CHAPTER

8 APPLICATIONS OF THE INTEGRALS

Syllabus *Applications in finding the area under simple curves, especially lines, circles/parabolas/ellipses (in standard form only).*

In this chapter you will study

How to calculate the areas enclosed by different curves, lines circles, parabolas and ellipse.

Revision Notes

➢ **Area Under Simple Curves :**

(i) Let us find the area bounded by the **curve** $y = f(x)$, X-axis and the ordinates $x = a$ and $x = b$. Consider the area under the curve as composed by large number of thin vertical strips.

Key Words

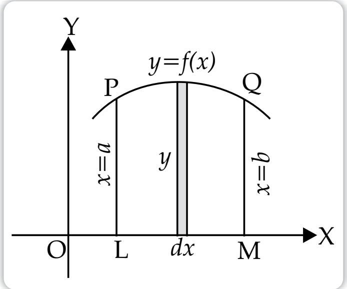
Curve: A **curve** is a continuous and smooth flowing line without any sharp turns. One way to recognize a curve is that it bends and changes its direction at least once.

- A **open curve** does not enclose any area within itself and it has two endpoints. Some of the open curves are given in the figure below.
- A **closed curve** has no end points and encloses an area (or a region). It is formed by joining the end points of an open curve together. *e.g.*: Circles, ellipses are formed from closed curves.
- A **simple curve** changes direction but does not cross itself while changing direction. A simple curve can be open and closed both.
- A **non-simple curve** crosses its own path.

Let there be an **arbitrary** strip of height y and width dx.

Area of elementary strip $dA = y\, dx$, where $y = f(x)$. Total area A of the region between X-axis ordinates $x = a$, $x = b$ and the curve $y = f(x)$ = sum of areas of elementary thin strips across the region $PQML$.

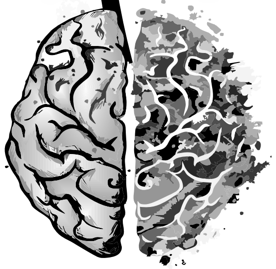
$$A = \int_a^b y\, dx = \int_a^b f(x)\, dx$$



Key Words

Arbitrary: In mathematics, "arbitrary" just means "for all".
For example: "For all $a, b, a + b = b + a$".
Another way to say this would be "$a + b = b + a$ for arbitrary a, b."

Applications of the Integrals



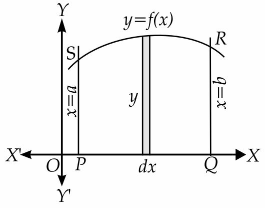
Area under simple curves

The area of the region bounded by the curve $y = f(x)$, X- *axis* and the lines $x=a$ and $x=b\,(b>a)$ is given by

$$A = \int_a^b y\,dx \quad \text{or} \quad \int_a^b f(x)dx$$

eg : The area bounded by $y=x^2$, X- axisin I quadrant and the lines $x=2$ and $x=3$ is -

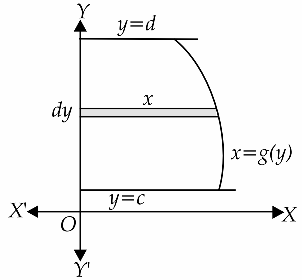
$$A = \int_2^3 y\,dx = \int_2^3 x^2\,dx = \left[\frac{x^3}{3}\right]_2^3 = \frac{1}{3}(27-8) = \frac{19}{3} \text{ Sq.units.}$$



The area of the region bounded by the curve $x = f(y)$, $Y-axis$ and the lines $y=c$ and $y=d\,(d>c)$ is given by $A = \int_c^d x\,dy$ or $\int_c^d f(y)dy$

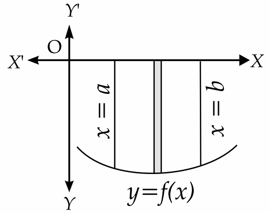
eg : The area bounded by $x = y^3$, $Y-axis$ in the I quadrant and the lines $y=1$ and $y=2$ is

$$\int_a^b x dy = \int_a^b y^3\,dy = \left[\frac{1}{4}y^4\right]_1^2 = \frac{1}{4}(2^4-1^4) = \frac{15}{4} \text{ Sq. units}$$



If the curve under consideration lies below X-axis, then $f(x) < 0$ from $x=a$ to $x=b$, the area bounded by the curve $y=f(x)$ and the ordinates $x=a$, $x=b$ and X-axis is negative

$$A = \left|\int_a^b f(x)dx\right|$$

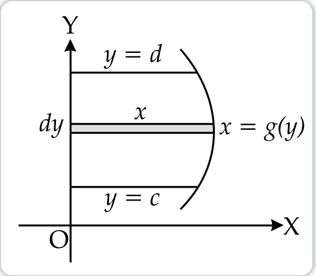


Trace the Mind Map

- First Level
- Second Level
- Third Level

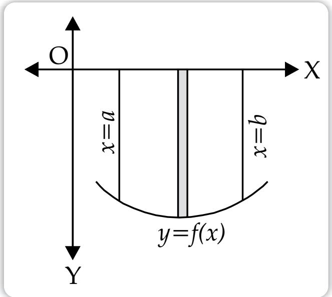
(ii) The area A of the region bounded by the curve $x = g(y)$, y-axis and the lines $y = c$ and $y = d$ is given by

$$A = \int_c^d x\,dy = \int_c^d g(y)\,dy$$



(iii) If the curve under consideration lies below X-axis, then $f(x) < 0$ from $x = a$ to $x = b$, the area bounded by the curve $y = f(x)$ and the ordinates $x = a$, $x = b$ and X-axis is negative. But, if the numerical value of the area is to be taken into consideration, then

$$\text{Area} = \left|\int_a^b f(x)dx\right|$$



Key Words

Ordinate: The Cartesian coordinate obtained by measuring parallel to the Y-axis.

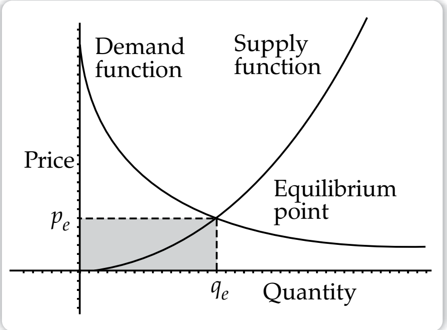
(iv) It may also happen that some portion of the curve is above X-axis and some portion is below X-axis as shown in the figure. Let A_1 be the area below x-axis and A_2 be the area above the X-axis. Therefore, area bounded by the curve $y = f(x)$, X-axis and the ordinates $x = a$ and $x = b$ is given by

$$A = |A_1| + |A_2|$$

Amazing Fact

Application of Integration in Economics and Commerce

Integration helps us to find out the total cost function and total revenue function from the marginal cost. It is possible to find out consumer's surplus and producer's surplus from the demand and supply function. Cost and revenue functions are calculate through indefinite integral.



OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. The area of the region bounded by the curve $x^2 = 4y$ and the straight-line $x = 4y - 2$ is

(A) $\frac{3}{8}$ sq. units

(B) $\frac{5}{8}$ sq. units

(C) $\frac{7}{8}$ sq. units

(D) $\frac{9}{8}$ sq. units

Ans. Option (D) is correct.

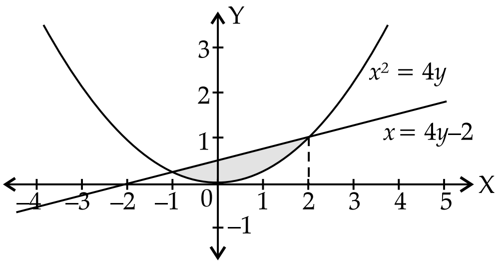
Explanation:

$$\begin{aligned} x^2 &= x+2 \\ x^2 - x - 2 &= 0 \\ (x-2)(x+1) &= 0 \\ x &= -1,\ 2 \end{aligned}$$

For $x = -1$, $y = \frac{1}{4}$ and for $x = 2$, $y = 1$

Points of intersection are $(-1, \frac{1}{4})$ and $(2, 1)$.

Graphs of parabola $x^2 = 4y$ and $x = 4y - 2$ are shown in the following figure:



$$A = \int_{-1}^{2}\left[\frac{x+2}{4}-\frac{x^2}{4}\right]dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\left[2+4-\frac{8}{3}-\frac{1}{2}+2-\frac{1}{3}\right]$$

$$= \frac{1}{4}\left[8-\frac{1}{2}-3\right]$$

$$= \frac{1}{4}\left[8-\frac{1}{2}-3\right]$$

$$= \frac{9}{8} \text{ sq. units}$$

Q. 2. The area of the region bounded by the circle $x^2 + y^2 = 1$ is

(A) 2π sq. units **(B)** π sq. units

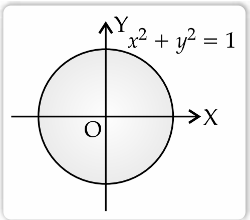
(C) 3π sq. units **(D)** 4π sq. units

Ans. Option (B) is correct.

Explanation : We have, $x^2+y^2=1$, which is a circle having centre at (0, 0) and radius '1' unit.

$$\Rightarrow \quad y^2 = 1-x^2$$

$$y = \sqrt{1-x^2}$$



From the figure, area of the shaded region,

$$A = 4\int_{0}^{1}\sqrt{1^2-x^2}\,dx$$

$$= 4\left[\frac{x}{2}\sqrt{1^2-x^2}+\frac{1^2}{2}\sin^{-1}\frac{x}{1}\right]_0^1$$

$$= 4\left[0+\frac{1^2}{2}\times\frac{\pi}{2}-0-0\right]$$

$$= \pi \text{ sq. units}$$

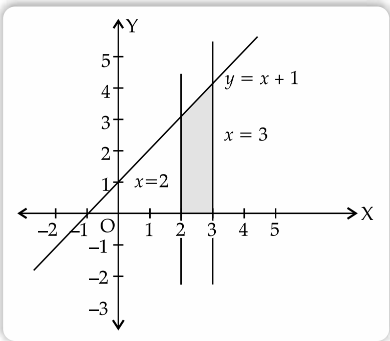
Q. 3. The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$ and $x = 3$ is

(A) $\frac{7}{2}$ sq. units **(B)** $\frac{9}{2}$ sq. units

(C) $\frac{11}{2}$ sq. units **(D)** $\frac{13}{2}$ sq. units

Ans. Option (A) is correct.

