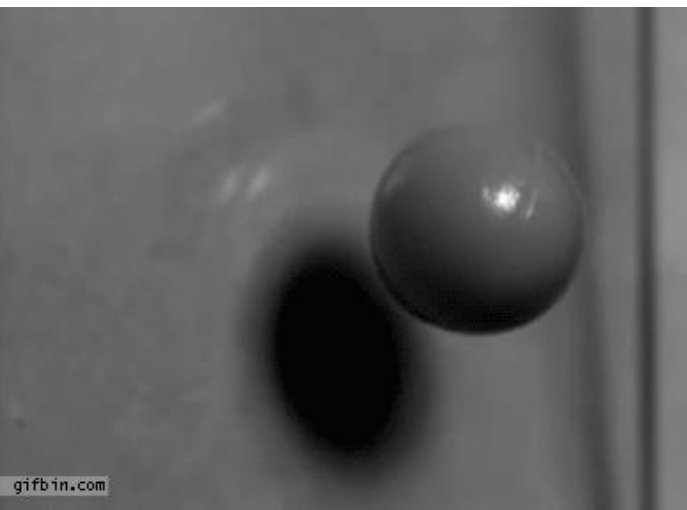


COM, MOMENTUM & COLLISION



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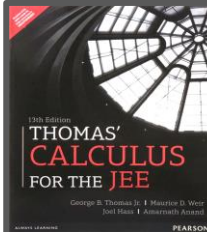
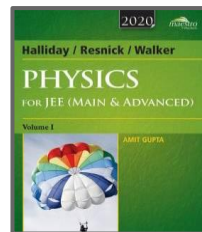
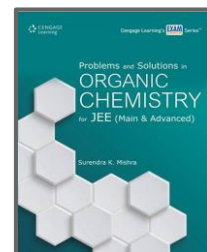
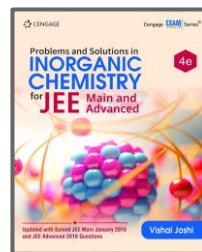
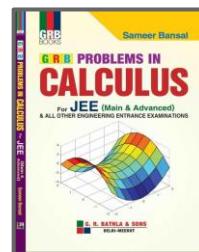
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Lesson 1 • Apr 2, 2021 12:30 PM

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- Coverage of Class 11 JEE syllabus
- Enhance conceptual understanding of JEE Main & JEE Advanced subjects
- Systematically designed courses
- Strengthen JEE problem-solving ability



Prashant Jain

Mathematics Maestro



Nishant Vora

Mathematics Maestro



Ajit Lulla

Physics Maestro



Abhilash Sharma

Physics Maestro



Sakshi Vora

Chemistry Maestro



Megha Khandelwal

Chemistry Maestro



12th

Evolve Batch

for Class 12th JEE Main and Advanced 2023

Code: **ABHILASH**

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- Top Educators from Unacademy Atoms
- Complete preparation for class 12th syllabus of JEE Main & Advanced
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Nishant Vora
Mathematic Maestro



Ajit Lulla
Physics Maestro



Sakshi Ganotra
Organic & Inorganic
Chemistry Maestro



Megha Khandelwal
Chemistry Maestros



Prashant Jain
Mathematics Maestro



Abhilash Sharma
Physics Maestro



Achiever Batch 2.0

for IIT JEE Main and Advanced 2023 Droppers

Code: ABHILASH

Batch highlights:

- Learn from India's Top Educators
- Coverage of Class 11 & 12 syllabus of JEE
- Deep dive at a conceptual level for JEE Main and JEE Advanced
- Systematic course flow of subjects and related topics
- Strengthening the problem-solving ability of JEE level problems

For more details, contact **8585858585**



Nishant Vora
Mathematics Maestros



Prashant Jain
Mathematics Maestros



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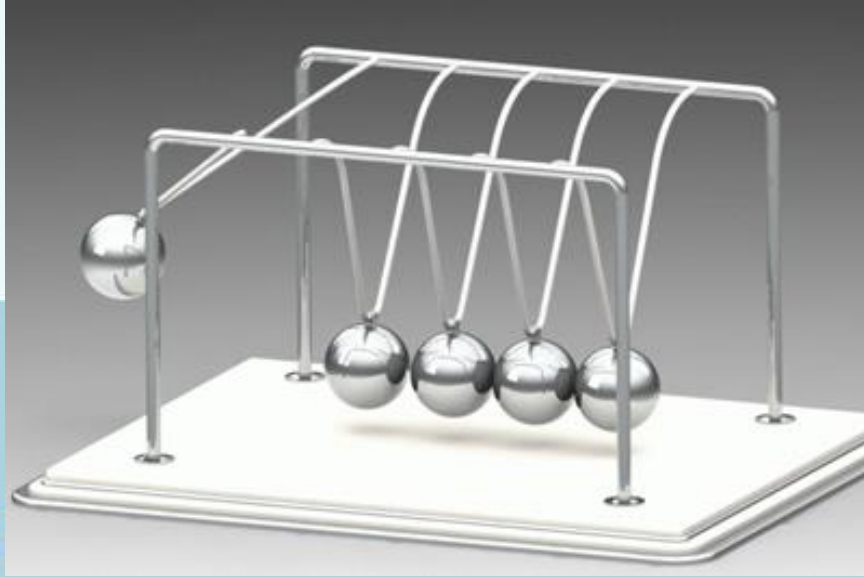
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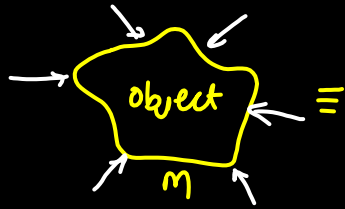
ABHILASH



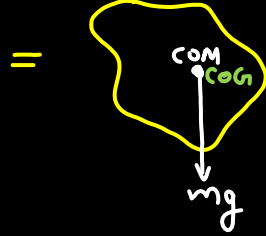
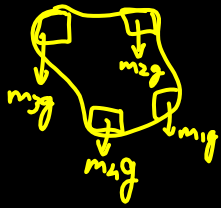


COM, Momentum & Collision

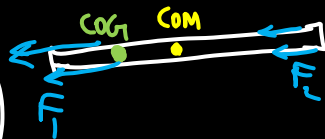
Centre of Mass



Centre of mass
•
m



near earth surface
COM is same as
COG (Centre of Gravity)



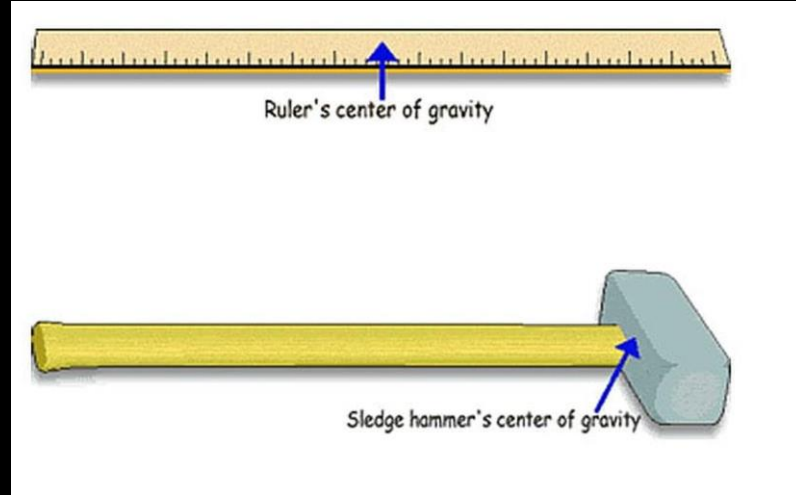
Uniform Rod

$$F_1 > F_2$$



Centre of Mass

- 1) COM
 - 2) CGM
 - 3) COB
 - 4) COP
- } \Rightarrow Nearer to the heavier side





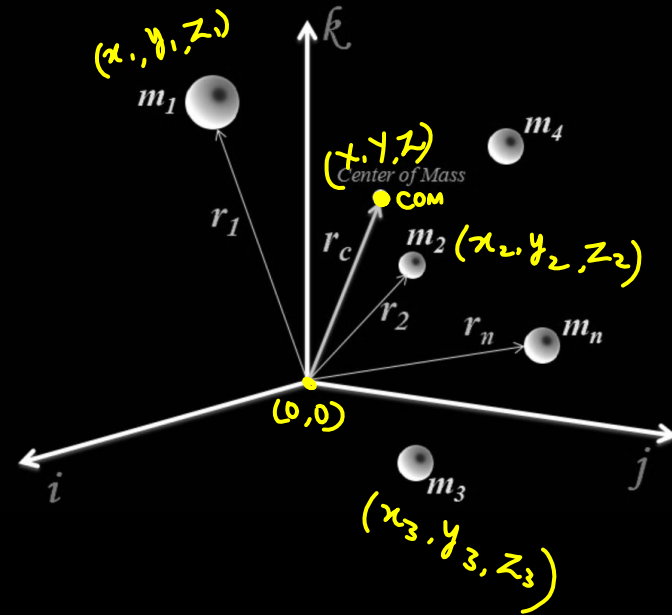
Centre of Mass

Discrete Mass System (Point masses)

$$X = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots}$$

$$Y = \frac{m_1 y_1 + m_2 y_2 + \dots}{m_1 + m_2 + \dots}$$

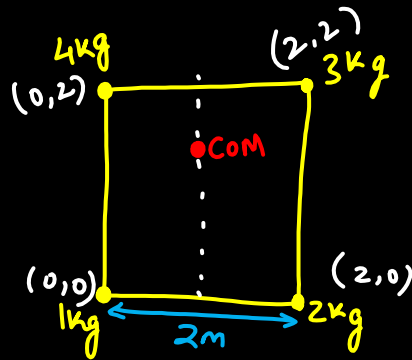
$$Z = \frac{m_1 z_1 + m_2 z_2 + \dots}{m_1 + m_2 + \dots}$$





Centre of Mass

1) Discrete Mass System



$$X = \frac{1 \times 0 + 2 \times 2 + 3 \times 2 + 4 \times 0}{10}$$

$$X = 1\text{m}$$

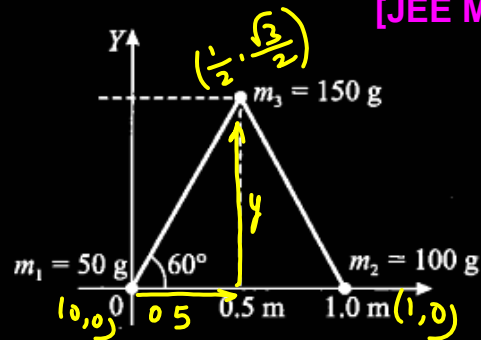
$$Y = \frac{1 \times 0 + 2 \times 0 + 3 \times 2 + 4 \times 2}{10}$$

$$Y = 1.4\text{m}$$



Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be

[JEE Main 2019]



(+4)

$$X = \frac{50 \times 0 + 100 \times 1 + 150 \times \frac{1}{2}}{300}$$

$$X = \frac{175}{300} = \frac{35}{60} = \frac{7}{12}$$

$$\tan 60^\circ = \frac{y}{0.5}$$

$$y = \frac{\sqrt{3}}{2} \text{ m}$$

$$Y = \frac{50 \times 0 + 100 \times 0 + 150 \times \frac{\sqrt{3}}{2}}{300}$$

$$= \frac{150}{300} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} \text{ m}$$

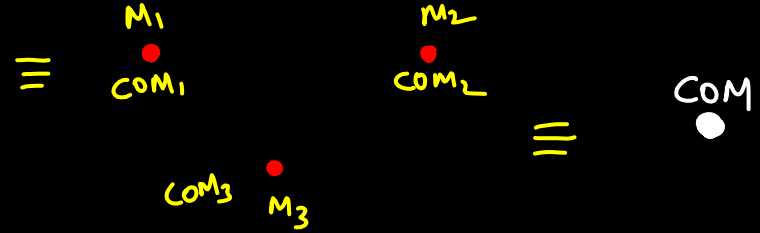
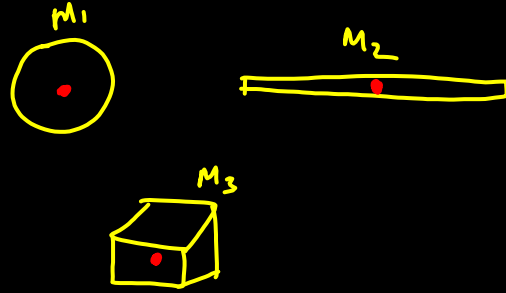
(a) $\left(\frac{7}{12} \text{ m}, \frac{\sqrt{3}}{8} \text{ m} \right)$

(b) $\left(\frac{\sqrt{3}}{4} \text{ m}, \frac{5}{12} \text{ m} \right)$

(c) $\left(\frac{7}{12} \text{ m}, \frac{\sqrt{3}}{4} \text{ m} \right)$

(d) $\left(\frac{\sqrt{3}}{8} \text{ m}, \frac{7}{12} \text{ m} \right)$

3) System of objects





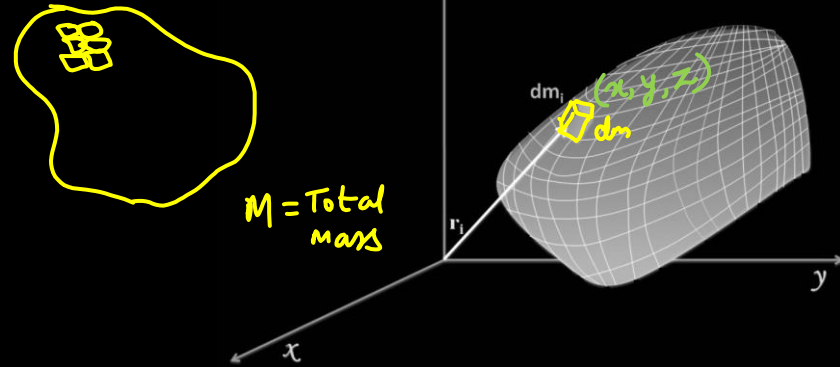
Centre of Mass

2) Continuous Body

$$x = \frac{\int dm \, x}{\int dm} = \frac{1}{M} \int dm \, x$$

$$y = \frac{1}{M} \int dm \, y$$

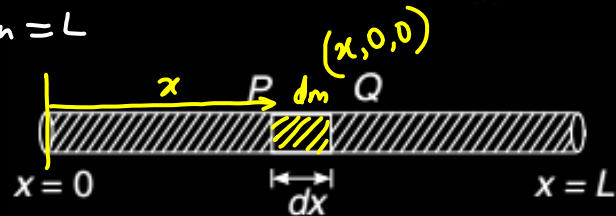
$$z = \frac{1}{M} \int dm \, z$$



Example

$$\begin{aligned}
 X &= \frac{1}{M} \int dm \, x \\
 &= \frac{1}{M} \int \left(\frac{M}{L} dx \right) x \\
 &= \frac{1}{L} \int_0^L x \, dx \\
 &= \frac{1}{L} \left[\frac{x^2}{2} \right]_0^L = \frac{1}{L} \cdot \frac{L^2}{2} \\
 \boxed{X = L/2}
 \end{aligned}$$

