CHAPTER 3 MATRICES

Syllabus

Concept, notation, order, equality, types of matrices: zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices : Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here, all matrices will have real entries).

In this chapter you will study

- *Different types of matrices*
- *Different operations on matrices*
- *Invertible matrices*

List of Topics

Topic-1 Matrices and Operations

Concepts Covered • *Basic concept of matrices,* • *Types of matrices,* • *Operations on matrices*

Revision Notes

1. MATRIX - BASIC INTRODUCTION:

A matrix is an ordered rectangular **array** of numbers (real or complex) or functions which are known as elements or the entries of the matrix. It is denoted by the uppercase letters *i.e.* A, B, C etc.

Key Words

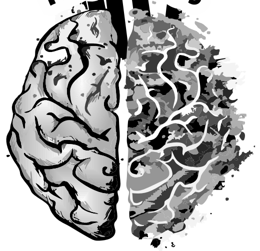
Array: An array is a rectangular arrangement of objects in equal rows (horizontal) and equal columns (vertical). Everyday example of arrays include a muffin tray and an egg carton.

Consider a matrix A given as,

Here in matrix A the horizontal lines of elements are said to constitute **rows** and vertical lines of elements are said to constitute **columns** of the matrix. Thus, matrix A has m **rows** and n **columns**. The array is enclosed by square brackets [ ], the parentheses ( ) or the double vertical bars || ||.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix}_{m\times n}$$

Matrices



Definition and its types

A matrix of order $m \times n$ is an ordered rectangular array of numbers or functions having 'm' rows and 'n' columns. The matrix $A = [a_{ij}]_{m \times n}$, $1 \le i \le m$, $1 \le j \le n$; $i, j \in N$ is given by

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots a_{1n} \\ a_{21} & a_{22} & \dots a_{2n} \\ a_{m_1} & a_{m_2} & \dots a_{mn} \end{bmatrix}_{m \times n}$$

- **Column matrix :** It is of the form $[a_{ij}]_{m \times 1}$
- **Row matrix :** It is of the form $[a_{ij}]_{1 \times n}$
- **Square matrix :** Here, $m = n$ (no. of rows = no. of columns)
- **Diagonal matrix :** All non-diagonal entries are zero *i.e* $a_{ij} = 0 \forall i \ne j$
- **Scalar matrix :** $a_{ij} = 0, i \ne j$ and $a_{ij} = k$ (Scalar), $i = j$, for some constant k.
- **Identity matrix :** $a_{ij} = 0, i \ne j$ and $a_{ij} = 1, i = j$
- **Zero matrix :** All elements are zero.

Equality of two matrices

$A = [a_{ij}] = [b_{ij}] = B$ if, A and B are of same order and $a_{ij} = b_{ij} \forall$ i and j; $i, j \in N$

Transpose of a Matrix

If $A = [a_{ij}]_{m \times n}$, then its transpose $A' = (A^T) = [a_{ji}]_{n \times m}$ *i.e.* if $A = \begin{pmatrix} 2 & 1 \end{pmatrix}$ then $A^T = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

Also, $(A')' = A$, $(kA)' = kA'$, $(A+B)' = A' + B'$, $(AB)' = B'A'$.

- A is symmetric matrix if $A = A'$ *i.e.* $A' = A$.
- A is skew – symmetric if $A = -A'$ *i.e.* $A' = -A$.
- A is any square matrix, then–

$$A = \frac{1}{2}\{(A + A') + (A - A')\} = \text{sum of a symmetric and a skew-symmetric matrix.}$$

($A + A'$: S.M.; $A - A'$: Skew S.M.)

For example if $A = \begin{pmatrix} 2 & 8 \\ 6 & 4 \end{pmatrix}$, then $A = \frac{1}{2}\left\{\begin{pmatrix} 4 & 14 \\ 14 & 8 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}\right\}$.

Properties for applying the operations

$R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$

$R_i \to kR_i$ or $C_i \to kC_i$

$R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$

Operations on matrices

Multiplication

If $A = [a_{ij}]_{m \times n}$ and $B = [b_{jk}]_{n \times p}$, then $AB = C = [C_{ik}]_{m \times p}$, $[C_{jk}] = \sum_{j=1}^{n} a_{ij} b_{jk}$.

Also, $A(BC) = (AB)C$, $A(B + C) = AB + AC$ and $(A + B)C = AC + BC$, but $AB \ne BA$ (always).

Addition

If A, B are two matrices of same order, then $A + B = [a_{ij} + b_{ij}]$. The addition of A and B follows: $A + B = B + A$, $(A + B) + C = A + (B + C)$, $-A = (-1)A$, $k(A + B) = kA + kB$, k is scalar and $(k + 1)A = kA + IA$, k and I are constants.

- If $A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$ then $A + B = \begin{bmatrix} -1 & 5 \\ -2 & 9 \end{bmatrix}$
- If $A = [2\ 3]_{1 \times 2}$, $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}_{2 \times 1}$, then $AB = [2 \times 4 + 3 \times 5] = [23]_{1 \times 1}$

Trace the Mind Map

- First Level
- Second Level
- Third Level

- A matrix having m rows and n columns is called a matrix of order $m \times n$ (read as 'm by n' matrix). A matrix A of order $m \times n$ is depicted as $A = [a_{ij}]_{m \times n}$; $i, j \in N$.
- Also in general, a_{ij} means an element lying in the i^{th} row and j^{th} column.
- Number of elements in the matrix $A = [a_{ij}]_{m \times n}$ is given as mn.

2. TYPES OF MATRICES:

(i) Column matrix: A matrix having only one column is called a **column matrix** or **column vector**.

$$e.g: \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}_{3\times1}, \begin{bmatrix} 4 \\ 5 \end{bmatrix}_{2\times1}$$

General notation : $A = [a_{ij}]_{m\times1}$

Key Facts

- The term matrix was introduced by the 19th century English Mathematician James Sylvester, but it was his friend the Mathematics Arthur Cayley who developed the algebraic aspect of matrices in two papers in the 1850s.
- The English Mathematician Cuthbert Edmund Cullis was the first to use modern bracket notation for matrices in 1913.

(ii) Row matrix: A matrix having only one row is called a **row matrix** or **row vector**.

$$e.g: \begin{bmatrix} 2 & 5 & -4 \end{bmatrix}_{1\times3}, \begin{bmatrix} \sqrt{2} & 4 \end{bmatrix}_{1\times2}$$

General notation : $A = [a_{ij}]_{1\times n}$

(iii) Square matrix: It is a matrix in which the number of rows is equal to the number of columns *i.e.*, an $n\times n$ matrix is said to constitute a square matrix of order $n\times n$ and is known as a **square matrix of order 'n'.**

$$e.g: \begin{bmatrix} 1 & 2 & 5 \\ 3 & 7 & -4 \\ 0 & -1 & -2 \end{bmatrix}_{3\times3}$$ is a square matrix of order.

General notation : $A = [a_{ij}]_{n\times n}$

(iv) Diagonal matrix: A square matrix $A = [a_{ij}]_{m\times m}$ is said to be a **diagonal matrix** if all the elements, except those in the leading diagonal are zero *i.e.*, $a_{ij} = 0$, for all $i \neq j$.

$$e.g: \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times3}$$ is a diagonal matrix of order 3.

- Also there are **more notations** specifically used for the diagonal matrices. For instance, consider the matrix given above, it can also be written as diag (2, 5, 4) or diag [2, 5, 4].
- Note that the elements $a_{11}, a_{22}, a_{33}, \ldots, a_{mm}$ of a square matrix $A = [a_{ij}]_{m\times m}$ of **order** m are said to constitute the **principal diagonal** or simply **the diagonal of the square matrix** A. These elements are known as **diagonal elements of matrix** A.

