CHAPTER

2

INVERSE TRIGONOMETRIC FUNCTIONS

Syllabus

Definition, range, domain, principal value branch, Graphs of inverse trigonometric functions.

In this chapter you will study

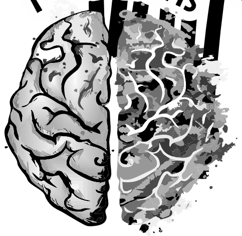
- *Domain of inverse trigonometric functions.*
- *Range of inverse trigonometric functions.*
- *Principal branch value of inverse trigonometric functions.*
- *Graph of different inverse trigonometric functions.*

Revision Notes

As we have learnt in class XI, the domain and range of trigonometric functions are given below:

S. No.	Function	Domain	Range
(i)	sine	R	$[-1, 1]$
(ii)	cosine	R	$[-1, 1]$
(iii)	tangent	$R-\left\{x : x=(2n+1)\frac{\pi}{2} ; n \in Z\right\}$	R
(iv)	cosecant	$R-\{x : x=n\pi, n \in Z\}$	$R-(-1, 1)$
(v)	secant	$R-\left\{x : x=(2n+1)\frac{\pi}{2} ; n \in Z\right\}$	$R-(-1, 1)$
(vi)	cotangent	$R-\{x : x=n\pi, n \in Z\}$	R

Inverse Trigonometric Functions



Domain and range of inverse trigonometric functions

(i) $y = \sin^{-1}x$. Domain = $[-1,1]$, Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii) $y = \cos^{-1}x$. Domain = $[-1,1]$ Range = $[0, \pi]$

(iii) $y = \text{cosec}^{-1}x$. Domain = $R-(-1,1)$, Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(iv) $y = \sec^{-1}x$. Domain = $R-(-1,1)$, Range = $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

(v) $y = \tan^{-1}x$. Domain = R, Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(vi) $y = \cot^{-1}x$. Domain = R, Range = $(0, \pi)$.

$\sin^{-1}x \neq (\sin x)^{-1} = \frac{1}{\sin x}$

Trigonometric functions

(i) $\sin : R \to [-1,1]$

(ii) $\cos : R \to [-1,1]$

(iii) $\tan : R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\} \to R$

(iv) $\cot : R - \{x : x = n\pi, n \in Z\} \to R$

(v) $\sec : R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\} \to R - (-1,1)$

(vi) $\text{cosec} : R - \{x : x = n\pi, n \in Z\} \to R - (-1,1)$

Some important relations

(i) $y = \sin^{-1}x \Rightarrow x = \sin y$

(ii) $x = \sin y \Rightarrow y = \sin^{-1}x$

(iii) $\sin(\sin^{-1}x) = x, -1 \leq x \leq 1$

(iv) $\sin^{-1}(\sin x) = x, \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$

(v) $\sin^{-1}\frac{1}{x} = \text{cosec}^{-1}x$

(vi) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

(vii) $\cos^{-1}\frac{1}{x} = \sec^{-1}x$

(viii) $\cot^{-1}(-x) = \pi - \cot^{-1}x$

(ix) $\tan^{-1}\frac{1}{x} = \cot^{-1}x, x > 0$

(x) $\sec^{-1}(-x) = \pi - \sec^{-1}x$

(xi) $\sin^{-1}(-x) = -\sin^{-1}x$

(xii) $\tan^{-1}(-x) = -\tan^{-1}x$

(xiii) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, -1 \leq x \leq 1$

(xiv) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

(xv) $\text{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}, |x| \geq 1$

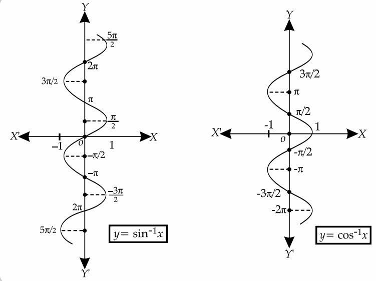
(xvi) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}, xy < 1$

(xvii) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}, -1<x<1$

(xviii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}, xy > 1$

(xix) $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}$

Graphs of trigonometric functions



Principal value branch and principal value

The range of an inverse trigonometric function is the principal value branch and those values which lies in the principal value branch is called the principal value of that inverse trigonometric function.

If $x > = 0$	If $x > 0$
$0 \leq \sin^{-1}x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \sin^{-1}x < 0$
$0 \leq \cos^{-1}x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cos^{-1}x \leq \pi$
$0 \leq \tan^{-1}x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \tan^{-1}x < 0$
$0 \leq \cot^{-1}x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cot^{-1}x < \pi$
$0 \leq \sec^{-1}x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \sec^{-1}x \leq \pi$
$0 \leq \text{cosec}^{-1}x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \text{cosec}^{-1}x < 0$

How to understand Mind Map?

- First Level
- Second Level
- Third Level

1. Inverse function

We know that if function $f : X \rightarrow Y$ such that $y = f(x)$ is **one-one** and **onto**, then we define another function $g : Y \rightarrow X$ such that $x = g(y)$, where $x \in X$ and $y \in Y$, which is also one-one and onto.

Key Words

One-one function: One to one function or one to one mapping states that each element of one set, say set A is mapped with a unique element of another set, say set B, where A and B are two different sets.

In terms of function, it states as if $f(x) = f(y) \Rightarrow x = y$, then f is one to one.

Onto function: If A and B are two sets, if for every element of B, there is atleast one or more element matching with set A, it is called onto function.

In such a case, Domain of g = Range of f

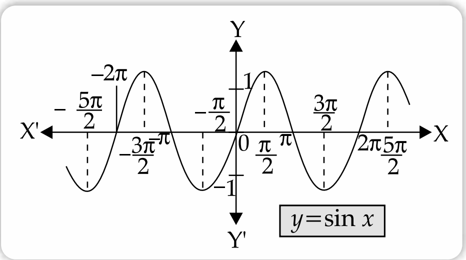
and Range of g = Domain of f

g is called the inverse of f

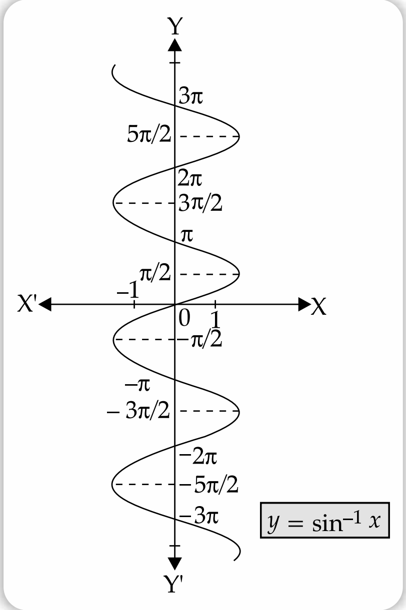
$$g = f^{-1}$$

or Inverse of $g = g^{-1} = (f^{-1})^{-1} = f$

The graph of sine function is shown here:

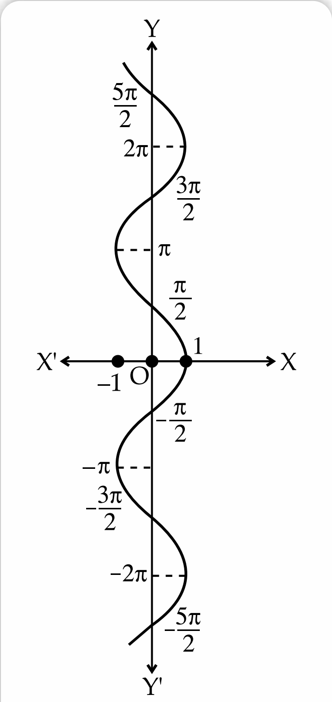


Principal value of branch function $\sin^{-1}$: It is a function with domain [– 1, 1] and range $\left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right], \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ or $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ and so on corresponding to each interval, we get a branch of the function $\sin^{-1} x$. The branch with range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is called the principal value branch. Thus, $\sin^{-1} : [-1, 1] \rightarrow \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

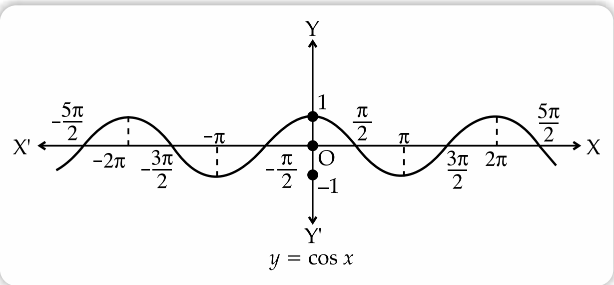


Principal value branch of function $\cos^{-1}$: The graph of the function $\cos^{-1}$ is as shown in figure. Domain of the function $\cos^{-1}$ is [–1, 1]. Its range in one of the intervals $(-\pi, 0)$, $(0, \pi)$, $(\pi, 2\pi)$, etc. is one-one and onto with the range [– 1, 1]. The branch with range $(0, \pi)$ is called the principal value branch of the function $\cos^{-1}$.

