ sec
(B) $\Delta t = 12.6°C$
(C) $t_0 = 0.84$ sec
(D) $\Delta t = 0.106°C$

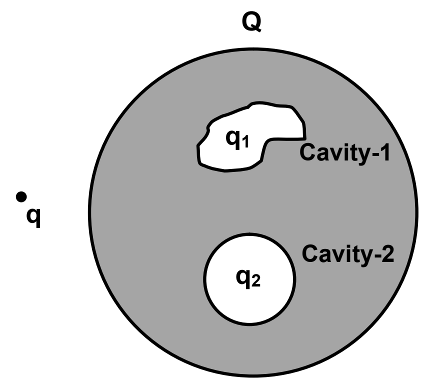
8. A particle of mass 'm' and charge 'q' is projected from origin with initial velocity $\left(v_0\hat{i} + v_0\hat{j}\right)$ in a constant magnetic field $B_0\hat{i}$ and constant electric field $E_0\hat{i}$. Now, choose the correct option(s) regarding the motion of the particle.
(A) The path of the particle is a uniform helix.
(B) The maximum magnitude of the y co-ordinate that the particle will have is $\frac{mv_0}{qB_0}$.
(C) The minimum magnitude of the z co-ordinate that the particle will have is zero.
(D) The maximum magnitude of the z co-ordinate that the particle will have is $\frac{2mv_0}{qB_0}$.

9. A smooth adiabatic massless piston divides a thermally insulated cylinder in two section of equal volume initially. Section-1 containing two moles of an ideal diatomic gas and in section-2 a vacuum is maintained. An ideal compressed spring is connected in section-2 as shown in figure. The natural length of spring is equal to the length of cylinder. Neglecting heat capacity of cylinder, piston and spring. Now gas is heated in section-1. Choose the correct options.



(A) Change in the internal energy of the gas during heating of the gas is 10 times the work done by the gas for the same time interval.
(B) Change in internal energy of the gas during heating of the gas is 5 times the work done by the gas for the same time interval.
(C) Heat capacity of gas is 3R.
(D) Heat capacity of gas is 6R.

10. A charged conductor has two cavities. A point charge q_1 is placed inside the cavity-1 and q_2 inside the cavity-2. 'Q' charge is given to the conductor. A point charge 'q' is also placed outside the conductor. Choose the correct alternative(s).



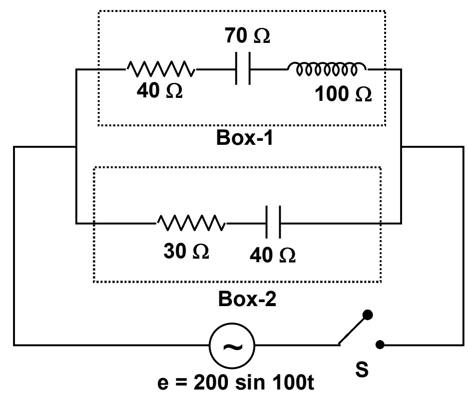
(A) A charge $(Q + q_1 + q_2)$ is uniformly distributed on the outer surface of the conductor.

(B) If charge q_2 is placed at the centre of cavity-2 then the magnitude of electric field due to the induced charge on the surface of cavity-2 at any point inside the conductor is zero.

(C) Electric field inside the conductor at a distance 'r' from point charge q due to the total charge on the outer surface of conductor is $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ towards the point charge q.

(D) Electric field outside the conductor at a distance 'r' from point charge q_1 due to induced charge on the surface of cavity-1 is $\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$ towards the charge q_1.

11. Two boxes are connected with AC source e = 200 sin 100t as shown in the figure. Box-1 contains a resistor, capacitor and inductor while Box-2 contains a resistor and capacitor. At t = 0 switch S is closed, then choose the correct alternatives regarding the given circuit.



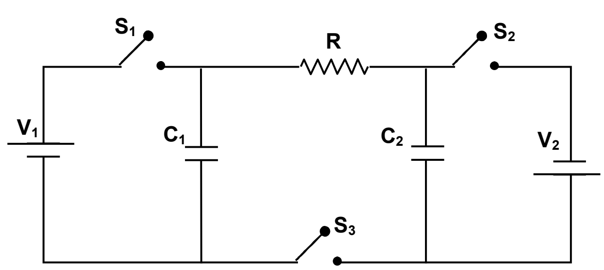
(A) Ratio of power factor of Box-1 and Box-2 is 4/3.

(B) Ratio of power factor of Box-1 and Box-2 is 3/4.

(C) Current passing through the voltage source is $4\sqrt{2}$ sin(100t + 8°) amp.

(D) Current passing through the voltage source is $4\sqrt{2}$ sin(100t – 8°) amp.

12. Two cells of potential difference V_1 = 20 Volt and V_2 = 10 Volt, two capacitors $C_1 = 3\mu F$ and $C_2 = 6\ \mu F$ and a resistance $R = 5k\Omega$ are connected in a circuit with the help of three switches S_1, S_2 and S_3 as shown in the figure. Initially switches S_1 and S_2 are closed and S_3 is opened so that capacitor C_1 and C_2 are charged. Now at t = 0, S_1 and S_2 are opened and S_3 is closed simultaneously. Choose the correct alternative(s).



(A) Ratio of work done by cell V_1 and cell V_2 is 2 : 1.

(B) The charge passed through the switch S_3 at $t = \frac{1}{100}$ sec is $60\left(1-\frac{1}{e}\right)\mu C$.

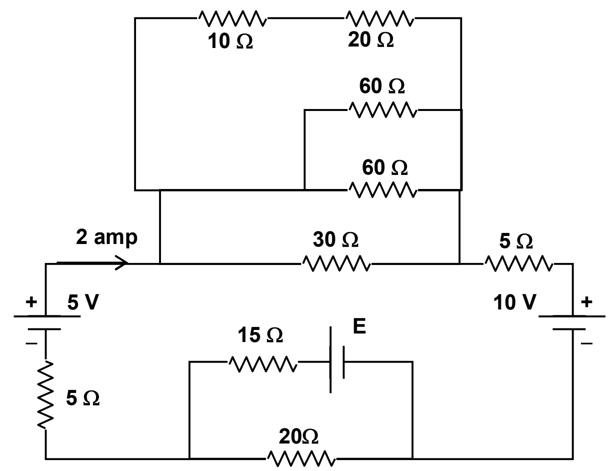
(C) The current passing through the switch S_3 at $t = \frac{1}{100}$ sec is $\frac{6}{e}$ milli-ampere.

