FRICTION & CIRCULAR MOTION

Abhilash Sharma

B.Tech - NIT Calicut

- Experience - 8+ years
- 700+ selections in JEE Advanced
- 6000+ selections in JEE Mains

BounceBack

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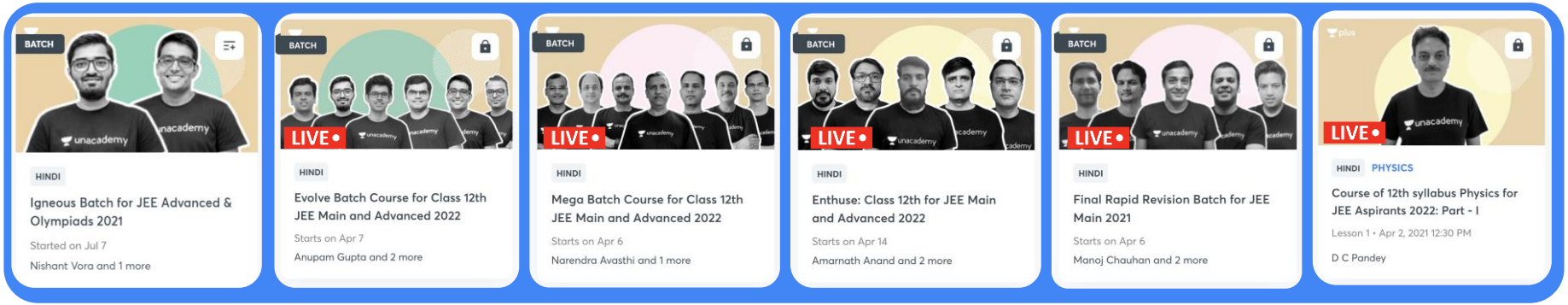

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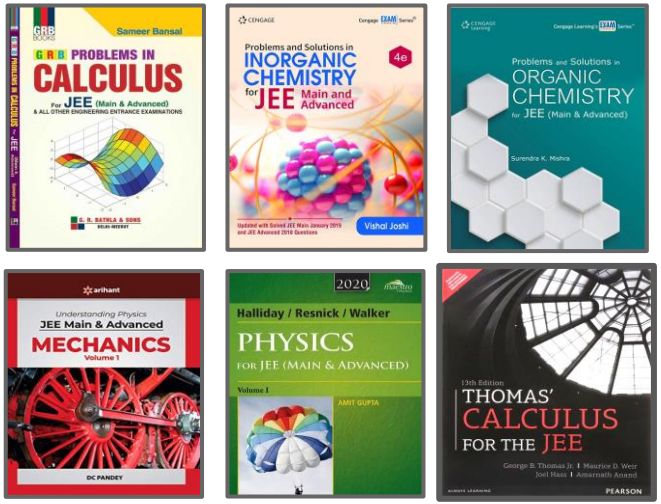

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Nurture Batch 11th

for IIT JEE Main and Advanced 2024

Code: ABHILASH

Batch highlights:

- Curated by India's Top Educators
- Coverage of Class 11 JEE syllabus
- Enhance conceptual understanding of JEE Main & JEE Advanced subjects
- Systematically designed courses
- Strengthen JEE problem-solving ability

Prashant Jain
Mathematics Maestro

Nishant Vora
Mathematics Maestro

Ajit Lulla
Physics Maestro

Abhilash Sharma
Physics Maestro

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Chemistry Maestro

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Organic & Inorganic Chemistry Maestro

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- Systematic course flow of subjects and related topics
- Strengthening the problem-solving ability of JEE level problems

For more details, contact **8585858585**

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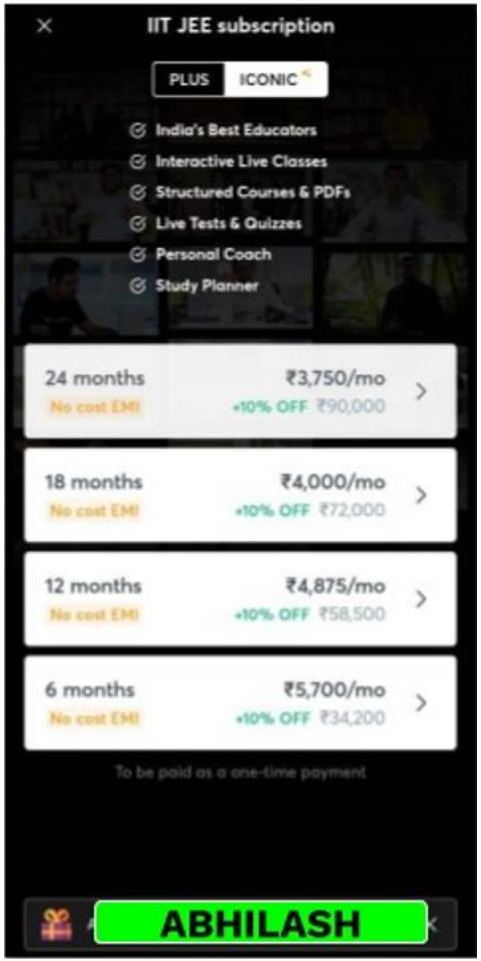
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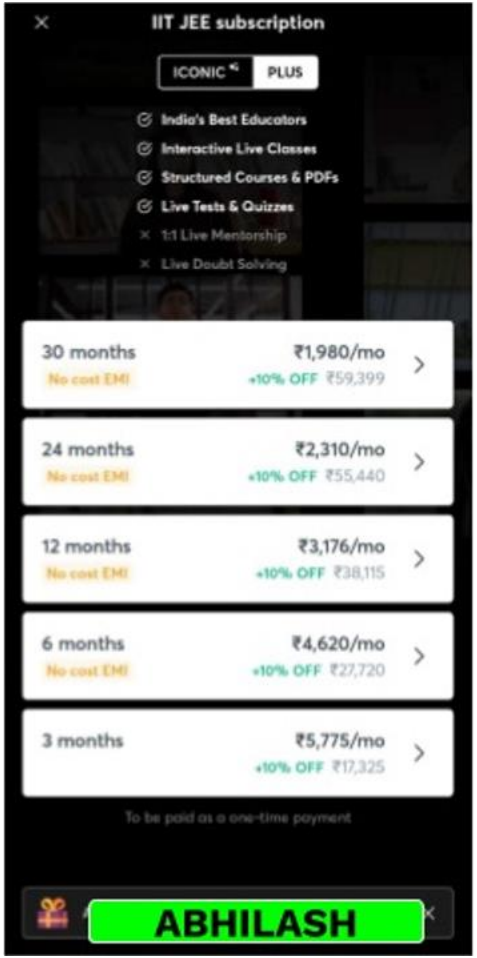
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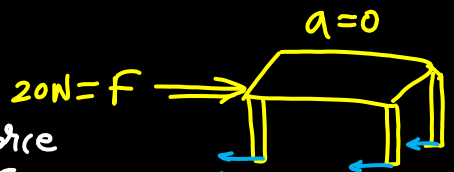
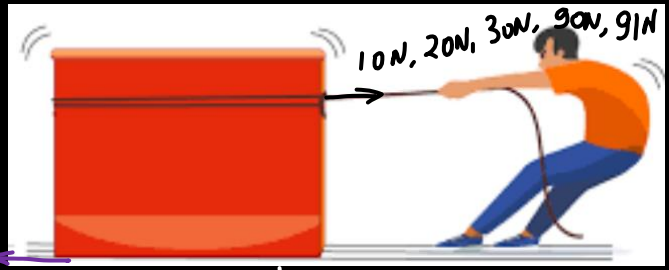
Friction & Circular Motion

Frictional Force



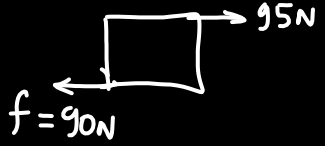
⇒ Reaction Force
⇒ Adjusting Force
⇒ Has a maximum/limiting value

