KINEMATICS 2D

Abhilash Sharma

B.Tech - NIT Calicut

- Experience - 8+ years
- 700+ selections in JEE Advanced
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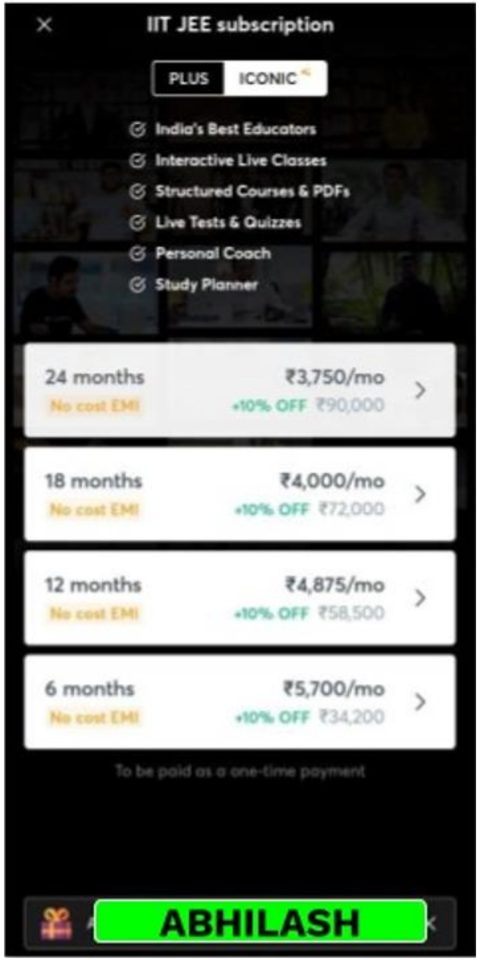
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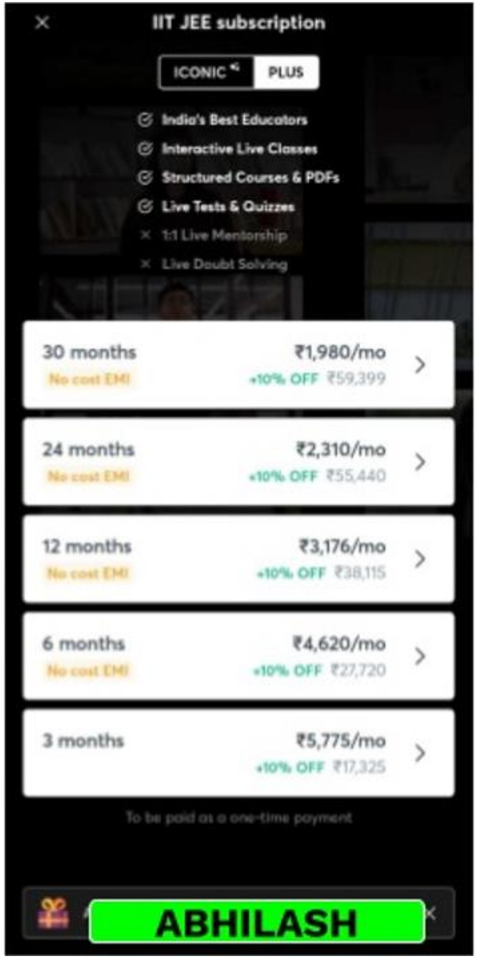
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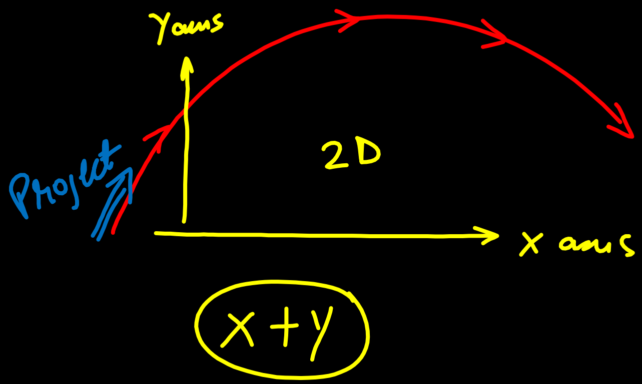
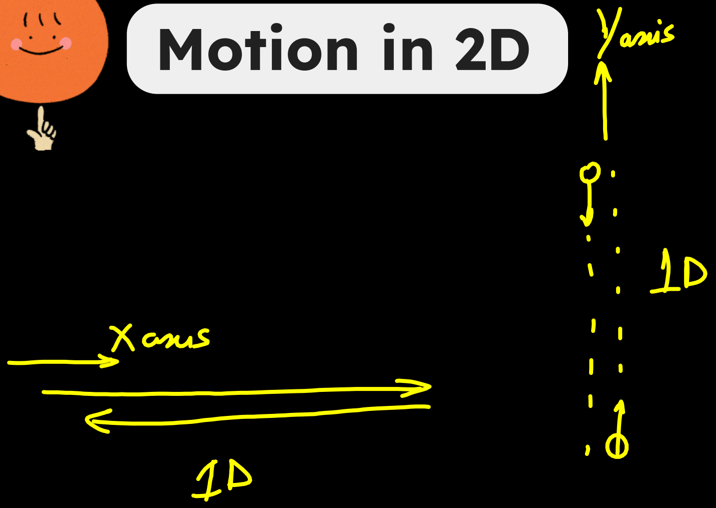
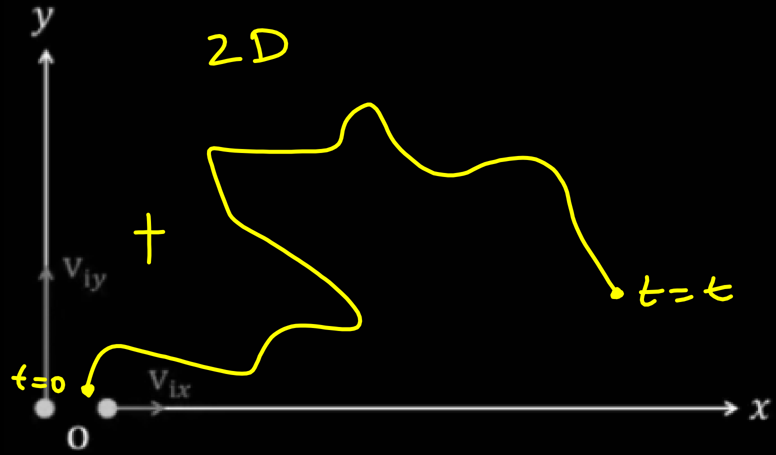
Kinematics 2D

2D Motion



Motion in 2D

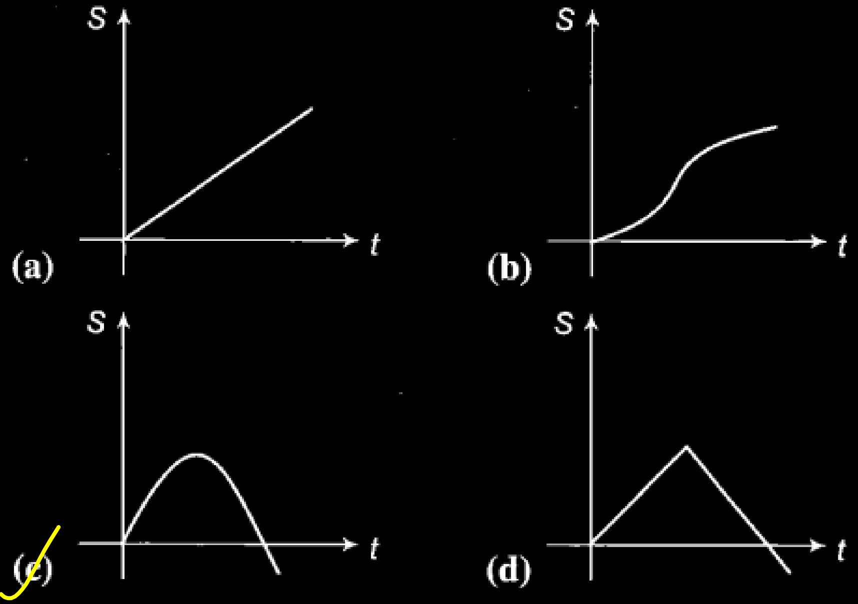




2D ⇒ Split into two 1D Motion

* time will be the connecting factor

A particle in x-y plane with $y = \frac{x}{2}$ and $v_x = 4 - 2t$. The displacement versus time graph of the particle would be