(D) The ratio of charges on the capacitors C_1 and C_2 at $t = \frac{1}{100}$ sec is 1 :1

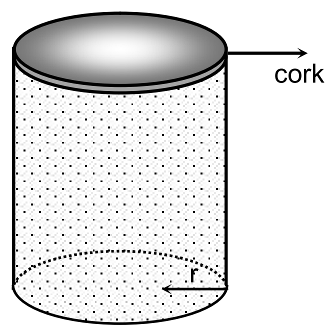
SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

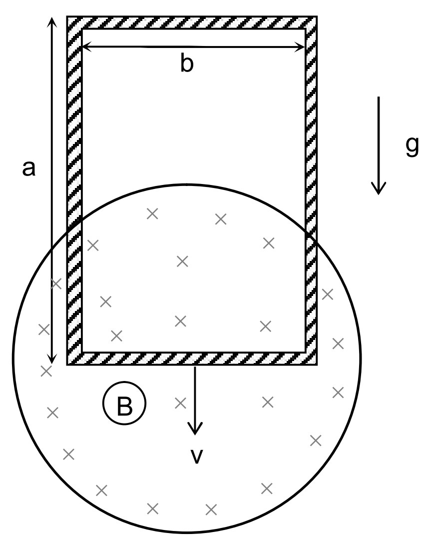
13. 9 resistors and 3 cells are connected in a circuit as shown in the figure. What should be the value of emf E (in volt) of the battery in order to flow a current of 2 ampere through the 5 volt cell.



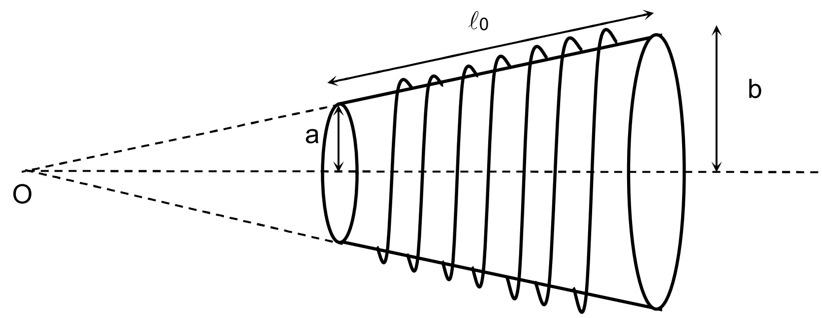
14. An ideal gas is enclosed in a cylindrical container of radius r, fitted with a light cork on its top. The coefficient of friction between the cork and the container wall is 'μ'. Initially the gas inside the container is at atmosphere pressure 'P_0'. Now gas inside the container is slowly heated, the cork is pushed out when the temperature of gas becomes four times as it was initially. Find the normal force per unit length (in kilo-newton/meter) exerted by the cork on the wall of the container. (Given $P_0 = 1.01 \times 10^5$ N/m^2, r = 40 cm, $\mu = 0.5$)



15. A conducting rectangular loop of dimension ($a \times b$) falls in a magnetic field of magnitude 120 Tesla which is directed inward the plane of paper. The magnetic field exist in a circular region of radius 2m. The mass of conducting loop is 60 gm and it has resistance equal to 60 Ω. The acceleration due to gravity (g = 10 m/s^2) is vertically downward. Conducting loop is released, while only the bottom edge of the loop is immersed in the magnetic field. It is assumed that loop attains a terminal velocity during the motion. It is also given that a = 10 cm and b = 5 cm. Find the terminal velocity of the conducting loop (in m/s).



16. A frustum of cone has N = 1000 turns closely wound on its surface carrying current $i = \frac{70}{22}$ amp as shown in the figure. The smaller and bigger radius of frustum are a = 10 cm and b = 20 cm respectively. The slant length of frustum is $\ell_0 = \frac{1}{2}$ m. Find the magnetic field (in micro tesla) at the vertex 'O' of the cone. (Given that $\pi = \frac{22}{7}$, $\ell n2 = 0.693$)



17. A blackened sphere is maintained at a 327°C by means of a heater. The blackened sphere is enveloped by a thin spherical shell black screen. The radius of blackened sphere and spherical shell are nearly same. The atmospheric temperature is considered at 0K. The nature of blackened sphere and spherical shell are assumed to be black body. Find the temperature of the spherical shell black screen in Kelvin. (Given $(2)^{1/4} = 1.19$).

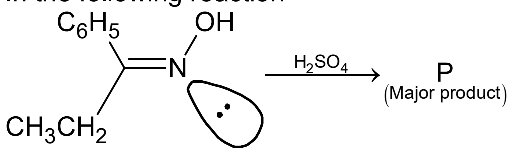
18. The resistance of a conductor is given by $R = R_0 (1 + at + bt^2 + ct^3)$, where a, b and c are positive constants, R_0 is the resistance at 0°C and 't' is the temperature in °C. The values of a, b and c are 2×10^{-2} (°C)$^{-1}$, 2×10^{-4} (°C)$^{-2}$ and 2×10^{-6} (°C)$^{-3}$ respectively. Find the temperature coefficient of resistance in (°C)$^{-1}$ at temperature 100°C.

Chemistry PART – II

SECTION – A
(One Options Correct Type)

This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

19. In the following reaction



The major product (P) is

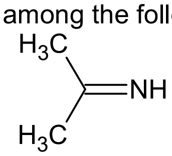
(A) $C_6H_5-\overset{\overset{\displaystyle O}{\parallel}}{C}-NH-CH_2CH_3$

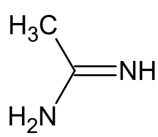
(B) $C_6H_5-NH-\overset{\overset{\displaystyle O}{\parallel}}{C}-CH_2CH_3$

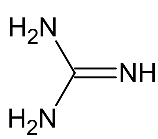
(C) $C_6H_5-CH=N-CH_2CH_3$

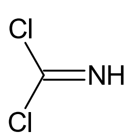
(D) $C_6H_5-N=CH-CH_2CH_3$

20. Which among the following compounds is most basic?

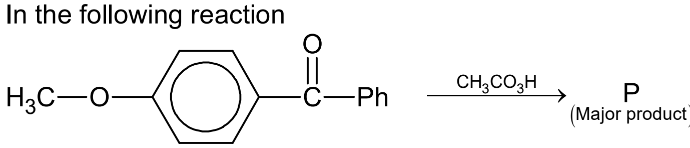
(A) 

(B) 

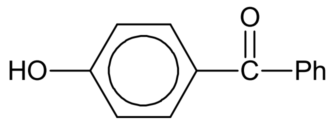
(C) 

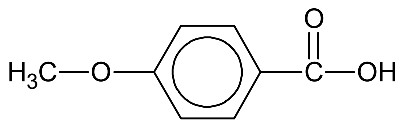
(D) 

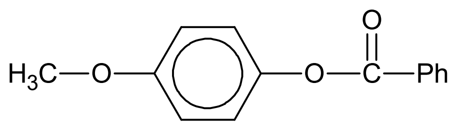
21. In the following reaction

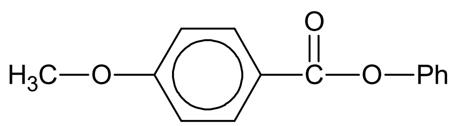


The major product (P) is

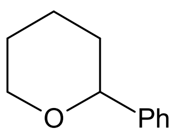
(A) 

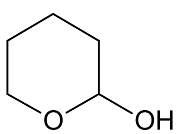
(B) 

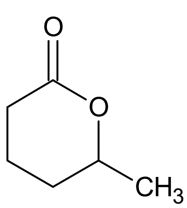
(C) 