⇒ frictional force ≤ Applied Force

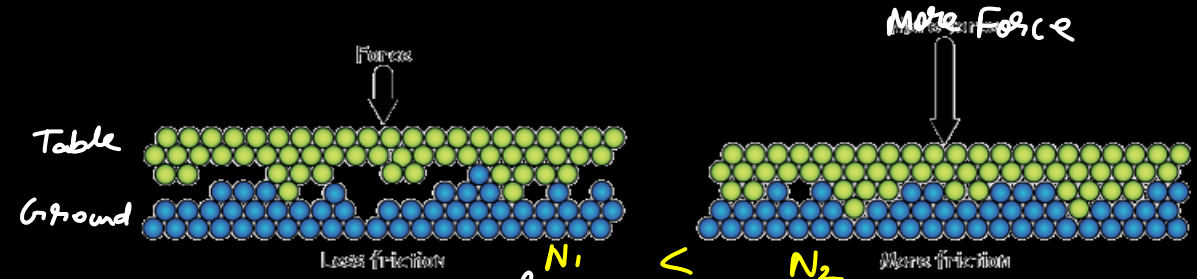


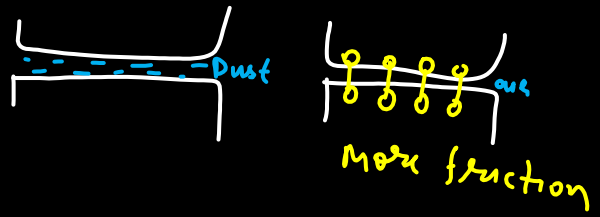
friction → 10N, 20N, 30N, 90N

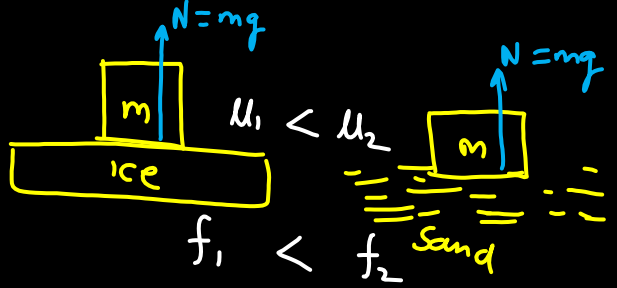
Limit/Max Value



Origin of Friction



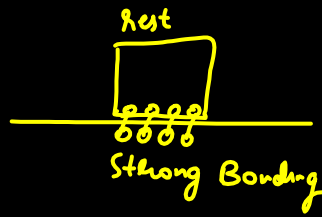




Types of Friction

Static Friction

$a = 0 \quad v = 0$

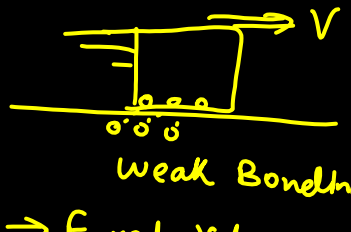


⇒ Variable Force

$\mu_s \Rightarrow$ coefficient of static friction

Kinetic/Sliding Friction

$v \neq 0$



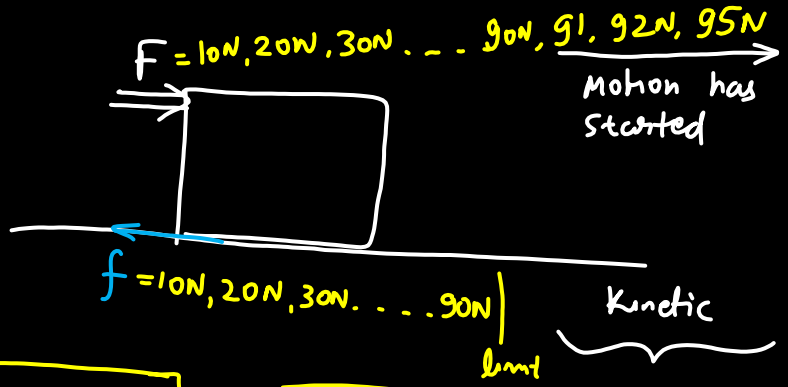
⇒ Fixed Value

$\mu_k \Rightarrow$ Coefficient of Kinetic Friction

$$(f_s)_{max} > f_k$$

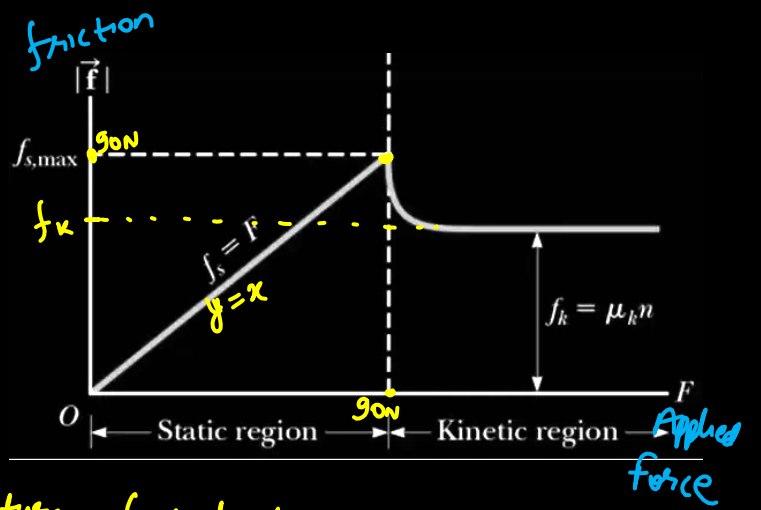
$$\mu_s > \mu_k$$

Types of Friction



$(f_s)_{max} = \mu_s N$

$f_k = \mu_k N$



$\mu \Rightarrow$ Nature of contact surfaces

$N \Rightarrow$ Pressing of the surface

Unacademy Atoms

Q

$N + 40\sin 37^\circ = 100$

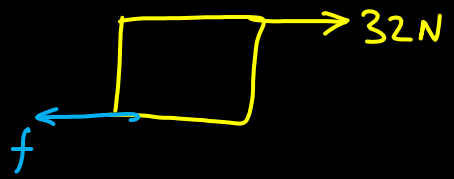
$N = 100 - 40 \times \frac{3}{5}$

$N = 76\,N$

$F = 40\cos 37^\circ$

$= 40 \times 4/5$

$= 32\,N$

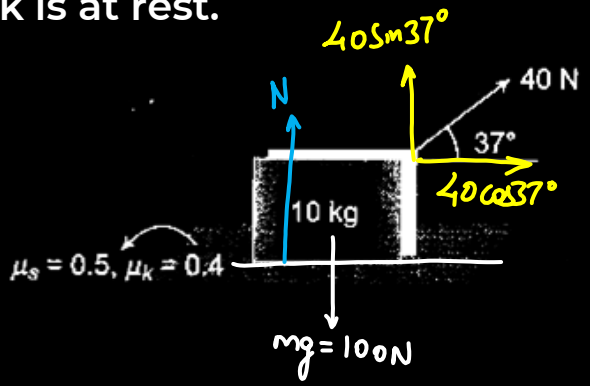


$(f_s)_{max} = \mu_s N = 0.5 \times 76$

$(f_s)_{max} = 38\,N$

Find out acceleration of the block.

Initially the block is at rest.

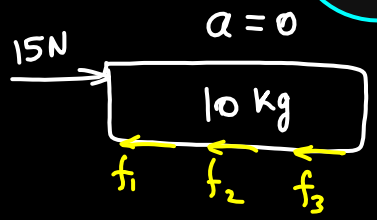


A 2.8 m/s2

B 0.2 m/s2

C 4 m/s2

D zero ✓



$(f_1)_{max} = \mu N_1 = (0.2)(50) = 10\,N$

$(f_2)_{max} = \mu N_2 = (0.2)(30) = 6\,N$

$(f_3)_{max} = \mu N_3 = (0.2)(20) = 4\,N$

$(f_1 + f_2 + f_3)_{max} = 20\,N$

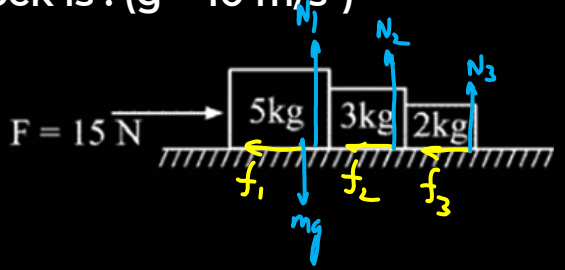
Three blocks of masses 5 kg, 3 kg and 2 kg are placed on a rough surface as shown in figure. Coefficient of friction between block and surface is same for all blocks and is equal to μ = 0.2 A horizontal force of 15 N kg block. The force exerted by 3 kg block on 2 kg block is : (g = 10 m/s²)

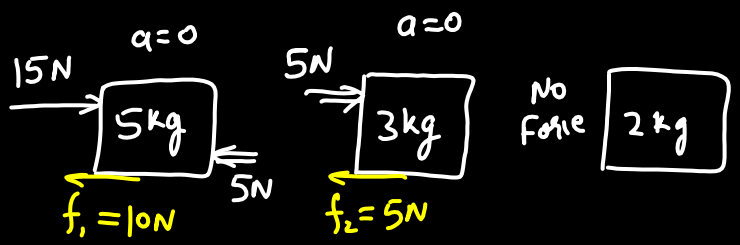
A. 15 N

B. 5 N

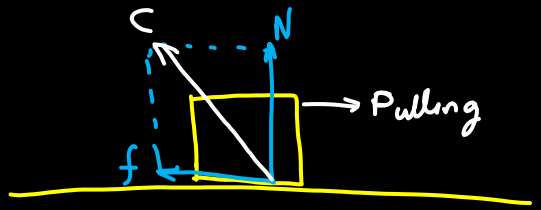
C. Zero ✓

D. None of these

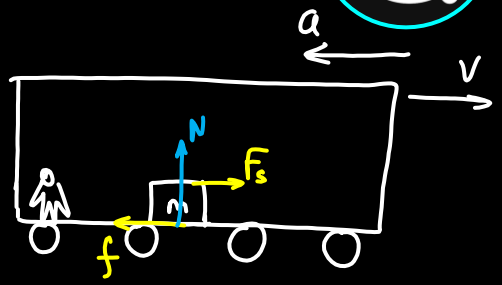


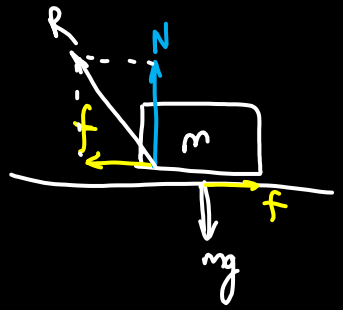


Contact Force (C)

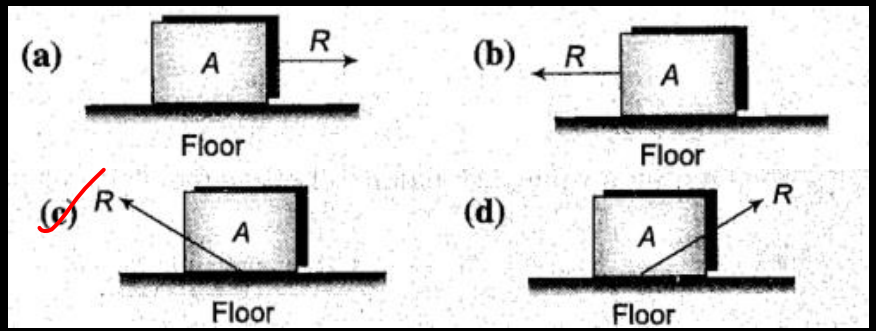


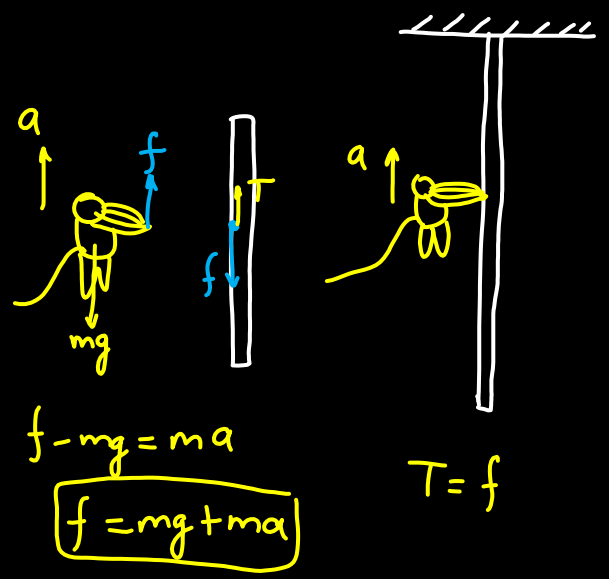
$$C = \sqrt{N^2 + f^2}$$





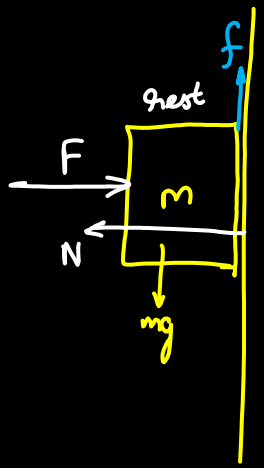
A box ‘A’ is lying on the horizontal floor of the compartment of a train running along horizontal rails from left to right. At time t , it decelerates. Then the reaction R by the floor on the box is given best by





A monkey of mass m is climbing a rope hanging from the roof with acceleration a. The coefficient of static friction between the body of the monkey and the rope is m. Find the direction and value of friction force on the monkey.

A. Upward, F = m(g + a)
B. Downward, F = m(g + a)
C. Upward F = mg
D. Downward F = mg



$f = mg \Rightarrow (f_s)_{max} \geq mg$

$N = F$

$\mu N \geq mg$

$\mu F \geq mg$

$$\boxed{F \geq \frac{mg}{\mu}}$$

$$F_{min} = \frac{mg}{\mu}$$

Unacademy Atoms

Q

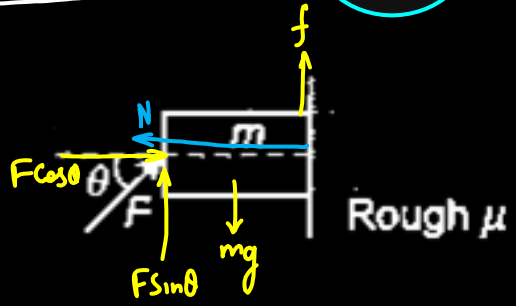
If the external force F is applied at an angle Θ with the horizontal as shown in the figure, then the minimum value of F required to keep the block stationary is

(a) $\frac{mg}{\mu\cos\theta}$

(b) $\frac{mg}{\sin\theta+\mu\cos\theta}$

(c) $\frac{mg}{\sin\theta-\mu\cos\theta}$

(d) $\frac{mg}{\mu\tan\theta}$



X axis

$F\cos\theta = N$

Y axis

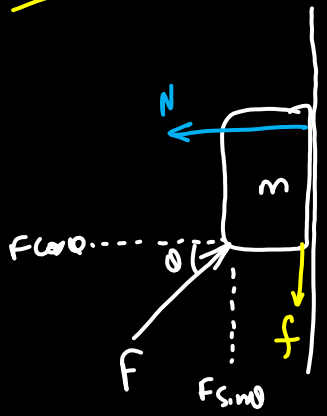
$F\sin\theta + (f_s)_{max} \geq mg$

$F\sin\theta + \mu N \geq mg$

$F\sin\theta + \mu(F\cos\theta) \geq mg$

$F \geq \frac{mg}{\sin\theta+\mu\cos\theta}$

Maximum case



Replace μ with $-\mu$

$$F_{max} = \frac{mg}{\sin\theta - \mu\cos\theta}$$

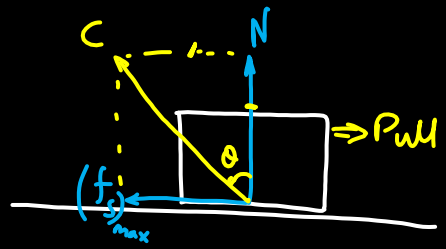
Angle of Friction

The angle of friction between any two surfaces in contact is defined as the angle which the resultant of the force of limiting friction f and normal reaction R makes with the direction of normal reaction R. It is represented by θ.

