CHAPTER

4

DETERMINANTS

Syllabus

Determinant of a square matrix (up to 3 × 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples. Solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

In this chapter you will study

- *Determinant of matrix*
- *Minors, Cofactors of matrix*
- *Area of triangle using determinant*
- *Inverse of matrix using adjoint method*
- *Solution of Linear equations in two or three variables*
- *Consistency or inconsistency of system of linear equations.*

List of Topics

Topic-1 Determinants, Minors & Co-factors

Concepts Covered • *Determinant value of a matrix,* • *Co-factor and Minor of a matrix* • *Inverse of matrix using Adjoint method,* • *Area of triangle with the help of determinant*

Revision Notes

Determinants, Minors & Co-factors

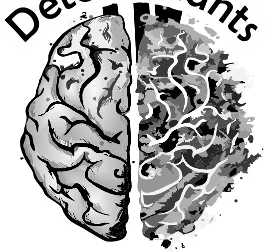
(a) Determinant: A unique number (real or complex) can be associated to every square matrix $A = [a_{ij}]$ of order m. This number is called the determinant of the square matrix A, where $a_{ij} = (i, j)^{th}$ element of A.

For instance, if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then, determinant of matrix A is written as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \det(A)$ and its value is given by $ad - bc$.

(b) Minors: Minors of an element a_{ij} of a determinant *(or a determinant corresponding to matrix A)* is the determinant obtained by deleting its i^{th} row and j^{th} column in which a_{ij} lies. Minor of a_{ij} is denoted by M_{ij}. Hence, we can get 9 minors corresponding to the 9 elements of a third order (*i.e.*, 3 × 3) determinant.

(c) Co-factors : Cofactor of an element a_{ij}, denoted by A_{ij}, is defined by $A_{ij} = (-1)^{(i+j)} M_{ij}$, where M_{ij} is minor of a_{ij}. Sometimes C_{ij} is used in place of A_{ij} to denote the co-factor of element a_{ij}.

Determinants



Applications of determinants & matrices

- If $a_1x+b_1y+c_1z=d_1, a_2x+b_2y+c_2z=d_2, a_3x+b_3y+c_3z=d_3$ then we can write $AX=B$,

  where $A=\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$
- Unique solution of $AX=B$ is $X=A^{-1}B$, $|A|\neq 0$.
- $AX=B$ is consistent or inconsistent according as the solution exists or not.
- For a square matrix A in $AX=B$, if
  - (i) $|A|\neq 0$ then there exists unique solution.
  - (ii) $|A|=0$ and (adj. A) $B\neq 0$, then no solution.
  - (iii) if $|A|=0$ and (adj.A).$B=0$ then system may or may not be consistent.

Area of a triangle

If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of triangle, Area of $\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

e.g., if (1, 2), (3, 4) and (–2, 5) are the vertices, then area of the triangle is

$$\Delta=\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ -2 & 5 & 1 \end{vmatrix}=\frac{1}{2}\left|1(4-5)-2(3+2)+1(15+8)\right|=6 \text{ sq.units.}$$

we take positive value of the determinant because area is considered positive.

Minors and cofactors of a matrix

Minor of an element a_{ij} in a determinant of matrix A is the determinant obtained by deleting i^{th} row and j^{th} column and is denoted by M_{ij}. If M_{ij} is the minor of a_{ij} and cofactor of a_{ij} is A_{ij} given by $A_{ij}=(-1)^{i+j}M_{ij}$.

- If $A_{3\times3}$ is a matrix, then $|A|=a_{11}.A_{11}+a_{12}.A_{12}+a_{13}.A_{13}$.
- If elements of one row (or column) are multiplied with cofactors of elements of any other row (or column), then their sum is zero. For e.g., $a_{11}A_{21}+a_{12}A_{22}+a_{13}A_{33}=0$.

e.g., if $A=\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, then $M_{11}=4$ and $A_{11}=(-1)^{1+1}4=4$.

Determinant of a square matrix 'A', |A| is given by

(i) if $A=[a_{11}]_{1\times1}$, then $|A|=a_{11}$

(ii) if $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$ then $|A|=a_{11}a_{22}-a_{12}a_{21}$

(iii) if $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$, then $|A|=a_{11}(a_{22}.a_{33}-a_{23}.a_{32})-a_{12}(a_{21}.a_{33}-a_{23}.a_{31})+a_{13}(a_{21}a_{32}-a_{22}a_{31})$

e.g., If $A=\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$, then $|A|=2\times4-3\times2=2$

Adjoint and inverse of a matrix

If $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $Adj\ A=\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

,where A_{ij} is the cofactor of a_{ij}.

- A(adj.A)=(adj.A).$A=|A|I$, A – square matrix of order 'n'
- If $|A|=0$, then A is singular. Otherwise, A is non-singular.
- If $AB=BA=I$, where B is a square matrix, then B is called the inverse of A, $A^{-1}=B$ or $B^{-1}=A$, $(A^{-1})^{-1}=A$.

Inverse of a square matrix exists if A is non-singular *i.e.* $|A|\neq 0$, and is given by

$$A^{-1}=\frac{1}{|A|}(\text{adj.}A)$$

Trace the Mind Map

- First Level
- Second Level
- Third Level

1. ADJOINT OF A SQUARE MATRIX:

Let $A = [a_{ij}]$ be a square matrix. Also, assume $B = [A_{ij}]$, where A_{ij} is the cofactor of the elements a_{ij} in matrix A. Then the transpose B^T of matrix B is called the **adjoint of matrix** A and it is denoted by "*adj* **(**A**)**".

To find adjoint of a 2 × 2 matrix: Follow this, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ or $adj\ A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

For example, consider a square matrix of order 3 as

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 2 & 0 & 5 \end{bmatrix}$, then in order to find the adjoint matrix A, we find a matrix B (formed by the co-factors of elements of matrix A as mentioned above in the definition)

i.e., $B = \begin{bmatrix} 15 & -2 & -6 \\ -10 & -1 & 4 \\ -1 & 2 & -1 \end{bmatrix}$. Hence, $adj\ A = B^T = \begin{bmatrix} 15 & -10 & -1 \\ -2 & -1 & 2 \\ -6 & 4 & -1 \end{bmatrix}$

2. SINGULAR MATRIX AND NON-SINGULAR MATRIX:

(a) Singular matrix: A square matrix A is said to be singular if $|A| = 0$ *i.e.*, its determinant is zero.

e.g. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 12 \\ 1 & 1 & 3 \end{bmatrix}$

$= 1(15-12)-2(12-12)+3(4-5)$
$= 3-0-3 = 0$
$\therefore$ A is singular matrix.

$B = \begin{bmatrix} -3 & 4 \\ 3 & -4 \end{bmatrix} = 12 - 12 = 0$

$\therefore$ B is singular matrix.

(b) Non-singular matrix: A square matrix A is said to be non-singular if $|A| \neq 0$.

e.g. $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$= 0\,(0-1) - 1(0-1) + 1(1-0)$
$= 0 + 1 + 1 = 2 \neq 0$
$\therefore$ A is non-singular matrix.

- A square matrix A is **invertible** if and only if A is **non-singular**.

3. ALGORITHM TO FIND A^{-1} BY DETERMINANT METHOD:

STEP 1: Find $|A|$.

STEP 2: If $|A| = 0$, then, write "A is a singular matrix and hence not invertible". Else write "A is a non-singular matrix and hence invertible".

STEP 3: Calculate the co-factors of elements of matrix A.

STEP 4: Write the matrix of co-factors of elements of A and then obtain its transpose to get *adj.A* (*i.e.*, adjoint A).

STEP 5: Find the inverse A^{-1} of A by using the relation $\frac{1}{|A|}(adj\,A)$.