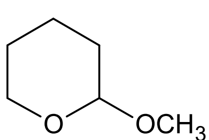
(D) 

22. Which of the following would give positive Fehling solution test?

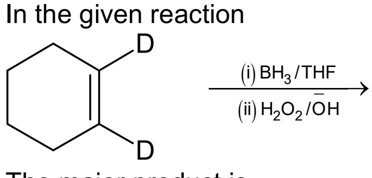
(A) 

(B) 

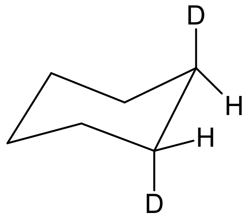
(C) 

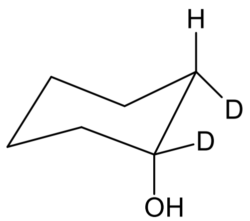
(D) 

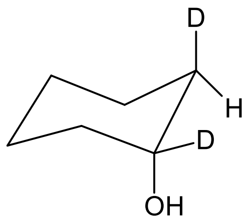
23. In the given reaction

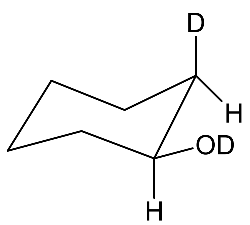


The major product is

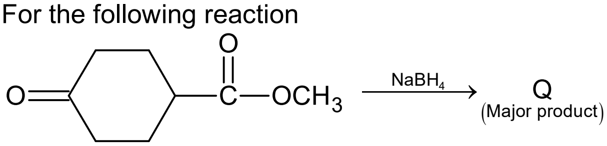
(A) 

(B) 

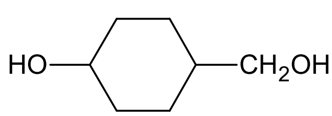
(C) 

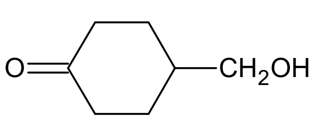
(D) 

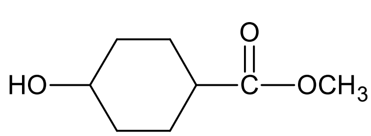
24. For the following reaction

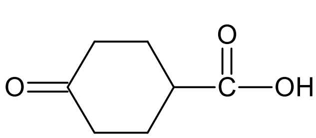


The major product (Q) is

(A) 

(B) 

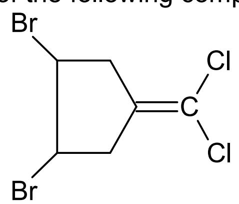
(C) 

(D) 

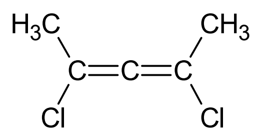
(One or More than one correct type)

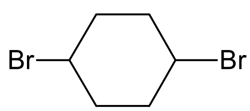
This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

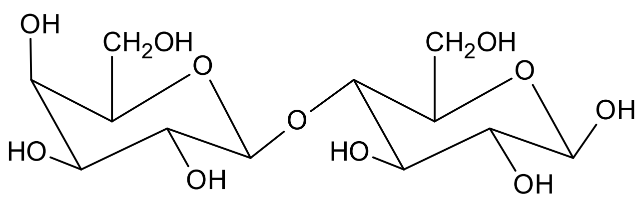
25. Which of the following compounds show geometrical isomerism?

(A) 

(B) $CH_3-N=N-CH_3$

(C) 

(D) 

26. 

Compound (P)

Which of the following statement (s) is/are correct about compound (P)?

(A) Compound (P) is Maltose

(B) Compound (P) on hydrolysis produces D-galactose and D-glucose.

(C) Compound (P) is a reducing sugar.

(D) Compound (P) shows mutarotation.

27. Which of the following statement(s) is/are correct about polymers?

(A) Neoprene is a polymer of isoprene.

(B) Glyptal is a condensation polymer of ethylene glycol and phthalic acid.

(C) Terylene is a condensation polymer of ethylene glycol and terephthalic acid.

(D) Nylon-6,6 is a condensation polymer of hexamethylenediamine and adipic acid.

28. Which of the following group(s) is/are ortho-para directing and deactivating for electrophilic aromatic substitution reaction?

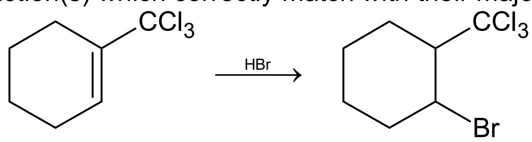
(A) $-CH=CH-NO_2$

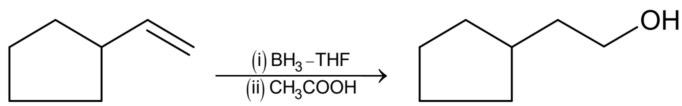
(B) $-Cl$

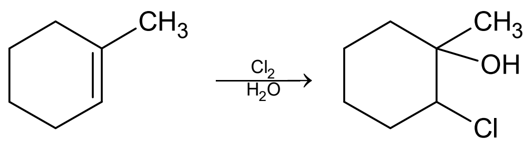
(C) $-SO_3H$

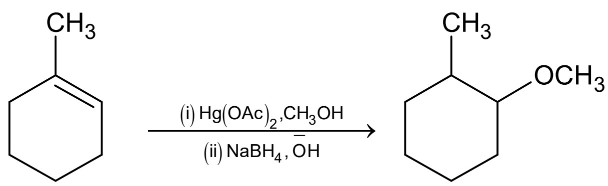
(D) $-CN$

29. The reaction(s) which correctly match with their major product is/are

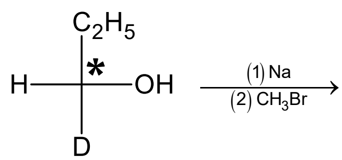
(A) 

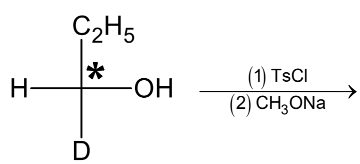
(B) 

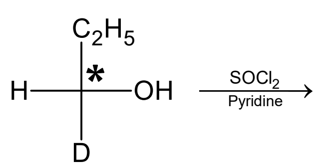
(C) 

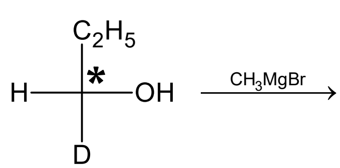
(D) 

30. In which of the following reaction (s) the configuration of the chiral carbon is retained in the product?

(A) 

(B) 

(C) 

(D) 

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

---

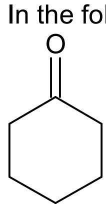
31. The observed rotation of 2 gram of a compound in 10 ml of solution in polarimeter tube of 25 cm long is +24.6°. The specific rotation of the compound (in degree) is