$$\tan\theta = \frac{(f_s)_{max}}{N}$$

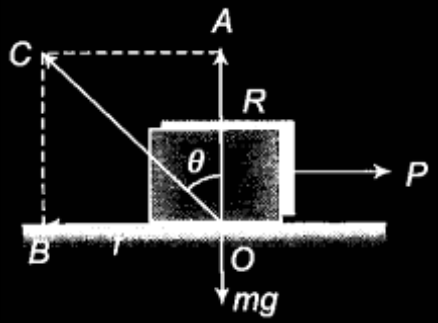
$$\tan\theta = \frac{\mu_s N}{N}$$

$$\tan\theta = \mu_s$$



$$C = \sqrt{N^2 + (f_s)_{max}^2}$$

$$C = \sqrt{N^2 + \mu_s^2 N^2}$$



Angle of Repose

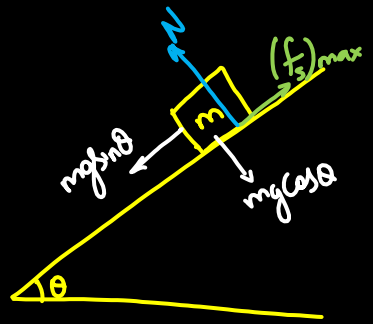
⇒ When sliding just starts

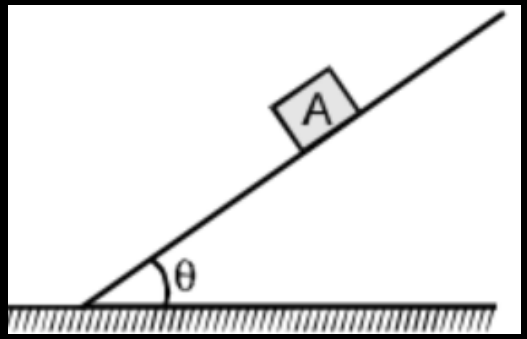
$mg\sin\theta = (f_s)_{max}$

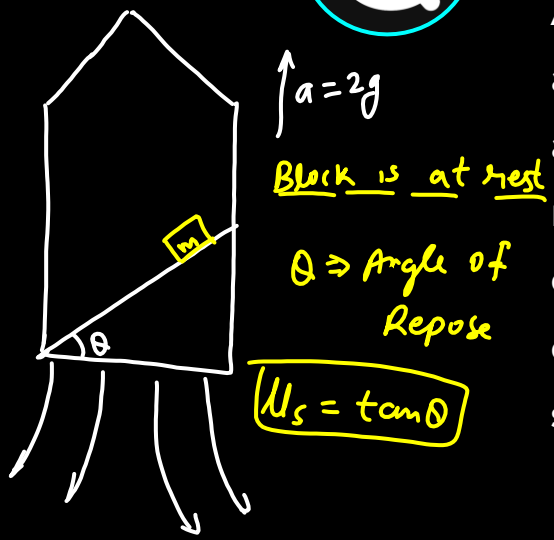
$mg\sin\theta = \mu_s N$

$mg\sin\theta = \mu_s(mg\cos\theta)$

$\boxed{\mu_s = \tan\theta}$







A rocket is fired vertically from the earth with an acceleration of 2g, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is :

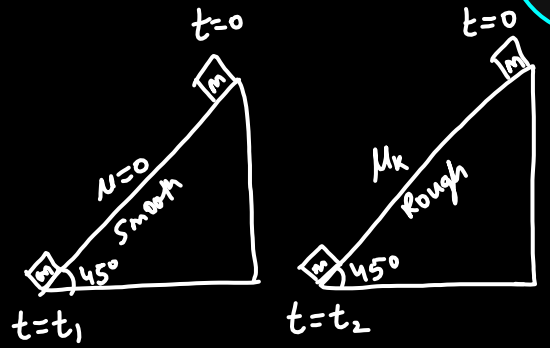
[JEE Main 2016]

A. $\tan 2\theta$

B. $\tan \theta$ ✓

C. $3 \tan \theta$

D. $2 \tan \theta$



Given $t_2 = nt_1$

A given object takes n times more time to slide down 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is:

[JEE Main 2018]

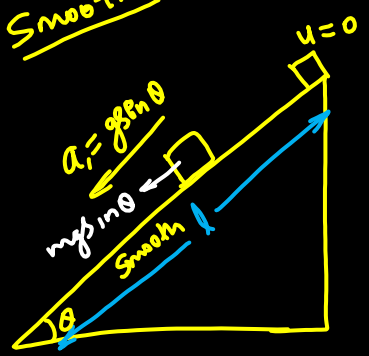
(a) $\sqrt{1-\frac{1}{n^2}}$

(b) $1-\frac{1}{n^2}$

(c) $\frac{1}{2-n^2}$

(d) $\sqrt{\frac{1}{1-n^2}}$

Smooth



$t = t_1$

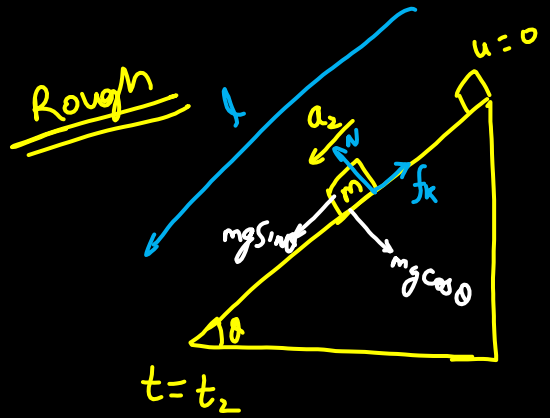
$u = 0,\ a_1 = g\sin\theta,\ s = l,\ t = t_1$

$$s = ut + \frac{1}{2}at^2$$

$$l = 0 + \frac{1}{2}(g\sin\theta)t_1^2$$

$$t_1 = \sqrt{\frac{2l}{g\sin\theta}}$$

Rough



$t = t_2$

$$mg\sin\theta - f_k = ma_2$$

$$mg\sin\theta - \mu_k N = ma_2$$

$$mg\sin\theta - \mu_k mg\cos\theta = ma_2$$

$$a_2 = g(\sin\theta - \mu_k\cos\theta)$$

$u=0,\quad a=a_2,\quad t=t_2,\quad s=l$

$$l = 0 + \frac{1}{2}a_2 t_2^2$$

$$\boxed{t_2 = \sqrt{\frac{2l}{g(\sin\theta - \mu_k\cos\theta)}}}$$

$t_2 = n t_1$

$$\sqrt{\frac{2l}{g(\sin\theta-\mu_k\cos\theta)}} = n\sqrt{\frac{2l}{g\sin\theta}}$$

$$\frac{1}{\sin\theta-\mu_k\cos\theta} = \frac{n^2}{\sin\theta}$$

$$\frac{1}{n^2} = \frac{\sin\theta-\mu_k\cos\theta}{\sin\theta}$$

$$\frac{1}{n^2} = 1 - \mu_k\cot\theta$$

$$\frac{1}{n^2} = 1 - \mu_k\cot 45^\circ$$

$$\boxed{\mu_k = 1 - \frac{1}{n^2}}$$

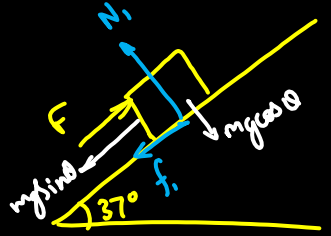
A man can just push a box on 37° slope. When he keeps it at the point where the angle increases to 53°, he can just hold it from sliding back. If the coefficient of friction between the box and the slope is μ , find 1/μ. Assume that man is applying same magnitude of force along the tangent to the curve only.

A. 1

B. 3

C. 5 ✓

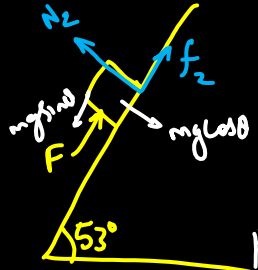
D. 7



$$F = f_1 + mg\sin 37°$$

$$F = \mu mg\cos 37° + mg\sin 37°$$

$$F = mg\left(\frac{4}{5}\mu + \frac{3}{5}\right)$$

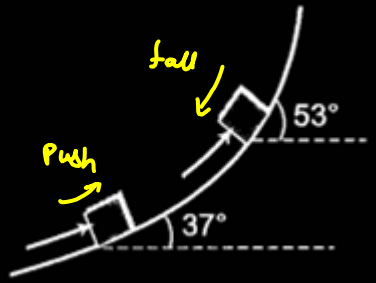


$$F + f_2 = mg\sin 53°$$

$$F + \mu N_2 = mg\left(\frac{4}{5}\right)$$

$$F + \mu mg\cos 53° = \frac{4}{5}mg$$

$$F = mg\left(\frac{4}{5} - \frac{3\mu}{5}\right)$$



$$mg\left(\frac{4}{5}\mu + \frac{3}{5}\right) = mg\left(\frac{4}{5} - \frac{3\mu}{5}\right)$$

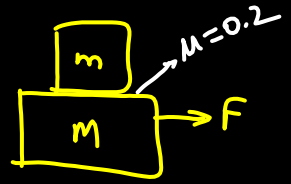
$$4\mu + 3 = 4 - 3\mu$$

$$7\mu = 1$$

$$\boxed{\mu = \frac{1}{7}}$$

Direction of Friction

Friction always acts opposite to the relative motion between two surfaces.



$a_{max} = \mu g$

$= 0.2 \times 10$

$= 2\ m/s^2$

$\uparrow f = ma \uparrow$

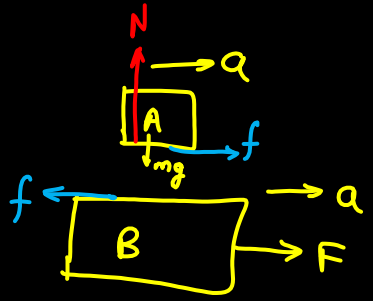
$f_{max} = ma_{max}$

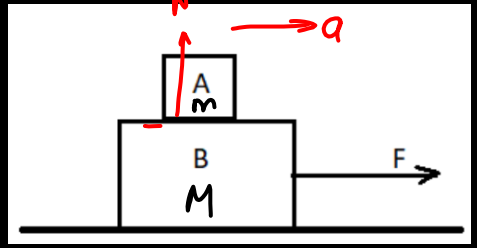
$\mu N = ma_{max}$

$\mu(mg) = ma_{max}$

$\boxed{a_{max} = \mu g}$

$\boxed{a = \frac{F}{m+M}}$





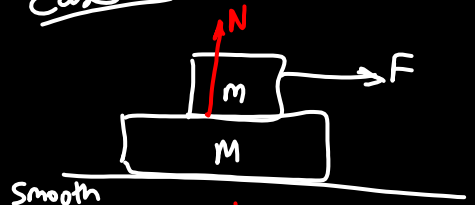
Direction of Friction

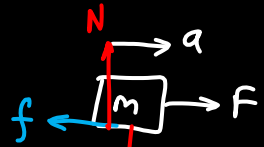
$$a = \frac{F}{M+m}$$

$$F_{max} = (M+m)a_{max}$$

$$F_{max} = (M+m)\mu g$$

Case 2:





$$a = \frac{F}{M+m}$$

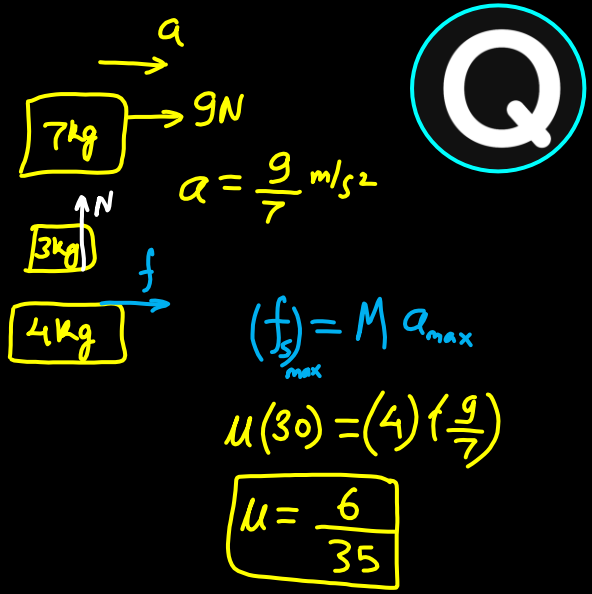
$$f = Ma$$

$$(f_s)_{max} = Ma_{max}$$

$$\mu N = Ma_{max}$$

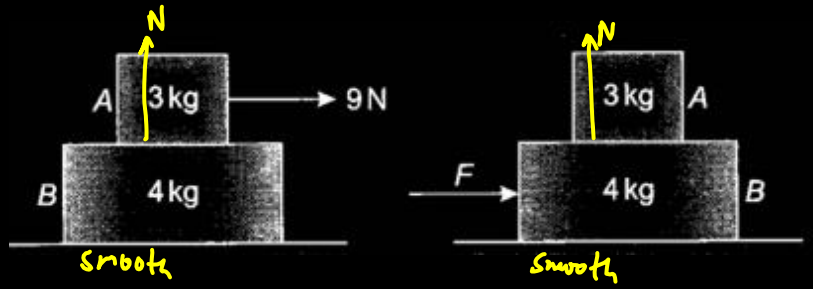
$$\mu(mg) = Ma_{max}$$

$$a_{max} = \mu \frac{m}{M} g$$

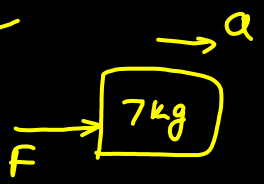


A 3 kg block A is placed on the top of a 4 kg block B as shown. To make the block A slip on B, assuming frictionless table, a horizontal force of 9N is to be applied to the top block. Find the minimum horizontal force (in N) that can be applied to lower block so that A slips on B.

