ALL INDIA TEST SERIES

TEST - 4

JEE (Advanced)

Time Allotted: 3 Hours **Maximum Marks: 198**

<u>General Instructions:</u>

- The test consists of total **54** questions.
- Each subject (PCM) has **18** questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Three Sections**: **Section-A, Section – B & Section-C.**

__Section-A (01 – 06, 19 – 24, 37– 42)__ this section contains __18 multiple__ choice questions.
Each question has FOUR options. __ONE OR MORE THAN ONE__ of these four option(s) is (are) correct answer(s).
For each question, choose the option(s) corresponding to (all) the correct answer(s)
Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+4** If only (all) the correct option(s) is (are) chosen:
Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen;
Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : **0** If none of the options is chosen (i. e. the question is unanswered);
Negative Marks : –**2** In all other cases

__Section-B (07 – 12, 25 – 30, 43– 48)__ contains __18 Numerical__ based questions with __Single digit integer__ as answer, ranging from __0 to 9__ and each question carries __+3 marks__ for correct answer and __–1 mark__ for wrong answer.

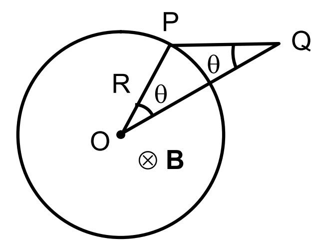
__Section-C (13 – 18, 31 – 36, 49– 54)__ contains __18 Numerical__ answer type questions with answer XXXXX.XX and each question carries __+4 marks__ for correct answer and __0 marks__ for wrong answer.

Physics PART – I

SECTION – A
(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

1. In a circular region centered at O and having radius R, magnetic field increases at a constant rate a_0 T/sec. A wire frame OPQ having resistance per unit length λ Ω/m is placed as shown in the figure. Choose the correct option(s).



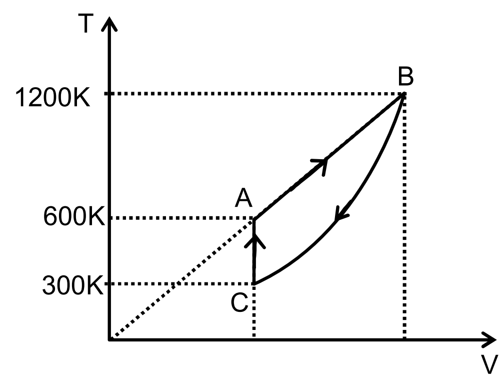
(A) $V_P = V_O$ for any θ as there is no induced emf across OP.

(B) $V_O - V_Q = \dfrac{a_0 \pi R^2}{8(\sqrt{2}+1)}$ for $\theta = 45°$

(C) $V_O - V_Q = \dfrac{a_0 \pi R^2}{2(\sqrt{2}-1)}$ for $\theta = 45°$

(D) $V_P - V_Q = \dfrac{a_0 \pi R^2}{8\sqrt{2}}$ for $\theta = 45°$

2. Two moles of an ideal monoatomic gas is taken through a cyclic process ABCA as shown in the T-V diagram. In the process BC, $T = \alpha V^2$, where α is a constant. Then choose the correct option(s). (R = ideal gas constant).



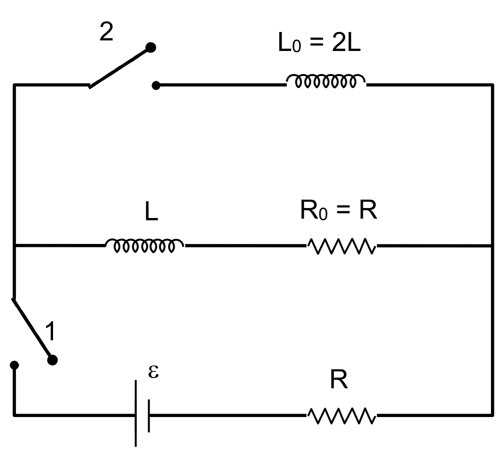
(A) The heat absorbed by the gas during the process AB is 3000 R.

(B) The heat rejected by the gas during the process BC is 3600 R.

(C) The heat rejected by the gas during the process BC is 1800 R.

(D) The total work done by the gas during the cyclic process is 300 R.

3. In the circuit shown, initially only switch-1 is closed for a long time. Then switch-2 is also closed. Choose the correct option(s).



(A) Current through the inductor 'L_0' in the steady state after closing the switch-2 will be $\frac{\varepsilon}{R}$.

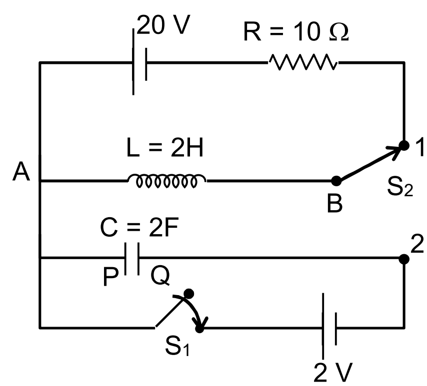
(B) Current through the inductor 'L_0' in the steady state after closing the switch-2 will be $\frac{\varepsilon}{2R}$.

(C) The total charge $\left(\frac{5\varepsilon L}{2R^2}\right)$ will flow through the resistor 'R_0' after closing the switch-2.

(D) The total charge $\left(\frac{3\varepsilon L}{2R^2}\right)$ will flow through the resistor 'R_0' after closing the switch-2.

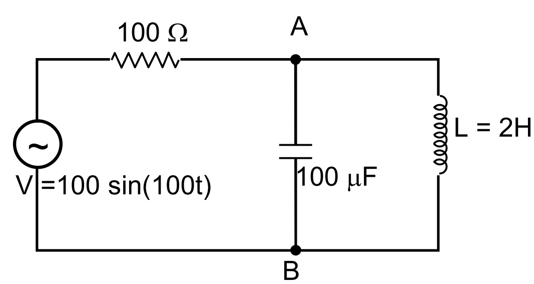
4. Choose the correct option(s).
(A) Coulomb's law can be used to calculate force between two charges when they are at rest only.
(B) Gauss's Law can be applied for moving charges as well as for charges at rest.
(C) Two solid spheres of radius R_1 and R_2 ($R_1 > R_2$) are made of same material and are heated to same temperature. Smaller sphere will cool faster if both are kept in the same environment.
(D) Kirchhoff's circuital Laws (KCL and KVL) can be used for AC circuit also.

5. A capacitor and an inductor are connected in a circuit as shown in the figure. Initially switch S_2 is at position-1 and switch S_1 both are closed for long time. Now S_1 is opened and S_2 is shifted to position-2 at t = 0. Then choose the correct option(s).



(A) At t = 2π sec, charge on the capacitor is 4C with plate P as positive plate
(B) At t = 2π sec, charge on the capacitor is 4C with plate Q as positive plate
(C) Maximum current through the inductor is $2\sqrt{2}$ A
(D) Current through the inductor at t = 3π seconds is 2A from B to A.

6. An AC source of voltage varying as V = 100 sin (100 t) volts is connected with resistance, capacitor and inductor as shown in the figure. Choose the correct option(s).



(A) Voltage across capacitor at time t = π/4 sec is 40 V.
(B) At t = 0 inductor will gain energy at rate 16 J/s
(C) At t = 0 capacitor will loose energy at rate 32 J/s
(D) Power consumption in circuit at t = 0 is 16 J/s.

SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

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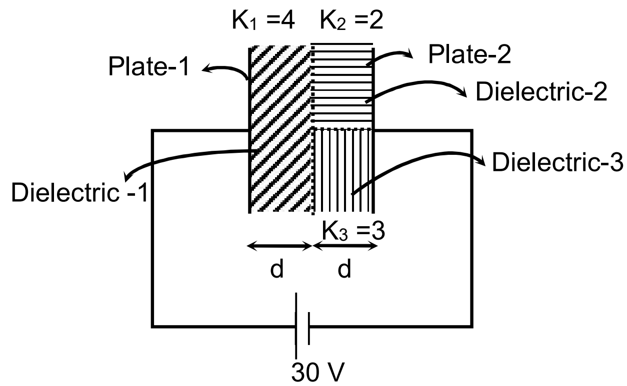
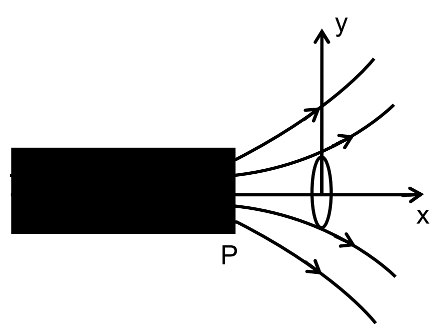
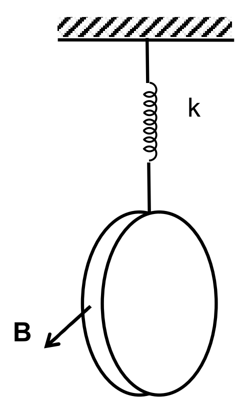
7. Three dielectrics are filled between parallel conducting plates, each plate having area 4 cm^2. Dielectrics 2 and 3 are of same size. Charge on the upper half of the plate-1 is q. Find the value of $\frac{q}{\varepsilon_0}$ (in volt meter).
(Given d = 1 mm)



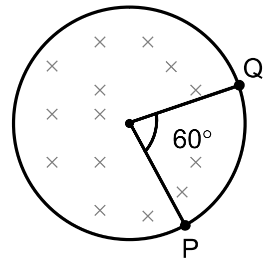
8. A circular ring of mass 100 gm and radius 0.5 m has zero resistance. Initially its plane is in y-z plane and co-axial with the magnet. It is given an initial velocity 1 m/s towards the magnet when its centre is at origin at a distance a_0 from end P of the magnet. The axial component of magnetic field is $B_x = B_0(1-x)T$ and the radial component $B_r = 2B_0$ T. The maximum distance travelled by coil towards magnet from its initial position is $\left(\frac{k}{10}\right)$ meter. Find the value of k. (Assuming initial current in coil to be zero. a_0 to be sufficiently large to avoid collision of coil with magnet and plane of coil always remains parallel to Y-Z plane. Given B_0 = 0.1 T, π^2 = 10, inductance of coil is 0.02 H)



9. A conducting disc of mass 'm' and volume 'V' is suspended with the help of a light spring of force constant 'k' from a fixed support. Thickness of the disc is much smaller than its radius. A uniform magnetic field of induction 'B' parallel to the plane of the disc is established. The disc is slightly pulled down from equilibrium position and then released. If the time period of small oscillations of the disc is $2\pi\sqrt{\frac{\alpha m}{k}}$. Find the value of α. (Take $\varepsilon_0 VB^2 = 2m$)



10. A circular coil is made from wire of uniform area of cross section and having resistance 12Ω. A voltmeter of resistance 10Ω is connected between the points P and Q of the coil and placed at the centre of the coil as shown in the figure. Given magnetic field within region, B = 2t tesla. Area of circular coil is 6 m^2 (Assume connecting wire has no resistance). Calculate the reading of the volt meter in volt.



11. A ball of mass 100 gm and carrying charge 1C is released from a point near the surface of earth, in a uniform horizontal magnetic field of 2T. Find the maximum speed attained by the ball in m/s. (Take g = 10 m/s^2, assume ball attains maximum speed before hitting ground)

12. Two rods each of length L_1 and one rod of length L_2 form isosceles triangle having base of length L_2. Co-efficient of linear expansion of base is 1×10^{-6} /°C while for rod of length L_1 is 4×10^{-6} /°C. Find the ratio $\left(\frac{L_2}{L_1}\right)$, if the length of latitude remains same when the temperature of the system is increased by 10°C.

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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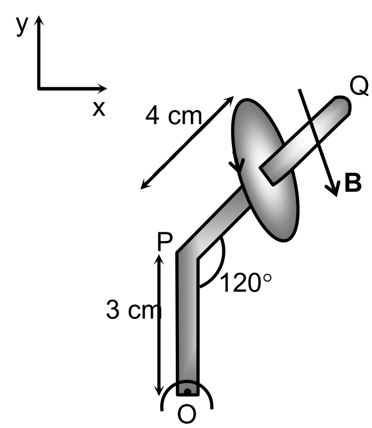
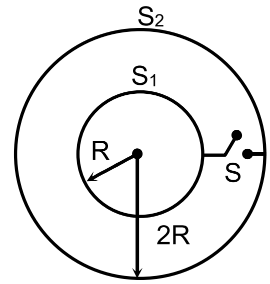
13. A ring of area 20 cm^2 is fixed to a rigid massless frame OPQ at its centre with the plane of ring perpendicular to PQ with the help of a non conducting rigid massless wires. Initially frame is in plane of paper and plane of ring is normal to the plane of paper. Frame can rotate about horizontal axis through O and normal to the plane of paper. A current of 2A flows through the ring. Find the initial angular acceleration of system in rad/s^2 about O. Mass of ring is 0.5 kg. A uniform magnetic field $\vec{B} = \left(4\hat{i} - \sqrt{3}\hat{j}\right)$ tesla is existing in the region.



14. A capacitor having capacitance 100 μF, filled with dielectric having resistance $\left(\frac{100}{\pi}\right)\Omega$, is connected in series with a resistance of $\frac{100}{\pi\sqrt{2}}\Omega$. Breakdown voltage of capacitor is 150 V. An AC voltage V = V_0sin(100πt) is applied across the combination. Find the maximum value of V_0 so that capacitors functions properly.

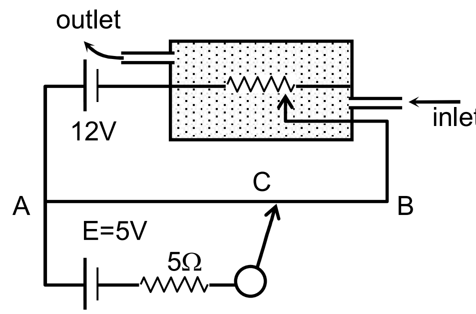
15. A body of mass 2 gm has specific heat capacity 0.25 cal/gm-°C and surface area 30 cm^2. It is kept in an enclosure which is maintained at temperature 127°C. Body is being supplied heat at rate 2 watt. If temperature of body is 27°C and its absorptive power is 0.5. Determine the rate at which its temperature is changing (in K/s) at the given instant. (Stefan's constant is 5.67×10^{-8} W/m^2-k^4, 1 cal = 4.2 J)

16. Two conducting spherical shells S_1 of radius R and S_2 of radius 2R are co-centric. A charge Q = 20 μC is kept at the centre and another same charge Q is given to S_1 and –Q to S_2. Determine the loss of energy(in Joule) when switch S is closed (Given R = 9 cm)



17. A thin conducting spherical shell of radius R and wall thickness t tears apart when placed in a uniform electric field E_0. Another thin spherical shell of same material but of radius 3R and wall thickness 2t is placed in a uniform electric field E. For what value of $\left(\frac{E}{E_0}\right)$ later shell will tear apart.

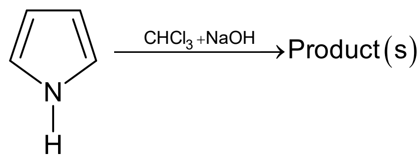
18. Rheostat is inside water kept in insulated box (ignore conductivity of the water). Water enters through inlet at 2mg/sec at temperature 15°C and comes out at same rate through outlet, at temperature 27°C. Wire of rheostat is cylindrical in shape having surface area 20 cm^2 and emissivity 0.5. Resistance of rheostat in the circuit is 100Ω at 0°C. Rheostat is connected in potentiometer arrangement along with driving cell of 12V and cell of emf 5V as shown in the figure. If balancing length AC is 40 cm at steady state. Find the length of wire AB in cm. Assume water in the box remains at temperature 27°C throughout, $\sigma = 5.65 \times 10^{-8}$ W/m^2-k^4. Thermal coefficient of resistivity of rheostat material = 4×10^{-3} /°C, specific heat of water = 4.2 J/gm-°C.

