ALL INDIA TEST SERIES

TEST – 9

JEE (Advanced)

Time Allotted: 3 Hours **Maximum Marks: 198**

General Instructions:

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-C.**

Section-A (01 – 06, 19 – 24, 37– 42) contains **18 multiple** choice questions which have **ONLY ONE CORRECT ANSWER**. Each question carries **+3 marks** for correct answer and **–1 mark** for wrong answer.

Section-A (07 – 12, 25 – 30, 43 – 48) *this section contains* ***18 multiple*** *choice questions.*
Each question has FOUR options. ***ONE OR MORE THAN ONE*** *of these four option(s) is (are) correct answer(s).*
For each question, choose the option(s) corresponding to (all) the correct answer(s)
Answer to each question will be evaluated according to the following marking scheme:

Full Marks **: +4** If only (all) the correct option(s) is (are) chosen:
Partial Marks **: +3** If all the four options are correct but ONLY three options are chosen;
Partial Marks **: +2** If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks **: +1** If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks **: 0** If none of the options is chosen (i. e. the question is unanswered);
Negative Marks **:** –**2** In all other cases.

Section-C (13 – 18, 31– 36, 49 – 54) *contains* ***18 Numerical*** *answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer and* ***0 marks*** *for wrong answer.*

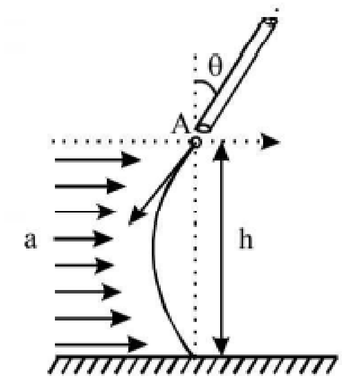
Physics

PART – I

SECTION – A
(One Options Correct Type)

This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. A particle is ejected from the tube at A with a velocity v at an angle θ with the vertical y-axis. A strong horizontal wind gives the particle a constant horizontal acceleration a in the x-direction. If the particle strikes the ground at a point directly under its released position and the downward y-acceleration is taken as g then



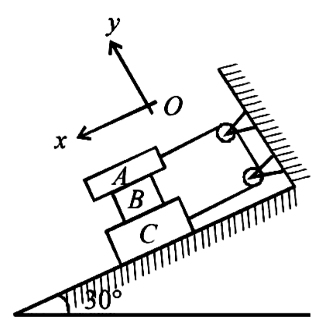
(A) $h = \frac{2v^2 \sin\theta \cos\theta}{a}$

(B) $h = \frac{2v^2 \sin\theta \cos\theta}{g}$

(C) $h = \frac{2v^2}{g} \sin\theta \left(\cos\theta + \frac{a}{g} \sin\theta \right)$

(D) $h = \frac{2v^2}{a} \sin\theta \left(\cos\theta + \frac{g}{a} \sin\theta \right)$

2. Three blocks A, B, C of weights 40 N, 30N, 80N respectively are at rest on an inclined plane as shown in figure. Determine the smallest value of coefficient of limiting friction (μ_s) for which equilibrium of system is maintained.



(A) 0.1757
(B) 0.2757
(C) 0.5757
(D) 0.8757

3. A particle falls from a height h on a fixed horizontal plane and rebounds. If e is the coefficient of restitution, the total distance travelled by the particle before it stops rebounding is

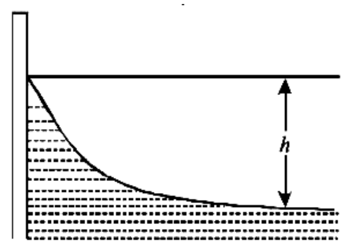
(A) $\frac{h}{2} \frac{[1-e^2]}{[1+e^2]}$

(B) $\frac{h[1-e^2]}{[1+e^2]}$

(C) $\frac{h[1+e^2]}{2[1-e^2]}$

(D) $\frac{h[1+e^2]}{[1-e^2]}$

4. Water of density ρ in a clean aquarium forms a meniscus, as illustrated in the figure. Calculate the difference in height h between the centre and the edge of the meniscus. The surface tension of water is γ.



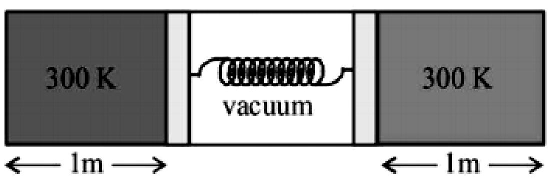
(A) $\sqrt{\frac{2\gamma}{\rho g}}$

(B) $\sqrt{\frac{\gamma}{\rho g}}$

(C) $\frac{1}{2}\sqrt{\frac{\gamma}{\rho g}}$

(D) $2\sqrt{\frac{\gamma}{\rho g}}$

5. Consider the shown diagram where the two chambers separated by piston-spring arrangement contain equal amounts of certain ideal gas. Initially when the temperatures of the gas in both the chambers are kept at 300 K the compression in the spring is 1m. The temperature of the left and the right chambers are now raised to 400 K and 500 K respectively. If the pistons are free to slide, the compression in the spring is about.



(A) 1.3 m
(B) 1.5 m
(C) 1.1 m
(D) 1.0 m

6. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is

(A) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$

(B) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$

(C) $300\left(\frac{2\rho}{2\rho-1}\right)$

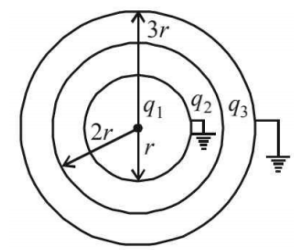
(D) $300\left(\frac{2\rho-1}{2\rho}\right)$

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

7. A horizontal disc rotates freely about a vertical axis through it centre. A ring, having the same mass and radius as the disc, is now gently placed on the disc. After some time, the two rotate with a common angular velocity. Select the correct statements from the following.
   (A) Some friction exists between the disc and the ring
   (B) The angular momentum of the 'disc plus ring' is conserved.
   (C) The final common angular velocity is (2/3)rd of the initial angular velocity of the disc.
   (D) (2/3)rd of the initial kinetic energy changes to heat.

8. Three concentric conducting spherical shells have radii r, 2r and 3r and charge q_1, q_2 and q_3 respectively as shown in the figure. Select the correct alternatives

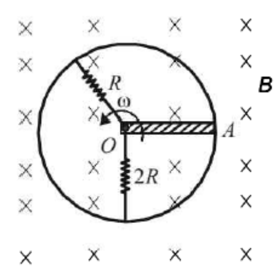


   (A) $q_1 + q_3 = -q_2$
   (B) $q_1 = -\frac{q_2}{4}$
   (C) $\frac{q_3}{q_1} = 3$
   (D) $\frac{q_3}{q_2} = -\frac{1}{3}$

9. Remote objects are viewed through a converging lens with a focal length F = 9 cm placed at a distance a = 36 cm in front of the eye. Assume that the radius r of the pupil is approximately 1.5 mm. Choose the correct options.
   (A) The minimum radius of the screen that should be placed behind the lens so that the entire field of view is covered is 0.5 mm.
   (B) The minimum radius of the screen that should be placed behind the lens so that the entire field of view is covered is 1.0 mm.
   (C) The screen must be placed in the plane S with its centre at point B.
   (D) The screen must be placed perpendicular to the plane S with its centre at point B.

10. Consider an attractive central force of the form $F(r) = -\frac{k}{r^n}, k$ is a constant. For a stable circular orbit to exist
   (A) n = 2
   (B) n < 3
   (C) n > 3
   (D) n = –1

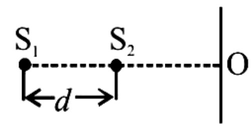
11. A rod OA of length l is rotating (about end O) over a conducting ring in crossed magnetic field B with constant angular velocity ω as shown in figure



   (A) Current flowing through the rod is $\frac{3B\omega\ell^2}{4R}$
   (B) Magnetic force acting on the rod is $\frac{3B^2\omega\ell^2}{4R}$
   (C) Torque due to magnetic force acting on the rod is $\frac{3B^2\omega\ell^4}{8R}$

(D) Magnitude of external force that acts perpendicularly at the end of the rod to maintain the constant angular speed is $\frac{3B^2\omega\ell^4}{8R}$

12. Two point monochromatic and coherent sources of light of wavelength λ are placed on the dotted line in front of an infinite screen. The source emit waves in phase with each other. The distance between S_1 and S_2 is d while their distance from the screen is much larger. Then



(A) If d is $\frac{3\lambda}{2}$, at O minima will be observed

(B) if d is $\frac{11\lambda}{6}$, then intensity at O will be $\frac{3}{4}$ of maximum intensity

(C) if d is 3λ, O will be a maxima

(D) if d is $\frac{7\lambda}{6}$, the intensity at O will be $\frac{3}{4}$ of maximum intensity

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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13. A free neutron at rest, decays into three particles: a proton, an electron and an anti-neutrino.