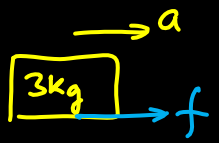
(a) 9 (b) 10 (c) 11 (d) 12



Case 2



$a = \frac{F}{7}$



$f = (3)(a)$

$f_{max} = 3a_{max}$

$\mu N = 3\left(\frac{F}{7}\right)$

$\mu(30) = 3\frac{F}{7}$

$\frac{6}{35} \times 30 = 3\frac{F}{7}$

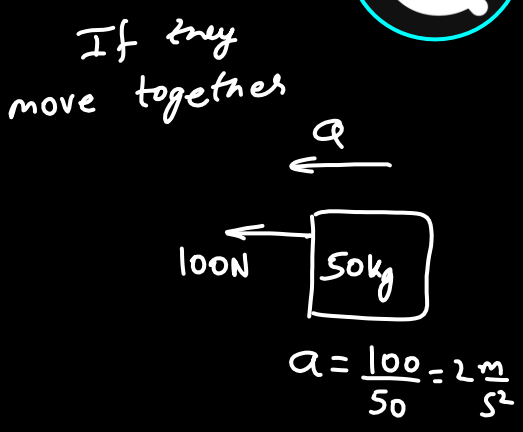
$\frac{6 \times 10}{5} = F$

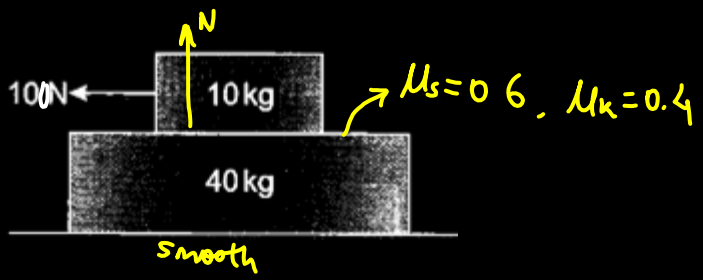
$F = 12N$

Very Good Question

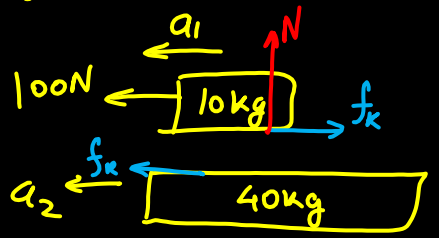
A 40 kg slab rests on a frictionless floor. A 10 kg block rests on top of the slab as shown in the figure. The coefficient of static friction between the block and slab is 0.60 and coefficient of kinetic friction is 0.40. The 10 kg block is acted upon by a horizontal force of 100 N. Find the resulting acceleration of slab (in m/s^2) (take g = 10 m/s^2).

(a) 0.2 (b) 1 (c) 1.5 (d) 2





Sliding is happening



$100 - f_k = 10a_1$

$100 - 40 = 10a_1$

$a_1 = 6\ m/s^2$

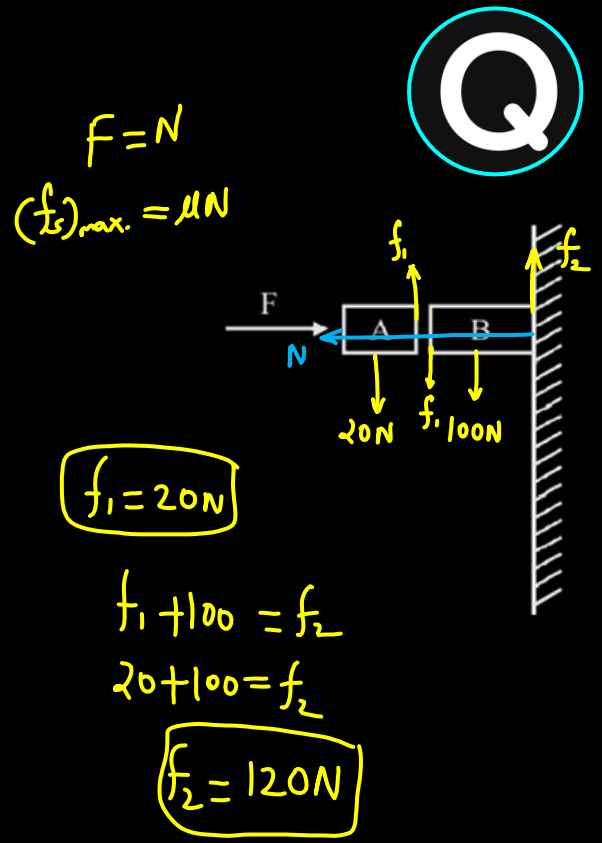
$f_k = Ma_2$

$\mu_k N = Ma_2$

$(0.4)(100) = 40a_2$

$40 = 40a_2$

$a_2 = 1\ m/s^2$



Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is

[JEE Main 2015]

A. 120 N

B. 150 N

C. 100 N

D. 80 N

HW Comment ⇒ Ans

In the following situation, the floor is smooth while coefficient of friction between the blocks is 0.5. If M=4 kg, m=1 kg, then what minimum force should be the minimum value of 'F' to keep smaller blocks at rest with respect to the bigger blocks.

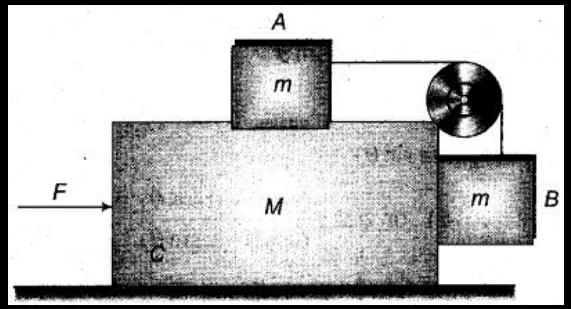
friction
Pseudo Force
Tension
Sliding Constraint

A. 30 N

B. 40 N

C. 20 N

D. 10 N



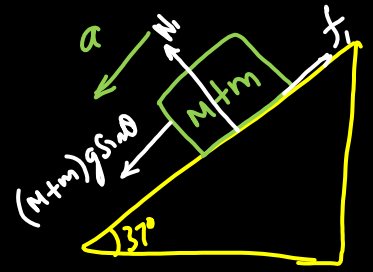
Unacademy Atoms

Q

Combined

A block of mass m is kept on the horizontal top surface of wedge of mass M = 4m which is kept on an incline plane of inclination θ = 37° as shown in the figure. Coefficient of friction between the wedge and incline is 1/3. Find the minimum coefficient of friction between m and M so that m does not slip on M when the system is released, from rest (g = 10 m/s²).

(a) 0.25 (b) 0.33 (c) 0.42 (c) 0.55



$(M+m)g\sin\theta - f_1 = (M+m)a$

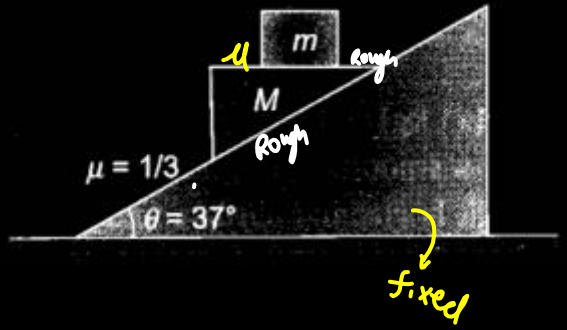
$(M+m)g\sin\theta - \mu(M+m)g\cos\theta = (M+m)a$

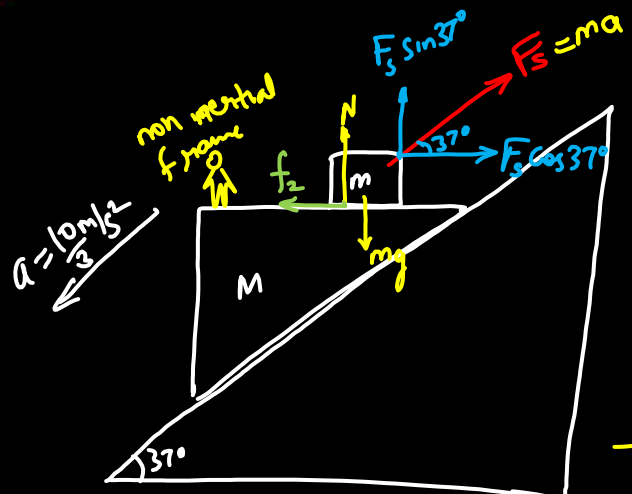
$g(\sin\theta - \mu\cos\theta) = a$

$\Rightarrow a = \frac{10}{3}\ m/s^2$

$10\left(\frac{3}{5} - \frac{1}{3}\times\frac{4}{5}\right) = a$

$2(3 - 4/3) = a$





Vertical

$$F_s \sin 37° + N = mg$$

$$ma\left(\frac{3}{5}\right) + N = mg$$

$$N = 10m - m \times \frac{10}{3} \times \frac{3}{5}$$

$$\boxed{N = 8m}$$

Horizontal

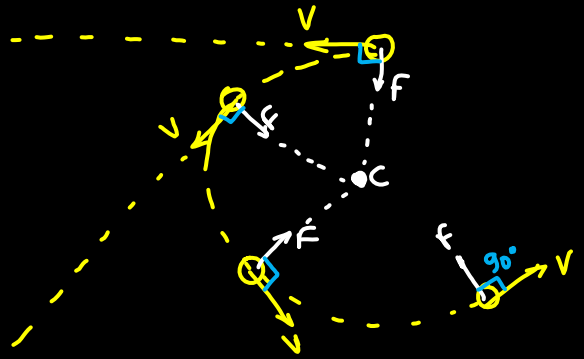
$$f_2 = F_s \cos 37°$$

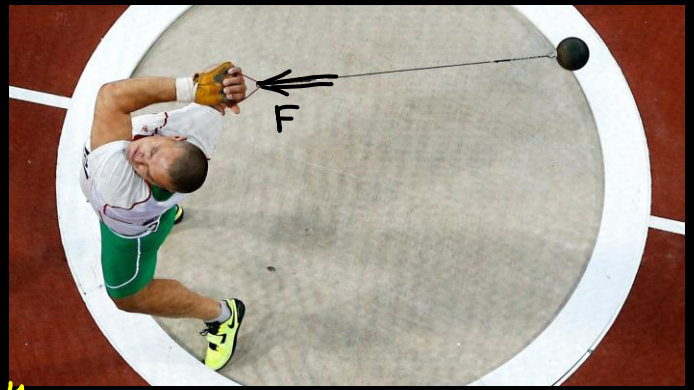
$$\mu N = ma\left(\frac{4}{5}\right)$$

$$\mu(8m) = m\left(\frac{10}{3}\right)\left(\frac{4}{5}\right)$$

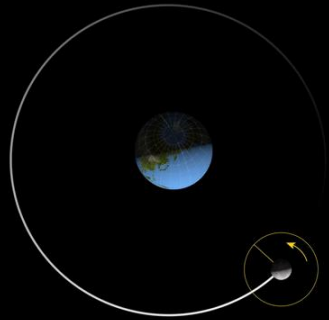
$$\boxed{\mu = \frac{1}{3}}$$

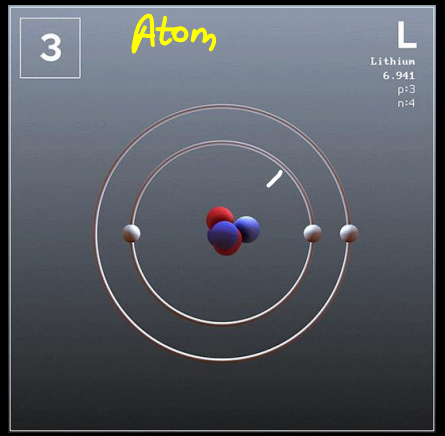
Circular Motion





Moon Around Earth





Circular Motion

Centripetal Acceleration (a_c)

