CHAPTER 11

THREE DIMENSIONAL GEOMETRY

Syllabus *Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.*

In this chapter you will study

- *Direction ratios and directions cosines of a line*
- *Equation of line in Cartesian and vector form*
- *Angle between two lines.*

List of Topics

Topic-1 Direction Ratios and Direction Cosines

Concepts Covered • *Direction Ratios,* • *Direction Cosines*

• *Relationship between DC's of a line.*

Revision Notes

1. Direction Cosines of a Line :

- If A and B are two points on a given line L, then direction cosines of vectors $\vec{AB}$ and $\vec{BA}$ are the direction cosines (d.c.'s) of line L. Thus if α, β, γ are the direction-angles which the line L makes with the positive direction of X, Y, Z-axis respectively, then its d.c.'s are $\cos\alpha, \cos\beta, \cos\gamma$.
- If direction of line L is reversed, the direction angles are replaced by their **supplements angles** *i.e.,* $\pi - \alpha, \pi - \beta, \pi - \gamma$ and so are the d.c.'s *i.e.,* the direction cosines become $-\cos\alpha, -\cos\beta, -\cos\gamma$.

Key Words

Supplement angles: Two angles or arcs whose sum is 180° degrees.

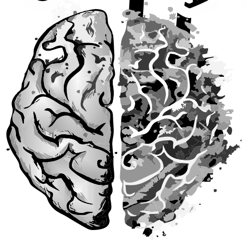
- So, a line in space has two set of d.c.'s *viz* $\pm\cos\alpha, \pm\cos\beta, \pm\cos\gamma$.
- The d.c.'s are generally denoted by l, m, n. Also $l^2 + m^2 + n^2 = 1$ and so we can deduce that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Also $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.
- The d.c.'s of a line joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are $\pm\frac{x_2 - x_1}{AB}, \pm\frac{y_2 - y_1}{AB}, \pm\frac{z_2 - z_1}{AB}$; where AB is the distance between the points A and B *i.e.,* $AB = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right|$

2. Direction Ratios of a Line :

Any three numbers a, b, c (say) which are proportional to d.c.'s *i.e.,* l, m, n of a line are called the **direction ratios** (d.r.'s) of the line. Thus, $a = \lambda l, b = \lambda m, c = \lambda n$ for any $\lambda \in R - \{0\}$.

Consider, $\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{1}{\lambda}$ (say)

Three Dimensional Geometry



Direction ratios and direction cosines of a line

D. Cs of a line are the cosines of the angles made by the line with the positive direction of the co-ordinate axes. If l, m, n are the D. Cs of a line, then $l^2 + m^2 + n^2 = 1$. D. Cs of a line joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$, where $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

D.Rs of a line are the no.s which are proportional to the D.Cs of the line if l, m, n are the D.Cs and a, b, c are D.Rs of a line, then

$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$, $m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$, $n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$

Parallel lines

(i) two skew lines are the line segment perpendicular to both the lines

(ii) $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is $\frac{|(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)|}{|(\vec{b}_1 \times \vec{b}_2)|}$

(iii) $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ is

$$= \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2 + (a_1 b_2 - a_2 b_1)^2}}$$

(iv) Distance between parallel lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is $\frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|}$

Skew lines

These are the lines in space which are neither parallel nor intersecting. They lie in different planes. Angle between skew lines is the angle between two intersecting lines drawn from any point (origin) parallel to each of the skew lines.

Angle between the two lines

If l_1, m_1, n_1, l_2, m_2, n_2 are the D.Cs and $a_1, b_1, c_1, a_2, b_2, c_2$ are the D.Rs of the two lines and 'θ' is the acute angle between them, then

$$\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2| = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

Equation of line Vector in 3D

Vector equation of a line passing through the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$

Vector equation of a line which passes through two points whose position vectors are $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

Equation of a line through point (x_1, y_1, z_1) and having D.Cs l, m, n is $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ Also, equation of a line that passes through two points is $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

Trace the Mind Map
- First Level
- Second Level
- Third Level

or $$l = \frac{a}{\lambda}, m = \frac{b}{\lambda}, n = \frac{c}{\lambda}$$

or $$\left(\frac{a}{\lambda}\right)^2 + \left(\frac{b}{\lambda}\right)^2 + \left(\frac{c}{\lambda}\right)^2 = 1 \text{ [Using } l^2 + m^2 + n^2 = 1]$$

or $$\lambda = \pm\sqrt{a^2 + b^2 + c^2}$$

Therefore, $$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}},$$

$$m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}, n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

- The d.r.'s of a line joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are $x_2 - x_1, y_2 - y_1, z_2 - z_1$ or $x_1 - x_2, y_1 - y_2, z_1 - z_2$.
- Direction ratios are sometimes called as **Direction Numbers.**

3. Relation Between the Direction Cosines of a Line :

Consider a line L with d.c's l, m, n. Draw a line passing through the origin and $P(x, y, z)$ and parallel to the given line L. From P draw a perpendicular PA on the X-axis, suppose $OP = r$

Now in ΔOAP, $\angle PAO = 90°$

we have, $$\cos\alpha = \frac{OA}{OP} = \frac{x}{r} \text{ or } x = lr.$$

Similarly we can obtain

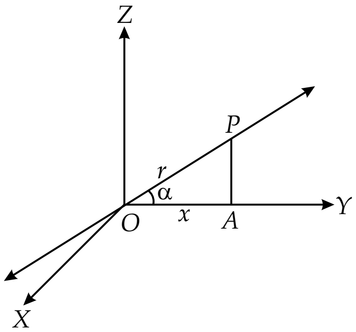
$$y = mr \text{ and } z = nr.$$

Therefore, $x^2 + y^2 + z^2 = r^2(l^2 + m^2 + n^2)$

But we know that

$$x^2 + y^2 + z^2 = r^2$$

Hence, $l^2 + m^2 + n^2 = 1.$



Key Formulae

1. Distance Formula :

The distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by the expression

$$AB = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right| \text{ units.}$$

2. Section Formula :

The co-ordinates of a point Q which divides the line joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in the ratio $m : n$

(a) internally, are $\left(\frac{(mx_2 + nx_1)}{m+n}, \frac{(my_2 + ny_1)}{m+n}, \frac{(mz_2 + nz_1)}{m+n}\right)$

(b) externally, are $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$.

Amazing Facts

- The largest 3D shape in the world is a Rhombicosidecahedron. It is an Archimedian solid. It has 20 faces that are triangular, 30 faces that are squares, and 12 are that are pentagons. This shape has 120 edges and 60 vertices.
- The Louvre pyramid is a beautiful installation that is perfect example of a 3D shape *i.e.*, square pyramid. It is situated in the city of Paris in the prestigious museum of the Louvre.

Mnemonics

Direction Cosines

1 glass **L** e **M** o **N** juice

$l^2 + m^2 + n^2 = 1$

Dance **C**horeographer **P**refer **D**ieting

Direction Cosines Positive Direction

Direction Ratios

Director Remo a **P**rofessional **D**ancer

Direction Ratios Proportional Direction

Choreographer created

Cosines

3 **L**ifetime **M**ovies with **N**ew faces **a b c**

l m n

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}}, m = \frac{b}{\sqrt{a^2+b^2+c^2}}, n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Interpretation :
Direction cosines of a line are the cosines of the angles made by the line with the positive directions of the co. ordinate axes. If *l* , *m, n* are the *D*. cs of a line, then $l^2+m^2+n^2=1$

OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. Distance of the point (α, β, γ) from *Y*-axis is

(A) β units (B) |β| units

(C) |β| + |γ| units (D) $\sqrt{\alpha^2+\gamma^2}$ units

Ans. Option (D) is correct.

Explanation:

The foot of perpendicular from point $P(\alpha,\beta,\gamma)$ on *Y*-axis is $Q(0,\beta,0)$.

∴ Required distance,

$$PQ = \left|\sqrt{(\alpha-0)^2+(\beta-\beta)^2+(\gamma-0)^2}\right|$$
$$= \left|\sqrt{\alpha^2+\gamma^2}\right| \text{ units}$$

Q. 2. If the direction cosines of a line are *k, k, k*, then

(A) $k > 0$ (B) $0 < k < 1$

(C) $k = 1$ (D) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

Ans. Option (D) is correct.

Explanation:

Since, direction cosines of a line are *k*, *k* and *k*.

∴ $l = k$, $m = k$ and $n = k$

We know that, $l^2+m^2+n^2 = 1$

⇒ $k^2+k^2+k^2 = 1$

⇒ $k^2 = \frac{1}{3}$

∴ $k = \pm\frac{1}{\sqrt{3}}$

Q. 3. If the direction ratios of a line are 2, 3 and –6, then direction cosines of the line making obtuse angle with Y-axis are:

(A) $\frac{2}{7},\frac{3}{7},\frac{6}{7}$ (B) $\frac{-2}{7},\frac{-3}{7},\frac{6}{7}$

(C) $\frac{-2}{7},\frac{3}{7},\frac{-6}{7}$ (D) $\frac{-2}{7},\frac{-3}{7},\frac{-6}{7}$

Ans. Option (B) is correct.

Explanation: Direction cosines of the line, whose direction ratios are 2, 3, –6 are:

$$\frac{2}{\sqrt{2^2+3^2+(-6)^2}}, \frac{3}{\sqrt{2^2+3^2+(-6)^2}}, \frac{-6}{\sqrt{2^2+3^2+(-6)^2}}$$

or $\frac{2}{7},\frac{3}{7},\frac{-6}{7}$

Since, line makes obtuse angle with Y-axis, then

$$\cos\beta < 0$$

Therefore, direction cosines are $\frac{-2}{7},\frac{-3}{7},\frac{6}{7}$.

Q. 4. If a line makes an angle α, β, γ with *X*-axis, *Y*-axis and *Z*-axis respectively, then cos 2α + cos 2β + cos 2γ is:

(A) –1 (B) 1

(C) 0 (D) 2

Ans. Option (A) is correct.

Explanation: We know that,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

⇒ $\left(\frac{1+\cos 2\alpha}{2}\right)+\left(\frac{1+\cos 2\beta}{2}\right)+\left(\frac{1+\cos 2\gamma}{2}\right) = 1$

⇒ $3 + \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2$

⇒ $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

Q. 5. The equations of Y-axis in spaces are:

(A) $x = 0, y = 0$ (B) $x = 0, z = 0$

(C) $y = 0, z = 0$ (D) None of these

Ans. Option (B) is correct.

Explanation: On Y-axis, coordinates of X-axis and Z-axis both are zero.

Q. 6. If the direction cosines of a line are $\frac{k}{3},\frac{k}{3},\frac{k}{3}$, then value of *k* is:

(A) $k = 1$ (B) $k = \frac{1}{3}$

(C) $k > 0$ (D) $k = \pm\sqrt{3}$

Ans. Option (D) is correct.

Explanation:

$\frac{k^2}{9}+\frac{k^2}{9}+\frac{k^2}{9} = 1$

⇒ $\frac{3k^2}{9} = 1$

⇒ $k^2 = 3$

⇒ $k = \pm\sqrt{3}$

Q. 7. The direction cosines of the line passing through the following points:
(–2, 4, –5), (1, 2, 3) is:

(A) $\frac{3}{\sqrt{77}},\frac{-2}{\sqrt{77}},\frac{8}{\sqrt{77}}$ (B) $\frac{-3}{\sqrt{77}},\frac{-2}{\sqrt{77}},\frac{8}{\sqrt{77}}$

(C) $\frac{3}{\sqrt{77}},\frac{-2}{\sqrt{77}},\frac{-8}{\sqrt{77}}$ (D) $\frac{-3}{\sqrt{77}},\frac{-2}{\sqrt{77}},\frac{-8}{\sqrt{77}}$

Ans. Option (A) is correct.