Thus, $\cos^{-1} : [-1, 1] \rightarrow [0, \pi]$



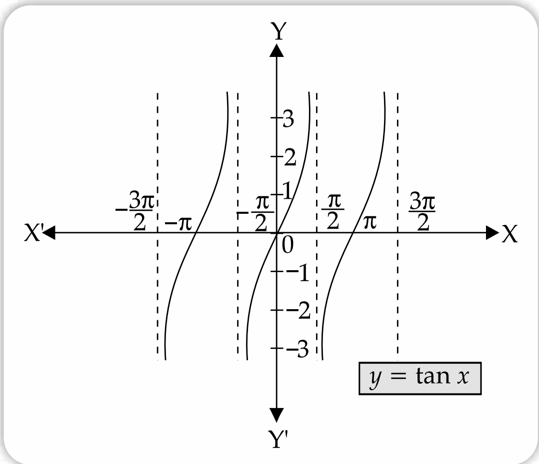
$y = \cos^{-1} x$

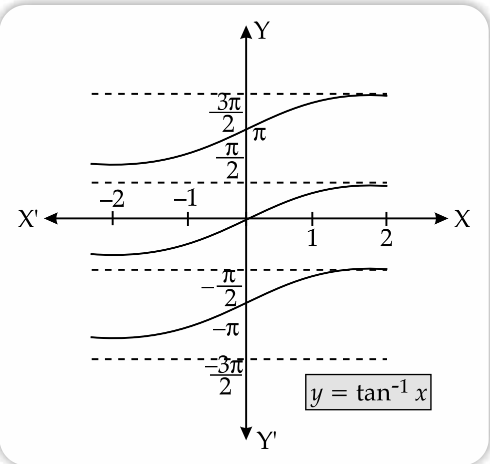


Principal value branch function $\tan^{-1}$: The function $\tan^{-1}$ is defined whose domain is set of real numbers and range is one of the intervals,

$$\left(\frac{-3\pi}{2},\frac{\pi}{2}\right),\left(\frac{-\pi}{2},\frac{\pi}{2}\right),\left(\frac{\pi}{2},\frac{3\pi}{2}\right),.....$$

Graph of the function is as shown in the figure:

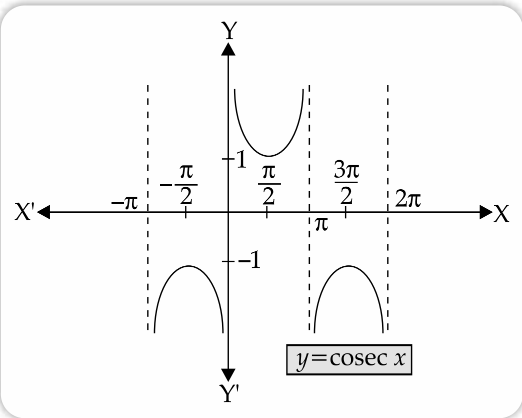


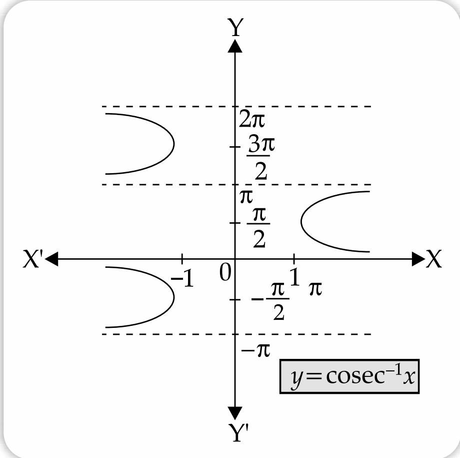


The branch with range $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is called the principal value branch of function $\tan^{-1}$. Thus, $\tan^{-1}: R \to \left(\frac{-\pi}{2},\frac{\pi}{2}\right)$.

Principal value branch of function cosec^{-1} : The graph of function cosec^{-1} is shown in the figure. The cosec^{-1} is defined on a function whose domain is $R - (-1, 1)$ and the range is any one of the interval,

$$\left[\frac{-3\pi}{2},\frac{-\pi}{2}\right]-\{\pi\},\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\},\left[\frac{\pi}{2},\frac{3\pi}{2}\right]-\{\pi\},...$$



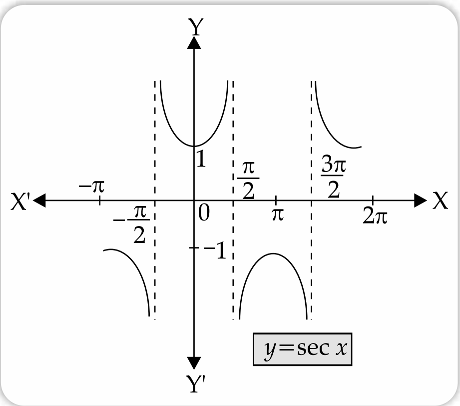


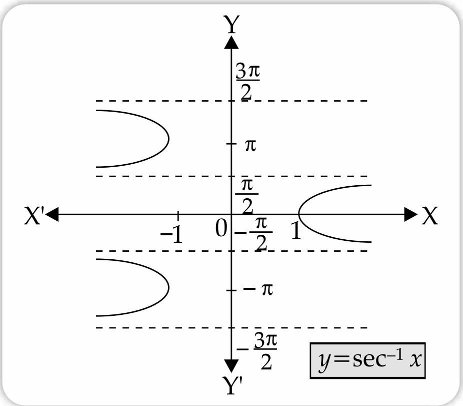
The function corresponding to the range $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$ is called the principal value branch of cosec^{-1}.

Thus, $\text{cosec}^{-1}: R-(-1,1) \to \left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$.

Principal value branch of function $\sec^{-1}$: The graph of function $\sec^{-1}$ is shown in figure. The $\sec^{-1}$ is defined as a function whose domain $R - (-1, 1)$ and range is $[-\pi, 0] - \left[\frac{-\pi}{2}\right]$, $[0, \pi]-\left\{\frac{\pi}{2}\right\}$, $[\pi, 2\pi]-\left\{\frac{3\pi}{2}\right\}$, etc. Function corresponding to range $[0, \pi]-\left\{\frac{\pi}{2}\right\}$ is known as the principal value branch of $\sec^{-1}$.

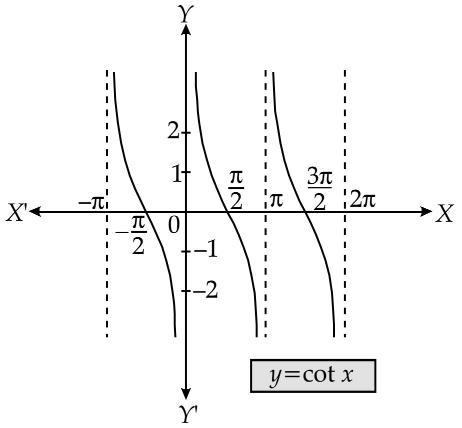
Thus, $\sec^{-1}: R-(-1,1) \to [0,\pi]-\left\{\frac{\pi}{2}\right\}$

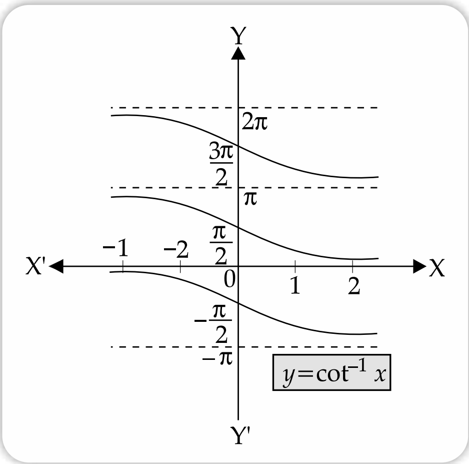




The principal value branch of function $\cot^{-1}$:

The graph of function $\cot^{-1}$ is shown below:





The $\cot^{-1}$ function is defined on function whose domain is R and the range is any of the intervals, $(-\pi, 0)$, $(0, \pi)$, $(\pi, 2\pi)$, ...

The function corresponding to $(0, \pi)$ is called the principal value branch of the function $\cot^{-1}$.

