LINES AND ANGLES

INTRODUCTION

In this chapter, you will study the properties of the angels formed when two lines intersect each other, and also the properties of the angles formed when a line intersects two or more parallel lines at distinct points. Further you will use these properties to prove some statements using deductive reasoning.

BASIC TERMS AND DEFINITIONS

(a) **LINE-SEGMENT :-** A part of portion of a line with two end points is called a line-segment. The line segment AB is denoted by AB and its length is denoted BY AB.

A B

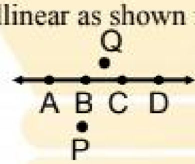
(b) **RAY : -** A part of a line with one end point is called a ray. Ray AB is denoted by AB.

A B

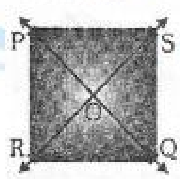
(c) **LINE :-** A line is the collection of infinite number of points and extends endlessly in both the directions. A line is generally denoted by small letters such as l,m,n,p,q,r etc.

(d) **COLLINEAR POINTS :-** If there or more points lie on the same line, then they are called collinear points. points A, B, C and D are collinear as shown is figure.

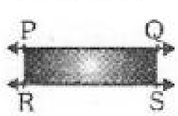
(e) **NON-COLLINEAR POINTS :-** If three or more points does not lie on the same line, they are called non-collinear points.
Points. A, B, C, P and Q are non-collinear as shown is figure.



(f) **INTERSECTING LINES :-** Two distinct lines are intersecting, if they have a common point. The common point is called the "point of intersection" of the two lines.

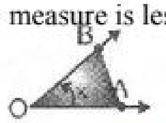


(g) **NON-INTERSECTING LINES (PARALLEL LINES):-** Two distinct lines which are not intersecting are said to be parallel lines. The parallel lines are always at a constant distance from each other.



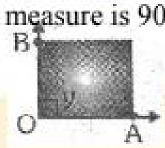
(h) **ANGLE :-** An angle is formed when two rays originate from the same end point. The rays making an angle are called the **'arms'** of the angle and the end point is called the **'vertex'** of the angle. The angles are of following types :-

(i) Acute angle :- An angle whose measure is less than 90^0 is called an acute angle.



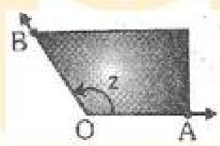
Acute angle : $0^0 < x < 90^0$

(ii) **RIGHT ANGLE :-** An angle whose measure is 90^0 is called a right angle.



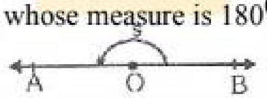
Right angle : $y = 90^0$

(iii) **OBTUSE ANGLE :-** An angle whose measure is more than 90^0 but less than 180^0 is called obtuse angle.



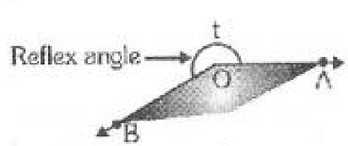
Obtuse angle : $90^0 < z < 180^0$

(v) **STRAIGHT ANGLE :-** An angle whose measure is 180^0 is called a straight angle.



Straight angle : $s = 180^0$

(v) **REFLREX ANGLE :-** An angle whose measure is more than 180^0 but less than 360^0 is called a reflex angle.



Reflex angle : $180^0 < t < 360^0$

(vi) COMPLEMENTRAY ANGLES :- Two angles, the sum of whose measures is 90^0 are called complementary angles.

AOC & BOC are complementary angles, as $x^0 + y^0 = 90^0$

(vii) SUPPLEMENTARY ANGLES :- Two angles, the sum of whose measures is 180^0 are called supplementary angles.

AOC & BOC are supplementary angle as $x^0 + y^0 = 180^0$.

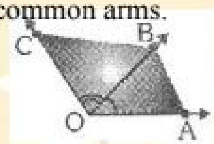
(viii) ADJACENT ANGLES :- Two angles are called adjacent angles is :

(i) they have the same vertex.

(ii) they have a common arm and

(iii) uncommon arms are on either side of the common arm.

In fig. AOB and BOC are adjacent angles. They have the common vertex O and the common arm OB. Ray OC and OA are non-common arms.



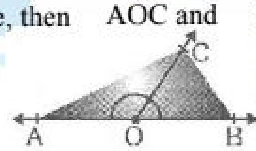
Adjacent angles.

When two angles are adjacent, then their sum is always equal to the angle formed by the two non-common arms. So we can write.

AOC = AOB + BOC

REMARK : - COA and COB are not adjacent angles beause their non common arms OB and OC lie on the same side of the common arm OB.

(ix) LINEAR PAIR OF ANGLES :- Let AOC & BOC be adjacent angles. If the noncommon arms OA and OB and form line, then AOC and BOC is said to form a linear pair of angles.



Let a ray OC stands on line AB.

Then, the angle formed at the point O are AOC, BOC and AOB.

When two angles are adjacent, then their sum is equal to angle formed by two non-common arms.

AOC + BOC = AOB [AOC and BOC are adjacent angles]

AOC + BOC = 180^0 [Straight angle = 180^0]

This result leads us to an axiom given below :

AXIOM-1 : ***If a ray stands on a line, then the sum of two adjacent angles so formed is*** $\boldsymbol{180^0}$

This given us another definition of linear pair angles - when the sum of two adjacent angles is 180^0, then they are called as linear pair of angles.

The above axiom can be stated in the reverse way as below :

AXIOM-2 : ***If the sum of two adjacent angles in*** $\boldsymbol{180^0}$***, ten the non-common arms of the angles form a line.***

(x) Vertically Opposite Angles :- If two lines intersect each other then, the pairs of opposite angles formed are called vertically opposite angles.

Two lines AB and CD intersect at point O.

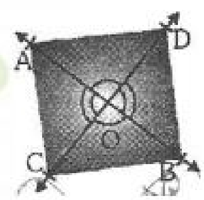
Then, there are two pairs of vertically opposite angles formed.

One pair is AOD and BOC. The other pair is AOC and BOD.

THEOREM-1 : ***If two lines intersect each other, then the vertically opposite angles are equal.***

Given : Two lines AB and CD intersect at a point O.

Two pairs of vertically opposite angles are :



(i) AOC and BOD

(ii) AOD and BOC

To Prove : **(i)** AOC = BOD

(ii) AOD = BOC

Proof:

	STATEMENT	REASON
1.	Ray OA stands of line CD	Linear pair of angles
2.	AOC + AOD = 180^0	Linear pair of angles
	Ray OD stands on line	From (2) and (3)
3.	AB	
	AOD + BOD = 180^0	
	AOC + AOD = AOD + BOD	
	AOC = BOD	

Similarly, we can prove that AOD = BOC.

Ex.1 Find the measure of the complementary angle of the following angles :-

(i) 22^0 **(ii)** 63^0

Sol. We know that the measure of the complementary angle of x^0 is equal to $(90^0 - x^0)$. Hence,

(i) Measure of the complementary angle of 22^0

$$= 90^0 - 22^0 = 68^0$$

(ii) Measure of the complementary angle of 63^0

$$= 90^0 - 63^0 = 27^0$$

Ex.2 How many degrees are there is an angle which equals two-third of its complement ?

Sol. Let the required angle be x^0.

Then its complementary angle = $90^0 - x^0$

$$x^0 = \frac{2}{3}(90^0 - x^0)X$$

$$3x^0 = 180^0 - 2x^0$$

$$3x^0 + 2x^0 = 180^0$$

$$5x^0 = 180^0$$

$$x^0 = \frac{180^0}{5} = 36^0$$

Hence, there are 36 degree in such an angle.

Ex.3 Find the measure of the supplementary angle of the following angles :

(i) 45^0 **(ii)** 57^0

Sol. We know that the measure of the supplementary angle of x^0 is equal to $(180^0 - x^0)$. Hence,

(i) Measure of the supplementary angle of 45^0

$$= 180^0 - 45^0 = 135^0$$

(ii) Measure of the supplementary angle of 57^0

$$= 180^0 - 57^0 = 123^0$$

Ex.4 Two supplementary angles are in the ratio of 3 : 7. Find the angles.

Sol. Let the two angles in the ratio of 3 : 7 be $3x^0$ and $7x^0$

These angles are supplementary.

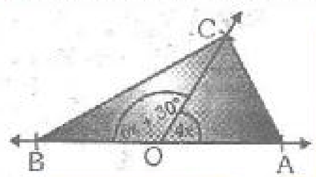
$3x^0 + 7x^0 = 180^0$

$10x^0 = 180^0$

$x^0 = \frac{180^0}{10} = 18^0$

Hence, the angles are $3x^0 = 3 \times 18^0 = 54^0$ and $7x^0 = 7 \times 18^0 = 126^0$.

Ex.5 What value of x would make AOB a line in figure, if AOC = 4x and BOC = $(6x + 30^0)$?



Sol. If AOB is a line, than

AOB = 180^0 [A straight angle = 180^0]

AOC + BOC = 180^0

$4x + (6x + 30^0) = 180^0$

$10x + 30^0 = 180^0$

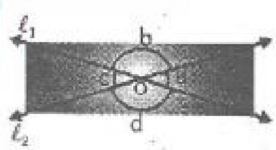
$10x = 180^0 - 30^0$

$10x = 150^0$

$x = \frac{150^0}{10} = 150^0$.

Ex.6 In fig, lines ℓ_1 and ℓ_2 intersect at O, forming angles as shown in the figure. If a = 35^0, find the value of a b, c and d.

Sol. Since lines ℓ_1 and ℓ_2 intersect at O.



a = c [Vertically opposite angles]

c = 35^0 [a = 35^0]

Clearly, a + b = 180^0 [Linear pair of angles]

35^0 + b = 180^0

b = $180^0 - 35^0$

b = 145^0

Since b and d are vertically opposite angles.

d = b d = 145^0 [b = 145^0]

Hence, b = 145^0, c = 35^0, d = 145^0

Ex.7 In fig, two straight lines PQ and RS intersect each other at O. If POT = 75^0, find the value of a, b and c.

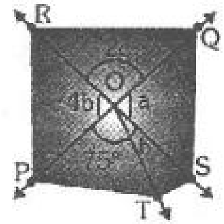
Sol. Since, ROS is a straight line.

ROS + POT + TOS = 180^0

$4b + 75^0 + b = 180^0$

$5b + 75^0 = 180^0$

$5b = 180^0 - 75^0$ $\quad 5b = 105^0$ $\quad b = \frac{105^0}{5}$ $\quad b = 21^0$



Since, PQ and RS intersect at O.

QOS = POR [Vertically opposite angles]

$a = 4b$

$a = 4 \times 21^0 = 84^0$ [$b = 21^0$]

Since, ROS is a straight line.

ROQ + QOS = 180^0 [Linear pair of angles]

$2c + a = 180^0$

$2c + 84^0 = 180^0$

$2c = 180.^0 - 84^0$

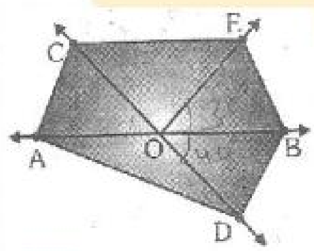
$2c = 96^0$ $\quad c = \frac{96^0}{2}$

$c = 48^0$

Hence, $a = 84^0$, $b = 21^0$ and $c = 48^0$.

