PRACTICE QUESTION PAPER - 2

Time: 3 Hours **MM: 70**

General Instructions:

(i) All questions are compulsory.

(ii) This question paper has five sections: Section A, Section B, Section C, Section D and Section E.

(iii) Section A contains twenty questions of one mark each, Section B contains eight questions of two marks each, Section C contains five questions of three marks each, Section D contains three questions of five marks each and Section E contains One Case-based of four marks.

(iv) There is no overall choice. However, internal choices have been provided in each section. You have to attempt the questions as per the choice.

SECTION-A

This section contains Multiple Choice Questions having one correct option. One Mark each

1. If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is

(A) reflexive **(B)** transitive **(C)** symmetric **(D)** None of these

2. The principal value of

$\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)+4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{6}$ **(C)** $\frac{4\pi}{3}$ **(D)** $\frac{3\pi}{4}$

OR

The domain of function $\cos^{-1}(2x - 1)$ is

(A) $[0, 1]$ **(B)** $[-1, 1]$ **(C)** $(-1, 1)$ **(D)** $[0, \pi]$

3. Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x=\frac{-1}{3}, y=7$ **(B)** Not possible to find **(C)** $y=7, x=\frac{-2}{3}$ **(D)** $x=\frac{-1}{3}, y=\frac{-2}{3}$

OR

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, then the value of α is:

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{3}$ **(C)** π **(D)** $\frac{3\pi}{2}$

4. If $A = [2\ \ -3\ \ 4]$, $B = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$, $X = [1\ \ 2\ \ 3]$ and $Y = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, then $AB + XY$ equals.

(A) $[28]$ **(B)** $[24]$ **(C)** 28 **(D)** 24

5. If $y=\sqrt{\sin x+y}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{\cos x}{2y-1}$ **(B)** $\frac{\cos x}{1-2y}$ **(C)** $\frac{\sin x}{1-2y}$ **(D)** $\frac{\sin x}{2y-1}$

6. The contentment obtained after eating x units of a new dish at a trial function is given by the function $f(x) = x^3 + 6x^2 + 5x + 3$. The marginal contentment when 3 units of dish are consumed is ____________.

(A) 60 **(B)** 68 **(C)** 24 **(D)** 48

7. If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ **(B)** $x^3 + \frac{1}{x^4} + \frac{129}{8}$ **(C)** $x^4 + \frac{1}{x^3} + \frac{129}{8}$ **(D)** $x^3 + \frac{1}{x^4} - \frac{129}{8}$

8. The order and degree of the differential equation

$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0$ respectively, are

(A) 2 and 4 **(B)** 2 and 2 **(C)** 2 and 3 **(D)** 3 and 3

9. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 2, |\vec{b}| = 3$ and $|\vec{c}| = 5,$ then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is

(A) 0 **(B)** 1 **(C)** –19 **(D)** 38

OR

If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to

(A) 0 **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{2}$ **(D)** π

10. If l, m, n are the direction cosines of a line, then;

(A) $l^2 + m^2 + 2n^2 = 1$ **(B)** $l^2 + 2m^2 + n^2 = 1$ **(C)** $2l^2 + m^2 + n^2 = 1$ **(D)** $l^2 + m^2 + n^2 = 1$

11. If $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B. State whether f is one-one or not.

12. Write the principal value of $\tan^{-1}(\sqrt{3}) + \cot^{-1}(-\sqrt{3})$.

13. If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A.

14. For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ write A^{-1}.

15. If the function f defined as $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ is continuous at $x = 3$, find the value of k.

OR

Find the slope of tangent to the curve $y = 3x^2 - 6$ at the point on it whose x-coordinate is 2.

16. If $[x]$ denotes the greatest integer function, then find $\int_0^{3/2} [x^2]dx$.

Read the following text and answer the following questions on the basis of the same. Attempt any four from Q. 17-Q. 21.

Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx} = k(50 - y)$ where x denotes the number of weeks and y the number of children who have been given the drops.

17. State the order of the above given differential equation.

(A) 2 **(B)** 1 **(C)** 0 **(D)** Can't define

18. Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = k(50 - y)$?