(v) Scalar matrix: A diagonal matrix $A = [a_{ij}]_{m\times m}$ is said to be a **scalar matrix** if its diagonal elements are equal. *i.e.*, $a_{ij} = \begin{cases} 0, & \text{when } i \neq j \\ k, & \text{when } i = j \text{ for some constant } k \end{cases}$

$$e.g: \begin{bmatrix} 17 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 17 \end{bmatrix}_{3\times3}$$ is a scalar matrix of order **3.**

(vi) Unit or Identity matrix: A square matrix $A = [a_{ij}]_{m\times m}$ is said to be an **identity matrix** if $a_{ij} = \begin{cases} 1, \text{ if } i = j \\ 0, \text{ if } i \neq j \end{cases}$

A **unit matrix** can also be defined as the **scalar matrix** in which all diagonal elements are equal to **unity**. We denote the identity matrix of order m by I_m or I.

$$e.g: I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}.$$

(vii) Zero matrix or Null matrix: A matrix is said to be a **zero matrix** or **null matrix** if each of its elements is '0'.

$$e.g., : \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times3}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times2}, [0 \;\; 0]_{1\times2}.$$

(viii) Horizontal matrix: A $m\times n$ matrix is said to be a **horizontal matrix** if $m < n$.

$$e.g: \begin{bmatrix} 1 & 2 & 5 \\ 4 & 8 & -9 \end{bmatrix}_{2\times3}$$

(ix) Vertical matrix: A $m\times n$ matrix is said to be a **vertical matrix** if $m > n$.

$$e.g: \begin{bmatrix} -5 & -1 \\ 8 & -9 \\ 4 & 0 \end{bmatrix}_{3\times2}$$

3. EQUALITY OF MATRICES:

Two matrices A and B are said to be equal and written as $A = B$, if they are of the **same order** and their **corresponding elements are identical** *i.e.* $a_{ij} = b_{ij}$ *i.e.,* $a_{11} = b_{11}, a_{22} = b_{22}, a_{32} = b_{32}$ etc.

4. ADDITION OF MATRICES:

If A and B are two $m \times n$ matrices, then another $m \times n$ matrix obtained by adding the corresponding elements of the matrices A and B is called the sum of the matrices A and B and is denoted by '$A + B$'.

Thus if $A = [a_{ij}]$, $B = [b_{ij}]$, or $A + B = [a_{ij} + b_{ij}]$.

Properties of matrix addition:

- Commutative property:

$$A + B = B + A$$

- Associative property:

$$A + (B + C) = (A + B) + C$$

- Cancellation laws:

  (i) Left cancellation: $A + B = A + C \Rightarrow B = C$

  (ii) Right cancellation: $B + A = C + A \Rightarrow B = C$.

5. MULTIPLICATION OF A MATRIX BY A SCALAR:

If a $m \times n$ matrix A is multiplied by a scalar k (say), then the new kA matrix is obtained by multiplying each element of matrix A by scalar k. Thus, if $A = [a_{ij}]$ and it is multiplied by a scalar k, then $kA = [ka_{ij}]$, *i.e.* $A = [a_{ij}]$ or $kA = [ka_{ij}]$. *e.g* :

$$A = \begin{bmatrix} 2 & -4 \\ 5 & 6 \end{bmatrix} \text{ or } 3A = \begin{bmatrix} 6 & -12 \\ 15 & 18 \end{bmatrix}$$

6. MULTIPLICATION OF TWO MATRICES:

Let $A = [a_{ij}]$ be a $m \times n$ matrix and $B = [b_{jk}]$ be a $n \times p$ matrix such that the number of columns in A is equal to the number of rows in B, then the $m \times p$ matrix $C = [c_{ik}]$ such that $[c_{ik}] = \Sigma_{j=1}^{n} a_{ij} b_{jk}$ is said to be the product of the matrices A and B in that order and it is denoted by AB *i.e.* "$C = AB$".

Properties of matrix multiplication:

- Note that the product AB is defined only when the number of columns in matrix A is equal to the number of rows in matrix B.
- If A and B are $m \times n$ and $n \times p$ matrices, respectively, then the matrix AB will be an $m \times p$ matrix *i.e.*, order of matrix AB will be $m \times p$.
- In the product AB, A is called the **pre-factor** and B is called the **post-factor**.
- If two matrices A and B are such that AB is possible then it is not necessary that the product BA is also possible.
- If A is a $m \times n$ matrix and both AB as well as BA are defined, then B will be a $n \times m$ matrix.
- If A is a $n \times n$ matrix and I_n be the unit matrix of order n, then $A I_n = I_n A = A$.
- Matrix multiplication is **associative** *i.e.*, $A(BC) = (AB)C$.
- Matrix multiplication is **distributive** over the **addition** *i.e.*, $A.(B + C) = AB + AC$.
- Matrix multiplication is not commutative.

7. IDEMPOTENT MATRIX:

A square matrix A is said to be an idempotent matrix if $A^2 = A$.

For example,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

are idempotent matrices.

8. TRANSPOSE OF A MATRIX:

If $A = [a_{ij}]_{m \times n}$ be a $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of matrix A is said to be a **transpose of matrix** A. The transpose of A is denoted by A' or A^T *i.e.*, if $A^T = [a_{ji}]_{n \times m}$.

For example,

$$\begin{bmatrix} 5 & -4 & 1 \\ 0 & \sqrt{5} & 3 \end{bmatrix}^T = \begin{bmatrix} 5 & 0 \\ -4 & \sqrt{5} \\ 1 & 3 \end{bmatrix}$$

PROPERTIES OF TRANSPOSE OF MATRICES:

(i) $(A + B)^T = A^T + B^T$

(ii) $(A^T)^T = A$

(iii) $(kA)^T = kA^T$, where k is any constant

(iv) $(AB)^T = B^T A^T$

(v) $(ABC)^T = C^T B^T A^T$

Mnemonics

Types of Matrices

Ram → Row Matrix, **C**haran → Column Matrix, **S**ays → Square Matrix, **D**rink → Diagonal Matrix, **S**prite → Scalar Matrix

and **N**escafe → Null Matrix, **I**ce → Identity Matrix, **T**ea → Triangular Matrix

Matrix Multiplication

No. of columns of first matrix = No. of rows of second matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} u & v \\ w & x \\ y & z \end{bmatrix}$$

R × **C**   **R** × C

Class **R**epresentative

OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. The number of all possible matrices of order 2 × 3 with each entry 1 or 2 is

(A) 16 **(B)** 6

(C) 64 **(D)** 24

[CBSE Board 2021]

Ans. Option (C) is correct.

Explanation: The order of the matrix = 2 × 3

The number of elements = 2 × 3 = 6

Each place can have either 1 or 2. So, each place can be filled in 2 ways.

Thus, the number of possible matrices = $2^6 = 64$

Q. 2. $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ **are equal, then value of** $ab - cd$ **is:**

(A) 4 **(B)** 16

(C) –4 **(D)** –16

[CBSE Board 2021]

Ans. Option (A) is correct.

Explanation: Given, $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$

$$\therefore \quad 3c + 6 = 12 \quad ...(i)$$
$$a - d = 2 \quad ...(ii)$$
$$a + d = -8 \quad ...(iii)$$
$$2 - 3b = -4 \quad ...(iv)$$

From eq. (i), we get $c = 2$

On solving eqs. (ii) and (iii), we get $a = -3$ and $d = -5$ from eq. (iv), we get $b = 2$

Now, $ab - cd = (-3)2 - 2(-5)$

$\Rightarrow ab - cd = -6 + 10 = 4$

Q. 3. For the matrix X = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$**, ($X^2$ – X) is:**

(A) 21 **(B)** 31

(C) 1 **(D)** 51

[CBSE Board 2021]

Ans. Option (A) is correct.

Explanation:

Here $$X^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow X^2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow X^2 - X = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow X^2 - X = 2\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 2I$$

Q. 4. For two matrices P = $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ **and** $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$**, P – Q is:**

(A) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$ **(B)** $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & 3 \\ -0 & -3 \\ -1 & -2 \end{bmatrix}$ **(D)** $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

[CBSE Board 2021]

Ans. Option (B) is correct.

Explanation:

Here, $$Q = (Q^T)^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

Now, $$P - Q = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Q. 5. A matrix A = $[a_{ij}]_{3 \times 3}$ **is defined by**

$$a_{ij} = \begin{cases} 2i + 3j & , \ i < j \\ 5 & , \ i = j \\ 3i - 2j & , \ i > j \end{cases}$$

The number of elements in A which are more than 5, is:

(A) 3 **(B)** 4

(C) 5 **(D)** 6 **[CBSE Board 2021]**

Ans. Option (B) is correct.

Explanation: Here, $$A = \begin{bmatrix} 5 & 8 & 11 \\ 4 & 5 & 13 \\ 7 & 5 & 5 \end{bmatrix}$$

Thus, number of elements more than 5, is 4.

Q. 6. If A is a square matrix such that A^2 = A, then $(I - A)^3$ + A is equal to:

(A) I **(B)** 0

(C) I – A **(D)** I + A

[CBSE Board 2021]

Ans. Option (A) is correct.

Explanation:

Topper Answer, 2020

Sol.

