PRACTICE QUESTION PAPER - 1

Time: 3 Hours **MM: 70**

General Instructions:

(i) All questions are compulsory.

(ii) This question paper has five sections: Section A, Section B, Section C, Section D and Section E.

(iii) Section A contains twenty questions of one mark each, Section B contains eight questions of two marks each, Section C contains five questions of three marks each, Section D contains three questions of five marks each and Section E contains One Case-based of four marks.

(iv) There is no overall choice. However, internal choices have been provided in each section. You have to attempt the questions as per the choice.

SECTION-A

This section contains Multiple Choice Questions having one correct option. One Mark each

1. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to $b \neq a, b \in T$. Then R **is**

(A) reflexive but not transitive **(B)** transitive but not symmetric
(C) equivalence relation **(D)** None of these

OR

The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ are

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5

2. The value of $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$ is

(A) $\frac{3+\sqrt{5}}{2}$ **(B)** $\frac{3-\sqrt{5}}{2}$ **(C)** $\frac{-3+\sqrt{5}}{2}$ **(D)** $\frac{-3-\sqrt{5}}{2}$

3. If $[x \ \ 1]\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$, then x equals

(A) 0 **(B)** –2 **(C)** –1 **(D)** 2

OR

$A = [a_{ij}]_{m\times n}$ is a square matrix, if

(A) $m < n$ **(B)** $m > n$ **(C)** $m = n$ **(D)** None of these

4. If A is a square matrix of order 3, such that $A(adj\ A) = 10l$, then $|adj\ A|$ is equal to

(A) 1 **(B)** 10 **(C)** 100 **(D)** 101

5. The function $f(x) = \frac{4-x^2}{4x-x^3}$

(A) discontinuous at only one point **(B)** discontinuous at exactly two points
(C) discontinuous at exactly three points **(D)** none of these

6. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s., then at $x = 3$ the slope of curve is changing at ________ units/s.

(A) –72 **(B)** –36 **(C)** 24 **(D)** 48

7. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta} dx$ is equal to

(A) $2(\sin x + x\cos\theta) + C$ **(B)** $2(\sin x - x\cos\theta) + C$
(C) $2(\sin x + 2x\cos\theta) + C$ **(D)** $2(\sin x - 2x\cos\theta) + C$

OR

The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\, dx$ is

(A) 0 **(B)** 2 **(C)** π **(D)** 1

8. The order of the differential equation

$$2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0 \text{ is}$$

(A) 2 **(B)** 1 **(C)** 0 **(D)** not defined

9. Which of the following statement is true.

(A) $\vec{a}$ and $\overrightarrow{-a}$ are collinear

(B) Two collinear vectors are always equal in magnitude

(C) Two vectors having same magnitude are collinear

(D) Two collinear vectors having the same magnitude are equal

10. The sum of the direction cosines of Z-axis is

(A) 1 **(B)** 0 **(C)** 3 **(D)** 2

11. Check whether the function $f : R \to R$ defined as $f(x) = x^3$ is one-one or not.

12. Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$.

13. Find the value of A^2, where A is a 2×2 matrix whose elements are given by $a_{ij} = \begin{cases} 1 & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$

14. Find $adj\ A$, if $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$.

15. If $y = x|x|$, find $\frac{dy}{dx}$ for $x < 0$.

16. Find $\int e^x(1 - \cot x + \operatorname{cosec}^2 x)dx$.

Read the following text and answer the following questions, on the basis of the same. Attempt any four from Q.17-Q. 21.

The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

17. The rate of growth of the plant with respect to sunlight is ______ .

(A) $4x - \left(\frac{1}{2}\right)x^2$ **(B)** $4 - x$ **(C)** $x - 4$ **(D)** $x - \frac{1}{2}x^2$

18. What is the number of days it will take for the plant to grow to the maximum height?

(A) 4 **(B)** 6 **(C)** 7 **(D)** 10

19. What is the maximum height of the plant?

(A) 12 cm **(B)** 10 cm **(C)** 8 cm **(D)** 6 cm

20. What will be the height of the plant after 2 days?

(A) 4 cm **(B)** 6 cm **(C)** 8 cm **(D)** 10 cm

21. If the height of the plant is 7/2 cm, the number of days it has been exposed to the sunlight is ______ .

(A) 2 **(B)** 3 **(C)** 4 **(D)** 1

SECTION-B

22. Let R be the relation in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$. Show that the relation R transitive? Write the equivalence class [0].

23. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $(A^2 - 5A)$.

OR

Find the inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$.

Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

24. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$.

OR

Find λ and μ if $(\hat{i} + 3\hat{j} + 9\hat{k}) \times (3\hat{i} - \lambda\hat{j} + \mu\hat{k}) = 0$.

25. If a line has the direction ratios –18, 12, – 4, then find direction cosines.

26. If $y = ae^{2x} + be^{-x}$, then show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.

OR

Find the equation of the normal to the curve $y = x + \frac{1}{x}$, $x > 0$ perpendicular to the line $3x - 4y = 7$.

27. One bag contains 3 red and 5 black balls. Another bag contains 6 red and 4 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is red.

28. Find: $\int \frac{2x}{\sqrt[3]{x^2 + 1}} dx$.

29. Write the sum of the order and degree of the following differential equation: $\frac{d}{dx}\left(\frac{dy}{dx}\right) = 5$.

SECTION-C

30. The random variable X has a probability distribution $P(X)$ of the following form, where $\boldsymbol{k}$ is some number:

$$P(X) = \begin{cases} k, \text{ if } x = 0 \\ 2k, \text{ if } x = 1 \\ 3k, \text{ if } x = 2 \\ 0, \text{ otherwise} \end{cases}$$

(i) Find the value of k

(ii) Find $P(X < 2)$

(iii) Find $P(X \le 2)$

(iv) Find $P(X \ge 2)$.

31. Using integration, find the area of the region in the first quadrant enclosed by the line $x + y = 2$, the parabola $y^2 = x$ and x-axis.

32. Find the intervals in which the function f given by $f(x) = \tan x - 4x$, $x \in \left(0, \frac{\pi}{2}\right)$ is

(a) strictly increasing

(b) strictly decreasing

OR

If $y = \frac{x\cos^{-1}x}{\sqrt{1-x^2}} - \log\sqrt{1-x^2}$, then prove that $\frac{dy}{dx} = \frac{\cos^{-1}x}{(1-x^2)^{3/2}}$.

33. Find the general solution of the differential equation. $xdy = (e^y - 1)dx$.

OR

Find the general solution of the following differential equation: $x\,dy - (y + 2x^2)dx = 0$.

34. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = 4\hat{i} - 7\hat{j} + \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 6$.

SECTION-D

35. An urn contains 4 balls. Two balls are drawn at random from the urn (without replacement) and are found to be white. What is the probability that all the four balls in the urn are white?

36. Solve the differential equation $x^2dy + (xy + y^2)\,dx = 0$ given $y = 1$, when $x = 1$.

37. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours a day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine, each unit of product A is sold at ₹ 7 profit and that of B at a profit of ₹ 4. Find the production level per day for maximum profit graphically.

OR

A retired person wants to invest an amount of ₹ 50,000. His broker recommends investing in two type of bonds '***A***' and '*B*' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least ₹ 20,000 in bond '*A*' and at least ₹ 10,000 in bond '*B*'. He also wants to invest at least as much in bond '***A***' as in bond '***B***'. Solve this linear programming problem graphically to maximise his returns.

SECTION-E

Read the following text and answer the questions on the basis of the same.

A motorcycle race was organized in a town, where the maximum speed limit was set by the organizers.

No participant are allowed to cross the specified speed limit, but

Two motorcycles A and B are running at the speed more than allowed speed on the road along the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - \mu(2\hat{i} + \hat{j} + \hat{k})$, respectively.

38. Find the Cartesian equation of the line along which motrocycle A is running. **2**

39. Find the shortest distance between the lines. **2**