$V_x = 4 - 2t$

$\frac{dx}{dt} = 4 - 2t$

$\int dx = 4\int dt - 2\int t \, dt$

$x = 4t - t^2$

$y = \frac{x}{2}$

$\Rightarrow y = 2t - \frac{t^2}{2}$

multiple choice

Starting at time $t = 0$ from the origin with speed $1\ \text{ms}^{-1}$, a particle follows a two-dimensional trajectory in the x-y plane so that its coordinates are related by the equation $y = \frac{x^2}{2}$. The x and y components of its acceleration are denoted by a_x and a_y, respectively. Then

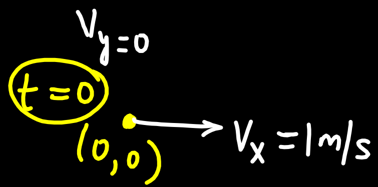
(A) $a_x = 1\ \text{ms}^{-2}$ implies that when the particle is at the origin, $a_y = 1\ \text{ms}^{-2}$

(B) $a_x = 0$ implies $a_y = 1\ \text{ms}^{-2}$ at all times

(C) at $t = 0$, the particle's velocity points in the x-direction

(D) $a_x = 0$ implies that at $t = 1$ s, the angle between the particle's velocity and the x axis is $45°$

[JEE Advanced 2020]



(c) $y = \frac{x^2}{2}$

$\frac{dy}{dt} = \frac{2x}{2}\frac{dx}{dt}$

$\boxed{V_y = x V_x}$

at origin $\Rightarrow x = y = 0$

$\Rightarrow V_y = 0$

$\frac{dV_y}{dt} = \frac{d}{dt}(x V_x)$

$a_y = x\frac{dV_x}{dt} + V_x \cdot \frac{dx}{dt}$

$\boxed{a_y = x a_x + V_x^2}$

(a) at origin $x = y = 0$

$\Rightarrow a_y = 0 + V_x^2$

$a_y = (1)^2 = 1\ m/s^2$

(b) $a_x = 0 \Rightarrow V_x = \text{const}$

$a_y = V_x^2 = \text{const.}$

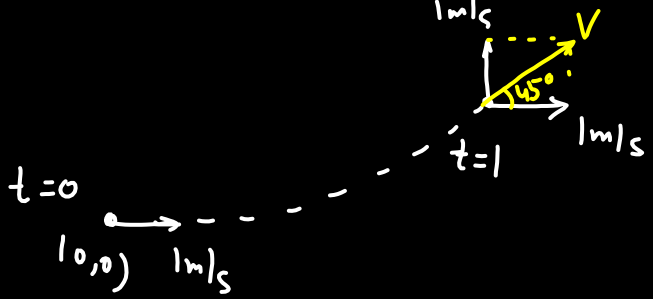
$a_y = 1\ m/s^2$

(d) $a_x = 0$, $V_x = \text{const}$

$V_x = 1\ m/s$

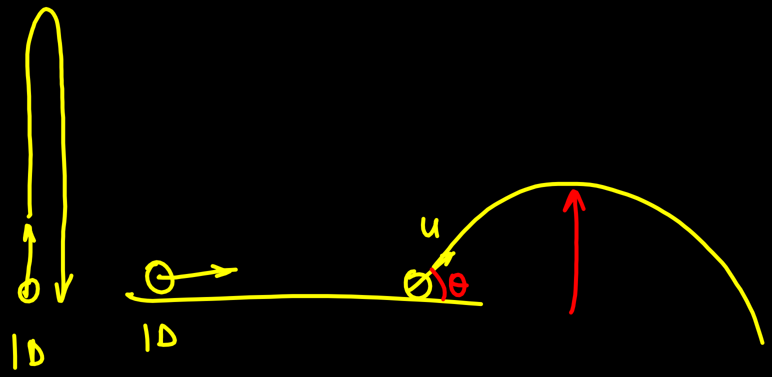
$a_y = 1\ m/s^2$, $u_y = 0$, $t = 1$ sec

$$V_y = u_y + a_y t = 0 + (1)(1) = 1\ m/s$$



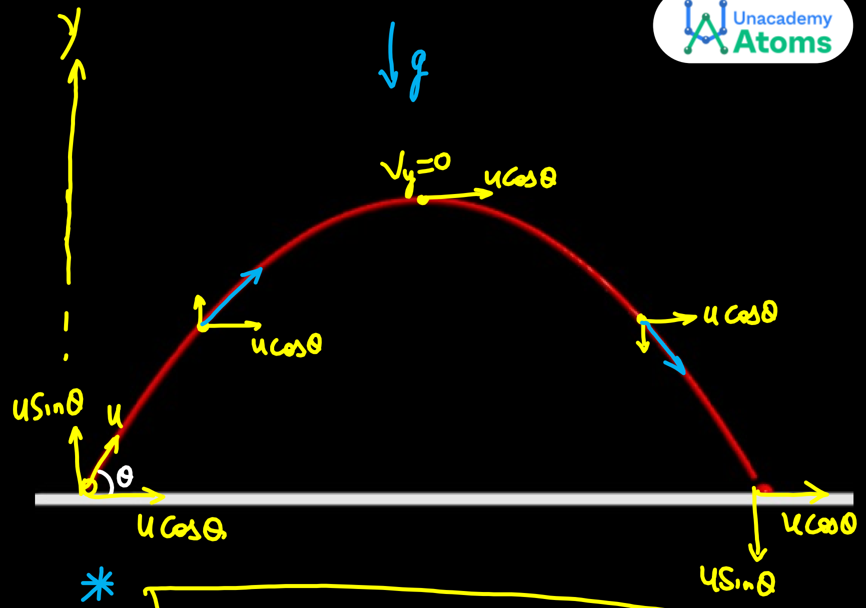
Projectile Motion (2D)

To throw



Projectile Motion

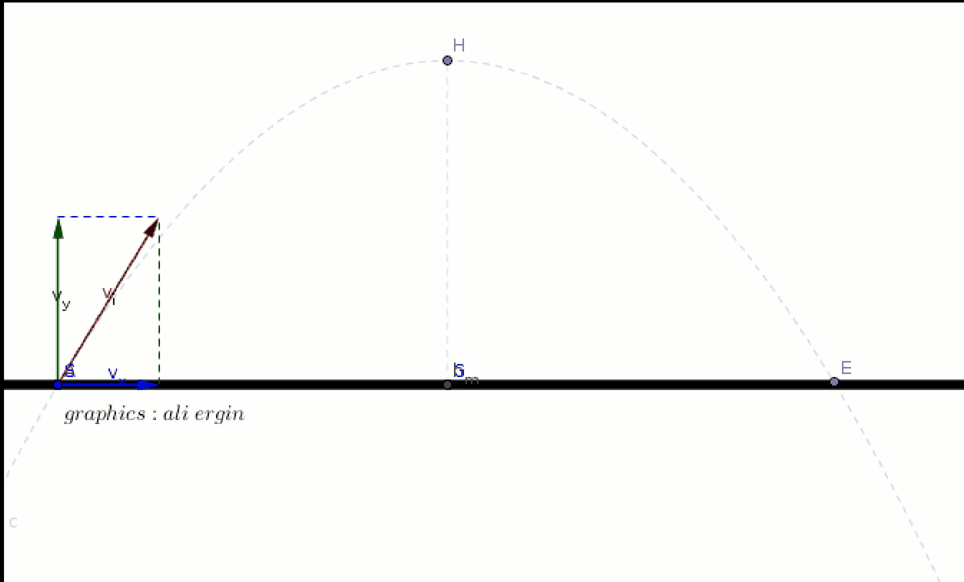
x	y
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$