Rod Mass = M
Length = L



Unitary Method

$ \begin{aligned} L &\rightarrow M \\ 1 &\rightarrow \frac{M}{L} \\ dm &\Rightarrow \frac{M}{L} x \, dx \end{aligned} $	$ \begin{aligned} 12 \text{ B} &\Rightarrow ₹ 36 \\ 1 \text{ B} &\Rightarrow ₹ \frac{36}{12} = ₹ 3 \\ 8 \text{ B} &\Rightarrow ₹ 3 \times 8 = ₹ 24 \end{aligned} $
---	--

Mass per unit length $= \rho(x) = a + b \frac{x^2}{L^2}$



$$dm = \rho dx$$

$$\int dm = \int \left(a + \frac{bx^2}{L^2} \right) dx$$

$$M = \int_0^L a dx + \frac{b}{L^2} \int_0^L x^2 dx$$

$$M = aL + \frac{b}{L^2} \times \frac{L^3}{3} = aL + \frac{bL}{3}$$

$$M = \left(a + \frac{b}{3} \right) L$$

A rod of length L has non-uniform linear mass density

given by $\rho(x) = a + b \left(\frac{x}{L} \right)^2$, where a and b are constants

and $0 \leq x \leq L$. The value of x for the centre of mass of the rod is at:

(a) $\frac{3}{2} \left(\frac{a+b}{2a+b} \right) L$

☒ (b) $\frac{3}{4} \left(\frac{2a+b}{3a+b} \right) L$

(c) $\frac{4}{3} \left(\frac{a+b}{2a+3b} \right) L$

(d) $\frac{3}{2} \left(\frac{2a+b}{3a+b} \right) L$

[JEE Main 2019]

$$X = \frac{1}{M} \int dm \cdot x$$

$$X = \frac{1}{M} \int (\rho dx) x$$

$$= \frac{1}{M} \int \left(a + \frac{bx^2}{L^2}\right) x dx$$

$$= \frac{1}{M} \left[\int_0^L a x dx + \int_0^L \frac{b}{L^2} x^3 dx \right]$$

$$= \frac{1}{M} \left[\frac{aL^2}{2} + \frac{b}{L^2} \frac{L^4}{4} \right]$$

$$X = \frac{1}{\left(a + \frac{b}{3}\right)L} \left[\frac{aL^2}{2} + \frac{bL^2}{4} \right]$$

$$= \frac{(2a+b)L/4}{(3a+b)/3}$$

$$X = \frac{3}{4} \frac{(2a+b)}{(3a+b)} L$$



Find the x -coordinate of the centre of mass of the bricks shown in figure:

$$\frac{L}{2} + \frac{L}{4} + \frac{L}{6} + \frac{L}{2}$$

$$L + \frac{(6+4)L}{24}$$

$$L + \frac{5}{12}L$$

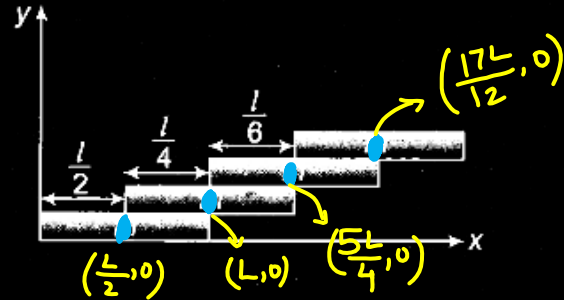
$$\frac{17L}{12}$$

(a) $\frac{24}{25}l$

(b) $\frac{25}{24}l$

(c) $\frac{15}{16}l$

(d) $\frac{16}{15}l$



$$\begin{array}{r|rrrr} 2 & 2 & 4 & 12 & \\ \hline 2 & 1 & 2 & 6 & \\ 3 & 1 & 1 & 3 & \end{array} \quad \begin{array}{r} 25 \\ +18 \\ \hline 43 \end{array}$$

$$X = \frac{mL/2 + mL + m\left(\frac{5L}{4}\right) + m\left(\frac{17L}{12}\right)}{4m}$$

$$\frac{6 + 12 + 15 + 17}{12} L = \frac{50L}{4 \times 12}$$

$$= \frac{25L}{24}$$





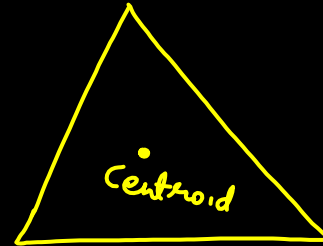
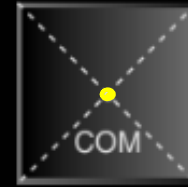
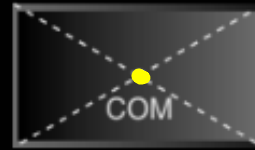
Centre of Mass

Uniform & symmetric Continuous Body

mass
distribution

shape

Geometrical
Centre = COM





Centre of Mass

Uniform & Asymmetric Continuous Body

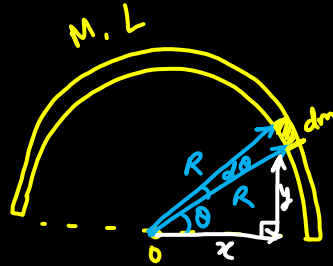
Semicircular Wire

$$X = \frac{1}{M} \int dm x$$

$$Y = \frac{1}{M} \int dm y$$

$$Y = \frac{1}{M} \int \frac{M}{L} dl (R \sin \theta)$$

$$\boxed{Y = \frac{2R}{\pi}}$$



$$x = R \cos \theta$$

$$y = R \sin \theta$$

element
Mass = dm
length = $dl = R d\theta$

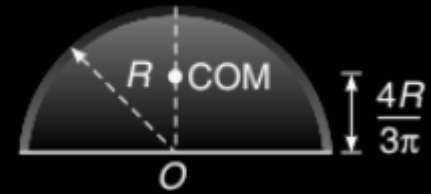
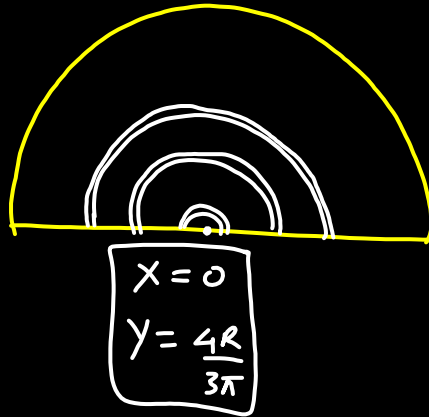
$$dm = \frac{M}{L} \times dl$$





Centre of Mass

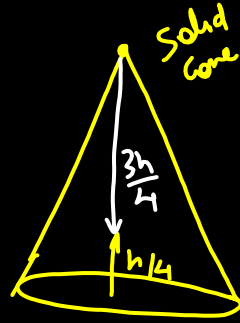
Semicircular Disc



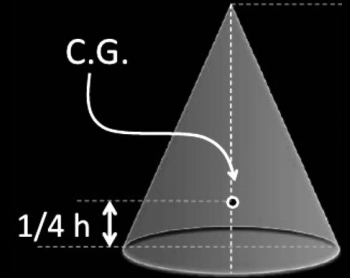


Centre of Mass

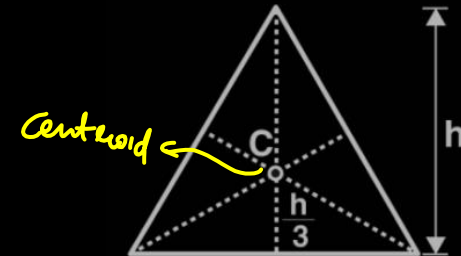
Cone



Hollow Cone
 $x=0, y=h/3$



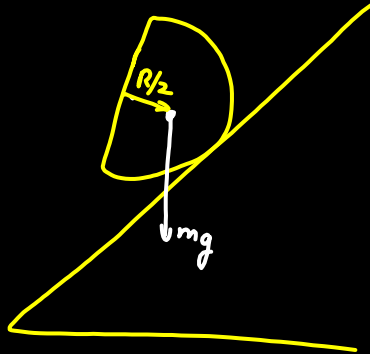
Solid cone
 $x=0, y=h/4$



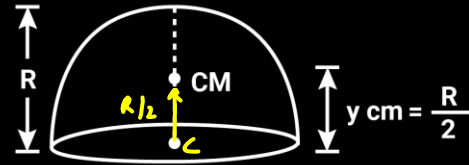
Triangle



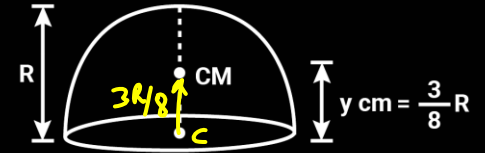
Centre of Mass



Hemi- Hollow Sphere



Solid Hemisphere



Q

The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then value of x is _____.

[Main Sep. 06, 2020 (II)]

$$R = 8 \text{ cm}$$



$$x = \frac{3R}{8} = 3 \text{ cm}$$



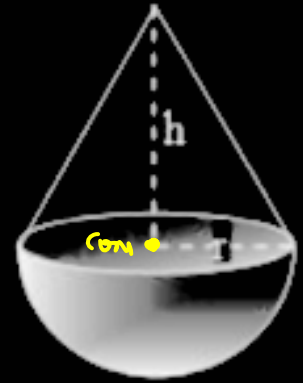
HW

Ans is comment

t.me/abhilashsharma_iitjee

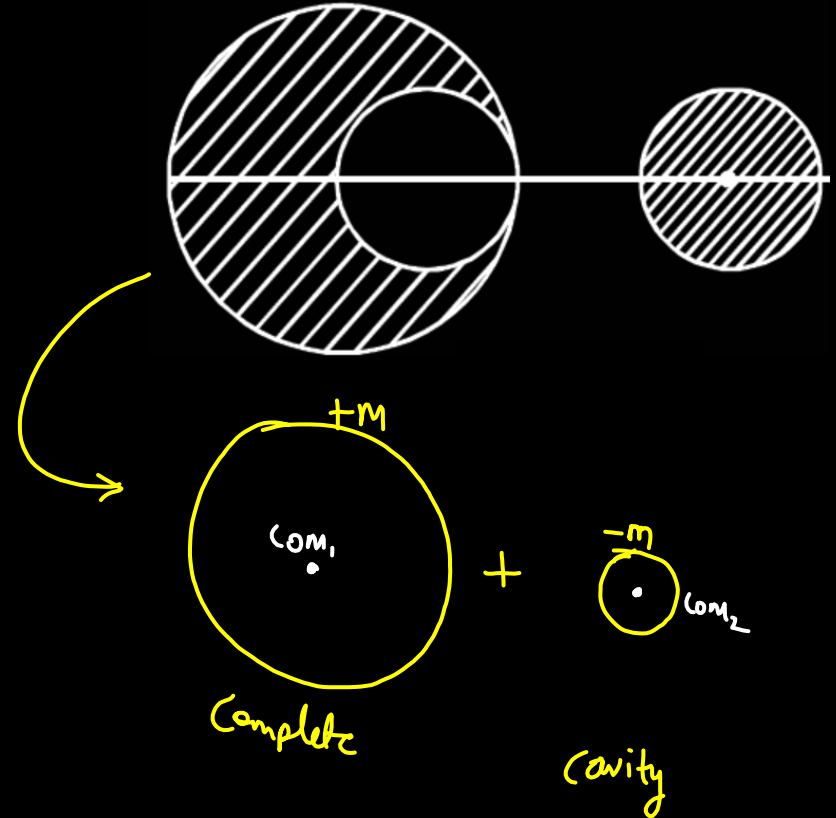
A uniform solid circular cone of base radius r is joined to a uniform solid hemisphere of radius r and of the same density, so as to have a common face. The centre of mass of the composite solid lies on the common face. The height of the cone is

- A. $2r$
- B. $\sqrt{3}r$
- C. $3r$
- D. $\sqrt{6}r$





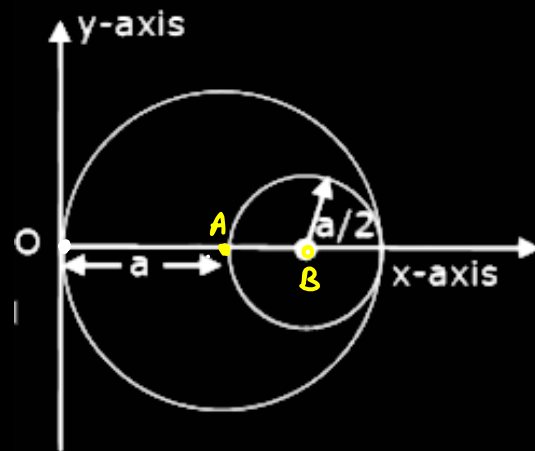
Centre of Mass of Objects with Cavity



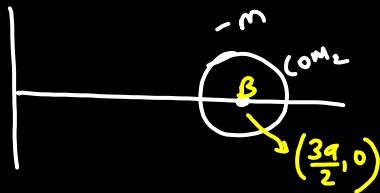
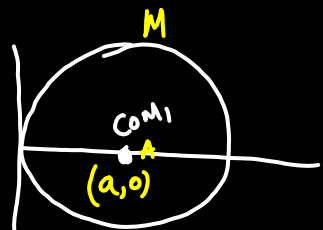
Q

A circular hole of radius $a/2$ is cut out of a circular disc of radius ' a ' shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be

[JEE Main 2021]