4. AREA OF TRIANGLE:

Area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \text{ sq. units}$$

- Since area is a positive quantity, we take absolute value of the determinant.
- If the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then $\Delta = 0$.
- The equation of a line passing through the points (x_1, y_1) and (x_2, y_2) can be obtained by the expression given here:

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Key Facts

- In mathematics, the determinant is a scalar value that is a function of the entries of a square matrix.
- There are 10 main properties of determinants which include reflection property, all-zero property, proportionality or repetition property, switching property, scalar multiple property, sum properly, invariance properly, factor properly, triangle properly and *w*-factor properly.

OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. Three points P(2x, x + 3), Q(0, x) and R(x + 3, x + 6) are collinear, then x is equal to:

(A) 0 **(B)** 2
(C) 3 **(D)** 1

[CBSE Term-I 2021]

Ans. Option (D) is correct.

Explanation: As points are collinear

⇒ Area of triangle formed by 3 points is zero.

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix}(x_1-x_2) & (x_2-x_3)\\(y_1-y_2) & (y_2-y_3)\end{vmatrix} = 0$$

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix}(2x-0) & \{0-(x+3)\}\\(x+3-x) & \{x-(x-6)\}\end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix}2x & -(x+3)\\3 & -6\end{vmatrix} = 0$$

$$\Rightarrow \quad -12x + 3(x+3) = 0$$
$$\Rightarrow \quad -12x + 3x + 9 = 0$$
$$\Rightarrow \quad -9x = -9$$
$$\Rightarrow \quad x = 1$$

Q. 2. If C_{ij} denotes the cofactor of element P_{ij} of the matrix P = $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & 2 & 4\end{bmatrix}$, then the value of $C_{31}.C_{23}$ is:

(A) 5 **(B)** 24
(C) –24 **(D)** –5

[CBSE Term-I 2021]

Ans. Option (A) is correct.

Explanation:

Here, $C_{31} = (-1)^{3+1}\begin{vmatrix}-1 & 2\\2 & -3\end{vmatrix} = 3 - 4 = -1$

and $C_{23} = (-1)^{2+3}\begin{vmatrix}1 & -1\\3 & 2\end{vmatrix} = -(2+3) = -5$

Thus, $C_{31}.C_{23} = (-1)(-5) = 5$

Q. 3. If A is a square matrix of order 3 and |A| = –5, then |adj A| is:

(A) 125 **(B)** –25
(C) 25 **(D)** ±25

[CBSE Term-I 2021]

Ans. Option (C) is correct.

Explanation: We know that,

$$|\text{adj } A| = |A|^{n-1}$$

where n is the order of the matrix

$$\therefore \quad |\text{adj } A| = [5]^{3-1}$$
$$= (-5)^2 = 25$$

Q. 4. If for the matrix A = $\begin{bmatrix}\alpha & -2\\-2 & \alpha\end{bmatrix}$, $|A^3|$ = 125, then the value of α is:

(A) ±3 **(B)** –3
(C) ±1 **(D)** 1

[CBSE Term-I 2021]

Ans. Option (A) is correct.

Explanation: $A = \begin{bmatrix}\alpha & -2\\-2 & \alpha\end{bmatrix}$

$\Rightarrow \quad |A| = \alpha^2 - 4$ ...(i)

Also, given $|A^3| = 125$

$\Rightarrow \quad |A|^3 = 125$

$\Rightarrow \quad |A| = 5$

$\Rightarrow \quad \alpha^2 - 4 = 5$ [from eq. (i)]

$\Rightarrow \quad \alpha^2 = 9$

$\Rightarrow \quad \alpha = \pm 3$

Q. 5. The inverse of the matrix X = $\begin{bmatrix}2 & 0 & 0\\0 & 3 & 0\\0 & 0 & 4\end{bmatrix}$ is:

(A) $24\begin{bmatrix}\frac{1}{2} & 0 & 0\\0 & \frac{1}{3} & 0\\0 & 0 & \frac{1}{4}\end{bmatrix}$ **(B)** $\frac{1}{24}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

(C) $\frac{1}{24}\begin{bmatrix}2 & 0 & 0\\0 & 3 & 0\\0 & 0 & 4\end{bmatrix}$ **(D)** $\begin{bmatrix}\frac{1}{2} & 0 & 0\\0 & \frac{1}{3} & 0\\0 & 0 & \frac{1}{4}\end{bmatrix}$

[CBSE Term-1 2021]

Ans. Option (D) is correct.

Explanation: The inverse of a diagonal matrix is obtained by replacing each element in the diagonal with its reciprocal.

Since, $X = \begin{bmatrix}2 & 0 & 0\\0 & 3 & 0\\0 & 0 & 4\end{bmatrix}$

Therefore $X^{-1} = \begin{bmatrix}\frac{1}{2} & 0 & 0\\0 & \frac{1}{3} & 0\\0 & 0 & \frac{1}{4}\end{bmatrix}$

Q. 6. If A is a 3 × 3 matrix such that $|A| = 8$, then $|3A|$ equals

(A) 8 **(B)** 24

(C) 72 **(D)** 216

[CBSE Delhi Set-I 2020]

Ans. Option (D) is correct.

Explanation:

Here $|A| = 8$

Then $|3A| = 3^3|A| = 27 \times 8 = 216$

Q. 7. If A is skew symmetric matrix of order 3, then the value of $|A|$ is:

(A) 3 **(B)** 0

(C) 9 **(D)** 27

[CBSE Delhi Set-III 2020]

Ans. Option (B) is correct.

Explanation: Determinant value of skew symmetric matrix is always '0'.

Q. 8. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is:

(A) 3 **(B)** 0

(C) – 1 **(D)** 1

[CBSE OD Set-I 2020]

Ans. Option (C) is correct.

Explanation:

$$\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$$

On expanding along R_1

$$2(x - 9x) - 3(x - 4x) + 2(9x - 4x) + 3 = 0$$

$$2(-8x) - 3(-3x) + 2(5x) + 3 = 0$$

$$-16x + 9x + 10x + 3 = 0$$

$$3x + 3 = 0$$

$$3x = -3$$

$$x = -\frac{3}{3}$$

$$x = -1$$

Q. 9. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then det ($adj\ A$) equals:

(A) a^{27} **(B)** a^9

(C) a^6 **(D)** a^2

[CBSE OD Set-III 2020]

Ans. Option (C) is correct.

Explanation:

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

$$\text{Det}(A) = a(a \times a - 0 \times 0) - 0 + 0 = a^3$$

$$\text{Det}(adj\ A) = (a^3)^2 = a^6$$

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. If A a non-singular square matrix of order 3 and $A^2 = 2A$, then find the value of $|A|$.

A I U [CBSE Delhi Set-II 2020]

Topper Answer, 2020

Sol.

$A^2 = 2A$

Premultiplying with A^{-1}

$A^{-1}A.A = 2A^{-1}A$

$A = 2I$

$|A| = 8|I|$

$|A| = 8$.

Q. 2. For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ write A^{-1}.

R [CBSE Delhi Set-I 2020]

Q. 3. Find the co-factors of all the elements of $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$.

R [CBSE Delhi Set-II 2020]

Sol. $A_{11} = 3, A_{12} = -4, A_{21} = 2, A_{22} = 1$ **1**

[CBSE Marking Scheme 2020]

Commonly Made Error

- Students may find minors instead of co-factors.

Answering Tip

- Learn to differentiate between minors and co-factors.

Q. 4. Find *adj A*, if $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$.

R [CBSE OD Set-I, II, III 2020]

Q. 5. Given that A is a square matrix of order 3 × 3 and |A| = –4. Find |adj A|.

R&U [CBSE SQP 2020-21]

Sol. $|\text{adj A}| = (-4)^{3-1} = 16$ **1**

[CBSE Marking Scheme 2020]

Detailed Solution:

$|\text{adj A}| = |A|^{n-1}$

where n = order of matrix A

$|\text{adj A}| = (-4)^{3-1} = (-4)^2 = 16$

Q. 6. Let A = $[a_{ij}]$ be a square matrix of order 3 × 3 and |A| = –7. Find the value of $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$ where A_{ij} is the cofactor of element a_{ij}.