32. The relative reactivity of $1^\circ : 2^\circ : 3^\circ$ hydrogen to bromination is 1 : 82 : 1600. In the following reaction

$$CH_3-CH_2-CH_3 \xrightarrow[h\nu]{Br_2} H_3C-\underset{\underset{Br}{|}}{CH}-CH_3 + CH_3CH_2CH_2Br$$

the percentage yield of 2-Bromopropane is

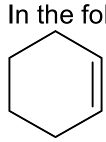
33. In the following reaction



$$+ Ph_3\overset{+}{P}-\overset{-}{C}H_2 \longrightarrow \underset{(Product)}{R} + Ph_3PO$$

the percentage mass of hydrogen in the product (R) is

34. In the following reaction



$$\xrightarrow[(2)\ Zn/H_2O]{(1)\ O_3} A \xrightarrow[\Delta]{dil.NaOH} \underset{(Major\ product)}{B}$$

the molecular weight of the major product (B) is x. The value of $\frac{x}{5}$ is

35. When formaldehyde is treated with ammonia a crystalline compound urotropine is formed. If the number of carbon atoms and nitrogen atoms in the compound are x and y respectively then the value of $\frac{x}{y}$ is

36. The weight of CH_4 (in grams) produced by reacting 4.6 grams of glycerol with excess of CH_3MgBr is

Mathematics PART – III

SECTION – A
(One Options Correct Type)

This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

37. In a triangle ABC, M is the point of intersection of median AD and BE of lengths 6 and 9 respectively and $\angle BMD = \frac{\pi}{6}$. If altitude from vertices A, B and C intersect the circumcircle in points H, I and J. Then area of hexagon AJBHCIA is equal to
(A) 18
(B) 72
(C) 36
(D) 108

38. Two perpendicular tangents are drawn to the ellipse $(x + y + 1)^2 + 2(x - y - 3)^2 = 4$ from a point. The chord of contact touches a circle concentric with the given ellipse. Then ratio of maximum and minimum radius of circle is
(A) 1
(B) 2
(C) 4
(D) $\sqrt{2}$

39. Number of integral values of 'a' such that $\left|\sin^{-1}\left(a^2-3\right)\right| + \left|\cos^{-1}\left(a^2-3\right)\right| = \left|\mathrm{cosec}^{-1}a^2\right| + \left|\sec^{-1}a^2\right|$
(A) 1
(B) 2
(C) 3
(D) 4

40. If $\cos\theta = \frac{ab+bc+ca}{a^2+b^2+c^2}$, where a, b, c are sides of a ΔABC, then θ can never be equal to
(A) $\frac{\pi}{12}$
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{3}$

41. A circle is drawn through the point of intersection A and B of the curve $\sqrt{2}x^2 + \sqrt{3}y + \sqrt{5}x + \sqrt{8} = 0$ and the line $y - \sqrt{7}x = 0$, the length of tangent drawn from origin to the circle is equal to
(A) 2
(B) $2\sqrt{2}$
(C) $4\sqrt{2}$
(D) 4

42. From a point Q on the tangent drawn to the parabola $y^2 - 4ax = 0$ at $P(3, 2\sqrt{3})$, let R and T be the feet of perpendiculars on SP and directrix respectively (S is the focus). Let PR = 6, then QT can be equal to
(A) 4
(B) 6
(C) 8
(D) 10

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

---

43. Let ABCD be a square of side length 1. E and F are two points on AB and BC respectively. If FD = 2FC and $\angle EDF = \frac{\pi}{4}$, then
(A) Ex-radius of ΔFBE opposite angle B is $\sqrt{2}$ unit
(B) DE < DF
(C) DEF is isosceles
(D) $\angle DFE = \frac{\pi}{3}$

44. A line touches a hyperbola at P and intersects the pair of asymptotes at points Q and R. If equation of pair of asymptotes is given by $4x^2 + 8xy - y^2 = 0$ and mid-point of QR is (1, 0), then
(A) equation of hyperbola is given by $4x^2 + 8xy - y^2 - 4 = 0$
(B) length of transverse axis of hyperbola is $\frac{\sqrt{89} - 3}{10}$
(C) circumcentre of ΔOQR is (a, b) then a + 9b = 4; (O is centre of hyperbola)
(D) circumcentre of ΔOQR is (a, b) then a – b = –1

45. Triangle is formed by the lines $y^3 - x^2y - 2y^2 + y + \lambda xy(y - 1) = 0$ (λ is a parameter), then
(A) locus of orthocentre of Δ is $y - 1 = 0$
(B) locus of circumcentre of Δ is $y = 0$
(C) locus of centroid of Δ is $3y - 1 = 0$
(D) locus of incentre of Δ is $x^2 - y^2 - 2y + 1 = 0$

46. Normals drawn at points A, B and C of a parabola are concurrent at N(4, 2). If A ≡ (3, 3) and B ≡ (1, –1). Then which of the following can be true?
(A) Focus is at $S \equiv \left(\frac{6}{5}, -\frac{3}{5}\right)$
(B) Slope of directrix is $-\frac{1}{2}$
(C) Number of parabolas possible is 1
(D) Number of parabolas possible is 2

47. A circle is drawn with centre (α, β) : α is rational and β is irrational, then
(A) there exist at most two rational points on the circle
(B) if there exist exactly two rational points P and Q on the circle then slope of PQ must be 0
(C) if there exist only one point $R(x_1, y_1)$ on circle with $(2, \sqrt{3})$ as centre then radius must be irrational
(D) for exactly one rational point $R(x_1, y_1)$ on circle with $(2, \sqrt{3})$ as centre then $x_1 = 1$

48. Let Q and Q′ be the feet of perpendiculars from foci S and S′ to the tangent at a point P on an ellipse with eccentricity = $\frac{1}{2}$. Given that SQ = 2S′Q′ and S′P = 4. If SP and S′Q′ intersect at R, then

(A) SP is equal to 2

(B) SQ is equal to $3\sqrt{6}$

(C) SR is equal to 12

(D) Q′Q is equal to $3\sqrt{5}$

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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49. Let $C_1 \equiv x^2 + y^2 - 169 = 0$ and $C_2 \equiv x^2 + y^2 - 24x - 32y + 111 = 0$. Let AB be their common chord. Maximum area of ΔABC if C lies on minor arc AB, is equal to

50. The ellipse $E : \frac{x^2}{25} + \frac{y^2}{16} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. A hyperbola (with same centre and transverse axis same as major axis of ellipse) intersects the ellipse orthogonally and passes through the vertices of the rectangle. Then eccentricity of hyperbola is

51. Sum of the series $\cot^{-1}(2\sqrt{2}) + \cot^{-1}(4\sqrt{2}) + \cot^{-1}(7\sqrt{2}) + \cot^{-1}(11\sqrt{2}) + ...$ upto infinite terms is $\tan^{-1}(k)$, then k is equal to

52. ABC is an isosceles triangle right angled at B. A point P lies inside such that PA = PB and $\angle PBA = \angle PCA = \alpha$. Then maximum value of $f(\theta) = \sin 2\alpha \sin \theta + \operatorname{cosec} 3\alpha \cos \theta : \theta \in R$, is equal to