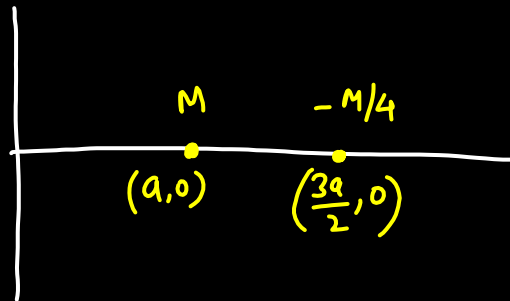


- A. $10a/11$
- B. $2a/3$
- C. $a/6$
- ☒ D. $5a/6$



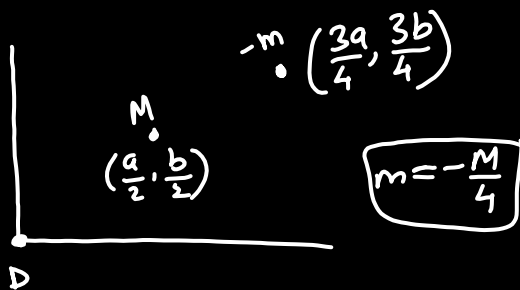
$$m = \frac{-M}{\pi a^2} \times \pi \left(\frac{a}{2}\right)^2$$

$$m = -\frac{M}{4}$$



$$X = \frac{Ma + \left(-\frac{M}{4}\right)\left(\frac{3a}{2}\right)}{M + \left(-\frac{M}{4}\right)}$$

$$X = \frac{a - \frac{3}{8}a}{\frac{3}{4}} = \frac{5a/8}{3/4} = \frac{5a}{6}$$

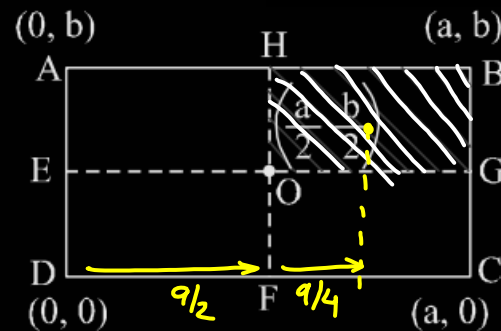


Q

A uniform rectangular thin sheet ABCD of mass M has length a and breadth b , as shown in the figure. If the shaded portion HBG is cut-off, the coordinates of the centre of mass of the remaining portion will be :

[JEE Main 2019]

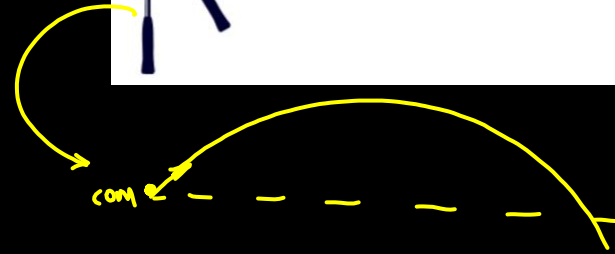
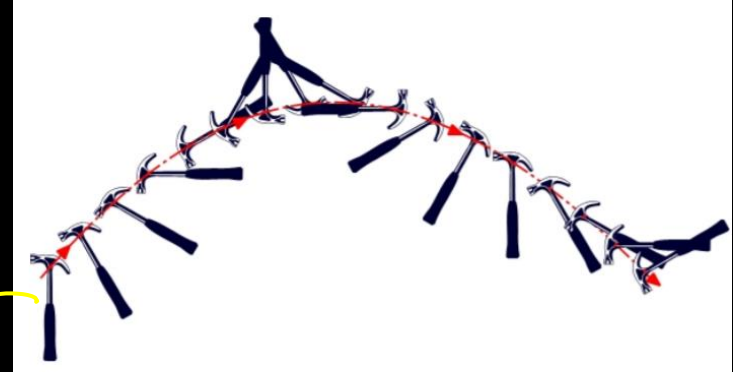
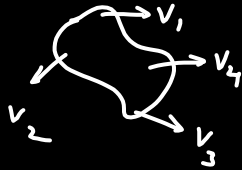
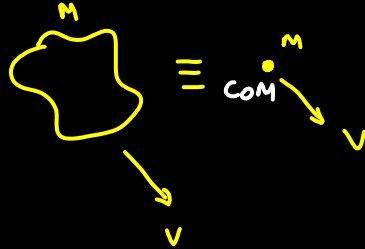
$$X = \frac{M(\frac{a}{2}) + (-\frac{M}{4})(\frac{3a}{4})}{M + (-\frac{M}{4})} = \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{\frac{5a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$



- (a) $(\frac{3a}{4}, \frac{3b}{4})$ (b) $(\frac{5a}{3}, \frac{5b}{3})$
(c) $(\frac{2a}{3}, \frac{2b}{3})$ (d) $(\frac{5a}{12}, \frac{5b}{12})$



Motion of Centre of Mass



Motion of Centre of Mass

$$\vec{V}_{\text{com}} = \frac{d\vec{r}_{\text{com}}}{dt}$$

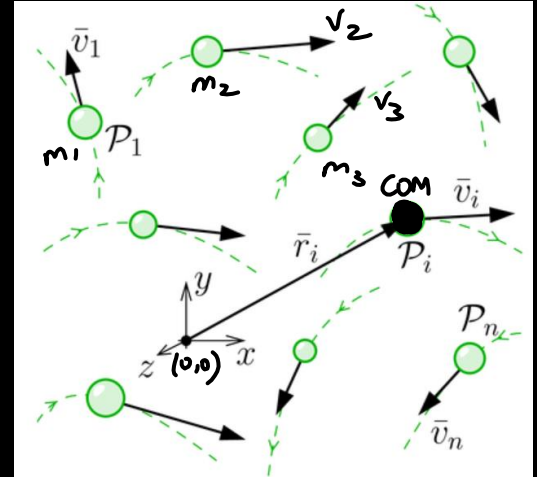
$$= \frac{d}{dt} \left[\frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots}{m_1 + m_2 + \dots} \right]$$

$$\vec{V}_{\text{com}} = \frac{1}{M} [m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots]$$

$$M \vec{V}_{\text{com}} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots$$

$$M \vec{V}_{\text{com}} = \vec{p}_1 + \vec{p}_2 + \dots$$

$$M \vec{V}_{\text{com}} = \vec{p}_{\text{system}}$$



$$\text{COM} \Rightarrow (x, y, z)$$

$$\vec{r}_{\text{com}} = x \hat{i} + y \hat{j} + z \hat{k}$$



Three particles of masses 1 kg, 2 kg and 3 kg are situated at the corners of an equilateral triangle move at speed 6 ms^{-1} , 3 ms^{-1} and 2 ms^{-1} respectively. Each particle maintains a direction towards the particle at the next corner symmetrically. Find velocity of CM of the system at this instant

$$\vec{V}_{\text{cm}} = \frac{m_1 \vec{V}_A + m_2 \vec{V}_B + m_3 \vec{V}_C}{m_1 + m_2 + m_3}$$

$$\vec{V}_{\text{cm}} = \frac{2(3\hat{i}) + 1(-3\hat{i} - 3\sqrt{3}\hat{j}) + 3(-\hat{i} + \sqrt{3}\hat{j})}{6}$$

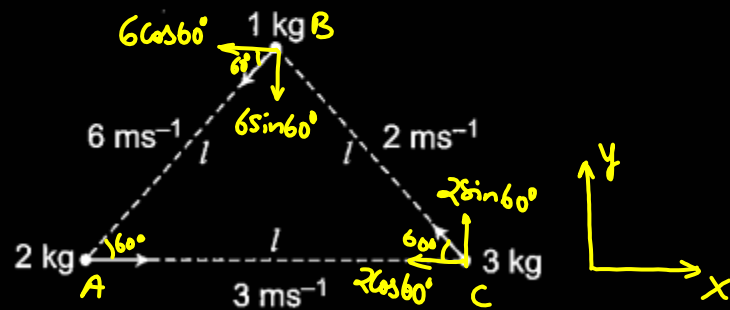
$$= \frac{6\hat{i} - 3\hat{i} - 3\hat{i} - 3\sqrt{3}\hat{j} + 3\sqrt{3}\hat{j}}{6}$$

$$= 0$$

$$\vec{V}_A = 3\hat{i}$$

$$\vec{V}_B = -3\hat{i} - 3\sqrt{3}\hat{j}$$

$$\vec{V}_C = -\hat{i} + \sqrt{3}\hat{j}$$



(a) 3 ms^{-1}

(b) 5 ms^{-1}

(c) 6 ms^{-1}

☒ (d) zero

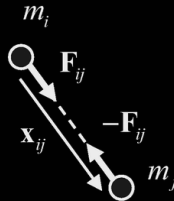
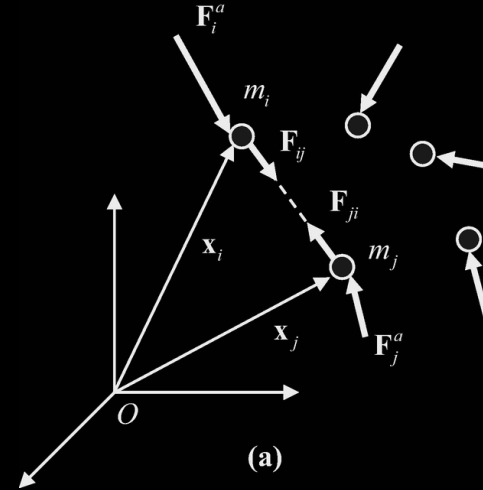


Acceleration of Centre of Mass

$$\vec{r}_{com} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots}{m_1 + m_2 + \dots}$$

$$\vec{v}_{com} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots}{m_1 + m_2 + \dots}$$

$$\vec{a}_{com} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots}{m_1 + m_2 + \dots}$$



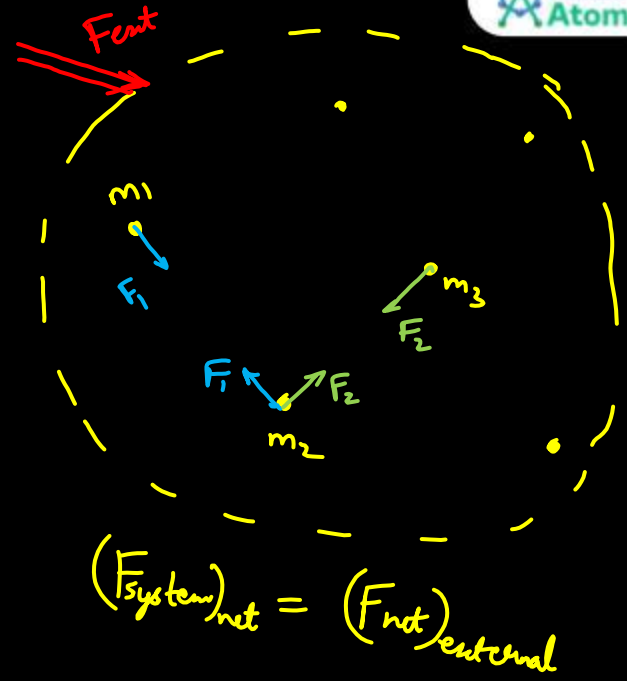
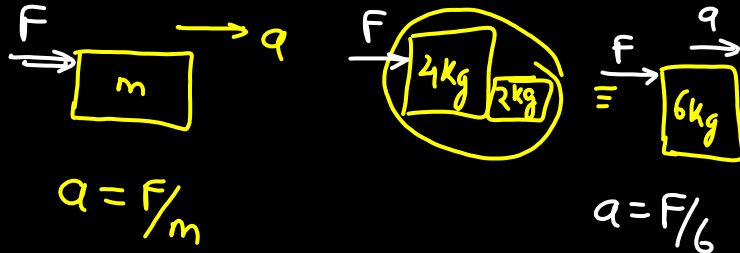
(a) (b)

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + \dots$$

$$= (\vec{F}_1)_{net} + (\vec{F}_2)_{net} + \dots$$

$$M\vec{a}_{cm} = (\vec{F}_{system})_{net}$$

$$M\vec{a}_{cm} = (\vec{F}_{net})_{external}$$





$$\vec{a}_{com} = \frac{m(a_A) + 2m(a_B) + 3m(a_C) + 4m(a_D)}{m + 2m + 3m + 4m}$$

$$\vec{a}_{com} = \frac{m(-a\hat{i}) + 2m(a\hat{j}) + 3m(a\hat{i}) + 4m(-a\hat{j})}{10m}$$

$$= \frac{(-a + 3a)\hat{i} + (2a - 4a)\hat{j}}{10}$$

$$= \frac{2(a\hat{i} - a\hat{j})}{10}$$

$$\vec{a}_A = -a\hat{i}$$

$$\vec{a}_B = +a\hat{j}$$

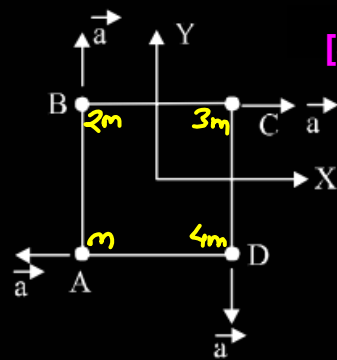
$$\vec{a}_C = a\hat{i}$$

$$\vec{a}_D = -a\hat{j}$$

$$(a) \frac{a}{5}(\hat{i} - \hat{j})$$

$$(c) \text{ Zero}$$

Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is :



[JEE Main 2019]

$$(b) a$$

$$(d) \frac{a}{5}(\hat{i} + \hat{j})$$



Acceleration of Centre of Mass

If $(\vec{F}_{\text{net}})_{\text{external}} = 0$

$$\Rightarrow \vec{a}_{\text{com}} = 0$$

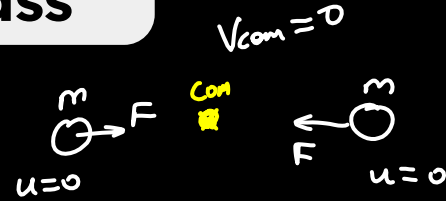
$$\Rightarrow \boxed{\vec{V}_{\text{com}} = \text{const}}$$

\Rightarrow If initially com was at rest, then it will remain at rest

$$M \vec{V}_{\text{com}} = \vec{P}_{\text{system}}$$

$$\Rightarrow \boxed{\vec{P}_{\text{system}} = \text{const}}$$

\Rightarrow Conservation of Linear Momentum of the system





Acceleration of Centre of Mass

Example

Very Important

$$X = \frac{m_1 \times 0 + m_2 \times h}{m_1 + m_2}$$

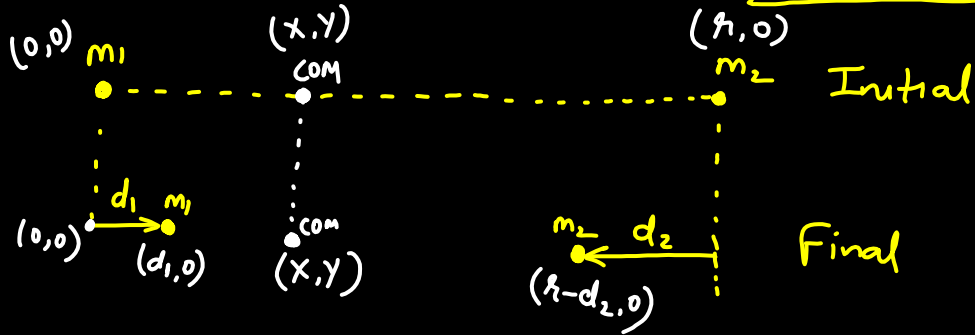
$$X = \frac{m_1 d_1 + m_2 (h - d_2)}{m_1 + m_2}$$

$$\frac{m_2 h}{m_1 + m_2} = \frac{m_1 d_1 + m_2 (h - d_2)}{m_1 + m_2}$$

$$m_2 h = m_1 d_1 + m_2 h - m_2 d_2$$

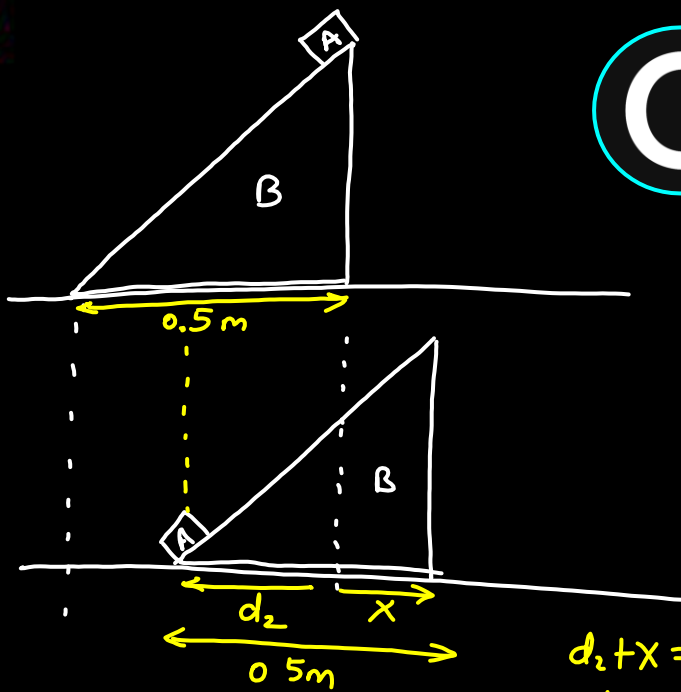
$$\boxed{m_1 d_1 = m_2 d_2} \quad \star$$

Consider a two-particle system with the particles having masses m_1 and m_2 . If the first particle is pushed towards the centre of mass through a distance d , by what distance should the second particle be moved so as to keep the centre of mass at the same position?



