CHAPTER 13 PROBABILITY

Syllabus *Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable.*

In this chapter you will study

- *Concept of conditional probability*
- *Bayes' theorem*
- *Probability distribution of different random variables.*

List of Topics

Topic-1 Conditional Probability and Multiplication Theorem on Probability

Concepts Covered • *Conditional Probability,* • *Multiplication Theorem of Probability*

Revision Notes

1. Basic Definition of Probability :

Let S and E be **the sample** space and **event** in an experiment respectively.

> **Key Words**
>
> **Sample Space:** A set in which all of the possible outcomes of a statistical experiment are represented as points.
>
> **Event:** Event is a subset of a sample space. *e.g.:* Event of getting odd outcome in a throw of a die.

Then, Probability

$$= \frac{\text{Number of Favourable Events}}{\text{Total number of Elementary Events}} = \frac{n(E)}{n(S)}$$

$$0 \le n(E) \le n(S)$$

$$0 \le P(E) \le 1$$

Hence, if $P(E)$ denotes the probability of occurrence of an event E, then $0 \le P(E) \le 1$ and $P(\bar{E}) = 1 - P(E)$ such that $P(\bar{E})$ denotes the probability of non-occurrence of the event E.

➲ **Note that $P(\bar{E})$ can also be represented as $P(E')$.**

2. Mutually Exclusive Or Disjoint Events :

Two events A and B are said to be mutually exclusive if occurrence of one prevents the occurrence of the other *i.e.*, they can't occur simultaneously.

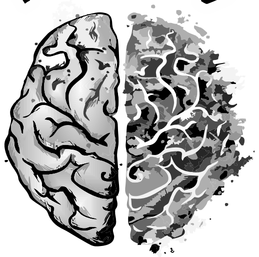
In this case, sets A and B are disjoint *i.e.*, $A \cap B = \phi$.

Consider an example of throwing a die. We have the sample space as, $S = \{1, 2, 3, 4, 5, 6\}$

Suppose A = the event of occurrence of a number greater than 4 = $\{5, 6\}$

B = the event of occurrence of an odd number = $\{1, 3, 5\}$

Probability



Conditional Probability

The probability of the event E is called the conditional probability of E given that F has already occurred, and is denoted by $P(E/F)$. Also,

$$P(E/F)=\frac{P(E\cap F)}{P(F)}, P(F)\neq 0.$$

Properties

(i) $0\leq P(E/F)\leq 1$, $P(E'/F)=1-P(E/F)$

(ii) $P((E\cup F)/G)=P(E/G)+P(F/G)-P((E\cap F)/G)$

(iii) $P(E\cap F)=P(E)P(F/E), P(E)\neq 0$

(iv) $P(F\cap E)=P(F)P(E/F), P(F)\neq 0$

eg : if $P(A)=\frac{7}{13}$, $P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$, then

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{4}{13}}{\frac{9}{13}}=\frac{4}{9}$$

Independent Event

If E and F are independent, then

$P(E\cap F)=P(E)P(F)$, $P(E-F)=P(E)$, $P(F)\neq 0$

and $P(E-F)=P(E)$, $P(F)$, $P(E)\neq 0$.

If A,B,C are mutually independent events then

(i) $P(A\cap B)=P(A).P(B)$

(ii) $P(A\cap C)=P(A).P(C)$

(iii) $P(B\cap C)=P(B).P(C)$

(iv) $P(A\cap B\cap C)=P(A).P(B).P(C)$

Theorem of total probability

Let, $\{E_1, E_2 \ldots E_n\}$ be a partition of a sample space 'S' and suppose that each of $E_1, E_2, \ldots E_n$ has non-zero probability. Let 'A' be any event associated with S, then $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \ldots + P(E_n)P(A/E_n)$.

$$P\left(\frac{E_i}{A}\right) = \sum_{i=1}^{n} P(E_i)P(A/E_i)$$

Probability Distribution

The probability distribution of a random variable x is the system of numbers

x: $x_1\ x_2 \ldots\ldots x_n$, $P(x)$: $p_1\ p_2 \ldots\ldots p_n$

where , $p_i>0$,

$$\sum_{i=1}^{n} p_i = 1, i = 1.2, \ldots, n.$$

Bayes' Theorem

If $E_1, E_2, \ldots, E_n$ are events which constitute a partition of sample space S, *i.e.*, $E_1, E_2, \ldots E_n$ are pairwise disjoint and $E_1\cup E_2\cup\ldots\cup E_n=S$ and A be any event with non-zero probability,

then $P(E_i/A)=\frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n}P(E_i)P(A/E_i)}, n=1,2,3\ldots\ldots n$

Mean of a random variable

Let x be *a* random variable whose possible values $x_1, x_2, \ldots, x_n$ occur with probabilities $p_1, p_2, \ldots, p_n$ respectively. Then, mean of x, $\mu=\sum_{i=1}^{n}x_ip_i$. It is also called the expectation of x, denoted by $E(x)$.

Random Variable

Real valued function whose domain is the sample space of a random experiment.

Trace the Mind Map

- First Level
- Second Level
- Third Level

and C = the event of occurrence of an even number = {2, 4, 6}

In these events, the events B and C are mutually exclusive events but A and B are not mutually exclusive events because they can occur together (when the number 5 comes up). Similarly A and C are not mutually exclusive events as they can also occur together (when the number 6 comes up).

3. Independent Events :

Two events are independent if the occurrence of one does not affect the occurrence of the other. Consider an example of drawing two balls one by one with replacement from a bag containing 3 red and 2 black balls.

Suppose A = the event of occurrence of a red ball in first draw

B = the event of occurrence of a black ball in the second draw.

Then, $P(A) = \frac{3}{5}, P(B) = \frac{2}{5}$

Here probability of occurrence of event B is not affected by the occurrence or non - occurrence of the event A.

Hence events A and B are independent events.

4. Exhaustive Events :

Two or more events say A, B and C of an experiment are said to be exhaustive events, if

(a) their union is the total sample space *i.e.*, $A \cup B \cup C = S$

(b) the event A, B and C are disjoint in pairs *i.e.*, $A \cap B = \phi$, $B \cap C = \phi$ and $C \cap A = \phi$.

(c) $P(A) + P(B) + P(C) = 1$.

Consider an example of throwing a die. We have $S = \{1, 2, 3, 4, 5, 6\}$

Suppose A = the event of occurrence of an even number = {2, 4, 6}

B = the event of occurrence of an odd number = {1, 3, 5}

and C = the event of getting a number multiple of 3 = {3, 6}

In these events, the events A and B are exhaustive events as $A \cup B = S$ but the events A and C or the events B and C are not exhaustive events as $A \cup C \neq S$ and similarly $B \cup C \neq S$.

➲ If A and B are mutually exhaustive events, then we always have

$P(A \cap B) = 0$ [As $n(A \cap B) = n(\phi) = 0$]

$\therefore$ $P(A \cup B) = P(A) + P(B)$.

➲ If A, B and C are mutually exhaustive events, then we always have

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

Mnemonics

Concept: Independent and Mutually exclusive events.

I **I**s not ME

ME **I**s not I

Here, **I**: Independent Events

ME: Mutually Exclusive events

5. Conditional Probability :

By the conditional probability, we mean the probability of occurrence of event A when B has already occurred.

The 'conditional probability of occurrence of event A when B has already occurred' is sometimes also called as probability of occurrence of event A w.r.t. B.

➲ $P(A|B) = \frac{P(A \cap B)}{P(B)}$, $B \neq \phi$ *i.e.*, $P(B) \neq 0$

➲ $P(B|A) = \frac{P(A \cap B)}{P(A)}$, $A \neq \phi$ *i.e.*, $P(A) \neq 0$

➲ $P(\bar{A}|B) = \frac{P(\bar{A} \cap B)}{P(B)}$, $P(B) \neq 0$

➲ $P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$, $P(\bar{B}) \neq 0$

➲ $P(\bar{A}|\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$, $P(\bar{B}) \neq 0$

➲ $P(A|B) + P(\bar{A}|B) = 1, B \neq \phi$.

Key Facts

- Probability originated from a gambler's dispute in 1654 concerning the division of a stake between two players whose game was interrupted before it close.
- Quantum physics is an inherently probabilistic theory in that only probabilities for measurement outcomes can be determined.

Key Formulae

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ i.e., $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

(b) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(c) $P(\overline{A} \cap B) = P(\text{only } B) = P(B - A) = P(B \text{ but not } A) = P(B) - P(A \cap B)$

(d) $P(A \cap \overline{B}) = P(\text{only } A) = P(A - B) = P(A \text{ but not } B) = P(A) - P(A \cap B)$

(e) $P(\overline{A} \cap \overline{B}) = P(\text{neither } A \text{ nor } B) = 1 - P(A \cup B)$

NOTE : EVENTS AND SYMBOLIC REPRESENTATIONS :

Verbal description of the event	Equivalent set notation
Event A	A
Not A	$\overline{A}$ or A'
A or B (occurrence of atleast one A or B)	$A \cup B$ or $A + B$
A and B (simultaneous occurrence of both A and B)	$A \cap B$ or AB
A but not B (A occurs but B does not)	$A \cap \overline{B}$ or $A - B$
Neither A nor B	$\overline{A} \cap \overline{B}$
Atleast one A, B or C	$A \cup B \cup C$
All the three A, B and C	$A \cap B \cap C$

Key Facts

- The probability of you being born was about 1 in 400 trillion.
- The probability of living 110 years or more is about 1 in 7 million.

- If you are in the group of 23 people, there is a 50% chance that 2 of them share a birthday. If you are in a group of 70 people, that probability jumps to over 99%.

OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. If A and B are two independent events with P(A) = $\frac{1}{3}$ and P(B) = $\frac{1}{4}$, then P(B′/A) is equal to

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$

(C) $\frac{3}{4}$ **(D)** 1

[CBSE Delhi Set II, 2020]

Ans. Option (C) is correct.

Topper Answer, 2020

Sol. $P(B'/A) = \dfrac{P(B' \cap A)}{P(A)} = \dfrac{\frac{3}{4} \times \frac{1}{3}}{\frac{1}{3}} = \dfrac{3}{4}$

c. 3/4

Q. 2. If A and B are two events such that $P(A) \neq 0$ and $P(B|A) = 1$, then

(A) $A \subset B$ **(B)** $B \subset A$

(C) $B = \phi$ **(D)** $A = \varphi$

Ans. Option (A) is correct.

Explanation:

$$P(A) \neq 0$$

and $$P(B|A) = 1$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$1 = \frac{P(B \cap A)}{P(A)}$$

$$P(A) = P(B \cap A)$$

$$\therefore \quad A \subset B$$

Q. 3. If $P(A|B) > P(A)$, then which of the following is correct:

(A) $P(B|A) < P(B)$ **(B)** $P(A \cap B) < P(A).P(B)$

(C) $P(B|A) > P(B)$ **(D)** $P(B|A) = P(B)$

Ans. Option (C) is correct.

Explanation:

$$P(A|B) > P(A)$$

$$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} > P(A)$$

$$\Rightarrow \quad P(A \cap B) > P(A).P(B)$$

$$\Rightarrow \quad \frac{P(A \cap B)}{P(A)} > P(B)$$

$$\Rightarrow \quad P(B|A) > P(B)$$

Q. 4. If A and B are any two events such that $P(A) + P(B) - P(A$ and $B) = P(A)$, then

(A) $P(B|A) = 1$ **(B)** $P(A|B) = 1$

(C) $P(B|A) = 0$ **(D)** $P(B|A) = 0$

Ans. Option (B) is correct.

Explanation:

$$P(A) + P(B) - P(A \text{ and } B) = P(A)$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A)$$

$$\Rightarrow \quad P(B) - P(A \cap B) = 0$$

$$\Rightarrow \quad P(A \cap B) = P(B)$$

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B)}{P(B)}$$

$$= 1$$

Q. 5. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

(A) 0 **(B)** $\frac{1}{3}$

(C) $\frac{1}{12}$ **(D)** $\frac{1}{36}$

Ans. Option (D) is correct.

Explanation: When two dices are rolled, the number of outcomes is 36. The only even prime number is 2. Let E be the event of getting an even prime number on each die.

$\therefore \quad E = \{(2, 2)\}$

$$\Rightarrow P(E) = \frac{1}{36}$$

Q. 6. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is equal to

(A) 0.24 **(B)** 0.3

(C) 0.48 **(D)** 0.96

Ans. Option (D) is correct.