⇒ x velocity always remains same

$V_x = u\cos\theta = \text{const}$

Projectile Motion



Projectile Motion

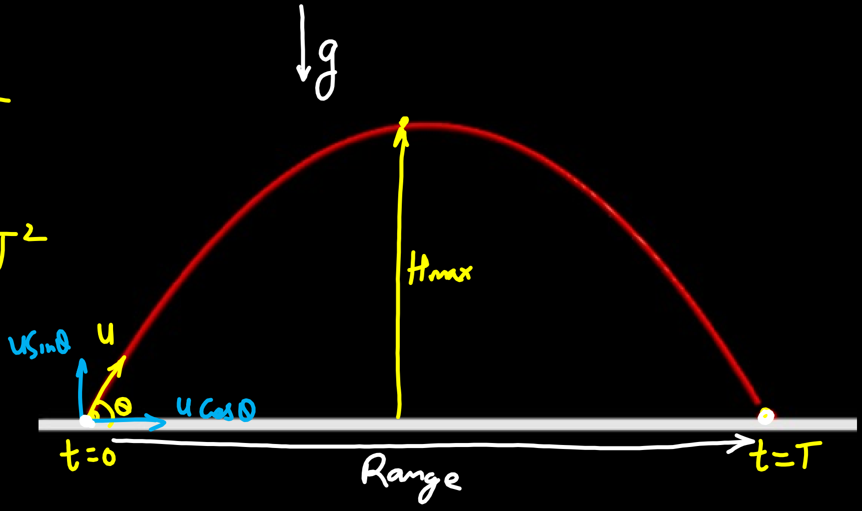
x	y
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$
$t = T$	$t = T$
$S_x = R$	$S_y = 0$

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$0 = (u\sin\theta)T + \frac{1}{2}(-g)T^2$$

$$0 = u\sin\theta - \frac{g}{2}T$$

$$T = \frac{2u\sin\theta}{g}$$



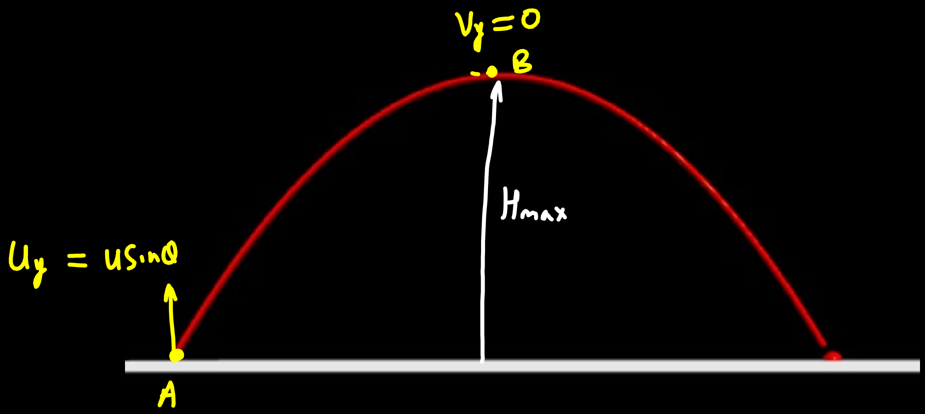
T ⇒ Time of flight

Projectile Motion

$$V_y^2 = u_y^2 + 2a_y S_y$$

$$0 = (u\sin\theta)^2 + 2(-g)(H_{max})$$

$$\boxed{H_{max} = \frac{u^2\sin^2\theta}{2g}}$$



Projectile Motion

$a_x = 0 \Rightarrow$ uniform Motion

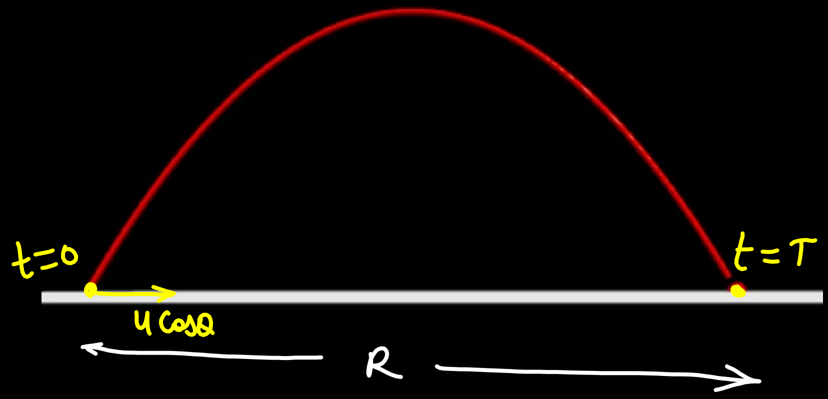
Distance = speed × time

$$R = (u\cos\theta)(T)$$

$$= u\cos\theta\left(\frac{2u\sin\theta}{g}\right)$$

$$R = \frac{u^2(2\sin\theta\cos\theta)}{g}$$

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$



Q

A particle is projected with a velocity v such that its range on the horizontal plane is twice the greatest height attained by it. The range of the projectile is (where g is acceleration due to gravity)

(a) $\frac{4v^2}{5g}$

(b) $\frac{4g}{5v^2}$

(c) $\frac{v^2}{g}$

(d) $\frac{4v^2}{\sqrt{5}g}$

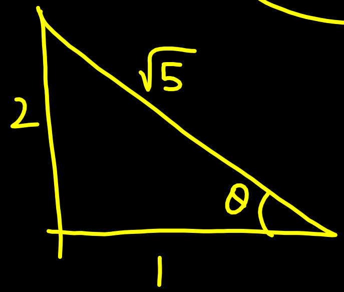
$$R = 2H_{max}$$

$$\frac{v^2 \sin 2\theta}{g} = 2\left(\frac{v^2 \sin^2\theta}{2g}\right)$$

$$\sin 2\theta = \sin^2\theta$$

$$2\sin\theta\cos\theta = \sin^2\theta$$

$$\tan\theta = 2$$

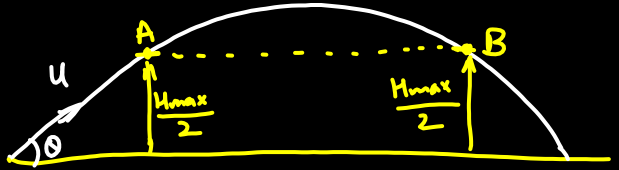


$$\sin\theta = \frac{2}{\sqrt{5}} \qquad \cos\theta = \frac{1}{\sqrt{5}}$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{v^2 (2\sin\theta\cos\theta)}{g}$$

$$R = \frac{v^2\left(2 \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}}\right)}{g} = \frac{4v^2}{5g}$$

Find the average velocity of a projectile between the instances it crosses half the maximum height. It is projected with speed ‘u’ at an angle $\boldsymbol{\theta}$ with the horizontal.



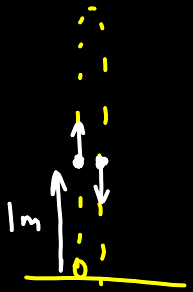
(A) $u\sin\theta$

(B) $u\cos\theta$ ✓

(C) u

(D) $u\tan\theta$

$$V_{avg} = \frac{\text{Total displacement}}{\text{Total Time}}$$



$S_y = 0$ $\quad (V_{avg})_y = 0$

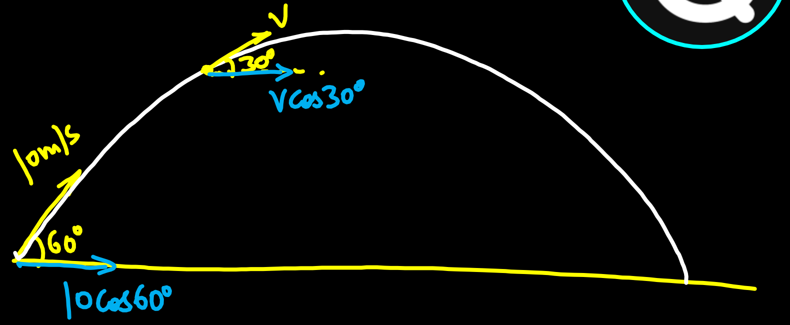
$S_x \neq 0$ $\quad (V_{avg})_x = u\cos\theta$

Q

A particle is projected at an angle 60° above the horizontal with a speed of 10 m/s. After some time the direction of its velocity makes an angle of 30° above the horizontal. The speed of the particle at this instant is