$(I-A)^3 + A$

$= I^3 - A^3 + 3A^2 I - 3A + A$

$= I - A^2 A + 3A^2 - 3A + A$

$= I - A^2 + 3A - 3A + A$

$= I - A + 3A - 3A + A$

$= I$

Q. 7. If $[x \ \ 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$, then x equals:

(A) 0 **(B)** –2

(C) –1 **(D)** 2

[CBSE Delhi Set - II 2020]

Ans. Option (D) is correct.

Explanation:

$$[x \ \ 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = [0 \ \ 0]$$

$$\Rightarrow \quad [x-2 \ \ 0] = [0 \ \ 0]$$

$$\Rightarrow \quad x - 2 = 0 \text{ [By def. of equality]}$$

$$\Rightarrow \quad x = 2$$

Q. 8. If $A = [2 \ \ -3 \ \ 4]$, $B = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$, $X = [1 \ \ 2 \ \ 3]$ and $Y = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, then $AB + XY$ equals:

(A) [28] **(B)** [24]

(C) 28 **(D)** 24

[CBSE OD Set - I 2020]

Ans. Option (A) is correct.

Explanation:

Given, $A = [2 \ \ -3 \ \ 4]$,

$$B = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix},$$

$$X = [1 \ \ 2 \ \ 3],$$

$$Y = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$AB + XY = [2 \ \ -3 \ \ 4]\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} + [1 \ \ 2 \ \ 3]\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= [6 - 6 + 8] + [2 + 6 + 12]$$

$$= [8] + [20] = [28]$$

Q. 9. Suppose P and Q are two different matrices of order $3 \times n$ and $n \times p$, then the order of the matrix $P \times Q$ is?

(A) $3 \times p$ **(B)** $p \times 3$

(C) $n \times n$ **(D)** 3×3

[CBSE SQP 2019-20]

Ans. Option (A) is correct.

Q. 10. $A = [a_{ij}]_{m \times n}$ is a square matrix, if

(A) $m < n$ **(B)** $m > n$

(C) $m = n$ **(D)** None of these

Ans. Option (C) is correct.

Explanation: It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore,

$A = [a_{ij}]_{m \times n}$ is a square matrix, if $m = n$.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. If A and B are matrices of order $3 \times n$ and $m \times 5$ respectively, then find the order of matrix 5A – 3B, given that it is defined. R **[CBSE SQP 2020-21]**

Sol. 3×5 1

[CBSE Marking Scheme 2020]

Detailed Solution:

$$5[A]_{3 \times n} - 3[B]_{m \times 5} = [5A - 3B]_{3 \times 5}$$

Q. 2. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then find A^3.

R **[CBSE OD SET-II 2020]**

Sol. $A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $A^3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ ½+½

[CBSE Marking Scheme 2020]

Detailed Solution:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 1+1 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

Commonly Made Error

- Some students cube the elements in A and obtain the answer.

Answering Tip

- Matrix multiplication is different from normal multiplication.

Q. 3. **Find the value of A^2, where A is a 2 × 2 matrix whose elements are given by**

$$a_{ij} = \begin{cases} 1 & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$

R&U [CBSE SQP 2020-21]

Q. 4. **If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A.** R&U [CBSE Delhi Set III-2019]

Sol. $A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ 1

[CBSE Marking Scheme 2019]

Detailed Solution:

Given $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

or $3A - \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$

or $3A = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ ½

or $3A = \begin{bmatrix} 9 & 3 \\ 3 & 6 \end{bmatrix}$

or $A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ ½

Q. 5. **If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, write the value of $(x + y)$.** R&U [CBSE Delhi Set II-2019]

Sol. $x + y = 6$ 1

[CBSE Marking Scheme 2019]

Detailed Solution:

Topper Answer, 2019

Sol.

Q3 $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

comparing corresponding element of each matrix,

$2+y=5$ $\quad$ $2x+2=8$

$y=3$ $\quad$ $x=3$

$x-y=3-3$

$x-y=0$

Q. 6. If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{pmatrix}$ **is a matrix satisfying AA′ = 9I, find** x.

A I R&U [S.Q.P. 2018-19]

[CBSE Delhi Comptt. Set I, II, III, 2018]

Q. 7. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

R&U **[Delhi Set I, II, III, 2016][NCERT Exemplar]**

Sol. Given $A^2 = I$

$\therefore (A - I)^3 + (A + I)^3 - 7A$

$= A^3 - I^3 - 3A^2I + 3AI^2 + A^3 + I^3 + 3A^2I + 3AI^2 - 7A$ ½

$= AI - I - 3I + 3A + AI + I + 3I + 3A - 7A$

$= A + 3A + A + 3A - 7A = A$ ½

Q. 8. Write the number of all possible matrices of order 2 × 2 with each entry 1, 2 or 3.

R&U [O.D. Set I, II, III 2016]

Sol. Number of elements of 2 × 2 matrix = 4 ½

$\therefore$ Number of ways to write 1, 2 or 3 at 4 places

$= 3^4$

$= 81$

$\therefore$ 81 matrices of order 2 × 2 are possible with each entry 1, 2 or 3. ½

Q. 9. If $\begin{bmatrix} 1 & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ -1 \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, **find** $x + y + z$.

R&U [Delhi Set I, II, III Comptt. 2016]

Sol. $x = 1, y = -2, z = 1$

or $x + y + z = 0$

[CBSE Marking Scheme 2016] 1

Commonly Made Error

- Mostly candidates commit errors while calculation as they confuse in multiplication of two matrices.

Answering Tip

- Give ample practice on problems based on multiplication of two matrices.

Q. 10. If $(2 \quad 1 \quad 3)\begin{pmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = A$, **then write the order of matrix A.**

U [Foreign 2016]

[NCERT Exemplar]

Sol. We have,

$$A = (2 \quad 1 \quad 3)\begin{pmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$= (-2-1 \quad 1+3 \quad -2+3)\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = (-3 \quad 4 \quad 1)\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$= (-3-1) = (-4)_{1\times 1}$$

$\therefore$ Order of matrix A is 1 × 1. 1

Q. 11. Write the element a_{23} of a 3 × 3 matrix $A = (a_{ij})$ whose elements a_{ij} are given by

$$a_{ij} = \frac{|i - j|}{2}$$

A [CBSE March 2015]

Sol. $a_{23} = \dfrac{|2-3|}{2}$ ½

$= \dfrac{1}{2}$ ½

[CBSE Marking Scheme 2015]

Q. 12. If $A = \begin{pmatrix} 0 & 3 \\ 2 & -5 \end{pmatrix}$ **and** $kA = \begin{pmatrix} 0 & 4a \\ -8 & 5b \end{pmatrix}$ **find the values of k and a.**

R&U [Outside Delhi Set II 2015]

Short Answer Type Questions-I (2 marks each)

Q. 1. Find a matrix A such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ **and** $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

R [CBSE All Set-III, 2019]

Sol.

$$2A - \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad 1$$

$$\Rightarrow A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix} \quad 1$$

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

Also, given $2A - 3B + 5C = O$

$\Rightarrow \quad 2A = 3B - 5C$ ½

$\Rightarrow 2A = 3\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$ ½

$\Rightarrow 2A = \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} -10 & 0 & 10 \\ -35 & -5 & -30 \end{bmatrix}$

$$\Rightarrow \quad 2A = \begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix} \quad \frac{1}{2}$$

$$\Rightarrow \quad A = \frac{1}{2}\begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$$

$$\Rightarrow \quad A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix} \quad \frac{1}{2}$$

Q. 2. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $(A^2 - 5A)$.

R [CBSE Delhi Set-II, 2019]

Q. 3. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, show that $(A - 2I)(A - 3I) = 0$.

R [CBSE OD Set-I, 2019]

Sol. $$(A-2I)(A-3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \quad 1$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \quad 1$$

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, $$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

$$\therefore \quad A - 2I = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-2 & 2+0 \\ -1+0 & 1-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \quad \frac{1}{2}$$

and $$A - 3I = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 2+0 \\ -1+0 & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \quad \frac{1}{2}$$

Now, $$\text{LHS} = (A - 2I)(A - 3I)$$

$$= \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \quad \frac{1}{2}$$

$$= \begin{bmatrix} 2-2 & 4-4 \\ -1+1 & -2+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \frac{1}{2}$$

$$= 0 = \text{RHS} \quad \frac{1}{2}$$

Hence Proved.