Explanation:



From the figure, area of the shaded region,

$$A = \int_{2}^{3}(x+1)dx$$

$$= \left[\frac{x^2}{2}+x\right]_2^3$$

$$= \left[\frac{9}{2}+3-\frac{4}{2}-2\right]$$

$$= \frac{7}{2} \text{ sq. units}$$

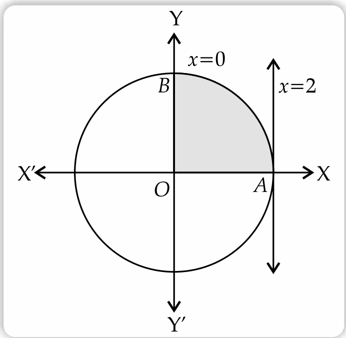
Q. 4. Area lying in the first quadrant and bounded by circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

(A) π **(B)** $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ **(D)** $\frac{\pi}{4}$

Ans. Option (A) is correct.

Explanation : The area bounded by the circle and the lines in the first quadrant is represented as :



$$A = \int_{0}^{2}y\,dx$$

$$= \int_{0}^{2}\sqrt{4-x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= \pi \text{ sq. units}$$

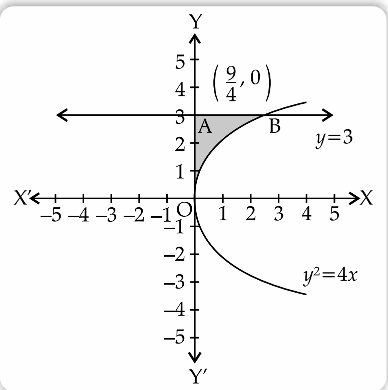
Q. 5. Area of the region bounded by the curve $y^2 = 4x$, Y-axis and the line $y = 3$ is

(A) 2 (B) $\frac{9}{4}$

(C) $\frac{9}{3}$ (D) $\frac{9}{2}$

Ans. Option (B) is correct.

Explanation: The area bounded by the curve, $y^2 = 4x$, Y-axis, and $y = 3$ is represented as :



$$\text{Area of } OAB = \int_0^3 x\,dy$$

$$= \int_0^3 \frac{y^2}{4}\,dy$$

$$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$$

$$= \frac{1}{12} \times 27$$

$$= \frac{9}{4} \text{ sq. units}$$

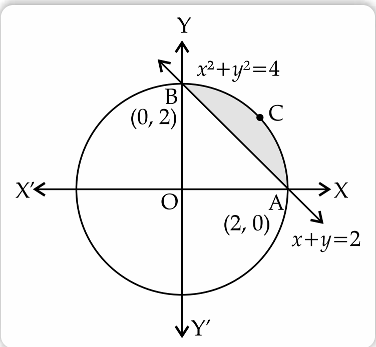
Q. 6. Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$

(A) $2(\pi - 2)$ (B) $\pi - 2$

(C) $2\pi - 1$ (D) $2(\pi + 2)$

Ans. Option (B) is correct.

Explanation: The smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line, $x + y = 2$ is represented by the shaded area ACBA as :



It can be observed that

Area of $ACBA$ = Area of $OACBO$ −Area of ΔAOB

$$A = \int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)\,dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$$

$$= \left[2 \times \frac{\pi}{2}\right] - [4-2]$$

$$= \pi - 2 \text{ sq. units}$$

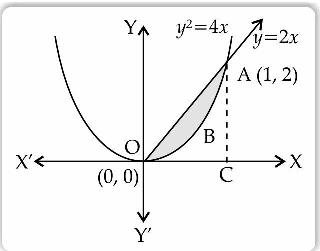
Q. 7. Area lying between the curve $y^2 = 4x$ and $y = 2x$

(a) $\frac{2}{3}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Ans. Option (B) is correct.

Explanation: The area lying between the curve $y^2 = 4x$ and $y = 2x$ is represented by the shaded area $OBAO$ as



The points of intersection of the curves are $O(0, 0)$ and $A(1, 2)$.

We draw AC perpendicular to X-axis such that coordinate of C is $(1, 0)$.

Area of $OBAO$=Area of ΔOCA −Area of $OCABO$

$$A = \int_0^1 2x\,dx - \int_0^1 2\sqrt{x}\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^1$$

$$= \left|\left[1 - \frac{4}{3}\right]\right|$$

$$= \left|-\frac{1}{3}\right|$$

$$= \frac{1}{3} \text{ sq. unit}$$

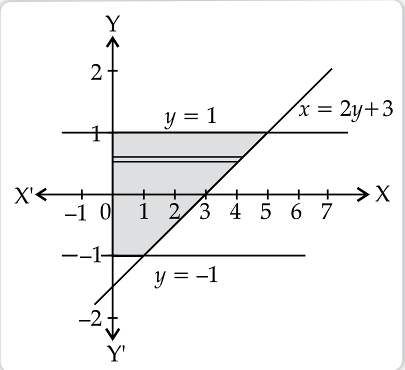
SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. Find the area bounded by $y = x^2$, the X-axis and the lines $x = -1$ and $x = 1$. R [CBSE SQP 2020-21]

Q. 2. Find the area bounded by the curve $x = 2y + 3$, Y-axis and the lines $y = 1$ and $y = -1$. A I

Sol.



From the figure, area of the shaded region,

$$A = \int_{-1}^{1} (2y+3)\,dy$$

$$= \left[y^2 + 3y\right]_{-1}^{1} = [1+3-1+3]$$

$= 6$ sq. units **2**

Short Answer Type Questions-I (2 marks each)

Q. 1. Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$.

A I R&U [CBSE SQP 2020-21]

Q. 2. Find the area of the region bounded by the parabola $y^2 = x$ and the line $2y = x$.

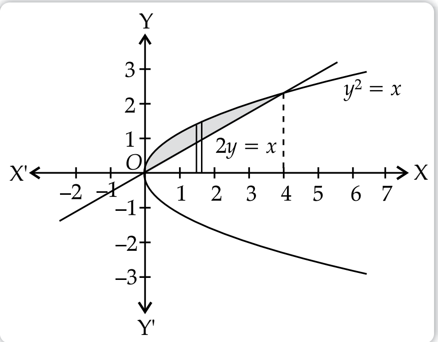
Sol. When $y^2 = x$ and $2y = x$

Solving we get $y^2 = 2y$

$\Rightarrow y = 0, 2$ and when $y = 2, x = 4$

So, points of intersection are (0, 0) and (4, 2).

Graphs of parabola $y^2 = x$ and $2y = x$ are as shown in the adjoining figure :



1

From the figure, area of the shaded region,

$$A = \int_0^4 \left[\sqrt{x} - \frac{x}{2}\right] dx$$

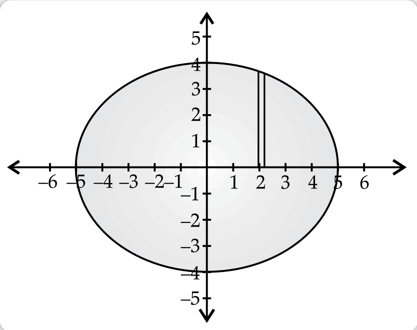
$$= \left[\frac{2}{3}x^{3/2} - \frac{1}{2}\cdot\frac{x^2}{2}\right]_0^4$$

$$= \frac{2}{3}.(4)^{3/2} - \frac{16}{4} - 0 = \frac{16}{3} - 4$$

$= \frac{4}{3}$ sq. units **1**

Q. 3. Find the area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

Sol. We have $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$, which is ellipse with its axes as coordinate axes.



$$\frac{y^2}{4^2} = 1 - \frac{x^2}{5^2}$$

$$y^2 = 16\left(1 - \frac{x^2}{25}\right)$$

$$y = \frac{4}{5}\sqrt{5^2 - x^2}$$

From the figure, area of the shaded region,

$$A = 4\int_0^5 \frac{4}{5}\sqrt{5^2 - x^2}\,dx$$

$$= \frac{16}{5}\left[\frac{x}{2}\sqrt{5^2 - x^2} + \frac{5^2}{2}\sin^{-1}\frac{x}{5}\right]_0^5$$ **1**

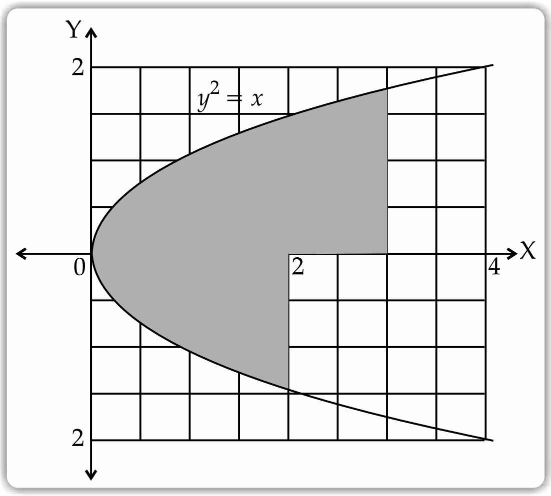
$$= \frac{16}{5}\left[0 + \frac{5^2}{2}\sin^{-1}1 - 0 - 0\right]$$

$$= \frac{16}{5}.\frac{25}{2}.\frac{\pi}{2}$$

$= 20\pi$ sq. units **1**

Short Answer Type Questions-II (3 or 4 marks each)

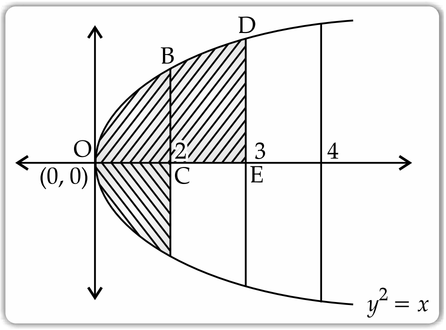
Q. 1. Shown below is a parabola.