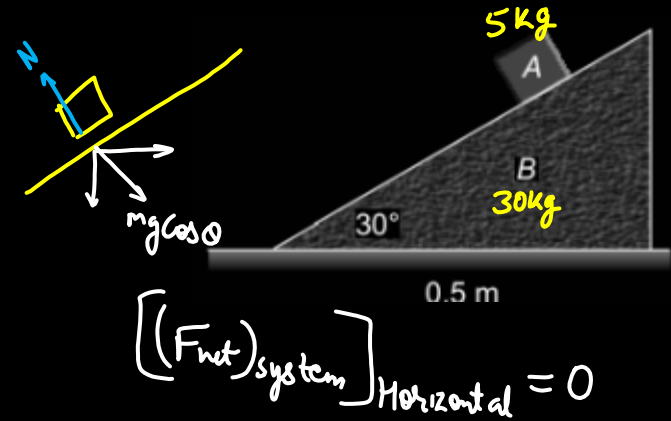
Q

Block A of mass 5 kg is placed on top of a smooth triangular block B having a mass of 30 kg. If the system is released from rest, determine the distance moved by B when A reaches the bottom. Neglect the size of block A.



- A. 1/12
- ☒ B. 1/14
- C. 1/16
- D. 1/20

$$\left. \begin{array}{l} d_1 + X = 0.5 \\ d_2 = 0.5 - X \\ m_2 = 5 \text{ kg} \end{array} \right\} \begin{array}{l} d_1 = X \\ M_1 = 30 \text{ kg} \end{array}$$



Along
Horizontal

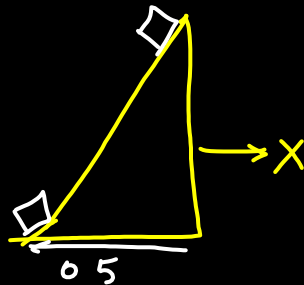
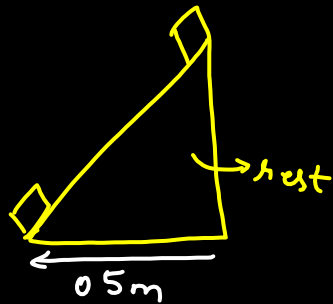
$$m_1 d_1 = m_2 d_2$$

$$30x = 5(0.5 - x)$$

$$6x = 0.5 - x$$

$$7x = 0.5$$

$$x = \frac{1}{14} \text{ m}$$



Net $0.5 - x$

Net
Motion of
Man $= 10 - x$

Q

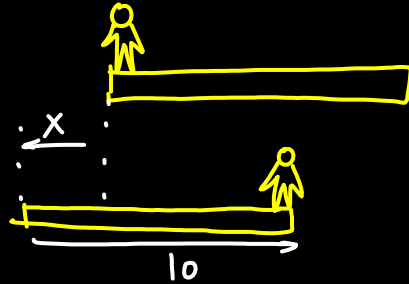
$$m_1 d_1 = m_2 d_2$$

$$60(10 - x) = 20x$$

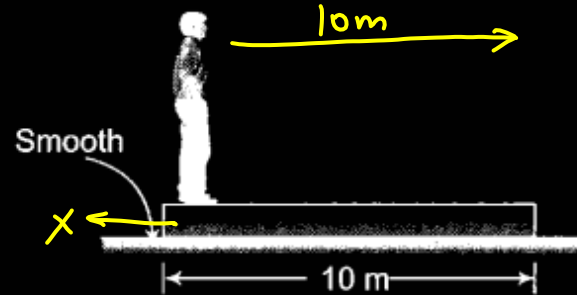
$$30 - 3x = x$$

$$x = 30/4$$

$$x = 7.5 \text{ m}$$



A wooden plank of mass 20 kg is resting on a smooth horizontal floor. A man of mass 60 kg starts moving from one end of the plank to the other end. The length of the plank is 10 m. Find the displacement of the plank over the floor when the man reaches the other end of the plank.



(a) 5 m

(c) 2.5 m

(b) 6.5 m

(d) 7.5 m



Q

$$(F_{net})_{horizontal} = 0$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(100)^2 \times 2 \times \frac{3}{5} \times \frac{4}{5}}{10}$$

$$R = 960 \text{ m}$$

$$(480, 0)$$

m

$$(960, 0)$$

COM

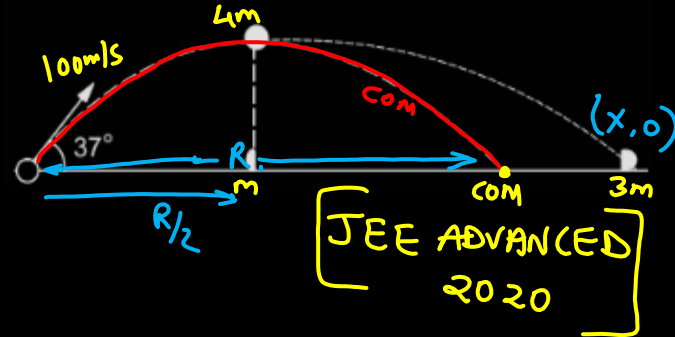
$$(x, 0)$$

3m

- A. 480 m
- B. 960 m
- C. 1120 m
- D. 2250 m

$$960 = \frac{m(480) + 3m(x)}{m + 3m}$$

A projectile is fired at a speed of 100 m/s at an angle of 37° above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1:3, the smaller coming to rest. Find the distance from the launching point to the point where the heavier piece lands



$$960 \times 4 = 480 + 3x$$

$$x = 320 \times 4 - 160$$

$$x = 1120 \text{ m}$$



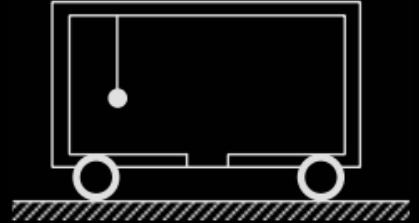
HW

Ans in comment

t.me/abhilashsharmaitjee

A cart of mass 6 kg is at rest on a frictionless horizontal surface and a pendulum bob of mass 2 kg hangs from the roof of the cart. The string breaks, the bob falls on the floor, makes several collisions on the floor and finally lands up in a small slot made in the floor. The horizontal distance between the string and the slot is 1 m. Find the displacement of the cart during this process

- A. 10 cm
- B. 25 cm
- C. 50 cm
- D. 75 cm





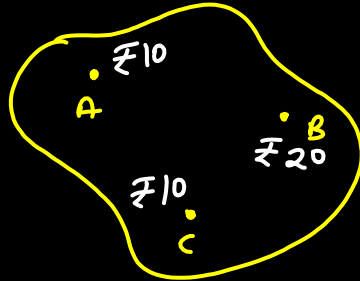
Linear Momentum Conservation

$$\text{Momentum} = \vec{p} = m\vec{v}$$

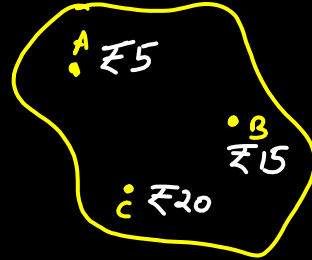
only if

$$(\vec{F}_{\text{net}})_{\text{external}} = 0$$

$$\Rightarrow \vec{p}_{\text{system}} = \text{const}$$



$$\text{System} = 40$$

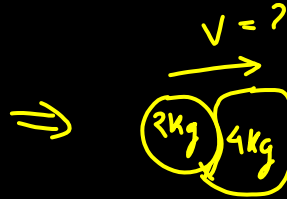


$$\text{System} = 20$$

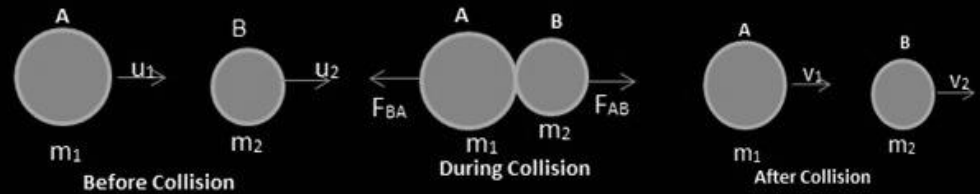
Diagram showing two masses: 2 kg moving at 4 m/s and 4 kg at rest.

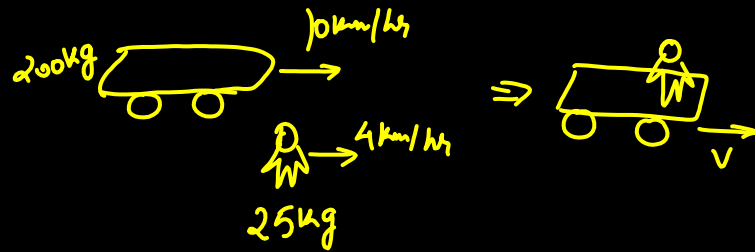
$$2 \times 4 + 0 = 6v$$

$$v = \frac{4}{3} \text{ m/s}$$



Linear Momentum Conservation





A bullock cart of mass 200 kg is moving at a speed of 10 km/h. As it overtakes a school boy walking at a speed of 4 km/h, the boy sits on the cart. If the mass of the boy is 25 kg, what will be the new velocity of the bullock cart ?

- A. $16/3$ km/h
- B. $25/3$ km/h
- ☒ C. $28/3$ km/h
- D. $32/3$ km/h

$$200 \times 10 + 25 \times 4 = 225 \times v$$

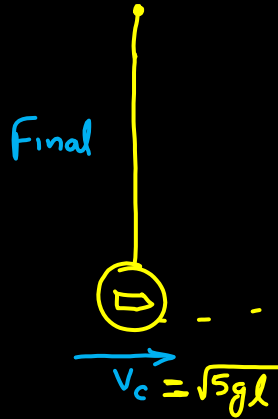
$$\frac{2100}{225} = v$$

$$\frac{420}{45} = \frac{84}{9} = \frac{28}{3} \frac{\text{km}}{\text{hr}}$$

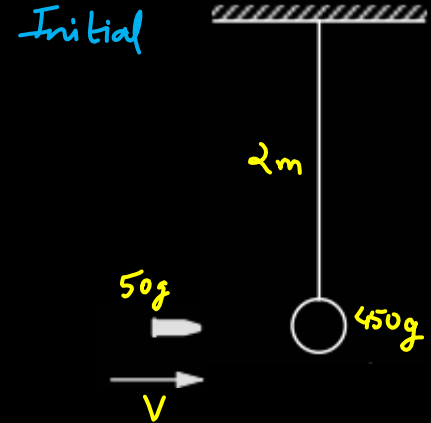


Q

A bullet of mass 50 g is fired horizontally into the bob of mass 450 g of a long simple pendulum with string length 2 m as shown in the figure. The bullet remains inside the bob and the bob just completes the vertical circular motion. Find the speed of the bullet



- A. 10 m/s
- ☒ B. 100 m/s
- C. 500 m/s
- D. 1000 m/s



$$50v = (50 + 450)v_c$$

$$50v = (500)\sqrt{5gl}$$

$$v = 10\sqrt{5 \times 10 \times 2}$$

$$v = 100 \text{ m/s}$$



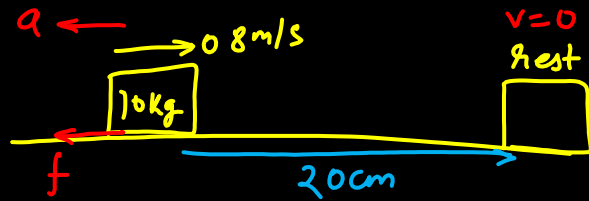
$$P_1 = P_2$$



$$500 \times (20 \times 10^{-3}) = 10v + 100 \times (20 \times 10^{-3})$$

$$10 - 2 = 10v$$

$$v = 0.8 \text{ m/s}$$



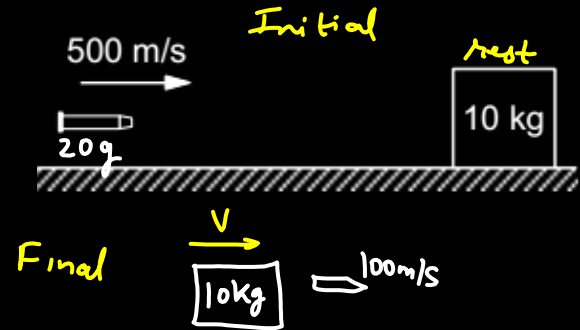
$$f = -ma$$

$$\mu(mg) = -ma$$

$$a = -\mu g$$

A bullet of mass 20 g travelling horizontally with a speed of 500 m/s passes through a wooden block of mass 10.0 kg initially at rest on a level surface. The bullet emerges with a speed of 100 m/s and the block slides 20 cm on the surface before coming to rest. Find the friction coefficient between the block and the surface