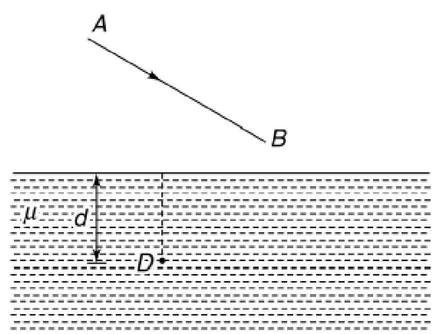
$${}_0^1n \rightarrow {}_1^1p + {}_{-1}^{0}e + \bar{v}$$

The rest masses are: $m_n = 939.5656\ MeV/c^2$

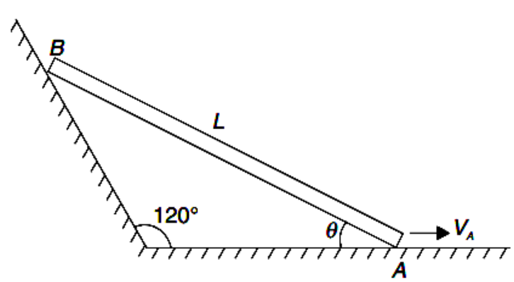
$m_p = 938.2723\ MeV/c^2$ $m_e = 0.5109\ MeV/c^2$

In a particular decay, the antineutrino was found to have a total energy (including rest mass energy) of 0.0004 MeV and the momentum of proton was found to be equal to the momentum of electron. Find the kinetic energy of the electron.

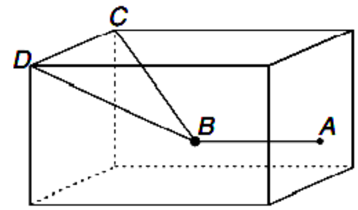
14. A diver D is still under water $\left(\mu = \frac{4}{3}\right)$ at a depth d = 10 m. A bird is diving along line AB at a constant velocity in air. When the bird is exactly above the diver he sees it at a height of 50 m from himself and velocity of the bird appears to be inclined at 45° to the horizontal. At what distance from the diver the bird actually hits the water surface.



15. A wall is inclined to a horizontal surface at an angle of 120° as shown. A rod AB of length L = 0.75 m is sliding with its two ends A and B on the horizontal surface and on the wall respectively. At the moment angle θ = 20° (see figure), the velocity of end A is v_A = 1.5 m/s towards right. Calculate the angular speed of the rod at this instant. [Take cos 40° = 0.766]



16. A room is in shape of a cube. A heavy ball (B) is suspended at the centre of the room tied to three inextensible strings as shown. String BA is horizontal with A being the centre point of the wall. Find the ratio of tension in the string BA and BC.



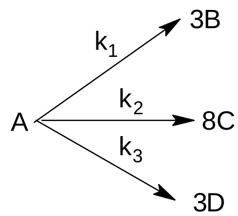
17. A parallel plate capacitor is to be constructed which can store q = 10 μC charge at V = 1000 volt. The minimum plate area of the capacitor is required to be A_1 when space between the plates has air. If a dielectric of constant K = 3 is used between the plates, the minimum plate area required to make such a capacitor is A_2. The breakdown field for the dielectric is 8 times that of air. Find $\frac{A_1}{A_2}$.

18. A 20 mm diameter copper pipe is used to carry heated water. The external surface of the pipe is at T = 80°C and its surrounding is at T_0 = 20°C. The outer surface of the pipe radiates like a black body and also loses heat due to convection. The convective heat loss per unit area per unit time is given by $h(T - T_0)$ where h = 6W $(m^2\ K)^{-1}$. Calculate the total heat lost by the pipe in unit time for one meter of its length.

Chemistry PART – II

SECTION – A
(One Options Correct Type)

This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

19. 

; At time t = 0, initial mole of A is 1.

Overall half life of the reaction 15 days. Then calculate the number of mole of C after 45 days if the raetio of $k_1 : k_2 : k_3$ is 4 : 2 : 1

(A) 1
(B) 2
(C) 3
(D) 4

20. For a complex (d^6 – configuration) having $\Delta_0 = 25000\ cm^{-1}$ and $P = 15000\ cm^{-1}$, the crystal field stabilisation energy is:

(A) $30,000\ cm^{-1}$
(B) $-60,000\ cm^{-1}$
(C) $-30,000\ cm^{-1}$
(D) $-60,000\ cm^{-1}$

21. When H_2S is passed in $Ba(OH)_2$ solution:

(A) milkyness is produced due to formation of insoluble salt
(B) no change is observed because H_2S does not reaction with $Ba(OH)_2$
(C) milkyness is produced due to the formation of $BaSO_3$
(D) no change is observed due to the formation of water soluble salt

22. A compound has the empirical formula $C_{10}H_8Fe$. A solution of 0.26g of the compound in 11.2 g of benzene (C_6H_6) boils at 80.26ºC. The boiling point of benzene is 80.10ºC; the K_b is 2.53ºC/molal. What is the molecular formula of the compound?

(A) $C_{30}H_{24}Fe_3$
(B) $C_{10}H_8Fe$
(C) C_5H_4Fe
(D) $C_{20}H_{16}Fe_2$

23. Two solid compounds X and Y dissociates at a certain temperature as follows

$X(s) \rightleftharpoons A(g) + 2B(g); K_{P_1} = 9 \times 10^{-3}\ atm^3$

$Y(s) \rightleftharpoons 2B(g) + C(g); K_{P_2} = 4.5 \times 10^{-3}\ atm^3$

The total pressure of gases over a mixture of X and Y is:

(A) 4.5 atm
(B) 0.45 atm
(C) 0.6 atm
(D) None of these

24. In the preparation of iron from haematite (Fe_2O_3) by the reduction with carbon

$Fe_2O_3 + C \rightarrow Fe + CO_2$

How much 80% pure iron may be produced from 120 kg of 90% pure Fe_2O_3?

(A) 94.5 kg
(B) 60.48 kg
(C) 116.66 kg
(D) 120 kg

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

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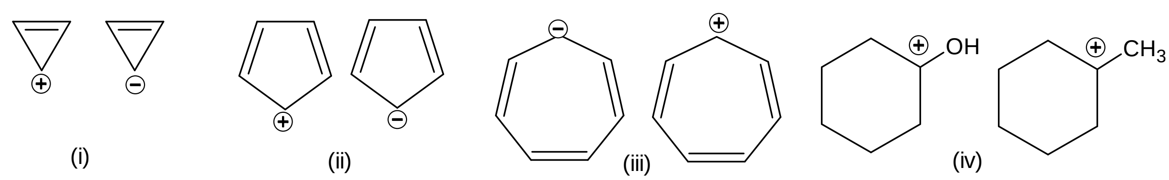
25. Consider the reactions given below. In which cases will the reaction proceed toward right by increasing the pressure?