(a) $\frac{5}{\sqrt{3}}$ m/s (b) $5\sqrt{3}$ m/s

(c) 5 m/s (d) $\frac{10}{\sqrt{3}}$ m/s



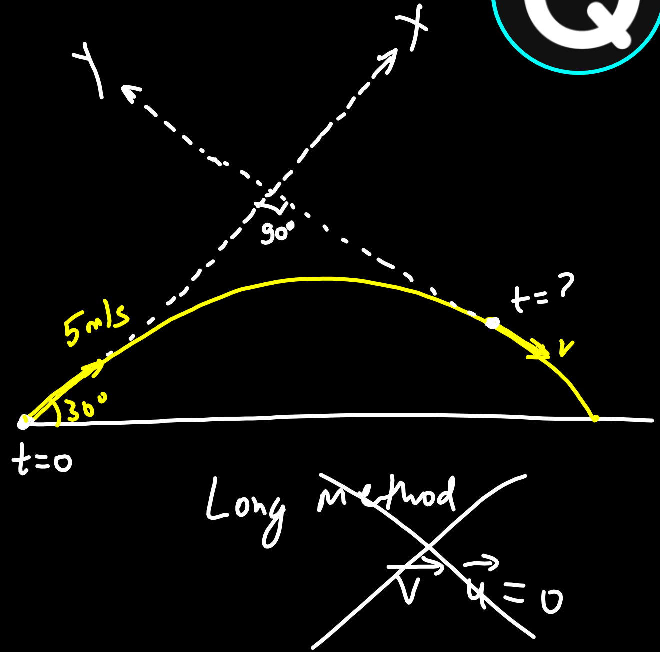
$V_x = const$

$v = u + at \Rightarrow$ st line

$10\cos 60° = V\cos 30°$

$10 \times \frac{1}{2} = V\frac{\sqrt{3}}{2}$

$V = \frac{10}{\sqrt{3}}$



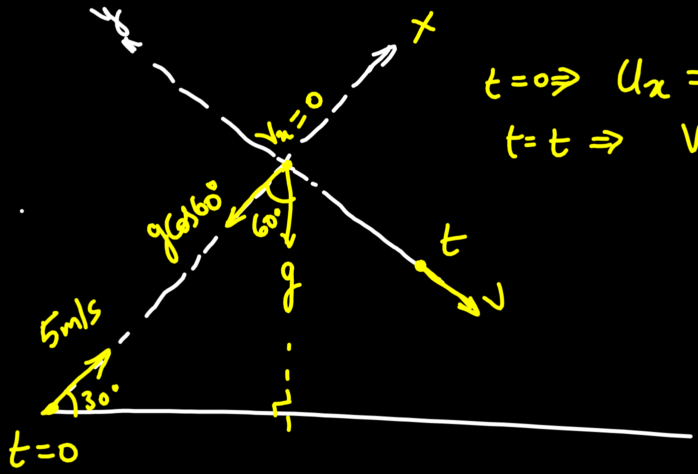
A particle is projected with a speed 5 m/s at an angle of 30° from the horizontal. The time at which the velocity of the projectile becomes perpendicular to the initial direction of projection is

(A) 0.25 s

(B) 0.5 s

(C) 1 s

(D) 1.5 s



$t=0 \Rightarrow U_x = 5\,m/s$

$t=t \Rightarrow V_x = 0 \quad , \quad a_x = -g\cos 60° = -g/2$

$V_x = U_x + a_x t$

$0 = 5 - \left(\frac{g}{2}\right) t \quad \Rightarrow \quad 5 = \frac{gt}{2}$

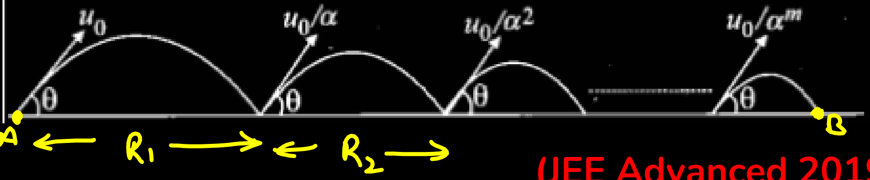
$5 \times 2 = gt$

$\frac{5\times 2}{g} = t$

$t = \frac{10}{g}$

$t = 1\,sec$

. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of $\frac{u_0}{\alpha}$. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is $0.8\,V_1$, the value of α is ______.



(JEE Advanced 2019)

$$V_{avg.} = \frac{\text{Total Displacement}}{\text{Total Time}}$$

$$= \frac{R_1 + R_2 + R_3 + \cdots}{T_1 + T_2 + \cdots}$$

$$= \frac{\left(\frac{u_0^2 \sin 2\theta}{g}\right) + \frac{(u_0/\alpha)^2 \sin 2\theta}{g} + \frac{(u_0/\alpha^2)^2 \sin 2\theta}{g} + \cdots}{\frac{2u_0 \sin\theta}{g} + \frac{2(u_0/\alpha)\sin\theta}{g} + \cdots}$$

$$V_{avg} = \frac{u_0^2 \sin 2\theta}{2u_0 \sin\theta}\left(\frac{1 + \frac{1}{\alpha^2} + \frac{1}{\alpha^4} + \frac{1}{\alpha^6} + \cdots}{1 + 1/\alpha + \frac{1}{\alpha^2} + \frac{1}{\alpha^3} + \cdots}\right)$$

$S_y = 0$

$$S_\infty = \frac{a}{1-r}$$

$$V_{avg} = \frac{u_0(2\sin\theta\cos\theta)}{2\sin\theta}\left[\frac{\dfrac{1}{1-\frac{1}{\alpha^2}}}{\dfrac{1}{1-\frac{1}{\alpha}}}\right]$$

$$V_{avg} = u_0\cos\theta\left[\frac{1-\frac{1}{\alpha}}{1-\frac{1}{\alpha^2}}\right]$$

$$0.8\,V_1 = u_0\cos\theta\,\frac{\left(1-\frac{1}{\alpha}\right)}{\left(1-\frac{1}{\alpha}\right)\left(1+\frac{1}{\alpha}\right)}$$

$$0.8(u_0\cos\theta) = u_0\cos\theta\,\frac{1}{1+\frac{1}{\alpha}}$$

$$0.8 + \frac{0.8}{\alpha} = 1$$

$$\frac{0.8}{\alpha} = 0.2$$

$$\boxed{\alpha = 4}$$

$S_y = 0 \quad (V_{avg})_y = 0$

$V_1 = (V_{avg})_x = u_0\cos\theta$

Projectile Motion

(Case of Equal Range)

$$R_1 = R_2$$

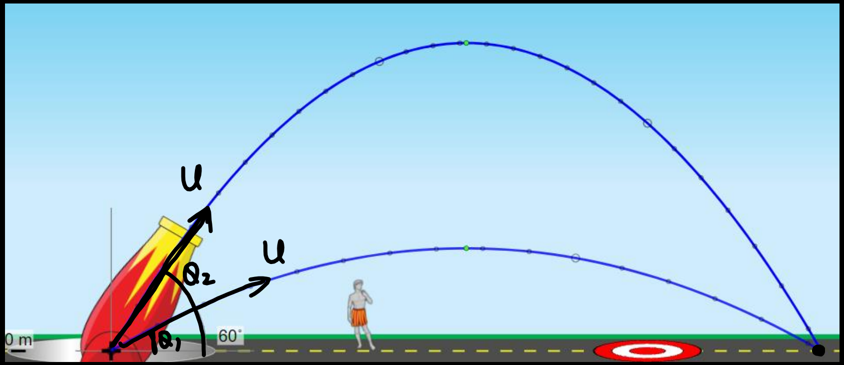
$$\frac{u^2 \sin(2\theta_1)}{g} = \frac{u^2 \sin(2\theta_2)}{g}$$

$$\sin(2\theta_1) = \sin(2\theta_2)$$

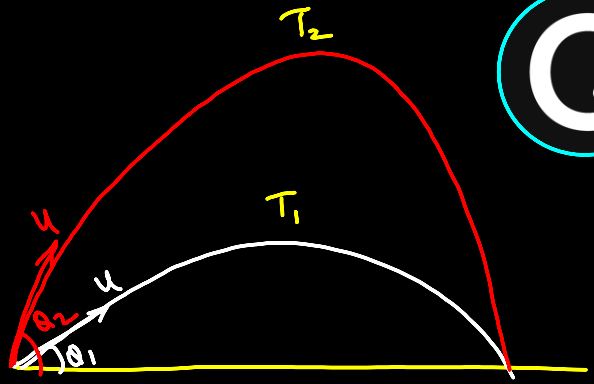
$$\sin(2\theta_1) = \sin(\pi - 2\theta_2)$$

$$2\theta_1 = \pi - 2\theta_2$$

$$\theta_1 + \theta_2 = \pi/2$$



$$\sin(\pi - \theta) = \sin\theta$$



Q

Two projectiles are fired from the same point with the same speed. They fall at the same point. Time of flight of the projectiles are 3 sec & 4 sec respectively. Their projection speed is