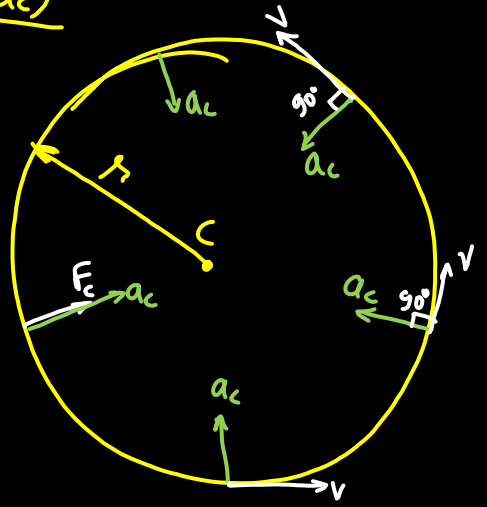
$$\vec{F_c} = m\vec{a_c}$$

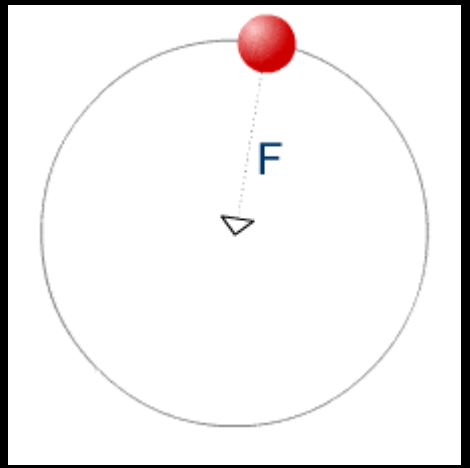
a = Rate of change of velocity

$|\vec{v}| = \text{const}$ Direction $\neq$ Const

Speed = const velocity $\neq$ const

$a_t = 0$





Kinematics of Circular Motion

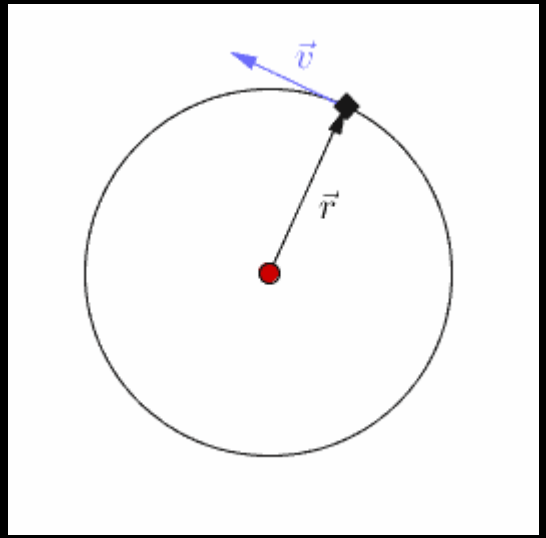
Centripetal Acceleration (a_c)

$$a_c = \frac{v^2}{r} \quad \frac{m}{s^2}$$

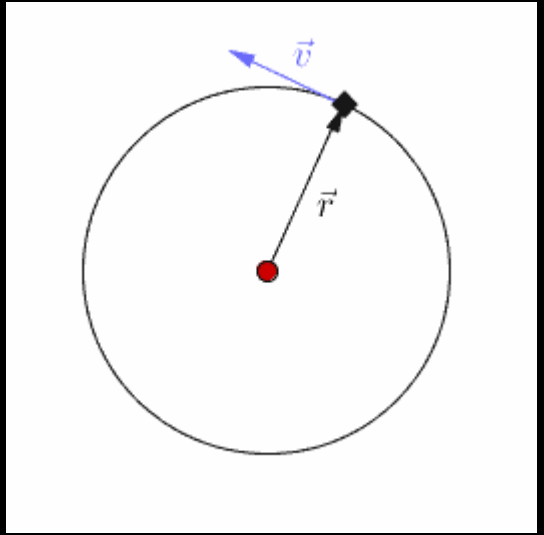
$|\vec{a_c}| = \text{const}$ if $|\vec{v}| = \text{const}$

$$a_c = \frac{(r\omega)^2}{r} = \frac{r^2\omega^2}{r}$$

$$a_c = \omega^2 r \quad \frac{m}{s^2}$$

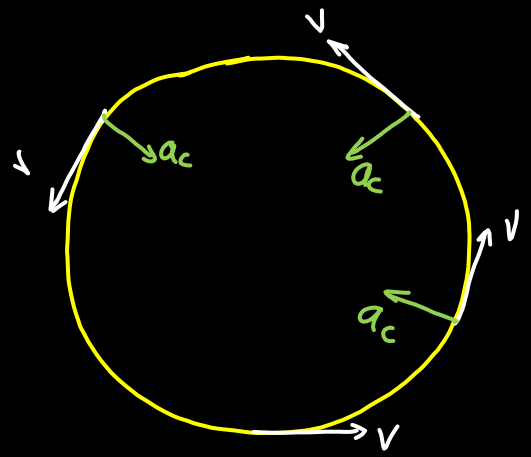


Kinematics of Circular Motion



An object follows a curved path. The following quantities may remain constant during the motion

(a) speed
(b) velocity
(c) acceleration
(d) magnitude of acceleration



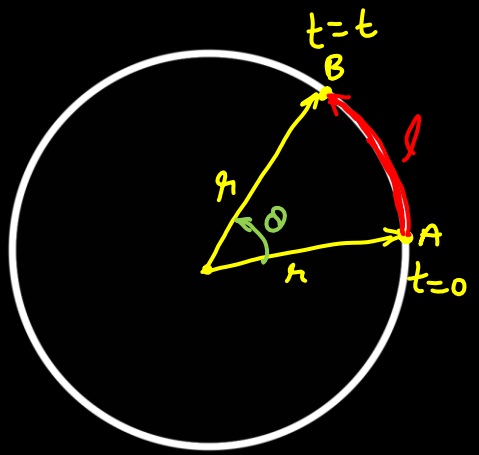
When a particle moves in a circle with a uniform speed

(a) its velocity and acceleration are both constant
(b) its velocity is constant but the acceleration changes
(c) its acceleration is constant but the velocity changes
(d) its velocity and acceleration both change.

Kinematics of Circular Motion

Linear Speed $= v = \frac{\text{distance}}{\text{time}}$

$v_{avg} = \frac{l}{t}$ $\quad$ $v = \frac{dl}{dt}$



Angular Displacement (θ)

$\theta \Rightarrow$ SI Unit Radians

# of Revolution $(n) = \frac{\theta}{2\pi}$

Kinematics of Circular Motion

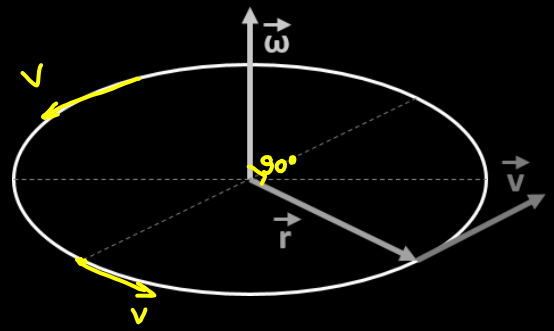
$\vec{\omega} \perp \vec{r} \perp \vec{v}$

Angular Velocity (ω)

$\omega_{avg} = \frac{\theta}{t}$ $\quad$ $\omega = \frac{d\theta}{dt}$

SI Unit ⇒ rad/sec

Direction ⇒ Curl your right hand fingers in the direc of motion of the object



$l = r\theta$

$\frac{dl}{dt} = r\frac{d\theta}{dt}$

$V = r\omega$ Scalar Form

$\vec{V} = \vec{\omega} \times \vec{r}$ Vector form

$$\vec{v} = \vec{\omega} \times \vec{r}$$

The angular velocity of a particle is w = 4i + j - 2k about the origin. If the position vector of the particle is 2i + 3j - 3k, then its linear velocity is

$\vec{v} = \vec{\omega} \times \vec{r}$

A. 5i + 8 j - 14k
B. 3i + 8j + 10k
C. 8i + 3j - 10k
D. -8i + 3j - 2k

$$\vec{v} = \begin{vmatrix} i & j & k \\ 4 & 1 & -2 \\ 2 & 3 & -3 \end{vmatrix}$$

$\vec{v} = i(-3+6) - j(-12+4) + k(12-2)$

$\vec{v} = 3\hat{i} + 8\hat{j} + 10\hat{k}$

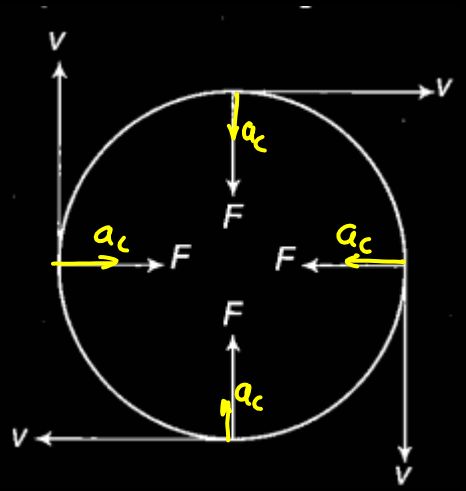
Dynamics of Circular Motion

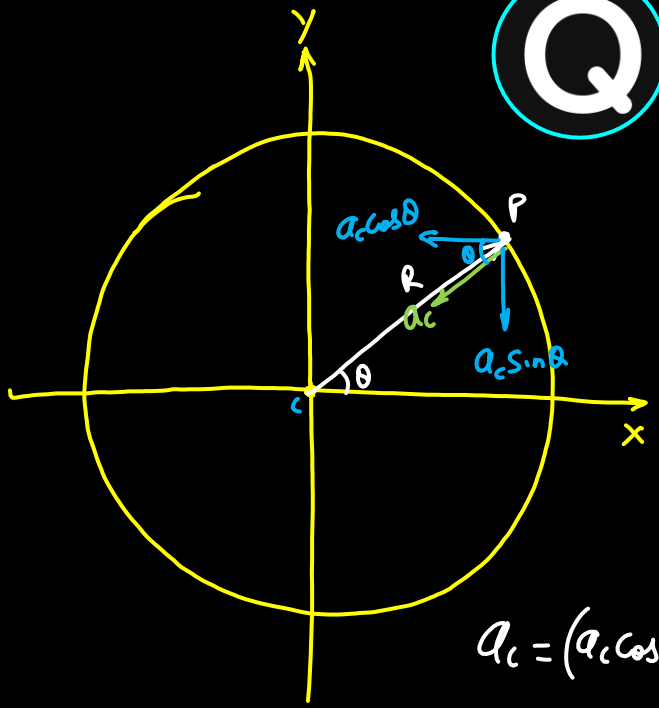
Centripetal Force (F_c)

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$F_c = m a_c$$

$$\boxed{F_c = \frac{mv^2}{r} = mr\omega^2}$$





For a particle uniform motion, the acceleration a at a point P(R,θ) on the circle of radius R is (here θ is measured from the y - axis) [JEE 2021]

(A) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

(B) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$

(C) $-\frac{v^2}{R}\sin\theta\,\hat{i} - \frac{v^2}{R}\cos\theta\,\hat{j}$

(D) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

$$a_c = (a_c\cos\theta)(-\hat{i}) + (a_c\sin\theta)(-\hat{j})$$

$$= -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$$

Equations of Motion in Circular Motion

$u \rightarrow \omega_0$
$v \rightarrow \omega$
$a \rightarrow \alpha$
$s \rightarrow \theta$
$t \rightarrow t$