- A. 0.16
- B. 0.32
- C. 0.50
- D. 0.72



$$u = 0.8 \text{ m/s}, \quad v = 0, \quad s = 0.2 \text{ m}$$

$$a = -\mu g$$

$$v^2 = u^2 + 2as$$

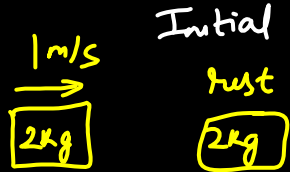
$$0^2 = (0.8)^2 + 2(-\mu g)(0.2)$$

$$4\mu = 0.64$$

$$\boxed{\mu = 0.16}$$



A block of mass 2 kg is moving on a frictionless horizontal surface with a velocity of 1 m/s towards another block of equal mass kept at rest. The spring constant of the spring fixed at one end is 100 N/m. Find the maximum compression of the spring



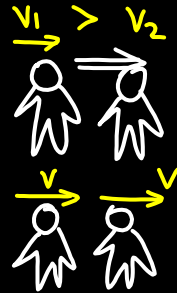
Final

$$P_i = P_f$$

$$2 \times 1 + 2 \times 0 = 2v + 2v$$

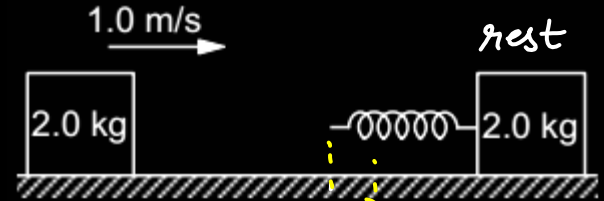
$$2 = 4v$$

$$v = 0.5 \text{ m/s}$$



(No Pushing
or Pulling)

- A. 5 cm
- ✓ B. 10 cm
- C. 15 cm
- D. 20 cm



$$E_1 = E_2$$

$$\frac{1}{2} \times 2 \times (1)^2 = \left(\frac{1}{2} \times 2 v^2 \right) \times 2 + \frac{1}{2} k x^2$$

$$1 = 2v^2 + \frac{1}{2} \times 100 x^2$$

$$1 - 2 \times \left(\frac{1}{2} \right)^2 = 50 x^2$$

$$\frac{1}{2} = 50 x^2$$

$$x = \frac{1}{\sqrt{100}} = \frac{1}{10} \text{ m}$$

$$x = 10 \text{ cm}$$

$$\frac{1 \text{ m/s}}{\boxed{2 \text{ kg}}}$$



$$\frac{1}{2} \times 2 \times v^2$$

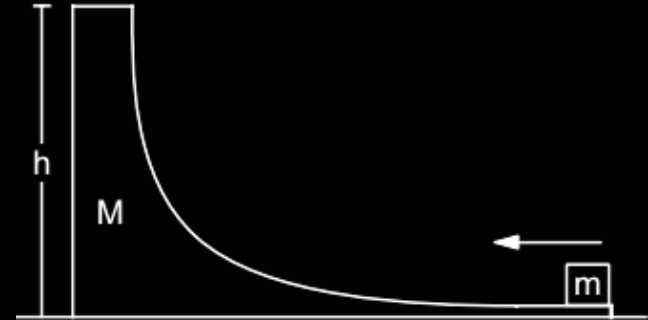


HW

As in comment

Figure shows a small body of mass 2 kg placed over a larger mass 8 kg whose surface is horizontal near the smaller mass and gradually curves to become vertical. The smaller mass is pushed on the longer one at a speed 10 m/s and the system is left to itself. Assume that all the surfaces are frictionless. Find the maximum height (from the ground) that the smaller mass ascends

- A. 2 m
- B. 4 m
- C. 6 m
- D. 8 m



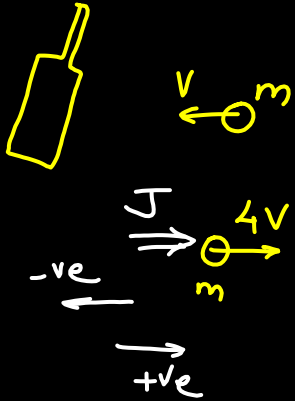




Impulse (J)

Impulse \Rightarrow A sudden momentum change.

$$\vec{J} = \vec{p}_2 - \vec{p}_1 \quad \text{kg} \cdot \frac{\text{m}}{\text{s}}$$



$$\vec{J} = \vec{p}_2 - \vec{p}_1$$

$$J = m(4v) - [-mv]$$

$$\boxed{J = 5mv}$$





Impulse

$$\vec{F} = \frac{d\vec{p}}{dt}$$

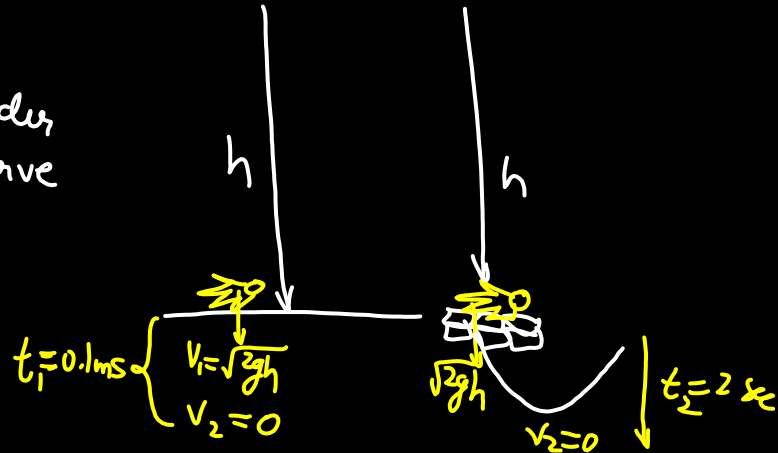
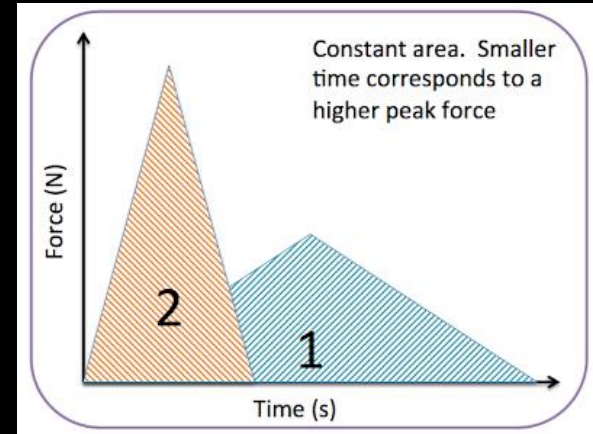
$$\int_{p_1}^{p_2} dp = \int_0^t F dt$$

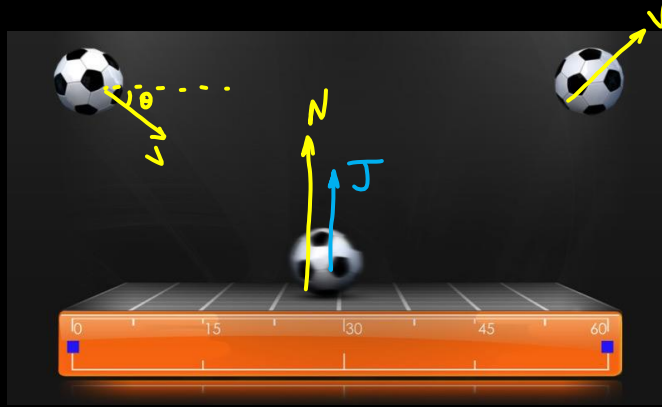
$$p_2 - p_1 = \int F dt$$

$$J = \int F dt$$

\Rightarrow Area under
F-t curve

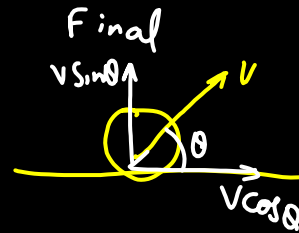
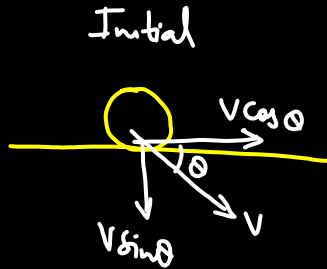
$$J = \int F \uparrow dt \downarrow$$





A ball of mass m moving with velocity v strikes the horizontal surface at an angle θ . Assuming no loss of energy, find the Impulse imparted to the ball

- A. $2mv$
- B. $2mv \cos \theta$
- ☒ C. $2mv \sin \theta$
- D. mv



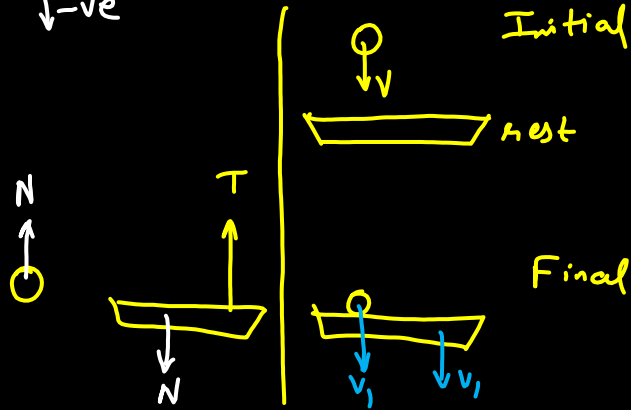
$$J_x = mv \cos \theta - mv \cos \theta = 0$$

$$J_y = (+mv \sin \theta) - (-mv \sin \theta) \\ = 2mv \sin \theta$$





↑ +ve
↓ -ve



A block of mass m and a pan of equal mass are connected by a string going over a smooth light pulley as shown in figure. Initially the system is at rest when a particle of mass m falls on the pan and sticks to it. If the particle strikes the pan with a speed v find the speed with which the system moves just after the collision.

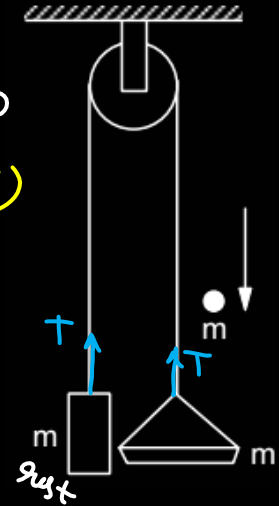
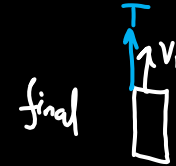
- A. $v/2$
- B. $3v$
- ☒ C. $v/3$
- D. $2v$

$$\text{Block} \Rightarrow \int T \cdot dt = mv_1 - 0$$

→ (3)

$$\text{Ball} \Rightarrow \int N \cdot dt = m(-v_1) - m(-v) \rightarrow (1)$$

$$\text{Pan} \Rightarrow \int T \cdot dt - \int N \cdot dt = m(-v_1) - 0 \rightarrow (2)$$



Add (1) & (2)

$$\int T dt = +mv - 2mv_i$$

Using (3)

$$mv_i = mv - 2mv_i$$

$$3mv_i = mv$$

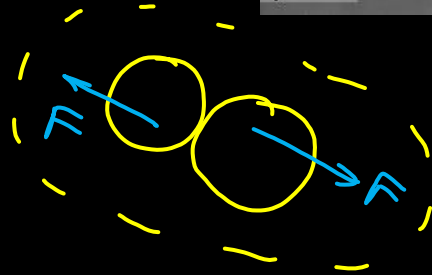
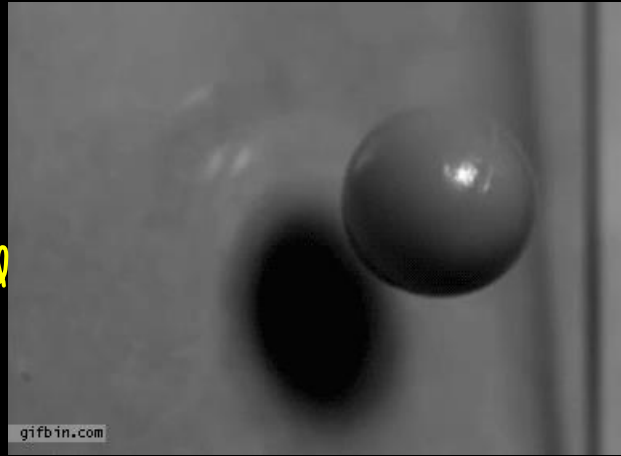
$$v_i = v/3$$



Collision

⇒ Involves
Impulsive
Forces

⇒ They are internal



$$(F_{\text{net}})_{\text{external}} = 0$$

$$\vec{P}_{\text{system}} = \text{const}$$

⇒ For all collisions



Collision

Tennis Ball \Rightarrow Elastic Collision

Plastic Ball \Rightarrow Inelastic Collision

Clay Ball \Rightarrow Completely Inelastic





Types of Collision

Elastic

Regains the shape

⇒ No Energy Loss

$$\Rightarrow p_1 = p_2$$

$$\Rightarrow E_1 = E_2$$

$$\Rightarrow e = 1$$

Inelastic

Deformation of shape

⇒ Energy Loss

$$\Rightarrow p_1 = p_2$$

$$\Rightarrow E_1 > E_2$$

$$\Rightarrow 0 < e < 1$$

Completely Inelastic

Loss of shape

⇒ Huge Energy Loss

$$\Rightarrow p_1 = p_2$$

$$\Rightarrow E_1 \gg E_2$$

$$\Rightarrow e = 0$$

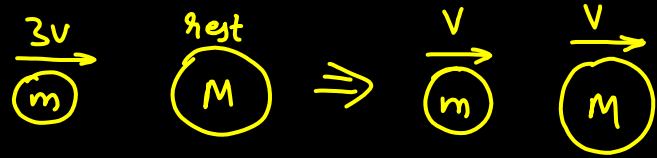


Types of Collision

Q

A ball of mass m moving with velocity $3v$ collides head on with a stationary ball of mass M . The velocity of both the balls become v after collision. The value of M/m

- A. 1
- ☒ B. 2
- C. 3
- D. 4



$$P_1 = P_2$$

$$3mv = (m+M)v$$

$$3m = m + M$$

$$2m = M$$

$$\boxed{\frac{M}{m} = 2}$$



Coefficient of Restitution (e)