Q. 4. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}. [CBSE SQP 2020-21]

Sol. $$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}, 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow \quad A^2 - 5A + 7I = \begin{bmatrix} O & O \\ O & O \end{bmatrix} = O \quad 1$$

$$\Rightarrow \quad A^{-1}(A^2 - 5A + 7I) = A^{-1}O$$

$$\Rightarrow \quad A - 5I + 7A^{-1} = O$$

$$\Rightarrow \quad 7A^{-1} = 5I - A$$

$$\Rightarrow \quad A^{-1} = \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$$

$$\Rightarrow \quad A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \quad 1$$

[CBSE SQP Marking Scheme 2020-21]

Commonly Made Error

- After proving the equation, some students find the inverse using formula.

Answering Tip

- Learn to find inverse from a matrix equation.

Short Answer Type Questions-II (3 marks each)

Q. 1. Find matrix X so that $X\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$

A I R&U [Delhi Set I, II, III 2017] [NCERT]

Sol. Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 1 then, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$ **1**

$a + 4b = -7$ $\quad 3a + 6b = -9$ $\quad c + 4d = 2$

$2a + 5b = -8$ $\quad 3c + 6d = 6$ $\quad 2c + 5d = 4$

equating and solving to get $a = 1, b = -2, c = 2, d = 0$ **1½**

$X = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$ ½

Commonly Made Error

- Some students take $X = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ but this is wrong find the order of X using the order of A and B.

$$XA = B$$
$$(X_{p\times q})\,(A)_{2\times3} = B_{2\times3}$$
$$p = 2, q = 2$$

Answering Tip

- Learn the properties of matrix multiplication.

Q. 2. A trust fund has ₹ 35,000 is to be invested in two different types of bonds. The first bond pays 8% interest per annum which will be given to orphanage and second bond pays 10% interest per annum which will be given to an N.G.O. (Cancer Aid Society). Use matrix multiplication, determine how to divide ₹ 35,000 among two types of bonds if the trust fund obtains an annual total interest of ₹ 3,200. R&U [O.D. Set I, II, III Comptt. 2015]

Sol. Let investment in first type of bond be ₹ x.

∴ The investment in second type of bond $= ₹\,(35{,}000 - x)$ **1**

$$\therefore \quad \begin{bmatrix} x & 35{,}000 - x \end{bmatrix}\begin{bmatrix} \frac{8}{100} \\ \frac{10}{100} \end{bmatrix} = [3{,}200]$$ **1½**

or $\frac{8}{100}x + (35{,}000 - x)\frac{10}{100} = 3{,}200$

or $x = ₹\,15{,}000$ **1**

∴ Investment in first bond = ₹ 15,000

and Investment in second bond

= ₹(35,000 – 15,000)

= ₹ 20,000 ½

[CBSE Marking Scheme 2015]

Commonly Made Error

- Students get wrong with the order while forming matrices.

Answering Tip

- Take the first matrix as a row matrix and the second one as a column matrix.

Q. 3. To promote the making of toilets for women, as organization tried to generate awareness through (i) house calls (ii) letters and (iii) announcements. The cost for each mode per attempt is given below :

(i) ₹ 50 (ii) ₹ 20 (iii) ₹ 40

The number of attempts made in three villages X, Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organization for the three villages separately, using matrices.

R&U [All India, 2015]

Long Answer Type Questions (5 marks each)

Q. 1. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$.

R&U [Delhi, 2015]

Sol. Getting $A^2 = A\cdot A$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$ **1½**

$$\therefore A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **1**

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$ **1½**

$\therefore$ $X = -(A^2 - 5A + 4I)$

or $X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$ **1**

[CBSE Marking Scheme 2015]

Commonly Made Error

- Most of the students attempt this question incorrectly. They make errors while calculating A^2. Sometimes they add two matrices to solve A^2.

Answering Tip

- Stress upon developing logical and reasoning skills to apply the correct property of matrix.

Q. 2. Find matrix *A*, if $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{pmatrix}$

R&U **[NCERT Exemplar] [Delhi Comptt. 2017]**

Q. 3. Find matrix *X* if: $X \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \\ 11 & 10 & 9 \end{pmatrix}$

R&U **[Delhi Comptt. set II 2017]**

Sol. Clearly order of X is 3×2

Let $X = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$ **1**

So $\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \\ 11 & 10 & 9 \end{pmatrix}$ **1**

$$\left.\begin{matrix} a + 4b = -7 & c + 4d = 2 & e + 4f = 11 \\ 2a + 5b = -8 & 2c + 5d = 4 & 2e + 5f = 10 \end{matrix}\right\}$$ **1**

$3a + 6b = -9, 3c + 6d = 6, 3e + 6f = 9$

Solving we get

$a = 1, \quad b = -2, \quad c = 2 \quad d = 0, \quad e = -5 \quad f = 4$ **2**

Thus $X = \begin{pmatrix} 1 & -2 \\ 2 & 0 \\ -5 & 4 \end{pmatrix}$ **1**

[CBSE Marking Scheme 2017]

Commonly Made Error

- Students fail to find the order of the matrix X correctly.

Answering Tip

- Learn the confirm ability of matrix multiplication with the order of the product matrix.

Topic-2

Symmetric, Skew Symmetric and Invertible Matrices

Concepts Covered • *Symmetric Matrix,* • *Skew Symmetric Matrix,* • *Invertible Matrix* • *Uniqueness Theorem*

Revision Notes

Symmetric matrix: A square matrix $A = [a_{ij}]$ is said to be a **symmetric matrix** if $A^T = A$. *i.e.,* if $A = [a_{ij}]$, then $A^T = [a_{ji}] = [a_{ij}]$ or $A^T = A$.

For example :

$$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}, \begin{bmatrix} 2+i & 1 & 3 \\ 1 & 2 & 3+2i \\ 3 & 3+2i & 4 \end{bmatrix}$$

Skew symmetric matrix: A square matrix $A = [a_{ij}]$ is said to be a **skew symmetric matrix** if $A^T = -[A]$ *i.e.,* if $A = [a_{ij}]$, then $A^T = [a_{ji}] = -[a_{ij}]$ or $A^T = -A$.

For example : $\begin{bmatrix} 0 & 1 & -5 \\ -1 & 0 & 5 \\ 5 & -5 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

Orthogonal matrix: A matrix A is said to be **orthogonal** if $A.A^T = I$, where A^T is transpose of A.

Invertible Matrix: An invertible matrix is a matrix for which matrix inversion operation exists, given that it satisfies the requisite conditions. Any given square matrix A of order $n \times n$ is called invertible if there exists another $n \times n$ square matrix B such that, AB = BA = I_n, where I_n is on identity matrix of order $n \times n$.

Example: Let matrix A = $\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ and matrix B = $\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$

Now, AB = $\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

and BA = $\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Hence, A^{-1} = B and B is called the inverse of A.

So, A can also be the inverse of B or B^{-1} = A.

Uniqueness of Inverse of Matrix

If there exists an inverse of a square matrix, it is always unique.

Proof: Let A be a square matrix of order $n \times n$. Let us assume matrices B and C be inverses of matrix A.

Now, AB = BA = I, since B is the inverse of matrix Δ.

Similarly, AC = CA = I

But, B = BI = B(AC) = (BA)C = IC = C

This proves B = C, or B and C are the same matrices.

Key Fact

- Note that $[a_{ji}] = -[a_{ij}]$ or $[a_{ii}] = -[a_{ii}]$ or $2[a_{ii}] = 0$ (Replacing j by i). *i.e.*, all the diagonal elements in a skew symmetric matrix are zero.
- For any matrices, AA^T and A^TA are symmetric matrices.
- For a square matrix A, the matrix $A + A^T$ is a symmetric matrix and $A - A^T$ is always a skew-symmetric matrix.
- Also note that any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix *i.e.*, $A = P + Q$ where $P = \frac{A + A^T}{2}$ is a symmetric matrix and $Q = \frac{A - A^T}{2}$ is a skew symmetric matrix.

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions

Q. 1. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, then the value of α is:

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{3}$

(C) π **(D)** $\frac{3\pi}{2}$

Ans. Option (B) is correct.

Explanation:

Given that, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Also $A + A' = I$

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating corresponding entries, we have

$$\Rightarrow 2\cos\alpha = 1$$

$$\Rightarrow \cos\alpha = \frac{1}{2}$$

$$\Rightarrow \cos\alpha = \cos\frac{\pi}{3}$$

$$\therefore \alpha = \frac{\pi}{3}$$

Q. 2. Matrices A and B will be inverse of each other only if:

(A) $AB = BA$ **(B)** $AB = BA = 0$

(C) $AB = 0, BA = I$ **(D)** $AB = BA = I$

Ans. Option (D) is correct.

Explanation: We know that if A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that $AB = BA = I$, then B is said to be the inverse of A.