(A) $4HCl(g)+O_2(g) \rightarrow 2Cl_2(g) + 2H_2O(g)$
(B) $Cl_2(g) + H_2O(g) \rightarrow 2HCl(g) + \frac{1}{2}O_2(g)$
(C) $CO_2(g) + 4H_2(g) \rightarrow CH_4(g) + 2H_2O(g)$
(D) $N_2(g) + O_2(g) \rightarrow 2NO(g)$

26. In which pair second ion is more stable than first?



(i) (ii) (iii) (iv)

(A) (i) and (ii)
(B) (ii) and (iii)
(C) (ii) and (iv)
(D) (iii) and (iv)

27. Which of the following is/are correct?

(A) $\Delta H = \Delta U + \Delta(PV)$ when P and V both changes
(B) $\Delta H = \Delta U + P\Delta V$ when pressure is constant
(C) $\Delta H = \Delta U + V\Delta P$ when volume is constant
(D) $\Delta H = \Delta U + P\Delta V + \Delta V\Delta P$ when P and V both changes

28. Ionisation energy order is CORRECT for:

(A) $Sc^{3+} > Sc^{2+} > Sc^{+}$
(B) $Sc^{3+} > Ti^{4+} > V^{5+}$
(C) $S_c > Y > La$
(D) $Sc > Ca > K$

29. Select which square planar complex(es) can show optical isomerism.

(A) Bis(en)platinum (II) ion
(B) bis (Gly)platinum (II)
(C) di(NH_3)(Gly)platinum (II) ion
(D) di(NH_3) (N-methyl N-ethylglycinato)platinum (II) ion

30. Reaction of R-2-butanol with p-toluenesulphonyl chloride in pyridine then LiBr gives:
(A) R-2-butyl bromide
(B) S-2-butyl tosylate
(C) R-2-butyl tosylate
(D) S-2-butyl bromide

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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31. 32 g of a sample of $FeSO_4.7H_2O$ were dissolved in dilute sulphuric acid and water and its volume was made up to 1 litre. 25 mL of this solution required 20 mL of 0.02 M $KMnO_4$ soluton for complete oxidation. Calculate the mass % of $FeSO_4.7H_2O$ in the sample.

32. A mixture of nitrogen and water vapours is admitted to a flask at 760 torr which contains a sufficient solid drying agent. After long time the pressure attained a steady value of 722 torr. If the experiment is done at 27°C and drying agent increases in mass by 0.9 gm, what is the volume of the flask? Neglect any possible vapour pressure of drying agent and volume occupied by drying agent.

33. Stearic acid $[CH_3(CH_2)_{16}CO_2H]$ is a fatty acid, the part of fat that stores most of the energy. 1.0 g of stearic acid was burned in a bomb calorimeter. The bomb had a heat capacity of 652 J/°C. If the temperature of 500g water (c = 4.18 J/g°C) rose from 25.0 to 39.3°C, how much heat was released when the stearic acid was burned?
[given $C_p = (H_2O) = 4.18\,J/g^oC$]

34. In neutral or faintly alkaline solution, 8 moles permanganate anion quantitatively oxidise thiosulphate anions to produce X moles of a sulphur containing product. The magnitude of X is:

35. In the Hall process, aluminum is produced by the electrolysis of molten Al_2O_3. How many second would it take to produce enough aluminum by the Hall process to make a case of 24 cans of aluminum soft-drink, if each can uses 5.0 g of Al, a current of 9650 amp is employed, and the current efficiency of the cell is 90.0%:

36. A 2.24L cylinder of oxygen at 1 atm and 273 K is found to develop a leakage. When the leakage was plugged the pressure dropped to 590 mm of Hg. The number of moles of gas that escaped will be:

Mathematics PART – III

SECTION – A
(One Options Correct Type)

This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

37. The results of 10 cricket matches (win, lose or draw) have to be predicted. How many different forecasting can contain exactly 7 correct results?
(A) 100
(B) 120
(C) 960
(D) None of these

38. The value of $\int \frac{1}{\sqrt[3]{x^2}\,\sqrt[3]{(2+3x)^4}}dx$ is
(A) $\frac{3}{2}\sqrt[3]{\frac{x}{2+3x}}+c$
(B) $\frac{1}{2}\sqrt[3]{\frac{x}{2+3x}}+c$
(C) $\frac{1}{2}\left(\frac{x}{2+3x}\right)^2+c$
(D) None of these
where c is integration costant

39. If A(– 1, 2, – 3), B(5, 0, – 6) and C(0, 4, – 1) are the vertices of Δ ABC, then direction ratios of the external bisector of $\angle$BAC are
(A) –11, 20, 23
(B) –11, 20, 20
(C) 11, 20, 21
(D) none of these

40. Number of solutions of |z – 1| + |z + i| = 4 and |2z – 1 + i| = $\sqrt{14}$ is
(A) 2
(B) 3
(C) 4
(D) none of these

41. In ΔABC, $\frac{\sum a\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)}{\sum \sin A}=nR$ where R is the radius of circumcircle, then n is equal to
(A) 1
(B) 2
(C) 3
(D) none of these

42. ABCD is a cyclic quadrilateral with $AC \perp BD$ and O is the centre of its circumcircle, then $\overrightarrow{OA}\cdot\overrightarrow{OB}+\overrightarrow{OB}\cdot\overrightarrow{OC}+\overrightarrow{OC}\cdot\overrightarrow{OD}+\overrightarrow{OD}\cdot\overrightarrow{OA}$ is equal to

(A) 1
(B) –1
(C) 0
(D) none of these

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

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43. Tangents are drawn from (-2, 0) to $y^2 = 8x$, radius of circle(s) that would touch these tangents and the corresponding chord of contact, can be equal to,

(A) $4\left(\sqrt{2}+1\right)$
(B) $4\left(\sqrt{2}-1\right)$
(C) $8\sqrt{2}$
(D) $4\sqrt{2}$

44. Let z_1, z_2 be the roots of $az^2 + bz + c = 0$ with a, b, c complex number and $a \neq 0$ and w_1, w_2 be roots of $(a+\bar{c})z^2+(b+\bar{b})z+(\bar{a}+c)=0$. If $|z_1|<1, |z_2|<1$, then

(A) $|w_1|<1$
(B) $|w_1|=1$
(C) $|w_2|<1$
(D) $|w_2|=1$

45. Let $[x]=$ the greatest integer less than or equal to x. The equation $\sin x=[1+\sin x]+[1-\cos x]$ has

(A) no solution in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
(B) no solution in $\left[\frac{\pi}{2},\pi\right]$
(C) no solution in $\left[\pi,\frac{3\pi}{2}\right]$
(D) no solution for $x \in R$

46. If in ΔABC, $a^4+b^4+c^4=2a^2\left(b^2+c^2\right)$, then $\angle A$ is

(A) 45°
(B) 60°
(C) 90°
(D) 135°

47. The solutions of $\theta \in [0, 2\pi]$ satisfying the equation $\log_{\sqrt{3}} \tan\theta\left(\sqrt{\log_{\tan\theta} 3 + \log_{\sqrt{3}} 3\sqrt{3}}\right) = -1$ are

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}, \frac{5\pi}{3}$

(C) has sum $\frac{4\pi}{3}$

(D) None of these

48. A drawer contains red and black balls. When two balls are drawn at random, the probability that they both are red is $\frac{1}{2}$. The number of balls in the drawer can be

(A) 21
(B) 11
(C) 4
(D) 3

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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49. The tangents are drawn from the points of a straight line 3x + 4y = 24 to the curve $x^2 + \frac{y^2}{4} = 1$. Then all the chords of contact passes through a fixed point (a, b), then 2a + 3b

50. Shortest distance between lines $\frac{x-6}{1} = \frac{y-2}{-2} = \frac{z-2}{2}$ and $\frac{x+4}{3} = \frac{y}{-2} = \frac{z+1}{-2}$ is d then $100d + \frac{1}{2} =$

51. Find the distance of the point P (3, 8, 2) from the line $\frac{1}{2}(x-1) = \frac{1}{4}(y-3) = \frac{1}{3}(z-2)$ measured parallel to the plane 3x + 2y – 2z + 15 = 0

52. Let f(x) = $\begin{vmatrix} x & 1 & 1 \\ \sin 2\pi x & 2x^2 & 1 \\ x^3 & 3x^4 & 1 \end{vmatrix}$. If f(x) be an odd function and its odd values is equal g(x), then find the value of λ.
If λf(1) g(1) = –2

53. If $A = \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix}$ and B = (adj A) and C = 5A, then find the value of |adj B|

54. Let $f_p(\alpha) = e^{\frac{i\alpha}{p^2}} . e^{\frac{2i\alpha}{p^2}} ....e^{\frac{i\alpha}{p}}$ $p \in N$ (where $i = \sqrt{-1}$, then find the value of $\left|\lim_{n\to\infty} f_n(\pi)\right|$

ALL INDIA TEST SERIES

TEST – 9

JEE (Advanced)

ANSWERS, HINTS & SOLUTIONS

Physics **PART – I**

SECTION – A

1. D

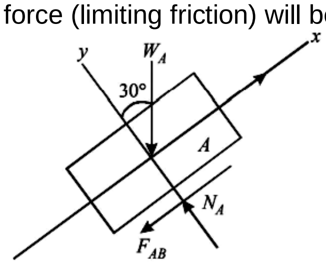
Sol. Since $0 = (v\sin\theta)t + \frac{1}{2}(-a)t^2 \Rightarrow t = \frac{2v\sin\theta}{a}$

Also, $h = (v\cos\theta)t + \frac{1}{2}gt^2$

$\Rightarrow h = \frac{2v^2}{a}\sin\theta\left(\cos\theta + \frac{g}{a}\sin\theta\right)$

2. A

Sol. For the impending motion, block A must slip up and block C down the inclined plane. Since the normal force between A and B is less than that between block B and C, the maximum frictional force (limiting friction) will be reached first between A and B while B and C will stay together.