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$e \Rightarrow +ve$$

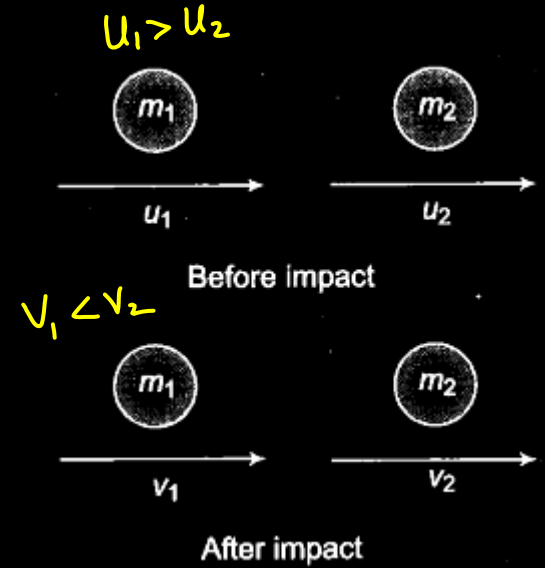
$$0 \leq e \leq 1$$

$$e = \frac{4-1}{5-2}$$

$$e = 1$$

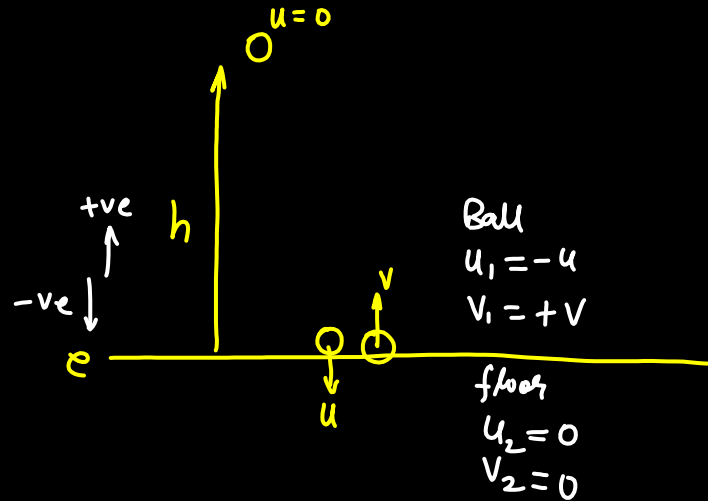
$$\begin{array}{c} 5\text{m/s} \\ \text{---} \text{---} \text{---} \end{array} \quad \begin{array}{c} 2\text{m/s} \\ \text{---} \text{---} \text{---} \end{array}$$

$$\begin{array}{c} 1\text{m/s} \\ \text{---} \text{---} \text{---} \end{array} \quad \begin{array}{c} 4\text{m/s} \\ \text{---} \text{---} \text{---} \end{array}$$



Example

A ball dropped from height h collides with the floor and bounces up. If the coefficient of restitution between ball and floor is e , find the velocity with which the ball rises.

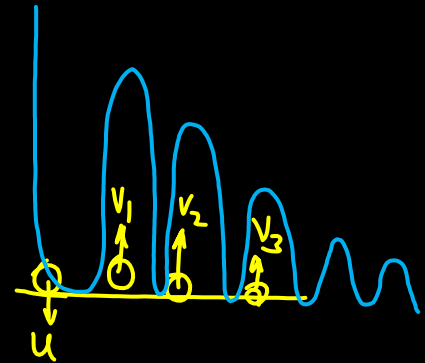


$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$e = \frac{0 - (-v)}{-u - 0}$$

$$e = v/u$$

$$\boxed{v = eu}$$



$$v_1 = eu$$

$$v_2 = e^2 u$$

$$v_3 = e^3 u$$

$$v_n = e^n u$$



Completely Inelastic Collision

$$\Rightarrow e = 0$$

$$\frac{v_2 - v_1}{u_1 - u_2} = 0$$

$$\boxed{v_2 = v_1}$$

\Rightarrow Objects stick together
after the collision





$$P_1 = P_2$$

$$mv = (16m) v_1$$

$$v_1 = \frac{v}{16}$$

$$\% \text{ loss in KE} = \frac{KE_2 - KE_1}{KE_1} \times 100 \%$$

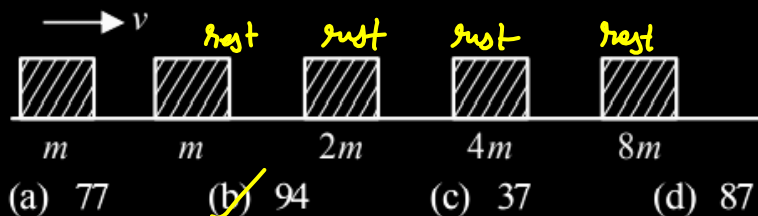
$$= \left(\frac{KE_2}{KE_1} - 1 \right) 100 \%$$

$$= \left(\frac{\frac{1}{2} \times 16m \times v_1^2}{\frac{1}{2} m v^2} - 1 \right) \times 100 \%$$

Blocks of masses m , $2m$, $4m$ and $8m$ are arranged in a line on a frictionless floor. Another block of mass m , moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass $8m$ starts moving the total energy loss is $p\%$ of the original energy. Value of ' p ' is close to :

Initial

[4 Sep. 2020 (I)]

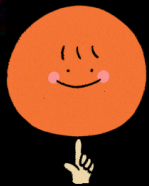


$$16m$$

$$\left(\frac{16 \left(\frac{v}{16} \right)^2}{v^2} - 1 \right) \times 100 \%$$

$$= \left(\frac{1}{16} - 1 \right) \times 100 \%$$

$$= -\frac{15}{16} \times 100 \%$$



Inelastic Collision

$$0 < e < 1$$

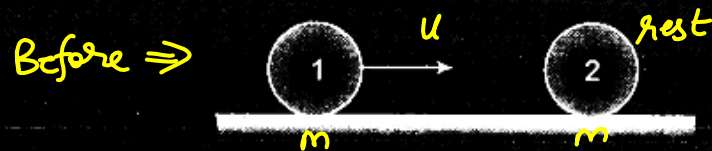
$$\Rightarrow p_1 = p_2$$

$$\Rightarrow e = \frac{v_2 - v_1}{u_1 - u_2}$$





Ball 1 collides with another identical ball 2 at rest as shown in figure.



For what value of coefficient of restitution e , the velocity of second ball becomes two times that of 1 after collision

(a) $1/3$

(b) $1/2$

(c) $1/4$

(d) $1/6$

$$P_1 = P_2$$

$$mu = mv + m(2v)$$

$$\boxed{u = 3v}$$

After \Rightarrow

Diagram showing the state after collision. Ball 1 is moving to the right with velocity v , and Ball 2 is moving to the right with velocity $2v$. Both balls have mass m .

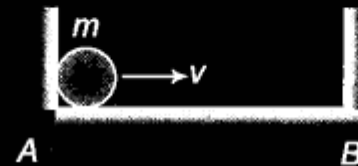
$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{2v - v}{u - 0} = \frac{v}{u} = \frac{1}{3}$$



**HW**

Ans in comment

A small ball moves towards right with a velocity v starting from A. It collides with the wall and returns back and continues to and fro motion. If the average speed for the first trip is $(2/3)v$, find the coefficient of restitution of impact (in 10^{-1}).



(a) 2

(b) 3

(c) 4

(d) 5







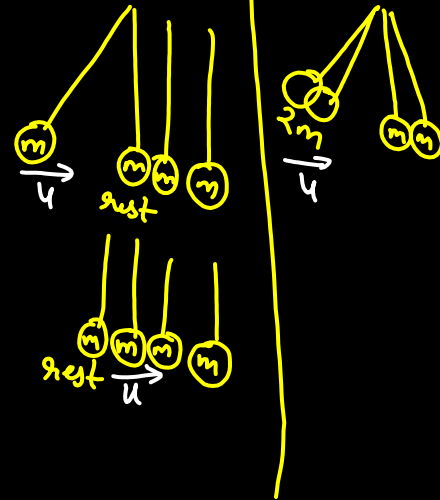
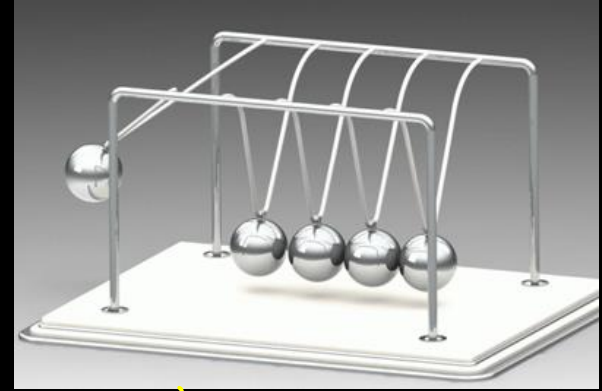
Elastic Collision

$$e = 1$$

$$\frac{v_2 - v_1}{u_1 - u_2} = 1$$

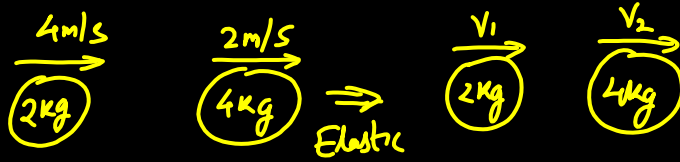
$$\Rightarrow v_2 - v_1 = u_1 - u_2$$

$$\Rightarrow p_1 = p_2$$





Elastic Collision



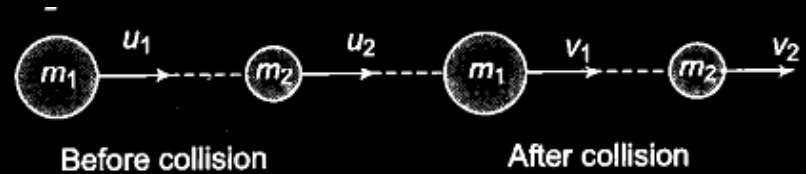
$$1) \quad v_2 - v_1 = 4 - 2$$

$$\boxed{v_2 - v_1 = 2} \rightarrow (1)$$

$$2) \quad 4 \times 2 + 2 \times 4 = 2v_1 + 4v_2$$

$$\boxed{8 = v_1 + 2v_2} \rightarrow (2)$$

$$\boxed{v_2 = 10/3 \text{ m/s}} \quad \boxed{v_1 = 4/3 \text{ m/s}}$$



Not Needed

$$\left\{ \begin{aligned} v_{1f} &= \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) u_{2i} \\ v_{2f} &= \left(\frac{2m_1}{m_1 + m_2} \right) u_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) u_{2i} \end{aligned} \right.$$



An object of mass m collides elastically with another object of mass M which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses M/m is

1) $e = 1$

$$v_2 - v_1 = u_1 - u_2$$

$$v - (-v) = u - 0$$

$$\boxed{2v = u} \rightarrow (1)$$

2) $mu = -mv + Mv$

$$m(2v) = (M - m)v$$

$$2m = M - m$$

$$\boxed{\frac{M}{m} = 3}$$

~~(a) 3~~

(b) 2

(c) $1/2$

(d) 1

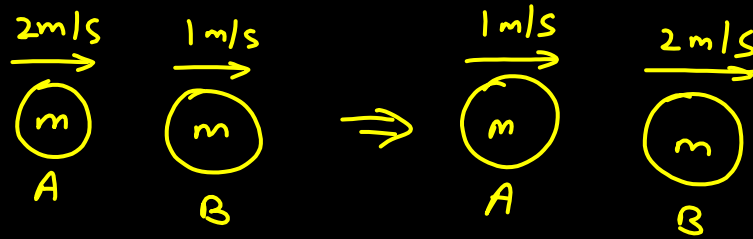




Elastic Collision

Special Cases

1) Equal Masses $m_1 = m_2$

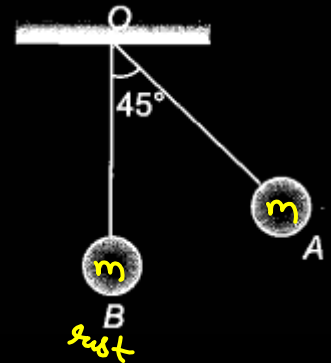
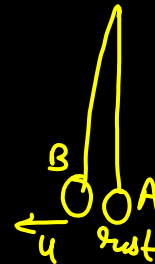


Exchange of velocity



The bob A of a simple pendulum is released when the string makes an angle 45° with the vertical. At the bottom most point of its trajectory it hits another bob B of same material and same mass kept at rest on the table. Assuming no loss of energy

- (a) Both A and B rise to the same height
- (b) Both A and B come to rest at B
- (c) Both A and B move with the same velocity of A
- ☒ (d) A comes to rest and B moves with the velocity of A

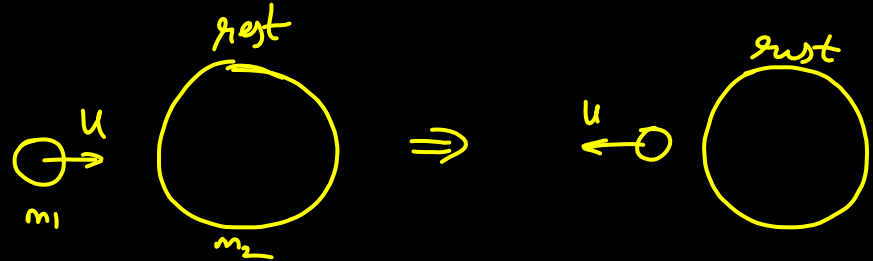




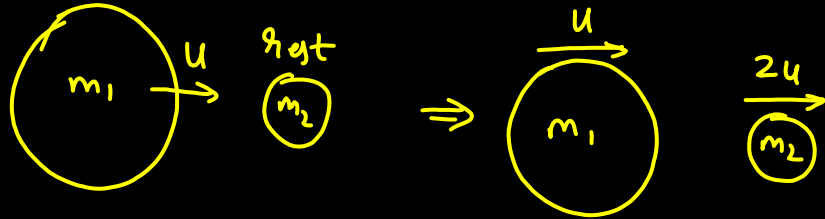
Elastic Collision

Special Cases

2) $m_1 \ll m_2$

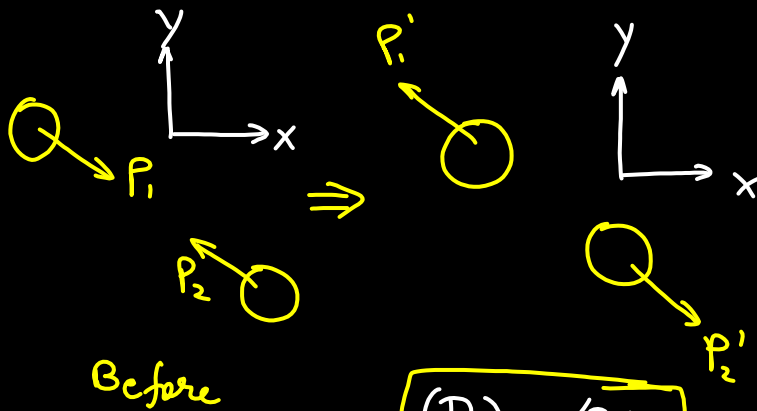


3) $m_1 \gg m_2$



Collision in 2D

Tom OP



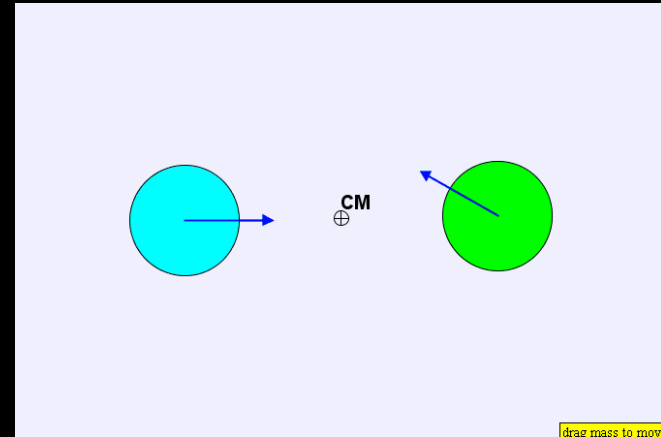
$$\begin{aligned}(P_1)_x &= (P_2)_x \\ (P_1)_y &= (P_2)_y\end{aligned}$$