Ex.8 In figure, lines AB and CD intersect at 0. If AOC + BOE = 70^0 and BOD = 40^0, find BOE and reflex COE.

(NCERT)



Sol. Lines AB and CD intersect at O.

AOC = BOD [Vertically Opposite Angles]

But, BOD = 40^0 ....(i) [Given]

AOC = 40^0 ....(ii)

Now, AOC + BOE = 70^0 [Given]

40^0 + BOE = 70^0 [Using (ii)]

BOE = $70^0 - 40^0$

BOE = 30^0

Again, Reflex COE = COD + BOD + BOE

= COD + 40^0 + 30^0 [Using (i) and (ii)]

= $180^0 + 40^0 + 30^0$ [Ray OA stands on line CD

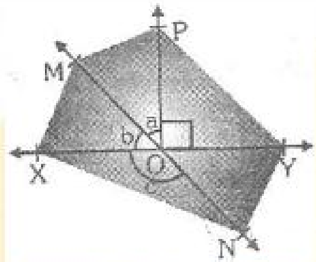
AOC + AOD = 180^0 (Linear pair of angles)

COD = 180^0]

= 250^0

Hence, BOE = 30^0 and reflex COE= 250^0

Ex.9 In figure, lines XY and MN intersect at O. If POY = 90^0 and a : b = 2 : 3, find c. (NCERT)



Sol. We have a : b 2 : 3

So, let a = 2x and b = 3x.

Clearly, ray OP stands on line XY.

XOP + POY = 180^0 [Linear pair of angles]

a + b + 90^0 = 180^0 [POY = 90^0 (given)]

a + b = 180^0 - 90^0

a + b = 90^0

2x + 3x = 90^0 5x = 90^0

x = $\frac{90^0}{5}$ x = 180^0

a = 2x a = 2 × 180^0

b = 3x

b = 3 × 18^0 b = 54^0

Ray OX stands on line MN.

MOX + XON = 180^0 [Linear pair of angles]

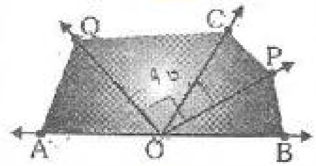
$b + c = 180^0$

$54^0 + c = 180^0$

$c = 180^0 - 54^0$

$c = 126^0$

Ex.10 In figure, OP bisects BOC and OQ, AOC. Prove that POQ = 90^0



Sol. **Given :** In fig, OP bisects BOC and OQ bisects AOC.

To Prove : POQ = 90^0

Proof :

	STATEMENT	REASON
1.	POC = $\frac{1}{2}$ BOC	OP bisects BOC
2.	COQ = $\frac{1}{2}$ AOC	OQ bisects AOC
3.	POQ = POC + COQ	
	= $\frac{1}{2}$ BOC + $\frac{1}{2}$ AOC	
	= $\frac{1}{2}$ (BOC + AOC)	
	= $\frac{1}{2} \times 180^0$	BOC + AOC = 18θ [linear pair of angles]
	POQ = 9θ	

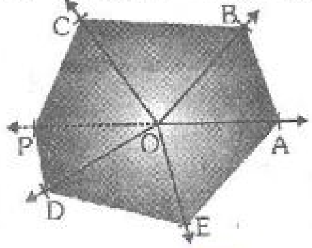
Hence, proved.

Ex.11 Prove that the sum of all angles round a point is equal to 360^0

OR

Rays OA, OB, OC, OD and OE have the common initial point O. Show that

AOB + BOC + COD + DOE = EOA = 360^0



Sol. **Given :** Rays OA, OB, OC, OD and OE have the common initial point O.

To Prove : AOB + BOC + COD + DOE + EOA = 360^0

Construction. Draw a ray OP opposite to ray OA.

Proof :

STATEMENT		REASON
1.	AOB + BOC + COP = 180^0	AOP is a line
2.	POD + DEO + EOA = 180^0	AOP is a line
3.	AOB + BOC + (COP + PD) + DEO + EOA = $180^0 + 180^0 = 360^0$	Adding (1) and (2)
4.	AOB + BOC + COD + DOE + EOA 360^0	COP + POD = COD

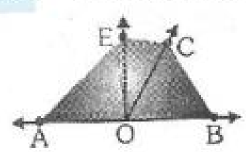
Hence, proved.

Ex.12 Prove that if a ray stands on a line, then the sum of two adjacent angles so formed is 180^0

Sol. **Given :** A ray OC stands on line AB then adjacent angle AOC and BOC are formed.

To Prove : AOC + BOC = 180^0.

Construction : Draw a ray OE $\perp$ AB.

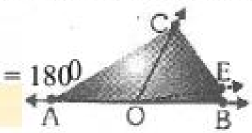


Proof :

STATEMENT		REASON
1.	AOC = AOE + EOC	
2.	BOC = BOE − EOC	
3.	AOC + BOC = AOE + EOC + BOE − EOC	Adding equation (1) and (2),
	AOC + BOC = AOE + BOE	OE ⊥ AB
	AOC + BOC = $90^0 + 90^0$	
	AOC + BOC = 180^0	

Hence, provd.

Ex.13 Prove that if the sum of two adjacent angles is 180^0, then the non-common arms are two opposite rays.



Sol. **Given :** Two adjacent angles are AOC and BOC and AOC + BOC = 180^0

To Prove : OA and OB are two opposite rays.

Construction : Let OA and OB are not two opposite rays.

Then, draw a ray OE opposite to OA such that AOE is a straight line.

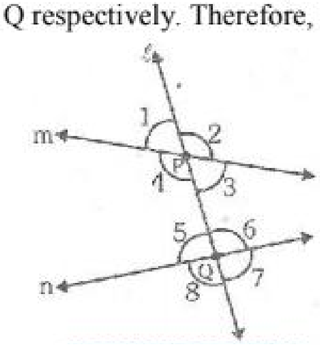
Proof :

STATEMENT		REASON
1.	AOC + BOC = 180^0	Given
2.	AOC + EOC = 180^0	Linear pair of angles
3.	AOC + EOC = AOC + BOC	From equation (1) and (2)
	EOC = BOC	

This is possible only if OE and OB coincide. Hence, OA and OB are two opposite rays.

ANGLES MADE BY A TRNSVERSAL WITH TWO LINES.

A line which intersects two or more lines at distinct points is called a transversal. Line ℓ intersects lines m and n at points P and Q respectively. Therefore, line ℓ is a transversal for lines m and n.



EXTERIOR ANGLES AND INTERIOR ANGLES :- We observe that four angles are formed at each of the points P and Q. Let us name these angles as 1, 2.... 8 as shown in above figure. 1, 2, 7 and 8 are called exterior angles, while 43, 4, 5 and 6 are called interior angles.

These eight angles can be classified into following groups:

(a) **Corresponding Angles :-** Two angles on the same side of transversal are known as corresponding angles, if both lie either above the two lines or below the two lines. The following pairs of angles are the pairs of corresponding angles :

(i) 1 and 5 **(ii)** 2 and 6 **(iii)** 4 and 8 **(i)** 3 and 7

(b) **Alternate interior Angles :** The following pairs of angles are the pairs of alternate interior angles :

(i) 4 and 6 **(ii)** 3 and 5

(c) **Alternate Exterior Angles :** The following pairs of angles are the pairs of alternate exterior angles :

(i) 1 and 7 **(ii)** 2 and 8

(d) **Consecutive Interior Angles or Co-interior Angles :** The pairs of angles on the same side of the transversal are called pairs of consecutive interior angles. The following pairs of angles are the pairs of consecutive interior angles :

(i) 4 and 5 **(ii)** 3 and 6

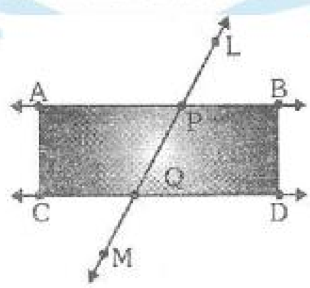
Consider, two parallel lines AB and CD and transversal LM intersecting AB and CD at P and Q respectively. By having a careful look at these three lines, it seems that :

(i) each pair of corresponding angles are equal

(ii) each pair of alternate interior angles are equal, and

(iii) each pair of consecutive interior angles are supplementary.

The converse of each of the above statements is also true.



Now for proving the above results, we assume that one of them always hold good i.e. it is an axiom. So, we take the following as an axiom.

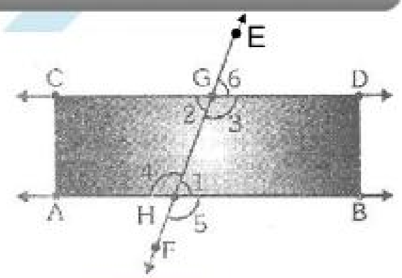
CORRESONDING ANGLES AXIOM : ***If a transversal intersects two parallel lines, then each pair of corresponding angles are equal.***

Conversely, if a transversal intersects two lines, making a pair of equal corresponding angles, then the lines are parallel.

But using the above axiom, we can now deduce the other facts about parallel lines and their transversal.

THEOREM -2 :- ***If a transversal interests two parallel lines, then each pair of alternate interior angles is equal.***

Given : AB and CD are two parallel lines and a transversal EF intersects them at point G and H respectively. Thus, the alternate interior angles are 2 and 1, and 3 and 4.



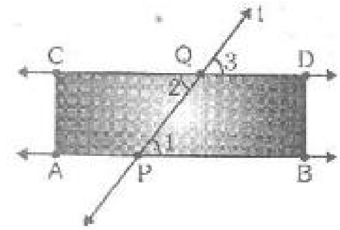
To Prove : 1 = 2 and 3 = 4

Proof :

	STATEMENT	REASON
1.	2 = 6	Vertically opposite angles
2.	1 = 6	Corresponding angels
3.	1 = 2	From equations (1) and (2)
4.	Similarly 4 = 5	Vertically opposite angles
5.	3 = 5	Corresponding angles
6.	3 = 4	From equation (3) and (4)

Hence, proved.

THEOREM -3 (converse of theorem of 2) :- ***If a transversal intersects two lines in such a way that a pair of alternate interior angles is equal, then the two lines are parallel.***



Given : A transversal t intersects two lines AB and CD at P and Q respectively such that 1 and 2 are a pair of alternate interior angles and 1 = 2.

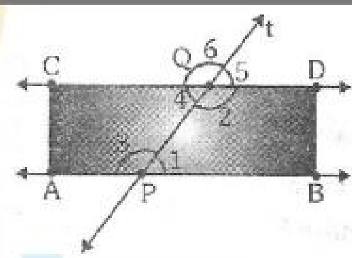
To Prove : AB || CD

Proof :

	STATEMENT	REASON
1.	2 = 3	Vertically opposite angles
2.	1 = 2	Given
3.	2 = 3	From (1) and (2)
4.	AB || CD.	By converse of corresponding angles axiom

Hence, proved.

THEOREM-4 :- ***If a transversal intersects two parallel lines, then each pair of consecutive interior angles is supplementary.***



Given :- AB and CD are two parallel lines. Transversal "t" intersects AB at P and CD at Q. making two pairs of consecutive interior angles, 1, 2 and 3, 4.