Then, $\cot^{-1} : R \longrightarrow (0, \pi)$

The principal value branch of trigonometric inverse functions is as follows:

Inverse Function	Domain	Principal Value Branch
$\sin^{-1}$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
cosec^{-1}	$R - (-1, 1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1}$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\tan^{-1}$	R	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}$	R	$(0, \pi)$

Key Facts

- Inverse trigonometric functions were considered early in the 1700s by Daniel Bernoulli, who use 'A.sin' for the inverse sine of a number, and in 1736. Euler wrote "At" for the inverse tangent.
- Inverse trigonometric functions one used to find the elevation of sun to the ground. The angle of tilt of the building can be found using inverse trigonometric functions.
- Inverse trigonometric functions help in identifying the angles of bridges to build scale models.
- Inverse trigonometric functions are often called 'arc functions', since given a value of a trigonometric function, they produce the length of arc needed to obtain that value.

(3) Principal Value:

Numerically smallest angle is known as the principal value.

Finding the principal value: For finding the principal value, following algorithm can followed :

STEP 1: First draw a trigonometric circle and mark the quadrant in which the angle may lie.

STEP 2: Select anti-clockwise direction for 1st and 2nd quadrants and clockwise direction for 3rd and 4th quadrants.

STEP 3: Find the angles in the first rotation.

STEP 4: Select the numerically least (magnitude wise) angle among these two values. The angle thus found will be the principal value.

STEP 5: In case, two angles one with positive sign and the other with the negative sign qualify for the numerically least angle then, it is the convention to select the angle with positive sign as principal value.

The principal value is never numerically greater than π.

(4) To simplify inverse trigonometric expressions, following substitutions can be considered:

Expression	Substitution
$a^2 + x^2$ or $\sqrt{a^2+x^2}$	$x = a \tan\theta$ or $x = a \cot\theta$
$a^2 - x^2$ or $\sqrt{a^2-x^2}$	$x = a \sin\theta$ or $x = a \cos\theta$
$x^2 - a^2$ or $\sqrt{x^2-a^2}$	$x = a \sec\theta$ or $x = a \operatorname{cosec}\theta$
$\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$	$x = a \cos 2\theta$
$\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ or $\sqrt{\frac{a^2+x^2}{a^2-x^2}}$	$x^2 = a^2 \cos 2\theta$
$\sqrt{\frac{x}{a-x}}$ or $\sqrt{\frac{a-x}{x}}$	$x = a\sin^2\theta$ or $x = a\cos^2\theta$
$\sqrt{\frac{x}{a+x}}$ or $\sqrt{\frac{a+x}{x}}$	$x = a\tan^2\theta$ or $x = a\cot^2\theta$

Note the following and keep them in mind:

- The symbol $\sin^{-1} x$ is used to denote the **smallest angle** whether positive or negative, such that the sine of this angle will give us x. Similarly $\cos^{-1} x$, $\tan^{-1} x$, $\operatorname{cosec}^{-1} x$, $\sec^{-1} x$ and $\cot^{-1} x$ are defined.
- You should note that $\sin^{-1} x$ can be written as **arcsin** x. Similarly, other Inverse Trigonometric Functions can also be written as arccos x, arctan x, arcsec x etc.
- Also note that $\sin^{-1} x$ (and similarly other Inverse Trigonometric Functions) **is entirely different from** $(\sin x)^{-1}$. In fact, $\sin^{-1} x$ is the measure of an angle in Radians whose sine is x whereas $(\sin x)^{-1}$ is $\frac{1}{\sin x}$ (which is obvious as per the laws of exponents).
- Keep in mind that these inverse trigonometric relations are **true only in their domains** *i.e.*, they are valid only for some values of 'x' for which inverse trigonometric functions are well defined.

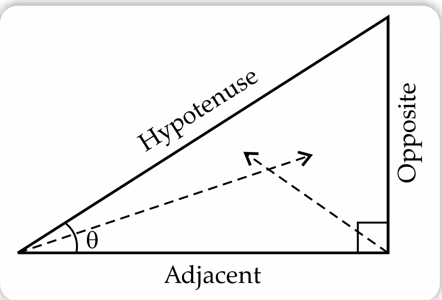
Mnemonics

Inverse trigonometric ratio can be used to find the angle of a right triangle when given two sides of the triangle.

SOH $\theta = \sin^{-1}\frac{\text{opposite}}{\text{hypotenuse}}$

CAH $\theta = \cos^{-1}\frac{\text{adjacent}}{\text{hypotenuse}}$

TOA $\theta = \tan^{-1}\frac{\text{opposite}}{\text{adjacent}}$



Key Formulae

TRIGONOMETRIC FORMULAE (ONLY FOR REFERENCE):

➢ **Relation between trigonometric ratios:**

(a) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ **(b)** $\tan\theta = \dfrac{1}{\cot\theta}$ **(c)** $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

(d) $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ **(e)** $\sec\theta = \dfrac{1}{\cos\theta}$

➢ **Trigonometric Identities:**

(a) $\sin^2\theta + \cos^2\theta = 1$ **(b)** $\sec^2\theta = 1 + \tan^2\theta$ **(c)** $\text{cosec}^2\,\theta = 1 + \cot^2\theta$

➢ **Addition/subtraction/ formulae & some related results:**

(a) $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ **(b)** $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

(c) $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

(d) $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

(e) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$ **(f)** $\cot(\text{A} \pm \text{B}) = \dfrac{\cot B\cot A \mp 1}{\cot B \pm \cot A}$

➢ **Multiple angle formulae involving *A*, 2 *A* & 3 *A*:**

(a) $\sin 2A = 2\sin A\cos A$ **(b)** $\sin A = 2\sin\dfrac{A}{2}\cos\dfrac{A}{2}$

(c) $\cos 2A = \cos^2 A - \sin^2 A$ **(d)** $\cos A = \cos^2\dfrac{A}{2} - \sin^2\dfrac{A}{2}$

(e) $\cos 2A = 2\cos^2 A - 1$ **(f)** $2\cos^2 A = 1 + \cos 2A$

(g) $\cos 2A = 1 - 2\sin^2 A$ **(h)** $2\sin^2 A = 1 - \cos 2A$

(i) $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$ **(j)** $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

(k) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ **(l)** $\sin 3A = 3\sin A - 4\sin^3 A$

(m) $\cos 3A = 4\cos^3 A - 3\cos A$ **(n)** $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

➢ **Transformation of sums/differences into products & vice-versa:**

(a) $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$ **(b)** $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

(c) $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$ **(d)** $\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

(e) $2\sin A\cos B = \sin(A + B) + \sin(A - B)$ **(f)** $2\cos A\sin B = \sin(A + B) - \sin(A - B)$

(g) $2\cos A\cos B = \cos(A + B) + \cos(A - B)$ **(h)** $2\sin A\sin B = \cos(A - B) - \cos(A + B)$

➢ **Relations in different measures of Angle:**

(a) Angle in Radian Measure = (Angle in degree measure) × $\dfrac{\pi}{180'}$rad

(b) Angle in Degree Measure = (Angle in radian measure) × $\dfrac{180°}{\pi}$

(c) θ (in radian measure) = $\dfrac{l}{r} = \dfrac{\text{arc}}{\text{radius}}$

Also following are of importance as well:

(a) 1 right angle = 90° **(b)** 1° = 60′, 1′ = 60″

(c) $1° = \dfrac{\pi}{180°}$ = 0·01745 radians (Approx.) **(d)** 1 radian = 57°17′45″ or 206265 seconds.

➢ **General Solutions:**

(a) $\sin x = \sin y$ Or, $x = n\pi + (-1)^n y$, where $n \in Z$.

(b) $\cos x = \cos y$ Or, $x = 2n\pi \pm y$, where $n \in Z$.

(c) $\tan x = \tan y$ Or, $x = n\pi + y$, where $n \in Z$.