Collision in 2D

$$(F_{\text{net}})_{\text{external}} = 0$$



Q

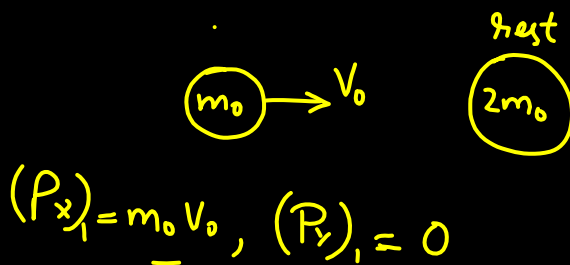
A particle of mass m_0 , travelling at speed v_0 , strikes a stationary particle of mass $2m_0$. As a result the particle of mass m_0 is deflected through 45° and has a final speed of $\frac{v_0}{\sqrt{2}}$. Then the speed of the particle of mass $2m_0$ after this collision is

(a) $\frac{v_0}{2}$

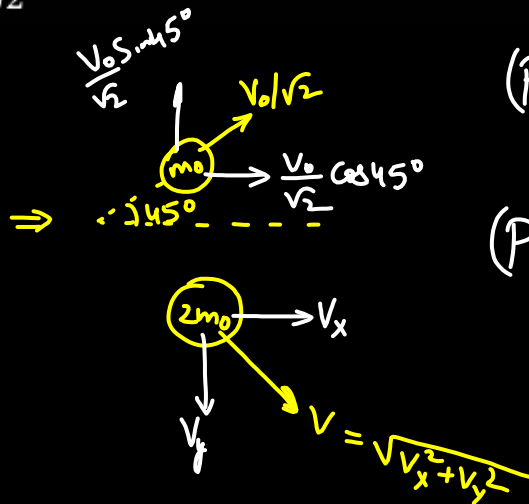
(b) $\frac{v_0}{2\sqrt{2}}$

(c) $\sqrt{2}v_0$

(d) $\frac{v_0}{\sqrt{2}}$



$$(P_x)_i = m_0 v_0, (P_y)_i = 0$$



$$(P_y)_f = (m_0 \frac{v_0}{\sqrt{2}}) - (2m_0 v_y)$$

$$(P_x)_f = m_0 \frac{v_0}{\sqrt{2}} + 2m_0 v_x$$

$$\underline{\underline{y}} \quad 0 = \frac{m_0 v_0}{2} - 2m_0 v_y$$

$$\boxed{v_y = \frac{v_0}{4}}$$

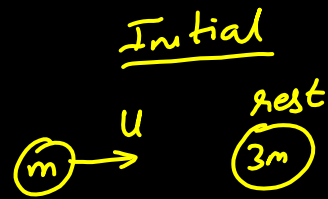
$$\underline{\underline{x}} \quad m_0 v_0 = \frac{m_0 v_0}{2} + 2m_0 v_x$$

$$\frac{m_0 v_0}{2} = 2m_0 v_x$$

$$\boxed{v_x = \frac{v_0}{4}}$$

$$v = \sqrt{\left(\frac{v_0}{4}\right)^2 + \left(\frac{v_0}{4}\right)^2}$$

$$v = \frac{\sqrt{2} v_0}{4} = \frac{v_0}{2\sqrt{2}}$$

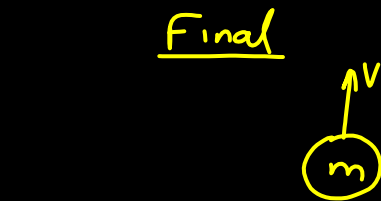


$(P_x)_1 = mu$
 $(P_y)_1 = 0$



A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass $3m$ at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by :

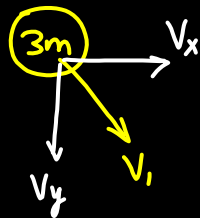
[JEE Main 2020]



$(P_y)_2 = mv - 3mv_y$

$0 = mv - 3mv_y$

$V_y = \frac{v}{3}$



$(P_x)_2 = 0 + 3mv_x$

$mu = 3mv_x$

$V_x = \frac{u}{3}$

(a) $v = \sqrt{\frac{2}{3}}u$

(c) $v = \frac{u}{\sqrt{2}}$

(b) $v = \frac{u}{\sqrt{3}}$

(d) $v = \frac{1}{\sqrt{6}}u$

$$E_1 = E_2$$

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)v_1^2$$

$$u^2 = v^2 + 3v_1^2$$

$$u^2 = v^2 + (\sqrt{v_x^2 + v_y^2})^2 \times 3$$

$$u^2 = v^2 + \left(\frac{u^2}{9} + \frac{v^2}{9}\right) \times 3$$

$$u^2 = v^2 + \frac{u^2}{3} + \frac{v^2}{3}$$

$$\frac{2}{3}u^2 = \frac{4v^2}{3}$$

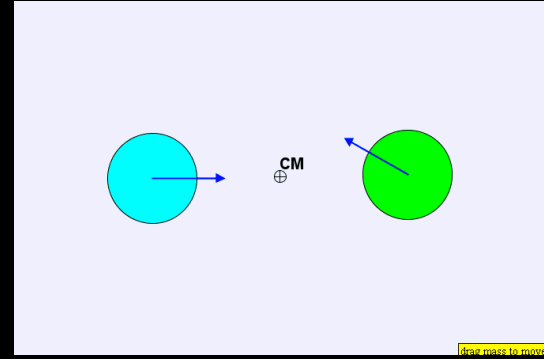
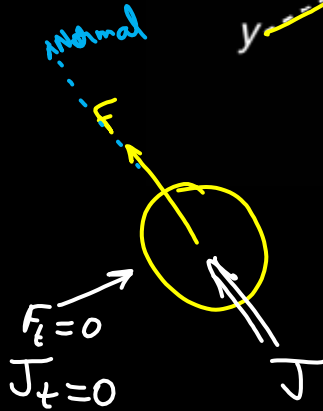
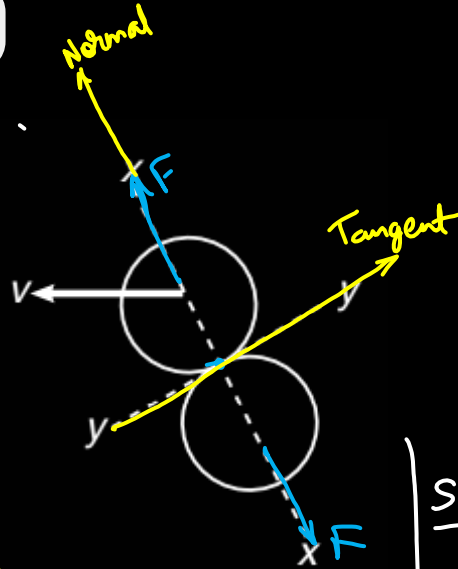
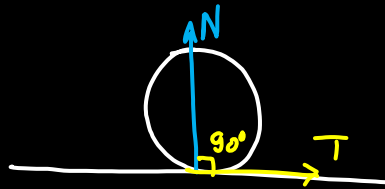
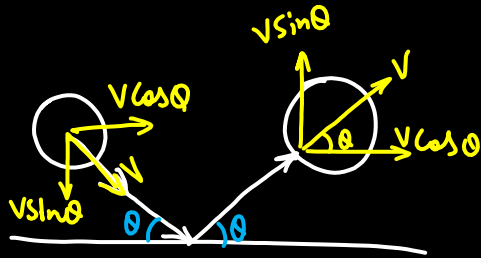
$$u = \sqrt{2}v$$

$$v = \frac{u}{\sqrt{2}}$$



Oblique Collision

(2D or 3D)



Step 1

Tangent ($J_t = 0$)
 $V_{1t} = V_{2t}$

Step 2

Normal ($J_n \neq 0$)

$e =$

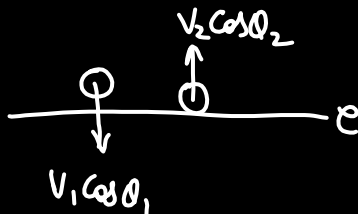
Example

Along Tangent
object's $p = \text{const.}$

$$v_1 \sin \theta_1 = v_2 \sin \theta_2$$

$\rightarrow (1)$

Normal



A ball of mass 2 grams hits the floor with a speed 10 m/s making an angle of incidence 45° with the normal. The coefficient of restitution is $1/\sqrt{3}$. The speed and the angle of reflection of the ball are

(a) 10 m/s, 45°

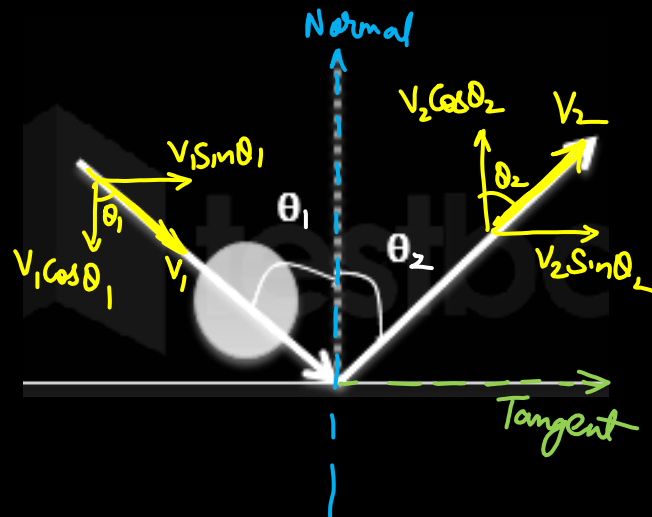
(c) $10\sqrt{2}$ m/s, 30°

(b) $10\sqrt{3}/2$ m/s, 60°

~~(d) $10\sqrt{2}/3$ m/s, 60°~~

$$v_2 \cos \theta_2 = e v_1 \cos \theta_1$$

$\rightarrow (2)$



$$\frac{(1)}{(2)} \quad \frac{v_1 \sin \theta_1}{e v_1 \cos \theta_1} = \frac{v_2 \sin \theta_2}{v_2 \cos \theta_2}$$

$$\frac{\tan \theta_1}{e} = \tan \theta_2$$

$$\frac{\tan 45^\circ}{1/\sqrt{3}} = \tan \theta_2$$

$$\boxed{\theta_2 = 60^\circ}$$

Using (1)

$$(10) \sin 45^\circ = v_2 \sin 60^\circ$$

$$\frac{10}{\sqrt{2}} = v_2 \frac{\sqrt{3}}{2}$$

$$\boxed{\frac{20}{\sqrt{6}} = v_2}$$

Q

$$3 \cos 60^\circ = v \cos 30^\circ$$

$$\frac{3}{2} = v \frac{\sqrt{3}}{2}$$

$$v = \sqrt{3} \text{ m/s}$$

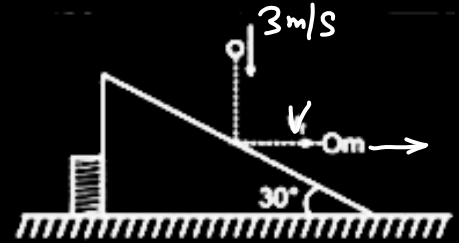
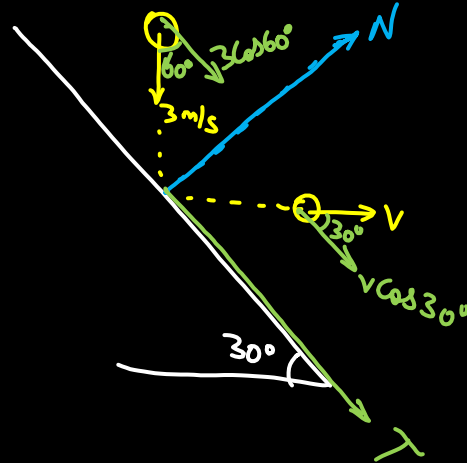
As shown in the figure a body of mass m moving with speed 3 m/s hits a smooth fixed incline plane and rebounds with a velocity v in the horizontal direction. If the angle of incline is 30° , then velocity v will be

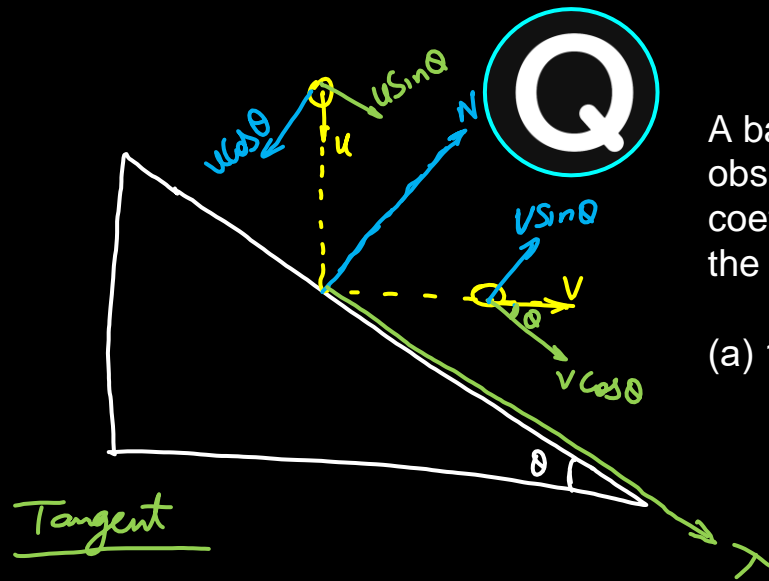
(a) 3

(b) $\sqrt{3}$

(c) $1/\sqrt{3}$

(d) Not possible





Tangent

$$u \sin \theta = v \cos \theta$$

$$\boxed{\tan \theta = \frac{v}{u}} \rightarrow (1)$$

A ball is dropped on a smooth inclined plane and is observed to move horizontally after the impact. The coefficient of restitution between the plane and ball is e . If the inclination of the plane is θ , then the value of $\tan \theta$ is

(a) 1

~~(b) \sqrt{e}~~

(c) $e/2$

(d) e

Normal

$$e(u \cos \theta) = v \sin \theta$$

$$e = \left(\frac{v}{u}\right) \tan \theta$$

$$e = \tan^2 \theta$$

$$\boxed{\tan \theta = \sqrt{e}}$$



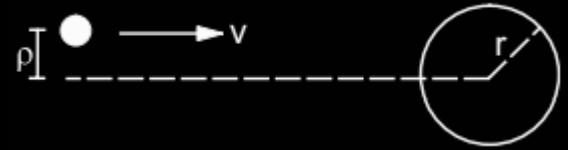


HW

Ans in comment

A small particle travelling with a velocity $v=10$ m/s collides elastically with a spherical body of equal mass and of radius $r=5$ m initially kept at rest. The centre of this spherical body is located a distance $\rho=4$ m away from the direction of motion of the particle. Find the final velocity of the sphere.