Explanation: DR's are 1 + 2, 2 – 4, 3 + 5, i.e. 3, – 2, 8.

Dividing by $\left|\sqrt{9+4+64}\right| = \sqrt{77}$

DC's are $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

Q. 8. If P(1, 5, 4) and Q(4, 1, –2), then direction ratios of is:

(A) <3, –4, –6> **(B)** <3, 4, 6>

(C) <3, 4, –6> **(D)** <3, –4, 6>

Ans. Option (A) is correct.

Explanation:

Direction ratio of = 4 – 1, 1 – 5, – 2 – 4, *i.e.*, <3, –4, –6>

Q. 9. The direction cosines of the Y-axis are:

(A) (1, 0, 0) **(B)** (1, 0, 0)

(C) (0, 1, 0) **(D)** (0, 0, 1)

Ans. Option (C) is correct.

Explanation: The direction cosines of the Y-axis are (0, 1, 0).

Q. 10. If *l, m, n* are the direction cosines of a line, then;

(A) $l^2 + m^2 + 2n^2 = 1$ **(B)** $l^2 + 2m^2 + n^2 = 1$

(C) $2l^2 + m^2 + n^2 = 1$ **(D)** $l^2 + m^2 + n^2 = 1$

Ans. Option (D) is correct.

Q. 11. The sum of the direction cosines of Z-axis is

(A) 1 **(B)** 0

(C) 3 **(D)** 2

Ans. Option (A) is correct.

Explanation: Z-axis makes an angle 90° with X-axis, 90° with Y-axis and 0° with Z-axis

∴ Direction cosines of Z-axis: cos90°, cos90°, cos0

i.e., 0, 0, 1

Therefore, sum of the direction cosines = 0 + 0 + 1 = 1

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. Find the direction cosines of a line which makes equal angles with the coordinate axes.

A I [CBSE O.D. Set-I 2019]

Sol. D.R.s are 1, 1, 1

∴ Direction cosines of the line are:

$$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \quad 1$$

[CBSE Marking Scheme, 2019]

Detailed Solution :

Direction cosines of a line making angle, α with *X*-axis, β with *Y*-axis and γ with *Z*-axis are *l, m, n*.

$$l = \cos\alpha, m = \cos\beta, n = \cos\gamma$$

Given, the line makes equal angles with coordinate axes.

So, $\alpha = \beta = \gamma$ ...(i)

Direction cosines are :

$l = \cos\alpha, m = \cos\alpha, n = \cos\alpha$

Since,

$$l^2 + m^2 + n^2 = 1$$

$$\therefore \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$

$$\Rightarrow 3\cos^2\alpha = 1$$

$$\Rightarrow \cos\alpha = \pm\sqrt{\frac{1}{3}}$$

or $$\cos\alpha = \pm\frac{1}{\sqrt{3}}$$

Therefore, direction cosines are.

$$l = \pm\frac{1}{\sqrt{3}}, m = \pm\frac{1}{\sqrt{3}}, \text{ and } n = \pm\frac{1}{\sqrt{3}}$$

Q. 2. If a line makes angles 90°, 135°, 45° with the *X, Y* and *Z* axes respectively, find its direction cosines.

[CBSE Delhi Set-I 2019]

Sol. d.c. 's = < cos 90°, cos 135°, cos 45°> ½

$$= < 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, > \quad ½$$

[CBSE Marking Scheme 2019]

Detailed Solution :

Direction cosines of a line making angle α with *X*-axis, β with *Y*-axis and γ with *Z*-axis are *l, m, n*

$$l = \cos\alpha, m = \cos\beta, n = \cos\gamma$$

Here, $\alpha = 90°, \beta = 135°, \gamma = 45°$

So, direction cosines are

$$l = \cos 90° = 0,$$

$$m = \cos 135° = \cos(90° + 45°)$$

$$= -\sin 45°$$

$$= -\frac{1}{\sqrt{2}}$$

and $$n = \cos 45° = \frac{1}{\sqrt{2}}$$

Therefore, direction cosines are 0, $-\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

Q. 3. Find the acute angle which the line with direction cosines $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}$, *n* makes with positive direction of *Z*-axis.

R&U [S.Q.P. 2018]

Sol. $$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 + n^2 = 1$$

$$\Rightarrow n = \frac{1}{\sqrt{2}} \quad ½$$

As, $\cos\gamma = n$

$\Rightarrow \quad \cos\gamma = \frac{1}{\sqrt{2}} \Rightarrow \gamma = 45° \text{ or } \frac{\pi}{4}$ ½

[CBSE Marking Scheme 2018]

Q. 4. Find the direction cosines of the line :

$$\frac{x-1}{2} = -y = \frac{z+1}{2}$$

R&U [S.Q.P. 2018]

Sol. Direction ratios of the given line are 2, –1, 2. ½

Hence, direction cosines of the line are:

$\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$ or $\frac{-2}{3}, \frac{1}{3}, \frac{-2}{3}$ ½

[CBSE Marking Scheme 2018]

Q. 5. If the points with position vectors $10\hat{i}+3\hat{j}, 12\hat{i}-5\hat{j}$ and $\lambda\hat{i}+11\hat{j}$ are collinear, find the value of λ.

R&U [Delhi Comptt. 2017]

Sol. Let A be $10\hat{i}+3\hat{j}$, B be $12\hat{i}-5\hat{j}$, C be $\lambda\hat{i}+11\hat{j}$

$$\vec{AB} = 2\hat{i}-8\hat{j}$$

$$\vec{AC} = (\lambda-10)\hat{i}+8\hat{j}$$ ½

As $\vec{AB}$ and $\vec{AC}$ are collinear

$$\frac{2}{\lambda-10} = \frac{-8}{8}$$ ½

So, $\lambda = 8$

[CBSE Marking Scheme, 2017]

Q. 6. Write the distance of the point (3, – 5, 12) from X-axis. R&U [Foreign 2017]

Q. 7. If a line makes angles 90°, 60° and θ with X, Y and Z-axis respectively, where θ is acute, then find θ.

R&U [Delhi 2017, 2015]

Q. 8. What is the distance of the point (p, q, r) from the X-axis ? R&U [S.Q.P. Dec. 2016-17]

Q. 9. A line passes through the point with position vector $2\hat{i}-3\hat{j}+4\hat{k}$ and makes angles 60°, 120°, and 45° with X, Y and Z-axis respectively. Find the equation of the line in the Cartesian form.

R&U [Delhi Set I, II, III, Comptt. 2016]

Sol. D-Cosines of line are $\frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}}$

Equation of line is :

$$\frac{x-2}{\frac{1}{2}} = \frac{y+3}{\frac{-1}{2}} = \frac{z-4}{\frac{1}{\sqrt{2}}}$$ ½

or $2x-4 = -2y-6 = \sqrt{2}(z-4)$ ½

[CBSE Marking Scheme 2016]

Q. 10. Write the direction ratios of the vector $3\vec{a}+2\vec{b}$ where $\vec{a} = \hat{i}+\hat{j}-2\hat{k}$ and $\vec{b} = 2\hat{i}-4\hat{j}+5\hat{k}$.

R&U [O.D. Set I, II, III Comptt. 2015]

Sol. Getting

$$3\vec{a}+2\vec{b} = 7\hat{i}-5\hat{j}+4\hat{k}$$ ½

∴ D.R's are 7, – 5, 4. ½

[CBSE Marking Scheme 2015]

Short Answer Type Questions-I (2 marks each)

Q. 1. Find the direction cosines of the following line:

$$\frac{3-x}{-1} = \frac{2y-1}{2} = \frac{z}{4}$$

R [SQP 2021-2022]

Q. 2. Let l_i, m_i, n_i; i = 1, 2, 3 be the direction cosines of three mutually perpendicular vector in space. Show that $AA' = I^3$, where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_2 & n_2 \end{bmatrix}$.

R&U [SQP Dec. 2016-17]

Sol.

$$AA' = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$ 1

because

$l_1^2 + m_1^2 + n_1^2 = 1$, for each i = 1, 2, 3 ½

$l_il_j + m_im_j + n_in_j = 0 (i \neq j)$ for each i, j = 1, 2, 3 ½

[CBSE Marking Scheme 2016]

Q. 3. If a line has the direction ratios –18, 12, –4, then find direction cosines.

Sol. Let $a = -18, b = 12$ and $c = -4$

Here, $a^2 + b^2 + c^2 = (-18)^2 + (12)^2 + (-4)^2$

$= 484$ 1

Now, $l = \frac{-18}{\sqrt{484}} = \frac{-18}{22} = \frac{-9}{11}$

$m = \frac{12}{\sqrt{484}} = \frac{12}{22} = \frac{6}{11}$

$n = \frac{-4}{\sqrt{484}} = \frac{-4}{22} = \frac{-2}{11}$ 1

Hence, direction cosines are $< \frac{-9}{11}, \frac{6}{11}, \frac{-2}{11} >$.

Q. 4. Find the direction cosines of the line passing through the two points (–2, 4, –5) and (1, 2, 3).

A I

Q. 5. Find the direction cosines of the line

$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$$

Sol. Given, line is $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$

or, $\frac{x-4}{-2}=\frac{y-0}{6}=\frac{z-1}{-3}$

Here, direction ratios are: <–2, 6, –3> **1**

Topic-2

Lines & Its Equations in Different forms

Concepts Covered • *Equation of line in cartesian and vector form,*
• *Shortest distance between lines*
• *Skew lines*
• *Condition of parallelism and perpendicularity of lines.*

Revision Notes

1. Equation of a Line passing through two given points :

Consider the two given points as $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ with position vectors $\vec{a}$ and $\vec{b}$ respectively. Also assume $\vec{r}$ as the position vector of any arbitrary point $P(x, y, z)$ on the line L passing through A and B.
Thus

$$\vec{OA}=\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k},\ \vec{OB}=\vec{b}=x_2\hat{i}+y_2\hat{j}+z_2\hat{k},$$
$$\vec{OP}=\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$

(a) Vector equation of a line : Since the points A, B and P all lie on the same line which means that they are all collinear points.

Further it means, $\vec{AP}=\vec{r}-\vec{a}$ and $\vec{AB}=\vec{b}-\vec{a}$ are collinear vectors, *i.e.*,

$$\vec{AP} = \lambda\vec{AB}$$

or $$\vec{r}-\vec{a} = \lambda(\vec{b}-\vec{a})$$

or $$\vec{r} = \vec{a}+\lambda(\vec{b}-\vec{a}),\ \text{where } \lambda\in R.$$

This is the vector equation of the line.