53. The value of $\sum_{p=1}^{10}\sum_{q=1}^{10}\sum_{r=1}^{10} p \tan^{-1}\left(\frac{q}{r}\right) = S$, then S is equal to

54. If value of $\sin 3°.\sin 9°.\ \sin 15°.\sin 21° \ ..... \ \sin 87°$ is $\frac{1}{2^x}$, then x is equal to

ALL INDIA TEST SERIES

TEST - 3

JEE (Advanced)

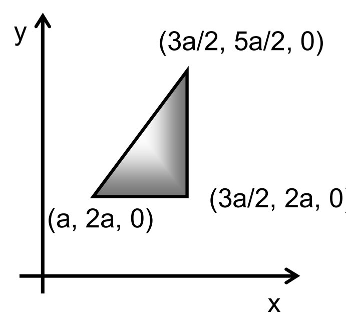
ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. C

Sol. If a point charge is at $\left(a, 2a, \frac{a}{2}\right)$ then given surface is $\frac{1}{8}$th of a square surface of side 'a'



2. D

Sol. In the steady state current through each bulb is

$$i_0 = \frac{400}{100+100} = 2\text{ A}$$

Now, when the switch is opened, the direction of current in bulb B_1 is reversed.

3. B

Sol. Initially

$P_1V_1 = nRT_1$ …(1)

and

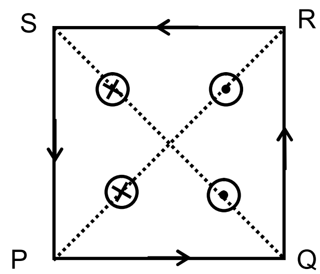
$$P_2 = P_0 + \frac{mg}{A} = \frac{7}{3} \times 10^5 \text{Pa} \qquad \ldots(2)$$

$P_2V_2 = nRT_2$ …(3)

So $\dfrac{P_2}{P_1} = \dfrac{T_2}{T_1}$

$T_2 = 700$ K = 427°C

4. C



Sol. Let $\int_P^Q \vec{B}_A \cdot d\vec{\ell} = -9\mu_0 = x$

$\int_R^S \vec{B}_B \cdot d\vec{\ell} = 5\mu_0 = y$

Now, apply amperes low in loop PQRS

$$2\left[\left(x - \frac{y}{2}\right) + (-2x + y) + (-8x + 4y) + (4x - 2y)\right] = \mu_0(5i_0)$$

$i_0 = 23$ amp

5. D

Sol. $d\phi = B.h.dr$

$$\phi = \int_{R_1}^{R_2} \frac{\mu_0 N}{4\pi} . \frac{2i}{r} h\, dr$$

$$= \frac{\mu_0 N h i}{2\pi} \ln \frac{R_2}{R_1}$$

$$= \left(\frac{\mu_0 N h}{2\pi} \ln \frac{R_2}{R_1}\right) i_0 \cos \omega t$$

$$\text{emf} = \frac{d\phi}{dt}$$

$$(\text{emf})_{\max} = \frac{\mu_0 h N \omega i_0}{2\pi} \ln \frac{R_2}{R_1}$$

6. B

Sol. At the state of impending motion, friction force has maximum value.

$$mg \sin\theta - qE \cos\theta - f_f = ma_0$$

$$a_0 = r\alpha$$

On solving 3qE = mg

7. A, D

Sol. $i^2 R_1 t = m_1 c_1 (T_1 - T_0)$

$$t_0 = \frac{c_1 d_1 S_1^2 (T_1 - T_0)}{i^2 \rho_1} \approx 0.095 \sec$$

Also $i^2 R_2 t = m_2 c_2 (\Delta T)$

$$\frac{R_1}{R_2} = \frac{m_1 c_1 (T_1 - T_0)}{m_2 c_2 \Delta T}$$

$$\Delta T = \frac{c_1 d_1 S_1^2 \rho_2}{c_2 d_2 S_2^2 \rho_1}\left(T_1 - T_0\right) \approx 0.106\,°C$$

8. B, C, D

Sol. Since the particle is having acceleration along x-axis so the path of the particle is a non uniform helix.

9. B, D

Sol.
$$\left\{\begin{aligned} &P_i\left(A\frac{\ell}{2}\right) = 2RT_i \\ &P_f\left(A\left(\frac{\ell}{2}+x\right)\right) = 2RT_f \end{aligned}\right\} \quad \ldots(1)$$

$$\left\{\begin{aligned} &P_i A = K\frac{\ell}{2} \\ &P_f A = K\left(\frac{\ell}{2}+x\right) \end{aligned}\right\} \quad \ldots(2)$$

Now

$$\left\{\begin{aligned} &K\left(\frac{\ell}{2}\right)^2 = 2RT_i \\ &K\left(\frac{\ell}{2}+x\right)^2 = 2RT_f \end{aligned}\right\} \quad \ldots(3)$$

Work done by gas,

dw = pdv = (ky)dy

$$W = K\int_{\ell/2}^{(\ell/2+x)} y\,dy = \frac{K}{2}\left[\left(\frac{\ell}{2}+x\right)^2 - \left(\frac{\ell}{2}\right)^2\right] \quad \ldots(4)$$

Change in internal energy

$$\Delta U = nC_V\,\Delta T = 2.\frac{5}{2}R\left(T_f - T_\ell\right) \quad \ldots(5)$$

$$\Delta Q = W + \Delta U = 6R\,\Delta T$$

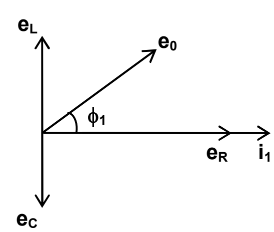
$$C = \frac{\Delta Q}{\Delta T} = 6R$$

10. C, D

Sol. Charges are non-uniformly distributed on the outer surface of the conductor due to point charge ‘q’.

11. A, C

Sol. Phasor diagram of Box-1



$$\tan\phi_1 = \frac{100-70}{40} = \frac{3}{4}$$

$$z_1 = \sqrt{(x_L - x_C)^2 + R^2} = 50\Omega$$

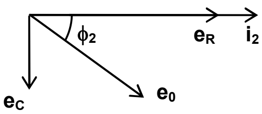
$$i_1 = \frac{200}{50} = 4\Omega$$

Phasor Diagram of Box-2

$$\tan\phi_2 = \frac{40}{30} = \frac{4}{3}$$

$$z_2 = \sqrt{x_C^2 + R^2} = 50\Omega$$

$$i_2 = \frac{200}{50} = 4 \text{ amp}$$



12. A, B, C, D

Sol. Let a charge 'q' flown from capacitor C_1 to C_2 at time 't'. Now for KVL.