(A) 15 m/s

(B) 20 m/s

(C) 25 m/s

(D) 30 m/s

$$T_1 = \frac{2u\sin\theta_1}{g} = 3$$

$$T_2 = \frac{2u\sin\theta_2}{g} = 4$$

$$\Rightarrow \theta_1 + \theta_2 = 90^\circ$$

$$\Rightarrow T_2 = \frac{2u\sin(90-\theta_1)}{g} = \frac{2u\cos\theta_1}{g}$$

$$T_1^2 + T_2^2 = \left(\frac{2u\sin\theta_1}{g}\right)^2 + \left(\frac{2u\cos\theta_1}{g}\right)^2$$

$$3^2 + 4^2 = \left(\frac{2u}{g}\right)^2 \left(\sin^2\theta_1 + \cos^2\theta_1\right)$$

$$25 = \left(\frac{2u}{g}\right)^2 (1)$$

$$5 = \frac{2u}{10}$$

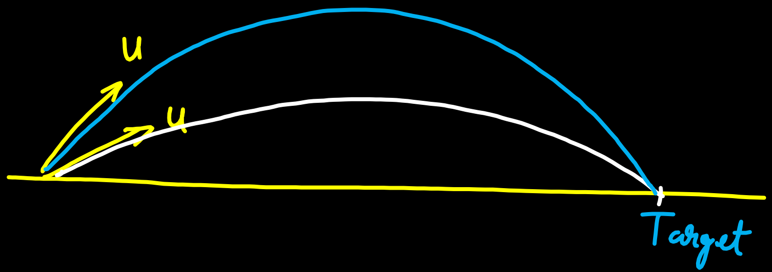
$$\boxed{u = 25\,m/s}$$

Q

HW

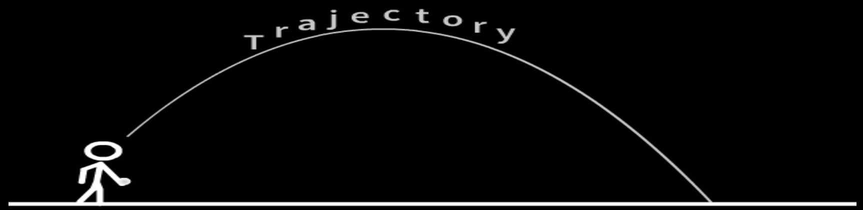
A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product $t_1 t_2$ is : [Main 12 April 2019 (I)]

(a) R/4g (b) R/g (c) R/2g (d) 2R/g



Trajectory

Eqⁿ b/w x & y



⇒ Parabolic Trajectory

Trajectory

X	Y
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$
$S_x = x$	$S_y = y$

Y

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

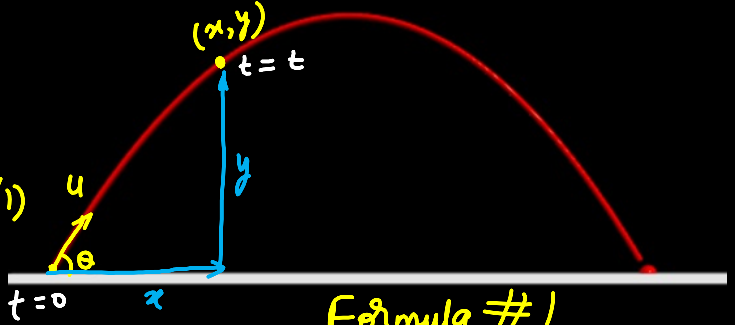
$$y = (u\sin\theta)t - \frac{g}{2}t^2 \quad \ldots\ldots (1)$$

X

$$d_x = v_x t$$

$$x = (u\cos\theta)t \quad \ldots\ldots (2)$$

$$\Rightarrow y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{g}{2}\left(\frac{x}{u\cos\theta}\right)^2$$



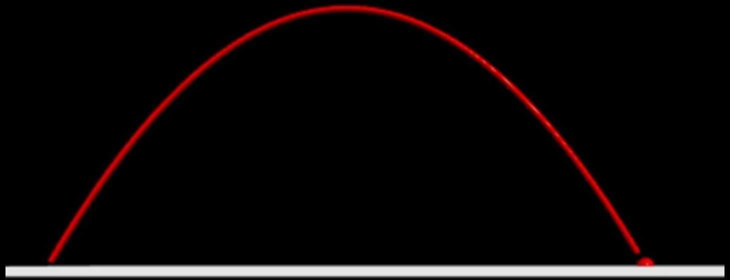
Formula #1

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Trajectory

Trajectory

Formula #2



$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

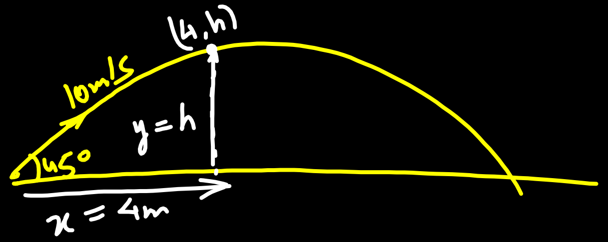
A projectile is fired with a speed of 10 m/s at an angle of 45° from the horizontal. What is the height achieved by the particle when it is at a horizontal distance of 4 m from the launching point.

(A) 1.6 m

(B) 2.4 m

(C) 2.5 m

(D) 3.4 m



$x = 4$, $y = h$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$h = 4\tan 45° - \frac{10 \times 16}{2 \times 100 \times \left(\frac{1}{\sqrt{2}}\right)^2} = 4 - 1.6 = 2.4\text{ m}$$

The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a and b are constants and x and y are respectively horizontal and vertical distances of the projectile from the point of projection. The maximum height attained by the particle and the angle of projection from the horizontal are

(a) $\frac{b^2}{2a}, \tan^{-1}(b)$

(b) $\frac{a^2}{b}, \tan^{-1}(2a)$

(c) $\frac{a^2}{4b}, \tan^{-1}(a)$

(d) $\frac{2a^2}{b}, \tan^{-1}(a)$

$$y = ax - bx^2$$

$$y = (\tan\theta)x - \frac{gx^2}{2u^2\cos^2\theta}$$

$\tan\theta = a$ $\quad$ $b = \frac{g}{2u^2\cos^2\theta}$

$$H_{max.} = \frac{u^2\sin^2\theta}{2g} = \frac{\left(\frac{g}{2b\cos^2\theta}\right)\sin^2\theta}{2g}$$

$$= \frac{\tan^2\theta}{4b} = \frac{a^2}{4b}$$

A particle is projected over a triangle from one extremity of its horizontal base. Grazing over the vertex, it falls on the other extremity of the base. If the base angles of the triangle are 30° & 60° then the projection angle is

(A) tan-1(1/√3)

(B) tan-1(2/√3)