Writing equilibrium equations:

$\Sigma F_y = 0;$

$N_A - W_A\cos 30° = 0$

$N_A = W_A\cos 30°$

$N_A = 20\sqrt{3}N$

Also, for impending motion if F_{AB} is frictional force between blocks A and B, then

$$F_{AB} = \mu_s . N_A = 20\sqrt{3}\mu_s N \qquad \ldots(1)$$

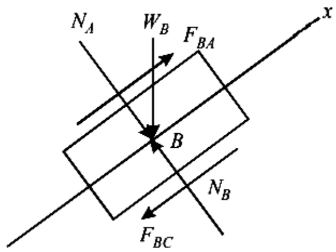
$$\Sigma F_x = 0;$$

$$T - W_A \sin 30° - F_{AB} = 0$$

$$T - 40\frac{1}{2} - 20\sqrt{3}\mu_s = 0$$

$$T = 20(1 + \sqrt{3}\mu_s)$$

From FBD of block B and C combined



Writing equilibrium equation

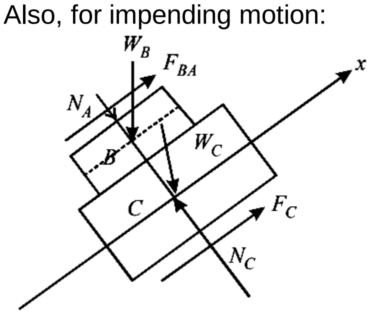
$$\Sigma F_y = 0;$$

$$N_C - N_A - (W_B + W_C)\cos 30° = 0$$

$$N_C - 20\sqrt{3} - 110\frac{\sqrt{3}}{2} = 0$$

$$N_C = 75\sqrt{3}N$$

Also, for impending motion:



$$F_C = \mu_s . N_C = 75\sqrt{3}\mu_s \qquad \ldots(3)$$

For $\Sigma F_x = 0,$ we have

$$T_A + (F_{BA} + F_C) - (W_B + W_C)\sin 30° = 0$$

$$T + [20\sqrt{3} + 75\sqrt{3}\mu_s] - \frac{110}{2} = 0$$

$$T = (55 - 95\sqrt{3}\mu_s)$$

Since tension is same, so from (2) and (4), we get

$$20(1 + \sqrt{3}\mu_s) = (55 - 95\sqrt{3}\mu_s)$$

Solving for μ_s we get, $115\sqrt{3}\mu_s = 35$

or $\qquad \mu_s = \frac{35}{115\sqrt{3}} = 0.1757$

$\therefore \qquad$ Minimum μ_s = 0.1757

3. D

Sol. The velocity of particle after falling through height h

$$u = \sqrt{2gh} \qquad \ldots(i)$$

$$v^2 = u^2 - 2gh$$

$\therefore$ $$0 = e^2u^2 - 2gh_1$$

or $$h_1 = \frac{e^2u^2}{2g} = \frac{e^2 2gh}{2} = e^2h \quad \text{[From Eq. (i)]}$$

$$0 = e^4u^2 - 2gh_2$$

or $$h_2 = e^4h$$

$$= h + 2h_1 + 2h_2 + \ldots\infty$$

$$= h + 2e^2h + 2e^2h + \ldots\infty$$

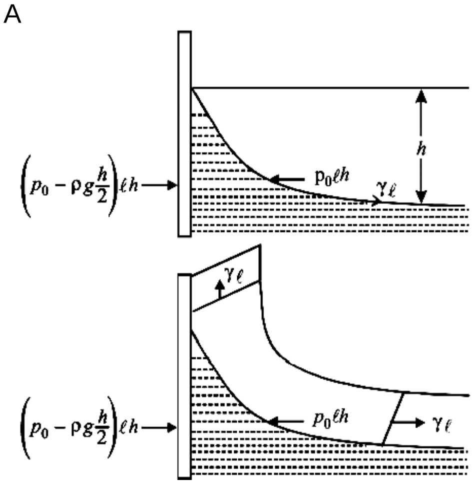
$$= h + 2e^2h(1 + e^2 + e^4 + \ldots\infty)$$

$$= h + 2e^2h\left(\frac{1}{1-e^2}\right)$$

$$= h\left(1 + \frac{2e^2}{1-e^2}\right) = \left(\frac{1+e^2}{1-e^2}\right)h$$

4. A

Sol.

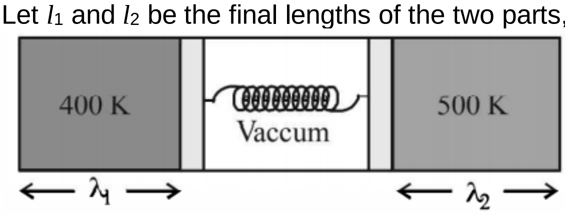


Balancing forces in horizontal direction

$$\left(p_0 - \rho g\frac{h}{2}\right)\ell h + \gamma\ell = p_0\ell h \Rightarrow h = \sqrt{\frac{2\gamma}{\rho g}}$$

5. A

Sol. Let l_1 and l_2 be the final lengths of the two parts, the from gas equation



$$\frac{P_0A\ell_0}{T_0} = \frac{PA\ell_1}{T_1} = \frac{PA\ell_2}{T_2} \qquad \ldots(i)$$

$P_0A = kx_0$ and $PA = kx$

or $\dfrac{P}{P_0} = \dfrac{x}{x_0}$ …(ii)

$\therefore$ $x - x_0 = \ell_1 + \ell_2 - 2\ell_0$ …(iii)

From equation (i),

$$\ell_1 = \frac{P_0\ell_0 T_1}{PT_0} \text{ and } \ell_2 = \frac{P_0\ell_0 T_2}{PT_0}$$

From equation (ii),

$$\ell_1 = \frac{x_0\ell_0 T_1}{xT_0} \text{ and } \ell_2 = \frac{x_0\ell_0 T_2}{xT_0}$$

Putting these in equation (iii),

$$x - x_0 = \frac{x_0\ell_0}{xT_0}[T_1 + T_2] - 2\ell_0$$

Substituting the values and solving for x, we get

$x \approx 1.3$ m

6. A

Sol. $f = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{m}}$;

In air : $T = mg = \rho Vg$

$\therefore f = \dfrac{1}{2\ell}\sqrt{\dfrac{\rho Vg}{m}}$ …(1)

In water : $T = mg - \text{upthrust}$

$= V\rho g - \dfrac{V}{2}\rho_\omega g = \dfrac{Vg}{2}(2\rho - \rho_\omega)$

$\therefore f' = \dfrac{1}{2\ell}\sqrt{\dfrac{\frac{Vg}{2}(2\rho - \rho_\omega)}{m}}$

$= \dfrac{1}{2\ell}\sqrt{\dfrac{Vg\rho}{m}}\sqrt{\dfrac{(2\rho - \rho_\omega)}{2\rho}}$

$\dfrac{f'}{f} = \sqrt{\dfrac{2\rho - \rho_\omega}{2\rho}}$

$f' = f\left(\dfrac{2\rho - \rho_\omega}{2\rho}\right)^{1/2}$

$300\left(\dfrac{2\rho - 1}{2\rho}\right)^{1/2}$ Hz

7. A, B, D

Sol. Let ω_1 = the initial angular velocity of the disc.