➢ **Relation in Degree & Radian Measures:**

Angles in Degree	0°	30°	45°	60°	90°	180°	270°	360°
Angles in Radian	0°	$\left(\frac{\pi}{6}\right)$	$\left(\frac{\pi}{4}\right)$	$\left(\frac{\pi}{3}\right)$	$\left(\frac{\pi}{2}\right)$	(π)	$\left(\frac{3\pi}{2}\right)$	(2π)

➢ **Trigonometric Ratio of Standard Angles:**

Degree	**0°**	**30°**	**45°**	**60°**	**90°**
sin x	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan x	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot x	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
cosec x	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec x	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞

➢ **Trigonometric Ratios of Allied Angles:**

Angles (→) **T – Ratios (↓)**	$\frac{\pi}{2}-\theta$	$\frac{\pi}{2}+\theta$	$\pi-\theta$	$\pi+\theta$	$\frac{3\pi}{2}-\theta$	$\frac{3\pi}{2}+\theta$	**2π – θ or – θ**	**2π + θ**
sin	cos θ	cos θ	sin θ	– sin θ	– cos θ	– cos θ	– sin θ	sin θ
cos	sin θ	– sin θ	– cos θ	– cos θ	– sin θ	sin θ	cos θ	cos θ
tan	cot θ	– cot θ	– tan θ	tan θ	cot θ	– cot θ	– tan θ	tan θ
cot	tan θ	– tan θ	– cot θ	cot θ	tan θ	– tan θ	– cot θ	cot θ
sec	cosec θ	– cosec θ	– sec θ	– sec θ	– cosec θ	cosec θ	sec θ	sec θ
cosec	sec θ	sec θ	cosec θ	– cosec θ	– sec θ	– sec θ	– cosec θ	cosec θ

✔ OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. The principal value of $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ **is**

(A) $\frac{\pi}{12}$ **(B)** π

(C) $\frac{\pi}{3}$ **(D)** $\frac{\pi}{6}$ **[CBSE Board 2021]**

Ans. Option (A) is correct.

Explanation: $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$= \cos^{-1}\left(\cos\frac{\pi}{3}\right)-\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $[\sin^{-1}(-\theta) = -\sin^{-1}\theta]$

$= \frac{\pi}{3}-\sin^{-1}\left(\sin\frac{\pi}{4}\right) = \frac{\pi}{3}-\frac{\pi}{4} = \frac{\pi}{12}$

Q. 2. The principal value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$ is:

(A) $\frac{\pi}{8}$ (B) $\frac{3\pi}{8}$

(C) $-\frac{\pi}{8}$ (D) $-\frac{3\pi}{8}$

[CBSE Board 2021]

Ans. Option (A) is correct.

Explanation: $\tan^{-1}\left(\tan\frac{9\pi}{8}\right) = \tan^{-1}\left[\tan\left(\pi+\frac{\pi}{8}\right)\right]$

$$= \tan^{-1}\left(\tan\frac{\pi}{8}\right) = \frac{\pi}{8}$$

$$\left[\because \frac{\pi}{8} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$$

Q. 3. What is the domain of the function $\cos^{-1}(2x - 3)$?

(A) $[-1, 1]$ (B) $(1, 2)$

(C) $(-1, 1)$ (D) $[1, 2]$

[CBSE Board 2021]

Ans. Option (D) is correct.

Explanation: Let, $f(x) = \cos^{-1}(2x - 3)$

$$-1 \leq 2x - 3 \leq 1$$

$$\Rightarrow \quad 2 \leq 2x \leq 4$$

$$\Rightarrow \quad 1 \leq x \leq 2$$

$\therefore x \in [1, 2]$ or domain of x is $[1, 2]$.

Q. 4. The principal value of $[\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})]$ is:

(A) π (B) $-\frac{\pi}{2}$

(C) 0 (D) $2\sqrt{3}$

[CBSE Board 2021]

Ans. Option (B) is correct.

Explanation: We have,

$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \pi + \cot^{-1}(\cot\sqrt{3})$$

$$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{2} \qquad [\cot^{-1}(-\theta) = \pi - \cot^{-1}\theta]$$

Q. 5. The simplest form of $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ is:

(A) $\frac{\pi}{4} - \frac{\pi}{2}$ (B) $\frac{\pi}{4} + \frac{\pi}{2}$

(C) $\frac{\pi}{4} - \frac{\pi}{2}\cos^{-1}x$ (D) $\frac{\pi}{4} + \frac{\pi}{2}\cos^{-1}x$

[CBSE Board 2021]

Ans. Option (C) is correct.

Explanation: We have, $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

Put $x = \cos 2\theta$, so that $\theta = \frac{1}{2}\cos^{-1}x$

$$\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right) \qquad \textbf{Divide by } \cos\theta$$

$$= \tan^{-1}\left|\frac{1-\tan\theta}{1+\tan\theta}\right|$$

$$= \left[\tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)\right]$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$$

Q. 6. The principal value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is:

(A) $\frac{13\pi}{6}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

[CBSE Board 2020]

Ans. Option (D) is correct.

Explanation:

Topper Answer, 2020

Sol. $\cos^{-1}\cos\left(\frac{13\pi}{6}\right) = \cos^{-1}\cos\left(2\pi+\frac{\pi}{6}\right) = \cos^{-1}\cos\left(\frac{\pi}{6}\right) = \pi/6$

D. $\pi/6$

Q. 7. The value of $\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$ is:

(A) $\frac{\pi}{10}$ (B) $\frac{3\pi}{5}$

(C) $-\frac{\pi}{10}$ (D) $\frac{-3\pi}{5}$

[CBSE OD Set-I 2020]

Ans. Option (C) is correct.

Explanation:

$$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right)$$

$$\left[\because \cos\left(\frac{\pi}{2}+x\right) = -\sin x\right]$$

$$= -\sin^{-1}\left(\sin\frac{\pi}{10}\right)$$

$[\because \sin^{-1}(-x) = -\sin^{-1} x]$

$$= -\frac{\pi}{10}$$

$$\left[\because \sin^{-1}(\sin x) = x, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$$

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$.

R&U [CBSE Delhi SET-I, 2020]

Sol. $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right] = -\sin^{-1}\left[\sin\left(2\pi+\frac{\pi}{8}\right)\right]$ ½

$= -\frac{\pi}{8}$ ½

[CBSE Marking Scheme 2020]

Commonly Made Error

- Most of the students write the answer as $\frac{-17\pi}{8}$.

Answering Tip

- $\sin^{-1}(\sin\theta) = \theta$, iff $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Q. 2. Write the principal value of

$\tan^{-1}(\sqrt{3}) + \cot^{-1}(-\sqrt{3})$.

R&U [Delhi & O.D., 2018] [NCERT]

Sol. $\frac{\pi}{3}+\left(\pi-\frac{\pi}{6}\right) = \frac{7\pi}{6}$ 1

[CBSE Marking Scheme 2018]

Detailed Answer:

$$\tan^{-1}\sqrt{3}+\cot^{-1}(-\sqrt{3})$$

$$= \tan\left(\tan\frac{\pi}{3}\right)+(\pi-\cot^{-1}\sqrt{3})$$

$$= \frac{\pi}{3}+\pi-\cot^{-1}\sqrt{3}$$

$$= \frac{4\pi}{3}-\frac{\pi}{6}=\frac{7\pi}{6}$$

Short Answer Type Questions-I (2 marks each)

Q. 1. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), \frac{-3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

R&U [CBSE SQP 2020-21]

Sol. $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right]$ ½

$$\tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$$

$\tan^{-1}\left[\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right] = \tan^{-1}\left[\tan\left\{\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}\right]$ 1

$\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right] = \frac{\pi}{4}+\frac{x}{2}$ ½

[CBSE SQP Marking Scheme 2020-21]

Q. 2. Prove that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x, \frac{1}{\sqrt{2}} \le x \le 1.$ **[CBSE Board 2020]**

Topper Answer, 2020

Sol. let $y = \sin^{-1}(2x\sqrt{1-x^2})$

let $x = \cos\theta$

$\Rightarrow \theta = \cos^{-1}x \qquad 0 \le \theta \le \pi/4$

$\Rightarrow y = \sin^{-1}(2\cos\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}\sin 2\theta$

$= 2\theta \qquad [\because 0 \le \theta \le \pi/4 \Rightarrow 0 \le 2\theta \le \pi/2]$

$= 2\cos^{-1}x$

Hence Proved

Q. 3. Prove that:

$$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

A I R&U [CBSE OD SET-I, II, III 2020]

Q. 4. Express $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$**; where** $-\frac{\pi}{4} < x < \frac{\pi}{4}$ **in the simplest form.** A [CBSE SQP 2020]

Sol. $\sin^{-1}\left(\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}}\right)$ if $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$= \sin^{-1}\left(\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}\right)$

if $-\frac{\pi}{4} + \frac{\pi}{4} < x + \frac{\pi}{4} < \frac{\pi}{4} + \frac{\pi}{4}$ **1**

$= \sin^{-1}\left(\sin\left(x + \frac{\pi}{4}\right)\right)$ if $0 < \left(x + \frac{\pi}{4}\right) < \frac{\pi}{2}$

i.e. principal values $= \left(x + \frac{\pi}{4}\right)$ **1**

[CBSE SQP Marking Scheme 2020]

Q. 5. Prove that 3 $\sin^{-1}x = \sin^{-1}[3x - 4x^3]$ $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

A [Delhi, O.D., 2018]

Sol. In RHS, put $x = \sin\theta$ ½

RHS $= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$ **1**

$= 3\theta = 3\sin^{-1}x =$ LHS ½

[CBSE Marking Scheme 2018]