Explanation:

Here,

$P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$

$$\because \quad P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow \quad P(B \cap A) = P(B|A).P(A)$$

$$= 0.6 \times 0.4 = 0.24$$

$$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.4 + 0.8 - 0.24$$

$$= 1.2 - 0.24 = 0.96$$

Q. 7. Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The probability, that both cards are queens, is

(A) $\frac{1}{13} \times \frac{1}{13}$ **(B)** $\frac{1}{13} + \frac{1}{13}$

(C) $\frac{1}{13} \times \frac{1}{17}$ **(D)** $\frac{1}{13} \times \frac{4}{51}$

Ans. Option (A) is correct.

Explanation:

$$\text{Required probability} = \frac{4}{52} \times \frac{4}{52} = \frac{1}{13} \times \frac{1}{13}$$

Q. 8. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3, is

(A) $\frac{1}{18}$ **(B)** $\frac{5}{18}$

(C) $\frac{1}{5}$ **(D)** $\frac{2}{5}$

Ans. Option (C) is correct.

Explanation: Let,

E_1 = Event that the sum of numbers on the dice was less than 6 and

E_2 = Event that the sum of numbers on the dice is 3.

$\therefore \; E_1 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2,3), (3, 1), (3, 2), (4,1)\}$

$$\Rightarrow \quad n(E_1) = 10$$

and $E_2 = \{(1, 2), (2, 1)\}$

$$\Rightarrow \quad n(E_2) = 2$$

$$\therefore \text{Required probability} = \frac{2}{10}$$

$$= \frac{1}{5}$$

Q. 9. A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number on the die and a spade card is

(A) $\frac{1}{2}$ **(B)** $\frac{1}{4}$

(C) $\frac{1}{8}$ **(D)** $\frac{3}{4}$

Ans. Option (C) is correct.

Explanation: Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$$\therefore \quad P(E_1) = \frac{3}{6} = \frac{1}{2}$$

and
$$P(E_2) = \frac{13}{52} = \frac{1}{4}$$

Then,
$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

Q. 10. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(A' \cap B')$ equals

(A) $\frac{4}{15}$ **(B)** $\frac{8}{45}$

(C) $\frac{1}{3}$ **(D)** $\frac{2}{9}$

Ans. Option (D) is correct.

Explanation: Since A and B are independent events, A' and B' are also independent. Therefore,

$$\begin{aligned} P(A' \cap B') &= P(A') \cdot P(B') \\ &= (1 - P(A)(1 - P(B)) \\ &= \left(1 - \frac{3}{5}\right)\left(1 - \frac{4}{9}\right) \\ &= \frac{2}{5} \cdot \frac{5}{9} \\ &= \frac{2}{9} \end{aligned}$$

Q. 11. If A and B are such events that $P(A) > 0$ and $P(B) \neq 1$, then $P\ A'/B'$ equals

(A) $1 - P(A|B)$ **(B)** $1 - P(A|B)$

(C) $\frac{1 - P(A \cup B)}{P(B')}$ **(D)** $P(A)|P(B)$

Ans. Option (C) is correct.

Explanation: We have,

$$P(A) > 0 \text{ and } P(B) \neq 1$$

$$\begin{aligned} P(A'/B') &= \frac{P(A' \cap B')}{P(B')} \\ &= \frac{1 - P(A \cup B)}{P(B')} \end{aligned}$$

Very Short Answer Type Questions (1 mark each)

Q. 1. The probabilities of A and B solving a problem independently are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If both of them try to solve the problem independently, what is the probability that the problem is solved?

A I R [CBSE SQP 2020-21]

Q. 2. Two cards are drawn at random and one by one without replacement from a well shuffled pack of 52 playing cards, find the probability that one card is red and the other is black. A I [CBSE Delhi Set II, 2020]

Topper Answer, 2020

Sol. let A be the event one card is red and other black.

$$P(A) = \frac{26}{52} \times \frac{26}{51} \times 2 = \frac{1}{2} \times \frac{26}{51} \times 2 = \frac{26}{51}$$

Q. 3. The probability that it will rain on any particular day is 50%. Find the probability that it rains only on first 4 days of the week.

R&U [CBSE SQP 2020-21]

Sol. $\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^7$ **1**

[CBSE Marking Scheme, 2020]

Detailed Solution :

P(rain on any particular day) = 50%

$$= \frac{50}{100} = \frac{1}{2}$$

$$\text{P(rain on first four days of week)} = \left(\frac{1}{2}\right)^4\left(1 - \frac{1}{2}\right)^3$$

$$= \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^7 = \frac{1}{128}$$

Commonly Made Error

- Some students forget to take the probability for not raining on the remaining three days and give the answer as $\frac{1}{16}$.

Answering Tips

- Read the question twice to sort out the hidden elements in it.

Short Answer Type Questions-I (2 marks each)

Q. 1. In the game of archery, the probability of Likith and Harish hitting the target are $\frac{2}{3}$ and $\frac{3}{4}$, respectively.

If both of them shoot an arrow, find the probability that the target is NOT hit by either of them. Show your steps. A I [Practice Questions 2021-22]

Q. 2. Two cards are drawn at random from a pick of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack? [SQP 2021-22]

Sol. The required probability = *P*[(The first is a red jack card and the second is a jack card) or (The first is a red non-jack card and the second is a jack card)] **1**

$$= \frac{2}{52}\cdot\frac{3}{51}+\frac{24}{52}\cdot\frac{4}{51} = \frac{1}{26} \quad \mathbf{1}$$

Q. 3. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three? R&U [CBSE Delhi Set-II, 2020]

Q. 4. Given that E and F are events such that $P(E) = 0.8$, $P(F) = 0.7$, $P(E \cap F) = 0.6$. Find $P(\bar{E}|\bar{F})$. R&U [CBSE SQP 2020-21]

Sol.
$$P(\bar{E}|\bar{F}) = P\frac{(\bar{E}\cap\bar{F})}{P(\bar{F})} = \frac{(\overline{E\cup F})}{P(\bar{F})}$$
$$= \frac{1-P(E\cup F)}{1-P(F)} \quad \text{...(i) } \mathbf{1}$$

Now $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.8 + 0.7 - 0.6 = 0.9$ ½

Substituting value of $P(E \cup F)$ in (i)

$$P(\bar{E}|\bar{F}) = \frac{1-0.9}{1-0.7} = \frac{0.1}{0.3} = \frac{1}{3} \quad ½$$

[CBSE SQP Marking Scheme 2020-21]

Commonly Made Error

- Students use the concept of independent events instead of applying De-Morgan's law.

Answering Tips

- The events are independent if and only if $P(A \cap B) = P(A).P(B)$

Q. 5. Find $[P(B/A) + P(A/B)]$, if $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$. R [CBSE OD Set III - 2020]

Sol.
$$P(A\cap B) = \frac{3}{10}+\frac{2}{5}-\frac{3}{5}=\frac{1}{10} \quad ½$$

Now, $P(B|A) + P(A|B)$
$$= \frac{P(A\cap B)}{P(A)}+\frac{P(A\cap B)}{P(B)} \quad ½$$
$$= \frac{1}{3}+\frac{1}{4}=\frac{7}{12} \quad \mathbf{1}$$

[CBSE Marking Scheme 2020]

Detailed Solution:

Given, $P(A) = \frac{3}{10}$

$P(B) = \frac{2}{5}$

and $P(A \cup B) = \frac{3}{5}$

Since,

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$\frac{3}{5} = \frac{3}{10}+\frac{2}{5} - P(A\cap B)$$
$$P(A\cap B) = \frac{7}{10}-\frac{3}{5}$$
$$P(A\cap B) = \frac{1}{10}$$
$$P\left(\frac{B}{A}\right)+P\left(\frac{A}{B}\right) = \frac{P(B\cap A)}{P(A)}+\frac{P(A\cap B)}{P(B)}$$
$$= \frac{\frac{1}{10}}{\frac{3}{10}}+\frac{\frac{1}{10}}{\frac{2}{5}} = \frac{1}{3}+\frac{1}{4} = \frac{7}{12}$$

Q. 6. Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$, find $P(A' \cap B')$. R [CBSE Delhi Set-I, II, III 2020]

Sol.
$$P(A'\cap B') = P(A')\,P(B') \quad \mathbf{1}$$
$$= (0.7)(0.4) = 0.28 \quad \mathbf{1}$$

[CBSE Marking Scheme 2020]

Commonly Made Error

- Student face problems in deciding when to add or when to multiply probability.

Answering Tips

- Thoroughly understand the cases of mutually exclusive events and independent events.

Q. 7. Three distinct numbers are chosen randomly from the first 50 natural numbers. Find the probability that all the three numbers are divisible by both 2 and 3. R&U [CBSE OD Set III - 2020]

Sol. Since there are only 8 numbers (in first 50 natural numbers) which are divisible by 6.

$\therefore$ favorable number of outcomes are ${}^{8}C_3$. ½

Total number of possible outcomes are ${}^{50}C_3$. ½

Required probability $= \dfrac{{}^{8}C_3}{{}^{50}C_3} = \dfrac{1}{350}$ 1

[CBSE Marking Scheme 2020]

Commonly Made Error

- Mostly students go wrong in problems involving permutations and combinations.

Answering Tips

- Practice more problems on elementary probability involving combinations.

Q. 8. A speaks truth in 80% cases and B speaks truth in 90% cases. In what percentage of cases are they likely to agree with each other in stating the same fact? A I R&U [CBSE SQP-2020]

Detailed Solution :

Sol. $P(A) = \dfrac{80}{100} = \dfrac{4}{5}, P(B) = \dfrac{90}{100} = \dfrac{9}{10}$ 1

$P(\text{Agree}) = P$ (Both speaking truth or both telling lie)

$= P(AB \text{ or } \overline{AB})$

$= P(A)\,P(B) \text{ or } P(\bar{A})\,P(\bar{B})$

$= \left(\dfrac{4}{5}\right)\left(\dfrac{9}{10}\right) + \left(\dfrac{1}{5}\right)\left(\dfrac{1}{10}\right)$

$= \dfrac{36+1}{50} = \dfrac{37}{50}$

$= \dfrac{74}{100} = 74\%$ 1

[CBSE SQP Marking Scheme, 2020]

Q. 9. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

A I R&U [O.D. Set-I, 2017] [CBSE Delhi Set III-2019]

Sol. Event A : Number obtained is even

B : Number obtained is red.

$P(A) = \dfrac{3}{6} = \dfrac{1}{2},$ ½

$P(B) = \dfrac{3}{6} = \dfrac{1}{2}$ ½

$P(A \cap B) = P$ (getting an even red number) $= \dfrac{1}{6}$ ½

Since $P(A) \cdot P(B) = \dfrac{1}{2} \cdot \dfrac{1}{2}$

$= \dfrac{1}{4} \neq P(A \cap B)$ which is $\dfrac{1}{6}$ ½

$\therefore$ A and B are not independent events.

[CBSE Marking Scheme, 2019]

Topper Answer, 2019

Sol. ~~P(A~~ $P(A) = \dfrac{3}{6} = \dfrac{1}{2}$ $\left[\because P(E) = \dfrac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}}\right]$

$P(B) = \dfrac{3}{6} = \dfrac{1}{2}$ [⇒ Sample space S = {1r, 2r, 3r, 4g, 5g, 6g}
A = {2r, 4g, 6g} A∩B = {2r}
B = {1r, 2r, 3r}]

$P(A \cap B) = P$(no. which is even and red) $= \dfrac{1}{6}$ [∵ No. which is even and red = {2r}]

$P(A) \cdot P(B) = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$

$P(A \cap B) \neq P(A) \cdot P(B)$

Events ~~are not~~ NOT INDEPENDENT.