In this case, it is clear that A is the inverse of B. Thus, matrices A and B will be the inverse of each other only if $AB = BA = I$.

Q. 3. If A and B are symmetric matrices of same order, then $AB - BA$ is a:

(A) Skew-symmetric matrix
(B) Symmetric matrix
(C) Zero matrix
(D) Identity matrix

Ans. Option (A) is correct.

Explanation: Given that,

A and B are symmetric matrices.

$\Rightarrow \quad A = A'$ and $B = B'$

Now, $(AB - BA)' = (AB)' - (BA)'$ ...(i)

$\Rightarrow (AB - BA)' = B'A' - A'B'$ [By reversal law]

$\Rightarrow (AB - BA)' = BA - AB$ [From Eq. (i)]

$\Rightarrow (AB - BA)' = -(AB - BA)$

$\Rightarrow (AB - BA)$ is a skew-symmetric matrix.

Q. 4. If the matrix A is both symmetric and skew-symmetric, then:

(A) A is a diagonal matrix
(B) A is a zero matrix
(C) A is a square matrix
(D) None of these **[CBSE Board 2021]**

Ans. Option (B) is correct.

Explanation: If A is both symmetric and skew-symmetric, then we have,

$A' = A$ and $A' = -A$

$\Rightarrow \quad A = -A$

$\Rightarrow \quad A + A = 0$

$\Rightarrow \quad 2A = 0$

$\Rightarrow \quad A = 0$

Therefore, A is a zero matrix.

Q. 5. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a:

(A) identity matrix
(B) symmetric matrix
(C) skew-symmetric matrix
(D) None of these

Ans. Option (B) is correct.

Explanation: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$\therefore A' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = A$

So, the given matrix is a symmetric matrix.

[Since, in a square matrix A, if $A' = A$, then A is called symmetric matrix.]

Q. 6. The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a:

(A) diagonal matrix
(B) symmetric matrix
(C) skew symmetric matrix
(D) scalar matrix

Ans. Option (C) is correct.

Explanation: We know that, in a square matrix, if $b_{ij} = 0$ when $i \neq j$ then it is said to be a diagonal matrix. Here, $b_{12}, b_{13}, \ldots \neq 0$ so the given matrix is not a diagonal matrix.

Now, $B = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$

$B' = \begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & 12 & 0 \end{bmatrix}$

$= -\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$

$= -B$

So, the given matrix is a skew-symmetric matrix, since we know that in a square matrix B, if $B' = -B$, then it is called skew-symmetric matrix.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. A square matrix A is said to be symmetric if .... **[CBSE Board 2020]**

Topper Answer, 2020

Sol. A square matrix A is said to be symmetric if $A^T = A$
i.e. $a_{ij} = a_{ji}$.

Q. 2. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric.

Find the values of 'a' and 'b'.

R&U [Delhi & O.D. 2018]

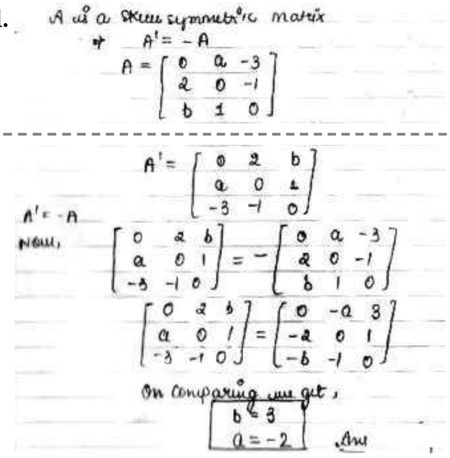
Sol. $a = -2, b = 3$ ½+½

[CBSE Marking Scheme 2018]

Detailed Solution:

Topper Answer, 2018

Sol.



A is a skew symmetric matrix

$\therefore A' = -A$

$A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$

$A' = \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}$

$A' = -A$

Now,

$\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$

On comparing we get,

$b = 3$

$a = -2$ Ans

Q. 3. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b.

R&U [Delhi Set I, II, III 2016]

Sol. As A is a symmetric matrix,

or $A' = A$ ½

$$\therefore \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

$\therefore$ By equality of matrices, $a = \frac{-2}{3}$ and $b = \frac{3}{2}$. ½

Commonly Made Error

- Few students commit error in solving the problems based on symmetric matrix.

Answering Tip

- Learn the difference between symmetric and skew symmetric matrices.

Q. 4. Write a 2 × 2 matrix which is both symmetric and skew symmetric. R&U [Delhi Set I Comptt. 2014]

Q. 5. Give an example of a skew symmetric matrix of order 3. R&U [CBSE S.Q.P, 2015-16]

Sol. $\begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$.

Short Answer Type Questions-I (2 marks each)

Q. 1. A is skew-symmetric matrix of order 3, then prove that det A = 0. [OD 2017]

Q. 2. Show that A'A' and AA' are both symmetric matrices for any matrix A.

Sol. Let, $P = A'A$

$\therefore P' = (A'A)'$

$= A'(A')'$ $[\because (AB)' = B'A']$

$= A'A = P$

So, A'A is symmetric matrix for any matrix A.

Similarly,

Let $Q = AA'$

$Q' = (AA')' = (A')'A'$

$= AA' = A$

So, AA' is symmetric matrix for any matrix A.

Q. 3. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ then find $3A + 4B$.

Sol. We have, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\text{Now, } 3A + 4B = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} + 4\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+8 & 0 & 0 \\ 0 & -3+12 & 0 \\ 0 & 0 & 6-4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 0 & 9 \\ 0 & 0 & 2 \end{bmatrix}$$

Q. 4. If $A = \begin{bmatrix} 4 & 4 \\ 2 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 \\ 3 & -5 \end{bmatrix}$, then find A – B – C.

Sol. Here, A, B and C are the three matrices of same order 2 × 2.

Now, $A - B - C = A + (-1)B + (-1)C$

$$= \begin{bmatrix} 4 & 4 \\ 2 & 7 \end{bmatrix} + (-1)\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + (-1)\begin{bmatrix} 1 & 4 \\ 3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 \\ 2 & 7 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} -1 & -4 \\ -3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4-1-1 & 4-2-4 \\ 2+1-3 & 7-3+5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ 0 & 9 \end{bmatrix}$$

Short Answer Type Questions-II (3 marks each)

Q. 1. Express the matrix $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

AI A [O.D. Set I, II, III Comptt. 2015]
[NCERT Exemplar]

Sol. $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$

Then $A' = \begin{bmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{bmatrix}$ **1**

Let $P = \frac{1}{2}(A + A')$

$$= \frac{1}{2}\begin{bmatrix} 4 & 11 & -5 \\ 11 & 6 & 3 \\ -5 & 3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & \frac{11}{2} & -\frac{5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 4 \end{bmatrix}$$ **1**

Since $P' = P$

$\therefore$ P is a symmetric matrix.

Let $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0 & -\frac{3}{2} & -\frac{7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & -\frac{7}{2} & 0 \end{bmatrix}$$

$$Q' = -\begin{bmatrix} 0 & -\frac{3}{2} & -\frac{7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & -\frac{7}{2} & 0 \end{bmatrix} = -Q$$ **1**

Since $Q' = -Q$

$\therefore$ Q is a skew symmetric matrix.

Also $P + Q = \begin{bmatrix} 2 & \frac{11}{2} & -\frac{5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 4 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{3}{2} & -\frac{7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & -\frac{7}{2} & 0 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix} = A$$ **1**

[CBSE Marking Scheme 2015]

Q. 2. Express the following matrix as a sum of a symmetric and skew-symmetric matrices and verify your result:

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$ A

Q. 3. Show that the matrix $B^T AB$ is symmetric or skew-symmetric accordingly when A is symmetric or skew-symmetric. AE [NCERT]

Sol. **Case I:** Let A be a symmetric matrix. Then $A^T = A$.

Now, $(B^T AB)^T = B^T A^T (B^T)^T$ [By reversal law]

$= B^T A^T B$ [$\because (B^T)^T = B$]

or $(B^T AB)^T = B^T AB$ [$\because A^T = A$]

$\therefore$ $B^T AB$ is a symmetric matrix. **2**

Case II: Let A be a skew-symmetric matrix. Then, $A^T = -A$.

Now, $(B^T AB)^T = B^T A^T (B^T)^T$ [By reversal law]

or $(B^T AB)^T = B^T A^T B$ [$\because (B^T)^T = B$]

or $(B^T AB)^T = B^T (-A) B$ [$\because A^T = -A$]

or $(B^T AB)^T = -B^T AB$ **2**

$\therefore$ $B^T AB$ is a skew-symmetric matrix.