Detailed Solution:

Topper Answer, 2018

Sol. $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

R.H.S. $\sin^{-1}(3x - 4x^3)$

Put $x = \sin\theta$ ½

$\sin^{-1}(3\sin\theta - 4\sin^3\theta)$ 1

$\sin^{-1}(\sin 3\theta)$

3θ

$3\sin^{-1}x$ ½

RHS = LHS

Hence Proved

$-\frac{1}{2} \le x \le \frac{1}{2}$

$-\frac{1}{2} \le \sin\theta \le \frac{1}{2}$

$\sin^{-1}\left(-\frac{1}{2}\right) \le \theta \le \sin^{-1}\left(\frac{1}{2}\right)$

$-\frac{\pi}{3} \le \theta \le \frac{\pi}{3}$

$\left[\frac{\pi}{3}, \frac{\pi}{3}\right] \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Q. 6. If 4 $\sin^{-1}x + \cos^{-1}x = \pi$**, then find the value of** x**.** R&U [S.Q.P. 2018]

Sol. $4\sin^{-1}x + \cos^{-1}x = \pi$

or $4\sin^{-1}x + \left(\frac{\pi}{2} - \sin^{-1}x\right) = \pi$ ½ or $4\sin^{-1}x - \sin^{-1}x = \pi - \frac{\pi}{2}$ ½

or $3\sin^{-1}x = \frac{\pi}{2}$

or $\sin^{-1}x = \frac{\pi}{6}$

$\therefore x = \frac{1}{2}$ **1**

Q. 7. Prove that $3\cos^{-1}x = \cos^{-1}[4x^3 - 3x]$, $x \in \left[\frac{1}{2}, 1\right]$

A [CBSE Comptt. Set I, II, III 2018]

Q. 8. Simplify : $\cot^{-1}\frac{-1}{\sqrt{x^2-1}}$ for $x < -1$.

R&U [S.Q.P. 2016-17]

Sol. Let $\sec^{-1}x = \theta$, then $x = \sec\theta$ and for $x < -1$,

$\frac{\pi}{2} < \theta < \pi$ ½

Given expression $= \cot^{-1}(-\cot\theta)$ ½

$= \cot^{-1}[\cot(\pi - \theta)] = \pi - \sec^{-1}x$ as $0 < \pi - \theta < \frac{\pi}{2}$ **1**

Short Answer Type Questions-II (3 marks each)

Q. 1. Solve the equation for x :

$$\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2} \ (x \neq 0)$$

R&U [CBSE Delhi Set III 2020]

Q. 2. Solve for $x : \sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.

AI A [NCERT] [CBSE Delhi Set I 2020]

Sol. $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$ **1**

$\Rightarrow (1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$

$\Rightarrow (1-x) = \cos(2\sin^{-1}x)$

$\Rightarrow 1 - x = 1 - 2x^2$ **1**

$\therefore 2x^2 - x = 0$

$\Rightarrow x = 0, x = \frac{1}{2}$ ½

Since, $x = \frac{1}{2}$ does not satisfy the given equation

$\therefore x = 0$ is the required solution. ½

[CBSE Marking Scheme 2020 (Modified)]

Commonly Made Error

- Errors are made by students while squaring, simplifying and solving higher degree algebraic equations.

Answering Tip

- Back substitute the answer in the given equation to obtain the final answer.

Q. 3. Find the value of $\sin\left(2\tan^{-1}\frac{1}{4}\right) + \cos\left(\tan^{-1}2\sqrt{2}\right)$.

R&U [CBSE SQP, 2018]

Sol. $\sin\left(2\tan^{-1}\frac{1}{4}\right) + \cos\left(\tan^{-1}2\sqrt{2}\right)$

Lets first evaluated, $\left(2\tan^{-1}\frac{1}{4}\right)$

Put $\tan^{-1}\frac{1}{4} = \theta$

$\Rightarrow \tan\theta = \frac{1}{4}$

Now, $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$= \frac{2\times\frac{1}{4}}{1+\left(\frac{1}{4}\right)^2} = \frac{8}{17}$ **1**

To evaluate $\cos(\tan^{-1}2\sqrt{2})$, put $\tan^{-1}2\sqrt{2} = \phi$

$\Rightarrow \tan\phi = 2\sqrt{2}$

$\Rightarrow \cos\phi = \frac{1}{3}$ **1**

$\sin\left(2\tan^{-1}\frac{1}{4}\right) + \cos\left(\tan^{-1}2\sqrt{2}\right) = \frac{8}{17} + \frac{1}{3} = \frac{41}{51}$ **1**

[CBSE Marking Scheme 2018 (Modified)]

Commonly Made Error

- Some errors are made by students in converting inverse trigonometric functions into simple trigonometric form and also algebraic calculations.

Answering Tip

- Use right triangles to convert the trigonometric ratios.

Q. 4. Prove that $\cos^{-1}(x) + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right) = \frac{\pi}{3}$.

AI R&U [SQP 2018]

Sol. Let, $\cos^{-1}x = \alpha$ or $x = \cos\alpha$

$$\text{LHS} = \alpha + \cos^{-1}\left[\cos\alpha\cos\left(\frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}\sqrt{1-\cos^2\alpha}\right] \quad \mathbf{1}$$

$$= \alpha + \cos^{-1}\left[\cos\left(\frac{\pi}{3}\right)\cos\alpha + \sin\frac{\pi}{3}\sin\alpha\right] \quad \mathbf{1}$$

$$= \alpha + \cos^{-1}\left[\cos\left(\frac{\pi}{3} - \alpha\right)\right]$$

$$= \alpha + \frac{\pi}{3} - \alpha \quad [\because \cos^{-1}(\cos\theta) = \theta \ \forall\ \theta \in (0, \pi)] \ ½$$

$$= \frac{\pi}{3} = \text{RHS} \quad ½$$

[CBSE Marking Scheme 2018 (Modified)]

Commonly Made Error

- Some students apply the formula for $\cos^{-1} x + \cos^{-1} y$ and gets confused.

Answering Tip

- Apply the properties of inverse trigonometric functions.

Q. 5. Solve for x : $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x,\ x > 0$.

R&U [O.D. Set I, II, III Comptt. 2017]
[NCERT][O.D. Set I, II, III Comptt. 2015]

Q. 6. Solve for x : $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\, x)$.

A [NCERT][Delhi Set I, II, III, 2016]
[NCERT Exemplar][Foreign 2015]

Sol. Given, $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\, x)$

or $\tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\,\text{cosec}\, x)$ ½

or $2\cot x\,\text{cosec}\, x = 2\,\text{cosec}\, x$ **1**

or $\cot x = 1$ ½

or $x = \cot^{-1}(1) = \frac{\pi}{4}$ **1**

[CBSE Marking Scheme 2016 (Modified)]

Q. 7. If $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1}x)$, then find x.

R&U [Delhi I, II, III 2015]

Sol. Given that $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1}x)$ ...(i)

We know that,

$$\cot^{-1}(A) = \sin^{-1}\frac{1}{\sqrt{1+A^2}}$$

Here, $A = x + 1$

Applying this identity in equation (i), we have

$$\sin\left[\sin^{-1}\frac{1}{\sqrt{1+(1+x)^2}}\right] = \cos(\tan^{-1}x) \quad \text{...(ii)}\ \mathbf{1}$$

Also, we know that

$$\tan^{-1}A = \cos^{-1}\frac{1}{\sqrt{1+A^2}}$$

Here, $A = x$

Applying this identity in equation (ii), we have

$$\sin\left[\sin^{-1}\frac{1}{\sqrt{1+(1+x)^2}}\right] = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) \quad \mathbf{1}$$

or
$$\frac{1}{\sqrt{1+(1+x)^2}} = \frac{1}{\sqrt{1+x^2}}$$

$$\left[\because \sin^{-1}(\sin\theta) = \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right.$$

and $\cos^{-1}(\cos\theta) = \theta\ \forall \in \theta \in [0, \pi]]$

Squaring and Reciprocating on both sides, we have

$$1 + (1+x)^2 = 1 + x^2 \quad ½$$

or $1 + 1 + x^2 + 2x = 1 + x^2$

or $1 + 2x = 0$

or $x = -\frac{1}{2}$ ½

[CBSE Marking Scheme 2015 (Modified)]

Commonly Made Error

- Generally students get confused with trigonometric identities of inverse trigonometric functions. They substitute wrong identities.

Answering Tip

- Students should do practice for the conversion like $\tan^{-1}\theta = \cos^{-1}\frac{1}{\sqrt{1+\theta^2}}$, $\sin^{-1}(\sin\theta) = \theta$ and $\cos^{-1}(\cos\theta) = -\theta$.