[CBSE SQP 2020-21]

Sol. 0

Q. 7. Check whether $(l + m + n)$ is a factor of the determinant $\begin{vmatrix} l+m & m+n & n+l \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$ or not. Give reason.

A [CBSE SQP-2020]

Sol. Apply $R_1 \rightarrow R_1 + R_2$

$$\begin{vmatrix} l+m+n & m+n+l & n+l+m \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$$

$= 2(l+m+n)\begin{vmatrix} 1 & 1 & 1 \\ n & l & m \\ 1 & 1 & 1 \end{vmatrix}$; yes $(l + m + n)$ is a factor. **1**

[CBSE SQP Marking Scheme 2020]

Q. 8. If A is a square matrix satisfying A′ A = I, write the value of |A|. AI U [CBSE OD Set-I, 2019]

Sol. $|A'|\,|A| = |I| \Rightarrow |A|^2 = 1$ ½

$\therefore$ $|A| = 1$ or $|A| = -1$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let the value of $|A| = x$

Since, $|A| = |A'|$ and $|I| = 1$

Given, $AA' = I$

$\therefore$ $|AA'| = |I|$

$\Rightarrow$ $|A||A'| = |I|$ $[\,|AA'| = |A||A'|\,]$

$\Rightarrow$ $x.x = 1$

$\Rightarrow$ $(x^2 - 1) = 0$

$\Rightarrow$ $(x-1)(x+1) = 0 \Rightarrow x = \pm 1$

$\therefore$ Value of $|A| = \pm 1$

Q. 9. If *A* and *B* are square matrices of the same order 3, such that $|A| = 2$ and $AB = 2I$, write the value of $|B|$.

U [CBSE Delhi Set-I, 2019]

Sol. $AB = 2I \Rightarrow |AB| = |2I| \Rightarrow |A| \cdot |B| = 2^3|I|$ ½

$\Rightarrow 2 \times |B| = 8 \Rightarrow |B| = 4$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, $AB = 2I$

or $AB = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\therefore$ $|AB| = 2(4-0) - 0 + 0 = 8$

Since, $|AB| = |A||B|$

$\therefore$ $|B| = \dfrac{|AB|}{|A|} = \dfrac{8}{2} = 4$

[given, $|A| = 2$]

Q. 10. Find |*AB*|, if A = $\begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and *B* = $\begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

A [CBSE OD Set-II, 2019]

Q. 11. If for any 2 × 2 square matrix A, A(adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of |A|.

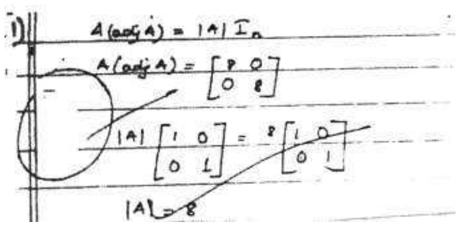
AI R [OD Set I 2017]

Sol. $|A| = 8$ **[CBSE Marking Scheme 2017]**

Detailed Solution:

Topper Answer, 2017

Sol.



Q. 12. Find the maximum value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$$

R&U [Delhi Set I, II, III 2016]

Sol. Let the maximum value be

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$$

Expanding, we get

$$\Delta = \sin\theta \cdot \cos\theta$$

$$= \frac{2\sin\theta.\cos\theta}{2} = \frac{1}{2}\sin 2\theta \qquad \frac{1}{2}$$

$$\therefore \text{ Maximum value} = \frac{1}{2}\times 1 = \frac{1}{2}$$

$$= \frac{1}{2} \qquad [\text{as} -1 \le \sin 2\theta \le 1] \qquad \frac{1}{2}$$

Q. 13. If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.

R&U [O.D. Set I, II, III 2016]

Q. 14. Evaluate x if : $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$.

R&U [Delhi Set I, II, III Comptt. 2016]

Sol. $2 - 20 = 2x^2 - 24$ **1**

or $x = \pm\sqrt{3}$

[CBSE Marking Scheme 2016]

Q. 15. Given $A = \begin{pmatrix} 4 & 2 & 5 \\ 2 & 0 & 3 \\ -1 & 1 & 0 \end{pmatrix}$, write the value of det. $(2AA^{-1})$. A [Outside Dec. Set I, II, III Comptt. 2016]

Sol. $|2AA^{-1}| = (2)^3$ $[\because AA^{-1} = I]$

$= 8$ **1**

[CBSE Marking Scheme 2016]

Q. 16. If A is a square matrix such that $|A| = 5$, write the value of $|AA^T|$. R&U [OD Set II 2016]

Sol.

Topper Answer, 2016

$|A| = 5$

$|A| = 5$

$|AA^T| = |A||A^T|$ (As A is a square matrix)

$= |A||A|$ (As $|A| = |A^T|$)

$= |A|^2 = 25$

Short Answer Type Questions-I (2 mark each)

Q. 1. Find the inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$.

Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

R&U [S.Q.P. 2017-18]

Q. 2. If A is a skew-symmetric matrix of order 3, then prove that det $A = 0$. R&U [OD Set I 2017]

Sol. Any skew symmetric matrix of order 3 is

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

or $\Rightarrow$ $|A| = -a(bc) + a(bc) = 0$ **1**

Since A is a skew-symmetric matrix $\therefore$ $A^T = -A$

$|A^T| = |-A| = (-1)^3. |A|$ ½

or $|A^T| = -|A|$

or $|A| = -|A|$

or $2|A| = 0$ or $|A| = 0.$ ½

[CBSE Marking Scheme 2017]

Detailed Solution:

Topper Answer, 2017

Sol. A is skew symmetric

$A = -A^T$

$|A| = (-1)|A^T|$

$|A| = -|A|$ $[\because |A| = |A^T|]$

$2|A| = 0$

$|A| = 0$

det $A = 0$

Q. 3. If A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$, then find the value of $|2AB|$.

R&U [Foreign 2017]

Q. 4. If $A = \begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3| = 125$, then find the values of p. R [CBSE OD Set-II, 2019]

Sol. $|A| = p^2 - 4$ ½

$|A^3| = 125 \Rightarrow |A|^3 = 125 \Rightarrow |A| = 5$ **1**

$\therefore\ p^2 - 4 = 5 \Rightarrow p = \pm 3$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, $A = \begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3| = 125$

Since, $|A^n| = |A|^n$

$\therefore$ $|A^3| = |A|^3$

Now, $|A| = \begin{vmatrix} p & 2 \\ 2 & p \end{vmatrix} = p^2 - 4$

According to given condition,

$$|A^3| = 125$$

$$\Rightarrow |A|^3 = 125$$

$$\Rightarrow (p^2 - 4)^3 = 125$$

$$\Rightarrow (p^2 - 4)^3 = 5^3$$

$$\Rightarrow p^2 - 4 = 5$$

$$\Rightarrow p^2 = 9$$

$$\Rightarrow p = \pm 3$$

Hence, values of $p = \pm 3$.

Commonly Made Error

- Some students find A^3 first and then take its determinant which is time consuming.

Answering Tip

- Learn all the properties of determinants thoroughly.

Q. 5. If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of $|A|$.

A I R&U [CBSE SQP 2020-21]

Sol. $A^2 = 2A$

$\Rightarrow |AA| = |2A|$

$\Rightarrow |A|\,|A| = 8|A|$

$(\because |AB| = |A|\,|B|$ and $|2A| = 2^3|A|)$ ½

$\Rightarrow |A|(|A| - 8) = 0$ **1**

$\Rightarrow |A| = 0$ or 8 ½

[CBSE SQP Marking Scheme 2020-21]

Commonly Made Error

- Students make mistakes in applying the property $|kA| = k^n|A|$. Instead they take it as $|kA| = k|A|$.

Answering Tip

- Practice more problems involving properties of determinants.