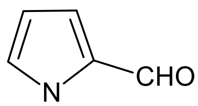


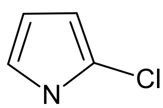
SECTION – A
(One or More than one correct type)

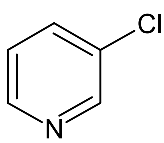
This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

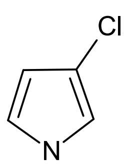
19. In the following reaction, the product (s) formed is/are



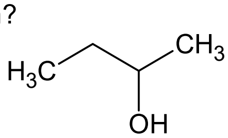
(A) 

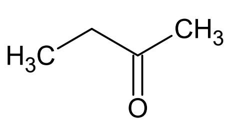
(B) 

(C) 

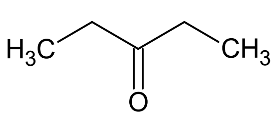
(D) 

20. Which of the following compound(s) form yellow precipitate on reaction with I_2 and NaOH solution?

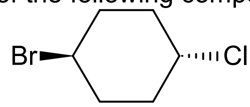
(A) 

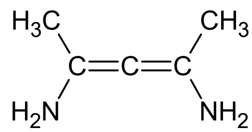
(B) 

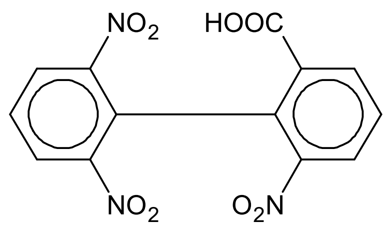
(C) $CH_3 – CH_2 – OH$

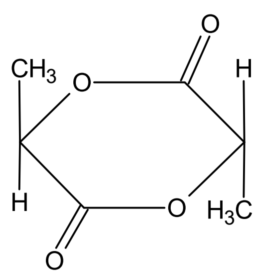
(D) 

21. Which of the following compound (s) is/are optically inactive?

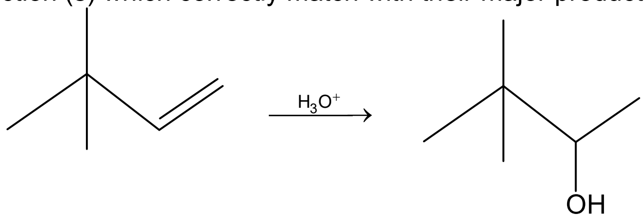
(A) 

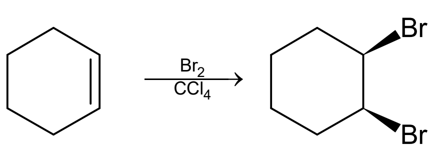
(B) 

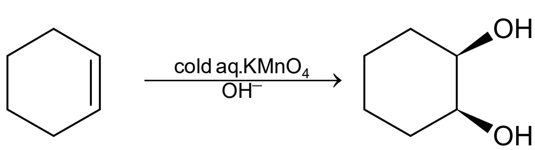
(C) 

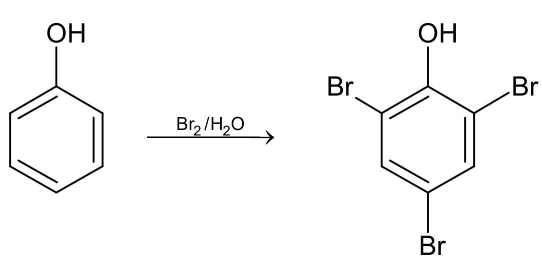
(D) 

22. The reaction (s) which correctly match with their major product is/are

(A) 

(B) 

(C) 

(D) 

23. Which of the following compounds will be oxidized by HIO_4?

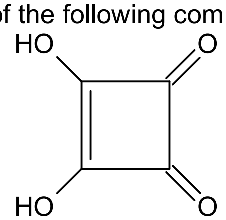
(A) H_2C-OH / $HC-OH$ / CH_3 (i.e. $H_2C(OH)-CH(OH)-CH_3$)

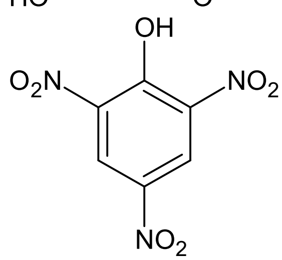
(B) H_2C-OCH_3 / $HC-OH$ / CH_3 (i.e. $H_2C(OCH_3)-CH(OH)-CH_3$)

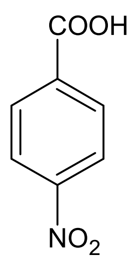
(C) CH_2OH / CH_2 / CH_2OH (i.e. $CH_2OH-CH_2-CH_2OH$)

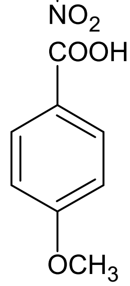
(D) CH_2OH / $C=O$ / CH_2OH (i.e. $CH_2OH-CO-CH_2OH$)

24. Which of the following compound(s) will liberate CO_2 on reaction with $NaHCO_3$?

(A) 

(B) 

(C) 

(D) 

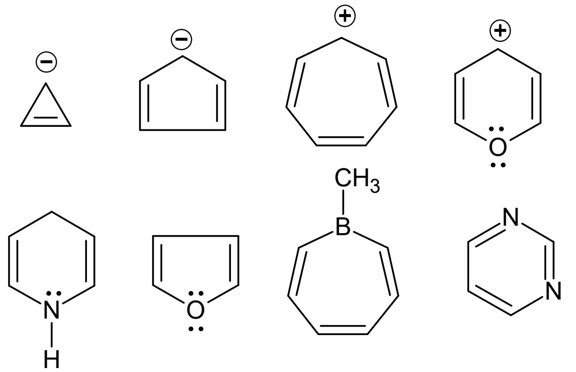
SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

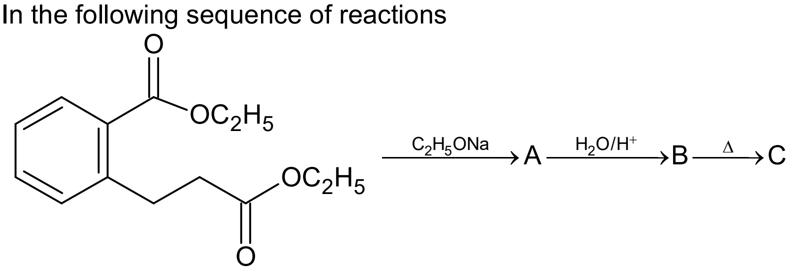
25. Total number of structural isomers of molecular formula C_4H_6 is

26. The total number of cyclic structural isomers of molecular formula C_4H_7Cl is

27. The total number of aromatic molecules/ions among the following is



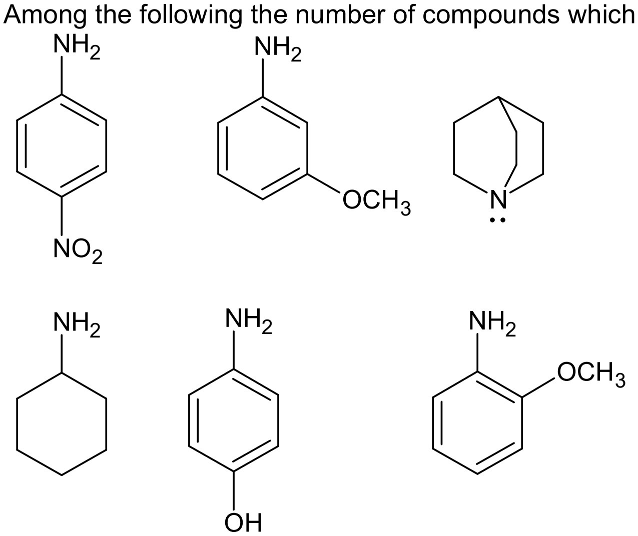
28. In the following sequence of reactions