Q. 10. If $P(\text{not } A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$. [CBSE O.D. Set-I, 2019]

Short Answer Type Questions-II (3 marks each)

Q. 1. ***A* and *B* throw a pair of dice alternately. *A* wins the game if he gets a total of 9 and *B* wins if he gets a total of 7. If *A* starts the game, find the probability of winning the game by *B*.**

R&U [Delhi Set I, II, III Comptt. 2016]

Sol. Let x = event getting a total of 9, y = event getting a total of 7

$$P(x) = \frac{4}{36} = \frac{1}{9};$$

and $$P(\bar{x}) = 1 - P(x) = \frac{8}{9}$$ ½

$$P(y) = \frac{6}{36} = \frac{1}{6};$$

and $$P(\bar{y}) = 1 - P(y) = \frac{5}{6}$$ ½

P(B wins the game)

$$= P(\bar{x}y + \overline{xyx}y + \overline{xyxyx}y + ...)$$

$$= P(\bar{x})\cdot P(y) + [P(\bar{x})]^2\cdot P(\bar{y})\cdot P(y) + [P(\bar{x})]^3\cdot[P(\bar{y})]^2\cdot P(y) + ...$$ **1**

$$= \frac{P(\bar{x})\cdot P(y)}{1 - P(\bar{x})\cdot P(\bar{y})} \quad \left[\begin{array}{l}\text{in G.P.}\\ S_\infty = \dfrac{a}{1-r}\end{array}\right]$$

$$= \frac{\frac{8}{9}\cdot\frac{1}{6}}{1 - \frac{8}{9}\cdot\frac{5}{6}}$$

$$= \frac{8}{54} \times \frac{54}{54-40} = \frac{4}{7}$$ **1**

[CBSE Marking Scheme, 2016] (Modified)

Q. 2. A bag contains $(2n + 1)$ coins. It is known that $(n - 1)$ of these coins have a head on both sides, whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n.

R&U [NCERT Exemplar]
[Outside Delhi Set I, II, III Comptt. 2016]

Sol. No. of coins with head on both sides = $(n - 1)$

No. of fair coins = $(n + 2)$

Let event

E_1 = Picking a coin with head on both sides

E_2 = Picking a fair coin

A : Getting a head on tossing the coin ½

$$P(E_1) = \frac{n-1}{2n+1},$$

$$P(E_2) = \frac{n+2}{2n+1}$$ ½

$$P(A/E_1) = 1,\ P(A/E_2) = \frac{1}{2}$$ ½

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$$

$$= \frac{n-1}{2n+1}\cdot 1 + \frac{n+2}{2n+1}\cdot\frac{1}{2}$$

$$= \frac{3n}{2(2n+1)}$$ **1**

or $$\frac{3n}{2(2n+1)} = \frac{31}{42} \text{ or } n = 31.$$ ½

[CBSE Marking Scheme, 2016] (Modified)

Q. 3. A problem in mathematics is given to 4 students *A, B, C, D*. Their chances of solving the problem, respectively, are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{2}{3}$. What is the probability that (i) the problem will be solved ? (ii) at most one of them solve the problem ?

A [SQP 2015-16]

Q. 4. Probability of solving specific problem independently by *A* and *B* are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem, independently, then find the probability that

(i) the problem is solved

(ii) exactly one of them solves the problem

R&U [NCERT] [Delhi Set I, II, III Comptt. 2015]

Sol. Let E_1 : Problem solved by A

E_2 : Problem solved by B

$\therefore$ $$P(E_1) = \frac{1}{2} \text{ and } P(E_2) = \frac{1}{3}$$

or $$P(\overline{E_1}) = \frac{1}{2} \text{ and } P(\overline{E_2}) = \frac{2}{3}$$ **1**

$$P(E_1 \cap E_2) = P(E_1)\cdot P(E_2) = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}$$

(i) P(Problem is solved)

$$P(E_1 \cup E_2) = 1 - P(\bar{E}_1)\cdot P(\bar{E}_2)$$

$$= 1 - \frac{1}{2} \times \frac{2}{3} = \frac{2}{3}$$ **1**

(ii) P(one of them is solved)

$$= P(E_1)P(\bar{E}_2) + P(E_2)P(\bar{E}_1)$$

$$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{2}$$

$$= \frac{1}{2}$$ **1**

[CBSE Marking Scheme, 2015] (Modified)

Long Answer Type Questions (5 marks each)

Q. 1. If A and B are two independent events such that $P(\bar{A}\cap B)=\frac{2}{15}$ and $P(A\cap\bar{B})=\frac{1}{6}$, then find $P(A)$ and $P(B)$. R&U [Delhi I, II, III, 2015]

Sol.

$$P(\bar{A}\cap B) = \frac{2}{15}$$

or $$P(\bar{A})\cdot P(B) = \frac{2}{15} \qquad \mathbf{1}$$

and $$P(A\cap\bar{B}) = \frac{1}{6}$$

or $$P(A)\cdot P(\bar{B}) = \frac{1}{6}$$

$\therefore$ $$[1-P(A)]\,P(B) = \frac{2}{15} \qquad \mathbf{1}$$

or $$P(B)-P(A)P(B) = \frac{2}{15} \qquad \text{..(i)}\ \mathbf{1}$$

Similarly

$$P(A)(1-P(B)) = \frac{1}{6}$$

or $$P(A)-P(A)P(B) = \frac{1}{6} \qquad \text{...(ii)}\ \mathbf{1}$$

From (i) and (ii),

$$P(A)-P(B) = \frac{1}{6}-\frac{2}{15}=\frac{1}{30}$$

Let $P(A) = x$,

and $P(B) = y$

$\therefore$ $$x=\left(\frac{1}{30}+y\right)$$

by, (i) $$y-\left(\frac{1}{30}+y\right)y = \frac{2}{15}$$

$\therefore$ $$30y^2-29y+4 = 0$$

$$(5y-4)(6y-1) = 0$$

On solving, we get

$$y = \frac{1}{6} \text{ or } y = \frac{4}{5}$$

$\therefore$ $$x = \frac{1}{5} \text{ or } x = \frac{5}{6} \qquad \tfrac{1}{2}$$

Hence, $$P(A) = \frac{1}{5},\ P(B) = \frac{1}{6}$$

Or

$$P(A) = \frac{5}{6},\ P(B) = \frac{4}{5}. \qquad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2015] (Modified)

Topic-2 Bayes' Theorem

Concept Covered • *Bayes' Theorem*

Revision Notes

BAYES' THEOREM :

If $E_1, E_2, E_3, \ldots\ldots, E_n$ are n non-empty events constituting a partition of sample space S *i.e.*, $E_1, E_2, E_3, \ldots, E_n$ are pair wise disjoint and $E_1\cup E_2\cup E_3\cup\ldots\cup E_n = S$ and A is any event of non-zero probability, then

$$P(E_i|A) = \frac{P(E_i).P(A|E_i)}{\sum_{j=1}^{n} P(E_j)P(A|E_j)},\ i=1, 2, 3, \ldots, n$$

For example, $P(E_1|A)$

$$= \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)+P(E_3).P(A|E_3)}$$

- Bayes' theorem is also known as the formula for the probability of causes.
- If $E_1, E_2, E_3, \ldots, E_n$ form a partition of S and A be any event, then

  $$P(A) = P(E_1).P(A|E_1) + P(E_2).P(A|E_2) + \ldots + P(E_n).P(A|E_n)$$

  $$[\because P(E_i\cap A) = P(E_i).P(A|E_i)]$$
- The probabilities $P(E_1)$, $P(E_2)$, ..., $P(E_n)$ which are known before the experiment takes place are called **prior probabilities** and $P(A|E_n)$ are called **posterior probabilities**.

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions

Q. 1. Method in which the previously calculated probability are revised with the values of new probability is known as:

(A) Addition Theorem **(B)** Updation Theorem
(C) Bayes' Theorem **(D)** Convension theorem

Ans. Option (C) is correct.

Explanation: Updation Theorem and Conversion theorem are not related to the concept to probability. In addition theorem, if A and B are any two events, then probability of happening of at least one of the event is defined by $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ Bayes' theorem is the method in which the calculated probabilities are revised with values of new probability.

Q. 2. Which of the formula is related to Bayes' theorem?

(A) $P\left(\frac{A}{B}\right) = \frac{P\left(\frac{B}{A}\right).P(A)}{P(B)}$

(B) $P\left(\frac{A}{B}\right) = \frac{P(A)}{P(B)}$

(C) $P\left(\frac{A}{B}\right) = \frac{P\left(\frac{B}{A}\right)}{P(B)}$

(D) $P\left(\frac{A}{B}\right) = \frac{1}{P(B)}$

Q. 3. In Bayes' theorem, previous probabilities are change with new available probabilities or information, are known as:

(A) Dependent probabilities
(B) Posterior probabilities
(C) Independent probabilities
(D) None of these

Q. 4. A group consists of equal number of girls and boys. Out of this group 20% of boys and 30% of the girls are unemployed. If a person is selected at random from this group, the probability of the selected person being employed is:

(A) 0.50 **(B)** 0.70
(C) 0.65 **(D)** 0.55

Ans. Option (C) is correct.

Explanation: $P(\text{girls}) = \frac{1}{2} = 0.5$

$P(\text{boys}) = \frac{1}{2} = 0.5$

$P(\text{Unemployed/boys}) = \frac{20}{100} = 0.2$

$P(\text{Unemployed/girls}) = \frac{30}{100} = 0.3$

$\therefore$ P(Unemployed) = P(boys) × P(Unemployed/boys) + P(girls) × P(Unemployed/girls)

[By Bayes' theroem]

$= 0.5 \times 0.2 + 0.5 \times 0.3 = 0.35$

$\therefore P(\text{Employed}) = 1 - 0.35 = 0.65$

Q. 5. A electric shop has two types of LED bulbs of equal quantity. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that on LED bulb chosen uniformly at random lasts more than 100 hours is:

(A) 0.75 **(B)** 0.65
(C) 0.55 **(D)** 0.45

Ans. Option (C) is correct.

Explanation:

Let $P(\text{LED of Type 1}) = P(T_1) = \frac{1}{2} = 0.5$

and $P(\text{LED of Type 2}) = P(T_2) = \frac{1}{2} = 0.5$

Also, $P\left(\frac{\text{LED lasting more than 100 hours}}{\text{LED of Type 1}}\right)$

$$= P\left(\frac{H}{T_1}\right) = 0.7$$

and $P\left(\frac{\text{LED lasting more than 100 hours}}{\text{LED of Type 2}}\right)$

$$= P\left(\frac{H}{T_2}\right) = 0.4$$

$$\therefore \quad P(H) = P(T_1) \times P\left(\frac{H}{T_1}\right) + P(T_2) \times P\left(\frac{H}{T_2}\right)$$

$$= 0.5 \times 0.7 + 0.5 \times 0.4$$
$$= 0.35 + 0.20 = 0.55$$

Very Short Answer Type Questions (1 mark each)

Q. 1. Write the rule of Total Probability R

Sol. Consider on event E which occurs via two different events A and B. The probability of E is given by the value of total probability as:

$$P(E) = P(A \cap E) + P(B \cap E)$$

$$P(E) = P(A)P\left(\frac{E}{A}\right) + P(B)\left(\frac{E}{B}\right) \quad 1$$

Q. 2. What is Bayes' theorem state?

Sol. Bayes' theorem states that the conditional probability of an event, based on the occurrence of another event, is equal to likelihood of second event given the first multiplied by the probability of the first event. 1

Q. 3. How is Bayes' theorem different from conditional probability? A I

Sol. Bayes' theorem defines the probability of an event based on the prior knowledge of the conditions related to the event whereas in case of the condition probability, we find the reverse probabilities using Bayes' theorem. **1**

Q. 4. Write the formula for the Bayes' theorem.

Sol. The formula for Bayes' theorem is:

$$P\left(\frac{A}{B}\right) = \frac{P\left(\frac{B}{A}\right).P(A)}{P(B)}$$

where $P(A)$ and $P(B)$ one the probabilities of event A and B. $P\left(\frac{B}{A}\right)$ is the probability of event A given B and $P\left(\frac{A}{B}\right)$ is the probability of event B given A. **1**

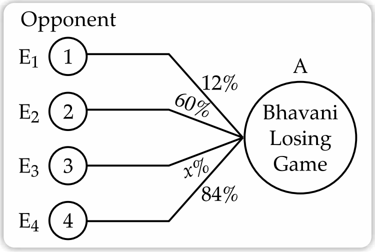
Short Answer Type Questions-I (2 marks each)

Q. 1. Bhavani is going to play a game of chess against one of four opponents in an inter-college sports competition. Each opponent is equally likely to be paired against her. The table below shows the chances of Bhavani losing, where paired against each opponent. A I

Opponent	Bhavani's Chances of Losing
Opponent 1	12%
Opponent 2	60%
Opponent 3	x%
Opponent 4	84%

If the probability that Bhavani Loses the game that day is $\frac{1}{2}$, find the probability for Bhavani to be losing when paired against opponent 3. Show your steps. **[Practice Questions 2021-22]**

Sol. Here,



Given, $P(A) = \frac{1}{2}$

Also, $P(E_1) = \frac{1}{4} = P(E_2) = P(E_3) = P(E_4)$

$$P\left(\frac{A}{E_1}\right) = 12\% = \frac{12}{100}$$

$$P\left(\frac{A}{E_2}\right) = 60\% = \frac{60}{100}$$

$$P\left(\frac{A}{E_3}\right) = x\% = \frac{x}{100}$$

$$P\left(\frac{A}{E_4}\right) = \frac{84}{100}$$ **1**

Using total probability theorem, we have

$$P(A) = P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right) + P(E_4).P\left(\frac{A}{E_4}\right)$$

$$\Rightarrow \frac{1}{2} = \frac{1}{4}\times\frac{12}{100} + \frac{1}{4}\times\frac{60}{100} + \frac{1}{4}\times\frac{x}{100} + \frac{1}{4}\times\frac{84}{100}$$

$$\Rightarrow \frac{1}{2} = \frac{12+60+x+84}{400}$$

$$\Rightarrow \frac{400}{2} = 156 + x$$

$$\Rightarrow x = 200 - 156$$

$$\Rightarrow x = 44$$

$$\therefore P\left(\frac{A}{E_3}\right) = 44\%$$ **1**

Q. 2. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. R&U **[Delhi, 2017]**

Sol. Let H_1 be the event that 6 appears on throwing a die H_2 be the event that 6 does not appear on throwing a die.