Long Answer Type Questions (5 marks each)

Q. 1. If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = 0$, Hence find A^{-1}.

AI R&U [CBSE OD Set I, II, III-2020]
[OD Set I, II, III COMPTT. 2016]

Sol. $A^2 = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$ **1**

$$A^3 = \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} \quad \mathbf{1\tfrac{1}{2}}$$

$$\text{LHS} = A^3 - 4A^2 - 3A + 11I$$

$$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - 4\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \quad \mathbf{1\tfrac{1}{2}}$$

Now,

$$A^{-1} = -\frac{1}{11}(A^2 - 4A - 3I)$$

$$= -\frac{1}{11}\begin{bmatrix} 2 & -5 & -3 \\ -7 & 1 & 5 \\ 4 & 1 & -6 \end{bmatrix} \quad \mathbf{1}$$

[CBSE Marking Scheme 2018]

Detailed Solution:

$$A^2 = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}$$

$$\therefore A^3 - 4A^2 - 3A + 11I$$

$$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - 4\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$$

$$-3\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - \begin{bmatrix} 36 & 28 & 20 \\ 4 & 16 & 4 \\ 32 & 36 & 36 \end{bmatrix}$$

$$- \begin{bmatrix} 3 & 9 & 6 \\ 6 & 0 & -3 \\ 3 & 6 & 9 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 28-36-3+11 & 37-28-9+0 & 26-20-6+0 \\ 10-4-6+0 & 5-16-0+11 & 1-4+3+0 \\ 35-32-3+0 & 42-36-6+0 & 34-36-9+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$$A^3 - 4A^2 - 3A + 11I = 0$$

Hence Proved.

$$A^3 \cdot A^{-1} - 4A^2 \cdot A^{-1} - 3A \cdot A^{-1} + 11IA^{-1} = 0 \cdot A^{-1}$$

$$A^2 I - 4AI - 3I + 11A^{-1} = 0$$

$$A^2 - 4A - 3I + 11A^{-1} = 0$$

$$11A^{-1} = 3I + 4A - A^2$$

$$= 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 4\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$$

$$11A^{-1} = \begin{bmatrix} 3+4-9 & 0+12-7 & 0+8-5 \\ 0+8-1 & 3+0-4 & 0-4-1 \\ 0+4-8 & 0+8-9 & 3+12-9 \end{bmatrix}$$

$$11A^{-1} = \begin{bmatrix} -2 & 5 & 3 \\ 7 & -1 & -5 \\ -4 & -1 & 6 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{2}{11} & \frac{5}{11} & \frac{3}{11} \\ \frac{7}{11} & -\frac{1}{11} & -\frac{5}{11} \\ -\frac{4}{11} & -\frac{1}{11} & \frac{6}{11} \end{bmatrix} = -\frac{1}{11}\begin{bmatrix} 2 & -5 & -3 \\ -7 & 1 & 5 \\ 4 & 1 & -6 \end{bmatrix}$$

Commonly Made Error

- After proving the matrix equation, students use the formula to find the inverse which is wrong.

Answering Tip

- Learn to find inverse from elementary operations, matrix multiplication, from equations and using formula.

COMPETENCY BASED QUESTIONS

Case based MCQs (4 marks each)

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
A	10,000	2,000	18,000
B	6,000	20,000	8,000

If the unit Sale price of Pencil, Eraser and Sharpener are ₹2.50, ₹1.50 and ₹1.00 respectively, and unit cost of the above three commodities are ₹2.00, ₹1.00 and ₹ 0.50 respectively, then, **[CBSE QB 2021]**

Q. 1. Total revenue of market *A*:

(A) ₹ 64,000 **(B)** ₹ 60,400

(C) ₹ 46,000 **(D)** ₹ 40600

Ans. Option (C) is correct.

Explanation: Total revenue of

$$= [10,000 \quad 2,000 \quad 18,000]\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$$

$$= 2.50 \times 10,000 + 1.50 \times 2,000 + 1.00 \times 18,000$$

$$= ₹46,000$$

Q. 2. Total revenue of market *B* is:

(A) ₹ 35,000 **(B)** ₹ 53,000

(C) ₹ 50,300 **(D)** ₹ 30,500

Ans. Option (B) is correct.

Explanation: Total revenue of market B

$$= [6,000 \quad 20,000 \quad 8,000]\begin{bmatrix}2.50\\1.50\\1.00\end{bmatrix}$$

$$= 2.50 \times 6,000 + 1.50 \times 20,000 + 1.00 \times 8,000$$

$$= ₹53,000$$

Q. 3. Cost incurred in market *A*:

(A) ₹ 13,000 **(B)** ₹ 30,100

(C) ₹ 10,300 **(D)** ₹ 31,000

Ans. Option (D) is correct.

Explanation: Cost incurred in market A

$$= [10,000 \quad 2,000 \quad 18,000]\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$$

$$= 2.00 \times 10,000 + 1.00 \times 2,000 + 0.50 \times 18,000$$

$$= ₹31,000$$

Q. 4. Profits in market A and B respectively are:

(A) (₹15,000, ₹17,000) **(B)** (₹17,000, ₹15,000)

(C) (₹51,000, ₹71,000) **(D)** (₹10,000, ₹20,000)

Ans. Option (A) is correct.

Explanation: Cost incurred in market B

$$= [6,000 \quad 20,000 \quad 8,000]\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$$

$$= [2.00 \times 6,000 + 1.00 \times 20,000 + 0.50 \times 8,000]$$

$$= [36,000]$$

Profit of market A & B = total revenue of A and B – Cost increased in market A and B

$$\begin{bmatrix}A\\B\end{bmatrix} = \begin{bmatrix}46,000\\50,000\end{bmatrix} - \begin{bmatrix}31,000\\36,000\end{bmatrix}$$

$$= \begin{bmatrix}15,000\\17,000\end{bmatrix}$$

i.e., (₹15,000, ₹17,000)

Q. 5. Gross profit in both markets is:

(A) ₹23,000 **(B)** ₹20,300

(C) ₹32,000 **(D)** ₹30,200

Ans. Option (C) is correct.

Explanation:

Gross profit in both markets = Profit in A + Profit in B

= 15,000 + 17,000 = ₹32,000

II. Read the following text and answer the following questions on the basis of the same:

Three schools DPS, CVC and KVS decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹25, ₹100 and ₹50 each respectively. The numbers of articles sold are given as **[CBSE QB 2021]**

School /Article	DPS	CVC	KVS
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Q. 1. What is the total money (in Rupees) collected by the school DPS?

(A) ₹700 **(B)** ₹7,000

(C) ₹6,125 **(D)** ₹7,875

Ans. Option (B) is correct.

Explanation: The funds collected by the schools can be obtained by matrix multiplication :

$$\begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}\begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

Funds collected by school DPS = 7000

Funds collected by school, CVC = 6125

Funds collected by school KVS = 7875

Q. 2. What is the total amount of money (in ₹) collected by schools CVC and KVS?

(A) 14,000 **(B)** 15,725

(C) 21,000 **(D)** 13,125

Ans. Option (A) is correct.

Explanation: Total amount of money collected by school

= 6125 + 7875

= 14000

Q. 3. What is the total amount of money collected by all three schools DPS, CVC and KVS?

(A) ₹15,775 **(B)** ₹14,000

(C) ₹21,000 **(D)** ₹17,125

Ans. Option (C) is correct.

Explanation: Total amount of money collected by all school DPS, CVC and KVS

= 7000 + 7875 + 6125

= 21000

Q. 4. If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all schools?

(A) ₹18,000 **(B)** ₹6,750

(C) ₹5,000 **(D)** ₹21,250

Ans. Option (D) is correct.

Q. 5. How many articles (in total) are sold by three schools?

(A) 230 **(B)** 130

(C) 430 **(D)** 330

Ans. Option (D) is correct.

Explanation: 110 + 95 + 125 = 330

III. Read the following text and answer the following questions on the basis of the same:

Two farmers Ramakishan and Gurucharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in ₹) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B **[CBSE QB 2021]**

September sales (in Rupees).

$$A = \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} \begin{matrix} \text{Ramakrishan} \\ \text{Gurcharan} \end{matrix}$$

October sales (in Rupees)

$$B = \begin{bmatrix} 5,000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix} \begin{matrix} \text{Ramakrishan} \\ \text{Gurcharan} \end{matrix}$$

Q. 1. The total sales in September and October for each farmer in each variety can be represented as _______.

(A) A + B **(B)** A – B

(C) A > B **(D)** A < B

Ans. Option (A) is correct.

