Probability

- **Experimental Probability:** The probability obtained from the result of an experiment when we actually perform the experiment is called experimental (or empirical) probability.
- **Theoretical Probability:** The probability we find through the theoretical approach without actually performing the experiment is called theoretical probability.
- The theoretical probability (or classical probability) of an event E, is denoted by P(E) and is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

- Experimental probability may or may not be equal to the theoretical probability.
- Formula of theoretical probability can be used to find the probabilities of various events.

Example 1: A dice is thrown once. What is the probability of getting 1 on the dice?
Solution: When a dice is thrown once, the possible outcomes are 1, 2, 3, 4, 5, 6.
Let A be the event of getting 1 on the dice.

$$\therefore P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Number of all possible outcomes}} = \frac{1}{6}$$

Example 2: A box contains 3 white, 5 green, and 6 red balls. A ball is drawn at random out of the box. Find the probability of drawing a red ball.
Solution: Total number of balls = 3 + 5 + 6 = 14
Therefore, total number of possible outcomes = 14
Let E be the event of drawing a red ball.
Number of outcomes favourable to E = 6

$$\therefore P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes}} = \frac{6}{14} = \frac{3}{7}$$

Example 3: A card is drawn at random from a deck of 52 cards. Find the probability that the card drawn is,

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    - A black card
    - An ace
    - Neither a black card nor an ace

Solution:
Since there are 52 cards in a deck, the number of all possible outcomes is 52.

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    - Number of black cards in the deck = 26

Therefore, the probability that the drawn card is black $= \frac{26}{52} = \frac{1}{2}$

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- Number of aces in the deck = 4

  Therefore, the probability that the drawn card is an ace $= \frac{4}{52} = \frac{1}{13}$

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- Number of cards which are neither black nor an ace = 26 – 2 = 24

  Therefore, the probability that the drawn card is neither black nor an ace $= \frac{24}{52} = \frac{6}{13}$

- **Complementary events**

  For an event E such that $0 \le P(E) \le 1$ of an experiment, the event $\bar{E}$ represents 'not E' , which is called the complement of the event E.
  We say, E and $\bar{E}$ are **complementary** events.

  $P(E) + P(\bar{E}) = 1$

  $\Rightarrow P(\bar{E}) = 1 - P(E)$

Example:

A pair of dice is thrown once. Find the probability of getting a different number on each die.

Solution:

When a pair of dice is thrown, the possible outcomes of the experiment can be listed as:

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

The number of all possible outcomes = 6 × 6 = 36

Let E be the event of getting the same number on each die.

Then, $\bar{E}$ is the event of getting different numbers on each die.

Now, the number of outcomes favourable to E is 6.

$\therefore P(\bar{E}) = 1 - P(E) = 1 - \frac{6}{36} = \frac{5}{6}$

Thus, the required probability is $\frac{5}{6}$.