To Prove : 1 + 2 = 180 and 3 + 4 = 180

Proof :

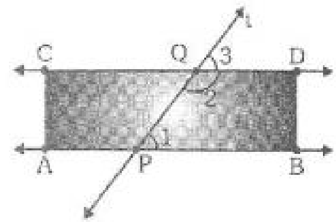
	STATEMENT	REASON
1.	AB || CD	
	1 = 5	Corresponding s axiom
2.	5 + 2 = 180	Linear pair of angles
3.	1 + 2 = 180	From (1) and (2)
4.	AB || CD	
	3 = 6	Corresponding s axiom
5.	6 + 4 = 180	Linear pair of angles
6.	3 + 4 = 180	From (4) and (5)

Hence, proved

THEOREM - 5 (converse of theorem 4) :- ***If a transversal intersects two lines in such a way that a pair of consecutive interior angles is supplementary, then the two lines are parallel.***

Given : A transversal intersect two lines AB and CD at P and Q respectively such that 1 and 2 are a pair of consecutive interior angles, and 1 + 2 = 180^0

To Prove : AB || CD

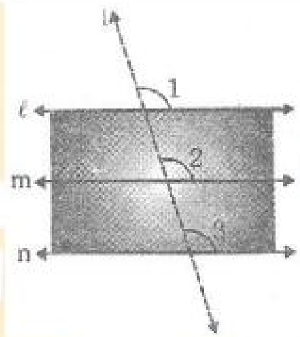


Proof :

	STATEMENT	REASON
1.	1 + 2 = 18θ	Given
2.	2 + 3 = 18θ	Linear pair of angles
3.	1 + 2 = 2 + 3	From (1) and (2)
	1 = 3	
4.	AB ‖ CD	By converse of corresponding angles axiom

Hence, proved.

THEOREM-6 :- ***If two lines are parallel to the same line, they will be parallel to each other.***



Given : Line m ‖ line ℓ and line n ‖ line ℓ.

To Prove : Line m ‖ line n.

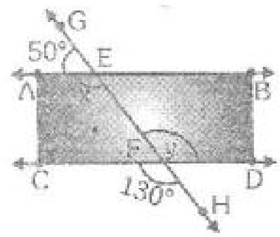
Construction : Draw a line t transversal for the lines ℓ, m and n.

Proof :

	STATEMENT	REASON
1.	m ‖ ℓ	
	1 = 2	Corresponding angles
2.	n ‖ ℓ	
	1 = 3	Corresponding angles
3.	2 = 3	From (1) and (2)
4.	m ‖ n	By converse of corresponding angles axiom.

Hence, proved.

Ex.14 In figure, find the values of x and y and then show that AB || CD. **(NCERT)**



Sol. Ray AE stands on lines GH

AEG + AEH = 180^0 (Linear pair of angles)

$50^0 + x = 180^0$

$x = 180^0 - 50^0 = 130^0$ ...(i)

$y = 130^0$ ...(ii) (Vertically opposite angles)

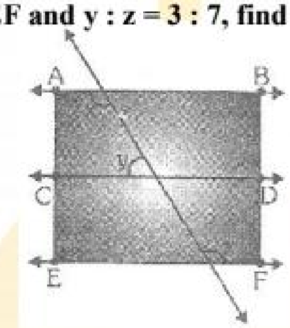
From (i) and (ii), we concluded that

$x = y$

But these are alternate interior angles and they are equal.

So, we can say that AB || CD.

Ex.15 In figure, if AB || CD, CD || EF and y : z = 3 : 7, find x. (NCERT)

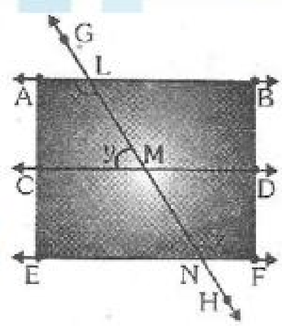


Sol. AB||CD and CD||EF

AB||EF (Lines parallel to the same line are parallel to each other)

$x = z$ ...(i) (Alternate Interior Angles)

$x + y = 180^0$ ...(ii) (Sum of the consecutive interior angles on the same side of the transversal GH is supplementary.)



From (i) and (ii),

$z + y = 180^0$

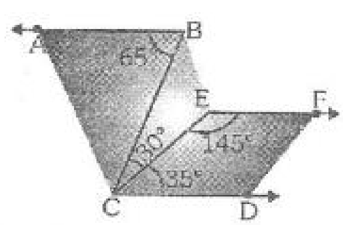
But, $y : z = 3 : 7$

Sum of the ratios = 3 + 7 = 10

$y = \frac{3}{10} \times 180^0 = 54^0$ and $z = \frac{7}{10} \times 180^0 = 126^0$

$x = z = 126^0$

Ex.16 In figure, ABC = 65^0, BCE = 30^0, DCE = 35^0 and CEF = 145^0. Prove that AB||EF.



Sol. ABC = 65^0

BCD= BCE + ECD = $30^0 + 35^0 = 65^0$

ABC = BCD (Alternate Angles)

AB||CD ...(i)

FEC + ECD = $145^0 + 35^0 = 180^0$

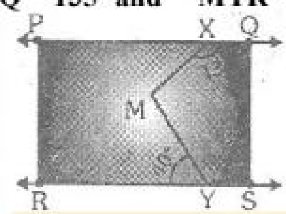
But these angles are consecutive interior angles formed on the same side of the transversal.

CD||EF ...(ii)

From (i) and (ii), AB||EF

Hence, proved.

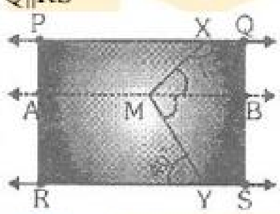
Ex.17 In figure, if PQ||RS, MXQ = 135^0 and MYR = 40^0, find XMY.



Sol. Through point M draw a line AB parallel to the line PQ.

AB||PQ and PQ||RS

AB||RS



Now, AB||PQ and QXM and XMB are interior angles on the same side of the transversal XM.

QXM + XMB = 180^0 (Sum of the interior angles on the same side of transversal XM is supplementary)

135^0 + XMB = 180^0

XMB = $180^0 - 135^0 = 45^0$

Now, AB||RS and BMY and MYR are alternate angles.

BMY = MYR

BMY = 40^0

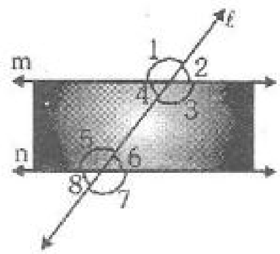
Hence, XMY = XMB + BMY = $45^0 + 40^0 = 85^0$

XMY = 85^0

Ex.18 In figure, if m||n and angle 1 and 2 are in the ratio 3 : 2, determine all the angles from 1 to 8.

Sol. Given 1 : 2 = 3 : 2.

Let $1 = 3x^0$ and $2 = 2x^0$



$1 + 2 = 180^0$ (Linear pair of angles)

$3x^0 + 2x^0 = 180^0$

$5x^0 = 180^0$

$x^0 = \frac{180^0}{5} = 36^0$

$1 = 3x^0 = (3 \times 36)^0 = 108^0$

and, $2 = 2x^0 = (2 \times 36)^0 = 72^0$

1 = 3, 2 = 4 [Vertically Opposite Angles]

$3 = 108^0$, $4 = 72^0$

6 = 2, 3 = 7 [Corresponding angles]

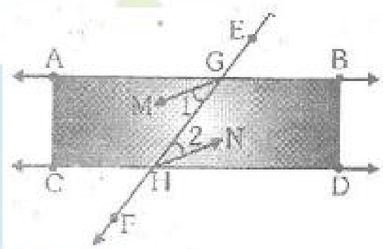
$6 = 72^0$, $7 = 108^0$

5 = 7, 8 = 6 [Vertically Opposite Angles]

$5 = 108^0$, $8 = 72^0$

Hence, $1 = 108^0$, $2 = 72^0$, $3 = 108^0$, $4 = 72^0$, $5 = 108^0$ $6 = 72^0$ $7 = 108^0$ and $8 = 72^0$.

Ex.19 Prove that if two parallel lines are intersected by a transversal, the bisector of any pair of alternate interior angles is parallel.



Sol. **Given :** AB and CD are two parallel lines and transversal EF intersects them at G and H respectively.

GM and HN are the bisectors of alternate angles AGH and GHD, respectively.

To Prove : GM||HN

Proof :

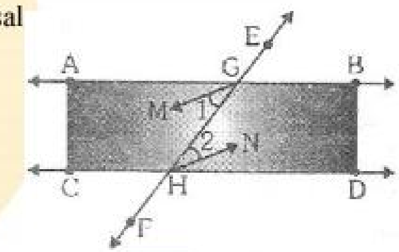
STATEMENT		REASON
1.	AB||CD	
	AGH = GHD	Alternate interior angles
	$\frac{1}{2}$ AGH = $\frac{1}{2}$ GHD	
	1 = 2	
2.	GM||HN	1 & 2 are alternate interior angels formed by transversal GH with GM and HN and are equal.

Hence, GM|HN

Ex.20 If the bisectors of a pair of alternate interior angle are parallel, prove that given lines are parallel.

Sol. **Given :** AB and CD are two straight lines cut by a transversal EF at G and H respectively.

GM and HN are the bisectors of alternate interior angles AGH and GHD respectively, such that GM||HN.



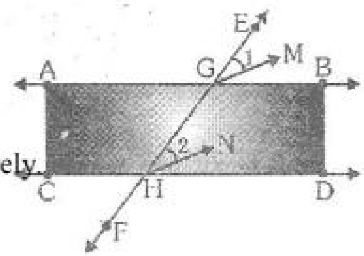
To Prove : AB||CD

Proof :

STATEMENT		REASON
1.	GM||HN	
	1 = 2	Alternate interior angles
	2 1 = 2 2	
	AGH = GHD	
2.	AB||CD	AGH & GHD are alternate interior angles formed by transversal EF with AB and CD and are equal.

Hence, proved.

Ex.21 Prove that if two parallel lines are intersected by a transversal, then bisectors of any two corresponding angles are parallel.



Sol. **Given :** AB and CD are two parallel lines and transversal EF intersects them at G and H respectively. GM and HN are the bisectors of two corresponding angles EGB and GHD respectively.

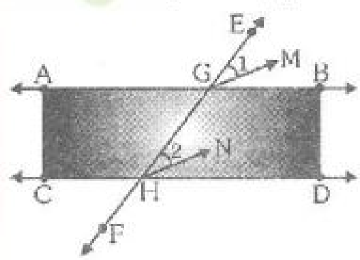
To Prove : GM||HN

Proof :

	STATEMENT	REASON
1.	AB||CD EGB = GHD $\frac{1}{2}$ EGB = $\frac{1}{2}$ GHD 1 = 2 GM||HN	Corresponding angles
2.		1 & 2 are corresponding angles formed by transversal GH with GN and HN and are equal.

Hence, proved.

Ex.22 If the bisectors of a pair of corresponding angles formed by transversal are parallel, prove that given lines are parallel.