(C) tan-1(4/√3)

tan-1(6/√3)

$$R = a + b$$

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

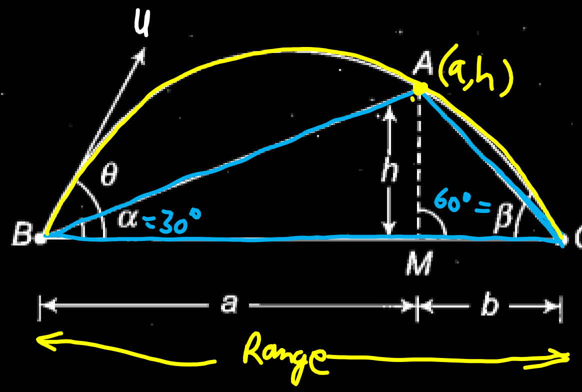
$$h = a\tan\theta\left[1 - \frac{a}{a+b}\right]$$

$$h = a\tan\theta\left[\frac{b}{a+b}\right]$$

$$\frac{(a+b)h}{ab} = \tan\theta$$

$$\left(\frac{1}{b} + \frac{1}{a}\right)h = \tan\theta$$

$$\left(\frac{h}{a} + \frac{h}{b}\right) = \tan\theta$$



ΔABM

$$\tan\alpha = \frac{h}{a}$$

ΔAMC

$$\tan\beta = \frac{h}{b}$$

$$\Rightarrow \tan\alpha + \tan\beta = \tan\theta$$

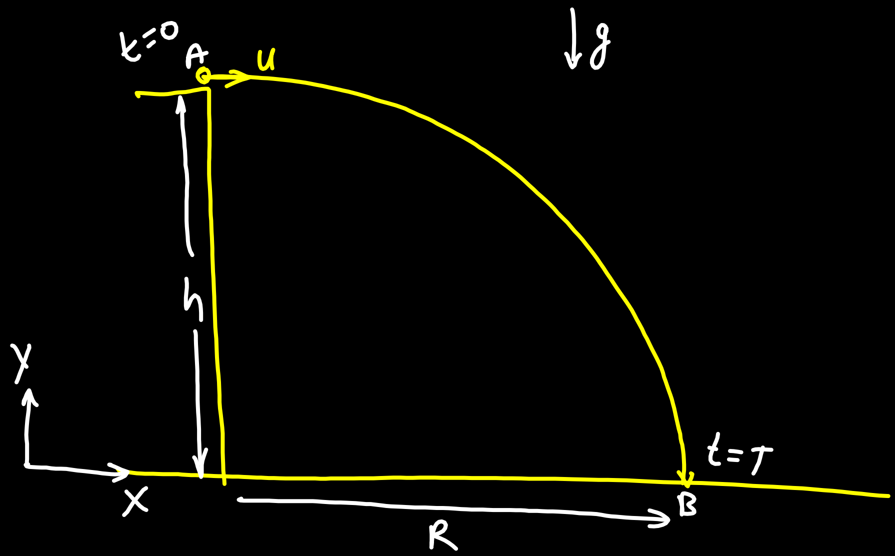
$$\tan 30^\circ + \tan 60^\circ = \tan\theta$$

$$\frac{1}{\sqrt{3}} + \sqrt{3} = \tan\theta$$

$$\frac{4}{\sqrt{3}} = \tan\theta$$

Projectile from a Height

R, H, T formula not applicable



x	y
$u_x = u$	$u_y = 0$
$S_x = R$	$S_y = -h$
$a_x = 0$	$a_y = -g$
$t = T$	$t = T$

Projectile from a Height

y

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$-h = 0 + \frac{1}{2}(-g)t^2$$

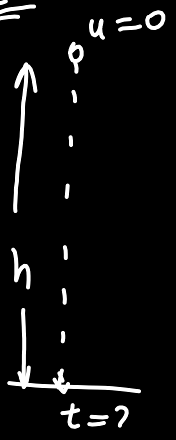
$$\boxed{t = \sqrt{\frac{2h}{g}}}$$

x

$$R = V_x t$$

$$\boxed{R = u\sqrt{\frac{2h}{g}}}$$

1D



$$S = ut + \frac{1}{2}at^2$$

$$h = 0 + \frac{g}{2}t^2$$

$$t = \sqrt{\frac{2h}{g}}$$

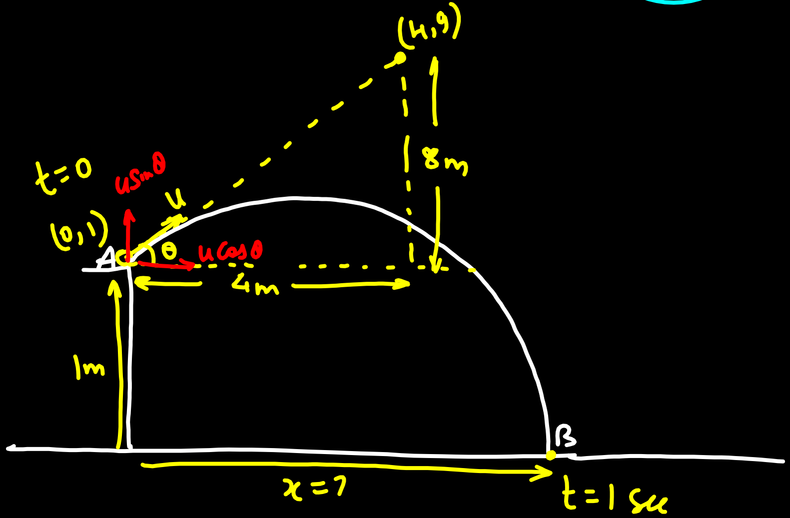
A particle is projected from a point (0,1) on Y-axis (assume $+Y$ direction vertically upwards) aiming towards a point (4, 9). It fell on ground along x-axis in 1 sec. Taking $g = 10\ \text{m/s}^2$ and all coordinates in metres. Find the X-coordinate where it fell.

(a) (3, 0)

(b) (4, 0)

(c) (2, 0) ✓

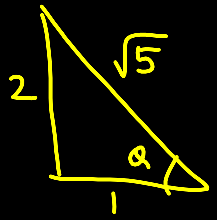
(d) $(2\sqrt{5}, 0)$



$\tan\theta = \frac{8}{4} = 2$

$\sin\theta = 2/\sqrt{5}$

$\cos\theta = \frac{1}{\sqrt{5}}$



x	y
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$S_x = x$	$S_y = -1\,m$
$a_x = 0$	$a_y = -g$
$t = 1$ sec	$t = 1$ sec

y

$$-1 = (u\sin\theta)(1) + \frac{1}{2}(-g)(1)^2$$

$$-1 = \frac{2u}{\sqrt{5}} - 5$$

$$4 = \frac{2u}{\sqrt{5}}$$

$$\boxed{u = 2\sqrt{5}\ m/s}$$

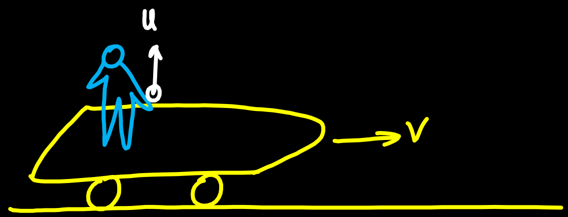
x

$$x = (u\cos\theta)(t)$$

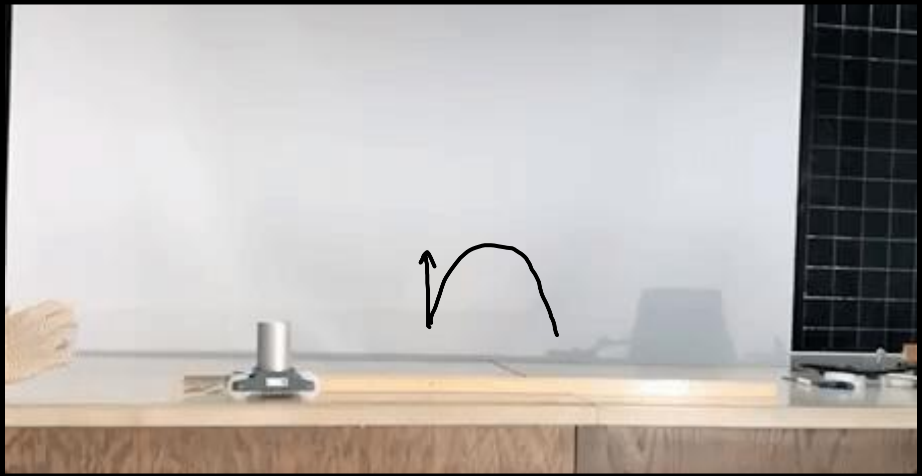
$$x = (2\sqrt{5})\left(\frac{1}{\sqrt{5}}\right)(1)$$