Degree of unsaturation in the product (C) is

29. The number of moles of formic acid formed by the oxidative cleavage of one mole of D-Glucose by excess of HIO_4 is

30. Among the following the number of compounds which are more basic than aniline is



SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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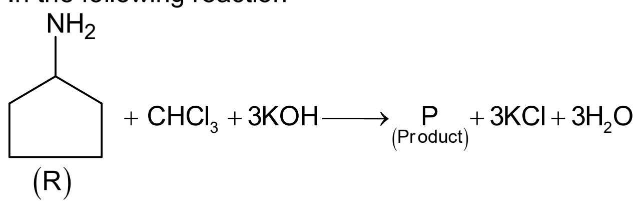
31. The pK_a values for the three ionisable groups x, y and z of Lysine are 2.2, 9.0 and 10.5 respectively.

$$\underset{(z)}{H_3\overset{+}{N}}-CH_2-CH_2-CH_2-CH_2-\underset{\underset{(y)}{\overset{+}{N}H_3}}{CH}-\underset{(x)}{COOH}$$

The isoelectric point of Lysine is

32. In the following reaction

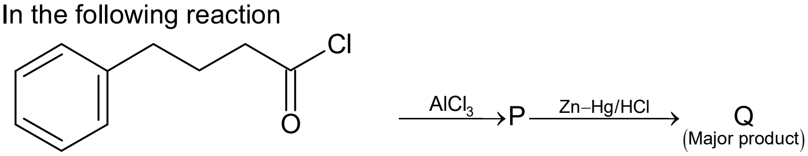


the percentage yield of the product (P) is 100%. The weight in grams of the product (P) formed by completely reacting 21.25 grams of reactant (R) is

33. Specific rotation of $(+)-2-$butanol is $+12.5^{\circ}$. A sample of 2-butanol containing both the enantiomers was found to have specific rotation value of – 2.8. The percentage of $(+)-2-$butanol in the sample is

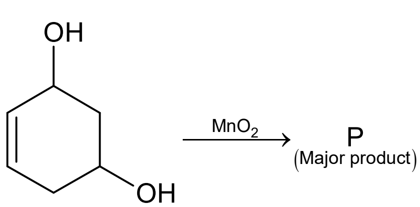
34. n-Butane is produced by monobromination of ethane followed by the Wurtz reaction. The weight of ethane (in grams) required to produce 7.25 g of n-Butane if the bromination and Wurtz reaction take place with 100% yield is [Atomic weight of Br = 80].

35. In the following reaction



the percentage yield of the product Q is 100%. The molecular weight of product (Q) is x. The value of $\frac{x}{8}$ is

36.



(R)

In this reaction the percentage yield of the product (P) is 100%. The weight in grams of the product (P) formed by completely reacting 22.8 grams of the reactant (R) is

Mathematics PART – III

SECTION – A

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

37. If $\sum_{P=0}^{2020} \sin(3^P\theta)\sec(3^{P+1}\theta) = a\tan(b\theta) + c\tan(d\theta)$, then which of the following is always true?

(A) $ac = -\frac{1}{4}$

(B) $ad = \frac{1}{2}$

(C) $cd = -\frac{1}{2}$

(D) $bd = 3^{2021}$

38. Given circles $C_1 : (x-1)(x-2) + (y-1)\left(y-\frac{1}{2}\right) = 0$, $C_2 : (x-1)(x-3) + (y-1)\left(y-\frac{1}{3}\right) = 0$, $C_3 : (x-2)(x-3) + \left(y-\frac{1}{2}\right)\left(y-\frac{1}{3}\right) = 0$. Let their points of intersection form ΔABC, then which of the following holds true?

(A) Centroid of ΔABC is $\left(2, \frac{11}{18}\right)$

(B) Orthocentre of ΔABC is $\left(-\frac{11}{18}, -4\right)$

(C) Circumcentre of ΔABC is $\left(\frac{37}{12}, \frac{81}{16}\right)$

(D) Centroid of triangle formed by centres of the circles C_1, C_2, C_3 is $\left(2, \frac{11}{18}\right)$

39. Consider an ellipse $\frac{x^2}{a^2} + \frac{(y-b)^2}{b^2} = 1$ resting on a plane ground where a and b are functions of time t (in seconds) given by a = 4 + 2t, b = 1 + t. There is a long rod one end of which is hinged at (10, 0). It rests freely on the ellipse with the other end in air, then which of the following is NOT true

(A) Eccentricity of ellipse attains a maxima after some time $t_1 > 0$

(B) At time t = 4 seconds, rod is still in contact with the ellipse

(C) Locus of the free end of the rod will be an arc of a circle

(D) Value of t for which rod breaks off contact from the ellipse is independent of its length

40. If a parabola has y-axis as directrix and line pair $x^2 - 4y^2 - 6x + 9 = 0$ is tangent to the parabola, then focus of the parabola lies inside the curve

(A) $x^2 + y^2 = 9$

(B) $y^2 = 4x + 9$

(C) $xy = 1$

(D) none of these

41. Consider sets $A = \{x \mid \sin(\sin x) = \cos(\cos x)\}$, $B = \{x \mid \sin(\sin x) = \tan(\tan x)\}$,

$C = \{x \mid \cos(\cos x) = \tan(\tan x)\}$, then for $x \in \left(0, \frac{\pi}{2}\right)$ which of the following hold true?

(A) n(A) = 0
(B) n(B) = n(C)
(C) n(A) + n(C) = n(B)
(D) n(C) > n(B)

42. Let $\sin^{-1} x : [-1, 1] \rightarrow \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ and $\cos^{-1} x : [-1, 1] \rightarrow [\pi, 2\pi]$. If $\frac{(\sin^{-1} x)^2}{3\pi} + \frac{(\cos^{-1} x)^2}{4\pi} = \frac{25}{28}\pi$, then

(A) $\sin^{-1} x \in \left[\pi, \frac{5\pi}{4}\right]$

(B) $\sin^{-1} x \in \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$

(C) $\cos^{-1} x \in \left[\pi, \frac{5\pi}{4}\right]$

(D) $\cos^{-1} x \in \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$

SECTION – B
(Single Digit Integer Type)

This section contains **06** questions. The answer to each question is a **Single Digit integer** ranging from **0** to **9**, both inclusive.

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43. Let L be set of all lines passing through (1, 1). Let A and B represent set of all points of intersection of tangents drawn at end points of chords, belonging to set L, to the curves $x^2 + y^2 = 4$ and $y^2 = 4x$ respectively. Then $n(A \cap B)$ equals

44. In a ΔABC, maximum value of 4(cos A + cos B + cos C) equals

45. Number of integral solutions of the equation $\sin^{-1}\left(\frac{4x}{4+x^2}\right) = \cos^{-1}\left(\frac{4-x^2}{4+x^2}\right)$ equals

46. Given are three chords of the hyperbola xy = 1
AB : x = y, BC : 2x – y + 1 = 0, AC : 2x + y – 3 = 0. If centroid of triangle formed by mid-points of the three chords is (a, b), then 3(a + b) equals

47. If in a ΔABC, a = 6, b = 7 and length of longest altitude is $\frac{12\sqrt{6}}{5}$, then third side equals

48. Number of real solutions for the equation $\begin{vmatrix} \sin^{-1} x & \cos^{-1} x & \tan^{-1} x \\ \cos^{-1} x & \tan^{-1} x & \sin^{-1} x \\ \tan^{-1} x & \sin^{-1} x & \cos^{-1} x \end{vmatrix} = 0$ equals

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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49. Let $f(a,b)=\sqrt{\left(\sqrt{1-a^2}-b\right)^2+(a+b-4)^2}$, then minimum possible value of f(a, b) $\forall$ a, b $\in$ domain of 'f' equals

50. Reflection of parabola $y^2 - 4x + 4 = 0$ in the line $x + y + 1 = 0$ is given by $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$, then g + f + c equals

51. If minimum eccentricity of ellipse that can rest on a rough inclined plane of angle 30° is equal to 'e', then the value of e^2 equals