Q. 8. Prove that:

$$\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right) = \frac{\pi}{4} - \frac{x}{2},$$

where $\pi < x < \frac{3\pi}{2}$. A [CBSE S.Q.P. 2016]

Sol. $\tan^{-1}\left(\dfrac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)$

$$= \tan^{-1}\left(\dfrac{\sqrt{2\cos^2\dfrac{x}{2}}+\sqrt{2\sin^2\dfrac{x}{2}}}{\sqrt{2\cos^2\dfrac{x}{2}}-\sqrt{2\sin^2\dfrac{x}{2}}}\right) \quad ½$$

$$= \tan^{-1}\left(\dfrac{-\sqrt{2}\cos\dfrac{x}{2}+\sqrt{2}\sin\dfrac{x}{2}}{-\sqrt{2}\cos\dfrac{x}{2}-\sqrt{2}\sin\dfrac{x}{2}}\right) \quad 1$$

$$\left(\text{As } \pi < x < \dfrac{3\pi}{2} \text{ or } \dfrac{\pi}{2} < \dfrac{x}{2} < \dfrac{3\pi}{4} \text{ or } \cos\dfrac{x}{2} < 0, \; \sin\dfrac{x}{2} > 0\right)$$

$$= \tan^{-1}\left(\dfrac{1-\tan\dfrac{x}{2}}{1+\tan\dfrac{x}{2}}\right) \quad ½$$

$$= \tan^{-1}\left(\tan\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)\right)$$

$$= \dfrac{\pi}{4}-\dfrac{x}{2} \qquad \left(\text{As, } -\dfrac{\pi}{4} > \dfrac{\pi}{4}-\dfrac{x}{2} > -\dfrac{\pi}{2}\right) \quad 1$$

[CBSE Marking Scheme 2015 (Modified)]

Long Answer Type Questions (5 marks each)

Q. 1. Prove that $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$

$$= \dfrac{\pi}{4}+\dfrac{1}{2}\cos^{-1}x^2; -1 < x < 1$$ **[Foreign set III 2017]**

R&U [NCERT Exemplar]

Q. 2. Prove that $\tan^{-1}\left(\dfrac{6x-8x^3}{1-12x^2}\right)-\tan^{-1}\left(\dfrac{4x}{1-4x^2}\right)$

$$= \tan^{-1}2x; |2x| < \dfrac{1}{\sqrt{3}}.$$

R&U [Outside Delhi Set I, II, III, Comptt. 2016]

Sol. Let, $2x = \tan\theta$ 1

$$\text{LHS} = \tan^{-1}\left(\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)-\tan^{-1}\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right) \quad 2$$

$= \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 2\theta)$ 1

$= 3\theta - 2\theta$

$= \theta$ or $\tan^{-1} 2x$ 1

$\therefore$ LHS = RHS

[CBSE Marking Scheme 2016 (Modified)]

Commonly Made Error

- Students apply the formula for $\tan^{-1} x + \tan^{-1} y$ and makes it complicated, even though the answer is obtained.

Answering Tip

- The right substitution gives the answer in a few steps.

Q. 3. If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \dfrac{5\pi^2}{8}$**, find** x.

R&U [Delhi I, II, III 2015]

Sol. $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \dfrac{5\pi^2}{8}$

or $(\tan^{-1}x)^2 + \left(\dfrac{\pi}{2}-\tan^{-1}x\right)^2 = \dfrac{5\pi^2}{8}$ 1

or $2(\tan^{-1}x)^2 - \pi\tan^{-1}x + \dfrac{\pi^2}{4} - \dfrac{5\pi^2}{8} = 0$

or $2(\tan^{-1}x)^2 - \pi\tan^{-1}x - \dfrac{3\pi^2}{8} = 0$ 2

or $\tan^{-1}x = \dfrac{\pi \pm \sqrt{\pi^2+3\pi^2}}{4}$ 1

or $\tan^{-1}x = \dfrac{3\pi}{4}, \dfrac{-\pi}{4}$

$\therefore$ $x = -1$ 1

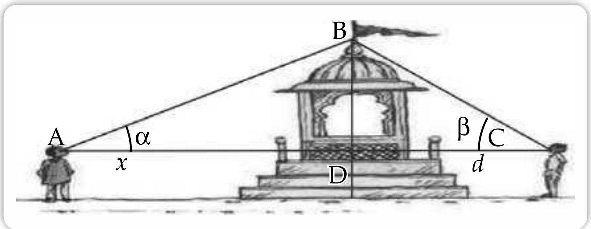
[CBSE Marking Scheme 2015 (Modified)]

COMPETENCY BASED QUESTIONS

Case based MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:



Two men on either side of a temple of 30 metres high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is $\mathbf{40\sqrt{3}}$ metres and the distance between the first person A and the temple is $\mathbf{30\sqrt{3}}$ meters. **[CBSE QB-2021]**

Q. 1. $\angle CAB = \alpha =$

(A) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ **(B)** $\sin^{-1}\left(\frac{1}{2}\right)$

(C) $\sin^{-1}(2)$ **(D)** $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Ans. Option (B) is correct.

Explanation: In Δ BDA

$$\sin\alpha = \frac{BD}{AB}$$

$$AB^2 = AD^2 + BD^2$$
$$= (30\sqrt{3})^2 + (30)^2$$
$$= (60)^2$$
$$AB = 60\,\text{m}$$

Now, $\sin\alpha = \frac{30}{60}$

$$\sin\alpha = \frac{1}{2}$$

i.e. $\angle \text{CAB} = \alpha = \sin^{-1}\left(\frac{1}{2}\right)$

Q. 2. $\angle CAB = \alpha =$

(A) $\cos^{-1}\left(\frac{1}{5}\right)$ **(B)** $\cos^{-1}\left(\frac{2}{5}\right)$

(C) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ **(D)** $\cos^{-1}\left(\frac{4}{5}\right)$

Ans. Option (C) is correct.

Explanation: In Δ *BDA*

$$\cos\alpha = \frac{AD}{AB}$$

$$\cos\alpha = \frac{30\sqrt{3}}{60}$$

$$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

∴ $\angle CAB = \alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Q. 3. $\angle BCA = \beta =$

(A) $\tan^{-1}\left(\frac{1}{2}\right)$ **(B)** $\tan^{-1}(2)$

(C) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **(D)** $\tan^{-1}\left(\sqrt{3}\right)$

Ans. Option (D) is correct.

Explanation:

$$DC = AC - AD$$
$$= 40\sqrt{3} - 30\sqrt{3}$$
$$= 10\sqrt{3}\,\text{m}$$

In Δ*BDC*

$$\tan\beta = \frac{BD}{DC} = \frac{30}{10\sqrt{3}} = \sqrt{3}$$
$$\angle BCA = \beta = \tan^{-1}(\sqrt{3})$$

Q. 4. $\angle ABC =$

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{6}$

(C) $\frac{\pi}{2}$ **(D)** $\frac{\pi}{3}$

Ans. Option (C) is correct.

Explanation: Since,

$$\sin\alpha = \frac{1}{2}$$

i.e., $\sin\alpha = \sin 30°$

$$\left[\because \sin 30° = \frac{1}{2}\right]$$

∴ $\alpha = 30°$

we, have $\tan\beta = \sqrt{3}$

$$\tan\beta = \tan 60°$$

∴ $\beta = 60°$

Now, In Δ*ABC*

$$\angle ABC + \angle BCA + \angle CAB = 180°$$
$$\angle ABC + 60° + 30° = 180°$$
$$\angle ABC = 90°$$

∴ $\angle ABC = \frac{\pi}{2}$

Q. 5. Domain and Range of $\cos^{-1}x$ is:

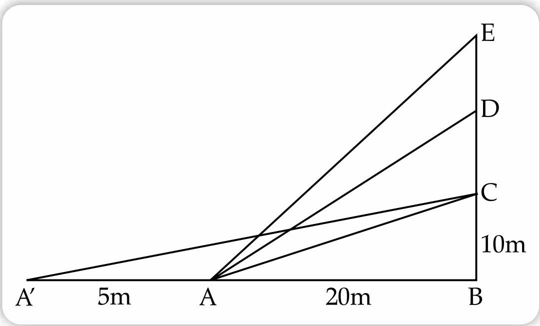
(A) $(-1, 1), (0, \pi)$ **(B)** $[-1, 1], (0, \pi)$

(C) $[-1, 1], [0, \pi]$ **(D)** $(-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ans. Option (C) is correct.