Long Answer Type Questions (5 & 6 mark each)

Q. 1. Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A(\text{adj } A) = |A|I_3$.

Sol. We have, $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

Let C_{ij} be the cofactor of the element a_{ij} of $|A|$.

Now, cofactros of $|A|$ are

$$C_{11} = (-1)^{1+1}\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = 1 - 4 = -3$$

$$C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -2(2 + 4) = -6$$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = -4 - 2 = -6$$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -(-2 - 4) = 6$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = -1 + 4 = 3$$

$$C_{23} = (-1)^{2+3}\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2 + 4) = -6$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = 4 + 2 = 6$$

$$C_{32} = (-1)^{3+2}\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2 + 4) = -6$$

$$C_{33} = (-1)^{3+3}\begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = -1 + 4 = 3$$

Now, the adjoint of the matrix A is given by

$$adj(A) = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

Now, $|A| = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix}$

$$= -1(1 - 4) + 2(2 + 4) - 2(-4 - 2)$$

$$= -1(-3) + 2(6) - 2(-6)$$

$$= 3 + 12 + 12 = 27$$

and $A\cdot(\text{adj } A) = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3+12+12 & -6-6+12 & -6+12-6 \\ -6-6+12 & 12+3+12 & 12-6-6 \\ -6+12-6 & 12-6-6 & 12+12+3 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix} - 27\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= 27I_3 = |A|I_3$$

Q. 2. Find the inverse of following matrices. **[NCERT]**

$$A = \begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{1} \\ \mathbf{2} & \mathbf{-1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Sol. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Then $|A| = \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} - 2\begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} + 1\begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix}$

$= 1(0) - 2(0) + 1(0 + 1)$ [expanding along C_1]

$= 1 \neq 0$

Thus, A is a non-singular matrix, so A^{-1} exists.

Now, co-factors corresponding to each element of determinant A are

$$C_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 1(-0-0) = 0$$

$$C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = -1(0-0) = 0$$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 1(0+1) = 1$$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} = -1(0-0) = 0$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1(0-1) = (-1)$$

$$C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -1(0+1) = -1$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} = 1(0+1) = 1$$

$$C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = -1(0-2) = 2$$

$$C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = 1(-1+2) = 1$$

Thus, matrix of co-factors

$$= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

and $\text{adj}(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$

[interchange rows and columns]

Now, $A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{1}\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

$\Rightarrow \quad A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

which is the required inverse of given matrix A.

Q. 3. Find the area of the triangle, whose vertices are (3, 8), (–4, 2) and (5, 1).

Sol. Given vertices of a triangle are (3, 8), (–4, 2) and (5, 1).

Let $(x_1, y_1) = (3, 8)$, $(x_2, y_2) = (-4, 2)$ and $(x_3, y_3) = (5, 1)$

Then, area of triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & x_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$

$$= \frac{1}{2}\left[3\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 8\begin{vmatrix} -4 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} -4 & 2 \\ 5 & 1 \end{vmatrix}\right]$$

$$= \frac{1}{2}[3(2-1) - 8(-4-5) + 1(-4-10)]$$

$$= \frac{1}{2}[3 + 72 - 14] = \frac{61}{2} \text{ sq units}$$

Q. 4. If the area of a triangle with vertices (–3, 0), (3, 0) and (0, *k*) is 9 sq units, then find the value of *k*.

Sol. Given area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq units.

We have, $\frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix} = \pm 9 \Rightarrow \begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix} = \pm 18$

$\Rightarrow -3(0 - k) - 0 + 1(3k - 0) = \pm 18$

$\Rightarrow \quad 3k + 3k = \pm 18 \Rightarrow 6k = \pm 18$

$\Rightarrow \quad k = \pm 3$

Q. 5. Find the equation of line joining *P*(11, 7) and Q(5, 5) using determinants. Also, find the value of *k*, if *R*(–1, *k*) is the point such that area of ΔPQR is 9 sq m.

Sol. Let $A(x, y)$ be any point on line PQ. Then, the points P, Q and A are collinear.

So, $\begin{vmatrix} 11 & 7 & 1 \\ 5 & 5 & 1 \\ x & y & 1 \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} 11 & 7 & 1 \\ -6 & -2 & 0 \\ x-11 & y-7 & 0 \end{vmatrix} = 0$$

Expanding determinant along C_3, we get

$$1\begin{vmatrix} -6 & -2 \\ x-11 & y-7 \end{vmatrix} = 0$$

$\Rightarrow \quad -6(y - 7) + 2(x - 11) = 0$

$\Rightarrow \quad -6y + 42 + 2x - 22 = 0$

$\Rightarrow \quad 2x - 6y + 20 = 0$

$\Rightarrow \qquad x - 3y + 10 = 0 \qquad$ [dividing by 2]

which is the required equation of line joining points P and Q.

Now, according to the question,

Area of ΔPQR = 9 sq m

$$\therefore \qquad \frac{1}{2}\begin{vmatrix} 11 & 7 & 1 \\ 5 & 5 & 1 \\ -1 & k & 1 \end{vmatrix} = \pm 9$$

$$\left[\because \text{ area of a triangle with vertices } (x_1, y_1), (x_2, y_2) \text{ and } (x_3, y_3) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right]$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\frac{1}{2}\begin{vmatrix} 11 & 7 & 1 \\ -6 & -2 & 0 \\ -12 & k-7 & 0 \end{vmatrix} = \pm 9$$

Expanding the determinant along C_3, we get

$$\frac{1}{2}\begin{vmatrix} -6 & -2 \\ -12 & k-7 \end{vmatrix} = \pm 9$$

$$\Rightarrow \qquad \frac{1}{2}[-6k + 42 - 24] = \pm 9$$

$$\Rightarrow \qquad (-6k + 18) = \pm 18 \Rightarrow -6k = 18 \pm 18$$

For positive sign, $k = \dfrac{-18+18}{-6} = 0$

and for negative sign, $k = \dfrac{-18-18}{-6} = 6$

Hence, the required values of k are 0 and 6.

Topic-2 Solutions of System of Linear Equations

Concepts Covered • *Unique Solution*, • *Consistent System*, • *Inconsistent System*

Revision Notes

SOLVING SYSTEM OF EQUATIONS BY MATRIX METHOD [INVERSE MATRIX METHOD]

(a) Homogeneous and Non-homogeneous system : A system of equations $AX = B$ is said to be a homogeneous system if $B = O$. Otherwise it is called a non-homogeneous system of equations.

$$a_1x + b_1y + c_1z = d_1,$$
$$a_2x + b_2y + c_2z = d_2,$$
$$a_3x + b_3y + c_3z = d_3$$

STEP 1 : Assume

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

STEP 2 : Find $|A|$. Now there may be following situations :

(i) $|A| \neq 0 \Rightarrow A^{-1}$ exists. It implies that the given system of equations is consistent and therefore, the system has **unique solution**. In that case, write

$$AX = B$$
$$\Rightarrow \qquad X = A^{-1}B$$
$$\left[\text{where } A^{-1} = \frac{1}{|A|}(adj\, A)\right]$$

Then by using the definition of equality of matrices, we can get the values of x, y and z.

(ii) $|A| = 0 \Rightarrow A^{-1}$ does not exist. It implies that the given system of equations may be **consistent** or **inconsistent**.

Key Words

Consistent System: A system is considered to be consistent if it has atleast one solution.

Inconsistent System: If a system has no solution, it is said to be inconsistent.

In order to check proceed as follow:

$\Rightarrow$ Find $(adj\ A)\ B$. Now, we may have either $(adj\ A)\ B \neq O$ or $(adj\ A)\ B = O$.

- If $(adj\ A)B = O$, then the given system may be consistent or inconsistent. To check, put $z = k$ in the given equations and proceed in the same manner in the new two variables system of equations assuming $d_i - c_ik$, $1 \le i \le 3$ as constant.
- And if $(adj\ A)\ B \neq O$, then the given system is inconsistent with no solutions.