Find the area of the shaded region. Show your steps.
(Note: Take $\sqrt{2}$ as 1.4 and $\sqrt{3}$ as 1.7.)

[CBSE Practice Questions 2021-22]

Sol.



Area bounded region

= 2 ar(OBC) + ar(CBDE)

$= 2\int_0^2 \sqrt{x}dx + \int_2^3 \sqrt{x}dx$ **1**

$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 + \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^3$

$= 2\times\frac{2}{3}\left[(2)^{\frac{3}{2}}\right] + \frac{2}{3}\left[(3)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right]$ **1**

$= \frac{4}{3}\times 2\sqrt{2} + \frac{2}{3}\left[3\sqrt{3} - 2\sqrt{2}\right]$

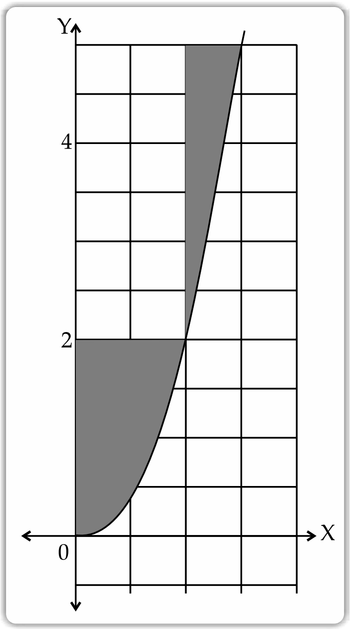
$= 2\sqrt{2}\left(\frac{4}{3} - \frac{2}{3}\right) + \frac{2}{3}\times 3\sqrt{3}$ **1**

$= 2\sqrt{2}\times\frac{2}{3} + 2\sqrt{3}$

$= \frac{4}{3}\sqrt{2} + 2\sqrt{3} = \frac{4}{3}\times 1.4 + 2\times 1.7$

= 1.87 + 3.4 = 5.27 sq. units (Approx.) **1**

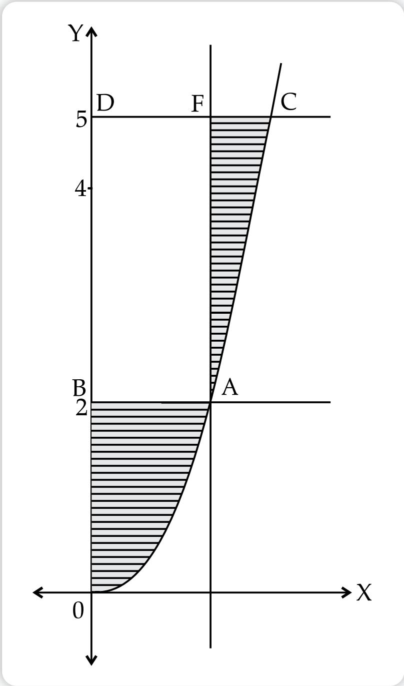
Q. 2. Shown below is the graph of $f(x) = 2x^2$ in the first quadrant.



Find the area of the shaded region. Show your steps.
(Note: You need not evaluate the square roots.)

[CBSE Practice Questions 2021-22]

Sol.



1

Area bounded region

= ar(OACD) – ar rect (ABDF)

$= \int_0^5 xdy$ – length × breadth

$= \int_0^5 \frac{\sqrt{y}}{\sqrt{2}}dy - (1\times 3)$ **2**

[Since, given $y = 2x^2$ or $x = \sqrt{\frac{y}{2}}$]

$$= \frac{1}{\sqrt{2}}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^5 - 3$$

$$= \frac{2}{3\sqrt{2}}(5)^{\frac{3}{2}} - 3$$

$$= \frac{2\times 5\sqrt{5}}{3\sqrt{2}} - 3 = \left(\frac{5\sqrt{10}}{3} - 3\right) \text{ sq. units.} \qquad \mathbf{1}$$

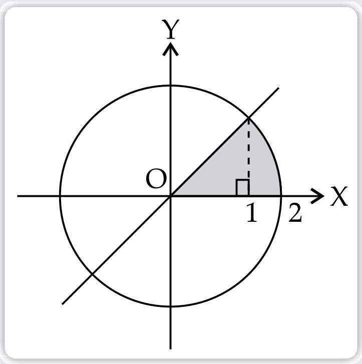
Q. 3. Find the area of the region bounded by the curves $x^2 + y^2 = 4$, $y = \sqrt{3}x$ and X-axis in the first quadrant. A I R&U [CBSE SQP 2020-21]

Sol. Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$

We get $x^2 + 3x^2 = 4$

$\Rightarrow$ $x^2 = 1$

$\Rightarrow$ $x = 1$ ½



½

Required Area $= \sqrt{3}\int_0^1 x dx + \int_1^2 \sqrt{2^2 - x^2}dx$ ½

$$= \frac{\sqrt{3}}{2}[x^2]_0^1 + \left[\frac{x}{2}\sqrt{2^2-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2 \qquad \mathbf{1}$$

$$= \frac{\sqrt{3}}{2} + \left[2\times\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2\times\frac{\pi}{6}\right]$$

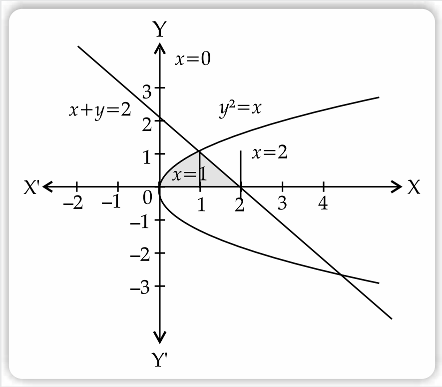
$$= \frac{2\pi}{3} \text{ sq. units} \qquad ½$$

[CBSE SQP Marking Scheme 2020-21]

Q. 4. Find the area of the ellipse $x^2 + 9y^2 = 36$ using integration. A I R&U [CBSE SQP 2020-21]

Q. 5. Using integration, find the area of the region in the first quadrant enclosed by the line $x + y = 2$, the parabola $y^2 = x$ and X-axis. [CBSE SQP 2021-22]

Sol. Solving $x + y = 2$ and $y^2 = x$ simultaneously, we get the points of intersection as (1, 1) and (4, –2). **1**



1

The required area = The shaded area

$$= \int_0^1 \sqrt{x}\,dx + \int_1^2 (2-x)\,dx \qquad \mathbf{1}$$

$$= \frac{2}{3}[x^{3/2}]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$$

$$= \frac{2}{3} + \frac{1}{2} = \frac{7}{6} \text{ square units} \qquad \mathbf{1}$$

[CBSE SQP Marking Scheme 2020-21]

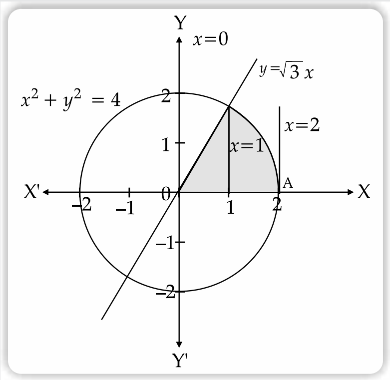
Q. 6. Using integration, find the area of the region

$$\{(x, y): 0 \le y \le \sqrt{3}x,\ x^2 + y^2 \le 4\}$$

[CBSE SQP 2021-22]

Sol. Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$, we get the points of intersection as $(1, \sqrt{3})$ and $(-1, -\sqrt{3})$

1



1

The required area = The shaded area

$$= \int_0^1 \sqrt{3}x\,dx + \int_1^2 \sqrt{4-x^2}\,dx$$

$$= \frac{\sqrt{3}}{2}\left[x^2\right]_0^1 + \frac{1}{2}\left[x\sqrt{4-x^2} + 4\sin^{-1}\frac{x}{2}\right]_1^2 \qquad \mathbf{1}$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}\left[2\pi - \sqrt{3} - 2\frac{\pi}{3}\right]$$

$$= \frac{2\pi}{3} \text{ square units} \qquad 1$$

[CBSE SQP Marking Scheme 2020-21]

Long Answer Type Questions-I (5 marks each)

Q. 1. Using integration find the area of the region bounded between the two circles $x^2 + y^2 = 9$ and $(x-3)^2 + y^2 = 9$.

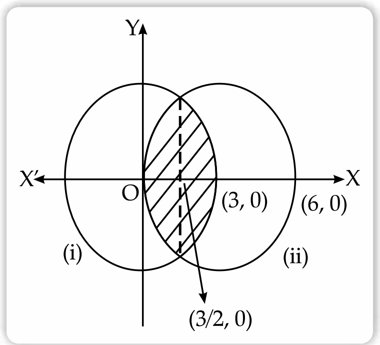
A [CBSE Delhi Set I, II, III-2020]

Sol. Point of intersection of,

$$x^2 + y^2 = 9; (x-3)^2 + y^2 = 9$$

$$\Rightarrow (x-3)^2 - x^2 = 0 \qquad 1$$

$$\Rightarrow x = \frac{3}{2}$$



1

Required area

$$= 2\left[\int_0^{3/2} \sqrt{9-(x-3)^2}\,dx + \int_{3/2}^{3} \sqrt{9-x^2}\,dx\right]$$

$$= 4\left[\int_{3/2}^{3} \sqrt{9-x^2}\,dx\right] \qquad 1$$

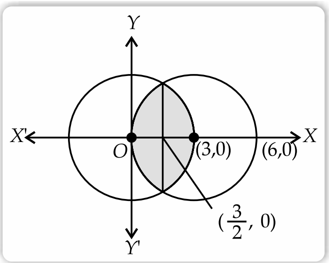
$$= 4\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_{3/2}^{3} \qquad 1$$

$$= \left(6\pi - \frac{9\sqrt{3}}{2}\right) \text{ sq. units} \qquad 1$$

[CBSE Marking Scheme 2020] (Modified)

Detailed Solution:

Let us consider the diagram



Here $\quad x^2 + y^2 = 9 \quad$ ...(i)

$(x-3)^2 + y^2 = 9 \quad$ ...(ii)

are two circles with centres (0, 0) and (3, 0) respectively.