(b) Cartesian equation of a line : By using the vector equation of the line $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$, we get

$$x\hat{i}+y\hat{j}+z\hat{k} = x_1\hat{i}+y_1\hat{j}+z_1\hat{k}+\lambda\left[(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}\right]$$

On equating the coefficients of $\hat{i},\hat{j},\hat{k}$, we get
$x = x_1+\lambda(x_2-x_1)$, $y = y_1+\lambda(y_2-y_1)$, $z = z_1+\lambda(z_2-z_1)$ ...(i)

On eliminating λ, we have

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

2. Angle between two lines :

(a) When d.r.'s or d.c.'s of the two lines are given : Consider two lines L_1 and L_2 with d.r.'s in proportion to a_1, b_1, c_1 and a_2, b_2, c_2 respectively ; d.c.'s as l_1, m_1, n_1 and l_2, m_2, n_2. Consider

$$\vec{b}_1=a_1\hat{i}+b_1\hat{j}+c_1\hat{k} \text{ and } \vec{b}_2=a_2\hat{i}+b_2\hat{j}+c_2\hat{k}.$$

These vectors $\vec{b}_1$ and $\vec{b}_2$ are parallel to the given lines L_1 and L_2. So in order to find the angle between the lines L_1 and L_2, we need to get the angle between the vectors $\vec{b}_1$ and $\vec{b}_2$.

So the acute angle θ between the vectors $\vec{b}_1$ and $\vec{b}_2$ (and hence lines L_1 and L_2) can be obtained as,

$$\vec{b}_1.\vec{b}_2 = |\vec{b}_1||\vec{b}_2|\cos\theta$$

Thus, $$\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

- Also, in terms of d.c.'s : $\cos\theta = |l_1l_2+m_1m_2+n_1n_2|$.
- Sine of angle is given as :

$$\sin\theta=\left|\frac{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|.$$

(b) When Vector equations of two lines are given : Consider vector equations of lines L_1 and L_2 as

$\vec{r}_1=\vec{a}_1+\lambda\vec{b}_1$ and

$\vec{r}_2=\vec{a}_2+\mu\vec{b}_2$ respectively.

Then, the acute angle θ between the two lines is given by the relation

$$\cos\theta=\left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|.$$

(c) When Cartesian equation of two lines are given: Consider the lines L_1 and L_2 in Cartesian form as,

$$L_1: \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

$$L_2: \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$$

Then the acute angle θ between the lines L_1 and L_2 can be obtained by,

$$\cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

Note :

- For two perpendicular lines : $a_1a_2 + b_1b_2 + c_1c_2 = 0$, $l_1l_2 + m_1m_2 + n_1n_2 = 0$.
- For two parallel lines :
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}; \quad \frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}.$$

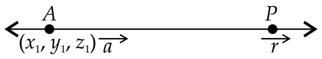
3. Shortest Distance between two Lines :

If two lines are in the same plane *i.e.,* they are coplanar, they will intersect each other if they are non-parallal. Hence, the shortest distance between them is zero. If the lines are parallel then the shortest distance between them will be the perpendicular distance between the lines *i.e.,* the length of the perpendicular drawn from a point on one line onto the other line. Adding to this discussion, in space, there are lines which are neither intersecting nor parallel. In fact, such pair of lines are non-coplanar and are called the skew lines.

Key Fact

1. Equation of a line in space passing through a given point and parallel to a given vector :

Consider the line L is passing through the given point $A(x_1, y_1, z_1)$ with the position vector $\vec{a}$, $\vec{d}$ is the given vector with d.r.'s a, b, c and $\vec{r}$ is the position vector of any arbitrary point $P(x, y, z)$ on the line.



Thus, $\vec{OA} = \vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$,

$\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{d} = a\hat{i} + b\hat{j} + c\hat{k}$.

(a) Vector equation of a line : As the line L is parallel to given vector $\vec{d}$ and points A and P are lying on the line so, $\overrightarrow{AP}$ is parallel to the $\vec{d}$.

or $\quad \overrightarrow{AP} = \lambda\vec{d}$,

where $\lambda \in R$ *i.e.,* set of real numbers

or $\quad \vec{r} - \vec{a} = \lambda\vec{d}$

or $\quad \vec{r} = \vec{a} + \lambda\vec{d}$.

This is the vector equation of line.

(b) Parametric equations : If d.r.'s of the line are a, b, c, then by using $\vec{r} = \vec{a} + \lambda\vec{d}$, we get

$$x\hat{i} + y\hat{j} + z\hat{k} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda\left(a\hat{i} + b\hat{j} + c\hat{k}\right)$$

Now, as we equate the coefficients of $\hat{i}, \hat{j}, \hat{k}$, we get the parametric equations of line given as,

$$x = x_1 + \lambda a, \; y = y_1 + \lambda b, \; z = z_1 + \lambda c.$$

- Co-ordinates of any point on the line considered here are $(x_1 + \lambda a, y_1 + \lambda b, z_1 + \lambda c)$.

Key Word

Parametric Equation: It is a type of equation that employs an independent variable called parameter (often denoted by t) and in which dependent variables are defined as continuous functions of the parameter and are not dependent on another existing variable.

(c) Cartesian equation of a line : If we eliminate the parameter λ from the parametric equations of a line, we get the Cartesian equation of line as

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

- If l, m, n are the d.c.'s of the line, then Cartesian equation of line becomes
$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$
- **Skew Lines :** Two straight lines in space which are neither parallel nor intersecting are known as the skew lines. They lie in different planes and are non-coplanar.
- **Line of Shortest distance :** There exists unique line perpendicular to each of the skew lines L_1 and L_2, and this line is known as the line of shortest distance (S.D.).

OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. The vector equation of the line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$ is:

(A) $\vec{r}=(5\hat{i}-4\hat{j}+6\hat{k})+\lambda(3\hat{i}+7\hat{j}-2\hat{k})$

(B) $\vec{r}=(3\hat{i}+7\hat{j}+2\hat{k})+\lambda(5\hat{i}-4\hat{j}+6\hat{k})$

(C) $\vec{r}=(5\hat{i}-4\hat{j}+6\hat{k})+\lambda(-3\hat{i}+7\hat{j}-2\hat{k})$

(D) $\vec{r}=(-3\hat{i}-7\hat{j}+2\hat{k})+\lambda(-5\hat{i}+4\hat{j}-6\hat{k})$

Ans. Option (A) is correct.

Explanation: Rewriting the given line as:

$$\frac{x-5}{3}=\frac{y-(-4)}{7}=\frac{z-6}{-2}$$

$\therefore$ Equation of line in vector form is

$\vec{r}=(5\hat{i}-4\hat{j}+6\hat{k})+\lambda(3\hat{i}+7\hat{j}-2\hat{k})$

Q. 2. The cartesian equation of a line is given by $\frac{2x-1}{\sqrt{3}}=\frac{y+2}{2}=\frac{z-3}{3}$

The direction cosines of the line is:

(A) $\frac{3}{\sqrt{55}},\frac{4}{\sqrt{55}},\frac{6}{\sqrt{55}}$ **(B)** $\frac{\sqrt{3}}{\sqrt{55}},\frac{4}{\sqrt{55}},\frac{6}{\sqrt{55}}$

(C) $\frac{-3}{\sqrt{55}},\frac{4}{\sqrt{55}},\frac{6}{\sqrt{55}}$ **(D)** $\frac{\sqrt{3}}{\sqrt{55}},\frac{-4}{\sqrt{55}},\frac{6}{\sqrt{55}}$

Ans. Option (B) is correct.

Explanation: Rewrite the given line as

$$\frac{2\left(x-\frac{1}{2}\right)}{\sqrt{3}}=\frac{y+2}{2}=\frac{z-3}{3}$$

or, $$\frac{x-\frac{1}{2}}{\sqrt{3}}=\frac{y+2}{4}=\frac{z-3}{6}$$

$\therefore$ DR's of line are $\sqrt{3}$, 4 and 6

Therefore, direction cosines are:

$$\frac{\sqrt{3}}{\sqrt{(\sqrt{3})^2+4^2+6^2}},\frac{4}{\sqrt{(\sqrt{3})^2+4^2+6^2}},\frac{6}{\sqrt{(\sqrt{3})^2+4^2+6^2}}$$

or, $\frac{\sqrt{3}}{\sqrt{55}},\frac{4}{\sqrt{55}},\frac{6}{\sqrt{55}}$

Q. 3. The point where the line joining the points (0, 5, 4) and (1, 3, 6) meets XY-plane is:

(A) (2, 9, 0) **(B)** (–2, 9, 0)

(C) (–2, –9, 0) **(D)** (2, –9, 0)

Ans. Option (B) is correct.

Explanation: The line joining the given point is:

$$\frac{x-1}{1}=\frac{y-3}{-2}=\frac{z-6}{2}=\lambda$$

Let $(\lambda + 1, -2\lambda + 3, 2\lambda + 6)$ be a point on the line. Given, the point meets at XY-plane, so Z-coordinate will be zero.

$\therefore \quad 2\lambda + 6 = 0$

$\Rightarrow \quad \lambda = -3$

$\therefore$ Point is (–2, 9, 0)

Q. 4. If a line makes angles $\frac{\pi}{4},\frac{3\pi}{4}$ with X-axis and Y-axis respectively, then the angle which it makes with Z-axis is:

(A) 0° **(B)** π

(C) both (A) and (B) **(D)** $\frac{\pi}{2}$

Ans. Option (D) is correct.

Explanation: We have,

$$\cos^2\frac{\pi}{4}+\cos^2\frac{3\pi}{4}+\cos^2\gamma=1$$

$$\Rightarrow \quad \frac{1}{2}+\frac{1}{2}+\cos^2\gamma = 1$$

$$\Rightarrow \quad \cos\gamma = 0$$

$$\Rightarrow \quad \gamma = \frac{\pi}{2}$$

Q. 5. The vector equation for the line passing through the points (–1, 0, 2) and (3, 4, 6) is:

(A) $\vec{r}=(\hat{i}+2\hat{k})+\lambda(4\hat{i}+4\hat{j}+4\hat{k})$

(B) $\vec{r}=(\hat{i}-2\hat{k})+\lambda(4\hat{i}+4\hat{j}+4\hat{k})$

(C) $\vec{r}=(-\hat{i}+2\hat{k})+\lambda(4\hat{i}+4\hat{j}+4\hat{k})$

(D) $\vec{r}=(-\hat{i}+2\hat{k})+\lambda(4\hat{i}-4\hat{j}-4\hat{k})$

Ans. Option (C) is correct.

Explanation: The vector equation of the line is given by:

$$\vec{r} = \vec{a}+\lambda(\vec{b}-\vec{a}),\ x\in R$$

Let $\vec{a} = -\hat{i}+2\hat{k}$

and $\vec{b} = 3\hat{i}+4\hat{j}+6\hat{k}$

$\therefore \quad \vec{b}-\vec{a} = 4\hat{i}+4\hat{j}+4\hat{k}$

Therefore, the vector equation is

$$\vec{r} = (-\hat{i}+2\hat{k})+\lambda(4\hat{i}+4\hat{j}+4\hat{k})$$

Q. 6.. The acute angle between the lines $x - 2 = 0$ and $\sqrt{3}x-y-2=0$ is

(A) 0° **(B)** 30°

(C) 45° **(D)** 60°

Ans. Option (B) is correct.

Explanation: Let the slope of the line $(x - 2 = 0)$ is m_1

So, $\quad m_1 = \infty$

And, the slope of the line $(\sqrt{3}x - y - 2 = 0)$ is m_2

$$\Rightarrow \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1.m_2}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{m_2}{m_1} - 1}{\frac{1}{m_1} + m_2}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = 30°$$

Q. 7. If the line $\frac{x-2}{2k} = \frac{y-3}{3} = \frac{z+2}{-1}$ and $\frac{x-2}{8} = \frac{y-3}{6} = \frac{z+2}{-2}$ are parallel, value of k is:

(A) –2 **(B)** $\frac{1}{2}$

(C) 2 **(D)** 4

Ans. Option (C) is correct.