Sol. **Given :** AB and CD are two straight lines cut by a transversal EF at G and H respectively.
GM and HN are the bisectors of corresponding angles EGB and GHD respectively such that GM||HN.

To Prove : AB || CD

Proof :

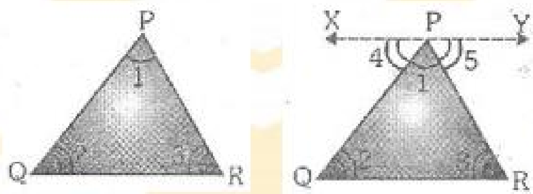
STATEMENT		REASON
1.	GM||HN	
	1 = 2	Corresponding angles
	2 1 = 2 2	
	EGB = GHD	
2.	AB||CD	EGB & GHD are corresponding angles formed by transversal EF with AB and CD and are equal.

Hence, proved

ANGLE SUM PROPERTY OF A TRIANGLE

In previous classes, we have learnt that the sum of the three angles is 180^0. In this section, we shall prove this fact a theorem.

THEOREM-7 : - *The sum of all the angles of a triangle is 180^0*



Given : In a triangle PQR,

1, 2 and 3 are the angles of PQR.

To Prove : 1 + 2 + 3 = 180^0

Construction :

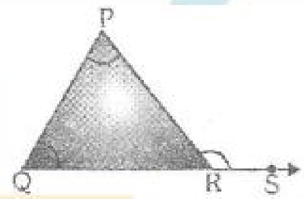
Draw a line XPY parallel to QR through the opposite vertex P.

Proof :

	STATEMENT	REASON
1.	4 + 1+ 5 = 180^0	XPY is a line
2.	XPY||QR	
	4 = 2 and 5 = 3	Alternate interior angles
3.	2 + 1 + 3 = 180^0	From (1) and (2)
	or 1 + 2 + 3 = 180^0	

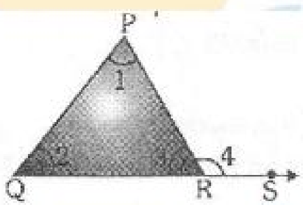
Hence, proved.

EXTERIOR ANGLE OF A TRIANGLE



Consider the PQR. If the side QR is produced to point S, then PRS is called an exterior angle of PQR. The PQR and QPR are called two interior opposite angles of the exterior PRS.

THEOREM - 8 :- ***If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.***



Given : In a triangle PQR, 1, 2 and 3 are the angles of PQR, side of QR is produced to S and exterior angle PRS = 4.

To Prove : 4 = 1 + 2

Proof :

	STATEMENT	REASON
1.	3 + 4 = 180^0	Linear pair of angles
2.	1 + 2 + 3 = 180^0	The sum of all the angles of a triangles is 180^0
3.	3 + 4 = 1 + 2 + 3	From (1) and (2)
	4 = 1 + 2.	

Hence, proved.

Ex.23 In a ABC, B = 105^0, c = 50^0. Find A.

Sol. We have,

$A + B + C = 180^0$

$A + 105^0 + 50^0 = 180^0$

$A = 180^0 - 155^0 = 25^0$

Ex.24 If the angles of a triangle are in the ratio 2 : 3 : 4, determine three angles.

Sol. Let the angles of the triangle be $2x^0$, $3x^0$ and $4x^0$. Then,

$2x^0 + 3x^0 + 4x^0 = 180^0$

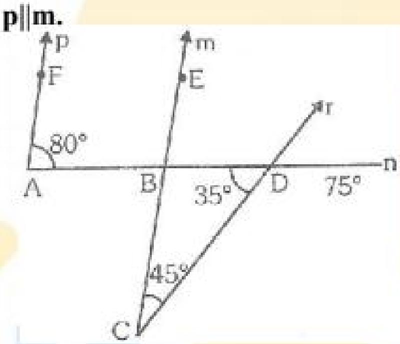
$9x^0 = 180^0$

$x^0 = \frac{180^0}{9}$

$x^2 = 20^0$

Hence, the angles of the triangle are 40^0, 60^0 and 80^0

Ex.25 In the fig, prove that p||m.



Sol. In BCD,

$B + C + D = 180^0$ [The sum of the angles of a triangle is 180^0]

$B + 45^0 + 35^0 = 180^0$

$B + 80^0 = 180^0$

$B = 180^0 - 80^0 = 100^0$ ...(i)

$EBD + CBD = 180^0$ [Linear pair of angles]

$EBD + 100^0 = 180^0$ [From (i)]

$EBD = 180^0 - 100^0 = 80^0$

EBD = FAB [Corresponding angles]

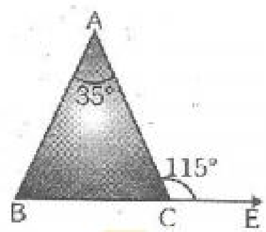
But these angles from a pair of equal corresponding angles for lines p and m and transversal n.

Hence, p||m.

Ex.26 An exterior angle of a triangle is 115^0 and one of the opposite angles is 35^0. Find the other two angles.

Sol. Let in ABC, exterior ACE = 115^0 and A = 35^0.

We know that, ACE = A + B [Exterior angle is equal to sum of interior opposite angles]



$115^0 = 35^0 +$ B

B = $115^0 - 35^0 = 80^0$

In ABC,

A + B + C = 180^0 [The sum of the three angles of a triangle is 180^0]

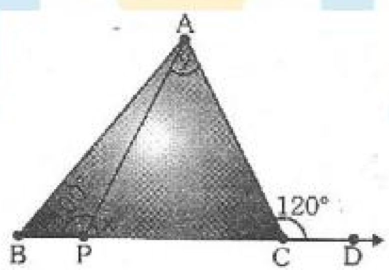
$35^0 + 80^0 +$ C = 180^0

$115^0 +$ C = 180^0

C = $180^0 - 115^0 = 65^0$

Hence, the other two angles are 80^0 and 65^0.

Ex.27 In figure, ACD = 120^0 and APB = 100^0, find x and y.



Sol. $x + 100^0 = 180^0$ [Linear pair of angles]

$x = 80^0$

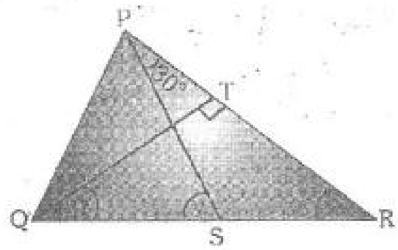
Now, we have

$x + y = 120^0$ [Sum of interior opposite angles is equal to the exterior angle]

$80^0 + y = 120^0 \quad y = 40^0$

Hence, $x = 80^0$, $y = 40^0$.

Ex.28 In figure, if OT ⊥ PR, TQR = 40^0 and SPR = 30^0 find x and y.



Sol. In TQR, we have TQR + QTR + QRT = 180^0

$40^0 + 90^0 + x = 180^0 \quad 130^0 + x = 180^0$

$x = 50^0$

Now, we have

$y = x + 30^0$ [Sum of interior opposite angles is equal to the exterior angle]

$= 50^0 + 30^0$

$y = 80^0$

Hence, $x = 50^0$, $y = 80^0$.

THINGS TO REMEMBER :

1. **If a ray stands on a line, then the sum of the adjacent angles so formed is 180^0**
2. **If the sum of two adjacent angles is 180^0, then their non common arms are two opposite rays.**
3. **The sum of all the angles round a point is equal to 360^0**
4. **If two lines intersect, then the vertically opposite angles are equal.**
5. **If a transversal intersects two parallel lines then the corresponding angles are equal, each pair of alternate interior angles is equal and each pair of consecutive interior angles is supplementary.**
6. **If a transversal intersects two lines in such a way that a pair of alternate interior angles is equal, then the two lines are parallel.**
7. **If a transversal intersects two lines in such a way that a pair of consecutive interior angels is supplementary, then the two lines are parallel.**
8. **If two parallel lines are intersected by a transversal, the bisectors of any pair of alternate interior angles are parallel and the bisectors of any two corresponding angles are also parallel.**

9. **If a line is perpendicular to one of two given parallel lines, then it is also perpendicular to the other line.**
10. **If two lines are parallel to the same line, they will be parallel to each other.**
11. **The sum of the angles of a triangle is 180^0**
12. **If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.**
13. **Two angles which have their arms parallel are either equal or supplementary.**
14. **Two angles whose arms are perpendicular are either equal or supplementary.**

CBSE BASED SOME IMPORTANT QUESTIONS.

Q.1 Find the angle which is complement of itself.

[Hint : Let the required angle be x^0. Its complementary angle = x^0.

$x^0 + x^0 = 90^0$] **[Ans. 45^0]**

Q.2 An angle is equal to five times its complement. Determine its measure.

[Hint : $x^0 = 5(90 - x)^0$] **[Ans. 75^0]**

Q.3 An angle is 20^0 less than its complement. Find its measure.

[Hint : $x^0 = (90 - x)^0 - 20^0$] **[Ans. 35^0]**

Q.4 Find the angle which is supplementary of itself.

[Hint : $x^0 + x^0 = 180^0$] **[Ans. 90^0]**

Q.5 Two supplementary angles differ by 34^0. Find the angles.

Sol. Let one angle be x^0. Then, the other angle is $(x + 34)^0$.

$x^0 + (x + 34)^0 = 180^0$

$2x^0 + 34^0 = 180^0$

$2x^0 = 180^0 - 34^0 \quad 2x^0 = 146^0 \quad x = 73^0$.

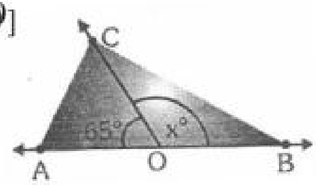
Thus, two angles are of measure 73^0 and $73^0 + 34^0 = 107^0$.

Q.6 An angle is equal to one-third of its supplement. Find its measure.

[Hint : $x^0 = \frac{1}{3}(180 - x)^0$] **[Ans. 45^0]**

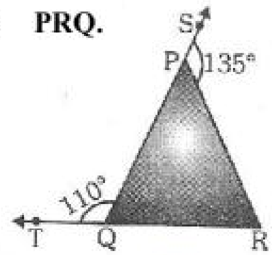
Q.7 In the adjoining figure, AOB is a straight line. Find the value of x.

[Hint : $AOC + BOC = 180^0$] **[Ans. 115^0]**



Q.8 In fig, sides QP and RQ of PQR are produced to points S and T respectively. If SPR = 135^0 and PQT = 110^0, find PRQ.

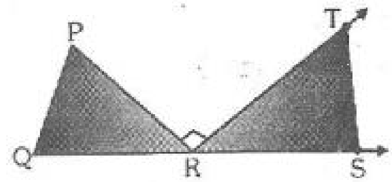
(NCERT)



[Hint : PQT + PQR = 180^0 [Linear pair of angles]

PQR + PRQ = 135^0] **[Ans. PRQ = 65^0]**

Q.9 In fig, side QR of $\triangle$PQR has been produced to S. If $\angle$P : $\angle$Q : $\angle$R = 3 : 2 : 1 and RT $\perp$ PR, find $\angle$TRS.