1) $\omega = \omega_0 + \alpha t$

2) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

3) $\omega^2 = \omega_0^2 + 2\alpha\theta$

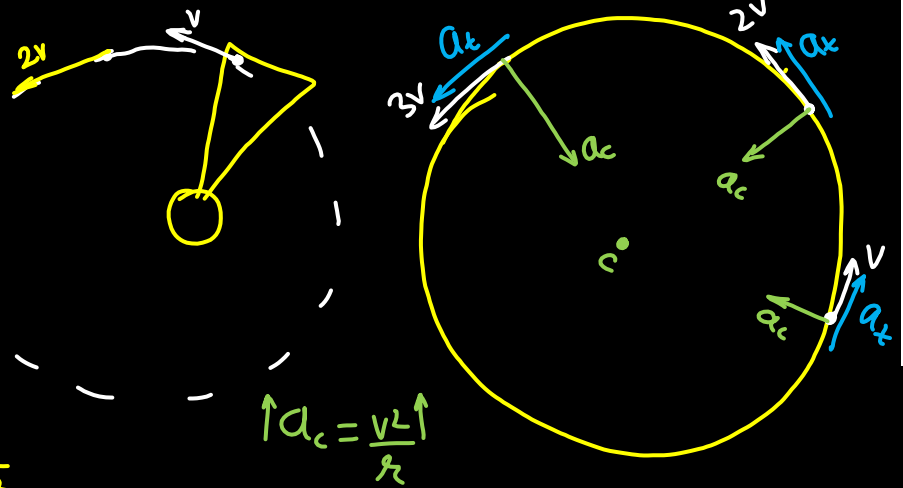
Circular Motion

Non Uniform Circular Motion

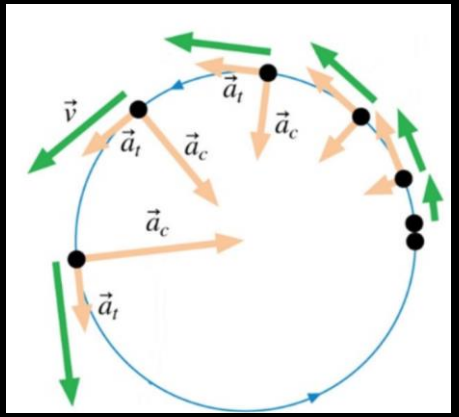
Speed ≠ Const

Tangential Acceleration (a_t)

$a_t \neq 0$



$$a_t = \frac{dv}{dt} \quad \frac{m}{s^2}$$



Circular Motion

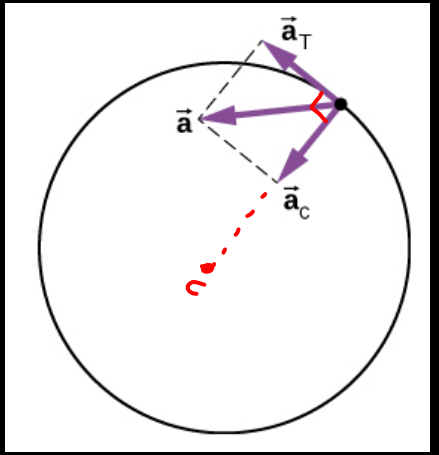
Non Uniform Circular Motion

Net Acceleration (a_{net})

$$\boxed{a_{net} = \sqrt{a_c^2 + a_t^2}}$$

$$a_{net} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

$$a_{net} = \sqrt{(\omega^2 r)^2 + (r\alpha)^2}$$



Circular Motion

$$v = r\omega$$

$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

$$a_t = r\alpha$$

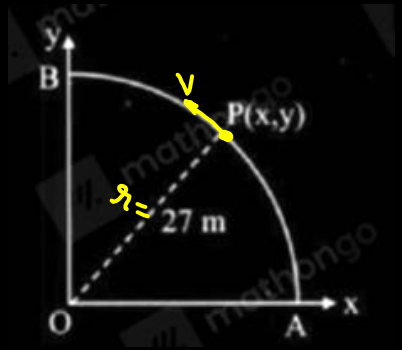
Angular Acceleration (α)

$$\alpha = \frac{d\omega}{dt} \quad \frac{rad}{s^2}$$

Uniform Circular Motion	Non Uniform Circular Motion
Velocity $\neq$ Const	Velocity $\neq$ Const
$a_c \neq 0$	$a_c \neq 0$
Speed = Const	Speed $\neq$ Const
$a_t = 0$	$a_t \neq 0$
ω = Const	$\omega \neq$ Const
$\alpha = 0$	$\alpha \neq 0$

Numerical Type

A point P moves in a counter - clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$ where s in metre and t is seconds. The radius of the path is 27 m . The acceleration of P when t = 3 s is __ m/s^2. (Take $\sqrt{13} = 3.6$)



$a_c = \frac{v^2}{r}$

$v = 3t^2 = 3(3)^2 = 27\ m/s$

$a_c = \frac{(27)^2}{27}$

$\boxed{a_c = 27\ m/s^2}$

$s = t^3 + 5$

$v = \frac{ds}{dt} = 3t^2$

Speed ↑ const

$a_t = \frac{dv}{dt} = 6t$

$\boxed{a_t = 6 \times 3 = 18\ m/s^2}$

$$a_{net} = \sqrt{(27)^2 + (18)^2}$$

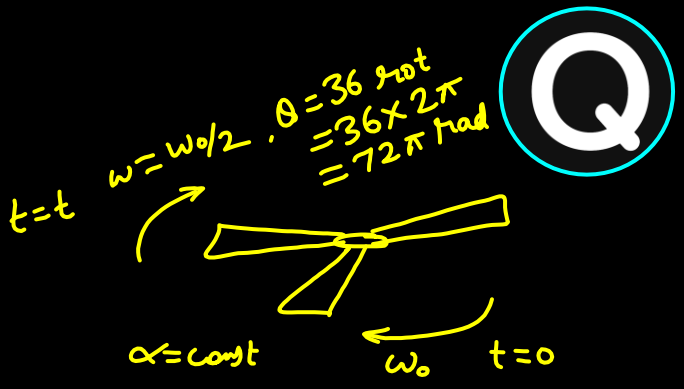
$$= \sqrt{729 + 324}$$

$$= \sqrt{1053}$$

$$= 9\sqrt{13}$$

$$= 9 \times 3\ 6$$

$$= 32\ 4\ m/s^2$$



$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 + 2\alpha(72\pi)$$

$$\frac{\omega_0^2}{4} - \omega_0^2 = 2\alpha(72\pi)$$

$$\alpha = \frac{-3\omega_0^2}{8(72\pi)}$$

When a ceiling fan is switched off, its angular velocity reduces to 50% of its initial value while it makes 36 rotations. If angular retardation of the fan is uniform , then how many more rotations will it make before coming to rest ?

A. 18
B. 12
C. 36
D. 48

$\omega_0' = \frac{\omega_0}{2}$, $\omega = 0$, $\theta' = ?$

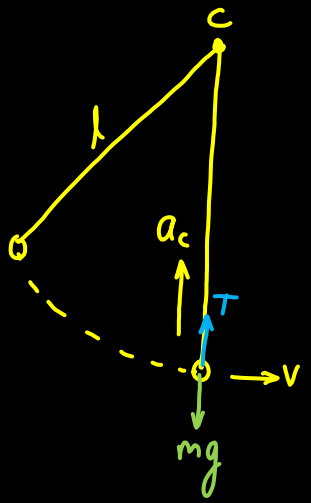
$$\omega^2 = \omega_0'^2 + 2\alpha\theta'$$

$$0^2 = \left(\frac{\omega_0}{2}\right)^2 + 2\left(\frac{-3\omega_0^2}{8\times 72\pi}\right)\theta'$$

$$\theta' = \frac{72\pi}{3} = 24\pi \text{ rad.}$$

$$\text{Rotation} = \frac{\theta'}{2\pi} = \frac{24\pi}{2\pi} = 12$$

Problem Solving Method



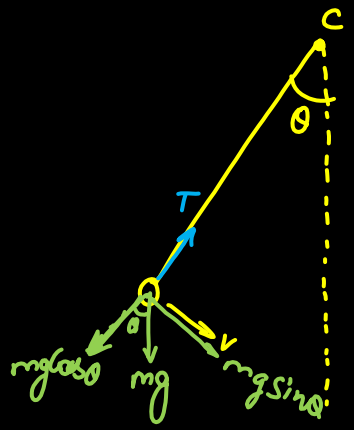
1. Draw the FBD
2. FInd the net force towards the centre
3. Equate Fnet =mv2/r

$$T - mg = ma_c$$

$$T - mg = \frac{mv^2}{l}$$

$$\boxed{T = mg + \frac{mv^2}{l}}$$

Find the tension in the string when the pendulum is making an angle θ with the vertical, given that the speed of particle at this instant is v



$$T - mg\cos\theta = \frac{mv^2}{l}$$

$$T = mg\cos\theta + \frac{mv^2}{l}$$

$$mg\sin\theta = ma_t$$

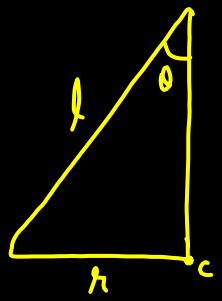
For the conical pendulum shown in the figure L=√2 m & r=1m, the speed v in m/s is

(a) √10 (b) √20 (c) √30 (d) √40

$\sin\theta = \frac{r}{l}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = 45°$



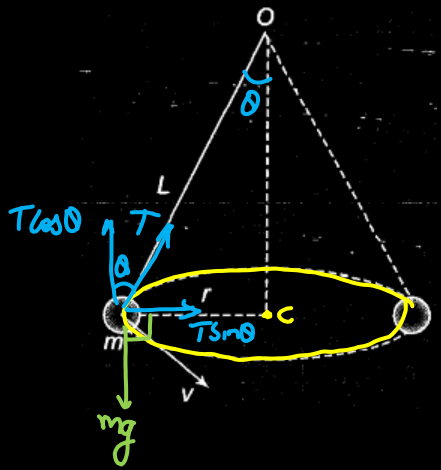
$T\cos\theta = mg$

$T\sin\theta = \frac{mv^2}{r}$

$\tan\theta = \frac{mv^2/r}{mg}$

$v = \sqrt{rg\tan\theta}$

$v = \sqrt{1\times10\times1} = \sqrt{10}$ m/s



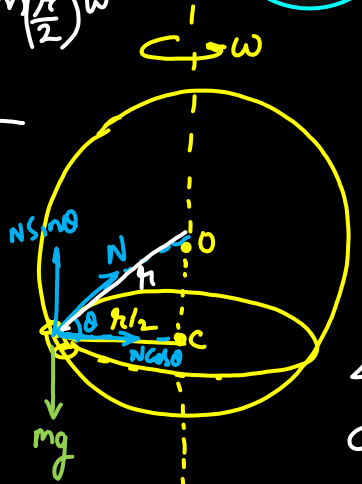
$N\sin\theta = mg$

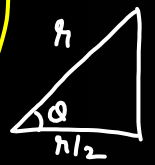
$N\cos\theta = m\left(\frac{r}{2}\right)\omega^2$

$\tan\theta = \frac{2g}{r\omega^2}$

$\sqrt{3} = \frac{2g}{r\omega^2}$

$\omega^2 = \frac{2g}{\sqrt{3}r}$

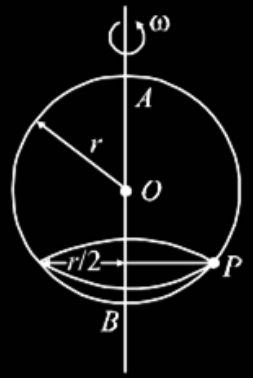




$\cos\theta = \frac{r/2}{r} = \frac{1}{2}$

A smooth wire of length 2πr is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure , the bead is at rest with respect to the circular ring at position P as shown. Then the value of $(\omega)^2$ is equal to :

[JEE Main 2019]



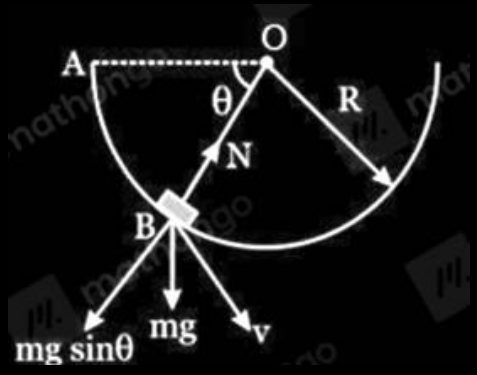
(a) $\frac{\sqrt{3g}}{2r}$

(b) $2g/(r\sqrt{3})$

(c) $(g\sqrt{3})/r$

(d) $2g/r$

Uses Energy Conservation

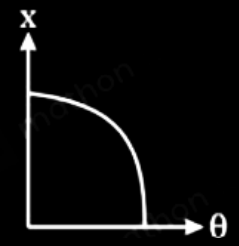


A small block of mass m is released from rest from point A inside a smooth hemisphere bowl of radius R, which is fixed on ground such that OA is horizontal. The ratio (x) of magnitude of centripetal force and normal reaction on the block at any point B varies with θ as :

(A)

(B)

(C)

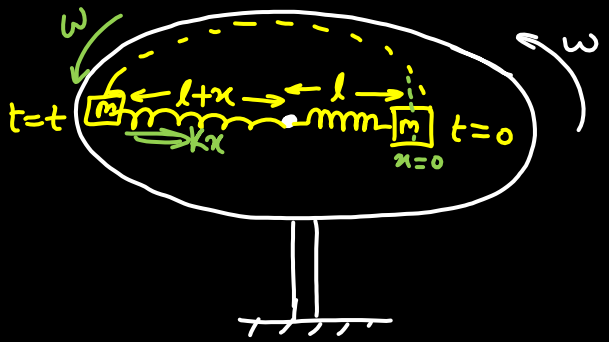


(D)

Q

A spring mass system (mass m, spring constant k and natural length l) rests in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about its axis with an angular velocity ω, ($k >> m\omega^2$) the relative change in the length of the spring is best given by the option

[JEE Main 2020]



$F_{net} = kx$

$mr\omega^2 = kx$

$m(l+x)\omega^2 = kx$

(a) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$

(b) $\frac{2m\omega^2}{k}$

(c) $\frac{m\omega^2}{k}$

(d) $\frac{m\omega^2}{3k}$

$$\frac{m l \omega^2}{K - m\omega^2} = x$$

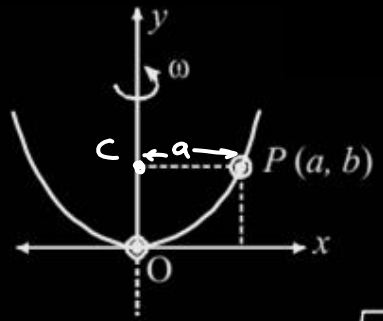
$$\text{Relative Change} = \frac{l_2 - l_1}{l_1} = \frac{l_2}{l_1} - 1 = \frac{l + x}{l} - 1 = \frac{x}{l}$$

$$= \frac{m\omega^2}{K - m\omega^2} \simeq \frac{m\omega^2}{K}$$

Good Question

A bead of mass m stays at point P(a, b) on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed w (see figure). The value of w is (neglect friction).