After solving (i) and (ii),

$$9 - 6x = 0$$

$$\Rightarrow x = \frac{3}{2}$$

Then required area $= 2\int_0^{3/2} \sqrt{9-(x-3)^2}\,dx + 2\int_{3/2}^{3} \sqrt{9-x^2}\,dx$

$$= 2\left[\frac{(x-3)}{2}\sqrt{9-(x-3)^2} + \frac{9}{2}\sin^{-1}\frac{(x-3)}{3}\right]_0^{3/2}$$

$$+2\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_{3/2}^{3}$$

$$\Rightarrow \text{Area} = 2\left\{\left[\frac{-3}{4}\sqrt{9-\frac{9}{4}} + \frac{9}{2}\sin^{-1}\left(\frac{-1}{2}\right)\right] - \left[0 + \frac{9}{2}\sin^{-1}(-1)\right]\right.$$

$$\left. + \left[0 + \frac{9}{2}\sin^{-1}1\right] - \left[\frac{3}{4}\sqrt{9-\frac{9}{4}} + \frac{9}{2}\sin^{-1}\frac{1}{2}\right]\right\}$$

$$\Rightarrow \text{Area} = 2\left\{-\frac{9\sqrt{3}}{8} - \frac{9}{2}\times\frac{\pi}{6} + \frac{9}{2}\times\frac{\pi}{2} + \frac{9}{2}\times\frac{\pi}{2} - \frac{9\sqrt{3}}{8} - \frac{9}{2}\times\frac{\pi}{6}\right\}$$

$$\Rightarrow \text{Area} = 2\left\{-2\times\frac{9\sqrt{3}}{8} - 9\times\frac{\pi}{6} + \frac{9}{2}\times\pi\right\}$$

$$\Rightarrow \text{Area} = 2\left\{3\pi - \frac{9\sqrt{3}}{4}\right\} \text{ sq. units}$$

Commonly Made Error

- Mostly students fail to correct integrate and apply the limits.

Answering Tip

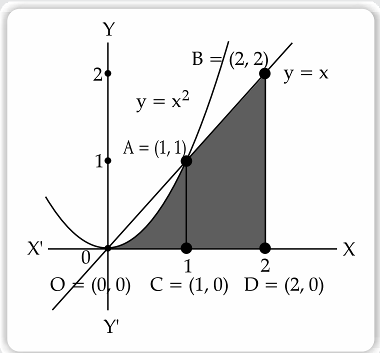
- Practice more problems in integration.

Q. 2. Using integration find the area of the region : $\{(x, y) : 0 \le y \le x^2,\ 0 \le y \le x, 0 \le x \le 2\}$

R&U [CBSE OD Set II-2020]

Sol. Parabola $y = x^2$ and line $y = x$ intersect at (0, 0) and (1, 1). 1

Correct Figure 1



Required Area $= \int_0^1 x^2 dx + \int_1^2 x\,dx$ **1**

$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2$ **1**

$= \frac{1}{3} + \frac{3}{2} = \frac{11}{6}$ sq. units **1**

[CBSE Marking Scheme 2020] (Modified)

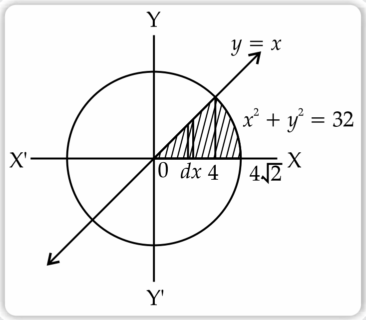
Q. 3. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

AI R&U [CBSE OD-2018] [NCERT]

[CBSE Set I-2020]

Sol. Correct figure : **1**

Pt. of intersection, $x = 4$ **1**



Area of shaded region

$= \int_0^4 y_1\,dx + \int_4^{4\sqrt{2}} y_2 dx$

$= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$ **1**

$= \left[\frac{x^2}{2}\right]_0^4 + \left\{\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right\}_4^{4\sqrt{2}}$ **1**

$= 8 + 16\frac{\pi}{2} - 8 - 4\pi = 4\pi$ sq. units **1**

[CBSE Marking Scheme 2018] (Modified)

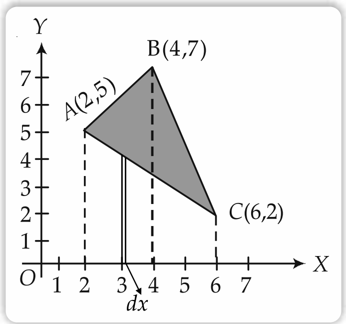
Q. 4. Find the area of the region $\{(x, y) : x^2 + y^2 \le 1 \le (x + y)\}$.

AI A [CBSE SQP-2020][Foreign 2017]

Q. 5. Using integration, find the area of triangle ABC, whose vertices are $A(2, 5)$, $B(4, 7)$ and $C(6, 2)$.

AI AE [CBSE Delhi Set-I 2019]

Sol.



1

Equation of AB : $y = x + 3$

Equation of BC : $y = \frac{-5x}{2} + 17$

Equation of AC : $y = \frac{-3x}{4} + \frac{13}{2}$ **1**

Required Area $= \int_2^4 (x+3)\,dx + \int_4^6 \left(\frac{-5x}{2} + 17\right)dx - \int_2^6 \left(\frac{-3x}{4} + \frac{13}{2}\right)dx$ **1**

$= \left[\frac{(x+3)^2}{2}\right]_2^4 + \left[\frac{-5x^2}{2} + 17x\right]_4^6 - \left[\frac{-3x^2}{8} + \frac{13x}{2}\right]_2^6$ **1½**

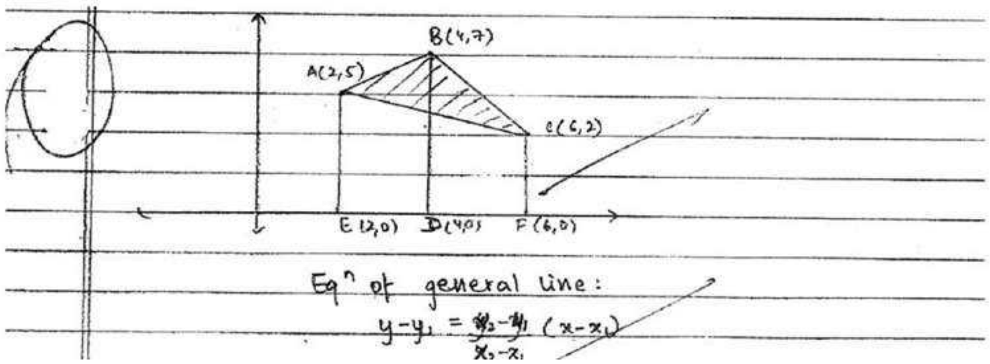
$= 7$ sq. units **½**

[CBSE Marking Scheme, 2019] (Modified)

OR

Topper Answer, 2019

Sol.



Eq^n of line AB : ~~Eq^n of li~~

$(y-5) = \frac{2}{2}(x-2)$

$\boxed{y_1 = x+3}$

Eq^n of line BC :

$(y-2) = \frac{5}{-2}(x-6)$

$\Rightarrow y-2 = -\frac{5}{2}x + 15$

$\Rightarrow \boxed{y_2 = -\frac{5x}{2} + 17}$

Eq^n of line AC

$(y-5) = \frac{3}{-4}(x-2)$

$y-5 = -\frac{3}{4}x + \frac{3}{2}$

$\boxed{y_3 = -\frac{3x}{4} + \frac{13}{2}}$

Area of shaded region is required area A = ar(ABDE) + ar(BCFD) − a

$$A = \int_2^4 y_1\,dx + \int_4^6 y_2\,dx - \int_2^6 y_3\,dx$$

$$= \int_2^4 (x+3)\,dx + \int_4^6 \left(-\frac{5x}{2}+17\right)dx - \int_2^6 \left(-\frac{3x}{4}+\frac{13}{2}\right)dx$$

$$A = \left[\frac{x^2}{2}+3x\right]_2^4 + \left[17x - \frac{5x^2}{4}\right]_4^6 - \left[\frac{13x}{2} - \frac{3x^2}{8}\right]_2^6$$

$$= [8+12-2-6] + [102-45-68+20] - \left[39 - \frac{27}{2} - 13 + \frac{3}{2}\right]$$

$= 12 + 9 - 14$

$\boxed{A = 7 \text{ sq. units}}$

Commonly Made Error

- Sometimes students do not apply correct limits or consider area twice the result.

Answering Tip

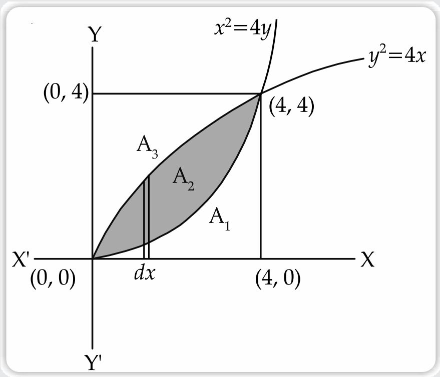
- Learn to apply limits correctly to avoid errors.

Q. 6. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ into three equal parts.

R&U [NCERT] [OD 2015,2016]

[CBSE Delhi Set III-2019]

Sol.



1

Point of intersection are (0, 0) and (4, 4) ½

Here, $A_1 = \int_0^4 \frac{x^2}{4}dx = \frac{16}{3}$ Sq. units ...(1) **1**

$A_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx = \frac{16}{3}$ Sq. units ...(2) **1**

$$A_3 = \int_0^4 \frac{y^2}{4} dy = \frac{16}{3} \text{ Sq. units} \quad ...(3) \ \mathbf{1}$$

From (1), (2) and (3), $A_1 = A_2 = A_3$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution :

Let *OABC* be the square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$.