Sol. In $\triangle$PQR,

$\angle P + \angle Q + \angle R = 180^0$ [$\because$ The sum of the angles of a triangle is 180^0]

$\angle P : \angle Q : \angle R = 3 : 2 : 1$ [Given]

Sum of the ratios = 3 + 2 + 1 = 6

$\angle R = \frac{1}{6} \times 180^0 = 30^0$

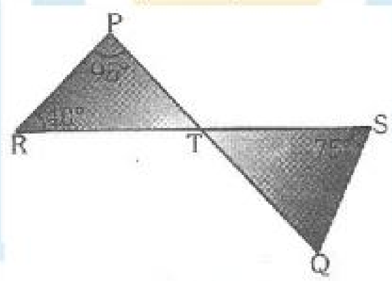
Now, $\angle PRQ + \angle PRT + \angle TRS = 180^0$ [Linear pair of angles]

$\Rightarrow 30^0 + 90^0 + \angle TRS = 180^0$

$\Rightarrow 120^0 + \angle TRS = 180^0$

$\Rightarrow \angle TRS = 180^0 - 120^0 = 60^0$

Q.10 In fig, if line PQ and RS intersect at point T, such that $\angle$PRT = 40^0, $\angle$RPT = 95^0 and $\angle$TSQ = 75^0, find $\angle$SQT.



Sol. In $\triangle$PRT,

$\angle PTR + \angle PRT + \angle RPT = 180^0$ [$\because$ The sum of the angles of a triangle is 180^0]

$\Rightarrow \angle PTR + 40^0 + 95^0 = 180^0$

$\Rightarrow \angle PTR + 135^0 = 180^0$

$\Rightarrow \angle PTR = 45^0$

$QTS = \quad PTR = 45^0$ [Vertically Opposite angles]

In TSQ,

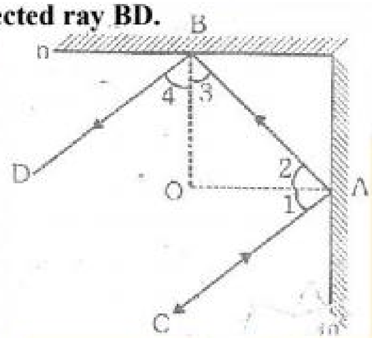
$QTS + \quad TSQ + \quad SQT = 180^0$ [The sum of the angles of a triangle is 180^0]

$45^0 + 75^0 + \quad SQT = 180^0$

$120^0 + \quad SQT = 180^0$

$SQT = 180^0 - 120^0 = 60^0$

Q.11 In fig, m and n are two plane mirrors perpendicular to each other. Prove that the incident ray CA is parallel to reflected ray BD.



Sol. **Given :** Two plane mirrors m and n, perpendicular to each other. CA is incident ray and BD is reflected ray.

To Prove : CA || DB

Construction : OQ and OB are perpendiculars to m and n respectively.

Proof :

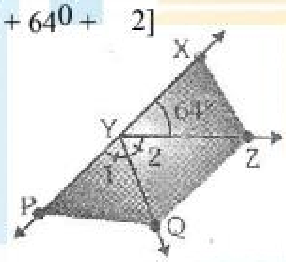
	STATEMENT	REASON
1.	m ⊥ n, OA ⊥ m and OB ⊥ n AOB = 90^0	Lines perpendicular to two perpendicular lines are also perpendicular.
2.	In AOB, AOB + OAB + OBA = 180^0 90^0 + 2 + 3 = 180^0 2 + 3 = 90^0 2(2 + 3) = 180^0 2(2) + 2(3) = 180^0 CAB + ABD = 180^0	The sum of the angles of a triangle is 180^0 Multiplying both sides by 2. Angle of incidence = Angle of reflection 1 = 2 and 3 = 4 2 2 = CAB and 2 3 = ABD
3.	CA || BD	CAB & ABD form a pair of consecutive interior angles and are supplementary.

Q.12 **It is given that XYZ = 64^0 and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ZYP, find XYQ and reflex QYP.**

[NCERT]

[Hint : 1 + 2 + 64^0 = 180^0 2 + 2 + 64^0 = 180^0

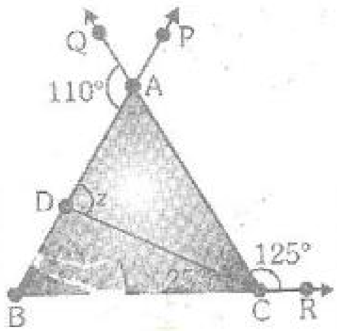
and reflex QYP = 180^0 + 64^0 + 2]



[Ans. 122^0, 302^0]

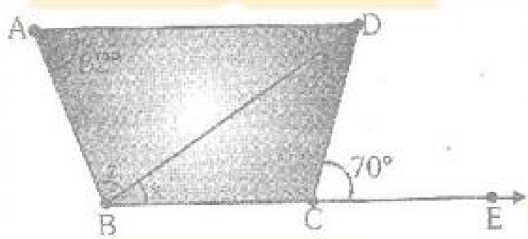
Q.13 In fig, sides BA, CA and BC are produced to points P, Q and R respectively.

Line CD meets BA at D such that BCD = 25^0. If BAQ = 110^0 and ACR = 125^0, then find x, y and z.



[Hint : $y + 25^0 + 125^0 = 180^0$; $z + y = 110^0$, $x + 25^0 = z$] **[Ans. $x = 55^0$, $y = 30^0$, $z = 80^0$]**

Q.14 In fig. if AD||BC, BAD = 62^0, BDC = 32^0 and BCE = 70^0, then find x, y and z.

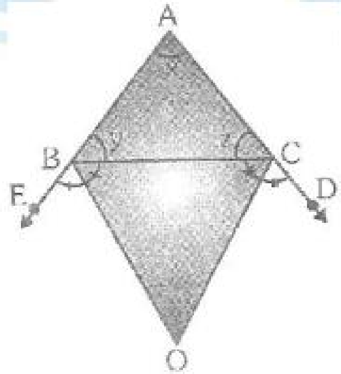


[Hint : $x + 32^0 = 70^0$; x = y (Alternate interior angles); $y + x + 62^0 = 180^0$]

[Ans. $x = 38^0$, $y = 38^0$, $z = 80^0$]

Q.15 In fig, the sides AB and AC of ABC are produced to point E and D respectively. If bisectors BO and CO of C BE and BCD respectively meet at point O, then prove tha BOC = 90^0 - $\frac{1}{2}$ BAC.

[NCERT]



Sol. **Given :** In fig. AB and AC of ABC are produced to points E and D respectively. BO and CO are bisectors of CBE and BCD respectively.

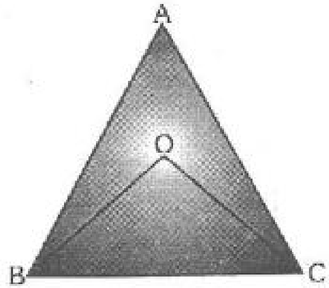
To Prove : $BOC = 90^0 - \frac{1}{2}$ BAC.

Proof :

	STATEMENT	REASON
1.	$CBO = \frac{1}{2}$ CBE $= \frac{1}{2}(180^0 - y)$ $= 90^0 - \frac{y}{2}$	OB bisects CBE CBE + y = 180^0, Linear pair of angles
2.	$BCO = \frac{1}{2}$ BCD $= \frac{1}{2}(180^0 - z)$ $= 90^0 - \frac{z}{2}$	Lines are also perpendicular. OC bisects BCD BCD + z = 180^0, Linear pair of angles
3.	In BOC, BOC + CBO + BCO = 180^0 $BOC + 90^0 - \frac{y}{2} + 90^0 - \frac{z}{2} = 180^0$ $BOC = \frac{z}{2} + \frac{y}{2}$ $BOC = \frac{1}{2}(y + z)$	Angles sum property of a triangle Using (1) and (2)
4.	$x + y + z = 180^0$ $y + z = 180^0 - x$	
5.	$BOC = \frac{1}{2}(180^0 - z)$ $BOC = 90^0 - \frac{x}{2}$ or, $BOC = 90^0 - \frac{1}{2}$ BAC	Angle sum property to a triangle From (3) and (4)

Hence, proved.

Q.16 If the bisectors of angles ABC and ACB of a triangle ABC meet at a point O, then prove that BOC = $90^0 + \frac{1}{2}$ A.



Sol. **Given :** A ABC such that the bisectors of ABC and ACB meet at a point O respectively.

To Prove : BOC = $90^0 + \frac{1}{2}$ A.

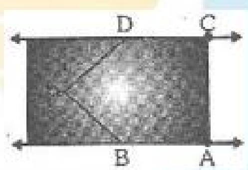
Proof :

	STATEMENT	REASON
1.	In ABC A + B + C = 180^0	Angle sum property of a triangle
2.	In OBC, 1 + 2 + BOC = 180^0 or 2 1 + 2 2 + 2 BOC = 360^0 or B + C + 2 BOC = 360^0 (180^0 - A + 2 BOC = 360^0 2 BOC = 360^0 - 180^0 + A BOC = $90^0 + \frac{1}{2}$ A.	Angle sum property of a triangle Multiplying both sides by 2 OB and OC bisect B and C respectively. Using (1), B + C = 180^0 - A

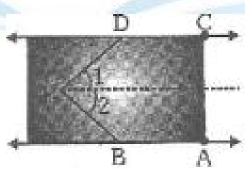
H

Q.17 I

ABP + BPD + CDP = 360^0



[Hint : Through P draw, PM||BA

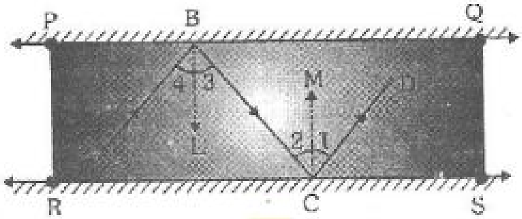


1 + CDP = 180^0 ...(i)

2 + ABP = 180^0 ...(ii)

Add (i) and (ii)]

Q.18 If figure PQ, and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



Sol. **Given :** In figure PQ, and RS are two mirrors placed parallel to each other. AB is incident ray and CD in reflected ray.

To Prove : AB || CD.

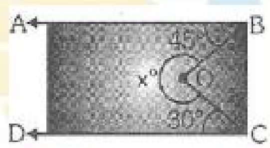
Construction : Draw perpendiculars at A and B to the plane mirrors.

Proof :

	STATEMENT	REASON
1.	BL ⊥ PQ, CM ⊥ RS and PQ || RS	Alternate interior angles.
2.	BL || CM 2 = 3 2 2 = 2 3 ABC = BCD	By law of reflection, Angle of incidence = Angle of reflection, ∴ 1 = 2 and 3 = 4. ∴
3.	AB||CD	ABC && BCD form a pair of alternate interior angles and are equal.

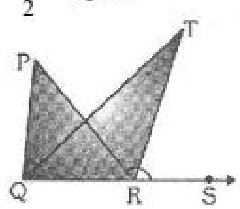
Hence, proved.

Q.19. Find the value of x, if in fig, AB||CD.



[Hint: Through O draw, OE||AB||CD.] **[Ans. 285^0]**

Q.20 In a fig, the side QR of PQR is produced to a point S. If the bisectors of PQR and PRS meet at point T, then prove that QTR = $\frac{1}{2}$ QPR.



