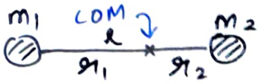
ROTATIONAL MOTION

CENTRE OF MASS :

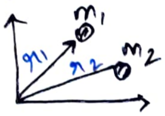
① 2-body system



$r_1 = \frac{m_2 l}{m_1+m_2}$ $\quad r_2 = \frac{m_1 l}{m_1+m_2}$

② Multiparticle system

$\vec{r} = \frac{m_1\vec{r_1} + m_2\vec{r_2}\dots}{m_1+m_2\dots}$



③ concept of mass moment vector ($\vec{z}$)/mm

$\vec{z} = m\vec{r}$

mass of particle

posit^n vector

- About com sum of mm of all particle = 0

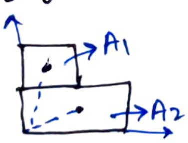
④ COM of subtractive body

$\vec{r}_{com} = \frac{A_1 r_1 - A_2 r_1}{A_1 - A_2}$ (complete area, area removed)

- You can measure x_1, x_2 from any point of your choice

⑤ Lamina type (2-D) with uniform + negligible thickness.

$r_{CM} = \frac{M_1\vec{r_1} + M_2\vec{r_2}}{M_1+M_2}$



$M = \rho \times A \times t$ $\quad t \approx 0$, ρ = same

$r_{CM} = \frac{A_1\vec{r_1} + A_2\vec{r_2}}{A_1+A_2}$

⑥ COM of continuous extended body :

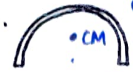
$\vec{r}_{com} = \frac{\int r\,dm}{\int dm}$ → $= M_{Total}$

1.  Rod

uniform → $\frac{L}{2}$

$\lambda \propto x$ → $\cancel{\frac{2L}{3}}$ $\frac{2L}{3}$

• put $dm = \lambda\, dr$

2.  Circular ring $y = \frac{2R}{\pi}$  Quadrant $\frac{2R}{\pi}, \frac{2R}{\pi}$

3.  Semicircular disc $= \frac{4R}{3\pi} = \frac{2}{3}\left(\frac{2R}{\pi}\right)$

4.  Hemispherical shell $= \frac{R}{2}$

5.  Solid sphere $= \frac{3R}{8}$

6.  Right triangular sheet $\frac{2a}{3}, \frac{2b}{3}$

7.  Cone hollow $\frac{H}{3}$ solid $\frac{H}{4}$ (from base)

⑦ $\vec{V}_{com} = \frac{m_1\vec{v_1} + m_2\vec{v_2} + \dots}{m_1+m_2+\dots}$

$\vec{a}_{CM} = \frac{m_1\vec{a_1} + m_2\vec{a_2}}{m_1+m_2}$

$\vec{F}_{net} = M_{Total}\vec{a}_{CM} = \vec{F_1} + \vec{F_2} + \dots$

If $F_{net} = 0$ (Does not mean $F_1 = 0$)

⇓

$\vec{a}_{CM} = 0$ (Doesn't ⇒ individual acc = 0)

⇓

$\vec{V}_{CM}$ = const (Does not mean $V_{CM} = 0$ also individual V is const)

⇓

P_{net} = const

⑧ $P_{net} = M\vec{V}_{com}$

⑨ Displacement of COM :

$\Delta\vec{r}_{CM} = \frac{m_1\vec{x_1} + m_2\vec{x_2} + \dots}{m_1+m_2+\dots}$

$\vec{x_1}, \vec{x_2}$ ⇒ wrt ground.