52. Tangents are drawn from variable point P to a fixed parabola having length of latus rectum equal to 'ℓ'. At points of tangency, normals are drawn to the parabola intersecting each other at point Q. If the quadrilateral so formed is a rectangle, then locus of point Q is a conic section having length of latus rectum equal to '$\lambda\ell$' where λ equals

53. In a triangle ΔPQR, PQ = 2 and QR = 1. If for a fixed value of $\angle QPR$, two triangles are possible such that area of one is twice the other, then $\sin^2 P$ equals

54. Number of solution of the equation $(\tan x)^{\tan x}=\frac{2019}{2020}$ $\forall$ x $\in \left(0,\frac{\pi}{2}\right)$ are

ALL INDIA TEST SERIES

TEST - 4

JEE (Advanced)

ANSWERS, HINTS & SOLUTIONS

Physics **PART – I**

SECTION – A

1. B, D

Sol. Induced emf in the loops, $\varepsilon = \frac{R^2\theta}{2}\frac{dB}{dt} = \frac{a_0R^2\theta}{2}$

Induced current in the loop,

$$I = \frac{\varepsilon}{2R(1+\cos\theta)\lambda} = \frac{a_0R\theta}{4\lambda(1+\cos\theta)}$$

$$V_O - V_Q = I\lambda 2R\cos\theta = \frac{a_0R^2\theta\cos\theta}{2(1+\cos\theta)}$$

At $\theta = 45°$,

$$V_O - V_Q = \frac{a_0R^2\pi}{8(\sqrt{2}+1)}$$

$$V_P - V_Q = \varepsilon - I\lambda R = \frac{a_0R^2\theta}{2} - \frac{a_0R^2\theta}{4(1+\cos\theta)} = \frac{a_0R^2\theta}{4}\left(\frac{1+2\cos\theta}{1+\cos\theta}\right)$$

For $\theta = 45°$,

$$V_P - V_Q = \frac{a_0R^2\pi}{8\sqrt{2}}$$

2. A, B, D

Sol. $\Delta Q_{AB} = nC_P\Delta T = 2\times\frac{5R}{2}(1200-600) = 3000\ R$

In the process BC, $T = \propto V^2$

PV^{-1} = constant

Molar heat capacity in the process BC,

$$C = C_V + \frac{R}{(1-x)} = \frac{3R}{2}+\frac{R}{2}$$

C = 2R

$\Delta Q_{BC} = nC\Delta T = 2 \times 2R\ (300 - 1200) = -3600\ R$

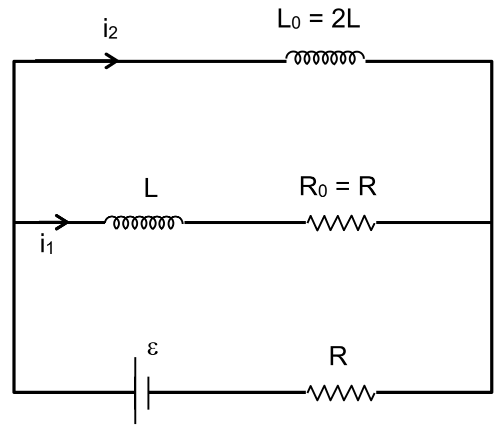
$\Delta Q_{CA} = nC_V\Delta T = 2 \times \frac{3R}{2}\ (600 - 300) = 900\ R$

The total work done by the gas during the cyclic process.

$\Delta W_{cycle} = \Delta Q_{cycle} = 3000\ R - 3600\ R + 900\ R = 300\ R$

3. A, C

Sol. Current through the inductor 'L_0' in the steady state after closing the switch-2 will be $I_S = \frac{\varepsilon}{R}$



Now, $2L\frac{di_2}{dt} = L\frac{di_1}{dt} + i_1R_0$

$$2L\int_0^{\varepsilon/R} di_2 - L\int_{\varepsilon/2R}^{0} di_1 = R_0\int i_1dt$$

$$2L\frac{\varepsilon}{R} - L\left(0-\frac{\varepsilon}{2R}\right) = R_0\Delta q$$

$$\frac{5L\varepsilon}{2R} = R_0\Delta q \Rightarrow \Delta q = \frac{5L\varepsilon}{2RR_0}$$

$$\Delta q = \frac{5L\varepsilon}{2R^2}$$

4. A, B, C, D

Sol. Cooling rate is inversely proportional to the radius of sphere.

5. B, C, D

Sol. At t = 0, current through the inductor = 2A, charge on the capacitor = 4C

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{2}\sec^{-1}$, $T = \frac{2\pi}{\omega} = 4\pi$ sec

$$\frac{q_{max}^2}{4} = \frac{1}{2}\times 2(2)^2 + \frac{1}{2}\times 2(2)^2 \Rightarrow q_{max} = 4\sqrt{2}C$$

$$q = 4\sqrt{2}\sin\left(\omega t + \frac{3\pi}{4}\right)$$

$$i = 2\sqrt{2}\cos\left(\omega t + \frac{3\pi}{4}\right)$$

6. A, B, C, D

Sol. $X_C = -100\hat{j}$, $x_L = 200\hat{j}$

$$\frac{1}{z_{AB}} = \frac{1}{200\hat{j}} + \frac{1}{-100\hat{j}}$$

$Z_{AB} = -200\ \hat{j}$

For circuit, $z = 100 - 200\,\hat{j}$

$$i_{0R} = \frac{V_0}{z} = \frac{100}{100(1-2j)} = \frac{1+2\hat{j}}{5}$$

$$i_{0R} = \frac{1}{\sqrt{5}}\sin(100t + \tan^{-1}2)$$

$$v_R = 20\sqrt{5}\sin(100t + \tan^{-1}2)$$

$$v_{AB} = \frac{1}{\sqrt{5}}\times(-200\hat{j}) = -40\sqrt{5}\hat{j}$$

$$V_L = V_C = 40\sqrt{5}\sin\left(100t + \tan^{-1}2 - \frac{\pi}{2}\right)$$

$$i_{0C} = \frac{V_C}{X_c} = \frac{40\sqrt{5}}{-100\hat{j}} = \frac{2}{\sqrt{5}}\hat{j}$$

$$i_C = \frac{2}{\sqrt{5}}\sin(100t + \tan^{-1}2)$$

$$i_{0L} = \frac{40\sqrt{5}}{200\hat{j}} = -\frac{1}{\sqrt{5}}\hat{j}$$

$$i_L = \frac{1}{\sqrt{5}}\sin(100t + \tan^{-1}2 - \pi)$$

At t = 0, $i_L = \frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{5}} = -\frac{2}{5}\text{A}$, $i_C = +\frac{2}{\sqrt{5}}\times\frac{2}{\sqrt{5}} = \frac{4}{5}$

$$V_C = V_L = -40\sqrt{5}\times\frac{1}{\sqrt{5}} = -40\text{ Volt}$$

We gained by inductor = $\left(-\frac{2}{5}\right)\times(-40) = 16\text{ J/S}$

Power gained by capacitor = $\left(\frac{4}{5}\right)\times(-40)$

At t = 0, $i_R = \frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{5}} = \frac{2}{5}\text{A}$

$$V_R = 20\sqrt{5}\times\frac{2}{\sqrt{5}} = 40\text{V}$$

Power consumed = $\frac{2}{5}\times 40 = 16\text{ J/s}$

SECTION – B

7. 8

Sol. $$\frac{\sigma}{4\varepsilon_0}\times 10^{-3} + \frac{\sigma}{2\varepsilon_0}\times 10^{-3} = 30$$

$$\Rightarrow \frac{\sigma}{\varepsilon_0} = 4\times 10^4$$

$$\frac{q}{\varepsilon_0} = 4\times 10^{+4}\times 2\times 10^{-4} = 8\ \text{V}-\text{m}$$