$Ar(OABC) = 4 \times 4$

$= 16$ sq. units

From, $y^2 = 4x$ and $x^2 = 4y$

$$\left(\frac{x^2}{4}\right)^2 = 4x$$

or $$\frac{x^4}{16} = 4x$$

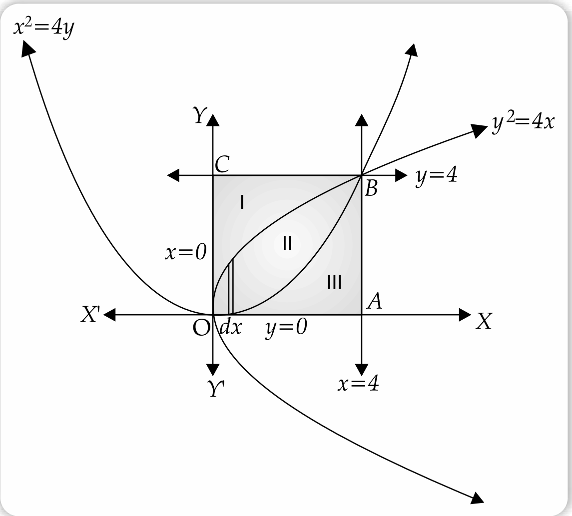
or $$x^4 - 64x = 0$$

or $$x(x^3 - 64) = 0$$

or $$x = 0 \text{ or } x = 4$$

When $$x = 0, y = 0$$

$$x = 4, y = 4$$



∴ Point of intersection of the two parabolas is (0, 0) and (4, 4).

$$\text{Area of part III} = \int_0^4 y\,dx (\text{parabola } x^2 = 4y)$$

$$= \int_0^4 \frac{x^2}{4} dx = \left[\frac{1}{4}\frac{x^3}{3}\right]_0^4$$

$$= \frac{1}{12}(64 - 0) = \frac{64}{12}$$

$$= \frac{16}{3} \text{ sq. units}$$

$$\text{Area of I} = \int_0^4 x\,dy$$

$$= \int_0^4 \frac{y^2}{4} dy = \left[\frac{y^3}{12}\right]_0^4$$

$$= \frac{16}{3} \text{ sq. units}$$

Area of II = Area of square – Area of I – Area of III

$$= 16 - \frac{16}{3} - \frac{16}{3} \text{ sq. units}$$

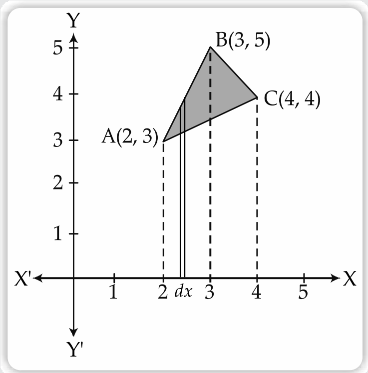
$$= \frac{16}{3} \text{ sq. units}$$

∴ The two curves divide the square into three equal parts.

Q. 7. Using integration, find the area of the triangle whose vertices are (2, 3), (3, 5) and (4, 4).

A I AE **[CBSE Delhi Set-III 2019]**

Sol.



1

Equation of AB: $y = 2x - 1$

Equation of BC: $y = -x + 8$

Equation of AC: $y = \frac{1}{2}(x + 4)$ 1½

$$\text{Required Area} = \int_2^3 (2x-1)dx + \int_3^4 (-x+8)dx - \int_2^4 \left(\frac{x+4}{2}\right)dx$$

1

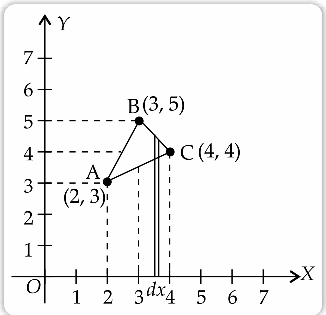
$$= \left[x^2 - x\right]_2^3 + \left[\frac{-x^2}{2} + 8x\right]_3^4 - \frac{1}{2}\left[\frac{x^2}{2} + 4x\right]_2^4 \quad \mathbf{1}$$

$$= 4 + \frac{9}{2} - 7 = \frac{3}{2} \text{ Sq. units} \quad ½$$

[CBSE Marking Scheme, 2019] (Modified)

Detailed Solution :

Given, vertices of triangle are (2, 3), (3, 5), (4, 4)



Equation of line AB,

$$y-3 = \frac{5-3}{3-2}(x-2)$$

[Using formula, $y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$]

$$\Rightarrow \quad y-3 = \frac{2}{1}(x-2)$$

$$\Rightarrow \quad y = 2x-1$$

Equation of line BC,

$$y-5 = \frac{4-5}{4-3}(x-3)$$

$$\Rightarrow \quad y-5 = \frac{-1}{1}(x-3)$$

$$\Rightarrow \quad y = -x+8$$

Equation of line AC,

$$y-3 = \frac{4-3}{4-2}(x-2)$$

$$\Rightarrow \quad y-3 = \frac{1}{2}(x-2)$$

$$\Rightarrow \quad y = \frac{x}{2}+2$$

Hence, required area is

$$= \int_2^3 (2x-1)dx + \int_3^4 (-x+8)dx - \int_2^4 \left(\frac{x}{2}+2\right)dx$$

$$= \left[2\left(\frac{x^2}{2}\right)-x\right]_2^3 + \left[\frac{-x^2}{2}+8x\right]_3^4 - \left[\frac{1}{2}\frac{x^2}{2}+2x\right]_2^4$$

$$= [(9-3)-(4-2)]+\left[\left(-\frac{16}{2}+32\right)-\left(-\frac{9}{2}+24\right)\right] - \left[\left(\frac{16}{4}+8\right)-\left(\frac{4}{4}+4\right)\right]$$

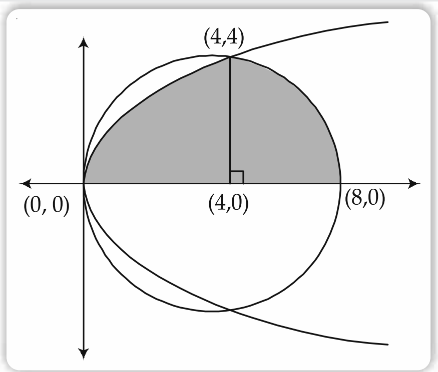
$$= 4+\frac{9}{2}-7$$

$$= -3+\frac{9}{2} = \frac{-6+9}{2} = \frac{3}{2} \text{ sq. units}$$

Q. 8. Find the area of the region lying above X-axis and included between the circle $x^2+y^2=8x$ and inside of the parabola $y^2=4x$.

AE **[CBSE Delhi Set-I-2019]**

Sol.



Correct Figure **1**

Given circle $x^2-8x+y^2=0$

Correct Figure **1**

Given circle $x^2-8x+y^2=0$

or $(x-4)^2+y^2=4^2$

Point of intersection (0, 0) and (4, 4) **1**

$$\text{Required Area} = \int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{4^2-(x-4)^2}\,dx \quad \textbf{1}$$

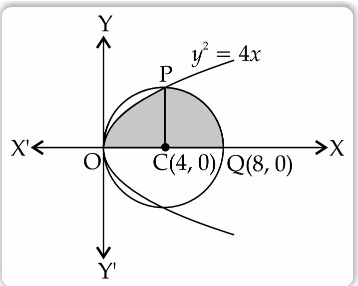
$$= \left[\frac{4}{3}x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8 \quad \textbf{1}$$

$$= \left(4\pi+\frac{32}{3}\right) \text{ sq. units} \quad \textbf{1}$$

[CBSE Marking Scheme, 2019] (Modified)

Detailed Solution :

The equation of circle is $x^2+y^2=8x$ ...(i)



The equation of parabola is $y^2=4x$ ...(ii)

eq. (i) can be re-written as

$$(x^2-8x)+y^2 = 0$$

or $\quad (x^2-8x+16)+y^2 = 16$

or $\quad (x-4)^2+y^2 = (4)^2$ ...(iii)

Which is a circle with centre $C(4, 0)$ and radius $=4$.

From eq. (i) & (ii), we get

$$x^2+4x = 8x$$

or $\quad x^2-4x = 0$

or $\quad x(x-4) = 0$

$\therefore \quad x = 0, 4$

$\therefore$ Points of intersection of circle (i) and parabola (ii) and $O(0, 0)$ and $P(4, 4)$, above the x-axis.

Therefore required area = area of region $OPQCO$

= (area of region $OCPO$) + (area of region $PCQP$)

$$= \int_0^4 y\,dx + \int_4^8 y\,dx$$

$$= 2\int_0^4 \sqrt{x}\,dx + \int_4^8 \sqrt{(4)^2-(x-4)^2}\,dx$$

$$= 2\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{(4)^2-(x-4)^2} + \frac{(4)^2}{2}\sin^{-1}\frac{x-4}{4}\right]_4^8$$

$$= \frac{4}{3}\left[x^{3/2}\right]_0^4 + \left[\left\{\frac{8-4}{2}\right\}\sqrt{16-16}+8\sin^{-1}(1)\right]$$

$$-[0+8\sin^{-1}0]$$

$$= \frac{4}{3}\left[4^{3/2} - 0\right] + \left[0 + 8 \times \frac{\pi}{2}\right] - (0 - 0)$$

$$= \frac{4}{3} \times 8 + 4\pi = \frac{32}{3} + 4\pi = \frac{4}{3}(8 + 3\pi) \text{ sq. units}$$

Commonly Made Error

- Students fail to identify the correct figure and go wrong.

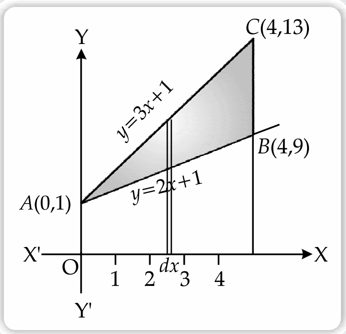
Answering Tip

- Familiarize with the different equations and forms of conics.

Q. 9. Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

A I R&U [CBSE OD Set II-2019]

Sol.