[JEE Main 2019]



$y = 4cx^2$

$\frac{dy}{dx} = 8cx$

(a) $2\sqrt{2gC}$ (b) $2\sqrt{gC}$ (c) $\sqrt{\frac{2gC}{ab}}$ (d) $\sqrt{\frac{2g}{C}}$

$N\cos\theta = mg$

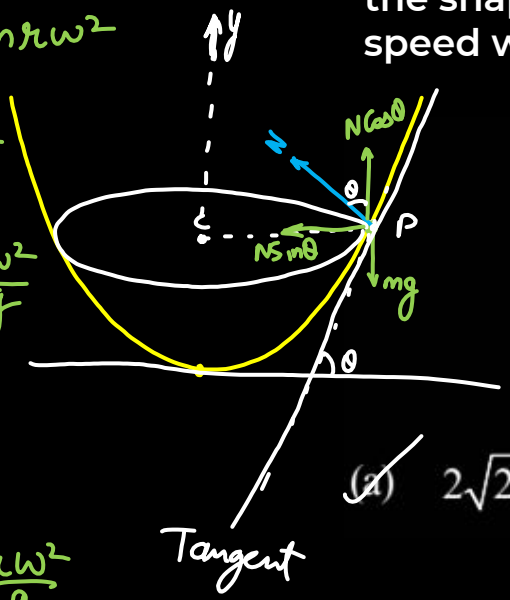
$N\sin\theta = mr\omega^2$

$\tan\theta = \frac{r\omega^2}{g}$

Slope of tangent $= \frac{r\omega^2}{g}$

$\frac{dy}{dx} = \frac{r\omega^2}{g}$

$8cx = \frac{r\omega^2}{g}$



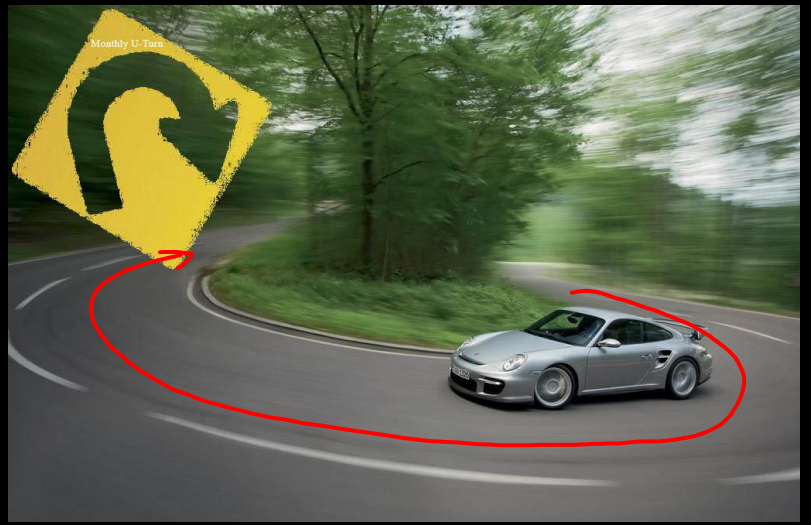
$R = a$, $x = a$

$$8ca = \frac{a\omega^2}{g}$$

$$\omega = \sqrt{8cg}$$

$$\boxed{\omega = 2\sqrt{2cg}}$$

Vehicles Taking Turn



Vehicles Taking Turn

Vehicles Taking Turn

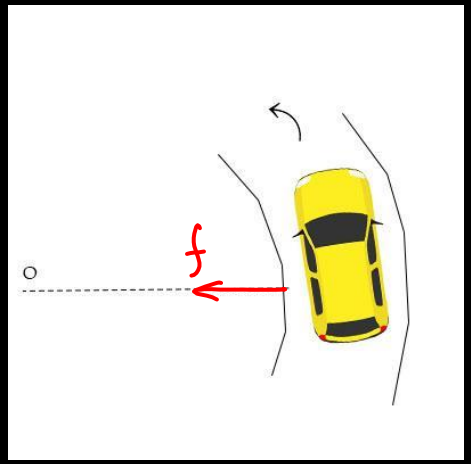
$$f = \frac{mv^2}{r}$$

$$(f_s)_{max} \geq \frac{mv^2}{r}$$

$$\mu N \geq \frac{mv^2}{r}$$

$$\mu(mg) \geq \frac{mv^2}{r}$$

$$\boxed{v \leq \sqrt{\mu r g}}$$



$r = 15\,ft = 5\,met$

$\mu = 0.5$

$v \leq \sqrt{(0.5)(5)(10)}$

$v \leq 5\ m/s = 18\ km/hr$

Well of Death

$V \Rightarrow$ more

Well of Death

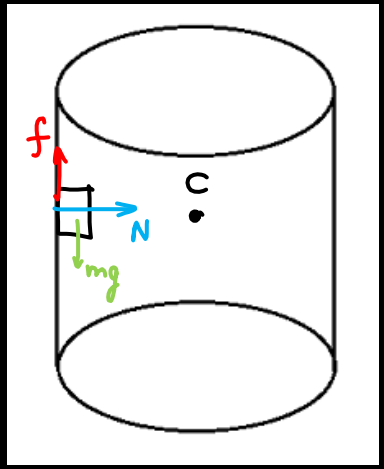
$$(f_s)_{max} \geq mg$$

$$\mu N \geq mg$$

$$N = \frac{mv^2}{r}$$

$$\mu\left(\frac{mv^2}{r}\right) \geq mg$$

$$\boxed{v \geq \sqrt{\frac{rg}{\mu}}}$$

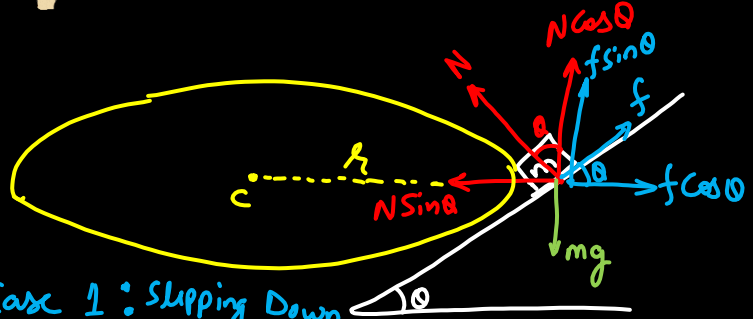


$r = 5\,m$

$\mu = 0.5$

$$v \geq \sqrt{\frac{50}{0.5}} = 10\,m/s = 36\,km/hr$$

Banking of Roads

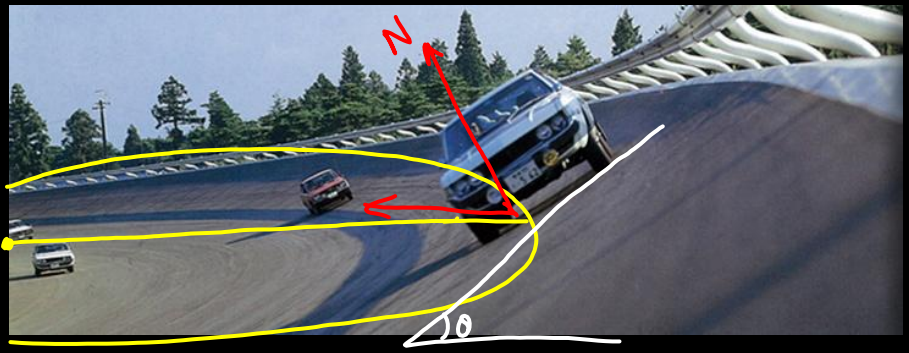


Case 1 : Slipping Down
(Minimum velocity)

Vertical

$N\cos\theta + f\sin\theta = mg$

$N = \frac{mg}{\cos\theta + \mu\sin\theta}$



Banking of Roads

Horizontal

$$N\sin\theta - f\cos\theta = \frac{mv^2}{r}$$

$$N(\sin\theta - \mu\cos\theta) = \frac{mv^2}{r}$$

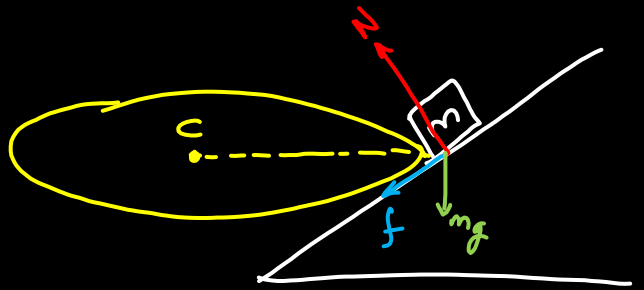
$$\frac{mg(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta} = \frac{mv^2}{r}$$

$$v_{min} = \sqrt{rg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)}$$

Banking of Roads

Case 2 : Slipping Up
(Maximum Velocity)

$$V_{max} = \sqrt{gr\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)}$$



A road is banked at an angle of 30° to the horizontal for negotiating a curve of radius $10\sqrt{3}$m. At what velocity will a car experience no friction while negotiating the curve ? Take g = 10 ms^{-2}

[JEE Main 2022]

A. 54 km/hr
B. 72 km/hr
C. 36 km/hr
D. 18 km/hr

$\mu = 0$

$V = \sqrt{gr \tan\theta}$

$V = \sqrt{10 \times 10\sqrt{3} \times \frac{1}{\sqrt{3}}}$

$V = 10 m/s$

$V = 36 km/hr$

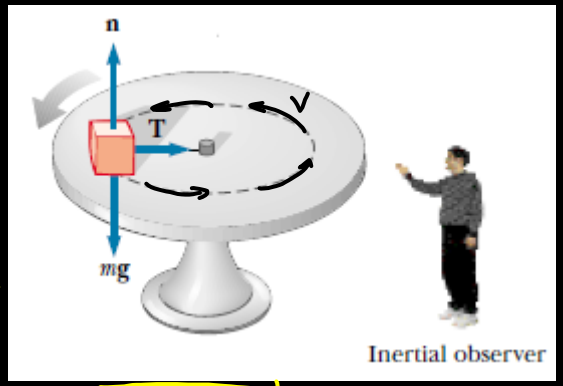
Centrifugal Force

⇒ Pseudo Force

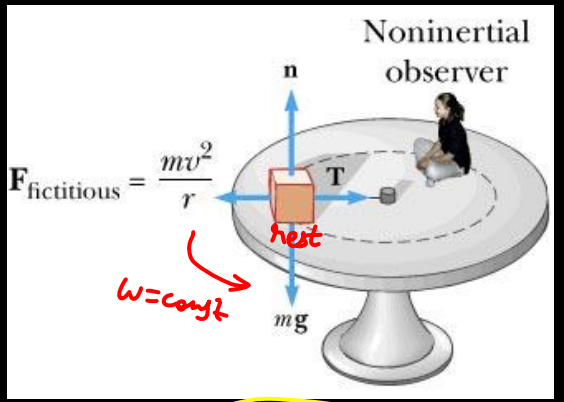
⇒ Only considered in non inertial frame

⇒ Acts away from the centre

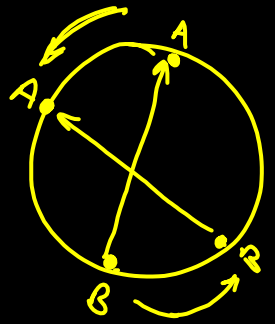
⇒ Its Magnitude is equal to Centripetal



$T = \frac{mv^2}{r}$



$T = \frac{mv^2}{r}$



Inertial Frame

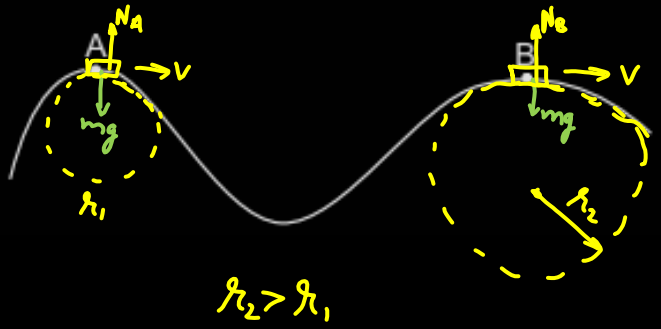
$mg - N_A = \frac{mv^2}{R_1}$

$N_A = mg - \frac{mv^2}{R_1}$

$N_B = mg - \left(\frac{mv^2}{R_2}\right)$

A car moves at a constant speed on a road as shown in figure. The normal force by the road on the car is N_A and N_B when it is at the points A and B respectively.