Sol. **Given :** In a fig, the side QR of PQR is produced to a point S. The bisectors of PQR and PRS meet at point T.

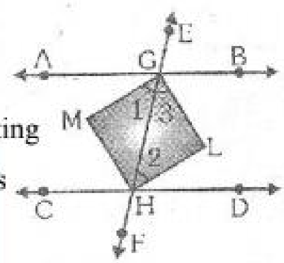
To Prove : QTR $= \frac{1}{2}$ QPR.

Proof :

	STATEMENT	REASON
1.	PRS = PQR + QPR	Sum, of interior opposite angles is equal to the exterior angle.
2.	TRS = TQR + QTR 2 TRS = 2 TQR + 2 QTR	Sum of interior opposite angles is equal to the exterior angle.
3.	PRS = PQR + 2 QTR PQR + QPR = PQR + 2 QTR QPR = 2 QTR or, QTR $= \frac{1}{2}$ QPR	OT bisects PQR and RT bisects PRS From (1) and (2)

Hence, proved.

Q.21 Prove that if two parallel lines are intersected by a transversal, then prove that the bisectors of the interior angles from a rectangle.



Sol. **Given :** Two parallel lines AB and CD and a transversal EF intersecting them at G and H respectively. GM, HM, GL and HL are the bisectors of the two pairs of interior angles.

To Prove : GMHL is a rectangle.

Proof :

	STATEMENT	REASON
1.	AB‖CD AGH = DHG $\frac{1}{2}$ AGH = $\frac{1}{2}$ DHG 1 = 2 GM‖HL	 Alternate interior angles GM & HL are bisectors of AGH and DHG respectively.
2.	Similarly, GM‖HL So, GMHL is a parallelogram	1 and 2 from a pair of alternate interior angles and are equal.
3.	AB‖CD BCH + DHG = 180^0 $\frac{1}{2}$ BGH + $\frac{1}{2}$ DHG = 90^0 3 + 2 = 90^0	Sum of interior angles on the same side of the transversal = 180^0 GL & HL are bisectors of BGH and DHG respectively.
4.	In GLH, 2 + 3 + 1 = 180^0 90^0 + 1 = 180^0 L = 180^0 - 90^0 L = 90^0	Angle sum property of a triangle. Using (3)

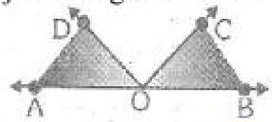
Thus, in parallelogram GMHL, L = 90^0. Hence, GMHL is a rectangle.

EXERCISE

SUBJECTIVE TYPE QUESTION

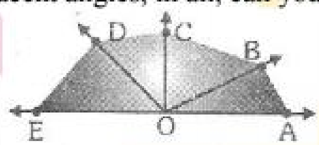
(A) ***SHORT ANSWER TYPE QUESTIONS :***

1. In fig. write all pairs of adjacent angles and all the linear pairs.



2. How many pairs of adjacent angles are formed when two lines intersect in a point ?

3. How many pairs of adjacent angles, in all, can you name in fig.



4. Find the measure of the complementary angle of each of the following angles :

(i) 20^0 (ii) 77^0 (iii) 90^0

5. Find the measure of the supplementary angle of each of the following angles :

(i) 132^0 (ii) 54^0(iii) 138^0

6. If an angle is 28^0 les than its complement, find its measure.

7. If an angle is 30^0 more than one half of its complement find the measure of the angle.

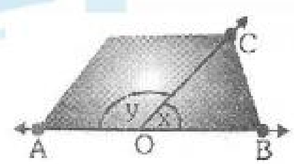
8. Two supplementary angles are in the ratio 4 : 5. Find the angles.

9. Two supplementary angle differ by 48^0. Find the angles.

10. If the angle $(2x - 10)^0$ and $(x - 5)^0$ are complementary angles, find x .

11. If the complement of an angle is equal to the supplement of the thrice of it, find the measure of the angle.

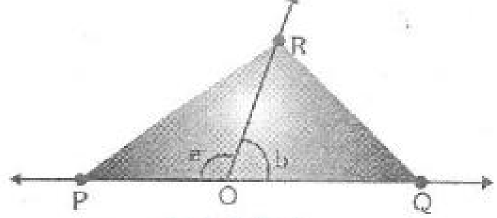
12. OA and OB are opposite rays.



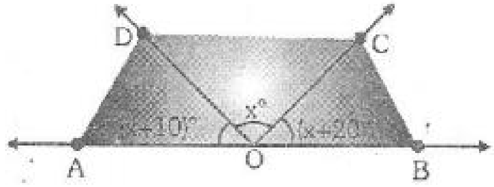
(i) If $x = 75^0$, what is the value of y ?

(ii) If $y = 110^0$, what is the value of x ?

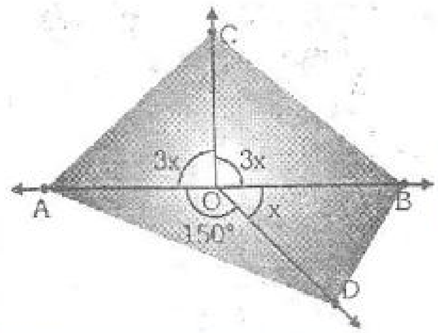
13. POR and QOR form a linear pair. If a - b = 80, find the values of a and b.



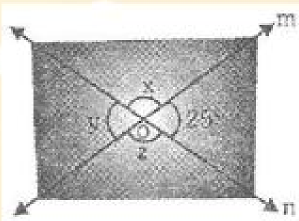
14. Find x, further find BOC, COD and AOD



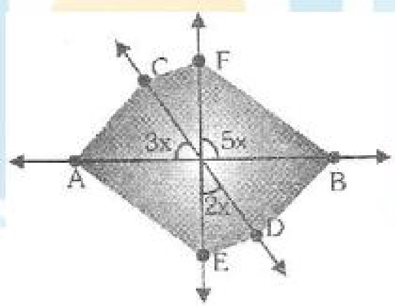
15. Determine the value of x.



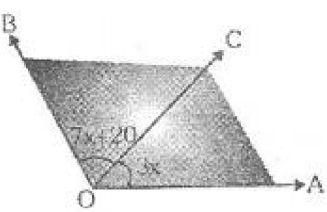
16. In fig. find the values of x, y and z.



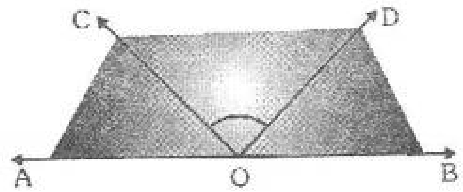
17. In fig. find the value of x.



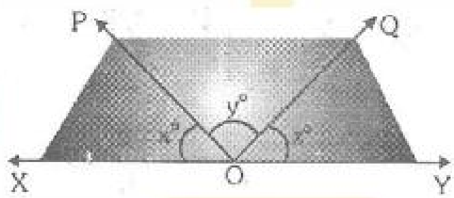
18. In figure, if BOC = $7x + 20^0$ and COA = 3x, then find the value of a x for which AOB becomes a straight line.



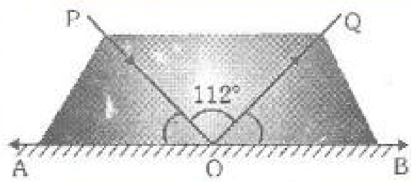
19. In figure, find COD when AOC + BOD = 100^0.



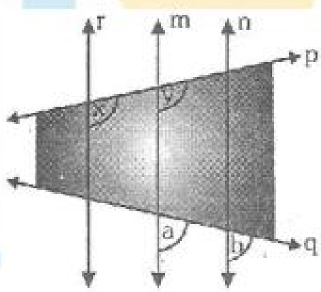
20. In figure, x : y : z = = 5 : 4 : 6. If XOY is a straight line find the values of x, y and z.



21. In the given figure, AB is a mirror, PO is the incident ray and OR, the reflected ray. If POR = 112^0 find POA.



22. In fig. if x = y and x = b, prove that r|| n.



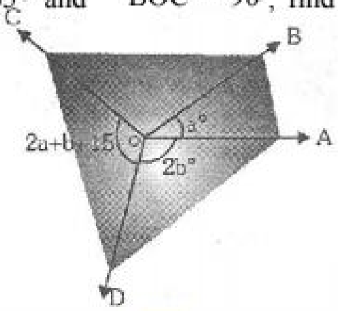
23. If p, m, n are three lines such that p||m and n ⊥ p, prove that n ⊥ m.

24. A transversal intersects two given lines in such a way that the interior angles on the same side of the transversal are equal. Is it always true that the given lines are parallel ? If not, state the condition(s) under which the two lines will be parallel.

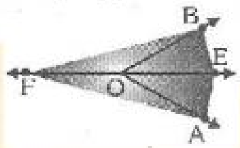
25. If the angles of a triangle are in the ratio 2 : 3 : 4, find the three angles.

(B) LONG ANSWER TYPE QUESTIONS :

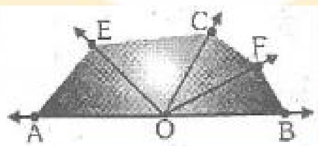
1. In the given figure, $2b - a = 65^0$ and BOC = 90^0, find the measure of AOB, AOD and COD.



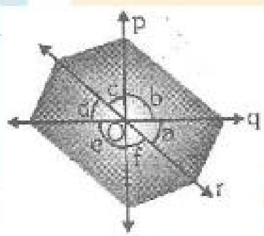
2. In fig. ray OE bisect AOB and OF is a ray opposite to OE. Show that FOB = FOA.



3. In fig. ray OE bisects AOC and OF bisects COB and OE⊥ OF. Show that A, O, B are collinear.



4. In fig, three lines p, q and r are concurrent at O. If $a = 50^0$ and $b = 90^0$, find c, d, e and f.



5. AB, CD and EF are three concurrent lines passing through the point O such that OF bisects BOD. If BOF = 35^0 find BOC and AOD.

6. Prove that the bisectors of a pair of vertically opposite angles are in the same straight line.

OR

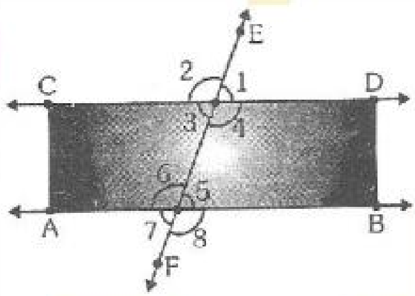
AB and CD are two intersecting lines. OP and OQ are respectively bisectors of BOD and AOC. Show that OP and OQ are opposite rays.

7. One of the four angles formed by two intersecting lines is a right angle. Show that the other three angles will also be right angles.

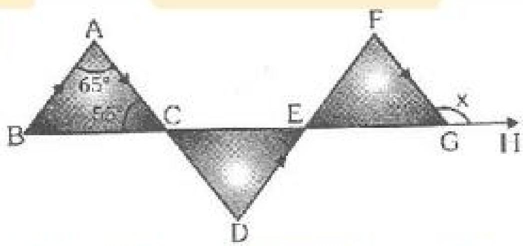
8. In fig. given that AB||CD.