1

Getting the points of intersection as $A(0, 1)$, $B(4, 9)$ and $C(4, 13)$ 1½

$$\text{Area of } \Delta ABC = \int_0^4 (3x+1)dx - \int_0^4 (2x+1)dx \quad 1$$

$$= \int_0^4 x\,dx = \left[\frac{x^2}{2}\right]_0^4 \quad 1$$

= 8 sq. units. ½

[CBSE Marking Scheme 2019] (Modified)

Detailed Solution :

Points of intersections of :

(i) Line AB ($y = 3x + 1$) and AC ($y = 2x + 1$) is $A(0, 1)$.

(ii) Line AB ($y = 3x + 1$) and BC ($x = 4$) is B (4, 13).

(iii) Line AC ($y = 2x + 1$) and BC ($x = 4$) is C (4, 9).

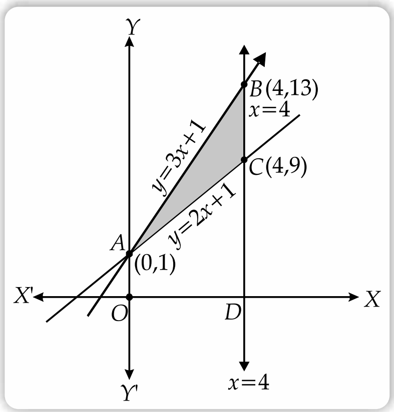
Required area of shaded portion

= Area of trapezium $AODB$

– Area of trapezium $AODC$

$$= \int_{AB} y\,dx - \int_{AC} y\,dx$$

$$= \int_0^4 (3x+1)\,dx - \int_0^4 (2x+1)\,dx$$



$$= \left[\frac{3x^2}{2} + x\right]_0^4 - \left[x^2 + x\right]_0^4$$

= (24 + 4) – (16 + 4)

= 8 sq. units.

Q. 10. Find the area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and x – axis. A [S.Q.P. 2018-19]

Sol. The given curves are

$$y = \sqrt{x} \quad ...(1)$$

$$2y + 3 = x \quad ...(2)$$

Solving equation (1) and (2), we get

$$2y + 3 = y^2$$

$$y^2 - 2y - 3 = 0$$

$$(y - 3)(y + 1) = 0$$

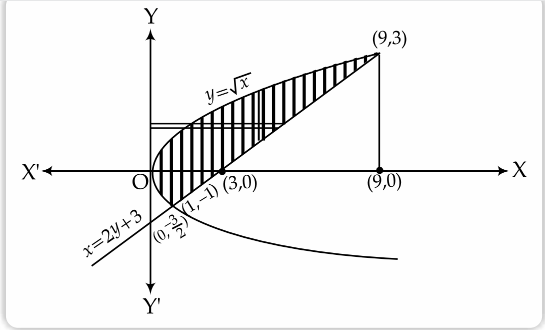
$\Rightarrow$ $y = -1, 3$

$\Rightarrow$ $y = 3$ (as $y > 0$) ½

substituting value of $y = 3$ in (2) we get

$$x = 2(3) + 3 = 9$$

i.e., (1) and (2) intersects at (9, 3) 1



1

$$\text{Required Area} = \int_0^3 (2y+3)dy - \int_0^3 y^2 dy \quad 1$$

$$= \left[y^2 + 3y - \frac{y^3}{3}\right]_0^3 \quad 1$$

= 9 + 9 – 9

= 9 sq units. ½

[CBSE Marking Scheme 2018] (Modified)

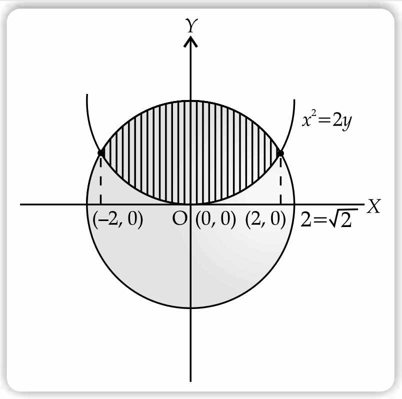
Q. 11. Find the area of the region.

$\{(x, y) : x^2 + y^2 \leq 8, x^2 \leq 2y\}$ A [S.Q.P. 2018-19]

Sol. The given curves are

$$x^2 + y^2 = 8 \quad ...(1)$$

$$x^2 = 2y \quad ...(2)$$



1

Solving (1) and (2)

$8 - y^2 = 2y \Rightarrow y = 2, -4 \Rightarrow y = 2$ (as $y > 0$)

Substituting $y = 2$ in (2) we get $x^2 = 4 \Rightarrow x = -2$ or 2 **1**

$$\text{Required Area} = \int_{-2}^{2} \sqrt{8 - x^2}\, dx - \int_{-2}^{2} \frac{x^2}{2} dx \quad \mathbf{1}$$

$$= 2\left[\int_0^2 \sqrt{(2\sqrt{2})^2 - x^2}\, dx - \int_0^2 \frac{x^2}{2} dx\right]$$

$$= 2\left[\frac{x}{2}\sqrt{8 - x^2} + \frac{8}{2}\sin^{-1}\left(\frac{x}{2\sqrt{2}}\right)\right]_0^2 - \frac{1}{3}\left[x^3\right]_0^2 \quad \mathbf{1}$$

$$= 2\left[2 + 4\left(\frac{\pi}{4}\right) - 0\right] - \frac{1}{3}[8 - 0]$$

$$= 4 + 2\pi - \frac{8}{3}$$

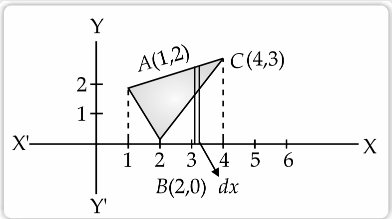
$$= \left(2\pi + \frac{4}{3}\right) \text{ sq. units} \quad \mathbf{1}$$

[CBSE Marking Scheme 2018] (Modified)

Q. 12. Using integration, find the area of the region bounded by the line $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$. R&U [NCERT]

[Foreign 2017]

Sol. Let the line AB, BC and CA have equations $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$ respectively. $B(2, 0)$, $C(4, 3)$ and $A(1, 2)$



2

$$\text{Area} = \int_1^4 \frac{1}{3}(x+5)dx - \int_1^2 (4-2x)dx$$

$$-\int_2^4 \frac{1}{2}(3x-6)dx \quad \mathbf{1}$$

$$= \frac{1}{3}\left[\frac{(x+5)^2}{2}\right]_1^4 + 2\left[\frac{(2-x)^2}{2}\right]_1^2 - \frac{3}{2}\left[\frac{(x-2)^2}{2}\right]_2^4 \quad \mathbf{1}$$

$$= \left(\frac{81}{6} - \frac{36}{6}\right) + (0-1) - \frac{3}{4}\cdot 4$$

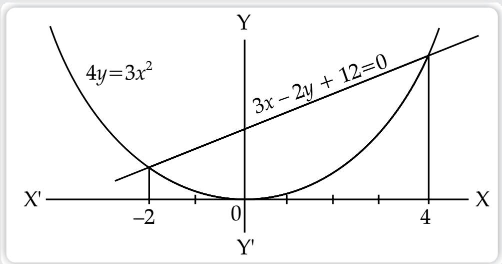
$$= \frac{15}{2} - 1 - 3 = \frac{7}{2} \text{ sq. units.} \quad \mathbf{1}$$

[CBSE Marking Scheme 2017] (Modified)

Q. 13. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$. AI R&U [O.D, 2017]

[NCERT] [O.D. Set I, II, III, Comptt. 2015]

Sol.



1

$4y = 3x^2$ and $3x - 2y + 12 = 0$ or $4\left(\frac{3x+12}{2}\right) = 3x^2$

or $3x^2 - 6x - 24 = 0$ or $x^2 - 2x - 8 = 0$ or $(x - 4)(x + 2) = 0$

or x-coordinates of points of intersection are $x = -2, x = 4$ **1**

$$\therefore \quad \text{Area } (A) = \int_{-2}^{4}\left[\frac{1}{2}(3x+12) - \frac{3}{4}x^2\right]dx \quad \mathbf{1}$$

$$= \left[\frac{1}{2}\frac{(3x+12)^2}{6} - \frac{3}{4}\frac{x^3}{3}\right]_{-2}^{4} \quad \mathbf{1}$$

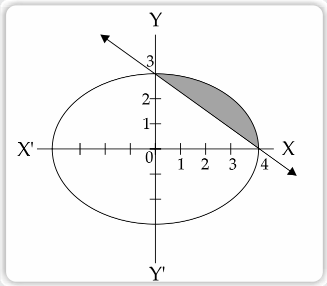
$$= 45 - 18 = 27 \text{ sq. units} \quad \mathbf{1}$$

[CBSE Marking Scheme 2017] (Modified)

Q. 14. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the straight line $3x + 4y = 12$.

R&U [Outside Delhi Set I, II, III, Comptt. 2016]

Sol.



1

Getting the points of intersection as (4, 0), (0, 3). **1**

$\therefore$ Required area

$$= \int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx - \frac{1}{4}\int_0^4 (12-3x)dx \quad \mathbf{1}$$

$$= \left[\frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2} + 8\sin^{-1}\frac{x}{4}\right] - \frac{1}{4}\left(12x - \frac{3x^2}{2}\right)\right]_0^4 \quad \mathbf{1}$$

$$= \left(\frac{3}{4}\cdot 8\cdot\frac{\pi}{2} - 6\right)$$

$$= (3\pi - 6) \text{ sq. units} \quad \mathbf{1}$$

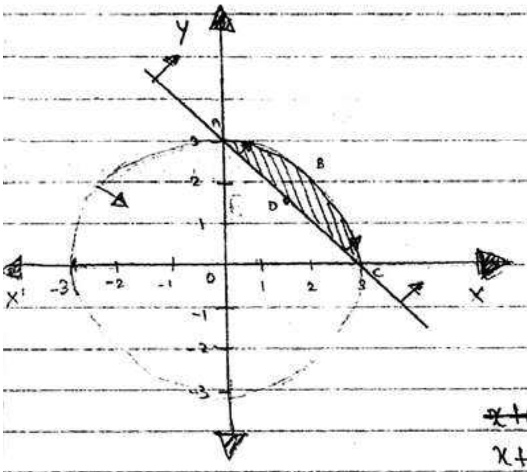
[CBSE Marking Scheme 2016] (Modified)

Long Answer Type Questions-II (6 marks each)

Q. 1. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \le 9, x + y \ge 3\}$ **.** **[Board CBSE 2020]**

Topper Answer, 2017

Sol.