ω_2 = the final common angular velocity of the disc and the ring.

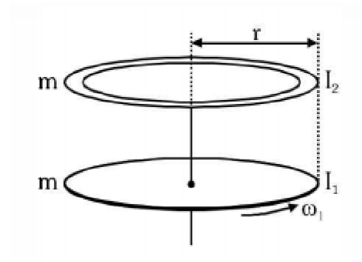
For the disc, $I_1 = \dfrac{1}{2}mr^2$

For the ring, $I_2 = mr^2$

By conservation of angular momentum,

$L = I_1\omega_1 - (I_1 + I_2)\omega_2$

or $\quad \omega_2 = \dfrac{I_1\omega_1}{I_1 + I_2} = \omega_1 / 3$



Initial kinetic energy = $E_1 = \dfrac{1}{2} I_1 \omega_1^2$

Final kinetic energy = $E_2 = \dfrac{1}{2}(I_1 + I_2)\omega_2^2$

Heat produced = loss in kinetic energy = $E_1 - E_2$

Ratio of heat produced to initial kinetic energy = $\dfrac{E_1 - E_2}{E_1} = \dfrac{2}{3}$

8. A, B, C

Sol. Potential on innermost shell is zero

$\dfrac{q_1}{r} + \dfrac{q_2}{2r} + \dfrac{q_3}{3r} = 0$

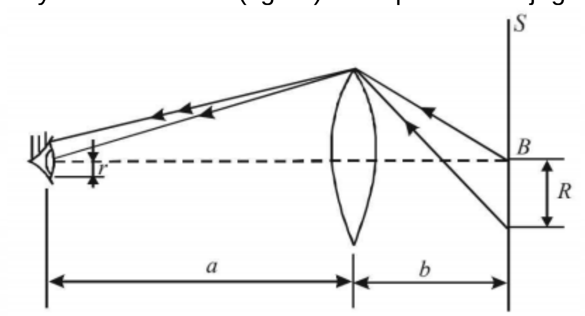
$\Rightarrow 6q_1 + 3q_2 + 2q_3 = 0 \qquad \ldots(1)$

Potential on outermost shell is zero

$\dfrac{q_1}{3r} + \dfrac{q_2}{3r} + \dfrac{q_3}{3r} = 0 \Rightarrow q_1 + q_3 = -q_2 \qquad \ldots(2)$

9. A, C

Sol. Let us first neglect the size of pupil, assuming that it is point-like. Obviously, only those of the beams passing through the lens will get into the eye which have passed through point B before they fall on the lens (figure). This point is conjugate to the point at which the pupil is located.



$\dfrac{1}{F} = \dfrac{1}{a} + \dfrac{1}{b}, b = \dfrac{aF}{a - F} = 12\text{ cm}$

$R = \dfrac{b}{a} r \simeq 0.5\text{mm},$ and the screen must be placed in the plane S with its centre at point B.

10. A, B, D

Sol. $F(r) = -\dfrac{k}{r^n}$

$\Rightarrow U(r) = -\int F(r)dr = -\dfrac{k}{(n-1)} \cdot \dfrac{1}{r^{n-1}}$

Kinetic energy $= \dfrac{L^2}{2I} = \dfrac{L^2}{2mr^2} = K(r)$

Since total energy $E(r) = U(r) + K(r)$

$$\Rightarrow E(r) = -\frac{k}{(n-1)} \cdot \frac{1}{r^{n-1}} + \frac{L^2}{2mr^2}$$

$$\Rightarrow \left.\frac{\partial E}{ar}\right|_{r=r_0} = 0 \text{ and } \left.\frac{\partial^2 E}{\partial r^2}\right|_{r=r_0} > 0,$$

Using both conditions, $(3-n)\frac{L^2}{m} > 0$

11. A, B, C, D

Sol. $I = \frac{\varepsilon}{\frac{2R}{3}} = \frac{3\varepsilon}{2R}$

$$= \frac{3}{2R} \times \frac{1}{2} B\omega\ell^2 = \frac{3B\omega\ell^2}{4R}$$

Magnetic force $F = \frac{3B\omega\ell^2}{4R} \times \ell \times B$

$$= \frac{3B^2\omega\ell^2}{4R}$$

$$\tau = \frac{3B^2\omega\ell^3}{4R} \times \frac{\ell}{2} = \frac{3B^2\omega\ell^4}{8R}$$

$\therefore$ Force to be applied at the end $= \frac{3B^2\omega\ell^3}{8R}$.

12. A, B, C, D

Sol. For maxima d = nλ

For minima d = (n + 1/2)λ

For intensity $\frac{3}{4}$ th of maximum $d = \left(n \pm \frac{1}{3}\right)\frac{\lambda}{2}$

SECTION – C

13. 00000.78

Sol. From conservation of energy

$$m_n c^2 = m_p c^2 + k_p + m_e c^2 + k_e + m_v c^2 + k_v$$

939.5656 = 938.2723 + 0.5109 + 0.0004 + (k_P + k_e)

[$\because$ $m_v c^2$ + k_v = 0.0004 MeV]

$\Rightarrow$ k_P + k_e = 0.0004 MeV

$$\Rightarrow \frac{P^2}{2m_P} + \frac{P^2}{2m_e} = 0.7820 \text{ MeV}$$

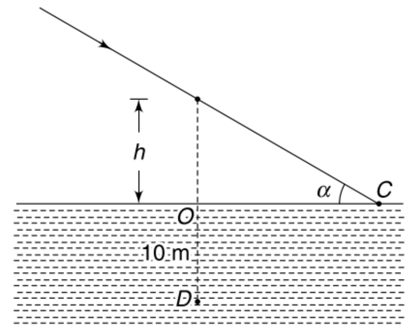
$$\Rightarrow \frac{P^2}{2m_e}\left[1 + \frac{m_e}{m_p}\right] = 0.7820$$

$$k_e = \left(\frac{m_P}{m_P + m_e}\right) \times 0.7820 \simeq 0.7820 \text{ MeV}$$

14. 00024.62

Sol. $h_{app} = \mu h$

$h_{app} = 50 - d = 40$ m and m = μ = 4/3



$\therefore$ h = 30 m

Now, $h_{app} = \mu h$

$V_{y\,app} = \mu V_y$

$V_{x\,app} = V_x$

Given $\dfrac{V_{yapp}}{V_{xapp}} = \tan 45° = 1$

$\therefore \dfrac{\mu V_y}{V_x} = 1 \Rightarrow \tan\alpha = \dfrac{1}{\mu} = \dfrac{3}{4}$

$\alpha = 37°$

α is the true angle that the line of motion of the bird makes with horizontal.

$\therefore OC = h\tan a = 30.\dfrac{3}{4}$

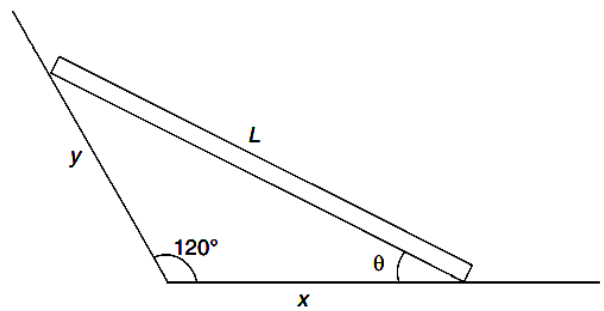
= 22.5 m

$\therefore DC = \sqrt{(22.5)^2 + 10^2}$

= 24.62 m

15. 00002.26

Sol. Sine Rule



$\dfrac{x}{\sin(60-\theta)} = \dfrac{y}{\sin\theta} = \dfrac{L}{\sin 120°}$

$\therefore x = \dfrac{2L}{\sqrt{3}}\sin(60-\theta)$

$\therefore \dfrac{dx}{dt} = \dfrac{2L}{\sqrt{3}}\cos(60-\theta)\left(-\dfrac{d\theta}{dt}\right)$

Note $-\dfrac{d\theta}{dt} = \omega =$ angular speed.

