WAVE

Wave: disturbance from eqb position
can be displacement, Press., dencity, electric/Mag. field

> Energy/Momentum is transmitted from one point to other

> particle oscillate, do not travel

Wave → Mechanical wave, Electro magnetic wave, Matter wave

- Mechanical wave > Medium
- Electro magnetic wave > no medium
- Matter wave > e, p

Mechanical wave → Transverse, Longitudnal wave

Transverse ($\perp$)
> require elasticity
> Solid, liq surface
eg: EMW, string wave

Longitudnal wave
> medium oscillate // to wave directn
> Solid, liq., gas
eg: sound wave, air column

Eqn of wave :

- y → depend on x and t (displacement)

eg: $y = A\log(at + bx)$
$y = \sqrt{ax + bt}$
$y = (ax - bt)^2$
$y = A\sin(ax - bt)^2$
$y = a\cos^2(wt - kx)$

- conditn for wave :
(i) y defined for all x & t
(ii) $\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$ (v = wave vel.)

Wave speed :

$v = \frac{\text{coefficient of } t}{\text{co.eff. of } x} = \frac{w}{k} = f\lambda$

Harmonic wave :

all particle execute SHM

$y = A\sin(wt \pm kx)$
- phase
- wave no.
- ± : wave travelling along x axis
- y : disp. of particle in y directn
- A : Amplitude
- $w = \frac{2\pi}{T} = 2\pi f$

$\boxed{k = \frac{2\pi}{\lambda}}$ → wave no. ; λ → wave length

w, k same sign ⇒ wave travel in −x axis
i.e both +ve

Wave v/s particle velocity :

$y = A\sin(wt - kx)$

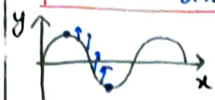
(wave velocity) $v = \frac{w}{k} = f\lambda \ +\hat{i}$

(particle velocity) → perform SHM
$V_p = \frac{dy}{dt} = Aw\cos(wt - kx)$

- $(V_p)_{max} = Aw$ (mean position)

$v_p^2 = w^2(A^2 - x^2)$

- $\boxed{V_p = -v\frac{dy}{dx}} = -v$ (slope of y wrt x)



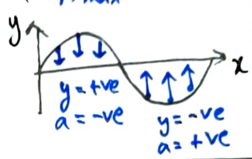
tangent ↑ slope = +ve ⇒ vp down (-ve)

slope = −ve ⇒ vp up (+ve) ↑

Accelerat^n of particle

particle → SHM
$a_p = -w^2 y$
always towards mean position

$(a_p)_{max} = w^2 A$



Phase diff :

> for same particle at diff time
$\Delta\phi = w\Delta t = \frac{2\pi}{T}\Delta t$

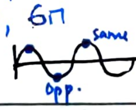
↳ for diff particle at give time :
$\Delta\phi = k\Delta x = \frac{2\pi}{\lambda}\Delta x$

phase diff → path diff.

Same phase : nλ
- same displ. (or wave ke bad) vel., acc.

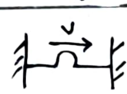
Opp phase : odd λ/2
- disp, speed, acc. magnitude in same BUT opp. directn

- After
T, 2T, 3T | T/2, 3T/2, 5T/2
2π, 4π, 6π | π, 3π, 5π
λ, 2λ | λ/2, 3λ/2, 5λ/2



SPEED OF WAVE ON STRING :

$\boxed{V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}}}$



T = Tension in string
μ = mass per unit length

> velocity, wavelength depend on medium BUT frequency depends only on source.

INTENSITY OF WAVE :

$\boxed{I = 2\pi^2 f^2 A^2 \rho V} = \frac{Eng}{Area \times t}$

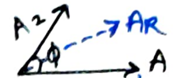
f → frequency, A → Amplitude, ρ → ρ of medium, V → wave vel.

> Variation of intensity with distance from source :
(i) Point source $I \propto \frac{1}{r^2}$
(ii) linear/planar/cylindrical source $I \propto \frac{1}{r}$

INTERFERENCE OF WAVE

$y_1 = A_1\sin(wt + kx)$
$y_2 = A_2\sin(wt + kx + \phi)$

$\vec{y}_{net} = \vec{y_1} + \vec{y_2}$



> $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$ (vector addn of Amplitude)

> $\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$ ($I \propto A^2$)

Constructive	Destructive
> same phase	opp. phase
> A, I → max, cos ϕ → max +1	> A, I → min, cos ϕ → −1
> $\phi = 0, 2\pi, 4\pi = 2n\pi$	> $\phi = \pi, 3\pi, 5\pi$, $\phi = (2n+1)\pi$
> $\Delta x = 0, \lambda, 2\lambda = n\lambda$	> $\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} = (2n+1)\frac{\lambda}{2}$
> $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$	> $I = (\sqrt{I_1} - \sqrt{I_2})^2$
> $A_{max} = A_1 + A_2$	$A = A_1 - A_2$