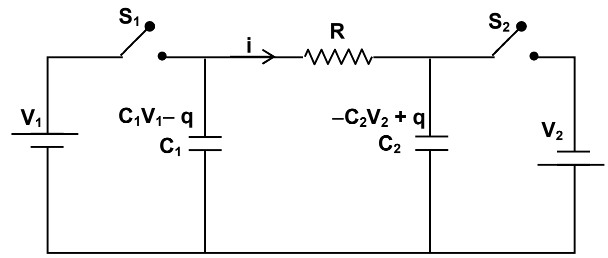
$$\frac{C_1V_1 - q}{C_1} - iR + \frac{C_2V_2 - q}{C_2} = 0 \qquad \ldots(i)$$

Let $V_1 + V_2 = V$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

After solving

$$q = CV\left(1 - e^{-\frac{t}{RC}}\right)$$



SECTION – C

13. 00108.75

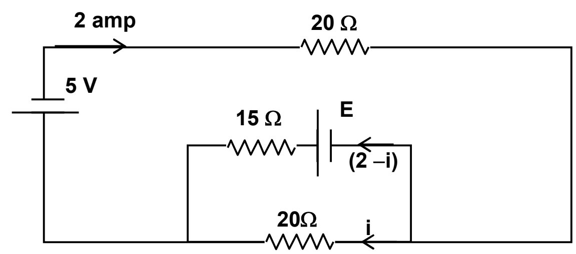
Sol. Circuit can be redrawn as shown.

$$-5 - 40 - 20i = 0$$

$$i = -\frac{9}{4} \qquad \ldots(i)$$

$E - 15(2 - i) + 20i = 0$

$E = 108.75$



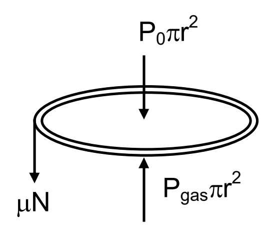
14. 00121.20

Sol. F.B.D. of cork

$\mu N + P_0\pi r^2 = 4P_0\pi r^2$

$$N = \frac{3P_0\pi r^2}{\mu}$$

So $\dfrac{N}{2\pi r} = \dfrac{3}{2}\dfrac{P_0 r}{\mu} = 121.2 \times 10^3$ N/m



15. 00001.00

Sol. induced emf, $\varepsilon = Bbv_T$

$$i = \frac{Bbv_T}{R}$$

In equilibrium

mg = ibB

$$v_T = \frac{mgR}{B^2b^2}$$

16. 00110.88

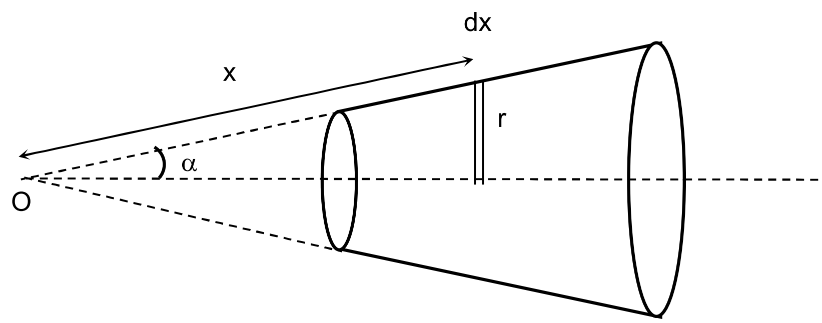
Sol. Consider a strip of length dx at x from apex along the slant length

$$di = \left(\frac{N}{\ell_0}dx\right)i \qquad \ldots(i)$$

$$dB = \frac{\mu_0}{2}\frac{(di)r^2}{x^3} \qquad \ldots(ii)$$

$$\sin\alpha = \frac{r}{x} = \frac{b-a}{\ell_0} \qquad \ldots(iii)$$

$$\text{So, } B = \frac{\mu_0 Ni(b-a)^2}{2\ell_0^3}\ell n\left(\frac{b}{a}\right) = 110.88\times10^{-6}\text{ Tesla}$$



17. 00504.20

Sol. At equilibrium

$$\sigma A\left(T^4 - T_1^4\right) = \sigma A T_1^4$$

$$T_1^4 = \frac{T^4}{2}$$

18. 00000.02

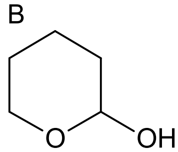
Sol. $\alpha = \frac{1}{R}\frac{dR}{dt} = \frac{a+2bt+3ct^2}{1+at+bt^2+ct^3} = 0.02\ (^\circ C)^{-1}$

SECTION – A

19. A
Sol. Beckmann rearrangement.

20. C
Sol. Guanidine is the most basic compound among the given compounds. It's conjugate acid is most stable due to resonance.

21. C
Sol. Bayer Villiger oxidation.

22. B
Sol. 

is hemiacetal. It gives positive Fehling solution test.

23. C
Sol. Hydroboration oxidation is a *syn* addition.

24. C
Sol. $NaBH_4$ reduces ketones but does not reduce esters.

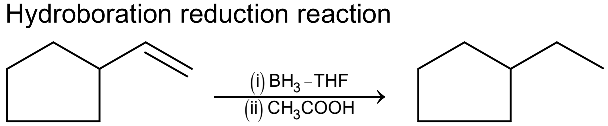
25. A, B, D
Sol. Cumulenes with even number of double bond do not show geometrical isomerism.

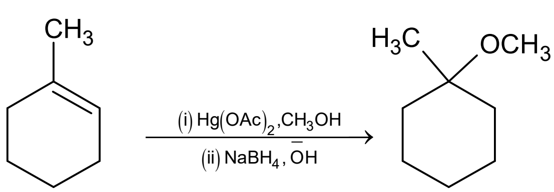
26. B, C, D
Sol. Compound (P) is Lactose.

27. B, C, D
Sol. Neoprene is a polymer of chloroprene.

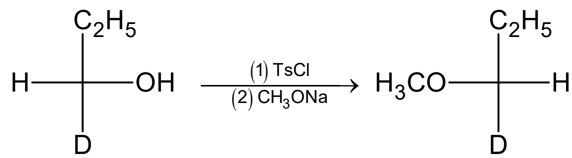
28. A, B
Sol. $-SO_3H$ and $-CN$ are metadirecting and deactivating groups.

29. A, C
Sol. (B) Hydroboration reduction reaction



(D)


30. A, D

Sol. (B) 

(C) When alcohol reacts with $SOCl_2$ in presence of pyridine inversion takes place.