Explanation: Given lines are $\frac{x-2}{2k} = \frac{y-3}{3} = \frac{z+2}{-1}$

and $\frac{x-2}{8} = \frac{y-3}{6} = \frac{z+2}{-2}$

The direction ratio of the first line is (2k, 3, –1) and the direction ratio of second line is (8, 6, –2)

Lines are parallel;

So, $\frac{2k}{8} = \frac{3}{6} = \frac{-1}{-2}$

$$\Rightarrow \quad \frac{k}{4} = \frac{1}{2} = \frac{1}{2}$$

$$\therefore \quad k = 2$$

Q. 8. If lines $\frac{2x-2}{2k} = \frac{4-y}{3} = \frac{z+2}{-1}$ and $\frac{x-5}{1} = \frac{y}{k} = \frac{z+6}{4}$ are at right angles, then the value of k is:

(A) –2 **(B)** 0

(C) 2 **(D)** 4

Ans. Option (A) is correct.

Explanation: Given lines are $\frac{2x-2}{2k} = \frac{4-y}{3} = \frac{z+2}{-1}$

and $\frac{x-5}{1} = \frac{y}{k} = \frac{z+6}{4}$

Writing the above equation in standard form, we get

$$\Rightarrow \quad \frac{2(x-1)}{2k} = \frac{-(y-4)}{3} = \frac{z+2}{-1}$$

$$\Leftrightarrow \quad \frac{(x-1)}{k} = \frac{y-4}{-3} = \frac{z+2}{-1}$$

Now, the direction ratio of the first line is (k, –3, –1) and the direction ratio of second line is (1, k, 4)

Since, lines are perpendicular,

$\therefore (k \times 1) + (-3 \times k) + (-1 \times 4) = 0$

$$\Rightarrow \quad k - 3k - 4 = 0$$

$$\Rightarrow \quad -2k - 4 = 0$$

$$\therefore \quad k = -2$$

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. Find the coordinates of the point where the line $\frac{x+3}{3} = \frac{y-1}{-1} = \frac{z-5}{-5}$ cuts the XY plane.

A I R&U [CBSE SQP 2020-21]

Sol. (0, 0, 0) 1

[CBSE SQP Marking Scheme 2020-21]

Detailed Solution:

In the XY plane, $z = 0$.

Hence the given equation becomes

$$\frac{x+3}{3} = \frac{y-1}{-1} = 1$$

$$\frac{x+3}{3} = 1$$

$$\Rightarrow \quad x = 0$$

$$\frac{y-1}{-1} = 1$$

$$\Rightarrow \quad y = 0$$

$\therefore$ The required point is (0, 0, 0).

Commonly Made Error

- Mostly students do not know how to find the point of intersection of a line and a plane.

Answering Tip

- Learn the concepts of lines and planes thoroughly.

Q. 2. Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.

R [CBSE Delhi Set-III, 2019]

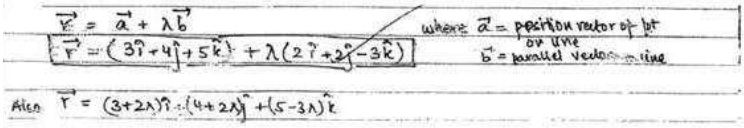
Sol. $\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$ 1

[CBSE Marking Scheme 2019]

OR

Topper Answer, 2019

Sol.


$\vec{r} = \vec{a} + \lambda\vec{b}$ where $\vec{a}$ = position vector of pt on line, $\vec{b}$ = parallel vector to line

$\vec{r} = (3\hat{i}+4\hat{j}+5\hat{k}) + \lambda(2\hat{i}+2\hat{j}-3\hat{k})$

Also $\vec{r} = (3+2\lambda)\hat{i} + (4+2\lambda)\hat{j} + (5-3\lambda)\hat{k}$

Q. 3. A line passes through the point with position vector $2\hat{i}-\hat{j}+4\hat{k}$ and is in the direction of the vector $\hat{i}+\hat{j}-2\hat{k}$. Find the equation of the line in cartesian form. [CBSE O.D. Set-I, 2019]

Sol. Equation of line are :

$$\frac{x-2}{1}=\frac{y+1}{1}=\frac{z-4}{-2} \quad 1$$

[CBSE Marking Scheme, 2019]

Detailed Solution :

Equation of a line passing through (x_1, y_1, z_1) and parallel to line having direction ratios *a, b, c* is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Since, the line passes through a point with position vector $2\hat{i}-\hat{j}+4\hat{k}$

$$x_1 = 2, y_1 = -1, z_1 = 4$$

Also, line is in the direction of $\hat{i}+\hat{j}-2\hat{k}$

Direction ratios : $a = 1, b = 1, c = -2$

Equation of line in cartesian form is :

$$\frac{x-2}{1} = \frac{y-(-1)}{1}=\frac{z-4}{(-2)}$$

$$\Rightarrow \quad \frac{x-2}{1} = \frac{y+1}{1}=\frac{z-4}{(-2)}$$

Q. 4. Find the cartesian equation of the line which passes through the point (– 2, 4, – 5) and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

A I R&U [SQP 2016-17] [NCERT]

Sol. Given line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$

[If two lines are parallel, then they both have proportional direction ratio]

or $$\frac{x-(-3)}{3} = \frac{y-4}{-5}=\frac{z-(-8)}{6} \quad ½$$

Here, given point is (– 2, 4, – 5) with D.R's. 3, – 5, 6

Therefore, cartesian equation of line will be :

$$\frac{x+2}{a} = \frac{y-4}{b}=\frac{z+5}{c}$$

or $$\frac{x+2}{3} = \frac{y-4}{-5}=\frac{z+5}{6} \quad ½$$

Q. 5. Find the vector equation for the line which passes through the point (1, 2, 3) and is parallel to the line $\frac{x-1}{-2}=\frac{1-y}{3}=\frac{3-z}{-4}$.

R&U [Outside Delhi Set I, II, III Comptt. 2016]

Q. 6. If the lines $\frac{x-1}{-2}=\frac{y-4}{3p}=\frac{z-3}{4}$ and $\frac{x-2}{4p}=\frac{y-5}{2}=\frac{z-1}{-7}$ are perpendicular to each other, then find the value of *p*.

R&U [S.Q.P. 2015]

Q. 7. The equation of a line is $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line. R&U [All India 2015]

Sol. Given equations of a line is ...(i)

$$5x-3 = 15y + 7 = 3-10z$$

Let us first convert the equation in standard form

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \quad ...(ii)$$

Let us divide Eq. (i) by LCM (coefficients of *x*, *y* and *z*), *i.e.*, LCM (5, 15, 10) = 30

Now, the Eq. (i) becomes

$$\frac{5x-3}{30} = \frac{15y+7}{30} = \frac{3-10z}{30}$$

or $$\frac{5\left(x-\frac{3}{5}\right)}{30} = \frac{15\left(y+\frac{7}{15}\right)}{30} = \frac{-10\left(z-\frac{3}{10}\right)}{30}$$

or $$\frac{x-\frac{3}{5}}{6} = \frac{y+\frac{7}{15}}{2} = \frac{z-\frac{3}{10}}{-3} \quad ½$$

On comparing the above equation with Eq. (ii), we get 6, 2, –3 are the direction ratios of the given line.

Now, the direction cosines of given line are

$$\frac{6}{\sqrt{6^2+2^2+(-3)^2}}, \frac{2}{\sqrt{6^2+2^2+(-3)^2}} \text{ and}$$

$$\frac{-3}{\sqrt{6^2+2^2+(-3)^2}}$$

i.e., $\left(\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}\right)$. ½

Short Answer Type Questions-I (2 marks each)

Q. 1. A(–1, 3, 2), B(–2, 3, –1), C(–5, –4, *p*) and D(–2, –4, 3), are four points in space. Lines AB and CD are parallel.
Find the value of *p*. Show your work and give valid reason. A I [CBSE Practice Questions 2022]

Sol. Given, lines AB and CD are parallel

$$\overrightarrow{AB} = (-2-(-1))\hat{i}+(3-3)\hat{j}+(-1-2)\hat{k}$$

or $\overrightarrow{AB} = -\hat{i}-3\hat{k}$ ...(i) ½

and $\overrightarrow{CD} = (-2-(-5))\hat{i}+(-4-(-4))\hat{j}+(3-p)\hat{k}$

or, $\overrightarrow{CD} = 3\hat{i}+(3-p)\hat{k}$ ...(ii) ½

We know that, if lines $\vec{a}=a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and $\vec{b}=a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$, are parallel then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, from eqs. (i) & (ii), we get

$$\frac{-1}{3} = \frac{-3}{3-p}$$

$\Rightarrow \quad 3-p = 9$

$\Rightarrow \quad p = 3-9$

$\Rightarrow \quad p = -6$ 1

Q. 2. Find the shortest distance between the following lines:

$$\vec{r} = (\hat{i}+\hat{j}-\hat{k})+s(2\hat{i}+\hat{j}+\hat{k})$$

$$\vec{r} = (\hat{i}+\hat{j}+2\hat{k})+t(4\hat{i}+2\hat{j}+2\hat{k})$$ A I R [SQP 2021-22]

Sol. Here, the lines are parallel. The shortest distance

$$= \left|\frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}}{|\vec{b}|}\right|$$

$$= \frac{|(3\hat{k})\times(2\hat{i}+\hat{j}+\hat{k})|}{\sqrt{4+1+1}}$$ 1 + ½

$$(3\hat{k})\times(2\hat{i}+\hat{j}+\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 3\\ 2 & 1 & 1\end{vmatrix} = -3\hat{i}+6\hat{j}$$ 1

Hence, the required shortest distance

$$= \frac{3\sqrt{5}}{\sqrt{6}} \text{ units}$$ ½

[CBSE Marking Scheme 2022]

Q. 3. Find the value of *k*, so that the lines $x = -y = kz$ and $x-2 = 2y+1 = -z+1$ are perpendicular to each other. U [CBSE Delhi Set II, 2020]

Topper Answer, 2020

Sol. $\frac{x}{k} = \frac{y}{-k} = \frac{z}{1}$ – line (i)

$\frac{x-2}{1} = \frac{y+1/2}{1/2} = \frac{z-1}{-1}$ – line (ii)

dR of line (i) $\langle k, -k, 1\rangle$

dR of line (ii) $\langle 1, 1/2, -1\rangle$

As they are perpendicular

$k - \frac{k}{2} - 1 = 0$

$\frac{k}{2} = 1 \Rightarrow k = 2$

Q. 4. Find the acute angle between the lines and

$$\frac{x-1}{4} = \frac{y+1}{-3} = \frac{z+10}{5}$$

R&U [CBSE SQP, 2020]