E be the event that he reports it is six.

$$P(H_1) = \frac{1}{6}, P(H_2) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$P(E/H_1) = \frac{4}{5}, P(E/H_2) = \frac{1}{5}$$ ½

$$P(H_1/E) = \frac{P(H_1)\cdot P(E/H_1)}{P(H_1).P(E/H_1) + P(H_2)P(E/H_2)}$$ ½

$$= \frac{\frac{1}{6}\times\frac{4}{5}}{\frac{1}{6}\times\frac{4}{5}+\frac{5}{6}\times\frac{1}{5}} = \frac{4}{9}$$ **1**

[CBSE Marking Scheme, 2017] (Modified)

Q. 3. Bag A contains 3 red and 2 black balls, while bag B contains 2 red and 3 black balls. A ball drawn at random from bag A is transferred to bag B and then one ball is drawn at random from bag B. If this ball was found to be a red ball, find the probability that the ball drawn from bag A was red.

R&U **[O.D. Comptt. 2017]**

Sol. Let the events be

E_1 : transferring a red ball from A to B

E_2 : transferring a black ball from A to B

A : Getting a red ball from bag B ½

$$P(E_1) = \frac{3}{5}, P(E_2) = \frac{2}{5} \qquad ½$$

$$P(A/E_1) = \frac{1}{2}, P(A/E_2) = \frac{1}{3}$$

½

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\frac{3}{5}\cdot\frac{1}{2}}{\frac{3}{5}\cdot\frac{1}{2}+\frac{2}{5}\cdot\frac{1}{3}} = \frac{9}{13} \qquad 1½$$

[CBSE Marking Scheme, 2017] (Modified)

Short Answer Type Questions-II (3 marks each)

Q. 1. There are two bags, I and II, Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black is colour, then find the probability that the transferred ball is also black.

[CBSE Delhi Set-II, 2020]

Topper Answer, 2020

Sol.

I	II
3R	4R
5B	3B

let B_1 be the event that a black ball is transferred to Bag 2.

let R_1 be the event that a red ball is transferred to Bag 2.

let B_2 be the event that a black ball is picked from Bag 2.

$P(B_1) = \frac{5}{8}$ $P(R_1) = \frac{3}{8}$ $P(B_2/B_1) = \frac{4}{8}$. $P(B_2/R_1) = \frac{3}{8}$

According to Bayes theorem

$$P(B_1|B_2) = \frac{P(B_1)P(B_2/B_1)}{P(B_1)P(B_2/B_1) + P(R_1)P(}$$

$$= \frac{\frac{5}{8}\times\frac{4}{8}}{\frac{5}{8}\times\frac{4}{8}+\frac{3}{8}\times\frac{3}{8}} = \frac{20}{20+9} = \frac{20}{29}$$

Q. 2. In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y.

A I R&U **[CBSE Delhi Set I, II, III-2020]**

Q. 3. Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator.

R&U **[CBSE OD Set I, II, III-2020]**

Sol. Let M be an event of choosing a man and N be an event of choosing a women. A be an event of choosing a good orator. ½

$$P(M) = P(W) = \frac{1}{2};$$

$$P(A|M) = \frac{5}{100} = \frac{1}{20},$$

$$P(A|W) = \frac{25}{1000} = \frac{1}{40} \qquad 1$$

$$P(A) = P(A|M).\ P(M) + P(A|W).\ P(W)$$

$$= \frac{1}{20}\times\frac{1}{2}+\frac{1}{40}\times\frac{1}{2} = \frac{3}{80} \qquad 1+½$$

[CBSE Marking Scheme 2020] (Modified)

Detailed Solution:

$$P(\text{Men good orator}) = \frac{5}{100}$$

$$P(\text{Women good orator}) = \frac{25}{1000}$$

P(of choosing good orator) = ?

Let E_1 and E_2 denote the events that the person is a good orator.

E_1 = Man orator, $P(E_1) = \frac{1}{2}$

E_2 = Woman orator, $P(E_2) = \frac{1}{2}$

A = Selecting good orator

$$P(A/E_1) = \frac{5}{100},\ P(A/E_2) = \frac{25}{1000}$$

Required Probability

$$P(A) = P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)$$

$$= \frac{1}{2} \times \frac{5}{100} + \frac{1}{2} \times \frac{25}{1000}$$

$$= \frac{3}{80}$$

Commonly Made Error

- Mostly students have a confusion whether to apply total probability theorem or Bayes' theorem.

Answering Tip

- Total probability theorem gives the total probability of an event whereas the Bayes' theorem is basically a conditional probability.

Q. 4. There are three coins, one is a two headed coin (having head on both the faces), another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows head, what is probability that it was the two headed coin?

A I R&U [CBSE SQP 2020]

Sol. Let E_1 = event of selecting a two headed coin

E_2 = event of selecting a biased coin, which shows 75% times Head

E_3 = event of selecting an unbiased coin.

A = event that tossed coin shows head. **1**

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$P\left(\frac{A}{E_1}\right) = P$(coin showing head given that it is two headed coin) ½

$= 1$

$P\left(\frac{A}{E_2}\right) = P$(coin showing head given that it is a biased coin)

$$= \frac{75}{100} = \frac{3}{4}$$ **1**

$P\left(\frac{A}{E_3}\right) = P$(coin showing head given that it is unbiased coin)

$$= \frac{1}{2}$$

By Bayes' theorem

P(getting two headed coin when it is known that it shows Head)

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + P(E_3)\,P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2}} = \frac{\frac{1}{3} \times 1}{\frac{1}{3}\left(1 + \frac{3}{4} + \frac{1}{2}\right)}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3} \times \frac{9}{4}} = \frac{4}{9}$$ **1**

Required probability $= \frac{4}{9}$

Commonly Made Error

- Students forget to define the events and apply Bayes' theorem directly and lose marks.

Answering Tip

- For Bayes' theorem and total probability theorem, events should be well defined.

Q. 5. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die ? [CBSE Delhi/OD, 2018]

Sol. E_1 : She gets 1 or 2 on die.

E_2 : She gets 3, 4, 5 or 6 on die.

A : She obtained exactly 1 tail

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3}$$ **1**

$$P(A/E_1) = \frac{3}{8}, P(A/E_2) = \frac{1}{2}$$ **1**

$$P(E_2/A) = \frac{P(E_2).P(A/E_2)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)} = \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{8}{11}$$ **1**

[CBSE Marking Scheme, 2018] (Modified)

OR

Topper Answer, 2018

Sol. Let E_1 = "Event that the girl threw 3, 4, 5 or 6"

E_2 = "Event that the girl threw 1, 2."

and A = "Event that the girl get exactly one tail"

$$P(E_1) = \frac{4}{6} = \frac{2}{3}, \quad P(E_2) = \frac{2}{6} = \frac{1}{3}$$

$$P(A/E_1) = \frac{1}{2}, \quad P(A/E_2) = 3/8$$

Now, $$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

$$P(E_1/A) = \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{2}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{3}{8}}$$

$$P(E_1/A) = \frac{\frac{2}{6}}{\frac{2}{6}+\frac{1}{8}} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{8}} = \frac{\frac{1}{3}}{\frac{8+3}{24}}$$

$$P(E_1/A) = \frac{1}{3}\times\frac{24}{11} = \boxed{\frac{8}{11}}$$ Ans.

Q. 6. Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they came from Bag III. R&U **[SQP 2017-18]**

Sol. Let E_1 = Bag I is chosen, E_2 = Bag II is chosen, E_3 = Bag III is chosen, A = The two balls drawn from the chosen bag are white and red.

$$P(E_1) = \frac{1}{3} = P(E_2) = P(E_3),$$

$$P(A/E_1) = \frac{1}{6}\times\frac{3}{6}\times 2, \; P(A/E_2) = \frac{2}{4}\times\frac{1}{4}\times 2,$$

$$P(A/E_3) = \frac{4}{9}\times\frac{2}{9}\times 2.$$ **1**

By Bayes' Theorem, the required probability =

$$P(E_3/A) = \frac{P(E_3)\times P(A/E_3)}{\sum_{i=1}^{3} P(E_i)\times P(A/E_i)}$$

$$= \frac{\frac{1}{3}\times\frac{4}{9}\times\frac{2}{9}\times 2}{\frac{1}{3}\times\frac{1}{6}\times\frac{3}{6}\times 2+\frac{1}{3}\times\frac{2}{4}\times\frac{1}{4}\times 2+\frac{1}{3}\times\frac{4}{9}\times\frac{2}{9}\times 2}$$

$$= \frac{\frac{16}{243}}{\frac{1}{18}+\frac{1}{12}+\frac{16}{243}}$$

$$= \frac{\frac{16}{243}}{\frac{108+162+128}{1944}} = \frac{16}{243}\times\frac{1944}{398} = \frac{64}{199}.$$ **2**

[CBSE Marking Scheme, 2018] (Modified)

Q. 7. **Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance ?** R&U [CBSE 2017]

Long Answer Type Questions (5 marks each)

Q. 1. ***A*** **manufacturer has three machine operators** ***A*****,** ***B*** **and** ***C*****. The first operator** ***A*** **produces 1% of defective items, whereas the other two operators** ***B*** **and** ***C*** **produces 5% and 7% defective items respectively.** ***A*** **is on the job for 50% of the time,** ***B*** **on the job 30% of the time and** ***C*** **on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by** ***A*** **?**

A I R&U [CBSE Delhi Set-I, 2019]

Sol. Let $\left.\begin{array}{l} E_1: \text{item is produced by } A \\ E_2: \text{item is produced by } B \\ E_3: \text{item is produced by } C \end{array}\right\}$ **1**

$$P(E_1)=\frac{50}{100}, P(E_2)=\frac{30}{100}, P(E_3)=\frac{20}{100} \quad \mathbf{1}$$

$$P(A/E_1)=\frac{1}{100},\ P(A/E_2)=\frac{5}{100},$$
$$P(A/E_3)=\frac{7}{100} \quad \mathbf{1}$$

$$P(E_1/A)=\frac{\frac{50}{100}\times\frac{1}{100}}{\frac{50}{100}\times\frac{1}{100}+\frac{30}{100}\times\frac{5}{100}+\frac{20}{100}\times\frac{7}{100}} \quad \mathbf{1}$$

$$=\frac{5}{34} \quad \mathbf{1}$$

[CBSE Marking Scheme, 2019] (Modified)

Topper Answer, 2019

Sol. A = ~~the~~ Event of choosing a defective item.

E_1 = Item produced by A

E_2 = Item produced by B

E_3 = Item produced by C.

$P(E_1) = \frac{50}{100} = \frac{1}{2}$, $P(E_2) = \frac{30}{100} = \frac{3}{10}$, $P(E_3) = \frac{20}{100} = \frac{1}{5}$

$P\left(\frac{A}{E_1}\right) = \frac{1}{100}$, $P\left(\frac{A}{E_2}\right) = \frac{5}{100} = \frac{5}{100}$; $P\left(\frac{A}{E_3}\right) = \frac{7}{100}$.