- (a) 10 m/s (b) 2m/s (c) 4m/s (d) 6m/s



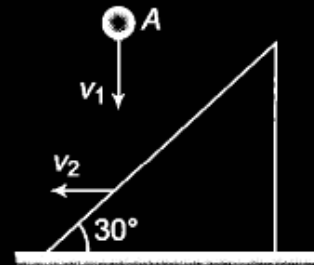


HW Ans in comment

A ball A is falling vertically downwards with velocity v_1 . It strikes elastically with a wedge moving horizontally with velocity v_2 as shown in figure. What must be the ratio $\frac{v_1}{v_2}$

so that the ball bounces back in vertically upwards direction relative to wedge?

t.me/abhilashsharma11tjee



(a) $\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) 2

(d) $\frac{1}{2}$

Variable Mass System

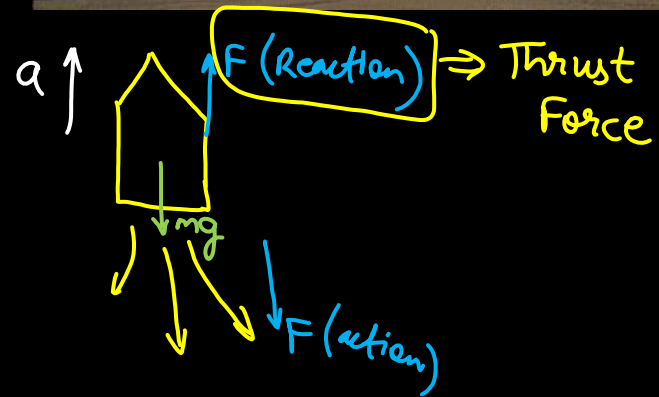
$F_{\text{net}} \Rightarrow \text{accelerate}$

$$\vec{F}_{\text{ext}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v})$$

$$\vec{F}_{\text{ext}} = m \frac{d\vec{v}}{dt} + \vec{v} \frac{dm}{dt}$$

$$\vec{F}_{\text{ext}} = m\vec{a} + \vec{v} \frac{dm}{dt}$$

$$m\vec{a} = \vec{F}_{\text{ext}} - \vec{v} \frac{dm}{dt} \rightarrow \text{Thrust Force}$$





Thrust Force

$$\vec{F}_t = \vec{v} \frac{dm}{dt}$$

$$\mu = \frac{dm}{dt} = \text{Rate of change of mass}$$

$$\boxed{F_t = \mu v}$$

Example

$\frac{dm}{dt}$ rest

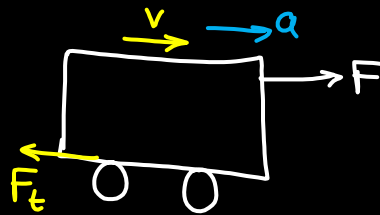
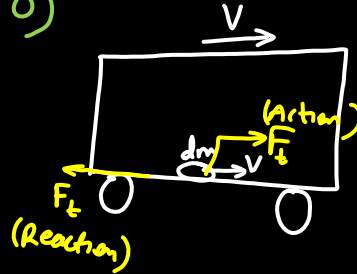
$\rightarrow v$

A constant force F is applied on a trolley of initial mass m_0 kept over a smooth surface at rest. Sand is poured gently over the trolley at a constant rate of μ kg/s. After time t , find the velocity of the trolley.

$$f \cdot dt = (dm \cdot v - 0)$$

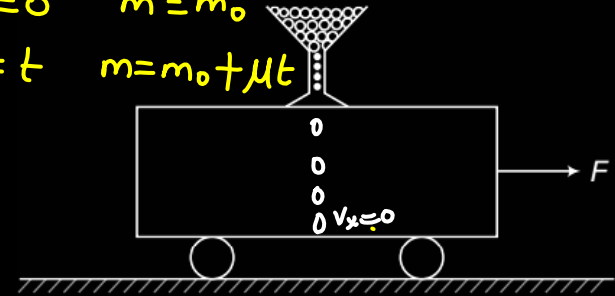
$$f = \frac{dm}{dt} v$$

$$f = \mu v$$



$$t=0 \quad m=m_0$$

$$t=t \quad m=m_0 + \mu t$$



Smooth

$$dp = (dm)(v) - 0$$

$$F_{\text{net}} = ma$$

$$F - F_t = ma$$

$$F - \mu v = m \frac{dv}{dt}$$

$$\int_0^t \frac{dt}{m} = \int_0^v \frac{dv}{F - \mu v}$$

$$\int_0^t \frac{dt}{m_0 + \mu t} = \int_0^v \frac{dv}{F - \mu v}$$

$$\frac{1}{\mu} \ln(m_0 + \mu t) \Big|_0^t = \frac{1}{-\mu} \ln(F - \mu v) \Big|_0^v$$

$$\frac{m_0 + \mu t}{m_0} = \frac{F}{F - \mu v}$$

$$\frac{m_0}{m_0 + \mu t} = 1 - \frac{\mu v}{F}$$

$$\frac{\cancel{\mu} v}{F} = \frac{\cancel{\mu} t}{m_0 + \mu t}$$

$$v = \frac{F t}{m_0 + \mu t}$$

Example

A trolley of initial mass m_0 is kept over a smooth surface as shown in figure. A constant force F is applied on it. Sand kept inside the trolley drains out from its floor at a constant rate of μ kg/s. After time t find the velocity of the trolley

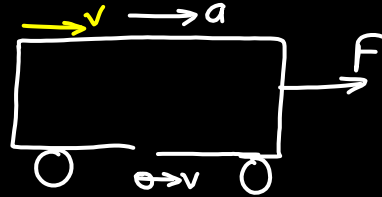
$$F = ma$$

$$F = (m_0 - \mu t) a$$

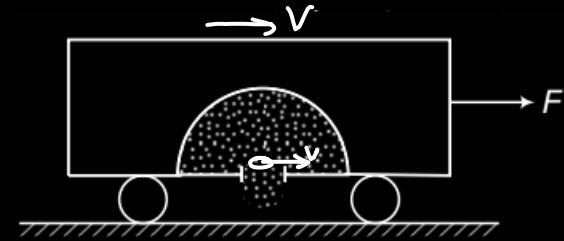
$$F = (m_0 - \mu t) \frac{dv}{dt}$$

$$F \int_0^t \frac{dt}{m_0 - \mu t} = \int_0^v dv$$

$$\frac{F}{-\mu} \ln(m_0 - \mu t) \Big|_0^t = v$$



$$F_t = 0$$



$$dp = dm v - dm v$$

$$dp = 0$$

$$v = \frac{F}{\mu} \ln \left(\frac{m_0}{m_0 - \mu t} \right)$$

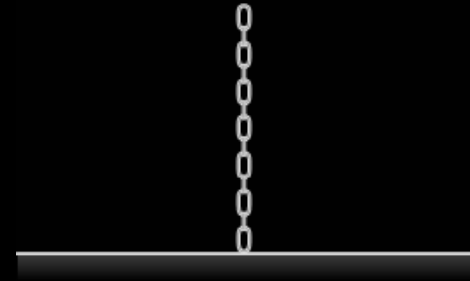


HW

A uniform chain of mass m and length l hangs on a thread and just touches the surface of a weighing scale by its lower end. Find the reading of the scale when half of its length has fallen. The fallen part does not form heap.

(a) $mg/2$ (b) mg (c) $3mg/2$

(d)





Variable Mass System

$$P_2 - P_1 = dP = [(m+dm)(v+dv)] - [dmu + mv]$$

$$dP = \cancel{mv} + m dv + dm v + \cancel{dm dv} - dm u - \cancel{mv}$$

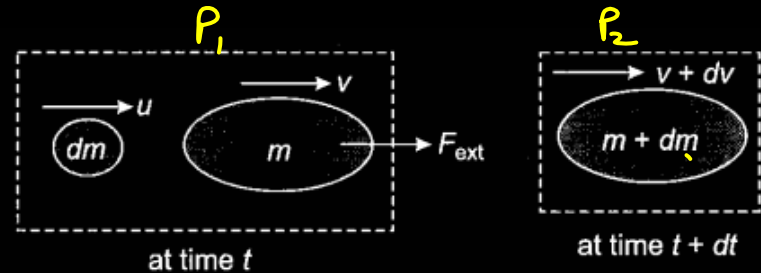
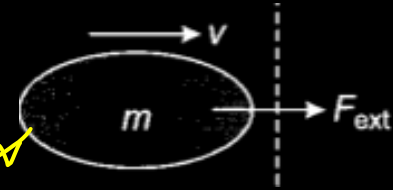
$$dP = m dv + dm(v-u)$$

$$dP - dm(v-u) = m dv$$

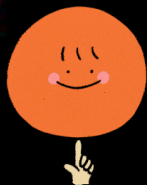
$$dP + (u-v) dm = m dv$$

$$\frac{dP}{dt} + (u-v) \frac{dm}{dt} = m \frac{dv}{dt}$$

$$\boxed{F_{\text{ext}} + (u-v) \frac{dm}{dt} = m \vec{a}}$$



$$\boxed{\vec{F}_{\text{ext}} + \vec{F}_{\text{thrust}} = \vec{F}_{\text{net}}}$$



Variable Mass System

$$\vec{F}_{ext} + \vec{V}_R \frac{dm}{dt} = \vec{F}_{net}$$

$$F_t = V_R \frac{dm}{dt}$$

$$F_t = \mu V_R$$

Rocket Propulsion

$$F_{\text{net}} = F_t - mg$$

$$ma = uV_r - mg$$

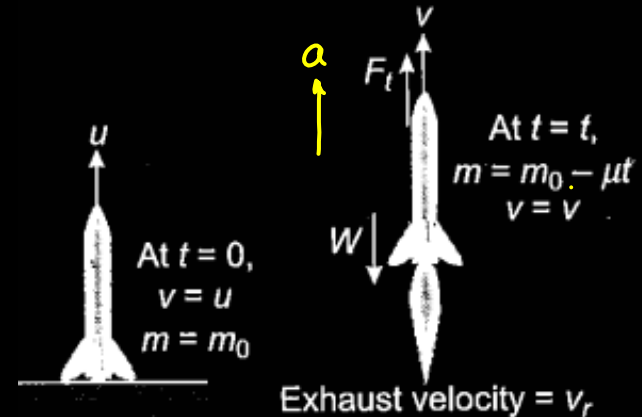
$$ma = \left(-\frac{dm}{dt}\right)V_r - mg$$

$$m \frac{dv}{dt} = -\frac{dm}{dt} V_r - mg$$

$$m dv = -dm V_r - mg dt$$

$$\int_u^v dv = -V_r \int_{m_0}^m \frac{dm}{m} - g \int_0^t dt$$

$$v - u = -V_r \ln\left(\frac{m}{m_0}\right) - gt$$



$$m = m_0 - \mu t$$

$$\frac{dm}{dt} = -\mu$$

$$-\frac{dm}{dt} = \mu$$



Rocket Propulsion

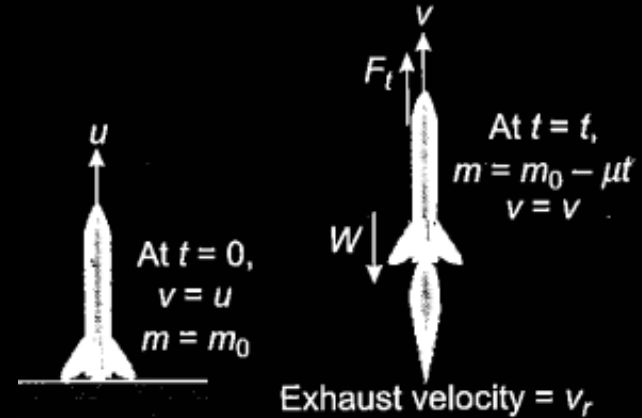
$$v = u + V_e \ln\left(\frac{m_0}{m}\right) - gt$$

$$v = u - gt + V_e \ln\left(\frac{m_0}{m_0 - \mu t}\right)$$

Generally

$u=0$ & Ignoring Gravity

$$v = V_e \ln\left(\frac{m_0}{m_0 - \mu t}\right)$$







Nurture Batch

11th

for IIT JEE Main and Advanced 2024

Code: **ABHILASH**

Batch highlights:

- Curated by India's Top Educators
- Coverage of Class 11 JEE syllabus
- Enhance conceptual understanding of JEE Main & JEE Advanced subjects
- Systematically designed courses
- Strengthen JEE problem-solving ability



Prashant Jain
Mathematics Maestro



Nishant Vora
Mathematics Maestro



Ajit Lulla
Physics Maestro



Abhilash Sharma
Physics Maestro



Sakshi Vora
Chemistry Maestro



Megha Khandelwal
Chemistry Maestro



12th

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Nishant Vora
Mathematic Maestro



Ajit Lulla
Physics Maestro



Sakshi Ganotra
Organic & Inorganic
Chemistry Maestro



Megha Khandelwal
Chemistry Maestros



Prashant Jain
Mathematics Maestro



Abhilash Sharma
Physics Maestro



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- Coverage of Class 11 & 12 syllabus of JEE
- Deep dive at a conceptual level for JEE Main and JEE Advanced
- Systematic course flow of subjects and related topics
- Strengthening the problem-solving ability of JEE level problems

For more details, contact **8585858585**



Nishant Vora
Mathematics Maestros



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Ajit Lulla
Physics Maestros



Abhilash Sharma
Physics Maestros



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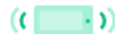
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12 months	₹4,875/mo	No cost EMI	+10% OFF	₹58,500
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24 months	₹2,310/mo	No cost EMI	+10% OFF	₹55,440
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6 months	₹4,620/mo	No cost EMI	+10% OFF	₹27,720
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