[θ is decreasing, hence a negative sign]

When $\theta = 20°, \dfrac{dx}{dt} = 1.5\ m/s.$

$\therefore \omega = \dfrac{1.5 \times 1.732}{2 \times 0.75 \times 0.766} = 2.26$ rad/s

16. 00001.15

Sol. Let tension in BC & BD be T_1 and that in string BA be T_2

The string CB and DB make an angle of $\alpha = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ with vertical because the diagonal of a cube makes $\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ angle with a side.

Line BM makes $\beta = 45°$ with vertical.

$CB = \dfrac{\sqrt{3}a}{2}, \quad CM = \dfrac{a}{2}$

$\therefore \sin\theta = \frac{1}{\sqrt{3}}$

Resultant of tension in CB and DB is along BM equal to

$2T_0 = 2T_1 \cos\theta = 2T_1\sqrt{\frac{2}{3}}$

Vertical component of T_0 balance Mg and its horizontal component is equal to T_2.

$\therefore T_0 \cos\beta = Mg$

$T_0 \sin\beta = T_2$

$\Rightarrow 2\sqrt{\frac{2}{3}}T_1 \sin\beta = T_2$

$\Rightarrow 2\sqrt{\frac{2}{3}}T_1 \frac{1}{\sqrt{2}} = T_2$

$\Rightarrow \frac{T_2}{T_1} = \frac{2}{\sqrt{3}}$

17. 00024.00

Sol. $C = \frac{K\epsilon_0 A}{d} =$ a constant

For A to be minimum, d must be minimum. The separation between the plates is limited by the breakdown strength of the dielectric.

For air capacitor $\frac{V}{d_{min}} = E_{air}$ [E_{air} = Breakdown field for air]

$\therefore \quad d_{min} = \frac{V}{E_{air}}$

Now $\frac{\epsilon_0 A_{min}}{d_{min}} = C$

$\Rightarrow \quad A_{min} = \frac{C}{\epsilon_0}\frac{V}{E_{air}}$

$\therefore \quad A_1 = \frac{CV}{\epsilon_0 E_{air}}$

With dielectric, similar calculation gives

$A_2 = \frac{CV}{K\epsilon_0 E_{dielec}}$

$\therefore \quad \frac{A_1}{A_2} = \frac{KE_{dielec}}{E_{air}} = 3 \times 8 = 24$

18. 00051.70

Sol. $q_{conv} = h(T - T_0) = 6(80 - 20) = 360\ Wm^{-2}$

For 1m length of the pipe

$Q_{conv} = q_{conv} A = q_{conv} \times 2\pi r$

$= 360 \times 2 \times 3.14 \times 0.01 = 22.6\ Wm^{-1}$

$q_{rad} = \sigma(T^4 - T_0^4) = 5.67 \times 10^{-8} (353^4 - 293^4) = 462\ Wm^{-2}$

For 1m length of the pipe

$Q_{rad} = q_{rad} A = 462 \times 2 \times 3.14 \times 0.01 = 29.1\ Wm^{-1}$

$\therefore \quad Q_{conv} + Q_{rad} = 22.6 + 29.1 = 51.7\ Wm^{-1}$

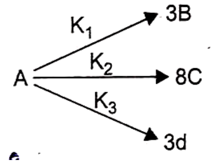
Chemistry

PART – II

SECTION – A

19. B

Sol. Mole of A remain after 45 days



$K_1 : K_2 : K_3 = 4 : 2 : 1$

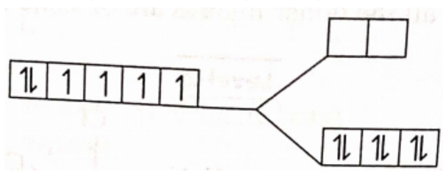
$$\frac{N_0}{2^n} = \frac{N_0}{2^{\frac{T}{t_{1/2}}}} \frac{1}{2^{45/15}} = \frac{1}{2^3} = \frac{1}{8}$$

Moles of A convert into product $\frac{7}{8}$mol

Moles of $[C] = \frac{K_2}{K_1 + K_2 + K_3} \times \frac{7}{8} \times \frac{8}{1} = 2$

20. C

Sol. CFSE (In octahedral) = $= (-0.4 \times n \times \Delta_0) + 2PE$



21. D

Sol. $H_2S + Ba(OH)_2 \rightarrow \underset{\text{(Water soluble)}}{BaS} + 2H_2O$

22. D

Sol. $T_b = 80.26, \quad \Delta T_b = 0.16;$

$$0.16 = 2.53 \times \frac{0.26 / M}{11.20} \times 1000; M \approx 367$$

That is almost molar mass of $C_{20}H_{16}Fe_2$.

23. B

Sol. $X(s) \rightleftharpoons \underset{x}{A(g)} + \underset{(2x+2y);}{2B(g)}$

$K_{P_1} = P_A . P^2_{B(total)}$

$Y(s) \rightleftharpoons \underset{y}{C(g)} + \underset{(2y+2x);}{2B(g)}$

$K_{p_2} = P_C . P^2_{B\ (total)}$

$$\frac{K_{P_1}}{K_{P_2}} = \frac{x}{y} \Rightarrow x = 2y$$

$K_{p_1} = x(2x + 2y)^2$

$\Rightarrow x = 0.1$ atm;

$\therefore$ y = 0.05 atm

Total pressure of gases $= P_A + P_B + P_C$

$= 3(x + y)$

$= 0.45$ atm.

24. A

Sol. Balanced reaction is

$2F_2O_3 + 3C \rightarrow 4Fe + 3CO_2$

Number of moles of Fe_2O_3

$= \left(\frac{120 \times 1000}{2 \times 56 + 48}\right) \times \frac{90}{100}$

Mass of 80% pure iron produced

$= \frac{120 \times 1000 \times 0.9}{2 \times 56 + 48} \times \frac{2 \times 56}{0.8}$

$= 94500$ gram or 94.5 kg

25. A, C

Sol. Apply Le chatlier principle.

26. B

Sol. (ii) and (iii)

2[nd] ion is aromatic.

27. A, B, C

Sol. $\Delta H = \Delta U + P.\Delta V + V.\Delta P$

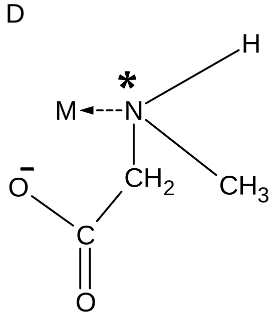
is correct relation.

28. A, C, D

Sol. Factual

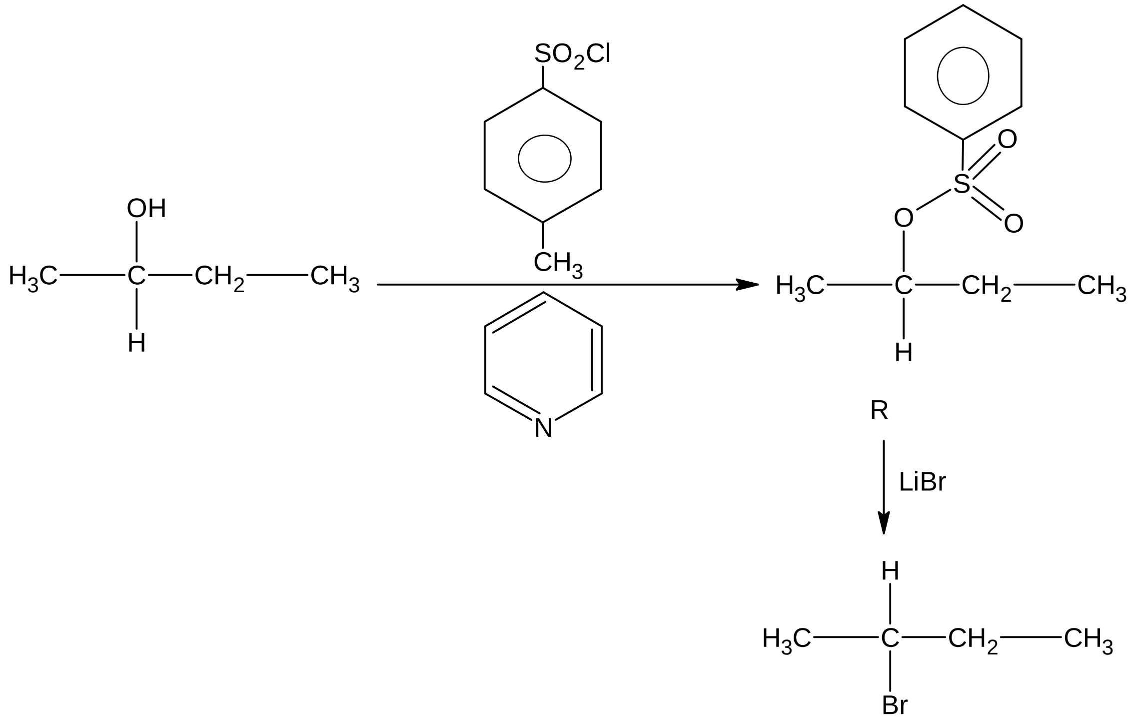
29. D

Sol.