Explanation: Combined sales in September and October for each farmer in each variety is given by $A + B =$

$$\begin{matrix} \text{Basmati} & \text{Permal} & \text{Naura} \end{matrix}$$
$$\begin{bmatrix} 15,000 & 30,000 & 36,000 \\ 70,000 & 40,000 & 20,000 \end{bmatrix} \begin{matrix} \text{Ramkrishan} \\ \text{Gurcharan singh} \end{matrix}$$

Q. 2. What is the value of A_{23}?

(A) 10,000 **(B)** 20,000

(C) 30,000 **(D)** 40,000

Ans. Option (A) is correct.

Explanation: $A_{23} = 10,000$

Q. 3. The decrease in sales from September to October is given by _______ .

(A) A + B **(B)** A – B

(C) A > B **(D)** A < B

Ans. Option (B) is correct.

Explanation: Change in sales from September to October is given by

$A - B =$

$$\begin{matrix} \text{Basmati} & \text{Permal} & \text{Naura} \end{matrix}$$
$$\begin{bmatrix} 5000 & 10,000 & 24,000 \\ 30,000 & 20,000 & 0 \end{bmatrix} \begin{matrix} \text{Ramkishan} \\ \text{Gurcharan Singh} \end{matrix}$$

Q. 4. If Ramkishan receives 2% profit on gross sales, compute his profit for each variety sold in October.

(A) ₹100, ₹200 and ₹120

(B) ₹100, ₹200 and ₹130

(C) ₹100, ₹220 and ₹120

(D) ₹110, ₹200 and ₹120

Ans. Option (A) is correct.

Explanation:

$$2\% \text{ of B} = \frac{2}{100} \times \text{B}$$
$$= 0.02 \times \text{B}$$
$$= 0.02$$

$$\begin{matrix} & \text{Basmati} & \text{Permal} & \text{Naura} \end{matrix}$$
$$\begin{bmatrix} 5000 & 10,000 & 6000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}\begin{matrix}\text{Ramkishan}\\ \text{Gurcharn Singh}\end{matrix}$$

Thus, in October Ramkishan receives ₹ 100, ₹ 200 and ₹ 120 as profit in the sale of each variety of rice, respectively.

Q. 5. If Gurucharan receives 2% profit on gross sales, compute his profit for each variety sold in September:

(A) ₹100, ₹200, ₹120 **(B)** ₹1000, ₹600, ₹200

(C) ₹400, ₹200, ₹120 **(D)** ₹1200, ₹200, ₹120

Ans. Option (B) is correct.

Explanation:

$$2\% \text{ of A} = \frac{2}{100} \times \text{A}$$
$$= 0.02 \times \text{A}$$
$$= 0.02$$

$$\begin{matrix} & \text{Basmati} & \text{Permal} & \text{Naura} \end{matrix}$$
$$\begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix}\begin{matrix}\text{Ramkishan}\\ \text{Gurcharn Singh}\end{matrix}$$

$$\begin{matrix} & \text{Basmati} & \text{Permal} & \text{Naura} \end{matrix}$$
$$= \begin{bmatrix} 200 & 400 & 600 \\ 1000 & 600 & 200 \end{bmatrix}\begin{matrix}\text{Ramkishan}\\ \text{Gurcharn Singh}\end{matrix}$$

Thus, in September Gurucharan receives ₹ 1000, ₹ 600 and ₹ 200 as Profit in the sale of each variety of rice, respectively.

IV. Read the following text and answer the following questions on the basis of the same:

On her birthday, Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in ₹).

[CBSE QB-2021]

Q. 1. The equations in terms x and y are:

(A) $5x - 4y = 40$, $5x - 8y = -80$ **(B)** $5x - 4y = 40$, $5x - 8y = 80$

(C) $5x - 4y = 40$, $5x + 8y = -80$ **(D)** $5x + 4y = 40$, $5x - 8y = -80$

Ans. Option (A) is correct.

Explanation: Let number of children = x

Amount distributed by Seema for one child = ₹y

Now, Total money = xy

and Total money will remain the same.

Given that, if there were 8 children less, everyone would have got ₹10 more.

Total money now = Total money before

$$(x - 8) \times (y + 10) = xy$$
$$\Rightarrow x(y + 10) - 8(y + 10) = xy$$
$$\Rightarrow xy + 10x - 8y - 80 = xy$$
$$\Rightarrow 10x - 8y - 80 = 0$$
$$\Rightarrow 10x - 8y = 80$$
$$\Rightarrow 5x - 4y = 40$$

Also, if there were 16 children more, everyone would have got ₹10 less.

Total money now = Total money before

$$(x + 16) \times (y - 10) = xy$$
$$\Rightarrow x(y - 10) + 16(y - 10) = xy$$
$$\Rightarrow xy - 10x + 16y - 160 = xy$$
$$\Rightarrow -10x + 16y - 160 = 0$$
$$\Rightarrow 10x - 16y + 160 = 0$$
$$\Rightarrow 5x - 8y = -80$$

Thus, required equations are:

$$5x - 4y = 40 \quad \text{...(i)}$$
$$5x - 8y = -80 \quad \text{...(ii)}$$

Q. 2. Which of the following matrix equations represent the information given above?

(A) $\begin{bmatrix} 5 & 4 \\ 5 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

(B) $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

(D) $\begin{bmatrix} 5 & 4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

Ans. Option (C) is correct.

Explanation: Writing eq. (i) & eq. (ii) in matrix form, we get

$$\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

Q. 3. The number of children who were given some money by Seema, is:

(A) 30 **(B)** 40

(C) 23 **(D)** 32

Ans. Option (D) is correct.

Explanation: On solving eqs. (i) & (ii) for x, we get $x = 32$.

Q. 4. How much amount is given to each child by Seema?

(A) ₹32 **(B)** ₹30

(C) ₹62 **(D)** ₹26

Ans. Option (B) is correct.

Explanation: On solving eqs. (i) & (ii) for y, we get $y = 30$ *i.e.*, y = ₹30

Q. 5. How much amount Seema spends in distributing the money to all the students of the Orphanage?

(A) ₹609 **(B)** ₹960

(C) ₹906 **(D)** ₹690

Ans. Option (B) is correct.

Explanation: Total amount = $xy = 32 \times 30$ = ₹960

Case based Subjective Questions (2 marks each)

I. Read the following text and answer the following questions on the basis of the same:

(Each Sub-part carries 2 marks)

In a city there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories type I, type II and type III. For boys the number of units of types I, II and III respectively are 80, 70 and 65 in factory A and 85, 65 and 72 are in factory B. For girls the number of units of types I, II and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

Q. 1. Write the matrices P and Q, if P represents the matrix of number of units of each type produced by factory A for both boys and girls; and Q represents the matrix of number of units of each type produced by factory B for both boys are girls.

Sol. In factory A, number of units of type I, II and III for boys are 80, 70, 65 respectively and for girls number of units of type I, II and III are 80, 75, 90 respectively.

$$\therefore \quad P = \begin{matrix} \\ I \\ II \\ III \end{matrix}\begin{matrix} \text{Boys} & \text{Girls} \\ \begin{bmatrix} 80 & 80 \\ 70 & 75 \\ 90 & 65 \end{bmatrix} \end{matrix} \quad \mathbf{1}$$

In factory B, number of units of type I, II and III for boys are 85, 65, 72 respectively and for girls number of units of type I, II and III are 50, 55, 80 respectively.

$$\therefore \quad Q = \begin{matrix} \\ I \\ II \\ III \end{matrix}\begin{matrix} \text{Boys} & \text{Girls} \\ \begin{bmatrix} 85 & 50 \\ 65 & 55 \\ 72 & 80 \end{bmatrix} \end{matrix} \quad \mathbf{1}$$

Q. 2. Find the total production of sports clothes of each type for boys and girls.

Sol. Let matrix X represent the number of units of each type produced by factory A for boys and matrix Y represents the number of units of each type produced by factory B for boys.

$$\therefore \quad X = \begin{matrix} I & II & III \\ [80 & 70 & 65] \end{matrix}$$

$$Y = \begin{matrix} I & II & III \\ [85 & 65 & 72] \end{matrix} \quad \mathbf{1}$$

Now, total production of sports clothes of each type for boys = X + Y

$= [80 \ \ 70 \ \ 65] + [85 \ \ 65 \ \ 72]$

$= [165 \ \ 135 \ \ 137]$

Similarly, for girls, let matrix S represents the number of units of each type produced by factory A and matrix T represents the number of units of each type produced by factory B.

$$\therefore \quad S = \begin{matrix} I & II & III \\ [80 & 75 & 90] \end{matrix}$$

$$T = \begin{matrix} I & II & III \\ [50 & 55 & 80] \end{matrix}$$

Now, required matrix = S + T

$= [80 \ \ 75 \ \ 90] + [50 \ \ 55 \ \ 80]$

$= [130 \ \ 130 \ \ 170]$ **1**

Solutions for Practice Questions (Topic 1)

Very Short Answer Type Questions

3. $$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{1}$$

[CBSE SQP Marking Scheme 2020-21]

Commonly Made Error

- Students fail to find the matrix A correctly.