II. Read the following text and answer the following questions on the basis of the same:

The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. "A" is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely C, D and E. "C" is the height of 10 metres from the ground level. For the viewer A, the angle of elevation of "D" is double the angle of elevation of "C". The angle of elevation of "E" is triple the angle of elevation of "C" for the same viewer. Look at the figure given and based on the above information answer the following:



[CBSE QB 2021]

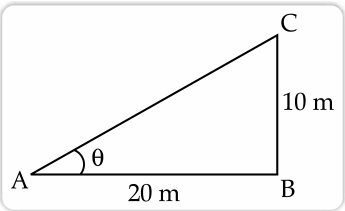
Q. 1. Measure of ∠CAB is:

(A) $\tan^{-1}(2)$ **(B)** $\tan^{-1}\left(\frac{1}{2}\right)$

(C) $\tan^{-1}(1)$ **(D)** $\tan^{-1}(3)$

Ans. Option (B) is correct.

Explanation: Let ∠CAB = θ, therefore ∠DAB = 2θ and ∠EAB = 3θ



In right ΔABC, we have

$$\tan\theta = \frac{BC}{AB} = \frac{10}{20} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \tan^{-1}\frac{1}{2} \quad ...(i)$$

$$\therefore \quad \angle CAB = \tan^{-1}\frac{1}{2}$$

Q. 2. Measure of ∠DAB is:

(A) $\tan^{-1}\left(\frac{3}{4}\right)$ **(B)** $\tan^{-1}(3)$

(C) $\tan^{-1}\left(\frac{4}{3}\right)$ **(D)** $\tan^{-1}(4)$

Ans. Option (C) is correct.

Explanation:

From eq. (i), we have

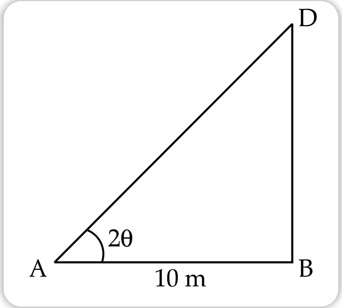
$$\tan\theta = \frac{1}{2}$$

$$\therefore \quad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

$$\Rightarrow \quad 2\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\therefore \quad \angle DAB = \tan^{-1}\left(\frac{4}{3}\right)$$



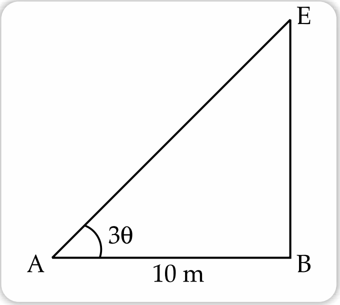
Q. 3. Measure of ∠EAB is:

(A) $\tan^{-1}(11)$ **(B)** $\tan^{-1}3$

(C) $\tan^{-1}\left(\frac{2}{11}\right)$ **(D)** $\tan^{-1}\left(\frac{11}{2}\right)$

Ans. Option (D) is correct.

Explanation: We have $\tan\theta = \frac{1}{2}$



$$\therefore \quad \tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$$

$$= \frac{3\times\frac{1}{2}-\left(\frac{1}{2}\right)^3}{1-3\left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{3}{2}-\frac{1}{8}}{1-\frac{3}{4}} = \frac{\frac{11}{8}}{\frac{1}{4}}$$

$$= \frac{11}{8}\times 4 = \frac{11}{2}$$

$$\therefore \quad 3\theta = \tan^{-1}\left(\frac{11}{2}\right)$$

$$\Rightarrow \quad \angle EAB = \tan^{-1}\left(\frac{11}{2}\right)$$

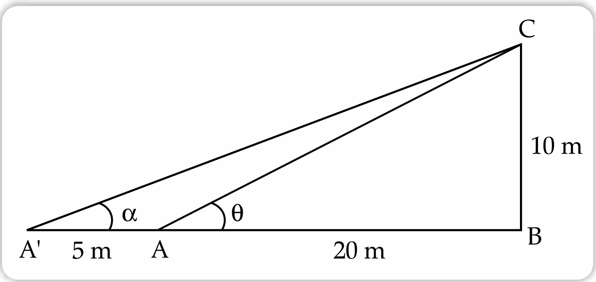
Q. 4. A' is another viewer standing on the same line of observation across the road. If the width of the road is 5 meters, then the difference between ∠CAB and ∠CA'B is:

(A) $\tan^{-1}\left(\frac{1}{12}\right)$ **(B)** $\tan^{-1}\left(\frac{1}{8}\right)$

(C) $\tan^{-1}\left(\frac{2}{5}\right)$ **(D)** $\tan^{-1}\left(\frac{11}{21}\right)$

Ans. Option (A) is correct.

Explanation: Let ∠CA'B = α



In ΔA'CB, $\tan\alpha = \frac{BC}{A'B} = \frac{10}{20+5}$

$\Rightarrow \tan\alpha = \frac{10}{25} = \frac{2}{5}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{2}{5}\right)$

$\Rightarrow \angle CA'B = \tan^{-1}\left(\frac{2}{5}\right)$

Also, $\angle CAB = \tan^{-1}\left(\frac{1}{2}\right)$ [Calculated above]

$$\therefore \angle CAB - \angle CA'B = \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{2}{5}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{2}-\frac{2}{5}}{1+\frac{1}{2}\times\frac{2}{5}}\right)$$

$$= \tan^{-1}\left(\frac{1}{12}\right)$$

Therefore, the required difference between ∠CAB and ∠CA'B in $\tan^{-1}\left(\frac{1}{12}\right)$.

Q. 5. Domain and Range of $\tan^{-1}x$ is:

(A) $R^+, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **(B)** $R^-, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(C) $R, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **(D)** $R, \left(0, \frac{\pi}{2}\right)$

Ans. Option (C) is correct.

Explanation: Domain and Range of $\tan^{-1}x$ are respectively $(-\infty, \infty)$ and $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ *i.e.,* $R, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

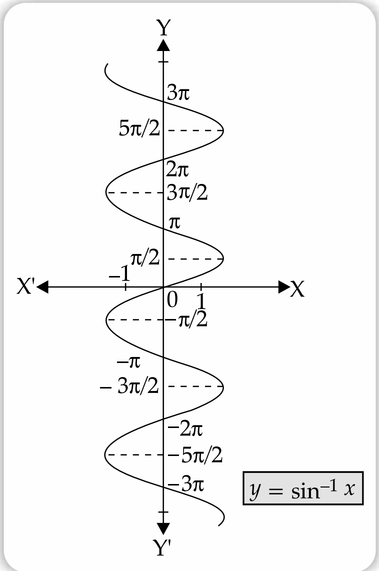
Case based Subjective Questions (4 marks each)

Read the following text and answer the following questions, on the basis of the same.

(Each Sub Part Caries 2 Marks)

Today in the class of Mathematics, Mrs. Agrawal in explaining the inverse trigonometric function. She draws the graph of the $\sin^{-1}x$ and and write down the following about the principal value of branch function $\sin^{-1}$:

Principal value of branch function $\sin^{-1}$: It is a function with domain [– 1, 1] and range $\left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right], \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ or $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ and so on corresponding to each interval, we get a branch of the function $\sin^{-1}x$. The branch with range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is called the principal value branch. Thus, $\sin^{-1} : [-1, 1] \rightarrow \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.



Q. 1. Find the value $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$. **[2]**

Sol. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

$= \sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ [$\because \sin^{-1}(-\theta) = -\sin\theta$]

$= \sin\left[\frac{\pi}{3} + \sin^{-1}\left(\sin\frac{\pi}{6}\right)\right]$ **1**

$= \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right]$

$= \sin\left[\frac{2\pi+\pi}{6}\right]$

$= \sin\left(\frac{3\pi}{6}\right)$

$= \sin\left(\frac{\pi}{2}\right) = 1$ **1**

Q. 2. Find the domain of $\sin^{-1}\sqrt{x-1}$ and $\sin^{-1}[x]$.

Sol. Let $f(x) = \sin^{-1}\sqrt{x-1}$

$\Rightarrow 0 \le x-1 \le 1$

$\Rightarrow \quad 1 \le x \le 2$

$\therefore \quad x \in [1, 2]$

$[\because \sqrt{x-1} \ge 0 \text{ and } -1 \le \sqrt{x-1} \le 1]$ **1**

We know that,

Domain of $\sin^{-x}$ is [–1, 1]

$\therefore$ Domain of $\sin^{-1}[x]$ is $\{x : -1 \le [x] \le 1\}$

But $\quad [x] = \begin{cases} -1 \ \forall \ -1 \le x < 0 \\ 0 \ \forall \ \ 0 \le x < 1 \\ 1 \ \forall \ \ 1 \le x < 2 \end{cases}$