SECTION – C

31. 00049.20

Sol. $[\alpha] = \dfrac{\theta}{\ell \times c} = \dfrac{24.6}{\frac{2}{10} \times \frac{25}{10}}$

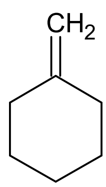
$= 49.20$

32. 00096.47

Sol. Percentage yield of 2-Bromopropane $= \dfrac{164}{170} \times 100$

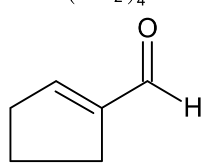
= 96.47

33. 00012.50

Sol. Product (R) is 

34. 00019.20

Sol. $A = OHC-(CH_2)_4-CHO$

B = 

MW of B = 96

$\therefore \dfrac{x}{5} = \dfrac{96}{5} = 19.20$

35. 00001.50

Sol. $6HCHO + 4NH_3 \longrightarrow \underset{\text{Urotropine}}{(CH_2)_6N_4} + 6H_2O$

Number of carbon atom = x = 6

Number of nitrogen atom = y = 4

$\therefore \dfrac{x}{y} = \dfrac{6}{4} = 1.5$

36. 00002.40

Sol.
$$\begin{array}{l} H_2C-OH \\ \quad | \\ HC-OH \\ \quad | \\ H_2C-OH \end{array} \xrightarrow[\text{excess}]{CH_3MgBr} 3CH_4$$

Weight of CH_4 produced $= \dfrac{48}{92} \times 4.6$ = 2.4 grams

SECTION – A

37. C

Sol. Area of hexagon = 2 · Area of ΔABC

= 2 · 6 · Area of ΔBMD

$= 12 \cdot \frac{1}{2} \times BM \cdot MD \cdot \sin\frac{\pi}{6}$

= 3 · 6 · 2 = 36 sq. units

38. B

Sol. Equation of ellipse can be written as $\frac{\left(\frac{x+y+1}{\sqrt{2}}\right)^2}{2} + \frac{\left(\frac{x-y-3}{\sqrt{2}}\right)^2}{1} = 1$ which is similar to

$\frac{x^2}{2} + \frac{y^2}{1} = 1$ whose director circle is $x^2 + y^2 = 3$

Let $P \equiv \left(\sqrt{3}\cos\theta, \sqrt{3}\sin\theta\right)$ equation of chord of contact is $x\sqrt{3}\cos\theta + 2y\sqrt{3}\sin\theta - 2 = 0$

It touches the circle $x^2 + y^2 = r^2 \Rightarrow r = \frac{|0+0-2|}{\sqrt{3\cos^2\theta + 12\sin^2\theta}} = \frac{2}{\sqrt{12 - 9\cos^2\theta}}$

$\Rightarrow r_{max} = \frac{2}{\sqrt{3}}$ and $r_{min} = \frac{2}{\sqrt{12}} \Rightarrow \frac{r_{max}}{r_{min}} = 2$

39. B

Sol. We have $\left|\sin^{-1}\left(a^2 - 3\right)\right| + \cos^{-1}\left(a^2 - 3\right) = \frac{\pi}{2}$; for $a^2 \geq 1$

$\Rightarrow \sin^{-1}\left(a^2 - 3\right) \geq 0 \Rightarrow 0 \leq a^2 - 3 \leq 1$

$\Rightarrow$ a has two integral values

40. D

Sol. In a triangle ABC, $\cos A < 1 \Rightarrow \frac{b^2 + c^2 - a^2}{2bc} < 1 \Rightarrow b^2 + c^2 - a^2 < 2bc$ ..... (1)

Similarly $c^2 + a^2 - b^2 < 2ca$ ..... (2)

$a^2 + b^2 - c^2 < 2ab$ ..... (3)

$\Rightarrow a^2 + b^2 + c^2 < 2(ab + bc + ca) \Rightarrow \frac{ab + bc + ca}{a^2 + b^2 + c^2} > \frac{1}{2}$

Also, $\frac{ab + bc + ca}{a^2 + b^2 + c^2} \leq 1 \Rightarrow \frac{1}{2} < \cos\theta \leq 1 \Rightarrow \theta \in \left[0, \frac{\pi}{3}\right)$

41. D

Sol. $A\left(x_1, \sqrt{7}x_1\right)$ and $B\left(x_2, \sqrt{7}x_2\right)$ satisfy the curve $\sqrt{2}x^2 + \sqrt{3}y + \sqrt{5}x + \sqrt{8} = 0$

$\Rightarrow \sqrt{2}x^2 + \left(\sqrt{21} + \sqrt{5}\right)x + \sqrt{8} = 0 \Rightarrow x_1x_2 = 2$

Length of tangent $OP = \sqrt{OA \cdot OB} = \sqrt{8x_1x_2} = 4$

42. D

Sol. R has two positions

$\Rightarrow$ Q and T have two positions

$\Rightarrow$ QT has two values 2 and 10

As $PQ_1 R_1$ and $PQ_1 S_1$ are congruent

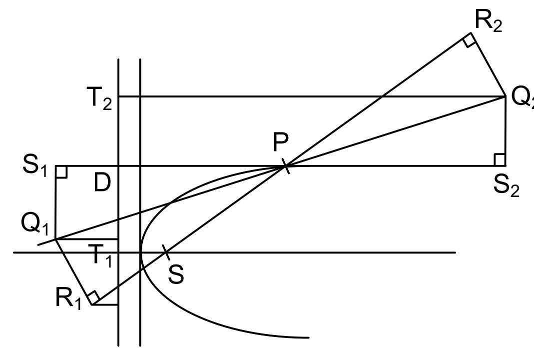
$\Rightarrow PR_1 = PS_1$ also PS = PD

$\Rightarrow Q_1T_1 = DS_1 = 2$

Similarly Δs PQ_2R_2

and PQ_2S_2 are also congruent

$\Rightarrow Q_2T_2 = 10$



43. B, D

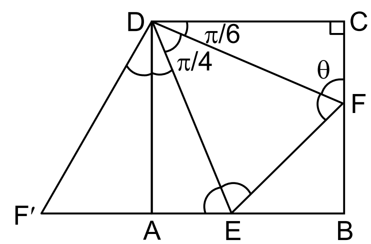
Sol. As FD = 2FC $\Rightarrow \frac{FC}{FD} = \frac{1}{2} \Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

Paste DCF such that C coincides with A and F goes to F′

Clearly triangles DEF′ and DEF are congruent

$\Rightarrow \angle DEF = \angle DEF' = \frac{5\pi}{12}$ and $\angle DFE = \frac{\pi}{3}$

Clearly D is excentre of Δ FBE $\Rightarrow$ Ex-radius = CD = 1 unit



44. A, C

Sol. Point of tangency is the mid-point of QR and hence lies on hyperbola $4x^2 + 8xy - y^2 + \lambda = 0$

$\Rightarrow \lambda = -4$

Let the circumcircle of ΔOQR be $x^2 + y^2 - 2ax - 2by = 0$

Homogenizing and comparing with pair of asymptotes, we get a – b = 1

45. B, C, D

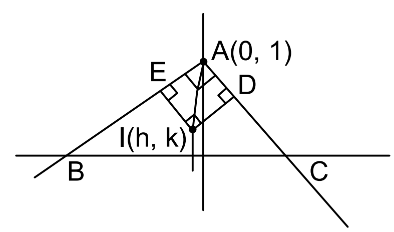
Sol. Two of the three lines represented are perpendicular to each other and intersecting at (0, 1) which is orthocentre a fixed point.