Answering Tip

- Practice questions on matrix formation and get familiar with the elemental notations.

6. $A' = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & x & -1 \end{pmatrix}$ and getting $x = -2$ ½+½

[CBSE Marking Scheme 2018]

Detailed Solution:

Given, $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{pmatrix}$

$A' = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & x & -1 \end{pmatrix}$

Since, $AA' = 9I$

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & x & -1 \end{pmatrix} = 9\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1+4+4 & 2+2+2x & -2+4-2 \\ 2+2+2x & 4+1+x^2 & -4+2-x \\ -2+4-2 & -4+2-x & 4+4+1 \end{pmatrix} = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 2x+4 & 0 \\ 2x+4 & x^2+5 & -x-2 \\ 0 & -x-2 & 9 \end{pmatrix} = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$ ½

Now, $x^2 + 5 = 9$

$x^2 = 9 - 5$

$x^2 = 4$

$x = \sqrt{4}$

$x = \pm 2$

Also, $2x + 4 = 0$

$2x = -4$

$x = -\frac{4}{2}$

So, $x = -2$ ½

12. $A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$

$kA = \begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix}$

But given $kA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$

$\therefore \begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix} = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$ ½

On equating individual terms,

$2k = -8 \Rightarrow k = -4$

$3k = 4a$

$3 \times (-4) = 4a \Rightarrow a = -3$ ½

Short Answer Type Questions-I

2. $A^2 = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$ 1

$$A^2 - 5A = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{pmatrix}$$

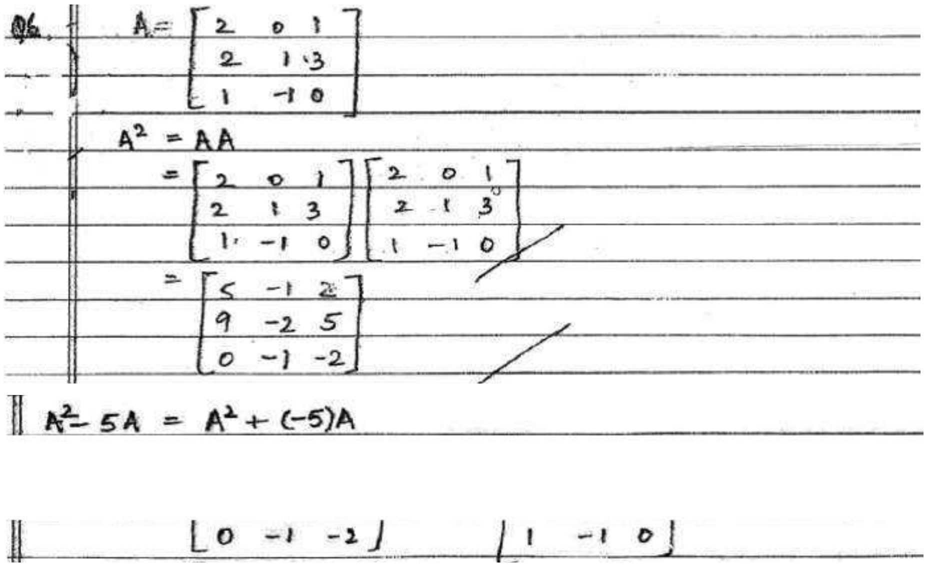
$$= \begin{pmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{pmatrix}$$ 1

[CBSE Marking Scheme, 2019]

Detailed Solution:

Topper Answer, 2020

Sol.



Q6. $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

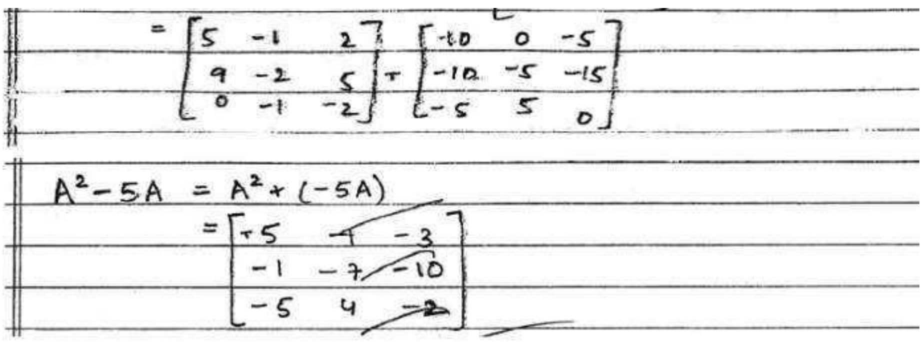
$A^2 = AA$

$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$

$A^2 - 5A = A^2 + (-5)A$

[0 -1 -2] [1 -1 0]



Short Answer Type Questions-II

3. Let A be the matrix that represent the number of attempts made in three villages X, Y and Z and let B be the matrix that represent the cost for each mode per attempt.

Then, the matrices A and B can be represented as

House Calls Letters Announcements

$$A = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}$$

and $$B = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$ **1**

Now, the cost incurred by the organisation for three villages can be represented as.

$$AB = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$ **1**

$$= \begin{bmatrix} 400\times 50 + 300\times 20 + 100\times 40 \\ 300\times 50 + 250\times 20 + 75\times 40 \\ 500\times 50 + 400\times 20 + 150\times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 20000 + 6000 + 4000 \\ 15000 + 5000 + 3000 \\ 25000 + 8000 + 6000 \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$$ **1**

Thus, the total cost incurred by the organisation for the three villages X, Y and Z separately are ₹ 30000, ₹ 23000 and ₹ 39000, respectively. **1**

Commonly Made Error

- Students get wrong when multiplying the matrices together.

Answering Tip

- The answer can be obtained by separately multiplying the matrices also.

Long Answer Type Questions

2. Clearly order of A is 2×3 **1**

Let $A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ **1**

So $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{pmatrix}$

gives

$$\left.\begin{matrix} 2a - d = -1, 2b - e = -8, 2c - f = -10 \\ a = 1, b = -2, c = -5 \end{matrix}\right\}$$ **2**

$\Rightarrow \quad d = 3, e = 4, f = 0$ **1**

Thus $A = \begin{pmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{pmatrix}$ **1**

Solutions for Practice Questions (Topic 2)

Very Short Answer Type Questions

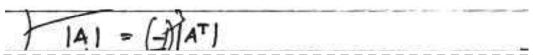
4. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is *a* 2×2 matrix which is symmetric as well as skew symmetric matrix.
1

[CBSE Marking Scheme 2014]

Short Answer Type Questions-I

Topper Answer, 2018

1.



$|A| = -|A| \quad [\because |A| = |A^T|]$

$2|A| = 0$

$|A| = 0$

$\det A = 0$

Short Answer Type Questions-II

2. We know that

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

Here, $\frac{1}{2}(A + A')$ is symmetric matrix and $\frac{1}{2}$ $(A - A')$ is skew symmetric matrix.

$$\text{Now, } A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$ **1**

$$\therefore \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 3+3 & -2+3 & -4-1 \\ 3-2 & -2-2 & -5+1 \\ -1-4 & 1-5 & 2+2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$ which is symmetric **1**

$$\frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 3-3 & -2-3 & -4+1 \\ 3+2 & -2+2 & -5-1 \\ -1+4 & 1+5 & 2-2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$ which is skew-symmetric. **1**

$$\therefore A = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$ **1**

REFLECTIONS

- Matrices are used in cryptography. In cryptography, the process of encryption is carried out with the help of invertible key. In this method, matrices are used.
- Wireless signals are modelled and optimized using matrices.
- In the realm of graphics, matrices are used to project three-dimensional images into two-dimensional planes.
- Matrices are applied in the study of electrical circuits, quantum mechanics and optics, in the calculation of battery power outputs and resistor conversion of electrical energy into another useful energy.

❐❐