$x^2+y^2 \le 9$ is the equation of a circle with centre at origin and radius = 3 cm, O(0,0) satisfies the inequation $x^2+y^2 \le 9$

$\therefore$ The area required is within the circle

~~x+y ≥ 3 is th~~

$x+y = 3$ is the equation of a line passing through (0,3) and (3,0)

O(0,0) does not satisfy the inequation.

The area required is the area above the line

$\therefore$ The required area is represented by ~~BCBD~~. ADCB

Required area $= \int_0^3 y_{circle} - y_{line}\,dx$

$= \int_0^3 \sqrt{9-x^2} - (3-x)\,dx$

$= \int_0^3 \sqrt{9-x^2} + x - 3\,dx$

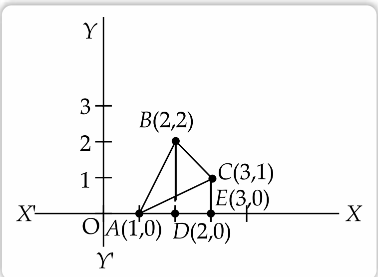
$= \left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{x^2}{2} - 3x\right]_0^3$

$= \frac{9\pi}{4}$ ~~square units~~ $\left(\frac{9\pi}{4} - \frac{9}{2}\right)$ square units

$= \frac{9}{2}\left(\frac{\pi}{2} - 1\right)$ square units

Q. 2. Using method of integration, find the area of the triangle whose vertices are (1, 0), (2, 2) and (3, 1).

Sol. ΔABC is a shown in figure



1

Equation of line between A(1, 0) and B(2. 2) is

$$\frac{y-0}{x-1} = \frac{2-0}{2-1}$$

$$\Rightarrow \quad y = 2(x-1) \qquad \text{½}$$

$$\therefore \quad \text{Area } \Delta ABD = \int_1^2 y\,dx$$

$$= 2\int_1^2 (x-1)dx$$

$$= 2\left[\frac{x^2}{2} - x\right]_1^2 = 1 \qquad \mathbf{1}$$

Equation of line between B(2, 2) and C(3, 1) is

$$\frac{y-2}{x-2} = \frac{1-2}{3-2}$$

$$\Rightarrow \quad y = 4 - x \qquad \text{½}$$

$$\therefore \quad \text{Area BDEC} = \int_2^3 (4-x)dx$$

$$= 4\int_2^3 dx - \int_2^3 x\,dx$$

$$= 4[x]_2^3 - \left[\frac{x^2}{2}\right]_2^3$$

$$= 4(3-2) - \left(\frac{3^2}{2} - \frac{2^2}{2}\right)$$

$$= \frac{3}{2} \qquad \mathbf{1}$$

Equation of lien between A(1, 0) and C(3, 1) is

$$\frac{y-0}{x-1} = \frac{1-0}{3-1}$$

$$\Rightarrow \quad \frac{y}{x-1} = \frac{1}{2}$$

$$\Rightarrow \quad y = \frac{1}{2}(x-1) \qquad \text{½}$$

$$\therefore \quad \text{Area DACE} = \int_1^3 y\,dx$$

$$= \frac{1}{2}\int_1^3 (x-1)dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2} - x\right]_1^3$$

$$= \frac{1}{2}\left[\left(\frac{3^2}{2} - 3\right) - \left(\frac{1^2}{2} - 1\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{9}{2} - 3\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= \frac{1}{2}\left[\frac{3}{2} + \frac{1}{2}\right] = \frac{1}{2} \times 2 = 1 \qquad \mathbf{1}$$

Hence, Required Area

= Area ABD + Area BDEC – Area ACE

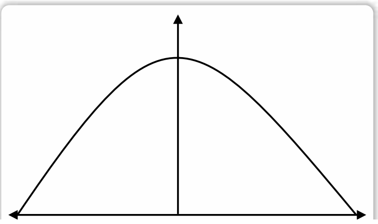
$= \left(1 + \frac{3}{2} - 1\right)$

Area of ΔABC = $\frac{3}{2}$ square units

COMPETENCY BASED QUESTIONS

Case based MCQs

I. Read the following text and answer the following questions on the basis of the same:



The bridge connects two hills 100 feet apart. The arch on the bridge is in a parabolic form. The highest point on the bridge is 10 feet above the road at the middle of the bridge as shown in the figure.

[CBSE QB-2021]

Q. 1. The equation of the parabola designed on the bridge is

(A) $x^2 = 250y$ **(B)** $x^2 = -250y$

(C) $y^2 = 250x$ **(D)** $y^2 = 250y$

Ans. Option (B) is correct.

Q. 2. The value of the integral $\int_{-50}^{50} \frac{x^2}{250} dx$ is

(A) $\frac{1000}{3}$ **(B)** $\frac{250}{3}$

(C) 1200 **(D)** 0

Ans. Option (B) is correct.

Explanation:

$$\int_{-50}^{50} \frac{x^2}{250} dx = \frac{1}{250}\left[\frac{x^3}{3}\right]_{-50}^{50}$$

$$= \frac{1}{250} \times \frac{1}{3}\left[(50)^3 - (-50)^3\right]$$

$$= \frac{1}{750}[125000 + 125000]$$

$$= \frac{1000}{3}$$

Q. 3. The integrand of the integral $\int_{-50}^{50} x^2\, dx$ is ________ function.

(A) Even **(B)** Odd

(C) Neither odd nor even **(D)** None of these

Ans. Option (A) is correct.

Explanation:

$$f(x) = x^2$$

$$f(-x) = x^2$$

$\therefore$ $f(x)$ is even function.

Q. 4. The area formed by the curve $x^2 = 250y$, X-axis, $y = 0$ and $y = 10$ is

(A) $\frac{1000\sqrt{2}}{3}$ **(B)** $\frac{4}{3}$

(C) $\frac{1000}{3}$ **(D)** 0

Ans. Option (C) is correct.

Explanation:

$$x^2 = 250y$$

$$y = \frac{1}{250}x^2$$

at $y = 0$ $x = 0$

at $y = 10$ $x = 50, -50$

$\therefore$ Area formed by curve

$$= \int_{-50}^{50} \frac{1}{250}x^2 dx$$

$$= \frac{1}{250} \times \frac{1}{3}\left[x^3\right]_0^{50}$$

$$= \frac{1}{750}[250,000]$$

$$= \frac{1000}{3} \text{ sq. units}$$

Q. 5. The area formed between $x^2 = 250y$, Y-axis, $y = 2$ and $y = 4$ is

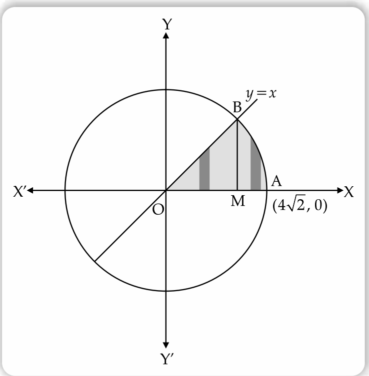
(A) $\frac{1000}{3}$ **(B)** 0

(C) $\frac{1000\sqrt{2}}{3}$ **(D)** None of these

Ans. Option (D) is correct.

II. Read the following text and answer the following questions on the basis of the same:

In the figure $O(0, 0)$ is the centre of the circle. The line $y = x$ meets the circle in the first quadrant at the point B.



Q. 1. The equation of the circle is _______.

(A) $x^2 + y^2 = 4\sqrt{2}$ **(B)** $x^2 + y^2 = 16$

(C) $x^2 + y^2 = 32$ **(D)** $(x - 4\sqrt{2})^2 + 0$

Ans. Option (C) is correct.

Explanation:

$$\text{Centre} = (0, 0),$$
$$r = 4\sqrt{2}$$

Equation of circle is

$$x^2 + y^2 = (4\sqrt{2})^2$$
$$\Rightarrow \quad x^2 + y^2 = 32$$

Q. 2. The co-ordinates of *B* are _______.

(A) (1, 1) **(B)** (2, 2)

(C) $(4\sqrt{2}, 4\sqrt{2})$ **(D)** (4, 4)

Ans. Option (D) is correct.

Explanation:

$$x^2 + y^2 = 32 \quad ...(i)$$
$$y = x \quad ...(ii)$$

Solving (i) and (ii),

$$\Rightarrow x^2 + y^2 = 32$$
$$\Rightarrow x^2 = 16$$
$$\Rightarrow x = 4,$$
$$\Rightarrow y = x = 4$$
$$\therefore B = (4, 4)$$

Q. 3. Area of ΔOBM is _______ sq. units

(A) 8 **(B)** 16

(C) 32 **(D)** 32π

Ans. Option (A) is correct.

Explanation:

$$Ar(\Delta OBM) = \int_0^4 x\,dx = \left[\frac{x^2}{2}\right]_0^4$$
$$= 8 \text{ sq. units}$$

Q. 4. *Ar* (*BAMB*) = _______ sq. units

(A) 32π **(B)** 4π

(C) 8 **(D)** $4\pi - 8$

Ans. Option (D) is correct.

Explanation:

$$Ar(BAMB) = \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$$
$$= \left[\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$
$$= (4\pi - 8) \text{ sq. units.}$$

Q. 5. Area of the shaded region is _______ sq. units.

(A) 32π **(B)** 4π

(C) 8 **(D)** $4\pi - 8$

Ans. Option (B) is correct.

Explanation:

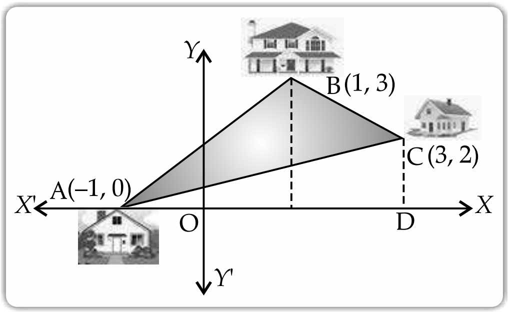
Area of shaded region

$$= Ar(\Delta OBM) + Ar(BAMB)$$
$$= 8 + 4\pi - 8$$
$$= 4\pi \text{ sq. units}$$

Case based Subjective Questions

I. Read the following text and answer the following questions on the basis of the same:

Three friends Amit, Sumit and Rahul living in a society. The location of their houses in the society forms a triangular shape. The location of three houses of that society is represented by the points A(–1, 0), B(1, 3) and C(3, 2) as shown in given figure.