(A) Variable separable method **(B)** Solving Homogeneous differential equation

(C) Solving Linear differential equation **(D)** all of the above

19. The solution of the differential equation $\frac{dy}{dx} = k(50 - y)$ is given by,

(A) $\log|50 - y| = kx + C$ **(B)** $-\log|50 - y| = kx + C$

(C) $\log|50 - y| = \log|kx| + C$ **(D)** $50 - y = kx + C$

20. The value of C in the particular solution given that $y(0) = 0$ and $k = 0.049$ is

(A) $\log 50$ **(B)** $\log \frac{1}{50}$ **(C)** 50 **(D)** –50

21. Which of the following solutions may be used to find the number of children who have been given the polio drops?

(A) $y = 50 - e^{kx}$ **(B)** $y = 50 - e^{-kx}$ **(C)** $y = 50(1 - e^{-kx})$ **(D)** $y = 50(e^{-kx} - 1)$

SECTION-B

22. How many equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) are there in all ? Justify your answer.

23. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, show that $(A - 2I)(A - 3I) = 0$.

OR

If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of $|A|$.

24. If $|\vec{a} + \vec{b}| = 60, |\vec{a} - \vec{b}| = 40$ and $|\vec{a}| = 22$, then find $|\vec{b}|$.

OR

Find the unit vector perpendicular to each of the vectors $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.

25. Find the area of the region bounded by the curves $x^2 + y^2 = 4$, $y = \sqrt{3}x$ and x-axis in the first quadrant.

26. Find the differential of $\sin^2 x$ w.r.t. $e^{\cos x}$.

OR

Show that the function $f(x) = \frac{x}{3} + \frac{3}{x}$ decreases in the intervals $(-3, 0) \cup (0, 3)$.

27. Find $[P(B/A) + P(A/B)]$, if $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$.

28. Find: $\int \frac{x}{x^2 + 3x + 2} dx$.

29. Solve the following differential equation: $\frac{dy}{dx} = x^3 \operatorname{cosec} y$, given that $y(0) = 0$.

SECTION-C

30. Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X.

31. Find the area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is.

32. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided.

OR

Find the values of $\boldsymbol{p}$ and $\boldsymbol{q}$, for which $f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\dfrac{\pi}{2}\\ p, & \text{if } x=\dfrac{\pi}{2}\\ \dfrac{q(1-\sin x)}{(p-2x)^2}, & \text{if } x>\dfrac{\pi}{2}\end{cases}$, is continuous at $x=\dfrac{\pi}{2}$.

33. Evaluate: $\displaystyle\int_{-1}^{1}\frac{x+|x|+1}{x^2+2|x|+1}.dx$.

OR

Find the general solution of the differential equation :

$x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

34. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

SECTION-D

35. Evaluate : $\displaystyle\int\frac{dx}{(x+1)(x+2)}$

36. A bag I contains 5 red and 4 white balls and a bag II contains 3 red and 3 white balls. Two balls are transferred from the bag I to the bag II and then one ball is drawn from bag II. If the ball drawn from the bag II is red, then find the probability that one red ball and one white ball are transferred from the bag I to the bag II.

37. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹40 and that of type B ₹50, find the number of units of each type that the company should produce to maximize profit. Formulate the above LPP and solve it graphically and also find the maximum profit.

OR

A manufacturer produces nuts and bolts. It takes 2 hours work on machine $\boldsymbol{A}$ and 3 hours on machine $\boldsymbol{B}$ to produce a package of nuts. It takes 3 hours on machine $\boldsymbol{A}$ and 2 hours on machine $\boldsymbol{B}$ to produce a package of bolts. He earns a profit of ₹ 24 per package on nuts and ₹ 18 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines both for at the most 10 hours a day. Make an LPP from above and solve it graphically ?

SECTION-E

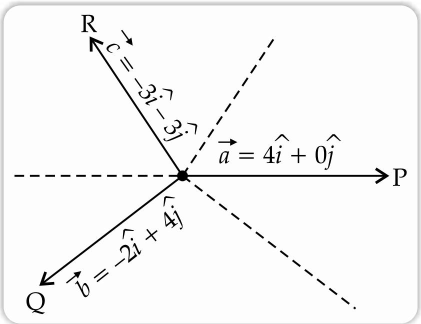
Read the following text and answer the following questions on the basis of the same.

Team P, Q, R went for playing a tug of war game. Teams P, Q, R have attached a rope to a metal ring and is trying to pull the ring into their own areas (team areas when in the given figure below).

Team pulls with force $F_1 = 4\hat{i}+0\hat{j}$ KN

Team Q pulls with force $F_2 = -2\hat{i}+4\hat{j}$ KN

Team R pulls with force $F_3 = -3\hat{i} - 3\hat{j}$ KN



38. What is the magnitude of the teams combined force? Also, find which team will win the game? **2**

39. In what direction is the ring getting pulled? **2**