$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{A^2_{max}}{A^2_{min}}$

$= \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$; $\sqrt{I_{max}} = \sqrt{I_1} + \sqrt{I_2}$

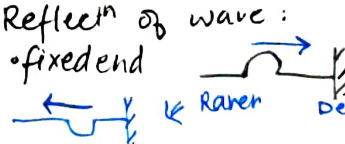
Reflectn & Transmission of wave on string :

Properties	Reflectn	Refractn/Transmision
frequency	same	same
velocity	same (directn change)	change
wavelength	same	change
		rarer → denser, V↓ λ↓
phase diff Δϕ	Δϕ = 0 (Denser → rarer); Δϕ = π opp (rare → dense)	Δϕ = 0

Reflectn of wave :

- fixed end

$y_i = A\sin(kx - wt)$



$y_r = -A\sin(kx + wt)$
− → opp phase Δϕ = π ; + → wave directn

- free end :

$y_i = A\sin(kx - wt)$; $y_r = A\sin(kx + wt)$

Transmission ⇒ same eqn (K change)
(No change in phase/directn of wave)

Reflected & Transmitted Amplitude (A_r & A_t):

• no. erg wasted $A_i = A_r = A$ (only reflectⁿ happen)

• Both happen: v_1 v_2

$A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) A_i$ $\quad A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_i$

STANDING / STATIONARY WAVE:

Standing wave are formed by superimpose of two wave with equal freq, amplitude travelling in opp. directⁿ

$y_1 = A\sin(kx - \omega t)$

$y_2 = A\sin(kx + \omega t)$

$$\boxed{y_{net} = \underbrace{2A\sin(kx)}_{\text{Amplitude}}\ \underbrace{\cos\omega t}_{\text{phase}}}$$

> Not a wave! (No energy/momentum transfer)

> All particle execute SHM with diff. amplitude

> Antinode → A_{max} ⇒ $\sin kx = +1$

$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ $\quad \boxed{\text{odd } \frac{\lambda}{4}}$

$A_{net} = 2A$

> Node → $A = 0$ ⇒ $\sin kx = 0$

$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$ $\quad \boxed{\frac{n\lambda}{2}}$

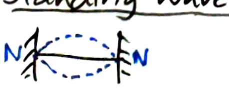
> Distance b/w two consecutive nodes / Antinode = $\frac{\lambda}{2}$

> " node and antinode = $\frac{\lambda}{4}$

> Do node ke biche vale sare particle are in same phase to each other & with bagal vale opp. phase ($\Delta\phi = \pi$)

★ velocity of wave is not zero! $v = \frac{\omega}{k}$

Standing wave on string:



Fixed end → node
free end → antinode

$v = \lambda f$ $\quad \boxed{\frac{\lambda}{2} = L}$ $\quad v = \sqrt{\frac{T}{\mu}}$

⇒ $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ fundamental / I^st harmonic freq.

First overtone / II^nd harmonic

$\lambda = L$ ⇒ $f = \frac{v}{L} = \frac{1}{L}\sqrt{\frac{T}{\mu}}$

> No. of segment = No. of antinode (gola)

> No. of nodes = No. of segment + 1

$$\boxed{f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}} = \boxed{\frac{nv}{\lambda}}$$ For n harmonic

SOUND WAVE

> longitudinal wave

Infrasonic $f < 20$; Audible 20 – 20kHz ; Ultrasonic wave $f > 20$ kHz

> sound wave: $V_{solid} > V_{liq} > V_{gas}$

light wave: $V_{gas} > V_{liq} > V_{solid}$

> sound wave travel due to pressure & density variatⁿ

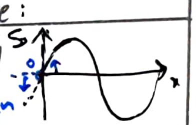
> compression → P, ρ → max

Rarefactⁿ P, ρ → min

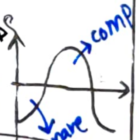
> eqⁿ of sound wave:

$\boxed{s = A\sin(kx - \omega t)}$

displacement in x directⁿ



At O point ← O → ⇒ $\Delta P = -ve$



$\boxed{\Delta P = -\Delta P_0 \cos(kx - \omega t)}$

$\boxed{\Delta P = \Delta P_0 \sin\left(kx - \omega t - \frac{\pi}{2}\right)}$

$\Delta P_0 = BKA$ (B → Bulk modulus, K → wave no., A → Amplitude)

$\Delta\rho = \Delta\rho_0 \sin\left(kx - \omega t - \frac{\pi}{2}\right)$

> Disp. node = Press. antinode

> Phase diff b/w disp. & press = $\frac{\pi}{2}$

> speed of sound wave

$v = \sqrt{\frac{E}{\rho}}$ (E → coefficient of elasticity of medium; ρ → density of medium)