(i) If $1 = (120 - x)^0$ and $5 = 5x^0$, find the measures of 1 and 5.

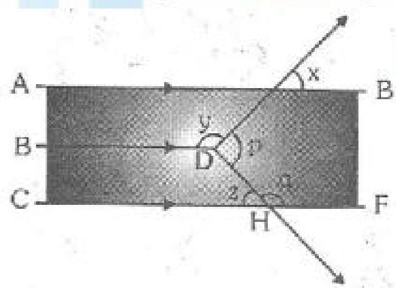
(ii) If $2 = (3x - 10)^0$ and $8 = (5x - 30)^0$, find the measures of 2 and 8



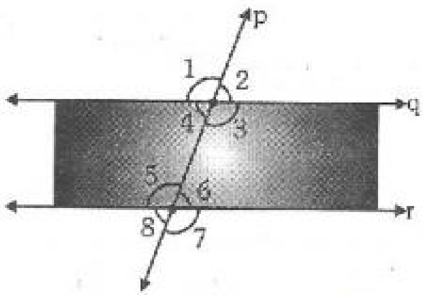
9. In figure if BA || DF, AD || FG, BAC = 65^0 and ACB = 55^0, then find FGH.



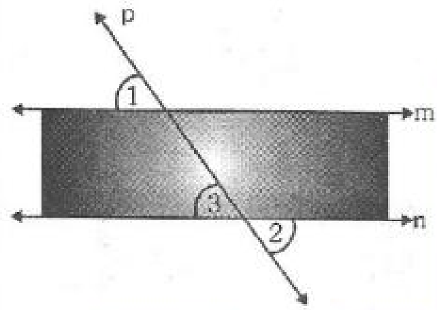
10. In figure, AB, CD and EF are three parallel lines, if 4y = 5x and z = y - 30, find q.



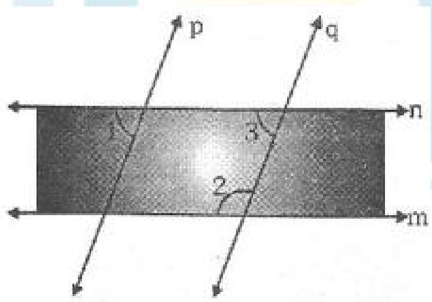
11. In fig, if p is transversal to lines q and r, q||r and angles 1 and 2 are in the ratio 3 : 2, find all the angles.



12. In fig. p is a transversal to lines m and n, $1 = 60^0$ and $2 = \frac{2}{3}$ of a right angle. Prove that m||n.

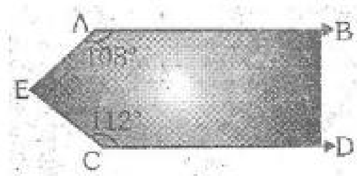


13. In fig, n||m and p||q. If $1 = 75^0$, prove that $2 = 1 + \frac{1}{3}$ of a right angle.

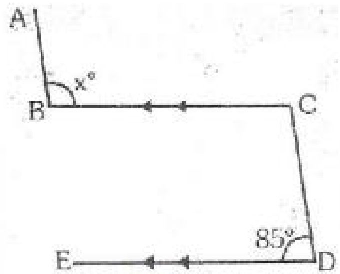


14. Find the value of x, if

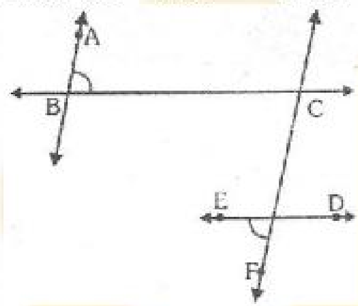
(i) In fig, AB || CD



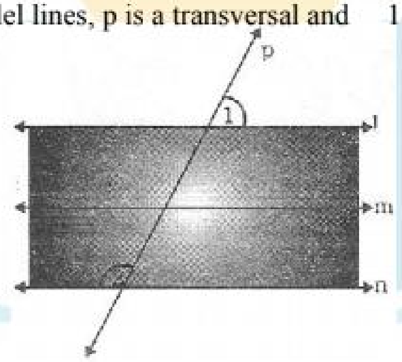
(ii) In fig, AB||CD and BC||DE



15. In fig, AB||CF and BC||ED. Prove that ABC = FDE.

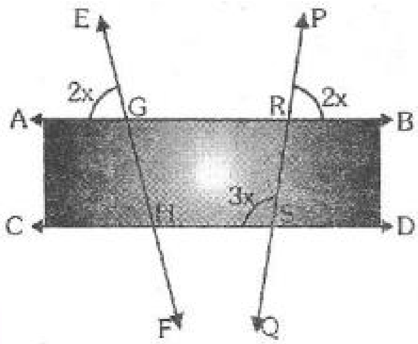


16. If fig. if l, m and n are parallel lines, p is a transversal and $1 = 60^{0}$, then find 2.

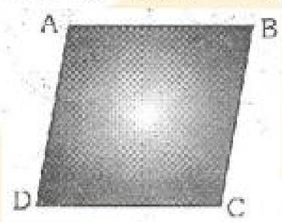


17. In fig, if AB||CD then find the value of

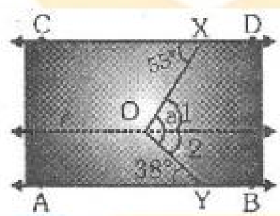
(i) x **(ii)** GHS **(iii)** PRG **(iv)** CHF **(v)** RSD



18. If fig AB||DC and AD||BC. Prove that DAB = DCB.



19. In fig AB||CD. Determine a.



20. One of the angles of a triangle is 65^0. Find the remaining two angles, if their deference is 25^0.

21. Prove that is one angle of a triangle is equal to the sum of the other two angles, the triangle is right angled.

22. Side BC and a ABC is produced is both the directions. Prove that the sum of the two exterior angles so formed is greater than 180^0.

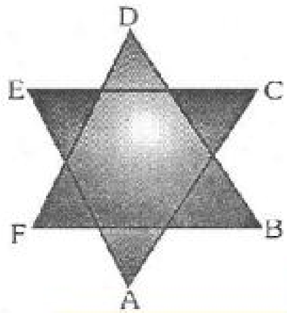
23. The side EF, EF and DE of a triangle DEF are produced in order forming three exterior angles DFP, EDQ and FER respectively. Prove that DFP + EDQ + FER = 360

24. In ABC, B = 45^0, C = 55.0 and bisector of A meets BC at a point D. Find ADB and ADC.

25. Prove that if two parallel lines are intersected by transversal, then the bisectors of the interior angles on the same side of the transversal intersect at right angles.

26. If two angles of a triangle are equal and complementary, what kind of triangle is it ?

27. If fig, show that A + B + C + D + E + F = 360^0



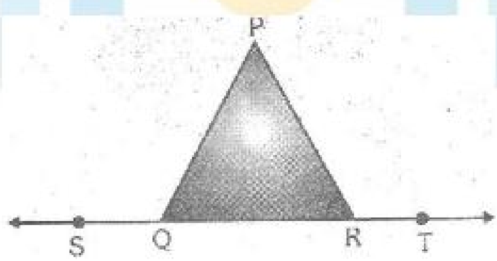
28. The side BC of a triangle ABC is produced to ray BC such that D is on ray BC. The bisector of A meets BC is L. Prove that ABC + ACD = 2 ALC.

29. Two angles of a triangle are equal and the third angle is greater than each of these angles by 30^0. Find all the angles of the triangle.

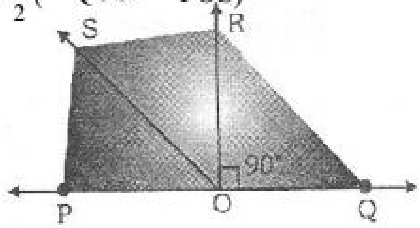
30. The side BC of a triangle ABC has bee produced both ways to D and E. If ABD = 125^0 and ACE = 130^0, then find BAC.

(C) NCERT QUESTIONS :

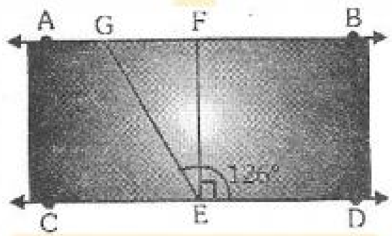
1. If fig PQR = PRQ, then prove that PQS = PRT



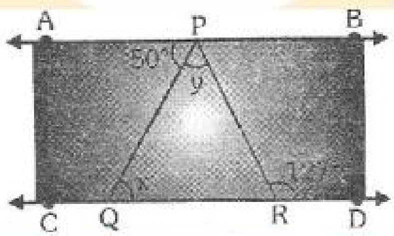
2. In fig, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ROS = $\frac{1}{2}$(QOS- POS)



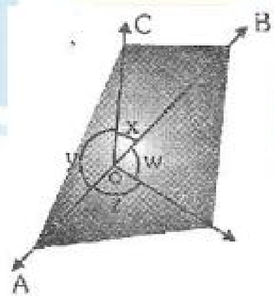
3. In fig. if AB || CD, EF ⊥ CD and GED- 126^0, find AGE, GEF and FGE.



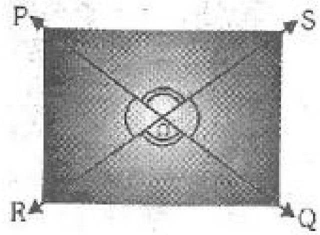
4. In fig AB || CD, APQ = 50^0 and PRD = 127^0, find x and y.



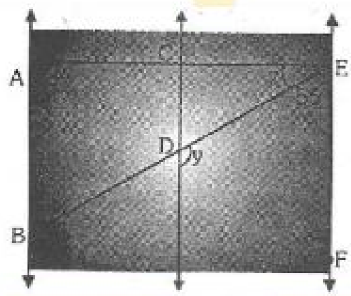
5. In figure, if x + y = w + z then prove that AOB is a straight line.



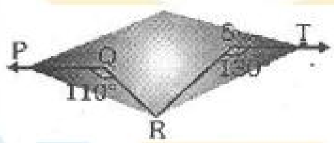
6. In fig lines PQ and RS intersect each other at point O. If   POR :   ROQ = 5 : 7, find all the angles.



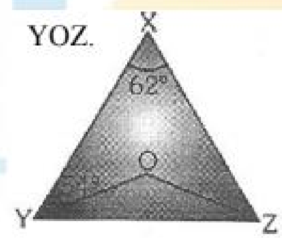
7. In fig AB || CD and CD || EF. Also EA ⊥ AB. If   BEF = 55^0, find the value of x, y and x:



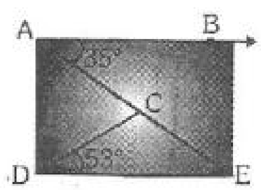
8. In fig, if PQ||ST,   PQR = 110^0 and   RST = 130^0, find   QRS.



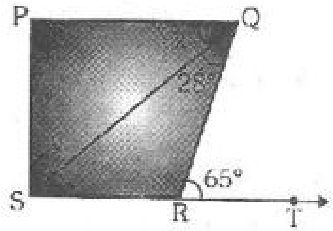
9. In fig,   x = 62^0,   XYZ = 54^0. If YO and ZO are the bisectors of   XZY and   XZY respectively of   XYZ, find   OZY and   YOZ.