8. 2

Sol. Resistance of coil is zero $\Rightarrow$ emf

Developed will be zero so ϕ = constant

or $Li + B_0(1 - x)\pi R^2$ = constant

at t = 0, i = 0 $\Rightarrow \phi = B_0 \pi R^2$

or $Li + B_0(1 - x)\pi R^2 = B_0\pi R^2$

$\Rightarrow Li = B_0 \pi R^2 x$

$$\text{or } i = \frac{B_0 \pi R^2}{L} x$$

acceleration of coil

$$ma_2 = i \times 2\pi R \times 2B_0 = -\frac{4B_0^2 \pi^2 R^3}{L} x$$

$$a_x = -\frac{4B_0^2 \pi^2 R^3}{mL} x$$

As $a_x \alpha - x \Rightarrow$ motion in S.H.M. and $\omega^2 = \dfrac{4B_0^2\pi^2R^3}{mL}$

$x = A\sin(\omega t + \pi)$

$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \pi)$$

at t = 0, $v = -v_0$

$\Rightarrow v_0 = A\omega$ or $A = \dfrac{v_0}{\omega}$

9. 3

Sol. $eE = evB$

$E = Bv$ …(i)

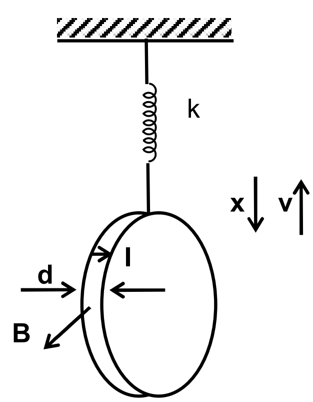
$$q = \left(\frac{\varepsilon_0 A}{d}\right)V_0 \quad (V_0 = \text{potential drop})$$

$$q = \left(\frac{\varepsilon_0 A}{d}\right)Ed$$

$q = \varepsilon_0 AE$

$q = \varepsilon_0 ABv$

$$I = \frac{dq}{dt} = \varepsilon_0 AB\frac{dv}{dt} \quad \text{…(ii)}$$



Now,

$$m\frac{dv}{dt} = kx - BId$$

$$m\frac{dv}{dt} = kx - Bd\left(\varepsilon_0 AB\frac{dv}{dt}\right)$$

$$m\frac{dv}{dt} = kx - \varepsilon_0 VB^2\frac{dv}{dt}$$ (where V = Ad = volume of the disc)

$$\left(m + \varepsilon_0 VB^2\right)\frac{dv}{dt} = kx$$

$$\frac{dv}{dt} = \frac{kx}{(m + \varepsilon_0 VB^2)}$$

$$\frac{d^2x}{dt^2} = -\frac{kx}{(m+\varepsilon_0 VB^2)}$$

Time period, $T = 2\pi\sqrt{\frac{m+\varepsilon_0 VB^2}{k}}$

10. 0

Sol. $2 - 2i - 10(i_1) = 0$

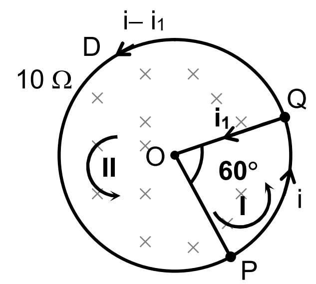
$i + 5i_1 = 1$ …(i)

$10 - 10(i - i_1) + 10i_1 = 0$

$i - 2i_1 = 1$ …(ii)

Solving (i) and (ii), $i_1 = 0$

$\Rightarrow$ Reading of voltmeter is zero.



11. 1

Sol. Work done by magnetic field zero.

$\Rightarrow mgy = \frac{1}{2}mv^2$

$v = \sqrt{2gy}$ …(i)

$ma_x = F_x$

$m\frac{dv_x}{dt} = qvB\cos\theta = qBv_y$

$\Rightarrow m\frac{dv_x}{dt} = qB\frac{dy}{dt}$

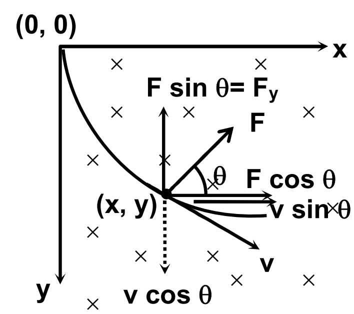
$mv_x = qBy$

$v_x = \frac{qBy}{m}$ …(ii)

Speed will be maximum at lowest point $v = v_x$ at lowest point.

$v = \frac{qB}{m} \times \frac{v^2}{2g}$

$\Rightarrow v = \frac{2mg}{qB} = \frac{2\times 0.1\times 10}{1\times 2} = 1\,m/s$



12. 4

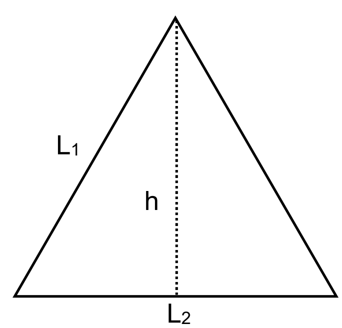
Sol. $h^2 = L_1^2 - \frac{L_2^2}{4}$

Differentiating and using $dL = L\,\alpha\,\Delta t$

$0 = 2L_1\alpha_1 L_1\Delta t - \frac{1}{4}2L_2\alpha_2 L_2\Delta t$

$L_1 L_1\alpha_1\Delta t = \frac{1}{4}L_2 L_2\alpha_2\Delta t$

$\Rightarrow \frac{L_2}{L_1} = 2\sqrt{\frac{\alpha_1}{\alpha_2}} = 2\sqrt{\frac{4\times 10^{-6}}{1\times 10^{-6}}} = 4$



SECTION – C

13. 00006.97

Range 6.90 to 7.00

Sol. $\vec{\tau} = \vec{M} \times \vec{B}$

$= (2 \times 20 \times 10^{-4})\left\{\cos 30\hat{i} + \sin 30\hat{j}\right\} \times (4\hat{i} - \sqrt{3}\hat{j})$

$= 20 \times 10^{-4}\left\{-3\hat{k} + 4(-\hat{k})\right\} = -140 \times 10^{-4}\hat{k}$ N-m

$I_0 = \dfrac{mR^2}{2} + md^2$

$= \dfrac{1}{4} \times \dfrac{20 \times 10^{-4}}{\pi} + \dfrac{1}{2} \times 37 \times 10^{-4}$

$= \dfrac{(10 + 37\pi)}{2\pi} \times 10^{-4}$

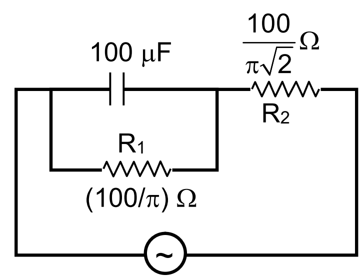
$\alpha = \dfrac{140 \times 10^{-4} \times 2\pi}{(10 + 37\pi) \times 10^{-4}} = \dfrac{140 \times 2\pi}{10 + 37\pi}$

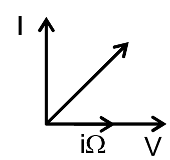
$\dfrac{879.646}{126.239} = 6.968 \text{ rad/s}^2$

14. 00391.96

Range 390.96 to 392.96

Sol. $\dfrac{1}{Z_1} = \sqrt{\left(\dfrac{\pi}{100}\right)^2 + \left(\dfrac{\pi}{100}\right)^2} = \dfrac{\pi\sqrt{2}}{100}$

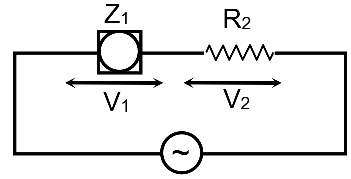




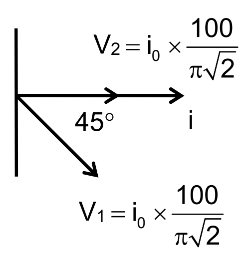
$Z_1 = \dfrac{100}{\pi\sqrt{2}}$

$i_0 \times \dfrac{100}{\pi\sqrt{2}} \le 150\sqrt{2}$

$i_0 \le \dfrac{300\pi}{100} = 3\pi$



$v_0 = i_0\sqrt{\left(\dfrac{100}{\pi\sqrt{2}}\right)^2 + \left(\dfrac{100}{\pi\sqrt{2}}\right)^2 + 2\left(\dfrac{100}{\pi\sqrt{2}}\right)\left(\dfrac{100}{\pi\sqrt{2}}\right) \times \dfrac{1}{\sqrt{2}}}$