In liq/gas ↳ E = Bulk modulus

in solid ↳ E = Youngs modulus

> sound speed in air:

∘ Newton → Isothermal ($B = P$)
$v \approx 280$ m/s ✗

∘ Laplace → Adiabatic ($B = \gamma P$)
$v \approx 332$ m/s

$$\boxed{v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}}$$ (R → J/molK, T → K, M → kg)

Factor on which V_{sound} depend

(i) Temp. $V_{t°C} = V_{0°C} + 0.61t$

(ii) Pressure → if temp = const No effect.

(iii) Humidity

$\rho_{moist\ air} < \rho_{dry\ air}$ (water vapour are lighter than air)

∴ $V_{moist} > V_{dry\ air}$

> $\frac{V_{sound}}{V_{rms}} = \sqrt{\frac{\gamma}{3}}$ $\quad$ ∵ $\gamma < 3$ $\quad V_{rms} > V_{sound}$

Organ pipe

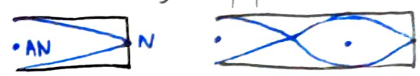
Open organ pipe (same as string)

$f_1 = \frac{v}{2L}$ (v → speed of sound) $\quad \frac{\lambda}{2} = L$ $\quad$ [nth overtone = (n+1)th harmonic]

$f_2 = 2f_1$ (II^nd harmonic / I^st overtone)

$f_3 = 3f_1$ (III^rd harmonic / II^nd overtone)

Closed organ pipe:



$\frac{\lambda}{4} = l$ $\quad \frac{\lambda}{4} + \lambda = l$

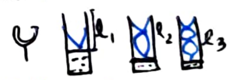
$\boxed{f_1 = \frac{v}{4L}}$ odd multiple

$f_3 = 3f_1$ (III^rd harmonic / I^st overtone)

$f_5 = 5f_1$ (V^th harmonic / II^nd overtone)

[nth overtone = (2n+1)th harmonic]

Resonance column expt.



sound of tuning fork ⇅ Resonance with air column

Since f = same

∴ Ratio of length $\boxed{l_1 : l_2 : l_3 = 1:3:5}$

Speed of sound calculatⁿ

$v = f\lambda$ $\quad$ successive resonance length diff $= \frac{\lambda}{2}$

∴ $\boxed{V = 2f\Delta l}$

$\Delta l = l_2 - l_1 = l_3 - l_2$

Sound level / Intensity level

$\beta = 10\log_{10}\frac{I}{I_0}$ dB (decibel)

I → measured intensity

I_0 → 10^{-12}

BEATS :

> superposit^n of two sound wave having small diff in freq. ($f_2 - f_1 < 10$ Hz)

Standing wave	Beats
→ opp. direct^n	→ same direct^n
→ freq. same	→ f diff.

$f_1 = 2$ $f_2 = 4$

Beat frequency $= \boxed{f_2 - f_1} = \dfrac{1}{\text{Time period of beat}}$

No. of beats per sec. / maxima per sec.

$\underset{\text{constructive}}{A} = A_1 + A_2$ $A_{destruct^n} = A_1 - A_2$

NOTE : scrap / peel off ⇒ f ↑
loaded with wax ⇒ f ↓

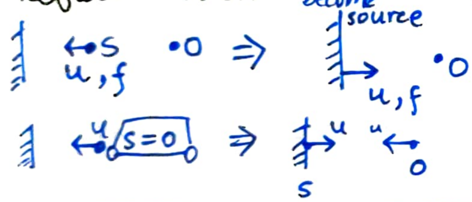
DOPPLER'S EFFECT :

$$\boxed{f_o = \left(\frac{V \pm V_o}{V \mp V_s}\right) f_s}$$

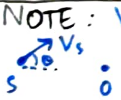
O → observer
S → source

o → s ⇒ f↑ ⇒ $+V_o$ | s → o ⇒ f↑ ⇒ $-V_s$

Reflect^n Problem

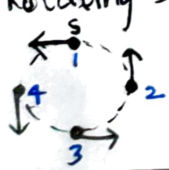


NOTE : V_s / V_o along line joing O & S



$V_s = V\cos\theta$

Rotating source



At 2, 4 $f_o = f_s$

O min^{m} f 1 ⇒ $+V_s$
3 ⇒ $-V_s$ ↳ max f

Apparent wavelength

when obs. at rest & source is moving

$$\frac{1}{\lambda'} = \left(\frac{V \pm V_o}{V \mp V_s}\right)\frac{1}{\lambda} \quad \left(\text{as } f \propto \frac{1}{\lambda}\right)$$