Mnemonics-1

Inverse of a Square Matrix

Det **m**ins **tr**a

↓ Transpose

↓ Minors along with signs

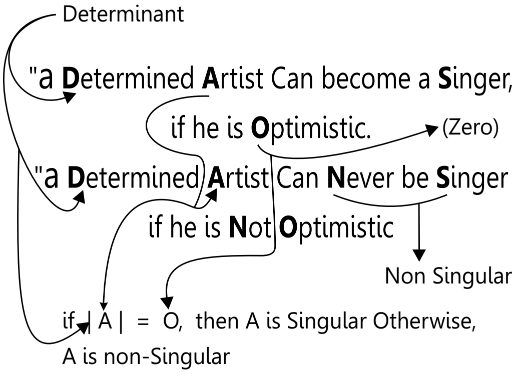
↓ Determinant

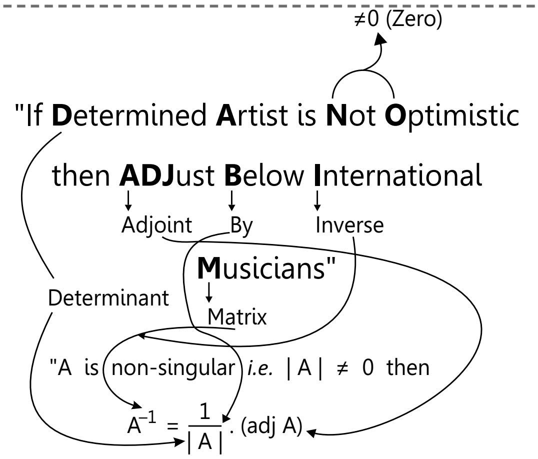
Mnemonics-2

Singular Matrix

A square matrix is said to be singular matrix if determinant of matrix denoted by |A| is zero otherwise it is non pingular matrix

Inverse Of a Matrix





Interpretation :

Singular & Non Singular Matrix -

if |A| = 0, then A is singular. Otherwise A is non-singular

Inverse of a Matrix–

Inverse of a Matrix exists if A is non- singular *i.e* |A| ≠ 0,and is given by

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. The system of linear equations

$5x + ky = 5,$

$3x + 3y = 5;$

will be consistent if:

(A) $k \neq -3$ **(B)** $k = -5$

(C) $k = 5$ **(D)** $k \neq 5$

[CBSE Term-1 2021]

Ans. Option (D) is correct.

Explanation: We have, $5x + ky - 5 = 0$

and $3x + 3y - 5 = 0$

For consistent system

$$\frac{5}{3} \neq \frac{k}{3}$$

$\Rightarrow$ $k \neq 5$

Q. 2. Consider the system, each consisting of *m* linear equations in *n* variables.

(i) If $m < n$, then all such system have a solution

(ii) If $m > n$, then none of these systems has a solution

(iii) If $m = n$, then there exists a system which has a solution which one of the following is correct?

(A) (i), (ii) and (iii) are true

(B) only (ii) and (iii) are true

(C) only (iii) is true

(D) none of them is true

Ans. Option (C) is correct.

Q. 3. The solution to the system of equation

$$\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -30 \end{bmatrix} \textbf{ is:}$$

(A) 6, 2 **(B)** –6, 2

(C) –6, –2 **(D)** 6, –2

Ans. Option (D) is correct.

Explanation:

$$\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -30 \end{bmatrix}$$

$2x + 5y = 2$ ...(i)

$-4x + 3y = -30$ ...(ii)

On solving eqs. (i) and (ii), we get

$x = 6$ and $y = -2$

Q. 4. The pair of equations $3x - 5y = 7$ and $6x - 10y = 14$ have:

(A) a unique solution

(B) infinitely many solution

(C) no solution

(D) two solutions

Ans. Option (B) is correct.

Explanation: Given equations are:

$3x - 5y = 7$ ...(i)

and $6x - 10y = 14$ or $3x - 5y = 7$ ...(ii)

Equations (i) and (ii) are same.

Hence it will have infinitely many solutions.

Q. 5. For what value of k system of equation $3x + 5y = 0$ and $kx + 10y = 0$ has a non-zero solution?

(A) 2 **(B)** 2

(C) 6 **(D)** 8

Ans. Option (C) is correct.

Explanation: for a non-zero solution

$$\begin{bmatrix} 3 & 5 \\ k & 10 \end{bmatrix} = 0$$

$$\Rightarrow \quad 30 - 5k = 0 \Rightarrow k = 6$$

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. For what values of k, the system of linear equations

$$x + y + z = 2$$
$$2x + y - z = 3$$
$$3x + 2y + kz = 4$$

has a unique solution ? A [Outside Delhi 2016]

Q. 2. If the system of equations : $x - ky - z = 0$, $kx - y - z = 0$, $x + y - z = 0$ has a non-zero solutions, then find the possible values of k A I

Sol. For the given homogeneous system to have non-zero solution determinant of coefficient matrix should be zero.

$$\begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 1(1 + 1) + k(-k + 1) - 1(k + 1) = 0$$

$$\Rightarrow \quad 2 - k^2 + k - k - 1 = 0$$

$$\Rightarrow \quad k^2 = 1 \Rightarrow k = \pm 1 \qquad \mathbf{1}$$

Q. 3. Find the value of λ, if the system of equations $2x + 3y = 8$, $7x - 5y + 3 = 0$, $4x - 6y + \lambda = 0$ is solvable. U

Sol. Given system of equations :

$2x + 3y = 8, 7x - 5y + 3 = 0, 4x - 6y + \lambda = 0$

Solving first two equations, we get $x = 1$, $y = 2$ substituting this value in third equation, we get

$$4 - 12 + \lambda = 0$$

$$\Rightarrow \quad \lambda = 8 \qquad \mathbf{1}$$

Q. 4. If $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find matrix A. A I

Sol. We know that if XAY = I, then $A = X^{-1}Y^{-1} = (YX)^{-1}$

In this case,

$$YX = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} \qquad ½$$

$$[YX]^{-1} = \begin{bmatrix} 8 & 5 \\ -11 & -7 \end{bmatrix}^{-1} = \begin{bmatrix} 7 & 5 \\ -11 & -8 \end{bmatrix} \qquad ½$$

Q. 5. Let $A = \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

If AB = C, then find A^2. A I

Sol. Here, $\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix} \quad 2x + y = 3 \text{ and } 3x + y = 2$$

On solving above equations, we get $x = -1$ and $y = 5$.

$$\therefore \quad A = \begin{bmatrix} -1+5 & 5 \\ 2(-1) & -1-5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix} \qquad ½$$

$$\text{Thus,} \quad A^2 = \begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -10 \\ 4 & -26 \end{bmatrix} \qquad ½$$

Short Answer Type Questions-I (2 marks each)

Q. 1. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

Q. 2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, be such that $A^{-1} = kA$, then find the value of k. A [Comptt. Set I, II, III, 2018]

Sol. Finding $A^{-1} = \dfrac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$ **1**

$$\Rightarrow \quad \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 2k & 3k \\ 5k & -2k \end{bmatrix} \qquad ½$$

$$\Rightarrow \quad k = \frac{1}{19} \qquad ½$$

[CBSE Marking Scheme 2018]

Short Answer Type Questions-II (3 marks each)

Q. 1. If $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ then using A^{-1}, solve the following system of equations : $x - 2y = -1, 2x + y = 2$.

R&U [S.Q.P. 2016-17]

Sol. $|A| = 5$

$$adj\,A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{adj\,A}{|A|} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \quad \frac{1}{2}$$

Given system of equations is $AX = B$, where

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \frac{1}{2}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1+4 \\ 2+2 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{4}{5} \end{bmatrix} \quad 1$$

$$x = \frac{3}{5} \text{ and } y = \frac{4}{5} \quad 1$$

[CBSE Marking Scheme 2016] (Modified)

Q. 2. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$. Then compute AB. Hence, solve the following system of equations : $2x + y = 4, 3x + 2y = 1$.