Using Baye's Thm,

$$A\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3).P(A/E_3)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{100}}{\frac{1}{2}\times\frac{1}{100}+\frac{3}{10}\times\frac{5}{100}+\frac{1}{5}\times\frac{7}{100}}$$

$$= \frac{\frac{1}{200}}{\frac{1}{200}+\frac{3}{200}+\frac{7}{500}} = \frac{\frac{1}{2}}{\frac{1}{2}+\frac{3}{2}+\frac{7}{5}} = \frac{\frac{1}{2}}{2+\frac{7}{5}} = \frac{\frac{1}{2}}{\frac{17}{5}}$$

$\boxed{P(E_1/A) = \frac{5}{34}}$ = Probability that defective item was produced by A

A I R&U [SQP 2018-19]

Q. 2. **The members of a consulting firm rent cars from three rental agencies: 50% from agency X, 30% from agency Y and 20% from agency Z. From past experience it is known that 9% of the cars from agency X need a service and tuning before renting, 12% of the cars from agency Y need a service and tuning before renting and 10% of the cars Z and a service and tuning before renting. If the rental car delivered to the firm needs service and tuning, find the probability that agency Z is not be blamed.**

Q. 3. **A bag** ***I*** **contains 5 red and 4 white balls and a bag** ***II*** **contains 3 red and 3 white balls. Two balls are transferred from the bag** ***I*** **to the bag** ***II*** **and then one ball is drawn from bag II. If the ball drawn from the bag** ***II*** **is red, then find the probability that one red ball and one white ball are transferred from the bag** ***I*** **to the bag** ***II*****.** R&U [SQP 2015-16]

Sol. Let, E_1 : Two white balls are transferred

E_2 : Two red balls are transferred

E_3 : One red and one white ball are transferred.

A : The ball drawn from the bag II is red. ½

$$P(E_1) = \frac{{}^4C_2}{{}^9C_2} = \frac{4\times3}{9\times8} = \frac{1}{6} \quad \mathbf{1}$$

$$P(E_2) = \frac{{}^5C_2}{{}^9C_2} = \frac{4\times5}{9\times8} = \frac{5}{18} \quad \mathbf{1}$$

$$P(E_3) = \frac{{}^5C_1\times{}^4C_1}{{}^9C_2} = \frac{4\times5\times2}{9\times8} = \frac{5}{9} \quad \mathbf{1}$$

$$P(A/E_1) = \frac{3}{8},\ P(A/E_2) = \frac{5}{8},$$

$$P(A/E_3) = \frac{4}{8} \quad ½$$

The required probability, $P(E_3/A)$, by Bayes' Theorem

$$= \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)\cdot P(A/E_1)+P(E_2)\cdot P(A/E_2)+P(E_3)\cdot P(A/E_3)}$$

$$= \frac{\frac{5}{9}\times\frac{4}{8}}{\frac{1}{6}\times\frac{3}{8}+\frac{5}{18}\times\frac{5}{8}+\frac{5}{9}\times\frac{4}{8}}$$

$$= \frac{20}{37} \quad \mathbf{1}$$

[CBSE Marking Scheme 2016] (Modified)

Q. 4. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{3}$ be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability $\frac{1}{3}$, what is the probability that the student knows the answer given that he answered it correctly ?

A I R&U [O.D. Comptt. 2015]

Sol. Let, E_1 : Student knows the answer

E_2 : Student guesses the answer

A : Answers correctly **1**

$$P(E_1) = \frac{3}{5},\ P(E_2) = \frac{2}{5} \quad \mathbf{1}$$

$$\therefore\ P(A/E_1) = 1,\ P(A/E_2) = \frac{1}{3} \quad \mathbf{1}$$

$$\therefore\ P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)} \quad \mathbf{1}$$

$$= \frac{\frac{3}{5}\cdot1}{\frac{3}{5}\cdot1+\frac{2}{5}\cdot\frac{1}{3}}$$

$$= \frac{9}{11} \quad \mathbf{1}$$

[CBSE Marking Scheme 2015] (Modified)

Q. 5. An urn contains 4 balls. Two balls are drawn at random from the urn (without replacement) and are found to be white. What is the probability that all the four balls in the urn are white ? [O.D. 2016]

Sol. Let E_1 : urn has 2 white balls

E_2 : urn has 3 white balls

E_3 : urn has 4 white balls

A : 2 balls drawn are white **1**

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3} \quad \mathbf{1}$$

$$P(A/E_1) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6},$$

$$P(A/E_2) = \frac{{}^3C_2}{{}^4C_2} = \frac{1}{2},$$

$$P(A/E_3) = \frac{{}^4C_2}{{}^4C_2} = 1 \quad \mathbf{1}$$

$\therefore\ P(E_3/A)$

$$= \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)} \quad \mathbf{1}$$

$$= \frac{\frac{1}{3}\cdot1}{\frac{1}{3}\cdot\frac{1}{6}+\frac{1}{3}\cdot\frac{1}{2}+\frac{1}{3}\cdot1}$$

$$= \frac{6}{10} = 0\cdot6 \quad \mathbf{1}$$

[CBSE Marking Scheme 2016] (Modified)

Topic-3 Random Variable and its Probability Distributions

Concepts Covered • *Random Variable,* • *Probability Distribution*

Revision Notes

1. RANDOM VARIABLE :

A random variable is a real valued function defined over the sample space of an experiment. In other words, a random variable is a real-valued function whose domain is the sample space of a random experiment. A random variable is usually denoted by uppercase letters X, Y, Z etc.

2. PROBABILITY DISTRIBUTION OF A RANDOM VARIABLE :

If the values of a random variable together with the corresponding probabilities are given, then this description is called a probability distribution of the random variable.

Key Terms

- **Discrete random variable :** It is a random variable which can take only finite or countable infinite number of values.
- **Continuous random variable :** A variable which can take any value between two given limits is called a continuous random variable.

Key Formulae

- Mean or Expectation of a random variable $X = \mu = \sum_{i=1}^{n} x_i P_i$

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions

Q. 1. Let a dice has property that the probability of a face with n dots showing up is proportional to n, then probability of face showing five dots is:

(A) $\frac{1}{7}$ (B) $\frac{1}{21}$

(C) $\frac{5}{21}$ (D) $\frac{6}{21}$

Q. 2. The probability distribution of a random variable X is given below:

X	0	1	2	3
P(X)	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

The value of k is:

(A) $\frac{1}{15}$ (B) $\frac{2}{15}$

(C) $\frac{6}{15}$ (D) $\frac{8}{15}$

Ans. Option (D) is correct.

Explanation: We know that:

$$\Sigma p_i = 1$$

$$\therefore \quad k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} = 1$$

$$\Rightarrow \quad \frac{15}{8}k = 1$$

$$\Rightarrow \quad k = \frac{8}{15}$$

Q. 3. For the probability distribution given in Q.3, find P(X ≤ 2).

(A) $\frac{11}{15}$ (B) $\frac{14}{15}$

(C) $\frac{17}{15}$ (D) $\frac{1}{15}$

Ans. Option (B) is correct.

Explanation: $P(X \le 2) = P(0) + P(1) + P(2)$

$$= \frac{8}{15} + \frac{8}{15} \times \frac{1}{2} + \frac{8}{15} \times \frac{1}{4}$$

$$= \frac{8}{15}\left(1+\frac{1}{2}+\frac{1}{4}\right)$$

$$= \frac{8}{15}\left(\frac{4+2+1}{4}\right)$$

$$= \frac{14}{15}$$

Q. 4. If z is a random variable defined as the number of heads minus number of tails in a simultaneous toss of 6 coins, the all possible values of Z are:

(A) {–6, –4, –2, 0, 2, 4, 6}
(B) {0, 2, 4, 6}
(C) {–6, –4, –2, 0, 2}
(D) {–4, –2, 0, 2, 4}

Ans. Option (A) is correct.

Explanation: Here, Z = no. of heads – no. of tails

No. of heads	No. of tails	Values of z
0	6	0 – 6 = –6
1	5	1 – 5 = –4
2	4	2 – 4 = –2
3	3	3 – 3 = 0
4	2	4 – 2 = 2
5	1	5 – 1 = 4
6	0	6 – 0 = 6

Q. 5. If a coin is tossed twice and z is a random variable as the no. of heads minus no. of tails, the probability distribution is:

(A)

–2	0	2
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(B)

–2	0	2
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$

(C)

–2	0	2
1	$\frac{1}{2}$	$\frac{1}{4}$

(D)

–2	0	2
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$

Ans. Option (A) is correct.

Explanation: Here, Z = no. of heads – no. of tails

No. of heads	No. of tails	Values of z
0	2	0 – 2 = –2
1	1	1 – 1 = 0
2	0	2 – 0 = 2

Now, $P(Z = -2) = P(\text{no head}) = P(TT) = \frac{1}{4}$

$P(Z = 0) = P(\text{one head}) = P(HT, TH)$
$= \frac{2}{4} = \frac{1}{2}$

$P(Z = 2) = P(\text{two heads}) = P(HH)$
$= \frac{1}{4}$

∴ Required probability distribution is:

X	–2	0	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Very Short Answer Type Questions (1 mark each)

Direction (Q.1 to Q. 5) State whether it is possible for a random variable to have any of the following probability distribution:

Q. 1.

X	0	1	2
P(X)	0.4	0.4	0.2

U

Q. 2.

Z	–1	0	1
P(Z)	0.3	0.4	0.3

A I

Sol. Here, we observe that $0 \le P(Z) \le 1 \ \forall\ Z$
and $\Sigma P(Z) = 0.3 + 0.4 + 0.3 = 1$
Thus, the given probability distribution is valid. **1**

Q. 3.

Y	1	2	3	4
P(Y)	0.2	0.3	0.2	0.2

A I

Sol. Here, we observe that $0 \le P(Y) \le 1 \ \forall\ Y$
But $\Sigma P(Y) = 0.2 + 0.3 + 0.2 + 0.3$
$= 0.9 \ne 1$
Hence, it is not possible for a random variable to have this distribution. **1**

Q. 4.

X	0	1	2	3
P(X)	0.3	–0.1	0.4	0.4

U

Sol. Here, we observe that P(X = 1) = –0.1. As the probability cannot be negative, it is not possible for a random variable to have the given distribution.

Q. 5. $P(x_i) = \left(\frac{1}{2}\right)^i$**, where i = 1, 2, 3 .... ∞** A I

Sol. Given, $P(x_i) = \left(\frac{1}{2}\right)^i$ for i = 1, 2, 3 ....

Thus, the probabilities are:
$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, ...$

All these probabilities lies between 0 to 1.

Also, $\Sigma P(X) = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}, ...$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1 \qquad \textbf{1}$$

[$\because$ Sum of infinite geometric series $a + ar + ar^2 +$ ..... is $S = \frac{a}{1-r}$]

Q. 6. A random variable X has following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	a	$4a$	$3a$	$7a$	$8a$	$10a$	$6a$	$9a$

Determine the value of a.

Q. 7. Find the mean of the given probability distribution:

A I

X	2	3	4
P(X)	0.2	0.5	0.3

Sol. Here,

x_i	p_i	p_ix_i
2	0.2	0.4
3	0.5	1.5
4	0.3	1.2
Total		**3.1**

$\therefore$ Mean $= \mu = \Sigma p_ix_i = 3.1$ **1**

Short Answer Type Questions-I (2 marks each)

Q. 1. A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement. A I [SQP 2021-22]

Sol. Let X be the random variable defined as the number of red balls.

Then X = 0, 1 **1**

$$P(X = 0) = \frac{3}{4}.\frac{2}{3} = \frac{6}{12} = \frac{1}{2} \qquad ½$$

$$P(X = 1) = \frac{1}{4}.\frac{3}{3}+\frac{3}{4}.\frac{1}{3} = \frac{6}{12} = \frac{1}{2} \qquad ½$$

Probability distribution Table:

X	0	1
P(X)	$\frac{1}{2}$	$\frac{1}{2}$

½

Q. 2. A refrigerator box contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random. Find the probability distribution of the number of milk chocolates. What is the most likely outcome? A I R&U [CBSE SQP 2020-21]

Sol. Let X denote the number of milk chocolates drawn

X	$P(X)$
0	$\frac{4}{6}\times\frac{3}{5}=\frac{12}{30}$
1	$\left(\frac{2}{6}\times\frac{4}{5}\right)\times 2=\frac{16}{30}$
2	$\frac{2}{6}\times\frac{1}{5}=\frac{2}{30}$

1½

Most likely outcome is getting one chocolate of each type. ½

[CBSE SQP Marking Scheme 2020-21]

Commonly Made Error

- Students get confused whether the items are drawn with replacement or without replacement.

Answering Tip

- At random means without replacement.