30. D

Sol.



S – 2 – Butylbromide

SECTION – C

31. 00069.50

Sol. M-eq. Of $FeSO_4.7H_2O$ in 1 litre

$$= 20 \times 0.02 \times 5 \times \left(\frac{1000}{25}\right) = 80$$

$$\therefore \frac{W}{278} \times 1 \times 1000 = 80 \Rightarrow W = 22.24$$

Mass & of $FeSO_4.7H_2O$ in given sample

$$= \frac{22.24}{32} \times 100 = 69.5$$

32. 00024.63

Sol. $P_{H_2O} = 760 - 722 = \frac{38}{760}\text{atm};\ n_{H_2O} = \frac{0.9}{18}$

$$V = \frac{0.9}{18} \times \frac{0.821 \times 300}{38} \times 760; V = 24.63L$$

33. 00039.21

Sol. $-q_{reaction} = q_{bomb} + q_{water}$

$$q_{reacton} = (C(bomb) + (m_{water} \times c))\Delta T$$

$$= (652 + 500 \times 4.18) \times 14.3$$

$$= 39210J \text{ or } 39.21 \text{ kJ}$$

34. 00000.60

Sol. $\overset{+7}{Mn}O_4^- + \overset{+2}{S_2}O_3^{2-} \rightarrow \overset{+4}{Mn}O_2 + \underset{X}{\overset{+6}{S}O_4^{2-}}$

Equivalents of MnO_4^- = equivalents of SO_4^{2-}

Moles of $MnO_4^- \times n$ -factor = moles of $SO_4^{2-} \times$ n – factor

$8 \times 3 = X \times 4$

$X = 6$

$\frac{X}{10} = 0.6$

35. 00148.14

Sol. No. of equivalent of aluminium, $\frac{W}{E} = \frac{I \times \eta \times t}{9600}$

$\frac{24 \times 5}{27} \times 3 = \frac{9650 \times 0.9 \times t}{96500}$

$t = 148.14$ sec

36. 00000.02

Sol. $P \propto n,$ initial mole $n_1 = \frac{PV}{RT} = 0.1$ mole

$\frac{P_1}{P_2} = \frac{n_1}{n_2}$

SECTION – A

37. C

Sol. Correct prediction can be given in 1 way, and Incorrect prediction can be in 2 ways

$\therefore$ Required number of ways = ${}^{10}C_7\ (1)^7\ (2)^3 = 960$.

38. A

Sol. $$\int \frac{1}{\sqrt[3]{x^2}\ \sqrt[3]{(2+3x)^4}}dx$$

$$= \int \frac{1}{x^{2/3}\ (2+3x)^{4/3}}\ dx$$

Let $\frac{2+3x}{x} = t \Rightarrow \frac{-2}{x^2}dx = dt$

$$= -\frac{1}{2}\int \frac{1}{t^{4/3}}dt = \frac{3}{2}t^{-1/3} = \frac{3}{2}\left[\frac{x}{2+3x}\right]^{1/3} + c\ .$$

39. A

Sol. If the external bisector of $\angle BAC$ meets BC at E, then E divides BC externally in the ratio of AB : AC.

AB = 7, AC = 3.

$\therefore$ Coordinates of E are $\left(\frac{-15}{4},\ 7,\ \frac{11}{4}\right)$

$\therefore$ The direction ratio of line AE are $\frac{-15}{4}+1,\ 7-2,\ \frac{11}{4}+3 = -\frac{11}{4},\ 5,\ \frac{23}{4}$

= – 11, 20, 23

40. A

Sol. |z – 1| + |z – i| = 4 represents ellipse with foci (1, 0) and (0, –1) and length of major axis = 4.

Centre of ellipse is $\left(\frac{1}{2},\ -\frac{1}{2}\right)$ and $\left(\frac{1-i}{2}\right)$

2a = 4, 2ae = $\sqrt{2} \Rightarrow e = \frac{1}{2\sqrt{2}}$

From $b^2 = a^2\ (1 - e^2)$, we get $b = \sqrt{\frac{7}{2}}$

Now $|2z - 1 + i| = \sqrt{14}$

$$\Rightarrow \left|z - \frac{1}{2} + \frac{i}{2}\right| = \sqrt{\frac{7}{2}}$$

$\Rightarrow \left|z - \left(\frac{1-i}{2}\right)\right| = \sqrt{\frac{7}{2}}$ represents a circle having centre at $\frac{1-i}{2}$ and radius $= \sqrt{\frac{7}{2}}$.

Since radius of circle = minor axis of ellipse.

Hence number of solution of given equation = 2.

41. A

Sol. We know $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{\sin A + \sin B + \sin C}{a+b+c} = \frac{1}{2R}$

$$\frac{\sum \sin A}{\sum a(\cos B + \cos C)} = \frac{1}{2R} \Rightarrow \frac{\sum \sin A}{\sum 2a\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)} = \frac{1}{2R}$$

$$\Rightarrow \frac{\sum \sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)}{\sum \sin A} = R \Rightarrow n = 1.$$

42. C

Sol. As OA = OC

so $\overrightarrow{OA} + \overrightarrow{OC}$ will be perpendicular to AC

$\therefore$ $\overrightarrow{OA} + \overrightarrow{OC} = \lambda\overrightarrow{BD}$ …..(1)

where λ is the real number.

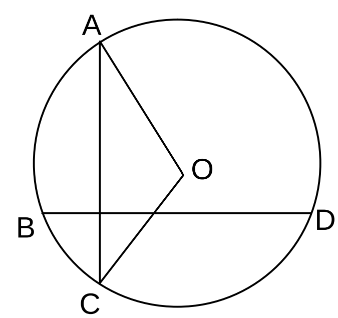
Similarly,

$\overrightarrow{OB} + \overrightarrow{OD} = \mu\overrightarrow{AC}$ ….(2)

where μ is the real number.

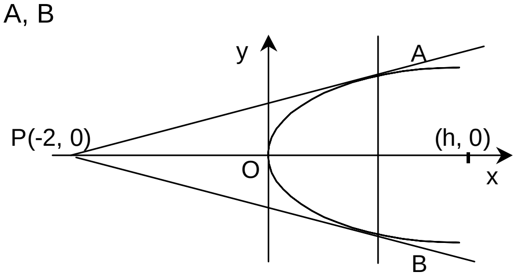
Now using (1) and (2), we get

$\left(\overrightarrow{OA} + \overrightarrow{OC}\right)\left(\overrightarrow{OB} + \overrightarrow{OD}\right) = \mu.\lambda\ \overrightarrow{AC}\cdot\overrightarrow{BD} = 0.$



43. A, B

Sol.



Point 'P' clearly lies on the directrix of $y^2 = 8x$.

Thus slope of PA and PB are 1 and -1 respectively.

Equation of PA : y = x + 2, equation of PB : y = -x – 2, equation of AB : x = 2.