Clearly circumcentre lies on x-axis

We have $AI = k\sqrt{2}$

$\Rightarrow (h - 0)^2 + (k - 1)^2 = \left(k\sqrt{2}\right)^2$

$\Rightarrow x^2 - y^2 - 2y + 1 = 0$



46. A, C

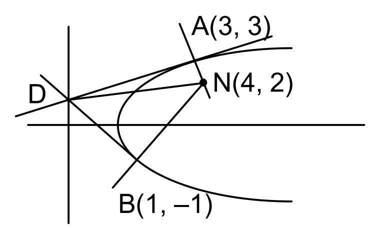
Sol. Clearly AN $\perp$ BN $\Rightarrow$ AB is a focal chord

$\Rightarrow D \equiv (0, 0)$

Directrix will be perpendicular to DN

$\Rightarrow$ Slope of directrix = –2. Focus will be foot of perpendicular from D on focal chord 2x – y – 3 = 0

$\Rightarrow S \equiv \left(\frac{6}{5}, -\frac{3}{5}\right)$



47. A, B, C

Sol. With centre at $\left(2, \sqrt{3}\right)$ equation of circle is $(x-2)^2 + \left(y - \sqrt{3}\right)^2 = r^2$

Let (x_2, y_2) and (x_3, y_3) be two rational points

$\Rightarrow x_2^2 + y_2^2 - 4x_2 - 2\sqrt{3}y_2 + 7 - r^2 = 0$ ..... (1)

and $x_3^2 + y_3^2 - 4x_3 - 2\sqrt{3}y_3 + 7 - r^2 = 0$ ..... (2)

Subtracting (1) and (2), we get

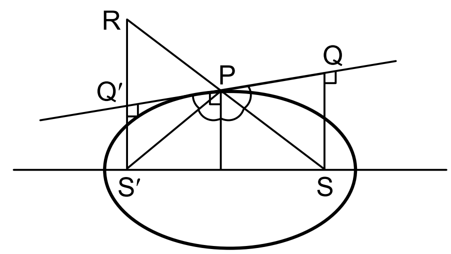
$(x_2 - x_3)(x_2 + x_3 - 4) + (y_2 - y_3)(y_2 + y_3 - 2\sqrt{3}) = 0$

For two rational points $\Rightarrow y_2 - y_3 = 0 \Rightarrow$ slope PQ = 0 for only one rational point

$\Rightarrow x_2 = x_3 \Rightarrow x_1 = 2$

48. B, C

Sol. Clearly Δs PSQ and PS′Q′ are similar



$\Rightarrow \frac{SP}{S'P} = \frac{SQ}{S'Q'} = 2 \Rightarrow SP = 8$

Also, $S'P = PR = 4 \Rightarrow SR = 12$

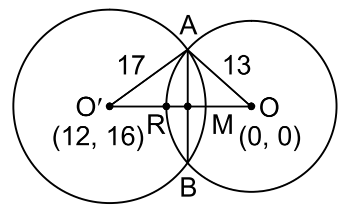
Also, $SQ.S'Q' = b^2 = a^2 - a^2e^2$

$\Rightarrow \frac{SQ^2}{2} = 36\left(1 - \frac{1}{4}\right) = 27 \Rightarrow SQ = 3\sqrt{6}$

SECTION – C

49. 00065.72

Sol. Let M be the mid-point of AB



$\Rightarrow AM^2 = 17^2 - x^2 = 13^2 - (20 - x)^2$

$\Rightarrow x = 13$

$\Rightarrow AB = 4\sqrt{30}$, clearly when C lies at R

Area is maximum $\Rightarrow$ Maximum area = $\frac{1}{2} \times 4\sqrt{30} \times 6$

= $12\sqrt{30}$ sq. units = 65.72

50. 00001.34

Sol. Let the equation of hyperbola be $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ ..... (1)

Eccentricity (e) of the ellipse $e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

As hyperbola intersects the ellipse orthogonally and passes through vertices

$\Rightarrow$ Their foci are coincident $\Rightarrow Ae_H = 3 \Rightarrow B^2 = A^2e_H^2 - A^2 = 9 - A^2$

Again hyperbola must pass through (5, 4) the vertex of rectangle

$\Rightarrow \frac{25}{A^2} - \frac{16}{9 - A^2} = 1 \Rightarrow A^4 - 50A^2 + 225 = 0 \Rightarrow A^2 = 5$ only

$\Rightarrow$ Equation of hyperbola is $\frac{x^2}{5} - \frac{y^2}{4} = 1,\ e = \sqrt{1 + \frac{4}{5}} = 1.34$

51. 00001.41

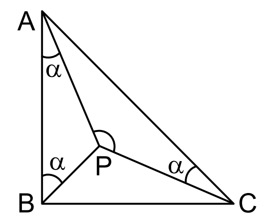
Sol. $S = \tan^{-1}\left(\frac{1}{2\sqrt{2}}\right) + \tan^{-1}\left(\frac{1}{4\sqrt{2}}\right) + \tan^{-1}\left(\frac{1}{7\sqrt{2}}\right) + ...$

$$= \tan^{-1}\left(\frac{\left(\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)}{1 + \frac{2}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}}\right) + \tan^{-1}\left[\frac{\frac{3}{\sqrt{2}} - \frac{2}{\sqrt{2}}}{1 + \frac{3}{\sqrt{2}} \cdot \frac{2}{\sqrt{2}}}\right] + ...\infty \text{ terms}$$

= $\frac{\pi}{2}-\tan^{-1}\frac{1}{\sqrt{2}}=\frac{\pi}{2}-\cot^{-1}\sqrt{2}=\tan^{-1}\left(\sqrt{2}\right)$

52. 00001.50

Sol. Applying sin rule in Δs PAB and PAC, we get



$\frac{PA}{\sin\alpha}=\frac{AB}{\sin 2\alpha}$ ..... (1)

$\frac{PA}{\sin\alpha}=\frac{AC}{\sin\frac{3\pi}{4}}$ ..... (2)

$\Rightarrow$ sin 2α = $\frac{1}{2}$ $\Rightarrow$ $\alpha=\frac{\pi}{12}$. Maximum value of $f(\theta)=\sqrt{\sin^2 2\alpha+\operatorname{cosec}^2 3\alpha}=\sqrt{\frac{1}{4}+2}=\frac{3}{2}$

53. 04319.69

Sol. $\sum_{q=1}^{10}\sum_{r=1}^{10}\sum_{p=1}^{10}p\tan^{-1}\left(\frac{q}{r}\right)$ = $\sum_{q=1}^{10}\sum_{r=1}^{10}\tan^{-1}\left(\frac{q}{r}\right)\times\frac{10\times 11}{2}=55.[25\pi]=4319.69$

54. 00014.50

Sol. Let P = sin 3°.sin 9°.sin 15° ..... sin 81°.sin 87°

Q = sin 6°.sin 12°.sin 18° ..... sin 84°

PQ = sin 3°.sin 6°.sin 9°.sin 12° ..... sin 81°.sin 84°.sin 87°

= $\frac{1}{2^{14}}\cdot(2\sin 3^\circ\cdot\cos 3^\circ)\cdot(2\sin 6^\circ\cdot\cos 6^\circ)(2\sin 9\cdot\cos 9^\circ).....(2\sin 42^\circ\cdot\cos 42^\circ)\cdot\sin 45^\circ$

= $\frac{1}{2^{14.5}}\cdot Q$

$\Rightarrow$ $P=\frac{1}{2^{14.5}}$

$\Rightarrow$ x = 14.50