Q. 3. The random variable X has a probability distribution $P(X)$ of the following form, where 'k' is some number.

$$P(X = x) = \begin{cases} k, \text{ if } x = 0 \\ 2k, \text{ if } x = 1 \\ 3k, \text{ if } x = 2 \\ 0, \text{ otherwise} \end{cases}$$

Determine the value of 'k'.

R&U [CBSE Delhi Set-III, 2019]

Q. 4. Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.

R&U [CBSE OD Set-I, 2019]

Sol. X = No. of heads in simultaneous toss of two coins.

X :	0	1	2	½
$P(x)$:	1/4	1/2	1/4	1½

[CBSE Marking Scheme, 2019]

Detailed Solution :

Let X be the number of heads.

Possible values of X are 0, 1, 2

$P(X=0)=\frac{1}{4}$, $P(X=1)=\frac{1}{2}$, $P(X=2)=\frac{1}{4}$. The probability distribution of X is :

X	0	1	2
$P(X)$	1/4	1/2	1/4

Short Answer Type Questions-II (3 marks each)

Q. 1. Find the mean of the number of doublets in three throws of a pair of dice.

Sol. Let X represent the random variable,

then $X = 0, 1, 2, 3$

$$p = \frac{1}{6}, q = 1 - p$$

$$q = \frac{5}{6} \qquad \text{½}$$

$$P(X=0) = P(r=0) = {}^3C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^3 = \frac{125}{216}$$

$$P(X=1) = P(r=1) = {}^3C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2 = \frac{75}{216}$$

$$P(X=2) = P(r=2) = {}^3C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1 = \frac{15}{216}$$

$$P(X=3) = P(r=3) = {}^3C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^0 = \frac{1}{216}$$

X_i	P_i	X_iP_i
0	$\frac{125}{216}$	0
1	$\frac{75}{216}$	$\frac{75}{216}$
2	$\frac{15}{216}$	$\frac{30}{216}$
3	$\frac{1}{216}$	$\frac{3}{216}$
Total		$\frac{1}{2}$

2

$$\text{Mean} = \Sigma X_i P_i = \frac{1}{2} \qquad \text{½}$$

Q. 2. A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence find the mean of the number of tails.

A I R&U [CBSE OD Set I, II, III-2020]

Sol. $P(\text{Head}) = \frac{3}{4}$, $P(\text{Tail}) = \frac{1}{4}$ **1**

Let X = number of tails. Clearly X can be 0, 1, 2 ½

Probability distribution is given by

X	0	1	2
$P(X)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

1

$$\text{Mean} = \sum X.P(X) = \frac{1}{2} \qquad \text{½}$$

[CBSE Marking Scheme 2020] (Modified)

Detailed Solution:

$$P(\text{Head}) = \frac{3}{4}, P(\text{Tail}) = \frac{1}{4}$$

Let X be the number of tails in two tosses :

X	0	1	2
	$\frac{3}{4}\times\frac{3}{4}$	$2\times\frac{3}{4}\times\frac{1}{4}$	$\frac{1}{4}\times\frac{1}{4}$
$P(X)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

$$\text{Mean} = \Sigma XP(X)$$

$$= 0\times\frac{9}{16}+1\times\frac{6}{16}+2\times\frac{1}{16}$$

$$= 0+\frac{6}{16}+\frac{2}{16}$$

$$= \frac{8}{16} = \frac{1}{2}$$

Commonly Made Error

- Mostly students have no idea in finding the probability of head and tail in case of biased coins.

Answering Tip

- Practice more problems with biased coins and dice.

Q. 3. Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples.

R&U [CBSE Delhi Set I, II, III-2020]

Q. 4. Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X. Also, find mean of the distribution.

AI R&U [CBSE SQP-2020]

Sol. Let X denotes the smaller of the two numbers obtained

So X can take values 1, 2, 3, 4, 5, 6 ½

P(X = 1 is smaller number)

$$P(X=1) = \frac{6}{^7C_2} = \frac{6}{21} = \frac{2}{7}$$

(Total cases when two numbers can be selected from first 7 numbers are $^7C_2$)

$$P(X=2) = \frac{5}{^7C_2} = \frac{5}{21}$$

$$P(X=3) = \frac{4}{^7C_2} = \frac{4}{21}$$

$$P(X=4) = \frac{3}{^7C_2} = \frac{3}{21} = \frac{1}{7}$$

$$P(X=5) = \frac{2}{^7C_2} = \frac{2}{21}$$

$$P(X=6) = \frac{1}{^7C_2} = \frac{1}{21}$$ 1

x_i	p_i	p_ix_i
1	$\frac{6}{21}$	$\frac{6}{21}$
2	$\frac{5}{21}$	$\frac{10}{21}$
3	$\frac{4}{21}$	$\frac{12}{21}$
4	$\frac{3}{21}$	$\frac{12}{21}$
5	$\frac{2}{21}$	$\frac{10}{21}$
6	$\frac{1}{21}$	$\frac{6}{21}$

½

$$\text{Mean} = \Sigma p_ix_i = \frac{6}{21}+\frac{10}{21}+\frac{12}{21}+\frac{12}{21}+\frac{10}{21}+\frac{6}{21}$$

$$= \frac{56}{21} = \frac{8}{3}$$ 1

[CBSE SQP Marking Scheme, 2020] (Modified)

Q. 5. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean of X.

R&U [CBSE Delhi/OD 2018]

Topper Answer, 2018

Sol. Let X denote the larger of the two numbers.

$X = 2, 3, 4, 5$

$P(X=2) = \frac{2}{20} = \frac{1}{10}$

$P(X=3) = \frac{4}{20} = \frac{2}{10}$

$P(X=4) = \frac{6}{20} = \frac{3}{10}$

$P(X=5) = \frac{8}{20} = \frac{4}{10}$

Probability Distribution :-

X	2	3	4	5
P(X)	1/10	2/10	3/10	4/10

Mean

$E(X) = \Sigma p_ix_i$

$\Sigma p_ix_i = \frac{2\times 1}{10} + \frac{3\times 2}{10} + \frac{4\times 3}{10} + \frac{5\times 4}{10}$

$\Sigma p_ix_i = \frac{2}{10} + \frac{6}{10} + \frac{12}{10} + \frac{20}{10} = \frac{40}{10} = \boxed{4}$ ans

Long Answer Type Questions (5 marks each)

Q. 1. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean of the number of kings.

A I R&U [CBSE Delhi Set I-2019]

Sol. X = no. of kings = 0, 1, 2 ½

$$P(X=0) = P(\text{no king}) = \frac{48}{52}\times\frac{47}{51} = \frac{188}{221}$$ ½

$P(X = 1) = P$(one king and one non-king)

$$= \frac{4}{52}\times\frac{48}{51}\times 2 = \frac{32}{221}$$ ½

$$P(X = 2) = P(\text{two kings}) = \frac{4}{52}\times\frac{3}{51} = \frac{1}{221}$$ ½

Probability distribution is given by 2

X	0	1	2
$P(X)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Thus, Mean $= \Sigma X \cdot P(X) = \frac{34}{221}$ or $\frac{2}{13}$ 1

Q. 2. Three numbers are selected at random (without replacement) from first six positive integers. Let X denote the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean of the distribution.

R&U [CBSE 2016] [NCERT]

Sol. The variance X takes values 3, 4, 5 and 6 1

$$P(X = 3) = \frac{1}{20};\ P(X = 4) = \frac{3}{20};$$ ½

$$P(X = 5) = \frac{6}{20};\ P(X = 6) = \frac{10}{20};$$ ½

X	**3**	**4**	**5**	**6**	
P(X)	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{10}{20}$	**2**
XP(X)	$\frac{3}{20}$	$\frac{12}{20}$	$\frac{30}{20}$	$\frac{60}{20}$	**1**

$$\text{Mean} = \Sigma XP(X) = \frac{105}{20} = \frac{21}{4}$$ 1

Q. 3. Three numbers are selected at random (without replacement) from first six positive integers. If X denotes the smallest of the three numbers obtained, find the probability distribution of X. Also find the mean and variance of the distribution.

R&U [O.D. Set II 2016]

Topper Answer, 2016

Sol.

X:	1	2	3	4	5	6
P(x:)	$\frac{10}{20} = \frac{1}{2}$	$\frac{6}{20} = \frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$	0	0

(as 5 & 6 can't be minimum values)

$P(x=1) = \frac{{}^5C_2}{{}^6C_3} = \frac{10}{20}$ (Numbers from 2-6 can be chosen) (Any two)

$P(x=2) = \frac{{}^4C_2}{{}^6C_3} = \frac{6}{20}$ (Numbers from 3-6 can be chosen) (Any two)

$P(x=3) = \frac{{}^3C_2}{{}^6C_3} = \frac{3}{20}$ (Any 2 out of 4, 5, 6 can be chosen)

$P(x:2) = \frac{{}^2C_2}{{}^6C_3} = \frac{1}{20}$ only 5, 6 can be chosen

Mean = E(x) = $\Sigma x_i p_i$

$= \frac{10}{20}\times 1 + \frac{6}{20}\times 2 + \frac{3}{20}\times 3 + \frac{1}{20}\times 4 + 0 + 0$

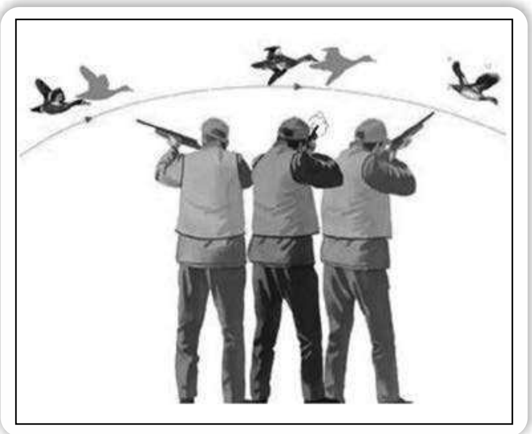
$= \frac{10}{20} + \frac{12}{20} + \frac{9}{20} + \frac{4}{20} = \frac{35}{20} = 1.75$

COMPETENCY BASED QUESTIONS (4 marks each)

Case based MCQs

I. Read the following text and answer any four questions from Q.1 to Q.5.

A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots. [CBSE QB 2021]



Q. 1. Let the target is hit by A, B, and C. Then, the probability that A, B and, C all will hit, is

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$

(C) $\frac{2}{5}$ (D) $\frac{1}{5}$

Ans. Option (C) is correct.

Explanation:

$$P(A) = \frac{4}{5},\ P(B) = \frac{3}{4},\ P(C) = \frac{2}{3}$$

Probability that A, B and C all will hit the target

$$= P(A \cap B \cap C)$$
$$= P(A)P(B)P(C)$$
$$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$$
$$= \frac{2}{5}$$

Q. 2. What is the probability that B, C will hit and A will lose?

(A) $\frac{1}{10}$ (B) $\frac{3}{10}$

(C) $\frac{7}{10}$ (D) $\frac{4}{10}$

Ans. Option (A) is correct.

Explanation:

$$P(\bar{A}) = 1 - \frac{4}{5}$$
$$= \frac{1}{5}$$

Probability that B, C will hit and A will lose

$$= P(\bar{A} \cap B \cap C)$$
$$= P(\bar{A}).P(B).P(C)$$
$$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3}$$
$$= \frac{1}{10}$$

Q. 3. What is the probability that 'any two of A, B and C will hit?

(A) $\frac{1}{30}$ (B) $\frac{11}{30}$

(C) $\frac{17}{30}$ (D) $\frac{13}{30}$

Ans. Option (D) is correct.

Explanation:

$$P(\bar{B}) = 1 - \frac{3}{4}$$
$$= \frac{1}{4},$$
$$P(\bar{C}) = 1 - \frac{2}{3}$$
$$= \frac{1}{3}$$

Probability that any two of A, B and C will hit

$$= P(\bar{A})P(B)P(C) + P(A)P(\bar{B})P(C) + P(A)P(B)P(\bar{C})$$
$$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}$$
$$= \frac{1}{10} + \frac{2}{15} + \frac{1}{5}$$
$$= \frac{3+4+6}{30}$$
$$= \frac{13}{30}$$

Q. 4. What is the probability that 'none of them will hit the target'?

(A) $\frac{1}{30}$ (B) $\frac{1}{60}$

(C) $\frac{1}{15}$ (D) $\frac{2}{15}$

Ans. Option (B) is correct.