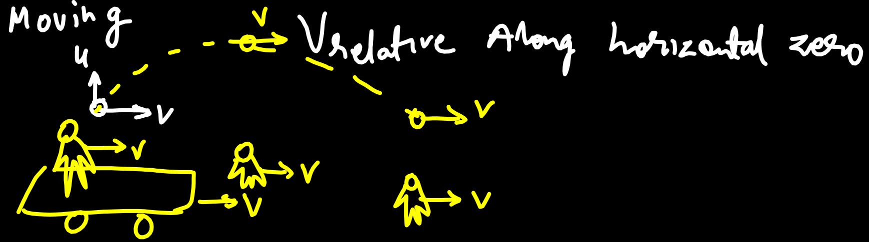
$$\boxed{x = 2\,m}$$

Projectile from a Moving Frame



From Ground





Q

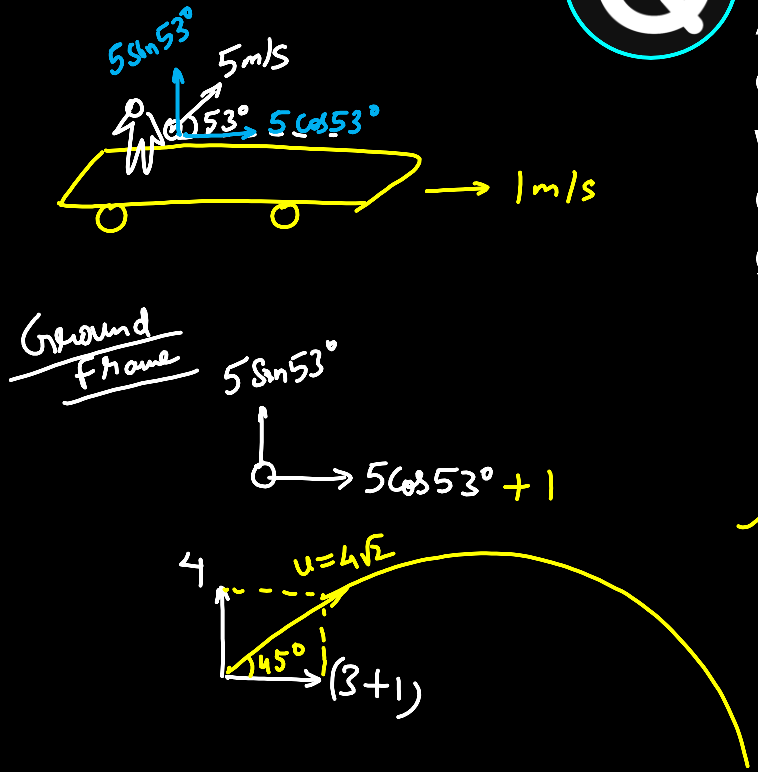
A ball is projected with a speed of 5 m/s at angle of 53° with respect to a trolley which is moving with 1 m/s on horizontal road. The horizontal distance covered by the ball with respect to ground is

(A) 0 m

(B) 2.4 m

(C) 3.2 m

(D) 4.6 m



$$R = \frac{u^2 \sin(2\theta)}{g}$$

$$R = \frac{(4\sqrt{2})^2 \sin(2 \times 45^\circ)}{10}$$

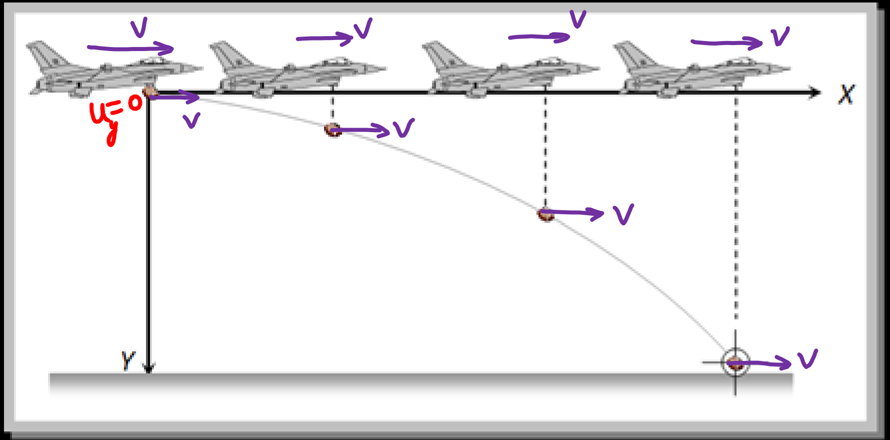
$$R = \frac{32 \times 1}{10}$$

$$\boxed{R = 3\ 2 m}$$

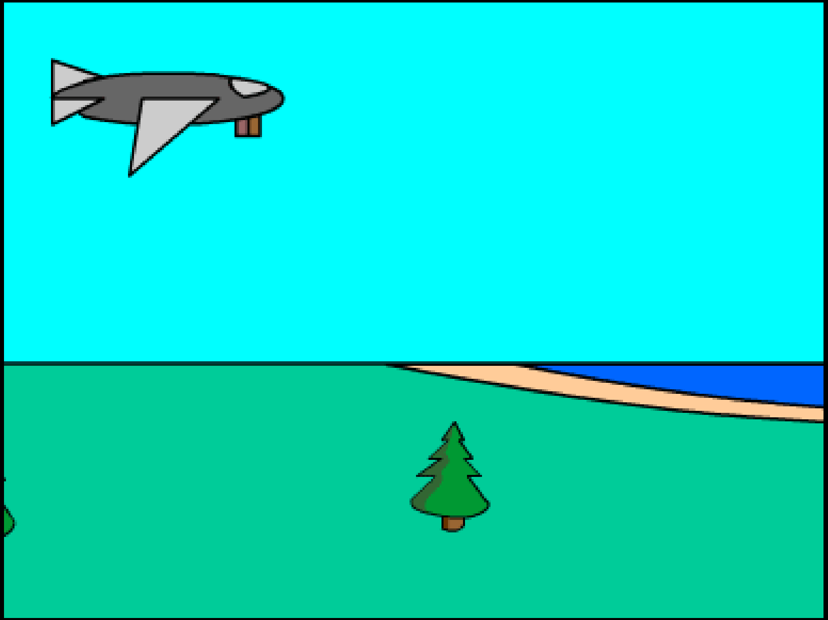
Unacademy Atoms

Projectile from a Height

Projectile from a Height



Projectile from a Height



Q HW {Comment}

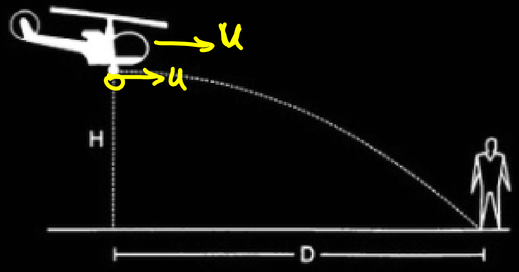
A helicopter on a flood relief mission flying horizontally with a speed ‘u=25 m/s’ at an altitude h=20 m has to drop a food packet for a victim standing on the ground. At what horizontal distance ‘D’ from the victim should the food packet be dropped?