10. In fig, if AB||DE,   BAC = 35^0 and   CDE = 53^0, find   DCE.



11. In fig, if PQ ⊥ PS, PQ||SR, SQR = 28^0 and QRT = 65^0, then find the value of x and y.



(D) ***WHICH OF THE FOLLOWNG STTEMETNS ARE TRUE (T) AND WHICH ARE FALSE (F):***

(a) Angles forming a linear pair are supplementary.

(b) If two adjacent angles are equal, then each angle measures 90^0.

(c) Angles forming a linear pair can both be acute angles.

(d) If angles forming a linear pair are equal, then each of these angles is of measure 90^0

(e) If two lines intersected by a transversal, then corresponding angles are equal.

(f) If two parallel lines are intersected by a transversal, then alternate interior angles are equal.

(g) Two lines perpendicular to the same line are perpendicular to each other.

(h) Two lines parallel to the same line are parallel to each other.

(i) If two parallel lines are intersected by a transversal, then the interior angles on the same side of the transversal are equal.

(j) Sum of the three angles of a triangle is 180^0

(k) An exterior angle of a triangle is less than either of its interior opposite angles.

(l) An exterior angle of a triangle is equal to the sum of the two interior opposite angles.

(m) An exterior angle of a triangle is greater than the opposite interior angles.

(n) Two distinct lines in a plane can have two points in common.

(o) If two lines intersect and if one pair vertically opposite angles is formed by acute angles, then the other pair of vertically opposite angles will be formed by obtuse angles.

(p) If two lines intersect and one of the angles so formed is a right angle, then the other three angles will not be right angles.

(q) Two lines that are respectively perpendicular to two intersecting lines always intersect each other.

(r) The two lines that are respectively perpendicular to two parallel lines are parallel to each other.

(s) Through a given point, we can draw only one perpendicular to a given line.

(t) Two lines that are respectively parallel to two intersecting lines intersect each other.

(E) FIL IN THE BLANKS :

(a) If one angle of a linear pair is acute, then its other angle will be ______.

(b) A ray stands on line, then the sum of the two adjacent angles so formed is ______.

(c) If the sum of two adjacent angles is 180^0, then the _____ arms of the two angles are opposite rays.

(d) If two parallel lines are intersected by a transversal, then each pair of corresponding angles are _______.

(e) If two parallel lines are intersected by a transversal, then interior angles on the same side of the transversal are ____.

(f) Two lines perpendicular to the same line are _____ to each other.

(g) Two lines parallel to the same line are ____ two each other.

(h) If a transversal intersects a pair of lines in such a way that a pair of alternate angles are equal, then lines are _____.

(i) If a transversal intersects a pair of lines in such a way that the sum of interior angles on the same side of transversal is 180^0, then the lines are ____.

(j) Sum of the angles of a triangle is ______.

(k) An exterior angle of a triangle is equal to the sum of two _____ opposite angles.

(l) An exterior angle of a triangle is always ____ than either of the interior opposite angles.

(m) Two distinct points in a plane determine a _____ line.

(n) Two distinct ____ in a plane cannot have more than one point in common.

(o) Given a line and a point, not on the line, there is one and only ____ line which passes through the given point and is ____ to the given line.

(p) A line separates a plane into _____ parts namely the two ____ and the ____ itself.

(q) Two angles which have their arms parallel are either ___ or ____.

(r) Two angle whose arms are perpendicular are either ____ or ____.

(s) A triangle cannot have more than ____ right angle(s).

(t) A triangle cannot have more than ____ obtuse angle(s).

OBJECTIVE TYPE QUESTIONS

1. If the supplement of an angle is three times its complement, then angle is :

(A) 40^0 (B) 35^0 (C) 50^0 (D) 45^0

2. Which of the following is true ?

(i) A triangle can have two right angles.

(ii) A triangle can have all angles less than 60^0

(iii) A triangle can have two acute angles.

(A) Only (ii) (B) Only (i) (C) Only (iii) (D) All are true

3. The angle between the bisectors of two adjacent supplementary angles is :

(A) Acute angle (B) Right angle (C) Obtuse angle (D) None of these

4. Which so the following is true ?

(i) A triangle can have two obtuse angles.

(ii) A triangle can have all angles equal to 60^0

(iii) A triangle can have all angles more than 60^0

(A) Only (ii) (B) Only (i) (C) Only (iii) (D) All are true

5. If two are intersected by a transversal, then each pair of corresponding angles so formed is :

(A) Equa (B) Complementary (C) Supplementary (D) None of these

6. If two angles are complementary of each other, then each angle is :

(A) An Obtuse angle (B) A Right angle

(C) An Acute angle (D) A supplementary angle.

7. X lies in the interior of BAC. If BAC = 70^0 and BAX = 42^0 then XAC = ?

(A) 280^0 (B) 29^0 (C) 27^0 (D) 30^0

8. Whish of the following statements is false ?

(A) A line segment can be produced to any desired length.

(B) Through a given point, only one straight line can be drawn.

(C) Through two given points, it is possible to draw one and only one straight line.

(D) Two straight lines can intersect in only one point.

9. An angle is 14^0 more than its complementary angle, then angle is :

(A) 38^0 (B) 52^0 (C) 50^0 (D) None of these

10. In the given figure, straight lines PQ and RS intersect at O. If the magnitude of θ is 3 times that of ϕ, then ϕ is equal to :

(A) 30^0 (B) 40^0 (C) 45^0 (D) 60^0

11. Two parallel lines have :

(A) A common point (B) Two common points

(B) No common point (D) Infinite common points

12. How many degrees are there in an angle which equals one-fifth of its supplement ?

(A) 15^0 (B) 30^0 (C) 75^0 (D) 150^0

13. Two angles whose measures are a & b are such that $2a - 3b = 60^0$ then $5b = ?$, if they form a linear pair :

(A) 120^0 (B) 300^0 (C) 60^0 (D) None of these

14. If two parallel lines are intersected by transversal then the bisectors of the interior angles form a :

(A) rhombus (B) ||gm (C) Square (D) Rectangle

15. If one angle of triangle is equal to the sum of the other two angles then triangle is :

(A) Acute triangle (B) Obtuse triangle (C) Right triangle (D) None of these

16. If the arms of one angle are respectively parallel to the arms of another angle, then the two angles are :

(A) Neither equal nor supplementary

(B) Not equal but supplementary

(C) Equal but not supplementary

(D) Either equal or supplementary

17. Which one of the following is not correct ?

(A) If two lines are intersected by a transversal, then alternate angles are equal.

(B) If two lines are intersected by a transversal then sum of the interior angles on the same side of transversal is 180^0.

(C) If two lines are intersected by a transversal then corresponding angles are equal.

(D) All of these.

18. If ℓ is any given line and P is any point not lying on ℓ, then the number of parallel lines than can be drawn through P, parallel to ℓ would be :

(A) One (B) Two (C) Infinite (D) None of these

19. Which one of the following statements is not false ?

(A) If two angles forms a linear pair, then each of these angles is of measure 90^0

(B) Angles forming a linear pair can both be acute angles.

(C) One of the angles forming a linear pair can be obtuse angles.

(D) Bisectors of the adjacent angles form a right angle.

20. There are four lines in a plane no two of which are parallel. The maximum number of points in which they can intersect is :

(A) 4 (B) 5 (C) 6 (D) 7

ANSWER KEY

(A) SHORT ANSWER TYPE QUESTIONS :

1. Adjacent angles : AOD, COD; BOC, COD, Linear pairs : AOD, BOD; AOC, BOC

2. 4 **3.** 10 **4. (i)** 70^0 **(ii)** 13^0 **(iii)** 0^0 **5. (i)** 48^0, **(ii)** 126^0, **(iii)** 42^0 **6.** 31^0 **7.** 50^0 **8.** 80^0, 100^0

9. 66^0, 114^0 **10.** 108^0, 72^0 **11.** 45^0 **12. (i)** $y = 105^0$, **(ii)** $x = 70^0$ **13.** $a = 130^0$ and $b = 50^0$

14. $x = 50^0$, BOC = 70^0, COD = 50^0, AOD = 60^0 **15.** $x = 30^0$ **16.** $x = 155^0$, $y = 25^0$ $z = 155^0$ **17.** $x = 18^0$ **18.** $x = 16^0$ **19.** COD = 80^0 **20.** $x = 60$.., $y = 48^0$, $z = 72^0$ **21.** POA = 34^0 **24.** No **25.** 40^0, 60^0, 80^0

(B) LONG ANSWER TYPE QUESTIONS :

1. AOB = 35^0, AOD = 100^0, COD = 135^0 **4.** $x = 40^0$, $d = 50^0$, $e = 90^0$, $f = 40^0$

5. BOC = 110^0, AOD = 110^0 **8. (i)** 1 = 100^0, 5 = 100^0, **(ii)** 2 = 20^0, 8 = 20^0

9. FGH = 125^0 **10.** $q = 110^0$

11. 1 = 108^0, 2 = 72^0, 3 = 108^0, 4 = 72^0, 5 = 108^0, 6 = 72^0, 7 = 108^0, 8 = 72^0

14. (i) $x = 140$, **(ii)** $x = 95$ **16.** 120^0 **17. (i)** 36^0, **(ii)** 108^0, **(iii)** 1008^0, **(iv)** 108^0, **(v)** 72^0

19. 93^0 **20.** 70^0, 45^0 **24.** ADB = 95^0, ADC = 85^0 **26.** Isosceles right angled triangle.

29. 50^0, 50^0, 80^0 **30.** 75^0

(C) NCERT QUESTIONS :

3. AGE = 126^0, GEF = 360^0, FGE = 54^0 **4.** $x = 50^0$, $y = 77^0$

6. POR = 75^0, SOQ = 75^0, ROQ = 105^0, POS = 105^0

7. $x = 125^0$, $y = 125^0$, $z = 35^0$ **8.** QRS = 60^0 **9.** OZY = 32^0, YOZ = 121^0

10. DCE = 92^0 **11.** $x = 37^0$, $y = 53^0$

(D) TRUE & FALSE :

(a) T **(b)** F **(c)** F **(d)** T **(e)** F **(f)** T **(g)** F **(h)** T **(i)** F **(j)** T **(k)** F **(l)** T **(m)** T
(n) F **(o)** T **(p)** F **(q)** T **(r)** T **(s)** T **(t)** T

(E) FILL IN THE BLANKS :

(a) Obtuse **(b)** 180^0 **(c)** Uncommon **(d)** Equal **(e)** Supplementary

(f) Parallel **(g)** Parallel **(h)** Parallel **(i)** Parallel **(j)** 180^0

(k) Interior **(l)** Greater **(m)** Unique **(n)** Lines

(o) One, Parallel (or Perpendicular) **(p)** Three, Half planes, line

(q) Equal, Supplementary **(r)** Equal, Supplementary **(s)** One**(t)** One

ANSWER KEYA

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	C	B	A	D	C	A	B	B	C
Que.	11	12	13	14	15	16	17	18	19	20
Ane.	C	B	B	D	C	D	D	A	C	C