Explanation: Probability that none of them will hit the target

$$= P(\bar{A} \cap \bar{B} \cap \bar{C})$$
$$= P(\bar{A}).P(\bar{B}).P(\bar{C})$$
$$= \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3}$$
$$= \frac{1}{60}$$

Q. 5. What is the probability that at least one of A, B or C will hit the target?

(A) $\frac{59}{60}$ (B) $\frac{2}{5}$

(C) $\frac{3}{5}$ (D) $\frac{1}{60}$

Ans. Option (A) is correct.

II. Read the following text and answer any four questions from Q.1 to Q.5.

The reliability of a COVID PCR test is specified as follows:

Of people having COVID, 90% of the test detects the disease but 10% goes undetected. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her as COVID positive. **[CBSE QB 2021]**

Q. 1. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID'?

(A) 0.001 **(B)** 0.1

(C) 0.8 **(D)** 0.9

Ans. Option (D) is correct.

Explanation:

E = Person selected has COVID

F = Does not have COVID

G = Test judge COVID positive

Probability of the person to be tested as COVID positive given that he is actually having COVID

$$= P(G/E) = 90\% = \frac{90}{100} = 0.9$$

Q. 2. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually not having COVID'?

(A) 0.01 **(B)** 0.99

(C) 0.1 **(D)** 0.001

Ans. Option (A) is correct.

Explanation: Probability of person to be tested as COVID positive given that he is actually not having COVID

$$= P(G/E) = 1\% = \frac{1}{100} = 0.01$$

Q. 3. What is the probability that the 'person is actually not having COVID'?

(A) 0.998 **(B)** 0.999

(C) 0.001 **(D)** 0.111

Ans. Option (B) is correct.

Explanation:

$$P(E) = 1 - P(E)$$

$$= 1 - 0.001 \quad \left[\because P(E) = 0.1\% = \frac{0.1}{100} = 0.001\right]$$

$$= 0.999$$

Q. 4. What is the probability that the 'person is actually having COVID given that 'he is tested as COVID positive'?

(A) 0.83 **(B)** 0.0803

(C) 0.083 **(D)** 0.089

Ans. Option (C) is correct.

Explanation:

$$P(E/G) = \frac{0.001 \times 0.9}{0.001 \times 0.9 + 0.999 \times 0.01}$$

$$= \frac{9 \times 10^{-4}}{9 \times 10^{-4} + 99.9 \times 10^{-4}}$$

$$= \frac{9 \times 10^{-4}}{10^{-4}(9 + 99.9)}$$

$$= \frac{9}{108.9}$$

$$= 0.083 \text{ (approx)}$$

Q. 5. What is the probability that the 'person selected will be diagnosed as COVID positive'?

(A) 0.1089 **(B)** 0.01089

(C) 0.0189 **(D)** 0.189

Ans. Option (B) is correct.

III. Read the following text and answer any four questions from Q.1 to Q.5.

In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1, E_2, E be the events that the student knows the answer, guesses the answer and answers correctly respectively. **[CBSE QB 2021]**

Q. 1. What is the value of $P(E_1)$?

(A) $\frac{2}{5}$ **(B)** $\frac{1}{3}$

(C) 1 **(D)** $\frac{3}{5}$

Ans. Option (D) is correct.

Q. 2. Value of $P(E|E_1)$ is

(A) $\frac{1}{3}$ **(B)** 1

(C) $\frac{2}{3}$ **(D)** $\frac{4}{5}$

Ans. Option (B) is correct.

Explanation: $P(E / E_1) = 1$

Q. 3. $\sum_{k=1}^{k=2} P(E \mid E_k)\, P(E_k)$ Equals

(A) $\frac{11}{15}$ **(B)** $\frac{4}{15}$

(C) $\frac{1}{5}$ **(D)** 1

Ans. Option (A) is correct.

Explanation:

$$\sum_{k=1}^{k=2} P(E \mid E_k)P(E_K) = P(E \mid E_1)P(E_1) + P(E \mid E_2)\,P(E_2)$$

$$= 1 \times \frac{3}{5} + \frac{1}{3} \times \frac{2}{5}$$

$$= \frac{11}{15}$$

Q. 4. Value of $\sum_{k=1}^{k=2} P(E_k)$

(A) $\frac{1}{3}$ **(B)** $\frac{1}{5}$

(C) 1 **(D)** $\frac{3}{5}$

Ans. Option (C) is correct.

Explanation:

$$\sum_{k=1}^{k=2} P(E_k) = P(E_1) + P(E_2)$$

$$= \frac{3}{5} + \frac{2}{5}$$

$$= \frac{5}{5}$$

$$= 1$$

Q. 5. What is the probability that the student knows the answer given that he answered it correctly?

(A) $\frac{2}{11}$ **(B)** $\frac{5}{3}$

(C) $\frac{9}{11}$ **(D)** $\frac{13}{3}$

Ans. Option (C) is correct.

Explanation:

$$P(E \mid E_1) = \frac{P(E_1).P(E_1 \mid E)}{P(E_1).P(E_1 \mid E) + P(E_2)P(E \mid E_2)}$$

$$= \frac{\frac{3}{5} \times 1}{\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{3}}$$

$$= \frac{\frac{3}{5}}{\frac{11}{15}}$$

$$= \frac{9}{11}$$

IV. Read the following text and answer any four questions from Q.1 to Q.5.

In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

[CBSE QB 2021]

Q. 1. The conditional probability that an error is committed in processing given that Sonia processed the form is:

(A) 0.0210 **(B)** 0.04

(C) 0.47 **(D)** 0.06

Ans. Option (B) is correct.

Q. 2. The probability that Sonia processed the form and committed an error is:

(A) 0.005 **(B)** 0.006

(C) 0.008 **(D)** 0.68

Ans. Option (C) is correct.

Explanation:

P (sonia processed the form and committed an error) = 20% × 0.4

$$= \frac{20}{100} \times 0.04$$

$$= \frac{1}{5} \times 0.04$$

$$= 0.008$$

Q. 3. The total probability of committing an error in processing the form is:

(A) 0 **(B)** 0.047

(C) 0.234 **(D)** 1

Ans. Option (B) is correct.

Explanation: Required Probability = P(Vinay processes the form)P(Vinay's error rate) + P(Sonia processes the form)P(Sonia's error rate) + P(Iqbal processes the rate) P(Iqbal's error rate)

$$= \frac{50}{100} \times 0.06 + \frac{20}{100} \times 0.04 + \frac{30}{100} \times 0.03$$

$$= \frac{470}{10000} = 0.047$$

Q. 4. The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is:

(A) 1 **(B)** $\frac{30}{47}$

(C) $\frac{20}{47}$ **(D)** $\frac{17}{47}$

Ans. Option (D) is correct.

Explanation: P(Error is committed in processing given that Vinay processed the form)

$$= \frac{\frac{5}{10} \times 0.06}{\frac{5}{10} \times 0.06} + \frac{2}{10} \times 0.04 + \frac{3}{10} \times 0.03$$

$$= \frac{30}{47}$$

P(Form has an error and it was not processed by Vinay)

$$= 1 - \frac{30}{47}$$

$$= \frac{17}{47}$$

Q. 5. Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i | A)$ is:

(A) 0 **(B)** 0.03

(C) 0.06 **(D)** 1

Ans. Option (D) is correct.

Explanation:

$$\sum_{i=1}^{3} P\left(\frac{E_i}{A}\right) = P\left(\frac{E_1}{A}\right) + P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right) = 1$$

[$\because$ sum of all occurrence of an event is equal to 1]

Case based Subjective Questions (2 mark each)

I. Read the following text and answer the questions.

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

- 960 of the total applications were the folk genre.
- 192 of the folk applications were for the below 18 category.
- 104 of the classical applications were for the 18 and above category.

Q. 1. What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work. 2

[CBSE Practice Paper 2021-22]

Sol. According to given information, we construct the following table.

Given, total applications = 2000

	Folk Genre	Classical Genre
	960 (given)	2000 – 960 = 1040
Below 18	192 (given)	1040 – 104 = 936
18 or Above 18	960 – 192 = 768	104 (given)

1

Let E_1 = Event that application for folk genre

E_2 = Event that application for classical genre

A = Event that application for below 18

B = Event that application for 18 or above 18

$$\therefore \quad P(E_2) = \frac{1040}{2000}$$

and $$P(B \cap E_2) = \frac{104}{2000}$$

$$\text{Required Probability} = \frac{P(B \cap E_2)}{P(E_2)}$$

$$= \frac{\frac{104}{2000}}{\frac{1040}{200}} = \frac{1}{10}$$ **1**

Q. 2. An application selected at random is found to be under the below 18 category. Find the probability that it is under the folk genre. Show your work. 2

Sol. Required probability $= P\left(\frac{\text{folk}}{\text{below 18}}\right)$

$$= P\left(\frac{E_1}{A}\right)$$

$$= \frac{P(E_1 \cap A)}{P(A)}$$ **1**

Now, $P(E_1 \cap A) = \frac{192}{2000}$

and $P(A) = \frac{192+936}{2000} = \frac{1128}{2000}$

$\therefore$ Required probability $= \dfrac{\frac{192}{2000}}{\frac{1128}{2000}} = \frac{192}{1128} = \frac{8}{47}$ **1**

Solutions for Practice Questions (Topic-1)

Very Short Answer Type Questions

1. $1-\frac{2}{3}\times\frac{3}{4}=\frac{1}{2}$ **1**

[CBSE SQP Marking Scheme 2020-21]

Commonly Made Error

- Mostly students find $P(A\cap B)$ instead of finding $P(A\cup B)$.

Answering Tip

- The problem is solved if at least one of them solves it, which is given by $P(A\cup B)$.

Short Answer Type Questions-I

1. Let Likith hit the target, $P(A) = \frac{2}{3}$

and Harish hit the target, $P(B) = \frac{3}{4}$

Required Probability $= P(\overline{A} \cap \overline{B})$

$= P(\overline{A}).P(\overline{B})$ **1**

[Since, events are independent]

$= [1 - P(A)].[1 - P(B)]$

$= \left(1-\frac{2}{3}\right).\left(1-\frac{3}{4}\right)$

$= \frac{1}{3}.\frac{1}{4} = \frac{1}{2}$ **1**

Topper Answer, 2020

3. let A be the event of finding a green signal on 2 consecutive days.

let B_i, $i=1,2,3$ be the probability of finding a green signal on i^{th} day.

$P(B_1)=P(B_2)=P(B_3)=\frac{30}{100}$.

$P(A) = P(B_1)P(B_2)P(\overline{B_3}) + P(\overline{B_1})P(B_2)P(B_3)$

$= \frac{30}{100}\times\frac{30}{100}\times\frac{70}{100} + \frac{70}{100}\times\frac{30}{100}\times\frac{30}{100}$

$= \frac{126\cancel{000}}{1000\cancel{000}} = 12.6\%$ or $\frac{126}{1000}$

10. $P(\overline{A})=0.7 \Rightarrow 1-P(A)=0.7 \Rightarrow P(A)=0.3$ ½

$P(A\cap B)=P(A)\cdot P(B\mid A)=0.3\times 0.5=0.15$ ½

$P(A\mid B)=\frac{P(A\cap B)}{P(B)}=\frac{0.15}{0.7}=\frac{15}{70}$ or $\frac{3}{14}$ **1**

[CBSE Marking Scheme, 2019]

Short Answer Type Questions-II

3. Let

E be the event $=$ A solves the problem

F be the event $=$ B solves the problem

G be the event $=$ C solves the problem

H be the event $=$ D solves the problem ½

$P(E) = \frac{1}{3}$ $\quad P(\overline{E}) = \frac{2}{3}$

$P(F) = \frac{1}{4}$ $\quad P(\bar{F}) = \frac{3}{4}$

$P(G) = \frac{1}{5}$ $\quad P(\bar{G}) = \frac{4}{5}$

$P(H) = \frac{2}{3}$ $\quad P(\bar{H}) = \frac{1}{3}$

(i) the probability $= P(E \cup F \cup G \cup H)$ ½

$$= 1 - P(\bar{E} \cap \bar{F} \cap \bar{G} \cap \bar{H})$$

$$= 1 - P(\bar{E}) \times P(\bar{F}) \times P(\bar{G}) \times P(\bar{H})$$

$$= 1 - \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{1}{3}$$

$$= \frac{13}{15}$$ **1**

(ii) the required probability $= P(\bar{E}) \times P(\bar{F}) \times P(\bar{G}) \times P(\bar{H}) + P(E) \times P(\bar{F}) \times P(\bar{G}) \times P(\bar{H}) + P(\bar{E}) \times P(F) \times P(\bar{G}) \times P(\bar{H}) + P(\bar{E}) \times P(\bar{F}) \times P(G) \times P(\bar{H}) + P(\bar{E}) \times P(\bar{F}) \times P(\bar{G}) \times P(H)$

$$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{1}{3} + \frac{1}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} \times \frac{1}{3} + \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} \times \frac{1}{3} + \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{2}{3}$$

$$= \frac{2}{15} + \frac{1}{15} + \frac{2}{45} + \frac{1}{30} + \frac{4}{15}$$ **1**

$$= \frac{7}{15} + \frac{1}{30} + \frac{2}{45}$$

$$= \frac{42 + 3 + 4}{90} = \frac{49}{90}$$

[CBSE Marking Scheme, 2016] (Modified)

Solutions for Practice Questions (Topic-2)

Multiple Choice Questions

2. Option (A) is correct.

Explanation: Since, $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$$\therefore \frac{P\left(\frac{B}{A}\right).P(A)}{P(B)} = \frac{P(B \cap A)}{P(A)} \times \frac{P(A)}{P(B)}$$

$$= \frac{P(B \cap A)}{P(B)}$$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= P\left(\frac{A}{B}\right)$$

3. Option (B) is correct.

Explanation: Posterior probabilities are the probabilities in Bayes' theorem which are changed with the new given information.