Sol. Vector in the direction of first line

$$\vec{b} = (3\hat{i}+4\hat{j}+5\hat{k})$$

Vector in the direction of second line

$$\vec{d} = (4\hat{i}-3\hat{j}+5\hat{k})$$

Angle θ between two lines is given by

$$\cos\theta = \frac{\vec{b}.\vec{d}}{|\vec{b}||\vec{d}|}$$

$$\cos\theta = \frac{(3\hat{i}+4\hat{j}+5\hat{k}).(4\hat{i}-3\hat{j}+5\hat{k})}{|(3\hat{i}+4\hat{j}+5\hat{k})|\,|(4\hat{i}-3\hat{j}+5\hat{k})|} \quad \mathbf{1}$$

$$\Rightarrow \quad \cos\theta = \frac{12-12+25}{\sqrt{9+16+25}\,\sqrt{9+16+25}}$$

$$\Rightarrow \quad \cos\theta = \frac{25}{\sqrt{50}\,\sqrt{50}} \quad ½$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3} \quad ½$$

Q. 5. Find the vector equation of the line passing through the point *A*(1, 2, – 1) and parallel to the line 5*x* – 25 = 14 – 7*y* = 35*z*. R&U **[Delhi 2017]**

Sol. Equation of given line is $\frac{x-5}{\frac{1}{5}} = \frac{y-2}{\frac{-1}{7}} = \frac{z}{\frac{1}{35}}$ ½

Its DR's $\left[\frac{1}{5}, -\frac{1}{7}, \frac{1}{35}\right]$ or $[7, -5, 1]$ ½

Equation of required line is

$$\hat{r} = (\hat{i}+2\hat{j}-\hat{k}) + \lambda(7\hat{i}-5\hat{j}+\hat{k}) \quad \mathbf{1}$$

[CBSE Marking Scheme, 2017]

Q. 6. Find the angle between the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x+1}{-3} = \frac{y-2}{2} = \frac{z-1}{0}$ R&U

Sol. Equation are

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} \quad ½$$

$$\frac{x+1}{-3} = \frac{y-2}{2} = \frac{z-1}{0}$$

$a_1 = 2, b_1 = 3, c_1 = 4$

$a_2 = -3, b_2 = 2, c_2 = 0$ ½

Now, $a_1a_2 + b_1b_2 + c_1c_2$

$= 2 \times -3 + 3 \times 2 + 4 \times 0$

$= -6 + 6 + 0 = 0$ **1**

∴ The lines are perpendicular to each other.

Q. 7. Find the equation of line passing through (1, 1, 2) and (2, 3, –1). R&U

Q. 8. Show that the points (2, 3, 4), (–1, –2, 1), (5, 8, 7) are collinear.

Sol. Let A(2, 3, 4), B(–1, –2, 1), C(5, 8, 7)

Direction ratios of line joining points A and B are

<– 1 – 2, – 2 – 3, 1 – 4> or <– 3, – 5, – 3> ½

Direction ratios of line joining points B and C are

< 5 – (–1), 8 – (–2), 7 – 1> or <6, 10, 6> ½

Let $a_1 = -3, b_1 = -5, c_1 = -3$

and $a_2 = 6, b_2 = 10, c_2 = 6$

Now, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

i.e., $\frac{-3}{6} = \frac{-5}{10} = \frac{-3}{6}$

or $\frac{-1}{2} = \frac{-1}{2} = \frac{-1}{2}$ **1**

Hence, given points are collinear.

Short Answer Type Questions-II (3 marks each)

Q. 1. The vector form of equations of two lines, l_1 and l_2 are

$$l_1 : \vec{r} = 2\hat{i} - \hat{k} + \lambda(-2\hat{j} + \hat{k})$$

$$l_2 : \vec{r} = \hat{i} + 3\hat{k} + 2\hat{k} + \mu(\hat{i} - 2\hat{k})$$

Show that l_1 and l_2 are skew lines.

[CBSE Practice Questions, 2022]

Sol. On comparing the given lines with

$\vec{r} = a_1 + \lambda b_1$ and $r_2 = a_2 + \lambda b_2$, we get

$a_1 = 2\hat{i} - \hat{k}$ and $b_1 = (-2\hat{i} + \hat{k})$

$a_2 = \hat{i} + 3\hat{j} + 2\hat{k}$ and $b_2 = (\hat{i} - 2\hat{k})$

Now, $\vec{a}_2 - \vec{a}_1 = (1-2)\hat{i} + (3-0)\hat{j} + (2-(-1))\hat{k}$

or, $\vec{a}_2 - \vec{a}_1 = -\hat{i} + 3\hat{j} + 3\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & 1 \\ 1 & 0 & -2 \end{vmatrix}$$

$$= \hat{i}(4-0) - \hat{j}(0-1) + \hat{k}(0-(-2))$$

$$= 4\hat{i} + \hat{j} + 2\hat{k} \quad \mathbf{1}$$

$$\therefore (\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = (-\hat{i} + 3\hat{j} + 3\hat{k}).(4\hat{i} + \hat{j} + 2\hat{k})$$

$$= -4 + 3 + 6 = 5$$

Since, $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) \neq 0$, then lines are not intersecting. **1**

Also, given lines are not parallel because

$$\frac{0}{1} \neq \frac{-2}{0} \neq \frac{1}{-2}$$

Since, the given lines are neither parallel nor intersecting, hence skew lines. **1**

Q. 2. Find the value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles.

Also, find whether the lines are intersecting or not.

R&U **[CBSE Delhi Set-III 2019]**

Sol. Given lines are :

$$\frac{x-1}{-3}=\frac{y-2}{\left(\frac{\lambda}{7}\right)}=\frac{z-3}{2} \text{ and } \frac{x-1}{-\left(\frac{3\lambda}{7}\right)}=\frac{y-5}{1}=\frac{z-6}{-5}$$ ½

As lines are perpendicular,

$$(-3)\left(\frac{-3\lambda}{7}\right)+\left(\frac{\lambda}{7}\right)(1)+2(-5)=0 \Rightarrow \lambda=7$$ ½

So, lines are

$$\frac{x-1}{-3}=\frac{y-2}{1}=\frac{z-3}{2} \text{ and } \frac{x-1}{-3}=\frac{y-5}{1}=\frac{z-6}{-5}$$ ½

Consider

$$\Delta=\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix}=-63$$ 1

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. ½

[CBSE Marking Scheme, 2019] (Modified)

Detailed Solution :

Topper Answer, 2019

Line I: $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2}$ Line II: $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$

$\Rightarrow \frac{x-1}{-3}=\frac{y-2}{\lambda/7}=\frac{z-3}{2}=\beta$ $\frac{x-1}{-3\lambda/7}=\frac{y-5}{1}=\frac{z-6}{-5}=\mu$

Direction ratios are $(-3, \frac{\lambda}{7}, 2)$ and $(\frac{-3\lambda}{7}, 1, -5)$ respectively
(a_1, a_2, a_3) (b_1, b_2, b_3)

For lines to be ⊥, $\vec{a}\cdot\vec{b}=0$ where $\vec{a}, \vec{b}$ are ||el vectors of lines

$a_1b_1+a_2b_2+a_3b_3=0$

$\frac{9\lambda}{7}+\frac{\lambda}{7}-10=0$

$\frac{10\lambda}{7}=10$

$\boxed{\lambda=7}$

∴ For λ = 7, lines are ⊥.

Any point of line I is $(-3\beta+1, \beta+2, 2\beta+3)$

line II is $(-3\mu+1, \mu+5, -5\mu+6)$

For lines to intersect, they should be equal.

$-3\beta+1=-3\mu+1$ $\mu+5=\beta+2$ $2\beta+3=-5\mu+6$

$3\mu-3\beta=0$ $\mu-\beta+3=0$ —(3) $2\beta+5\mu=3$ —(2)

$\mu=\beta$ —(1)

From (1), (2)

$7\mu=3$

$\boxed{\mu=\frac{3}{7}}$ $\boxed{\beta=\frac{3}{7}}$

In eqn(3)

$\frac{3}{7}-\frac{3}{7}+3 \neq 0$

∴ Values do not satisfy eqn(3)

Q. 3. If the lines $\frac{x-1}{-3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3\lambda}=\frac{y-1}{2}=\frac{z-6}{-5}$ are perpendicular, find the value of λ. Hence find whether the lines are intersecting or not.

[CBSE OD Set-I, 2019]

Q. 4. Find the vector equation of the line joining (1, 2, 3) and (– 3, 4, 3) and show that it is perpendicular to the *Z*-axis. R&U [CBSE SQP 2018-19]

Sol. Vector equation of the line passing through (1, 2, 3) and (–3, 4, 3) is $\vec{r} = \vec{a}+\lambda(\vec{b}-\vec{a})$ where

$\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b} = -3\hat{i}+4\hat{j}+3\hat{k}$

$$\Rightarrow \quad \vec{r} = \hat{i}+2\hat{j}+3\hat{k}+\lambda\left(-4\hat{i}+2\hat{j}\right) \quad ...(1) \; \mathbf{1}$$

Equation of Z-axis is

$$\vec{r} = \mu\hat{k} \quad ...(2) \; \mathbf{1}$$

Since $\left(-4\hat{i}+2\hat{j}\right).\hat{k} = 0$

$\therefore$ line (1) is $\perp$ to Z-axis. **1**

[CBSE Marking Scheme 2018-19] (Modified)

Q. 5. Find the equation of the line which intersects the lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and passes through the point (1, 1, 1).

R&U [SQP 2017-18]

Sol. General point on the first line is

$(\lambda-2, 2\lambda+3, 4\lambda-1)$.

General point on the second line is

$(2\mu+1, 3\mu+2, 4\mu+3)$. ½

Direction ratios of the required line are

$(\lambda-3, 2\lambda+2, 4\lambda-2)$. ½

Direction rations of the same line may be

$(2\mu, 3\mu+1, 4\mu+2)$. ½

Therefore, $\frac{\lambda-3}{2\mu} = \frac{2\lambda+2}{3\mu+1} = \frac{4\lambda-2}{4\mu+2}$ ....(1)

or $\frac{\lambda-3}{2\mu} = \frac{2\lambda+2}{3\mu+1} = \frac{2\lambda-1}{2\mu+1} = k$ (say)

or $\lambda-3 = 2\mu k$, $2\lambda+2 = (3\mu+1)k$, $2\lambda-1 = (2\mu+1)k$

or $\frac{\lambda-3}{2} = \mu k$, $2\lambda+2 = 3\left(\frac{\lambda-3}{2}\right)+k$,

$2\lambda-1 = 2\left(\frac{\lambda-3}{2}\right)+k$

or $k = \frac{4\lambda+4-3\lambda+9}{2} = \lambda+2$ or $\lambda = 9$, $\mu = \frac{3}{11}$,

which satisfy (1). ½

Therefore, the direction ratios of the required line are (6, 20, 34) or, (3, 10, 17). ½

Hence, the required equation of line is

$$\frac{x-1}{3} = \frac{y-1}{10} = \frac{z-1}{17}. \quad ½$$

[CBSE Marking Scheme, 2017] (Modified)

Q. 6. Find the value of *p*, so that the lines

$$l_1: \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$$

and $$l_2: \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

are perpendicular to each other.

Also find the equation of a line passing through a point (3, 2, – 4) and parallel to line l_1.