$= 3\pi \times \dfrac{100}{\pi\sqrt{2}}\sqrt{2 + \sqrt{2}}$

$= 300\sqrt{1 + \dfrac{1}{\sqrt{2}}} = 391.96$

390.96 to 392.96 volts

15. 00001.66
Range 1.60 to 1.70

Sol. $ms\left(\frac{dT}{dt}\right) = a\sigma AT_0^4 - e\sigma AT_b^4 + 2$

$= 2.177 - 0.6889 + 2$

= 3.4831 watt

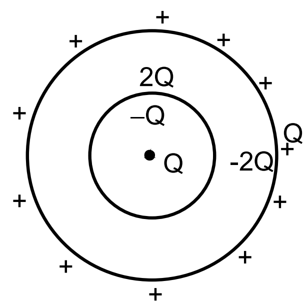
$2 \times 0.25 \times 4.2\left(\frac{dT}{dt}\right) = 3.4831$

$\frac{dT}{dt} = 1.66\ \text{K/sec}$

1.60 to 1.70 K/sec

16. 00040.00

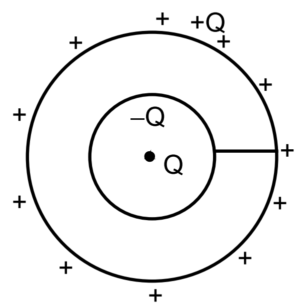
Sol. When switch S is open



Energy = $U_0 + \frac{kQ^2}{2R} + \frac{kQ^2}{4R} + \frac{kQ^2}{R} - \frac{kQ^2}{2R} - \frac{kQ^2}{2R}$

$= U_0 + \frac{3kQ^2}{4R}$

When switch S is closed



Energy $U_2 = U_0 + \frac{kQ^2}{2R} + \frac{kQ^2}{4R} - \frac{kQ^2}{R} + \frac{kQ^2}{2R} - \frac{kQ^2}{2R}$

$= U_0 - \frac{kQ^2}{4R}$

Loss of energy = $U_1 - U_2 = \frac{kQ^2}{R}$

$= \frac{9 \times 10^9 \times 400 \times 10^{-12}}{9 \times 10^{-2}} = 40\text{J}$

17. 00000.82
Range 0.80 to 0.83

Sol. Force acting on hemisphere

$F \propto \sigma SE$

Where σ is charge ensily

S $\rightarrow$ surface area

E $\rightarrow$ electric field present

as $\sigma \propto E$

$\Rightarrow F \propto SE^2$

$\propto R^2E^2$

For spherical shell of thickness 2t and radius 3R will require 6 timesforceto tear apart compared to first shell.

$6F \propto (3R)^2.E^2$

$F \propto R^2E_0^2$

$\Rightarrow 6 = 9\left(\frac{E}{E_0}\right)^2 \Rightarrow \frac{E}{E_0} = \sqrt{\frac{2}{3}}$

$E = \sqrt{\frac{2}{3}} E_0$

$\frac{E}{Eo} = 0.816$

0.80 to 0.83

18. 00068.48
Range 67.50 to 69.50

Sol. $2 \times 10^{-6} \times 4200 \times 12 = 4 \times \frac{1}{2} \times 20 \times 10^{-4} \times 5.67 \times 10^{-8} \times (300)^3 (\Delta T)$

$\Delta T = \frac{2 \times 4.2 \times 12 \times 10^{-3}}{40 \times 5.65 \times 27 \times 10^{-6}} = 16.5°C$

Temperature of rheostat wire = 27 + 16.5 = 43.5°C

Resistance of Rh at 43.5°

$R = R_0(1 + \alpha \Delta t)$

Potential drop across Rh = $\sqrt{\text{Power} \times \text{Resistance}}$

$= \sqrt{2 \times 10^{-3} \times 1 \times 12 \times 4.2 \times R}$

$= \sqrt{2 \times 12 \times 4.2 \times 10^{-3} \times 100} (1 + \Delta t)^{1/2}$

$= 3.18(1 + 2 \times 10^{-3} \times 4.35) = 3.44$ Volt

PD across AB = 12 – 3.44 = 8.56 volt

Balance length = 40 cm

$\Rightarrow \frac{V_{AB}}{AB} \times 40 = 5$

AB = 68.48 cm

Length of wire AB = 68.48 cm

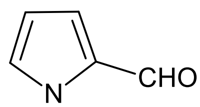
Range 67.5 to 69.5 cm

Chemistry PART – II

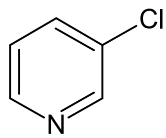
SECTION – A

19. A, C

Sol. Reimer-Tiemann reaction.



Normal RTR product



Abnormal RTR product

20. A, B, C

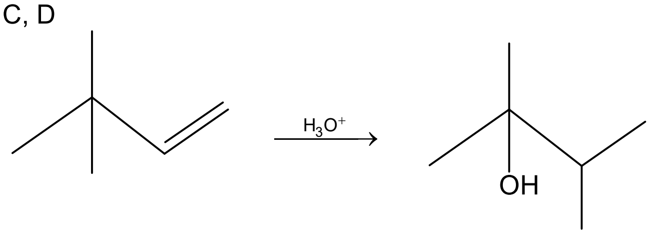
Sol. Ketones having $CH_3-\overset{O}{\overset{\|}{C}}-$ give haloform reaction. CH_3CH_2OH and secondary alcohol containing $CH_3-\overset{OH}{\overset{|}{C}}H-$ also give haloform reaction.

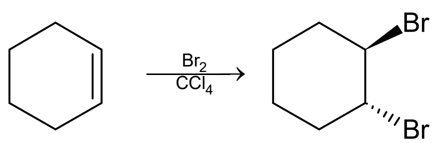
21. A, C, D

Sol. This cumulene is optically active because it is chiral.

22. C, D

Sol.





+ enantiomer

23. A, D

Sol. HIO_4 does not cleave compounds in which an intervening – CH_2 – group separate – OH groups and compounds in which – OH group is adjacent to – OR group.

24. A, B, C, D

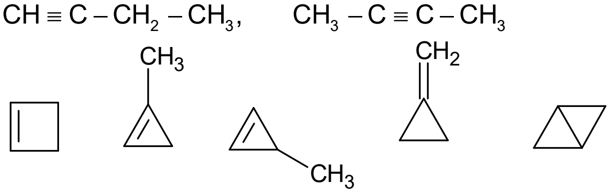
Sol. All the given compounds are more acidic than H_2CO_3.

SECTION – B

25. 9

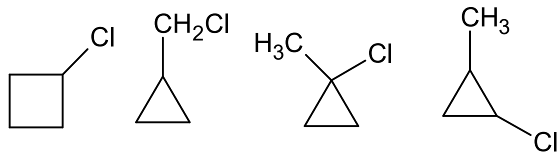
Sol. $CH_2=CH-CH=CH_2$, $CH_2=C=CH-CH_3$

$CH\equiv C-CH_2-CH_3$, $CH_3-C\equiv C-CH_3$



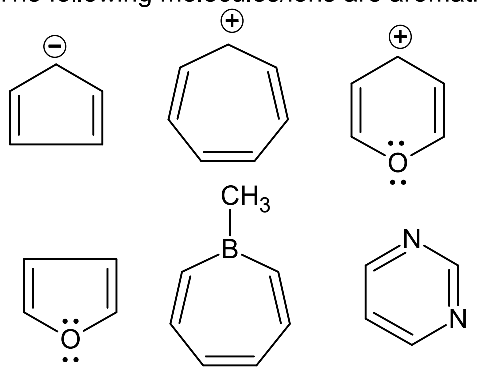
26. 4

Sol.