IMP CASE :

If initially COM is at rest ($V_{com}=0$) and no ext. force acts on system toh finally bhi com rest pr hoga (no shift)

$F_{net}=0$ $\quad a_{com}=0$ $\quad V_i = V_f = 0$

↳ Agar bodies ek dusre ki taraf chal rahi toh

$m_1 x_1 = -m_2 x_2$

effective distance travelled

$(x_1 = x - y)$

Linear		Angular
S	$\boxed{S = R\theta}$	θ
v	$\boxed{v = R\omega}$	ω
a_t	$\boxed{a_t = R\alpha}$	α
m	$\boxed{I = mr^2}$	I
F	$\boxed{\tau = \vec{r}\times\vec{F}}$	τ
$\frac{1}{2}mv^2$		$\frac{1}{2}I\omega^2$
$\vec{F} = m\vec{a}$		$\vec{\tau} = I\vec{\alpha}$
$m\vec{v}(\vec{P})$		$I\vec{\omega}(\vec{L})$
$F = \frac{\Delta \vec{P}}{\Delta t}$		$\tau = \frac{\Delta\vec{L}}{\Delta t}$

MOMENT OF INERTIA.

• system of particle.

$I = \Sigma m r^2 = m_1 r_1^2 + m_2 r_2^2$

Unit = $kg\,m^2$; ⊥ distance on axis.

• If axis = x + m at (y, z) take $r = \sqrt{y^2+z^2}$

• continuous body :

$I = \int dm\, r^2$

↳ Parallel axis theorem.

$I' = I_{com} + mr^2$

↳ ⊥ axis theorem (only 2D)

$I_\perp = I_1 + I_2$

⊥ to plane ; plane of body

$I_z = I_y + I_x$

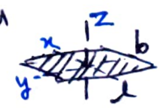
If body symmetry

$I_1 = I_2 \Rightarrow I_{\perp axis} = 2I_{plane}$

 Ring  Hollow cylinder $\quad I = MR^2$

 Disc, solid cylinder $\quad I = \frac{MR^2}{2}$

 Rod  plate $\quad I = \frac{ML^2}{12}$

Reactangular plate 

$I_z = I_x + I_y = \frac{ML^2}{12} + \frac{Mb^2}{12}$

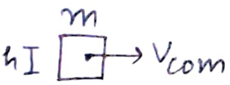
$I = \frac{ML^2}{12}$

θ → no effect

$\boxed{\text{solid sphere} = \frac{2}{5}MR^2}$ $\quad \boxed{\text{Hollow sphere} = \frac{2MR^2}{3}}$

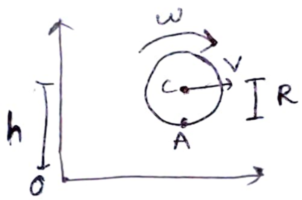
ANGULAR MOMENTUM

Case I : Pure translation



$$\boxed{\vec{L} = m\vec{V}_{com}h}$$

Case II : Rolling body

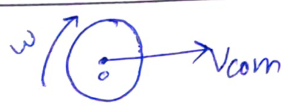


$\vec{L}_c = I_c\omega$

$\vec{L}_A = I_c\omega + M\vec{V}_{com}R$

$\vec{L}_0 = I_c\omega + MV_{com}h$

ROLLING OF A BODY



Forward slipping : $V_{com} > R\omega$

Backward slipping : $V_{com} < R\omega$

Pure rolling : $V_{com} = R\omega$

Kinetic energy of body during pure rolling :

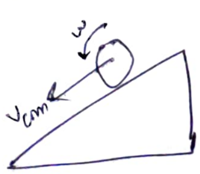
$K.E = KE_{translatn} + KE_{rotatn}$

$= \frac{1}{2}mV_{com}^2 + \frac{1}{2}I_{com}\omega^2$

$$\boxed{KE_{Total} = \frac{1}{2}mV_{com}^2\left(1 + \frac{K^2}{R^2}\right)}$$

(K = radius of gyration)

INCLINED PLANE :



By conservatn of energy

① $a_{com} = \dfrac{g\sin\theta}{1 + \frac{K^2}{R^2}}$

② $V_{cm} = \sqrt{\dfrac{2gh}{1 + \frac{K^2}{R^2}}}$

③ $f = \dfrac{mg\sin\theta}{1 + \frac{R^2}{K^2}}$

force of friction.