R&U [NCERT] [Delhi Comptt., 2017]

Sol. Given lines can be written as

$$l_1 = \frac{x-1}{-3} = \frac{y-2}{\frac{p}{7}} = \frac{z-3}{2} \quad ½$$

and $$l_2 = \frac{x-1}{\left(\frac{-3p}{7}\right)} = \frac{y-5}{1} = \frac{z-6}{-5} \quad ½$$

Since the lines are perpendicular,

$$\therefore \; (-3)\left(-\frac{3p}{7}\right)+\left(\frac{p}{7}\right)(1)+(2)(-5)=0$$

or $p = 7$ **1**

Equation or line passing through (3, 2, – 4) and parallel to l_1 is

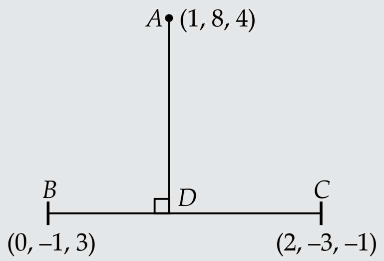
$$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2} \quad \mathbf{1}$$

[CBSE Marking Scheme 2017] (Modified)

Q. 7. Find the co-ordinates of the foot of perpendicular drawn from a point *A* (1, 8, 4) to the line joining the points *B*(0, – 1, 3) and *C*(2, – 3, –1).

R&U [NCERT Exemplar][O.D. Comptt. 2017]

Sol.



Equation of line passing through *B* and *C* is

$$\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4}$$

or $$\frac{x}{1} = \frac{y+1}{-1} = \frac{z-3}{-2} \quad \mathbf{1}$$

Any point *D* on *BC* can be $[\lambda, -\lambda-1, -2\lambda+3]$ for some value of λ.

∴ Direction ratios of AD are $(\lambda - 1, -\lambda - 9, -2\lambda - 1)$ ½

$AD \perp BC$ or $1(\lambda - 1) - 1(-\lambda - 9) - 2(-2\lambda - 1) = 0$ ½

or $\lambda = -\frac{5}{3}$ ½

∴ D is $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ ½

[CBSE Marking Scheme, 2017] (Modified)

Q. 8. Prove that the line through $A(0, -1, -1)$ and $B(4, 5, 1)$ intersects the line through $C(3, 9, 4)$ and $D(-4, 4, 4)$. R&U [Foreign 2016]

Sol. The equation of line through A (0, –1, –1) and B (4, 5, 1) is

$$\frac{x-0}{4-0} = \frac{y+1}{5+1} = \frac{z+1}{1+1}$$

i.e., $$\frac{x}{4} = \frac{y+1}{6} = \frac{z+1}{2} \quad ...(i)$$

and equation of line through C (3, 9, 4) and D (–4, 4, 4) is

$$\frac{x-3}{-4-3} = \frac{y-9}{4-9} = \frac{z-4}{0}$$

i.e., $$\frac{x-3}{-7} = \frac{y-9}{-5} = \frac{z-4}{0} \quad ...(ii)$$ **1**

We know that, the lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \text{ and}$$

$$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c^2} \text{ will intersect,}$$

if $$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

∴ The given lines will intersect, if

$$\begin{vmatrix} 3-0 & 9-(-1) & 4-(-1) \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix} = 0$$ **1**

Now consider,

$$\begin{vmatrix} 3-0 & 9-(-1) & 4-(-1) \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 10 & 5 \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}$$

$= 3(0 + 10) - 10(0 + 14) + 5(-20 + 42)$

$= 30 - 140 + 110 = 0$

Hence, the given lines intersect. **1**

Q. 9. Find the shortest distance between the lines :

$$\vec{r} = (t+1)\hat{i} + (2-t)\hat{j} + (1+t)\hat{k}$$

$$\vec{r} = (2s+2)\hat{i} - (1-s)\hat{j} + (2s-1)\hat{k}.$$

A I A [Delhi Set I, II, III, Comptt. 2016]

Q. 10. Find the vector and cartesian equations of line through the point (1, 2, – 4) and perpendicular to the two lines

$$\vec{r} = (8\hat{i} - 9\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k}) \text{ and}$$

$$\vec{r} = 15\hat{i} - 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$$ R&U [NCERT]

[OD 2015] [Delhi Set I, II, III 2016]

Sol. Given Lines :

$$\vec{r} = (8\hat{i} - 9\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

$$\vec{r} = 15\hat{i} - 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$$

The required line passes through the point (1, 2, – 4) and is perpendicular to the above two lines.

Let a, b, c denote the direction ratios of the required line, then using $a_1a_2 + b_1b_2 + c_1c_2 = 0$, we get

$3a - 16b + 7c = 0$ ½+½

and $3a + 8b - 5c = 0$

Solving,

$$\frac{a}{(80-56)} = \frac{-b}{(-15-21)} = \frac{c}{24+48} = \lambda \text{ (Let)}$$

or $$\frac{a}{24} = \frac{b}{36} = \frac{c}{72} = \lambda$$ ½

or $$\frac{a}{2} = \frac{b}{3} = \frac{c}{6} = (12\lambda) = \lambda'$$ ½

∴ The equation of the required line in the vector form :

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda'(2\hat{i} + 3\hat{j} + 6\hat{k})$$ ½

Cartesian from : $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ ½

Long Answer Type Questions (5 marks each)

Q. 1. Find the shortest distance between the lines

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

If the lines intersect find their point of intersection.

A I R&U [CBSE SQP 2020-21]

Sol. We have

$a_1 = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$b_1 = \hat{i} + 2\hat{j} + 2\hat{k}$

$a_2 = 5i - 2j$

$b_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$$\overrightarrow{a_2}-\overrightarrow{a_1} = 2\hat{i}-4\hat{j}+4\hat{k} \quad \mathbf{1}$$

$$\overrightarrow{b_1}\times\overrightarrow{b_2} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 2\\ 3 & 2 & 6\end{vmatrix}$$

$$= \hat{i}(12-4)-\hat{j}(6-6)+\hat{k}(2-6)$$

1

$$\overrightarrow{b_1}\times\overrightarrow{b_2} = 8\hat{i}+0\hat{j}-4\hat{k} = 8\hat{i}-4\hat{k}$$

$$\because \quad (\overrightarrow{b_1}\times\overrightarrow{b_2}).(\overrightarrow{a_2}-\overrightarrow{a_1}) = 16-16 = 0 \quad \mathbf{1}$$

$\therefore$ The lines are intersecting and the shortest distance between the lines is 0.

Now for point of intersection

$$3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$$

$$= 5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$$

$$\Rightarrow \quad 3+\lambda = 5+3\mu \quad ...(i)$$

$$2+2\lambda = -2+2\mu \quad ...(ii)$$

$$-4+2\lambda = 6\mu \quad ...(iii) \ \mathbf{1}$$

Solving (i) and (ii) we get $\mu = -2$ and $\lambda = -4$

Substituting in equation of line we get

$$\vec{r} = 5i-2j+(-2)(3\hat{i}+2\hat{j}-6\hat{k})$$

$$= -\hat{i}-6\hat{j}-12\hat{k}$$

Point of intersection is (–1, –6, –12) **1**

[CBSE SQP Marking Scheme 2020-21]

Commonly Made Error

- Some students write the wrong values of a_1, a_2 and b_1, b_2 that is why the answer of shortest distance is wrongly calculated.

Answering Tips

- First student should write the question in standard form $\vec{r}+\overrightarrow{a_1}+\lambda\overrightarrow{b_1}, \vec{r}=\overrightarrow{a_2}+\mu_1\overrightarrow{b_2}$ then solve the shortest distance.

Q. 2. Find the vector and cartesian equations of the line which is perpendicular to the lines with equations

$$\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4} \text{ and } \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

and passes through the point (1, 1, 1). Also find the angle between the given lines.

A I R&U [CBSE Delhi Set I, II, III-2020]

Sol. Let equation of required line is

$$\frac{x-1}{a} = \frac{y-1}{b}=\frac{z-1}{c} \quad ...(i)\ ½$$

Since the line is perpendicular to

$$\frac{x+2}{1} = \frac{y-3}{2}=\frac{z+1}{4}$$

and
$$\frac{x-1}{2} = \frac{y-2}{3}=\frac{z-3}{4},$$

$$a+2b+4c = 0 \quad ...(ii)$$

$$2a+3b+4c = 0 \quad ...(iii)\ ½$$

Solving (ii) and (iii),

$$\frac{a}{-4} = \frac{b}{4}=\frac{c}{-1}$$

$\therefore$ DR's of line in cartesian form is : –4, 4, –1 ½

Equation of line in Cartesian form is :

$$\frac{x-1}{-4} = \frac{y-1}{4}=\frac{z-1}{-1} \quad \mathbf{1}$$

Vector form of line is

$$\vec{r} = (\hat{i}+j+k)+\lambda(-4\hat{i}+4j-k) \quad \mathbf{1}$$

Let θ be the angle between given lines.

$$\cos\theta = \frac{1(2)+2(3)+4(4)}{\sqrt{1+4+16}\sqrt{4+9+16}}$$

$$= \frac{24}{\sqrt{21}\sqrt{29}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{24}{\sqrt{609}}\right) \quad \mathbf{1+½}$$

[CBSE Marking Scheme 2020] (Modified)

Commonly Made Error

- Most often students get wrong with the conditions for perpendicularity.

Answering Tips

- The angle between lines and their various conditions should be learned thoroughly.

Q. 3. Find the foot of perpendicular from $P(1, 2, -3)$ to the line $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}$. Also, find the image of P in the given line.

A I R&U [Outside Delhi Set I, II, III, Comptt. 2016]

COMPETENCY BASED QUESTIONS

Case based MCQs

I. Read the following text and answer the following questions on the basis of the same:

The equation of motion of a missile are $x = 3t$, $y = -4t$, $z = t$, where the time 't' is given in seconds, and the distance is measured in kilometres.

[CBSE QB 2021]

Q. 1. What is the path of the missile?

(A) Straight line **(B)** Parabola
(C) Circle **(D)** Ellipse

Ans. Option (A) is correct.

Q. 2. Which of the following points lie on the path of the missile at $t = 2$ second ?

(A) (6, 8, 2) **(B)** (6, –8, –2)
(C) (6, –8, 2) **(D)** (–6, –8, 2)

Ans. Option (C) is correct.

Explanation: (6, –8, 2) point lie on the path of the missile.

$\because \quad x = 3t,\ y = -4t,\ z = t$

at $\quad t = 2$

$x = 6,\ y = -8,\ z = 2$

i.e., (6, –8, 2)

Q. 3. At what distance will the rocket be from the starting point (0, 0, 0) in 5 seconds?

(A) $\sqrt{550}$ kms **(B)** $\sqrt{650}$ kms
(C) $\sqrt{450}$ kms **(D)** $\sqrt{750}$ kms

Ans. Option (B) is correct.

Explanation: Here,

$$t = 5 \text{ seconds}$$
$$x = 3t = 3 \times 5 = 15$$
$$y = -4t = -4 \times 5 = -20$$
$$z = t = 5$$
$$(x, y, z) \equiv (15, -20, 5)$$

Distance from starting point (0, 0, 0)

$$= \left|\sqrt{(15-0)^2 + (-20-0)^2 + (5-0)^2}\right|$$
$$= \left|\sqrt{225 + 400 + 25}\right|$$
$$= \sqrt{650} \text{ kms} = 5\sqrt{26} \text{ kms}$$

Q. 4. If the position of rocket at a certain instant of time is (5, –8, 10), then what will be the height of the rocket from the ground? (The ground is considered as the *XY*-plane).

(A) 12 km **(B)** 11 km
(C) 20 km **(D)** 10 km

Ans. Option (D) is correct.