A [S.Q.P. 2015-16]

Sol.

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

and $$B = \begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$$

then $$AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I \quad 1$$

$$\Rightarrow \quad A\left(\frac{1}{2}B\right) = I$$

On multiplying by A^{-1}

$$A^{-1} = \frac{1}{2}B$$

$$A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \quad 1$$

The given system of equations are equivalent to $A'X = C$,

where $$X = \begin{bmatrix} x \\ y \end{bmatrix} \quad 1$$

and $$C = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, A' = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$X = (A')^{-1}C = (A^{-1})'C$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \end{bmatrix}$$

$\therefore \quad x = 7$

and $\quad y = -10 \quad 1$

[CBSE Marking Scheme 2015] (Modified)

Q. 3. Ishan wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50m and breadth is increased by 50m, then its area will remain the same. But if length is decreased by 10m and breadth is decreased by 20m, then its area will decrease by 5300m². Using matrices, find the dimensions of the plot.

A I [CBSE OD Set-I, 2016]

Sol. Let length be x m and breadth be y m

$\therefore \quad (x-50)(y+50) = xy$

$\Rightarrow \quad 50x - 50y = 2500$ or $x - y = 50 \quad \frac{1}{2}$

and $\quad (x-10)(y-20) = xy - 5300$

$\Rightarrow \quad 2x + y = 550 \quad \frac{1}{2}$

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 50 \\ 550 \end{pmatrix} \quad \frac{1}{2}$$

$$\Rightarrow \quad \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 50 \\ 550 \end{pmatrix} \quad 1$$

$$\Rightarrow \quad \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 50+550 \\ -100+550 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 600 \\ 450 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 200 \\ 150 \end{pmatrix}$$

$\therefore x = 200$ m and $y = 150$ m

[CBSE Marking Scheme 2016]

Q. 4. On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However if there were 16 children more,

everyone would have got ₹10 less. Using matrix method, find the number of children and the amount distributed by Seema.

[CBSE Delhi Set-I, 2016]

Sol. Try yourself similar to Q. No. 3 of 3 marks.

Long Answer Type Questions (5 marks each)

Q. 1. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$**, find** A^{-1}**. Hence**

Solve the system of equations:

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

A I R&U **[CBSE SQP 2020-21]**

Q. 2. Evaluate the product AB, where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Hence solve the system of linear equations

$$x - y = 3$$
$$2x + 3y + 4z = 17$$
$$y + 2z = 7$$

R&U **[CBSE SQP 2020-21]**

Sol.
$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$ **1½**

$$\Rightarrow AB = 6I$$

$$\Rightarrow A\left(\frac{1}{6}B\right) = I$$

$$\Rightarrow A^{-1} = \frac{1}{6}(B)$$ **1**

The given equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$AX = D$$

where $D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$$\Rightarrow X = A^{-1}D$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$ **1**

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$x = 2$

$y = -1$

$z = 4$ **1½**

[CBSE SQP Marking Scheme 2020-21]

Commonly Made Error

- After finding the product, some students find the inverse using formula which is wrong.

Answering Tip

- Learn to find inverse from the product of two matrices.

Q. 3. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$**, find** A^{-1}**. Hence, solve the system of equations :**

$$x - y = 3;$$
$$2x + 3y + 4z = 17;$$
$$y + 2z = 7.$$

A I R&U **[CBSE SQP, 2020-21]**

Sol. $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

$|A| = 2(-2) - 3(2-0) + 4(1-0) = -6 \neq 0$ **½**

$\therefore A^{-1}$ exists

Co-factors

$A_{11} = -2 \quad A_{12} = -2 \quad A_{13} = 1$

$A_{21} = -2 \quad A_{22} = 4 \quad A_{23} = -2$

$A_{31} = 4 \quad A_{32} = 4 \quad A_{33} = -5$ **2**

$$\text{Adj } A = \begin{bmatrix} -2 & -2 & 1 \\ -2 & 4 & -2 \\ 4 & 4 & -5 \end{bmatrix}^T$$

$$\text{Adj } A = \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$$ **1**

System of equations can be written as $AX = B$

where $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$ ½

Now, $AX = B$

$\Rightarrow \quad X = A^{-1}B$

$$\Rightarrow \quad X = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}\begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$$

$$\Rightarrow \quad X = \frac{1}{-6}\begin{bmatrix} -34-6+28 \\ -34+12+28 \\ 17-6-35 \end{bmatrix}$$

$$\Rightarrow \quad X = \frac{1}{-6}\begin{bmatrix} -12 \\ 6 \\ -24 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$ ½

$\Rightarrow x = 2, \quad y = -1, \quad z = 4$ ½

[CBSE SQP Marking Scheme 2020] (Modified)

Q. 4. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations: $x + y + z = 6, x + 2z = 7, 3x + y + z = 12$. A I R&U [CBSE Delhi Set III 2019]

Sol. $|A| = 4 \neq 0 \Rightarrow A^{-1}$ exists. ½

$$\text{Adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ **2**

$$\therefore A^{-1} = \frac{1}{|A|}\cdot adj\ A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ ½

Given system of equations can be written as $AX = B$

where, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$\therefore \quad X = A^{-1} \cdot B$ ½

$$= \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$ **1**

$\therefore x = 3, y = 1, z = 2$ ½

[CBSE Marking Scheme, 2019] (Modified)

Q. 5. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$, find A^{-1}.

Hence solve the system of equations:

$x + 3y + 4z = 8$

$2x + y + 2z = 5$

and $5x + y + z = 7$

A I R&U [CBSE OD Set I 2019]

Q. 6. Given A = $\begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, compute $(AB)^{-1}$. R&U [Comptt. 2018 Set I, II, III]

Q. 7. In $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$ find A^{-1}. Using A^{-1} solve the system of equations: $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2;\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5;$ $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$ U&A [Delhi Set-II 2017]

Sol. Here $|A| = 1200$ **1**

Co-factors are

$$\left.\begin{aligned} C_{11} &= 75, C_{21} = 150, C_{31} = 75 \\ C_{12} &= 110, C_{22} = -100, C_{32} = 30 \\ C_{13} &= 72, C_{23} = 0, C_{33} = -24 \end{aligned}\right\}$$ **1**

$$A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$ ½

Given equation in matrix from is :

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$ **1**

or $\quad AX = B$

or $\quad X = A^{-1}B$

$$\text{or} \quad \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{-1}{3} \\ \frac{1}{5} \end{bmatrix}$$ **1½**

or $x = 2, y = -3, z = 5$

[CBSE Marking Scheme 2017] (Modified)

COMPETENCY BASED QUESTIONS

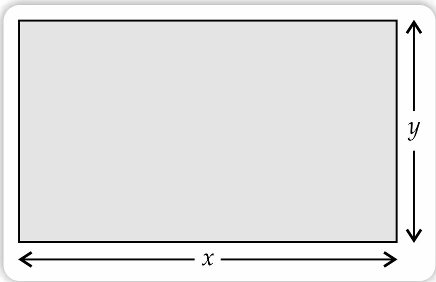
Case based MCQs (4 marks each)

Attempt any four sub-parts from each question. Each sub-part carry 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2

[CBSE QB 2021]



Q. 1. The equations in terms of x and y are:

(A) $x - y = 50, 2x - y = 550$
(B) $x - y = 50, 2x + y = 550$
(C) $x + y = 50, 2x + y = 550$
(D) $x + y = 50, 2x + y = 550$

Ans. Option (B) is correct.

Explanation: $(x - 50)(y + 50) = xy$

$$x - y = 50 \quad ...(i)$$

$$(x - y)(y - 20) = xy - 5300$$

$$2x + y = 550 \quad ...(ii)$$

Q. 2. Which of the following matrix equation is represented by the given information?

(A) $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -50 \\ -550 \end{bmatrix}$

Ans. Option (A) is correct.