Let the centre of the circle be (h, 0) and radius be 'r'

$\Rightarrow \frac{|h+2|}{\sqrt{2}} = \frac{|h-2|}{1} = r$

$\Rightarrow$ $h^2 + 4 + 4h = 2(h^2 + 4 – 4h) \Rightarrow h^2 – 12h + 4 = 0$

$h = \frac{12 \pm 8\sqrt{2}}{2} = 6 \pm 4\sqrt{2} \Rightarrow |h – 2| = 4(\sqrt{2} – 1), 4(\sqrt{2} + 1).$

44. B, D

Sol. $a + \bar{c} \neq 0$

$(a + \bar{c})w_1^2 + (b + \bar{b})w_1 + (\bar{a} + c) = 0$

$(\bar{a} + c)\bar{w}_1^2 + (\bar{b} + b)\bar{w}_1 + (a + \bar{c}) = 0$

$\because \bar{w}_1 \neq 0$ $\therefore (a + \bar{c})\frac{1}{\bar{w}_1^2} + (b + \bar{b})\frac{1}{\bar{w}_1} + (\bar{a} + c) = 0$

$\therefore \frac{1}{\bar{w}_1}$ is also a root but $w_2 \neq \frac{1}{\bar{w}_1}$

$\therefore w_1 = \frac{1}{\bar{w}_1} \Rightarrow |w_1| = 1$ and $|w_2| = 1$

45. A, B, C, D

Sol. At $x = -\frac{\pi}{2}, \frac{3\pi}{2}; [1+\sin x] = 0, [1-\cos x] = 1$

$\therefore$ $\sin x = 0 + 1 \Rightarrow -1 = 1$ (absurd)

At x = 0, $[1+\sin x] = 1, [1-\cos x] = 0$

$\therefore$ $\sin x = 1 + 0 \Rightarrow 0 = 1$ (absurd)

At $x = \frac{\pi}{2}, [1+\sin x] = 2, [1-\cos x] = 1$

$\therefore$ sin x = 2 + 1 = 3 (absurd)

At x = π, [1+ sin x]= 1, [1 – cos x] = 2

$\therefore$ sin x = 1 +2 = 3 (absurd)

In $\left(-\frac{\pi}{2}, 0\right), [1+\sin x] = 0, [1-\cos x] = 0$

$\therefore$ sin x = 0 + 0 = 0 (absurd)

In $\left(0, \frac{\pi}{2}\right), [1+\sin x] = 1, [1-\cos x] = 0$

$\therefore$ sin x = 1 + 0 = 1 (absurd)

In $\left(\frac{\pi}{2}, \pi\right), [1+\sin x] = 1, [1-\cos x] = 1$

$\therefore$ sin x = 1 + 1 = 2 (absurd)

In $\left(\pi, \frac{3\pi}{2}\right), [1+\sin x] = 0, [1-\cos x] = 1$

$\therefore$ $\sin x = 0 + 1 = 1$ (absurd)

46. A, D

Sol. We have $a^4 + b^4 + c^4 = 2a^2\left(b^2 + c^2\right)$

$a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2 = 2b^2c^2$

$\left(b^2 + c^2 - a^2\right)^2 = 2b^2c^2$

$b^2 + c^2 - a^2 = \sqrt{2}bc$ or $b^2 + c^2 - a^2 = -\sqrt{2}bc$, $\frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{\sqrt{2}}$

or $\frac{b^2 + c^2 - a^2}{2bc} = -\frac{1}{\sqrt{2}}$

$\cos A = \frac{1}{\sqrt{2}} = \cos 45°$

$\cos A = -\frac{1}{\sqrt{2}} = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

47. A, C

Sol. $\log_{\sqrt{3}} \tan\theta\left[\sqrt{\dfrac{\log_{\sqrt{3}} 3}{\log_{\sqrt{3}} \tan\theta}+\dfrac{\log\left(\sqrt{3}\right)^3}{\log\sqrt{3}}}\right]=-1$

$\Rightarrow \quad \log_{\sqrt{3}} \tan\theta\left[\sqrt{\dfrac{2}{\log_{\sqrt{3}} \tan\theta}+3}\right]=-1$

Let $\log_{\sqrt{3}} \tan\theta = y$

$\Rightarrow \quad y\sqrt{\dfrac{2}{y}+3}=-1 \Rightarrow \sqrt{\dfrac{2}{y}+3}=-\dfrac{1}{y} \Rightarrow \dfrac{2}{y}+3=\dfrac{1}{y^2}$ or $y^2(2+3y)=y \Rightarrow y\left[3y^2+2y-1\right]=0$

$\because \quad y < 0$

$y(3y-1)(y+1)=0$

$y = -1$ ($\because$ y cannot be positive)

$\Rightarrow \quad \log_{\sqrt{3}} \tan\theta = -1$

$\tan\theta = \dfrac{1}{\sqrt{3}}$

$\therefore \quad \theta = \dfrac{\pi}{6}$ and $\dfrac{7\pi}{6}$

$\therefore$ There are two value of θ in $[0, 2\pi]$

48. A, C

Sol. Let the drawer contains p balls of which 'm' are red.

Probability of drawing two red balls at random is $\dfrac{{}^{m}C_2}{{}^{p}C_2}=\dfrac{1}{2}$

$\Rightarrow 2m(m-1)=p(p-1)$

$\Rightarrow 2m^2-2m-p^2+p=0$

$\Rightarrow m=\dfrac{2\pm\sqrt{4-8\left(p-p^2\right)}}{4}=\dfrac{1\pm\sqrt{1-2p+2p^2}}{2}$

$\Rightarrow 1-2p+2p^2$ should be an odd perfect square.

i.e., $p = 21,\ 4$ but $p \neq 3$

when 3 balls out of 4 are red.

15 balls out of 21 are red.

SECTION – C

49. 00002.25

Sol. Let the coordinates of a point lying on the straight line 3x + 4y = 24 is $\left(t,\ \dfrac{24-3t}{4}\right)$

Equation of the chord of contact is $tx+\dfrac{y}{16}(24-3t)=1$

$\Rightarrow (24y-16)+t(16x-3y)=0$

$\Rightarrow$ this line always passes through the fixed point which is the point of intersection of the lines $24y-16=0$ and $16x-3y=0$

fixed point $\equiv \left(\dfrac{1}{8}, \dfrac{2}{3}\right)$, which lies on $16x-3y=0$, $9y^2=32x$ and $24x+24y=19$.

50. 00900.50

Sol. $$S.\,D = \frac{\begin{vmatrix} 10 & 2 & 3 \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}}{\sqrt{8^2+8^2+4^2}} = \frac{108}{12} = 9$$

51. 00007.00

Sol. Let general point of line be A (2λ + 1, 4λ + 3, 3λ + 2). Let this point lies at the same distance as the point p (3, 8, 2) from the plane 3x + 2y – 2z + 15 = 0

Therefore, $\frac{3.3+2.8-2.2+15}{\sqrt{17}} = \frac{3(2\lambda+1)+2(4\lambda+3)-2(3\lambda+2)+15}{\sqrt{17}}$

$\Rightarrow 36 = 8\lambda + 20 \Rightarrow \lambda = 2$

Therefore, A is (5, 11, 8)

$PA = \sqrt{(5-3)^2+(11-8)^2+(8-2)^2} = \sqrt{4+9+36} = 7$

52. 00000.50

Sol. f(–x) = – f(x) = g(x)

∴ $f(x)\,.\,g(x) = -(f(x))^2$ or $f(1)\,g(1) = -(f(1))^2 = -\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 3 & 1 \end{vmatrix}^2 = -4$

$\Rightarrow \quad \lambda f(1)\,g(1) = -2 \quad \Rightarrow \lambda(-4) = -2 \quad \Rightarrow \lambda = \frac{1}{2}$

53. 00125.00

Sol. $|adjB| = |adj(adjA)| - |A|^{(3-1)^2} = |A|^4$

$\because |n| = 5$

$= 125$

54. 00001.00

Sol. $f_p(\alpha) = e^{\frac{i\alpha}{p^2}(1+2+\ldots+p)} = e^{\frac{i\alpha}{2}\left(1+\frac{1}{p}\right)}$

$\lim_{n\to\infty} f_n(\pi) = \lim_{n\to\infty} e^{\frac{i\alpha}{2}\left(1+\frac{1}{n}\right)} = e^{\frac{i\alpha}{2}}$

$\left|\lim_{n\to\infty} f_n(\pi)\right| = \left|e^{\frac{i\alpha}{2}}\right| = 1$