Explanation: Height of the rocket from the ground (*i.e.*, *XY*-plane)

$= 10$ km

Q. 5. The equation of *Y*-axis in space are

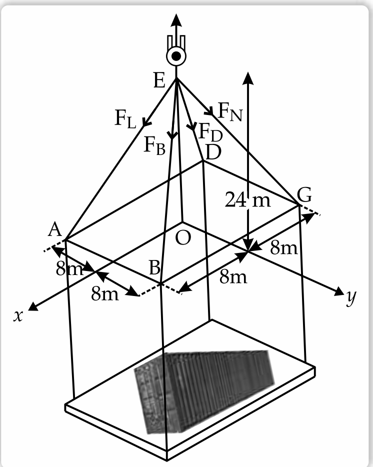
(A) $x = 0, y = 0$ **(B)** $x = 0, z = 0$
(C) $y = 0, z = 0$ **(D)** $y = 0$

Ans. Option (B) is correct.

Explanation: As on the *Y*-axis, *X*-coordinate and *Z*-coordinates are zeroes.

II. Read the following text and answer the following questions on the basis of the same:

Rohan wants to prepare a model for the science exhibition. He wanted to show something about the forces. He prepared a model for picking heavy object as shown below, where the forces in the cable are given.



Q. 1. The coordinates of points A and E are:

(A) (8, –6, 0) and (0, 0, 24)
(B) (8, 6, 0) and (0, 0, 24)
(C) (6, –8, 0) and (0, 24, 0)
(D) (6, –8, 0) and (8, 6, 24)

Ans. Option (A) is correct.

Explanation: From the given figure, it is clear that the coordinates of points A and E are (8, –6, 0) and (0, 0, 24) respectively.

Q. 2. The cartesian equation of line along EA is

(A) $\frac{x}{-4} = \frac{y}{3} = \frac{z}{12}$ **(B)** $\frac{x}{-4} = \frac{y}{3} = \frac{z-24}{12}$

(C) $\frac{x}{-3} = \frac{y}{4} = \frac{z-12}{12}$ **(D)** $\frac{x}{3} = \frac{y}{4} = \frac{z-24}{12}$

Ans. Option (B) is correct.

Explanation: Since, coordinates of A and E are (8, –6, 0) and (0, 0, 24).

Thus, the equation of line passing through (8, –6, 0) and (0, 0, 24) is:

$$\frac{x-0}{8-0} = \frac{y-0}{-6-0} = \frac{z-24}{0-24}$$

$$\Rightarrow \quad \frac{x}{8} = \frac{y}{-6} = \frac{z-24}{-24}$$

$$\Rightarrow \quad \frac{x}{-4} = \frac{y}{3} = \frac{z-24}{12}$$

Q. 3. The vector ED is

(A) $8\hat{i} - 6\hat{j} + 24\hat{k}$ (B) $-8\hat{i} - 6\hat{j} + 24\hat{k}$

(C) $-8\hat{i} - 6\hat{j} - 24\hat{k}$ (D) $8\hat{i} + 6\hat{j} + 24\hat{k}$

Ans. Option (C) is correct.

Explanation: Here, coordinates of D and E are (–8, –6, 0) and (0, 0, 24).

$\therefore$ Vector $\overrightarrow{ED}$ is $(-8-0)\hat{i} + (-6-0)\hat{j} + (0-24)\hat{k}$

i.e., $-8\hat{i} - 6\hat{j} - 24\hat{k}$

Q. 4. The length of the cable EB is

(A) 24 units (B) 26 units

(C) 27 units (D) 25 units

Ans. Option (B) is correct.

Explanation: Here, coordinates of B and E are (8, 6, 0) and (0, 0, 24).

$\therefore$ Length of cable,

$$EB = \left|\sqrt{(8-0)^2 + (6-0)^2 + (0-24)^2}\right|$$

$$= \left|\sqrt{64 + 36 + 576}\right| = 26 \text{ units}$$

Q. 5. The sum of all vectors along the cables is

(A) $96\hat{i}$ (B) $96\hat{j}$

(C) $-96\hat{k}$ (D) $96\hat{k}$

Ans. Option (C) is correct.

Explanation: Sum of all vectors along the cables

$$= \overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$$

$$= (8\hat{i} - 6\hat{j} - 24\hat{k}) + (8\hat{i} + 6\hat{j} - 24\hat{k}) + (-8\hat{i} + 6\hat{j} - 24\hat{k}) + (-8\hat{i} - 6\hat{j} - 24\hat{k})$$

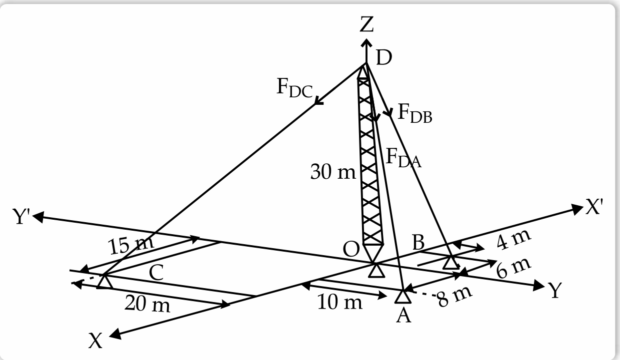
$$= -96\hat{k}$$

Case based Subjective Questions

I. Read the following text and answer the following questions on the basis of the same:

A pillar is to be constructed on a field. Mahesh is an Engineer for that project. This was Mahesh's first project after completing his Engineering. He draws the following diagram of that pillar for the approval.

Consider the following diagram, where the forces in the cable are given.



Q. 1. Find the equation of the line along the cable AD.

Sol. From the given figure, it is clear that the coordinates of points A and D are (8, 10, 0) and (0, 0, 30), respectively. **1**

Thus, the equation of line passing through (8, 10, 0) and (0, 0, 30) is:

$$\frac{x-0}{8-0} = \frac{y-0}{10-0} = \frac{z-30}{0-30}$$

$$\Rightarrow \quad \frac{x}{8} = \frac{y}{10} = \frac{z-30}{-30}$$

$$\Rightarrow \quad \frac{x}{4} = \frac{y}{5} = \frac{z-30}{-15}$$

$$\Rightarrow \quad \frac{x}{4} = \frac{y}{5} = \frac{30-z}{15}$$ **1**

Q. 2. Find the sum of the distances OA, OB and OC.

Sol. Here, Coordinates of O = (0, 0, 0)

Coordinates of A = (8, 10, 0)

Coordinates of B = (–6, 4, 0)

Coordinates of C = (15, –20, 0)

Also, $OA = \left|\sqrt{8^2 + 10^2}\right| = \sqrt{164}$

$OB = \left|\sqrt{6^2 + 4^2}\right|$

$= \left|\sqrt{36 + 16}\right| = \sqrt{52}$

and $OC = \left|\sqrt{15^2 + 20^2}\right|$

$= \left|\sqrt{225 + 400}\right|$

$= \sqrt{625} = 25$ **1**

$\therefore$ The sum of the distances OA, OB and OC

$= \sqrt{164} + \sqrt{52} + 25$ ½

II. Read the following text and answer the question on the basis of the same.

A motor cycle race was organized in a town, where the maximum speed limit was set by the organizers. No participant are allowed to cross the specified speed limit, but

Two motorcycles A and B are running at the speed more than allowed speed on the road along the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$, respectively.

Q. 1. Find the Cartesian equation of the line along which motorcycle A is running.

Sol. The line along which motorcycle A is running, is, $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$, which can be rewritten as

$$(x\hat{i} + y\hat{j} + z\hat{k}) = \lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$$

$\Rightarrow \quad x = \lambda, y = 2\lambda, z = -\lambda$

$\Rightarrow \quad \frac{x}{1} = \lambda, \frac{y}{2} = \lambda, \frac{z}{-1} = \lambda$ **1**

Thus, the required cartesian equation is $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$

1

Q. 2. Find the shortest distance between the lines. 2

Sol. Here, $\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}, \vec{a}_2 = 3\hat{i} + 3\hat{j}, \vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$,

$\vec{b}_2 = 2\hat{i} + \hat{j} + \hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$

and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 3\hat{k}$

Now, $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j}).(3\hat{i} - 3\hat{j} - 3\hat{k})$

$= 9 - 9 = 0$

Hence, shortest distance between the given lines is 0.

Solutions for Practice Questions (Topic-1)

Very Short Answer Type Questions

6. $\sqrt{(-5)^2 + (12)^2} = 13$ **1**

[CBSE Marking Scheme, 2017]

7. We know that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ½

or $\cos^2 90° + \cos^2 60° + \cos^2\theta = 1$

$0 + \frac{1}{4} + \cos^2\theta = 1$

$\cos\theta = \frac{\sqrt{3}}{2}$

or $\theta = 30°$ ½

[CBSE Marking Scheme 2017]

8. Distance of the point (p, q, r) from the X-axis

= Distance of the point (p, q, r) from the point$(p, 0, 0)$

$= \left|\sqrt{q^2 + r^2}\right|$ units **1**

[CBSE Marking Scheme 2016]

Short Answer Type Questions-I

1. The given line is

$$\frac{x-3}{1} = \frac{y - \frac{1}{2}}{1} = \frac{2}{4}$$ **1**

Its direction ratios are <1, 1, 4> ½

Its direction cosines are

$< \frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}} >$ ½

[CBSE Marking Scheme 2022]

4. Let A(–2, 4, –5) and B(1, 2, 3)

$\therefore \quad AB = \left|\sqrt{(1+2)^2 + (2-4)^2 + (3+5)^2}\right|$

$= \left|\sqrt{9+4+64}\right| = \sqrt{77}$ **1**

Therefore, direction cosines of the line joining two points are:

$\frac{1+2}{\sqrt{77}}, \frac{2-4}{\sqrt{77}}, \frac{3+5}{\sqrt{77}}$

or, $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$ **1**

Solutions for Practice Questions (Topic-2)

Very Short Answer Type Questions

5. $\vec{r} = \hat{i}+2\hat{j}+3\hat{k}+\lambda(-2\hat{i}-3\hat{j}+4\hat{k})$. **1**

[CBSE Marking Scheme 2016]

6. Using formula

$l_1l_2 + m_1m_2 + n_1n_2 = 0$ ½

or $-8p + 6p - 28 = 0$

or $-2p = 28$

$\therefore$ $p = -14$ ½

Short Answer Type Questions-I

7. Equation of line

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$ ½

$$\frac{x-1}{2-1} = \frac{y-1}{3-1} = \frac{z-2}{-1-2}$$ ½

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{-3}$$ **1**

Short Answer Type Questions-II

3. Lines are perpendicular

$\therefore -3(3\lambda) + 2\lambda(2) + 2(-5) = 0 \Rightarrow \lambda = -2$ **1**

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_3 \end{vmatrix} = \begin{vmatrix} 1-1 & 1-2 & 6-3 \\ -3 & 2(-2) & 2 \\ 3(-2) & 2 & -5 \end{vmatrix}$$ 1½

½

$$= -63 \neq 0$$

Lines are not intersecting

[CBSE Marking Scheme, 2019] (Modified)

Detailed Solution :

The equations of the given lines are :

$$\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$$

and $$\frac{x-1}{3\lambda} = \frac{y-1}{2} = \frac{z-6}{-5}$$

On comparing these lines with $\dfrac{x-x_1}{a_1} = \dfrac{y-y_1}{b_1} = \dfrac{z-z_1}{c_1}$ and $\dfrac{x-x_2}{a_2} = \dfrac{y-y_2}{b_2} = \dfrac{z-z_2}{c_2}$, we get

$a_1 = -3, b_1 = 2\lambda, c_1 = 2$ and $a_2 = 3\lambda, b_2 = 2, c_2 = -5$

Since, lines are perpendicular,

So, $(-3)(3\lambda) + (2\lambda)(2) + (2)(-5) = 0$

$\Rightarrow -9\lambda + 4\lambda - 10 = 0$

$\Rightarrow -5\lambda - 10 = 0$

$\Rightarrow -5\lambda = 10$

$\Rightarrow \lambda = -2$

Hence, for $\lambda = -2$ the given lines are perpendicular.