Q. 3. The value of x (length of rectangular field) is:

(A) 150 m **(B)** 400 m
(C) 200 m **(D)** 320 m

Ans. Option (C) is correct.

Explanation: We have,

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$,

$X = \begin{bmatrix} x \\ y \end{bmatrix}$

Now $AX = B$

$$X = A^{-1}B$$

$$Adj(A) = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$|A| = 1 - [2 \times (-1)] = 1 + 2 = 3$$

$$A^{-1} = \frac{Adj(A)}{|A|} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$X = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{50}{3} + \frac{550}{3} \\ \frac{-100}{3} + \frac{550}{3} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

$\Rightarrow$ $x = 200$

$y = 150$

Q. 4. The value of y (breadth of rectangular field) is:

(A) 150 m **(B)** 200 m
(C) 430 m. **(D)** 350 m

Ans. Option (A) is correct.

Q. 5. How much is the area of rectangular field?

(A) 60000 sq.m. **(B)** 30000 sq.m.
(C) 30000 m **(D)** 3000 m

Ans. Option (B) is correct.

Explanation: Area of rectangular field

$$= xy$$

$$= 200 \times 150$$

$$= 30000 \text{ sq m.}$$

II. Read the following text and answer the following questions on the basis of the same:

The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to kept the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. The sum of the number of awardees for honesty and supervision is twice the number of awardees for helping.

Q. 1. $x + y + z =$ _______.

(A) 3 **(B)** 5

(C) 7 **(D)** 12

Ans. Option (D) is correct.

Explanation:

$$x + y + z = 12 \quad \text{...(i)}$$

$$2x + 3y + 3z = 33 \quad \text{...(ii)}$$

$$x - 2y + z = 0 \quad \text{...(iii)}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$|A| = 1(3 + 6) - 1(2 - 3) + 1(-4 - 3)$$

$$= 9 + 1 - 7$$

$$= 3$$

$$A^{-1} = \frac{1}{|A|}(adj\ A)$$

$$= \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad x = 3, y = 4, z = 5$$

$$x + y + z = 12 \quad \text{[from (i)]}$$

Q. 2. $x - 2y =$ _______.

(A) z **(B)** $-z$

(C) $2z$ **(D)** $-2z$

Ans. Option (B) is correct.

Explanation: $x - 2y = -z$ [from (iii)]

Q. 3. The value of z is _______.

(A) 3 **(B)** 4

(C) 5 **(D)** 6

Ans. Option (C) is correct.

Explanation: $z = 5$

Q. 4. The value of $x + 2y$ = _______.

(A) 9 **(B)** 10

(C) 11 **(D)** 12

Ans. Option (C) is correct.

Explanation: $x + 2y = 3 + 8 = 11$

Q. 5. The value of $2x + 3y + 5z$ = _______.

(A) 40 **(B)** 43

(C) 50 **(D)** 53

Ans. Option (B) is correct.

Explanation:

$$2x + 3y + 5z = 6 + 12 + 25$$

$$= 43$$

Case based Subjective Questions (4 marks each)

I. Read the following text and answer the following questions on the basis of the same:

(Each Sub-part carries 2 marks)

The Palace of Peace and Reconciliation, also known as the Pyramid of peace and Accord is a 62-metre high Pyramid in Mursultan, the capital of Kazakhstan, that serves as a non-denominational national spiritual centre and an event venue. It is designed by faster and partners with a stained glass apex. It has 25 smaller equilateral triangles as shown in the figure.

Q. 1. If the vertices of one triangle are (0, 0), $\left(3, \sqrt{3}\right)$ and $\left(3, -\sqrt{3}\right)$ then find the area of such triangle area of and a face of the Pyramid.

Sol. Required Area $= \left|\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3 & \sqrt{3} & 1 \\ 3 & -\sqrt{3} & 1 \end{vmatrix}\right|$

$$= \left|\frac{1}{2}\left[1\left(-3\sqrt{3}-3\sqrt{3}\right)\right]\right|$$

$$= \frac{6\sqrt{3}}{2} = 3\sqrt{3} \text{ sq units}$$

Since, a face of the Pyramid consists of 25 smaller equilateral triangles.

$\therefore$ Area of a face of the Pyramid = 25 × $3\sqrt{3} = 75\sqrt{3}$ sq. units. **1**

Q. 2. Find the length of a altitude of a smaller equilateral triangle.

Sol. Area of equilateral triangle = $\frac{\sqrt{3}}{4}$ (side)^2

$$\therefore \quad 3\sqrt{3} = \frac{\sqrt{3}}{4} (\text{side})^2$$

[As calculated above area of equilateral triangle is $3\sqrt{3}$ sq. units]

$\Rightarrow$ (side)^2 = 12

$\Rightarrow$ side = $2\sqrt{3}$ units **1**

Let h be the length of the altitude of a smaller equilateral triangle.

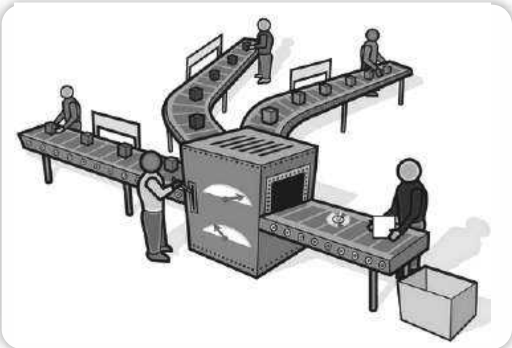
Then, $\frac{1}{2}$ × base × height = $3\sqrt{3}$

or, $\frac{1}{2}$ × side × height = $3\sqrt{3}$

or, $\frac{3\sqrt{3}\times 2}{2\sqrt{3}}$ = 3 units **1**

II. Read the following text and answer the following questions on the basis of the same:

A factory produces three products every day. Their production on a particular day is 45 tons. It is found that production of third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product.



Q. 1. It x, y and z respectively denotes the quantity (in tons) of first, second and third product produced, then construct the system of equations and write it in matrix form.

Sol. According to given information, the system of linear equations is:

$$x + y + z = 45$$
$$z = x + 8$$
$$x + z = 2y$$

or,
$$x + y + z = 45$$
$$-x + z = 8$$
$$x - 2y + z = 0 \quad \mathbf{1\tfrac{1}{2}}$$

In matrix form,

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix} \quad \tfrac{1}{2}$$

Q. 2. If $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix}^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & 2 \\ 3 & 0 & -3 \\ 1 & -2 & 1 \end{bmatrix}$, then find the inverse of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix}$

Then $A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & 2 \\ 3 & 0 & -3 \\ 1 & -2 & 1 \end{bmatrix}$

We know that,

$$(A')^{-1} = (A^{-1})'$$

$$\therefore \quad (A^{-1})' = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{1}{6} & \frac{-1}{3} & \frac{1}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{3} & 0 & \frac{-1}{3} \\ \frac{1}{3} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{3} & 0 & \frac{-1}{3} \\ \frac{1}{3} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix} \quad \mathbf{1}$$

or, $(A')^{-1} = (A^{-1})'$

Solutions for Practice Questions (Topic-1)

Very Short Answer Type Questions

2. $|A| = 1$ ½

$A^{-1} = \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$ ½

[CBSE Marking Scheme 2020]

Commonly Made Error

- Students use elementary operations for finding the inverse.

Answering Tip

- Find the inverse using formula for very short answer questions.

4. $adj\, A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$ 1

[CBSE Marking Scheme 2020]

Detailed Solution:

Given, $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

$C_{11} = 3, \quad C_{12} = -4$

$C_{21} = 1, \quad C_{22} = 2$

$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}^T$

$\text{adj A} = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$

Commonly Made Error

- Students take the minors instead of co-factors for finding adj A.

Answering Tip

- Learn the difference between minors and co-factors.