$\therefore$ Domain of $\sin^{-1}[x]$ is [–1, 2) **1**

Solutions for Practice Questions

Short Answer Type Questions-I

3. $$\text{LHS} = \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$

$$= \frac{9}{4}\left[\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right] = \frac{9}{4}\cos^{-1}\frac{1}{3} \quad \mathbf{1}$$

$$= \frac{9}{4}\sin^{-1}\left(\sqrt{1-\left(\frac{1}{3}\right)^2}\right)$$

$$= \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right) = \text{RHS} \quad \mathbf{1}$$

[CBSE Marking Scheme 2020]

Detailed Answer:

To prove:

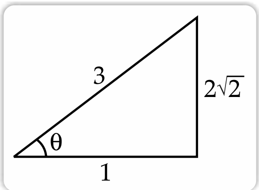
$$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$\text{L.H.S.} = \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$

$$= \frac{9}{4}\left[\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right]$$

[Using, $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$]

$$= \frac{9}{4}\cos^{-1}\left(\frac{1}{3}\right) \quad ...(i)$$



Let $\quad \cos^{-1}\frac{1}{3} = \theta$

$$\cos\theta = \frac{1}{3}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(\frac{1}{3}\right)^2 = 1$$

$$\sin^2\theta = 1 - \frac{1}{9}$$

$$\sin^2\theta = \frac{8}{9}$$

$$\sin\theta = \frac{2\sqrt{2}}{3}$$

$$\theta = \sin^{-1}\frac{2\sqrt{2}}{3}$$

$$\cos^{-1}\left(\frac{1}{3}\right) = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

Now, from equation (i) $= \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

= R.H.S. **Hence proved.**

Commonly Made Error

- Apply the properties to minimise the working.

Answering Tip

- Some students convert sin to tan and makes it complicated.

7. Put $x = \cos\theta$ in R.H.S. ½

as $\frac{1}{2} \le x \le 1$, RHS $= \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$= \cos^{-1}(\cos 3\theta) = 3\theta$ ½+½

$= 3\cos^{-1}x$ = LHS ½

[CBSE Marking Scheme 2018]

Short Answer Type Questions-II

1. Given equation can be written as

$$\sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{5}{x}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{12}{x}\right) = \cos^{-1}\left(\frac{5}{x}\right)$$

$$\therefore \sin^{-1}\left(\frac{12}{x}\right) = \sin^{-1}\left(\frac{\sqrt{x^2-25}}{x}\right) \quad \mathbf{1}$$

$$\Rightarrow \frac{12}{x} = \frac{\sqrt{x^2-25}}{x}$$

$\Rightarrow \quad x^2 - 25 = 144$

$\Rightarrow \quad x = \pm 13,$ **1½**

Since, $x = -13$ does not satisfy the given equation,

$\therefore$ required solution is $x = 13$. ½

[CBSE Marking Scheme 2020 (Modified)]

5. Given

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$$

or $\tan^{-1}1 - \tan^{-1}x = \frac{1}{2}\tan^{-1}x$ **1½**

or $\frac{3}{2}\tan^{-1}x = \frac{\pi}{4}$

or $\tan^{-1}x = \frac{\pi}{6}$ ½

or $x = \tan\left(\frac{\pi}{6}\right)$ ½

$\therefore \quad x = \frac{1}{\sqrt{3}}$ ½

[CBSE Marking Scheme 2015]

Long Answer Type Questions

1. Put $x^2 = \cos 2\theta$ or $\theta = \frac{1}{2}\cos^{-1}x^2$ 1

$$\text{LHS} = \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}\right)$$ **1**

$$= \tan^{-1}\left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right) = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$ **1½**

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right) = \frac{\pi}{4}+\theta$$ **1**

$$= \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2, -1 < x < 1 = \text{RHS}$$ ½

[CBSE Marking Scheme 2017 (Modified)]

REFLECTIONS

- Inverse trigonometric ratios are used by carpenters to create a desired cut angle.
- Inverse trigonometric ratio one used to measure the angle of depth and angle of inclination.
- Architects used the inverse trigonometric ratios to calculate the angle of bridge and the supports.

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SELF ASSESSMENT PAPER - 01

Time: 1 hour **MM: 30**

UNIT-I

(A) OBJECTIVE TYPE QUESTIONS:

I. Multiple Choice Questions **[1×6 = 6]**

Q. 1. The function $f(x) = x^2 + 4x + 4$ is

(A) odd function **(B)** even function

(C) neither odd or even function **(D)** periodic function

Q. 2. What is the domain of $\sin^{-1}(x + 1)$?

(A) $[-1, 1]$ **(B)** $[-2, 0]$ **(C)** $[-2, 0)$ **(D)** $[-2, 2]$

Q. 3. If $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}$, then $f(x)$ is

(A) $x^2 + 1$ **(B)** $2 - x^2$ **(C)** $x^2 - 2$ **(D)** $4x^2 - 2$

Q. 4. The value of $\tan^{-1}\{\tan(-6)\}$ is

(A) 2π **(B)** $2\pi - 6$ **(C)** $2\pi + 6$ **(D)** None of these

Q. 5. The value of $\sin\left(\sec^{-1}\frac{17}{15}\right)$ = ...

(A) $\frac{8}{17}$ **(B)** $\frac{15}{17}$ **(C)** $\frac{17}{8}$ **(D)** $\frac{8}{15}$

Q. 6. Principal value of $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is

(A) $\frac{\pi}{4}$ **(B)** $-\frac{\pi}{4}$ **(C)** $-\frac{\pi}{6}$ **(D)** $\frac{\pi}{6}$

II. Case-Based MCQs **[1×4 = 4]**

Attempt any 4 sub-parts from each questions. Each question carries 1 mark.

Read the following text and answer the questions on the basis of the same.

Reena is preparing notes on the topic 'Relations' which she studied in the last Mathematics class. She write down the following in her note book.

A relation R on a set A is said to be an equivalence relation on A iff it is

- Reflexive *i.e.*, $(a, a) \in R\ \forall\ a \in A$.
- Symmetric *i.e.*, $(a, b) \in R \Rightarrow (b, a) \in R \in a, b\ \forall\ A$
- Transitive *i.e.*, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R\ \forall\ a, b, c \in A$

Q. 7. If the relation R = {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} defined on the set A = {1, 2, 3}, then R is

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** Equivalence

Q. 8. If the relation R = {(1, 2), (2, 1), (1, 3), (3, 1)} defined on the set A = {1, 2, 3}, then R is

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** Equivalence

Q. 9. If the relation R on a set of natural numbers defined as $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$, then R is

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** Equivalence

Q. 10. If a relation R on set A = {1, 2, 3, ..., 13, 14} defined as $R = \{(x, y) : 3x - y = 0\}$, then R is

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** None of these

Q. 11. If the relation on set A = {1, 2, 3} defined as R = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}, then R is

(A) Reflexive **(B)** Symmetric **(C)** Transitive **(D)** Equivalence

(B) SUBJECTIVE TYPE QUESTIONS:

III. Very Short Answer Type Questions **[3×1 = 3]**

Q. 12. If $f(x) = 2x - x^2$, then find the value of $f(x + 2) - f(x - 2)$ at $x = 0$.

Q. 13. Check function $f : R \to R$ defined as $f(x) = 5x^2 - 8$ is one-one or not.

Q. 14. Prove that $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x \ \forall \ x \in (-\infty, -1] \cup [1, \infty)$.

IV. Short Answer Type Questions–I **[2×3 = 6]**

Q. 15. Consider a function $f : [0, \pi/2] \to R$, given by $f(x) = \sin x$ and $g : [0, \pi/2] \to R$, given by $g(x) = \cos x$. Show that f and g and are one-one, but $f = g$ is not one-one.

Q. 16. Find the principal value of the following.

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ **(ii)** $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Q. 17. If function λ, is defined by $\lambda(p, q) = \begin{cases} (p-q)^2, & \text{if} \quad p \geq q \\ p+q, & \text{if} \quad p < q \end{cases}$ then find the value of expression $\frac{\lambda(-(-3+2),(-2+3))}{(-(-2+1))}$.

V. Short Answer Type Questions-II **[3×2 = 6]**

Q. 18. Show that R = {$(a, b) : a, b \in Z$} and $a + b$ is even} is a equivalence relation.

Q. 19. If $y = \cot^{-1}\left(\sqrt{\cos x}\right) - \tan^{-1}\left(\sqrt{\cos x}\right)$, prove that $\sin y = \tan^2 \frac{x}{2}$.

VI. Long Answer Type Questions **[5×1 = 5]**

Q. 20. (a) Evaluate : $\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$ where $0 \leq \cos^{-1} x < \pi$ and $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$

(b) Prove that : $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}$

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