(a) $N_A = N_B$
(b) $N_A > N_B$
(c) $N_A < N_B$
(d) insufficient information to decide the relation of N_A and N_B .



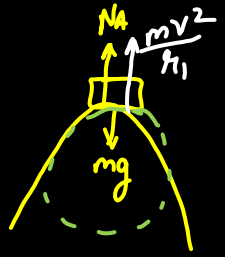
$R_2 > R_1$

Non inertial Frame

Car is at Rest

$$N_A + \frac{mv^2}{r_1} = mg$$

$$\boxed{N_A = mg - \frac{mv^2}{r_1}}$$



Vertical Circular Motion

$$F = mg + T$$

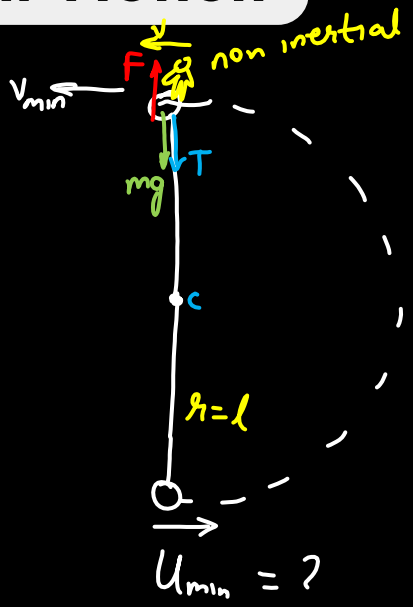
$$\frac{mv^2}{r} = mg + T$$

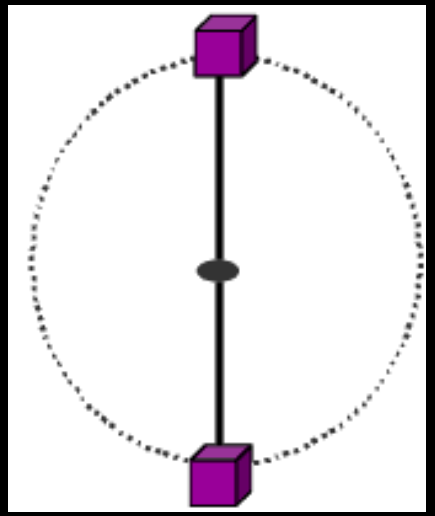
For $V_{min} \Rightarrow T = 0$

$$\frac{m V_{min}^2}{r} = mg$$

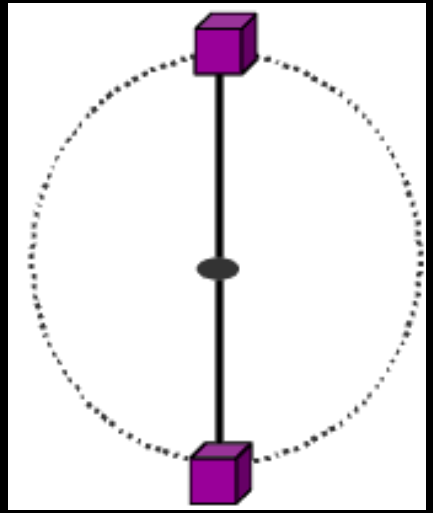
$$V_{min} = \sqrt{rg}$$

$$V_{min} = \sqrt{gl}$$





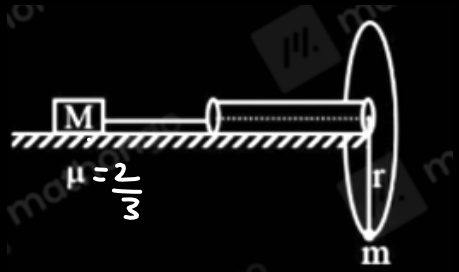
Vertical Circular Motion



Save it for WEP

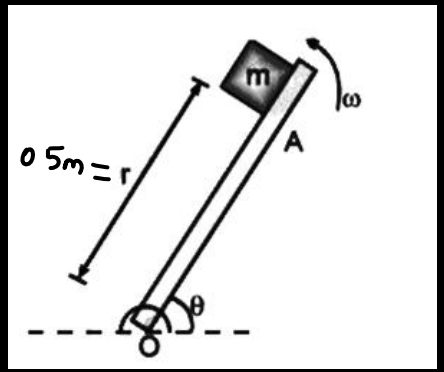
The figure below shows a block of mass M connected to an ideal string which passes through a thin fixed smooth pipe. On the other end, a particle of mass m is connected which revolves in a vertical circle of radius r. If the coefficient of friction between M and the surface is $\mu = 2/3$, then for what minimum value of M, the block of mass m can undergo complete vertical circular motion ?

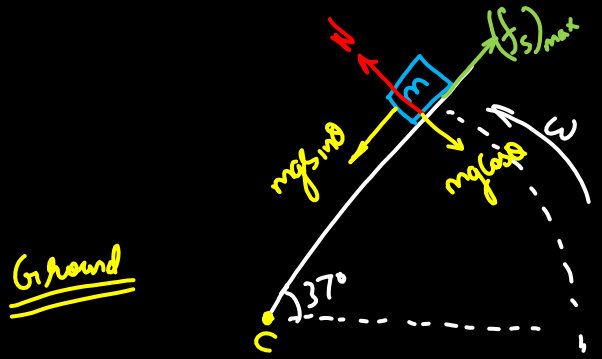
A. $M_{min} = 6m$
B. $M_{min} = 9m$
C. $M_{min} = 3m$
D. $M_{min} = 15m$



A rough platform OA rotates in a vertical plane about a horizontal axis through the point O with a constant counterclockwise velocity w = 3 rad s^{-1}. As it passes through the position θ = 0, a small mass m is placed upon it at a radical distance. If the mass is observed to slip at θ = 37°, then the coefficient to friction between the mass and the member is

A. 3/16
B. 9/16
C. 4/9
D. 5/9





$$mg\sin\theta - (f_s)_{max} = mr\omega^2$$

$$mg\sin\theta - \mu mg\cos\theta = mr\omega^2$$

$$g\left(\frac{3}{5} - \mu\frac{4}{5}\right) = 0.5 \times (3)^2$$

$$3 - 4\mu = \frac{9}{4}$$

$$3 - \frac{9}{4} = 4\mu$$

$$\frac{3}{4} = 4\mu$$

$$\boxed{\mu = \frac{3}{16}}$$

HW Comment ⇒ Ans

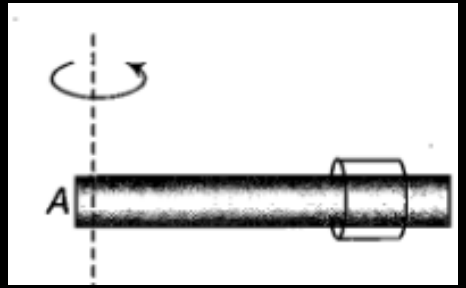
The figure shows a rod which starts rotating with angular acceleration α about vertical axis passing through one of its end (A) in horizontal plane. A bead of mass m just fits the rod and is situated at a distance 'r' from end A. Friction exists between rod and the bead with coefficient μ. As the angular velocity of rod increases the bead starts sliding over the rod (say after time t_o).
Friction force acting on bead at time t ($<t_0$) is given by

(a) μmg

(b) $mr(\alpha t)^2$

(c) $m\mu\sqrt{g^2+(r\alpha)^2}$

(d) $\mu m\sqrt{g^2+r^2(\alpha t)^4}$



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For more details, contact **8585858585**

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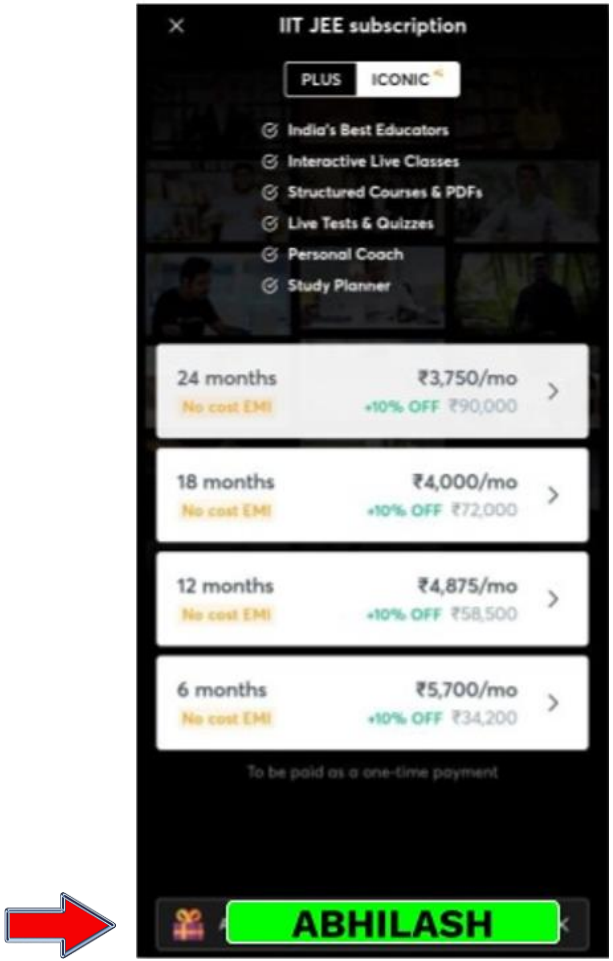
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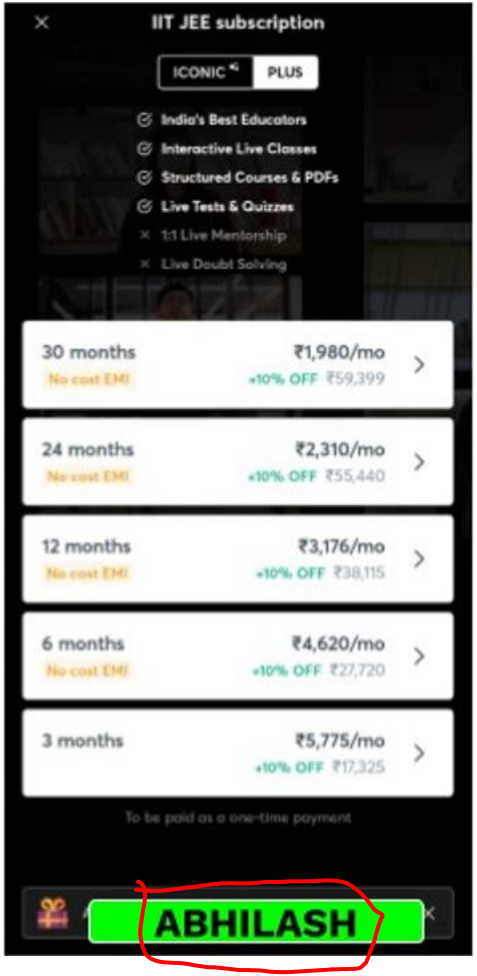
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