Short Answer Type Questions-II

2. E_1 : selecting shop X
E_2 : selecting shop Y
A : purchased tin is of type B ½

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A|E_1) = \frac{4}{7}, \; P(A|E_1) = \frac{6}{11}$$ **1**

$$P(E_2|A) = \frac{P^2(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{2} \cdot \frac{6}{11}}{\frac{1}{2} \cdot \frac{4}{7} + \frac{1}{2} \cdot \frac{6}{11}}$$ **1**

$$= \frac{21}{43}$$ ½

[CBSE Marking Scheme 2020] (Modified)

7. Let E_1 : Selecting a student with 100% attendance
E_2 : Selecting a student who is not regular ½
A : selected student attains A grade.

$$P(E_1) = \frac{30}{100} \text{ and } P(E_2) = \frac{70}{100}$$ ½

$$P(A/E_1) = \frac{70}{100} \text{ and } P(A/E_2) = \frac{10}{100}$$ ½

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{30}{100} \times \frac{70}{100}}{\frac{30}{100} \times \frac{70}{100} + \frac{70}{100} \times \frac{10}{100}} = \frac{3}{4}$$ **1½**

[CBSE Marking Scheme 2017] (Modified)

OR

Topper Answer, 2017

7. E₁ : Event that students have 100% attendance

$P(E_1) = \frac{30}{100}$

E_2 : Event that students are irregular

$P(E_2) = \frac{70}{100}$

A : Student has grade A

$P(A|E_1) = \frac{70}{100}$

$P(A|E_2) = \frac{10}{100}$

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{30}{100} \times \frac{70}{100}}{\frac{30}{100} \times \frac{70}{100} + \frac{70}{100} \times \frac{10}{100}}$$

$$= \frac{21\cancel{00}}{28\cancel{00}} = \frac{3}{4}$$

Yes regularity is required in schools for discipline as well as scoring well in academics.

Long Answer Type Questions

2. Let A be the event that car delivered to firm needs service and tuning. Also let E_1, E_2 and E_3 be the events of car being rented from agencies X, Y and Z respectively.

$$P(E_1) = \frac{50}{100}, P(E_2) = \frac{30}{100}, P(E_3) = \frac{20}{100} \quad \textbf{1}$$

$$P(A|E_1) = \frac{9}{100}, P(A|E_2) = \frac{12}{100}, P(A|E_3) = \frac{10}{100} \quad \textbf{1}$$

$P(E_3|A)$

$$= \frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)} \quad \textbf{1}$$

$$P(E_3|A) = \frac{\frac{20}{100}\times\frac{10}{100}}{\frac{50}{100}\times\frac{9}{100}+\frac{30}{100}\times\frac{12}{100}+\frac{20}{100}\times\frac{10}{100}} \quad \textbf{1}$$

$$= \frac{20}{101}$$

$$P(E'_3|A) = 1 - P(E_3|A) = 1 - \frac{20}{101} = \frac{81}{101} \quad \textbf{1}$$

[CBSE Marking Scheme, 2018] (Modified)

Solutions for Practice Questions (Topic-3)

Multiple Choice Questions

1. Option (C) is correct.

Explanation: Let P(n) = Proportional to n

where n = 1, 2, 3, ..., 6

Given, P(n) = nk

$\therefore$ P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1

$\Rightarrow k + 2k + 3k + 4k + 5k + 6k = 1$

$\Rightarrow \quad 21k = 1$

$\Rightarrow \quad k = \frac{1}{21}$

$\Rightarrow \quad P(5) = 5k = \frac{5}{21}$

Very Short Answer Type Questions

1. Here, we observe that $0 \le P(X) \le 1 \ \forall \ X$

and $\quad \Sigma P(X) = 0.4 + 0.4 + 0.2 = 1$

Hence, the given probability distribution is valid. **1**

6. We know that, $\Sigma P(X) = 1 \quad ...(i)$

Here, $\Sigma P(X) = a + 4a + 3a + 7a + 8a + 10a + 6a + 9a$

$\therefore \quad 1 = 48a \quad$ [from (i)]

or, $\quad a = \frac{1}{48}$ **1**

Detailed Solution :

Short Answer Type Questions-I

3. $k + 2k + 3k = 1$ **1**

$\Rightarrow k = \frac{1}{6}$ **1**

[CBSE Marking Scheme, 2019]

Topper Answer, 2019

Sol.

$\sum_{i=0}^{n} P(X_i) = 1 \quad \left[\because \sum_{i=0}^{n} P(x_i) = 1\right]$

[∵ sum of all probabilities = 1]

disjoint exhaustive & exclusive

$\Rightarrow \quad k + 2k + 3k = 1$

$\Rightarrow \quad 6k = 1$

$\boxed{k = \frac{1}{6}}$

Short Answer Type Questions-II

3. Let X represents the number of rotten apples drawn

X :	P(X) :	X.P(X)	
0	$\frac{7}{10}.\frac{7}{10}.\frac{7}{10} = \frac{343}{1000}$	0	½
1	$3.\frac{7}{10}.\frac{7}{10}.\frac{3}{10} = \frac{441}{1000}$	$\frac{441}{1000}$	
2	$3.\frac{7}{10}.\frac{3}{10}.\frac{3}{10} = \frac{189}{1000}$	$\frac{378}{1000}$	1
3	$\frac{3}{10}.\frac{3}{10}.\frac{3}{10} = \frac{27}{1000}$	$\frac{81}{1000}$	1

Mean $= \sum XP(X) = \frac{900}{1000} = \frac{9}{10}$ ½

[CBSE Marking Scheme 2020] (Modified)

Detailed Solution:

Given that rotten apples are 3 and fresh apples are 7.

Total apples $= 10$

Suppose X : no of rotten apples then X can take the values 0, 1, 2 and 3.

Now suppose E : getting a rotten apple

$\therefore \quad P(E) = \frac{3}{10}$ and $P(E') = \frac{7}{10}$

Then $\quad P(X = 0) = P(E')P(E')P(E') = \frac{343}{1000}$

$P(X = 1) = 3P(E)P(E')P(E') = \frac{441}{1000}$

$P(X = 2) = 3P(E)P(E)P(E') = \frac{189}{1000}$

$P(X = 3) = P(E)P(E)P(E) = \frac{27}{1000}$

Now, $\quad$ Mean $= \Sigma XP(X)$

$= 0 \times \frac{343}{1000} + 1 \times \frac{441}{1000} + 2 \times \frac{189}{1000} + 3 \times \frac{27}{1000}$

Mean $= \frac{900}{1000} = \frac{9}{10}$

REFLECTIONS

- Student correlate the different real life situation with the concepts of conditional probability and Bayes' theorem.
- Probability has many applications in different fields such as wether forecasting. sports and gaming strategies, buying or selling insurance, online shopping and online games, determining blood groups and analyzing political stragglers.

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SELF ASSESSMENT PAPER - 06

Time: 1 hour **MM: 30**

UNIT-VI

(A) OBJECTIVE TYPE QUESTIONS:

I. Multiple Choice Questions **[1×6 = 6]**

Q. 1. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'|A)$ is equal to

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{4}$ **(D)** 1

Q. 2. A bag contains 3 black, 4 red and 2 green balls. If three balls are drawn simultaneously at random, then the probability that the balls are of different colours is ______

(A) $\frac{2}{7}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{3}{4}$

Q. 3. A and B play with two dice on the condition that A wins if he thrown 6 before B throw 7, then the probability that A wins is

(A) $\frac{36}{1141}$ **(B)** $\frac{30}{61}$ **(C)** $\frac{172}{1141}$ **(D)** $\frac{191}{1141}$

Q. 4. If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{5}$, then $P(\bar{B}|\bar{A}) =$

(A) $\frac{37}{40}$ **(B)** $\frac{37}{45}$ **(C)** $\frac{23}{40}$ **(D)** $\frac{23}{45}$

Q. 5. If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{5}$, then

(A) $P(A|B) = \frac{1}{2}$ **(B)** $P(A|(A \cup B)) = \frac{5}{6}$ **(C)** $P((A \cap B)|(A' \cup B')) = 0$ **(D)** All of these

Q. 6. A coin is tossed three times in succession. If E is the event that there are at least two heads and F is the event in which first throw is a head, then P(E|F) is equal to

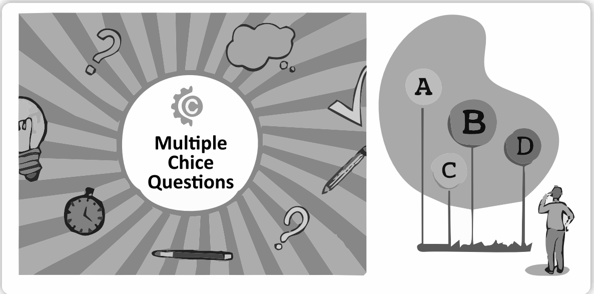
(A) $\frac{3}{4}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{8}$

II. Case-Based MCQs **[1×4 = 4]**

Attempt any 4 sub-parts from each questions. Each question carries 1 mark.

Read the following text and answer the questions on the basis of the same.

In a test, you either guesses or copies or knows the answer to a multiple-choice question with four choice. The probability that you make a guess is $\frac{1}{3}$, you copy the answer is $\frac{1}{6}$. The probability that your answer is correct, given that you guess it, is $\frac{1}{8}$. And also, the probability that you answer is correct, given that you copy it, is $\frac{1}{4}$.



Q. 7. The probability that you know the answer is

(A) 0 **(B)** 1 **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{4}$

Q. 8. The probability that your answer is correct given that you guess it, is

(A) $\frac{1}{2}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$

Q. 9. The probability that your answer is correct given that you known the answer, is

(A) $\frac{1}{7}$ **(B)** 1 **(C)** $\frac{1}{9}$ **(D)** $\frac{1}{10}$

Q. 10. The probability that you know the answer given that you correctly answered it, is

(A) $\frac{4}{7}$ **(B)** $\frac{5}{7}$ **(C)** $\frac{6}{7}$ **(D)** None of these

Q. 11. The total probability of correctly answered the question, is

(A) $\frac{7}{12}$ **(B)** $\frac{11}{12}$ **(C)** $\frac{5}{12}$ **(D)** None of these

(B) SUBJECTIVE TYPE QUESTIONS:

III. Very Short Answer Type Questions **[1×3 = 3]**

Q. 12. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

Q. 13. An unbiased coin is tossed 4 times. Find the probability of getting at least one head.

Q. 14. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.6$, then find $P(B' \cap A)$.

IV. Short Answer Type Questions–I **[2×3 = 6]**

Q. 15. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three ?

Q. 16. A purse contains 3 silver and 6 copper coins and a second purse contains 4 silver and 3 copper coins. If a coin is drawn at random from one of the two purses, find the probability that it is a silver coin.

Q. 17. A and B throw a pair of dice alternately till one of them gets the sum of the numbers as multiples of 6 and wins the game. If A starts first, find the probability of B winning the game.

V. Short Answer Type Questions-II **[3×6 = 6]**

Q. 18. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

Q. 19. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls ?

Given that:

(i) the youngest is a girl.

(ii) atleast one is a girl.

VI. Long Answer Type Questions **[1×5 = 5]**

Q. 20. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn randomly one-by-one without replacement and are found to be both kings. Find the probability of the lost card being a king.

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