Now, given lines can be written as after substituting value of $\lambda = -2$

$$\frac{x-1}{-3} = \frac{y-2}{-4} = \frac{z-3}{2}$$

and $$\frac{x-1}{-6} = \frac{y-1}{2} = \frac{z-6}{-5}$$

The coordinate of any point on first line are given by

$$\frac{x-1}{-3} = \frac{y-2}{-4} = \frac{z-3}{2} = r \quad \text{(say)}$$

or $x = -3r + 1, y = -4r + 2, z = 2r + 3$

So, the coordinates of a general point on first line are $(-3r + 1, -4r + 2, 2r + 3)$.

The coordinates of any point on second line are given by

$$\frac{x-1}{-6} = \frac{y-1}{2} = \frac{z-6}{-5} = s$$

or $x = -6s + 1, y = 2s + 1, z = -5s + 6$

So, the coordinates of a general point on second line are $(-6s + 1, 2s + 1, -5s + 6)$

If the lines intersect, then they have a common point. So, for some value of λ and μ, we must have

$-3r + 1 = -6s + 1, -4r + 2 = 2s + 1, 2r + 3 = -5s + 6$

or $r = 2s, -4r - 2s = -1, 2r + 5s = 3$

Solving first two of these two equations, we get $r = \dfrac{1}{5}$ and $s = \dfrac{1}{10}$. These values of r and s do not satisfy the third equation.

Hence, the given lines do not intersect.

Commonly Made Error

- Some students compute the shortest distance to show that the lines are intersecting.

Answering Tips

- Learn the concepts of parallel, perpendicular, skew and intersecting lines.

9. Equations of lines can be written as :

$$\vec{r} = (\hat{i}+2\hat{j}+\hat{k}) + t(\hat{i}-\hat{j}+\hat{k});$$

$$\vec{r} = (2\hat{i}-\hat{j}-\hat{k}) + s(2\hat{i}+\hat{j}+2\hat{k})$$ ½

Let, $\vec{a}_1 = \hat{i}+2\hat{j}+\hat{k}$,

$\vec{b}_1 = \hat{i}-\hat{j}+\hat{k}$,

$\vec{a}_2 = 2\hat{i}-\hat{j}-\hat{k}$,

$$\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$$ ½

Then, $\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k},$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= -3\hat{i} + 3\hat{k}$$ 1

$\therefore$ Shortest distance

$$= \left| \frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$= \left| \frac{-3-6}{\sqrt{9+9}} \right|$$

$$= \frac{3}{\sqrt{2}} \text{ or } \frac{3\sqrt{2}}{2} \text{ units}$$ 1

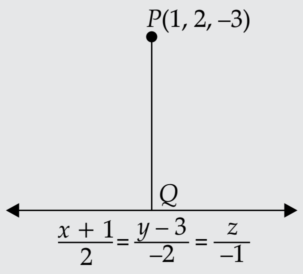
[CBSE Marking Scheme 2016] (Modified)

Long Answer Type Questions

3. Any point on the given line is

$(2\lambda - 1, -2\lambda + 3, -\lambda)$ *i* if this point is Q then ½

$$\vec{PQ} = (2\lambda - 2)\hat{i} + (-2\lambda + 1)\hat{j} + (-\lambda + 3)\hat{k}$$ ½



Since $\vec{PQ}$ is perpendicular to the line

$\therefore 2(2\lambda - 2) - 2(-2\lambda + 1) - 1(-\lambda + 3) = 0$

or $\lambda = 1$ 1

$\therefore$ Foot of perpendicular is $Q(1, 1, -1)$

Let $P'(x., y, z)$ be the image of P in the line, then

$$\frac{x+1}{2} = 1,$$ ½

$$\frac{y+2}{2} = 1$$ 1

$$\frac{z-3}{2} = -1$$

or $x = 1, y = 0, z = 1$ 1

or Image P is (1, 0, 1). ½

[CBSE Marking Scheme 2016] (Modified)

REFLECTIONS

- Recognize three dimensional shapes and their environment.
- Create composite shapes.
- Describe attributes of original and composite shapes.
- Compare and contrast original and created composite shapes.
- Can you easily recognize 3D shapes and their environment?
- Will you be able to create composite shapes?
- Will you be able to handle the complex situations of 3D geometry?

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SELF ASSESSMENT PAPER - 04

Time: 1 hour **MM: 30**

UNIT-IV

(A) OBJECTIVE TYPE QUESTIONS:

I. Multiple Choice Questions **[1×6 = 6]**

Q. 1. A point from a vector starts is called .... and where it ends is called its ....

(A) terminal point, end point

(B) initial point, terminal point

(C) origin, end point

(D) initial point, end point

Q. 2. The vector equation of line through the points(1, –1, 6) and (4, 3, – 1) is ……….

(A) $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$

(B) $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(\hat{i} - \hat{j} + 6\hat{k})$

(C) $\vec{r} = (\hat{i} - \hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 4\hat{j} - 7\hat{k})$

(D) $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(4\hat{i} + 3\hat{j} - \hat{k})$

Q. 3. The equation of a line, which is parallel to $2\hat{i} + \hat{j} + 3\hat{k}$ and which passes through the point (5, –2, 4) is ....

(A) $\frac{x+5}{2} = \frac{y-2}{1} = \frac{z-4}{3}$ **(B)** $\frac{x-5}{2} = \frac{y-2}{1} = \frac{z+4}{3}$

(C) $\frac{x+5}{2} = \frac{y-2}{1} = \frac{z+4}{3}$ **(D)** $\frac{x-5}{2} = \frac{y+2}{1} = \frac{z-4}{3}$

Q. 4. The angle between the unit vectors $\hat{a}$ and $\hat{b}$ so that $\sqrt{3}\hat{a} - \hat{b}$ is also a unit vector is ………..

(A) 90° **(B)** 60° **(C)** 30° **(D)** 45°

Q. 5. The area of a triangle formed by vertices *O*, *A* and *B*, where $\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is

(A) $3\sqrt{5}$ sq. units **(B)** $5\sqrt{5}$ sq. units **(C)** $6\sqrt{5}$ sq. units **(D)** 4 sq. units

Q. 6. The position vectors of two points A and B are $\vec{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively, The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is _________

(A) $2\hat{i} - \hat{j} + \hat{k}$ **(B)** $2\hat{i} + \hat{j} + \hat{k}$ **(C)** $2\hat{i} - \hat{j} - \hat{k}$ **(D)** $-2\hat{i} + \hat{j} + \hat{k}$

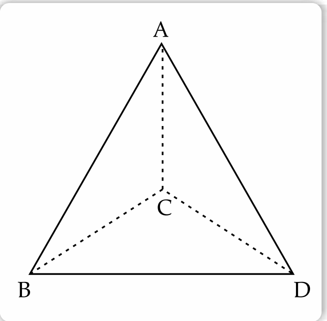
II. Case-Based MCQs **[1×4 = 4]**

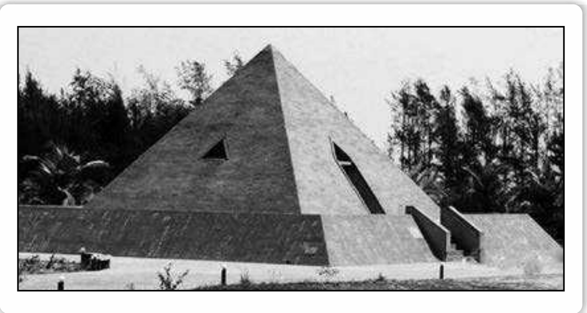
Attempt any 4 sub-parts from each questions. Each question carries 1 mark.

Read the following text and answer the following questions on the basis of the same.

A building of a multinational company is to be constructed in the form of a triangular pyramid, ABCD as shown in the figure.

A Shanghai temple is in the form of a triangular pyramid with floor ABD and vertex C.





Let its angular points are A(3, 0, 1), B(–1, 4, 1), C(5, 2, 3) and D(0, –5, 4) and G be the point of intersection of the medians of ΔBCD.

Based on the above data, answer the following.

Q. 7. The coordinates of points G are

(A) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ **(B)** $\left(0, \frac{1}{2}, \frac{1}{3}\right)$ **(C)** $\left(\frac{4}{3}, \frac{1}{3}, \frac{8}{3}\right)$ **(D)** $\left(\frac{4}{3}, \frac{8}{3}, \frac{1}{3}\right)$

Q. 8. The length of vector $\vec{AG}$ is

(A) $\sqrt{17}$ units **(B)** $\frac{\sqrt{51}}{3}$ units **(C)** $\frac{3}{\sqrt{6}}$ units **(D)** $\frac{\sqrt{59}}{4}$ units

Q. 9. Area of triangle ABC (in sq. units) is

(A) 24 **(B)** $8\sqrt{6}$ **(C)** $4\sqrt{6}$ **(D)** $5\sqrt{6}$

Q. 10. The sum of length of $\vec{AB}$ and $\vec{AC}$ is

(A) 4 units **(B)** 9.1 units **(C)** 8.7 units **(D)** 6 units

Q. 11. The length of the perpendicular from the vertex D on the opposite face is

(A) $\frac{14}{\sqrt{6}}$ units **(B)** $\frac{2}{\sqrt{6}}$ units **(C)** $\frac{3}{\sqrt{6}}$ units **(D)** $8\sqrt{6}$ units

(B) SUBJECTIVE TYPE QUESTIONS:

III. Very Short Answer Type Questions **[1×3 = 3]**

Q. 12. Write a unit vector in the direction of the sum of vectors $\vec{a} = 2\vec{i} + 2\vec{j} - 5\vec{k}$ and $\vec{b} = 2\vec{i} + \vec{j} - 7\vec{k}$.

Q. 13. Find the equation of line passing through (1, 1, 2) and (2, 3, –1).

Q. 14. Find the direction cosines of the line:

$$\frac{x-1}{2} = -y = \frac{z+1}{2}$$

IV. Short Answer Type Questions–I **[2×3 = 6]**

Q. 15. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

Q. 16. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find value of $|2\hat{a} + \hat{b} + \hat{c}|$.

Q. 17. Find the vector and vector of magnitude $3\sqrt{2}$ units which makes an angle of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with Y and Z-axes, respectively.

V. Short Answer Type Questions-II **[3×2 = 6]**

Q. 18. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a} + \vec{b} + \vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.

Q. 19. Show that the points A, B and C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $c = \hat{i} - 3\hat{j} - 5\hat{k}$ from the vertices of a right- angled triangle.

VI. Long Answer Type Questions **[1×5 = 5]**

Q. 20. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find the vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.

❑❑