Q. 1. Find the equation of line AB and BC. **2**

Sol. Equation of line AB is

$$y - 0 = \frac{3 - 0}{1 - (-1)}[x - (-1)]$$

[Since, equation of line passing through (x_1, y_1) and (x_2, y_2) is $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$]

$$\therefore \quad y = \frac{3}{2}(x + 1) \quad \mathbf{1}$$

Similarly, equation of line BC is

$$y - 3 = \frac{2 - 3}{3 - 1}(x - 1)$$
$$\Rightarrow y - 3 = \frac{-1}{2}(x - 1)$$
$$\Rightarrow y = \frac{-1}{2}x + \frac{7}{2} \quad \mathbf{1}$$

Q. 2. Find the area of region ABCD. **2**

Sol. Area of region ABCD

= Area of ΔABC + Area of ΔADC ...(i)

Now, area of ΔABC

$$= \frac{1}{2}|-1(3-2) + 1(2-0) + 3(0-3)|$$
$$= \frac{1}{2}|-1 + 2 - 9|$$
$$= \frac{1}{2} \times 8 = 4 \text{ sq. units} \quad \mathbf{1}$$

and Area of ΔADC $= \frac{1}{2} \times \text{base} \times \text{height}$

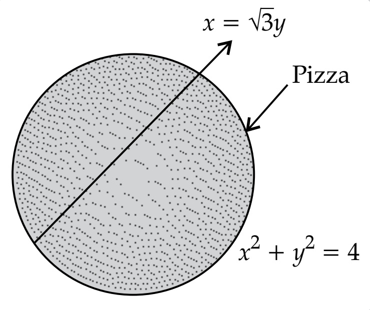
$$= \frac{1}{2} \times AD \times CD = \frac{1}{2} \times 4 \times 2$$
$$= 4 \text{ sq. units} \quad ½$$

From eq. (i), we get

Area of region ABCD = 4 + 4 = 8 sq. units ½

II. Read the following text and answer the question on the basis of the same.

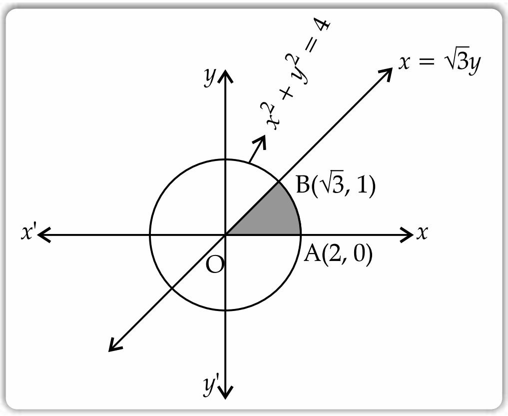
Riya loves to eat pizza. On her birthday, she invites her friends for pizza party in a famous pizza shop. She decided to cut pizza as her birthday cake. Riya cuts the pizza with a knife. Pizza is a circular in shape which is represented by $x^2 + y^2 = 4$ and the sharp edge of knife represents a straight line given by $x = \sqrt{3}y$.



Q. 1. Find the value of the region bounded by the circular pizza and the edge of the knife in the first quadrant. **2**

Sol. Required area $= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}dx + \int_{\sqrt{3}}^{2} \sqrt{4-x^2}dx$

$$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$$



$$= \frac{1}{\sqrt{3}}\left[\frac{3}{2} - 0\right] + \left[2\sin^{-1}(1) - \left(\frac{\sqrt{3}}{2} + 2\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{\sqrt{3}}{2} + \frac{2\pi}{2} - \frac{\sqrt{3}}{2} - \frac{2\pi}{3}$$

$$= \frac{\pi}{3} \text{ sq. units}$$ **2**

Q. 2. Find the area of the each slice of pizza when Riya cuts the pizza into 4 equal pieces. Also, find area of whole pizza. **2**

Sol. We have, $x^2 + y^2 = 4$

$\Rightarrow (x - 0)^2 + (y - 0)^2 = (2)^2$

$\therefore$ Radius = 2

Area of $\frac{1}{4}$ th slice of pizza $= \frac{1}{4}\pi(2)^2 = \pi$ sq. units

Area of whole pizza $= \pi(2)^2 = 4\pi$ sq. units

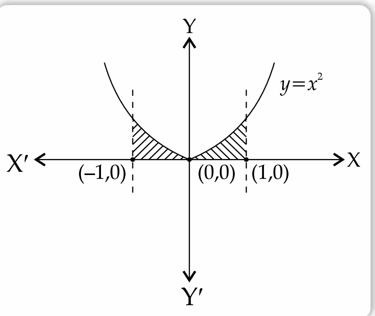
Solutions for Practice Questions

Very Short Answer Type Question

1. A $= 2\int_0^1 x^2dx = \frac{2}{3}[x^3]_0^1 = \frac{2}{3}$ sq. unit **1**

[CBSE Marking Scheme, 2020]

Detailed Answer :



Area of shaded region $= 2\int_0^1 x^2dx$

$$= \frac{2}{3}[x^3]_0^1 = \frac{2}{3} \text{ sq. unit}$$

Commonly Made Error

- Students fail to identify the figure correctly.

Answering Tip

- Learn to draw the graphs correctly.

Short Answer Type Question-I

1.

$$\text{Area} = 2\int_0^2 \sqrt{8x}\,dx$$

$$= 2\times 2\sqrt{2}\int_0^2 x^{\frac{1}{2}}dx \qquad \mathbf{1}$$

$$= 4\sqrt{2}\left[\frac{x^{3/2}}{3/2}\right]_0^2$$

$$= \frac{8}{3}\sqrt{2}[2^{\frac{3}{2}}-0]$$

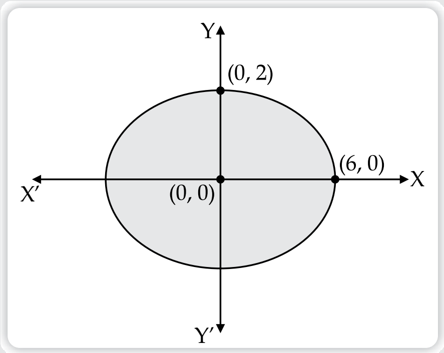
$$= \frac{8\sqrt{2}}{3}\times 2\sqrt{2} \qquad \tfrac{1}{2}$$

$$= \frac{32}{3}\text{ sq units} \qquad \tfrac{1}{2}$$

[CBSE SQP Marking Scheme, 2020-21]

Short Answer Type Question-II

4. Required Area $= \frac{4}{3}\int_0^6 \sqrt{6^2-x^2}\,dx$ ½



½

$$= \frac{4}{3}\int_0^6 \sqrt{6^2-x^2}\,dx \qquad \mathbf{1}$$

$$= \frac{4}{3}\left[\frac{x}{2}\sqrt{6^2-x^2}+\frac{36}{2}\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$$

$$= \frac{4}{3}\left[18\times\frac{\pi}{2}-0\right] = 12\pi \text{ sq.units} \qquad \mathbf{1}$$

[CBSE SQP Marking Scheme 2020-21]

Commonly Made Error

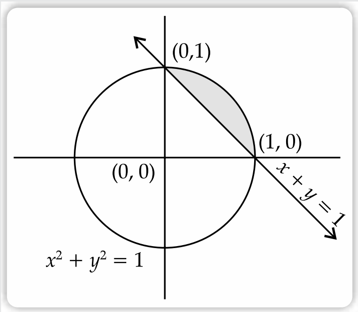
- Some students fail to find the standard equation of the ellipse and hence get wrong with the figure.

Answering Tip

- Learn to sketch the graphs of circle, ellipse and parabola from a standard equation.

Long Answer Type Question

4.



For correct figure
For correct shading } **1**

$$A = \int_0^1 (\sqrt{1-x^2}-(1-x))dx \qquad \mathbf{1}$$

$$= \left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x-\frac{x^2}{2}\right]_0^1 \qquad \mathbf{2}$$

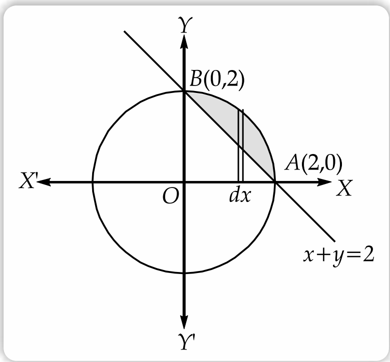
$$= \frac{1}{2}\sin^{-1}(1)-\frac{1}{2} = \left(\frac{\pi}{4}-\frac{1}{2}\right)\text{sq. units} \qquad \mathbf{1}$$

[CBSE SQP Marking Scheme 2020] (Modified)

Detailed Solution :

Given region $\{(x, y) : (x^2 + y^2) \le 1 \le (x + y)\}$

or The given region is bounded inside the circle $x^2 + y^2 = 1$ and above the line $x + y = 1$ as shown in the figure.



∴ Required area of the shaded portion

$$= \int_0^1 \sqrt{1-x^2}\,dx - \int_0^1 (1-x)\,dx$$

$$= \left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x-\frac{x^2}{2}\right]_0^1$$

$$= \left[0+\frac{1}{2}\sin^{-1}(1)-0-0\right]-\left[1-\frac{1}{2}-0+0\right]$$

$$= \frac{1}{2}\left(\frac{\pi}{2}\right)-\frac{1}{2} = \left(\frac{\pi}{4}-\frac{1}{2}\right) \text{ sq. units.}$$

REFLECTION

- Area of different figures can be find by using the concept and techniques given in his chapter.
- Proper drawing of given figure help to find the area easily.

❑❑❑