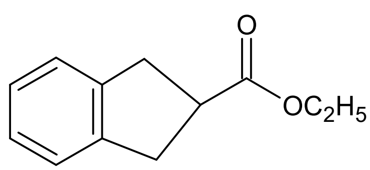
27. 6

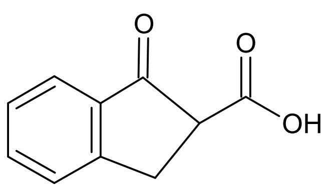
Sol. The following molecules/ions are aromatic



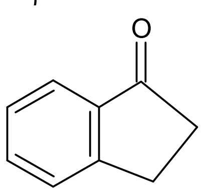
28. 6

Sol.

A = 

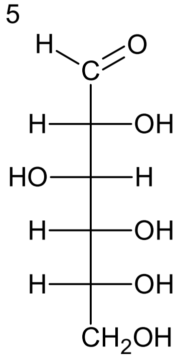
B = 

β – keto acid

C = 

29. 5

Sol.

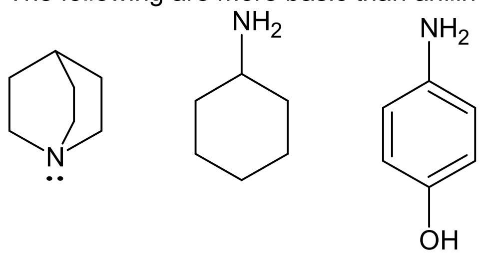


D – Glucose

$$\xrightarrow{HIO_4(excess)} 5H-\overset{\overset{O}{||}}{C}-OH + H-\overset{\overset{O}{||}}{C}-H$$

30. 3

Sol. The following are more basic than aniline



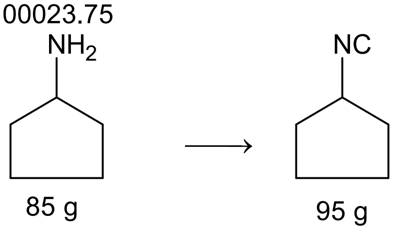
SECTION – C

31. 00009.75

Sol. $P^{I} = \dfrac{9 + 10.5}{2} = 9.75$

32. 00023.75

Sol.



Weight of product (P) formed $= \dfrac{95}{85} \times 21.25 = 23.75$ g

33. 00038.80

Sol. Optically purity $= \dfrac{-2.8}{12.5} \times 100$

$= 22.4(-)$

Amount of racemic mixture = 100 – 22.4

= 77.6%

% of $(-) = \dfrac{77.6}{2} + 22.4 = 61.20\%$

% of $(+) = 100 - 61.2 = 38.8\%$

34. 00007.50

Sol. $C_2H_6 + Br_2 \longrightarrow C_2H_5Br + HBr$

30 g 109 g

$2C_2H_5Br + 2Na \longrightarrow C_4H_{10} + 2NaBr$

2×109 58

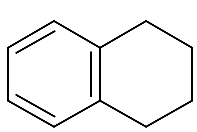
Weight of C_2H_6 required to produce 7.25 g $C_4H_{10} = \dfrac{60}{58} \times 7.25$

= 7.50 g

35. 00016.50

Sol.

Product(Q) =

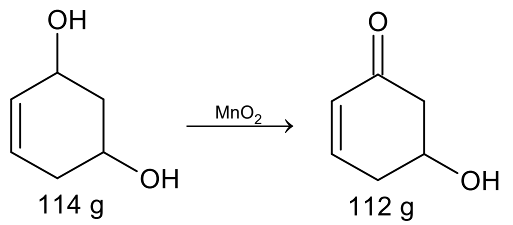


MW of Q = 132

$$\therefore \frac{x}{8} = \frac{132}{8} = 16.50$$

36. 00022.40

Sol.



Weight of product (P) $= \frac{112}{114} \times 22.8 = 22.40$ g

Mathematics PART – III

SECTION – A

37. A, D

Sol. $\sum_{P=0}^{2020} \sin(3^P\theta)\sec(3^{P+1}\theta) = \frac{1}{2}[\tan 3^{2021}\theta - \tan\theta]$

38. A, C, D

Sol. Circles intersect at (1, 1), $\left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right)$

So, centroid = $\left(2, \frac{11}{18}\right)$

Orthocentre = $\left(\frac{-1}{6}, -6\right)$

Circumcentre = $\left(\frac{37}{12}, \frac{81}{16}\right)$

39. A, B

Sol. $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\frac{de}{dt} < 0 \;\; \forall\ t > 0 \Rightarrow$ e(t) is a decreasing function $\forall$ t > 0

Rod will break off contact when 4 + 2t = 10 i.e. at t = 3s

40. A, B

Sol. Reflection of directrix in given tangents passes through focus

Reflection of directrix in given tangents will be 3x – 4y + 6 = 0 and 3x + 4y + 6 = 0

So, focus will be (–2, 0)

41. A, D

Sol. Draw graphs and check n(A) = 0, n(C) = n(B) + 1

42. A, D

Sol. $\sin^{-1}x + \cos^{-1} = \frac{5\pi}{2}$ and $\frac{\sin^{-1}x}{3\pi} = \frac{\cos^{-1}x}{4\pi}$

$\sin^{-1}x = \frac{15\pi}{14}$, $\cos^{-1}x = \frac{10\pi}{7}$

SECTION – B

43. 1

Sol. A : x + y = 4

B : y = 2x + 2

n(A $\cap$ B) = 1

44. 6

Sol. cos A + cos B + cos C = $1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{3}{2}$

45. 3

Sol. $\sin^{-1}\left(\dfrac{2\left(\frac{x}{2}\right)}{1+\left(\frac{x}{2}\right)^2}\right)=\cos^{-1}\left(\dfrac{1-\left(\frac{x}{2}\right)^2}{1+\left(\frac{x}{2}\right)^2}\right)$

Now, make cases based on x

Equation satisfies for x = 0, 1, 2

46. 5

Sol. Mid-point of segment of a chord between asymptotes coincides with mid-point of the chord

47. 5

Sol. Area of Δ = $\frac{1}{2}(\text{shortest side})\times(\text{longest altitude})$

48. 0

Sol. Either $\sin^{-1} x + \cos^{-1} x + \tan^{-1} x = 0$ or $\sin^{-1} x = \cos^{-1} x = \tan^{-1} x$

Both of which are not possible $\forall\, x \in R$

SECTION – C

49. 00001.83

Sol. f(a, b) represents distance between circle $x^2 + y^2 = 1$ and line x + y = 4

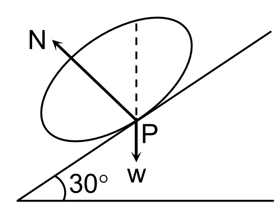
So, minimum value of f(a, b) is $2\sqrt{2}-1$

50. 00012.00

Sol. Reflection is $x^2 + 2x + 4y + 9 = 0$

51. 00000.67

Sol. For minimum eccentricity angle between normal at point of contact and vertical will be 30°



If ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then point of contact can be taken as

(a cos θ, b cos θ) then $\dfrac{\frac{a}{b}\tan\theta-\frac{b}{a}\tan\theta}{1+\tan^2\theta}=\tan 30^\circ$

$\Rightarrow \dfrac{e^2\sin 2\theta}{2\sqrt{1-e^2}}=\dfrac{1}{\sqrt{3}}$. For minimum eccentricity $e=\sqrt{\dfrac{2}{3}}$

52. 00000.25

Sol. P lies on directrix of the parabola

53. 00000.16

Sol. $PR^2 - 2PR\cdot PQ\cos(\angle QPR) + PQ^2 - QR^2 = 0$

$\Delta_1 = 2\Delta_2 \Rightarrow$ one root of the equation is twice the other

$\Rightarrow \sin(\angle QPR)=\dfrac{\sqrt{5}}{4\sqrt{2}}$

54. 00002.00

Sol. $f(x) = x^x$ is decreasing in $\left(0,\frac{1}{e}\right)$ and increasing in $\left(\frac{1}{e},\infty\right)$

So, if $x^x = c$ where $c \in \left(\left(\frac{1}{e}\right)^{\frac{1}{e}},1\right)$ then equation has two solutions