10. $\det A = \det B = 0$ ½

$\Rightarrow |AB| = 0$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$

$\therefore \quad AB = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$

or $\quad AB = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$

or $\quad AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Hence, $|AB| = 0$

13. Given $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$

or $2x(x + 3) - (-2)(-3x) = 8$

or $2x^2 + 6x - 6x = 8$

or $x^2 = \frac{8}{2}$ ½

or $x^2 = 4$

or $x = \pm 2$ ½

Since $x \in N$, so, $x = 2$

Short Answer Type Questions-I

1. $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1} = \frac{1}{9-10}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ 1 + ½

$\therefore P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 13 & 8 \\ 1 & 1 \end{bmatrix}$ ½

[CBSE Marking Scheme 2017-18]

3. $|2AB| = 2^3 \times |A| \times |B|$ 1

$= 8 \times (-1) \times 3 = -24$ 1

[CBSE Marking Scheme 2017]

Solutions for Practice Questions (Topic-2)

Very Short Answer Type Questions

1. For a unique solution

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$ 1

$k + 2 - 2k - 3 + 4 - 3 \neq 0$

$\Rightarrow \quad k \neq 0$

[CBSE Marking Scheme 2016]

Commonly Made Error

- Students get confused with the conditions for unique and infinite solutions.

Answering Tip

- Learn the solution of equations with the different conditions.

Short Answer Type Questions-I

1. $|A| = 2,$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \qquad \mathbf{1}$$

$$\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix},$$

$$\text{RHS} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

LHS = RHS **1**

[CBSE Marking Scheme 2018]

Detailed Solution:

Topper Answer, 2018

Sol.

$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$|A| = 14 - 12 = 2$

$|A| \neq 0$ hence inverse exists

Now, $C_{11} = 7$, $C_{12} = +4$

$C_{21} = +3$, $C_{22} = 2$

$\text{Adj}(A) = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ Ans

$2A^{-1} = 9I - A$

$2 \times \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ Hence Proved.

Long Answer Type Questions

1. $|A| = 1(-1-2) - 2(-2-0)$

$= -3 + 4 = 1 \neq 0$ ½

A is non-singular, therefore A^{-1} exists

$$\text{Adj } A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{|A|}(\text{Adj } A)$$

$$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \qquad \mathbf{1½}$$

The given equations can be written as :

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} \qquad ½$$

Which is of the form $A'X = B$

$$\Rightarrow \quad X = (A')^{-1}B = (A^{-1})'B \qquad \mathbf{1}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\Rightarrow \quad x = 0$

$y = -5$

$z = -3$ **1½**

[CBSE SQP Marking Scheme 2020-21]

Commonly Made Error

- Students get confused with how to use the given matrix to solve the equation.

Answering Tip

- The coefficient matrix is the transpose of the given matrix.

5. $|A| = 11;\ Adj(A) = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ **1+2**

$$\therefore A^{-1} = \frac{1}{|A|}.Adj\ A = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix} \qquad ½$$

Taking; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix};\quad B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

The system of equations in matrix form is

$A \cdot X = B \quad \therefore X = A^{-1} \cdot B$

$\therefore$ Solution is :

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ **1**

$\therefore \; x = 1, y = 1, z = 1$ ½

[CBSE Marking Scheme, 2019] (Modified)

Detailed Solution:

Try yourself similar to Q. No. 4 of LATQ.

6. $|A| = 5(-1) + 4(1) = -1$ **1**

$C_{11} = -1 \qquad C_{21} = 8 \qquad C_{31} = -12$

$C_{12} = 0 \qquad C_{22} = 1 \qquad C_{32} = -2$ **2**

$C_{13} = 1 \qquad C_{23} = -10 \qquad C_{33} = 15$

$$A^{-1} = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$ **1**

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$$ **1**

[CBSE Marking Scheme 2018] (Modified)

REFLECTIONS

- Here you practiced of finding area of triangle using determinants?
- Will you be able to find the volume of parallelopiped using determinants?

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SELF ASSESSMENT PAPER - 02

Time: 1 hour **MM: 30**

UNIT-II

(A) OBJECTIVE TYPE QUESTIONS:

I. Multiple Choice Questions **[1×6 = 6]**

Q. 1. A 2 × 2 matrix whose elements a_{ij} are given by $a_{ij} = \dfrac{(i-j)^2}{2}$ is :

(A) $\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ **(C)** $\begin{bmatrix} 0 & \frac{-1}{2} \\ \frac{-1}{2} & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ 0 & \frac{1}{2} \end{bmatrix}$

Q. 2. If $A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and $B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$, then value of $a_{11}b_{11} + a_{22}b_{22}$ is :

(A) 10 **(B)** 15 **(C)** 20 **(D)** 25

Q. 3. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, then values of x and y are

(A) 2, 3 **(B)** 2, 1 **(C)** 1, 2 **(D)** 3, 2

Q. 4. Value of $\begin{vmatrix} 1 & \log_b a \\ \log_a b & 1 \end{vmatrix}$ is :

(A) 1 **(B)** 0 **(C)** –1 **(D)** None of these

Q. 5. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, then $|A^2 - 2A|$ = ...

(A) 15 **(B)** 20 **(C)** 25 **(D)** –15

Q. 6. Find the value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$.

(A) 0 **(B)** 1 **(C)** –1 **(D)** 2

II. Case-Based MCQs **[1×4 = 4]**

Attempt any 4 sub-parts from each questions. Each question carries 1 mark.

Read the following text and answer the questions on the basis of the same.

A bolt manufacturing company produces three types of bolts x, y and z which he sells in two markets. Annual sales (in ₹) are indicated in the following table.

Markets	Products		
	x	y	x
I	10000	2000	18000
II	6000	20000	8000

It unit sales price of x, y and z are ₹2.50, ₹1.50 and ₹1.00 respectively, then answer the following questions using the concept of matrices.

Q. 7. The total revenue collected from market I is :

(A) ₹44,000 **(B)** ₹48,000 **(C)** ₹46,000 **(D)** ₹53,000

Q. 8. Total revenue collected from market II is :

(A) ₹46,000 **(B)** ₹51,000 **(C)** ₹53,000 **(D)** ₹49,000

Q. 9. If the unit costs of the above three types of bolts are ₹2, ₹1 and ₹0.50 respectively, then the gross profit from the market is :

(A) ₹46,000 **(B)** ₹25,000 **(C)** ₹32,000 **(D)** ₹20,000

Q. 10. If matrix $A = [a_{ij}]_{2 \times 2}$, where $a_{ij} = 1$ if $i \neq j$ and $a_{ij} = 0$ if $i = j$, then A^2 is :

(A) I **(B)** A **(C)** 0 **(D)** none of these

Q. 11. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, then AB is:

(A) $\begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0 & 3 \\ 22 & 43 \end{bmatrix}$ **(C)** $\begin{bmatrix} 43 & 22 \\ 0 & 3 \end{bmatrix}$ **(D)** $\begin{bmatrix} 22 & 43 \\ 3 & 0 \end{bmatrix}$

(B) SUBJECTIVE TYPE QUESTIONS:

III. Very Short Answer Type Questions **[1×3 = 3]**

Q. 12. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then find $A + A^T$.

Q. 13. Find the trace of matrix $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$.

Q. 14. If A is a skew symmetric matrix of odd order n, then $|A| = 0$.

IV. Short Answer Type Questions–I **[2×3 = 6]**

Q. 15. Using determinants, find the area of the triangle whose vertices are (–2, 4), (2, –6) and (5, 4). Are the given points collinear?

Q. 16. For the matrix $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y so that $A^2 + xI = yA$.

Q. 17. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then show that $A^{-1} = \frac{1}{19} A$.

V. Short Answer Type Questions-II **[3×2 = 6]**

Q. 18. Find the minors and cofactors of the elements of first row of determinant $\begin{vmatrix} 1 & 2 & 0 \\ 3 & 5 & -1 \\ 4 & 7 & 8 \end{vmatrix}$.

Q. 19. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^TA^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

VI. Long Answer Type Questions **[5×1 = 5]**

Q. 20. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equation :

$x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$

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