(A) 15 m

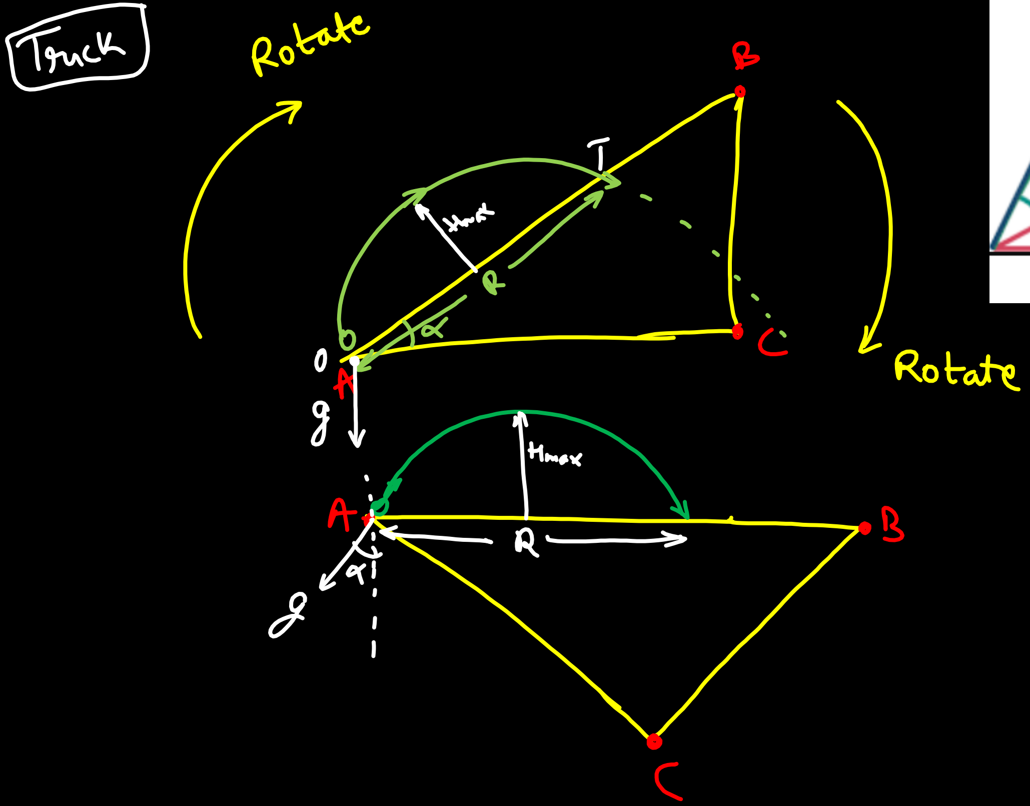
(B) 25 m

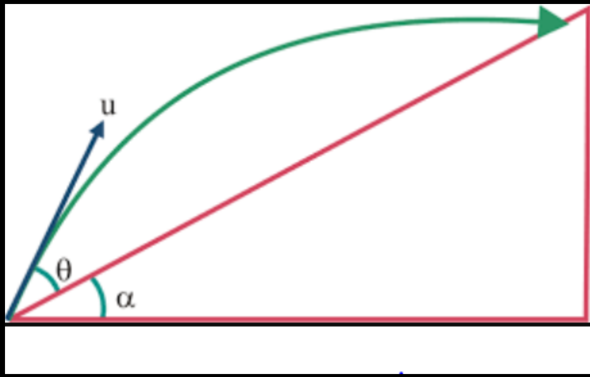
(C) 35 m

(D) 50 m

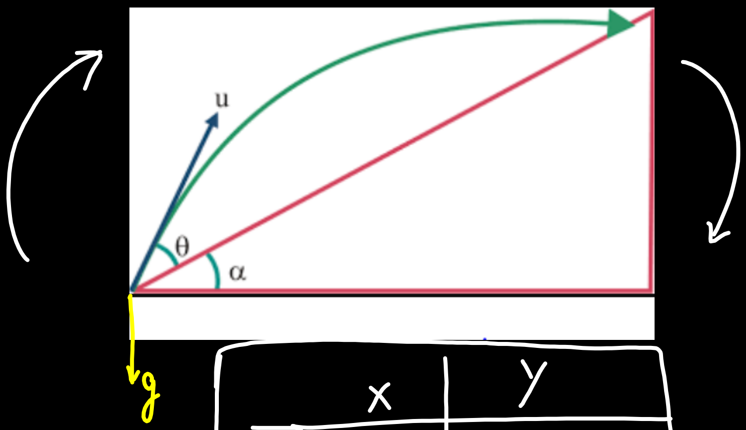


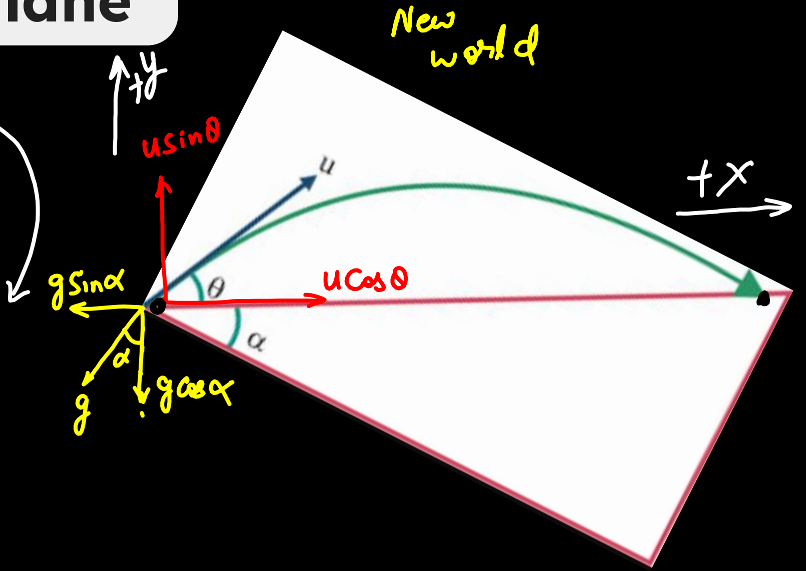
Projectile on an Inclined Plane





Projectile on an Inclined Plane





x	y
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = -g\sin\alpha$	$a_y = -g\cos\alpha$
$S_x = R$	$S_y = 0$

Projectile on an Inclined Plane

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$0 = (u\sin\theta) T + \frac{1}{2}(-g\cos\alpha) T^2$$

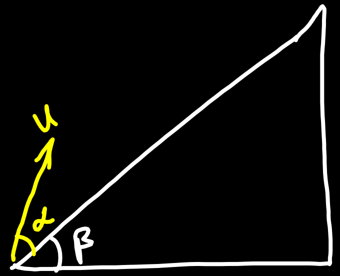
$$\boxed{T = \frac{2u\sin\theta}{g\cos\alpha}}$$

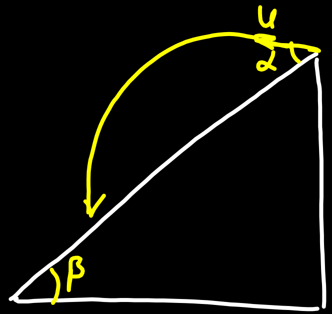
$$R = (u\cos\theta) T + \frac{1}{2}(-g\sin\alpha) T^2$$

$$=$$

Projectile on an Inclined Plane

Projectile on an Inclined Plane





	Up the Incline	Down the Incline
Range	$R = \dfrac{2u^2 \sin\alpha \cos(\alpha+\beta)}{g\cos^2\beta}$	$\dfrac{2u^2 \sin\alpha \cos(\alpha-\beta)}{g\cos^2\beta}$
Time of flight	$\dfrac{2u\sin\alpha}{g\cos\beta}$	$\dfrac{2u\sin\alpha}{g\cos\beta}$
Angle of projection for maximum range	$\dfrac{\pi}{4} - \dfrac{\beta}{2}$	$\dfrac{\pi}{4} + \dfrac{\beta}{2}$
Maximum Range	$\dfrac{u^2}{g(1+\sin\beta)}$	$\dfrac{u^2}{g(1-\sin\beta)}$

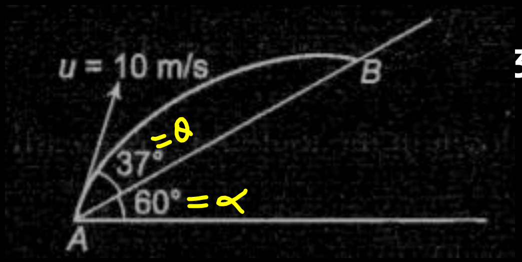
Where α- Angle of Projection $\quad$ β- Angle of Inclined Plane

A particle is projected at an angle of 37' with an inclined plane. Calculate its time of flight.

(A) 2.2 sec (B)

2.4 sec ✓

(C) 2.6 sec



3

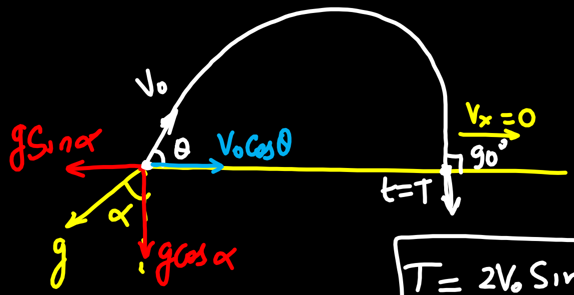
sec

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

$$= \frac{2\times(10)(3/5)}{10\times 1/2}$$

$$T = \frac{12}{5} = 2.4\text{ sec}$$

A ball is thrown at an angle θ up an inclined plane with a velocity V_0 such that it hits the incline normally. If angle of incline is α, then the value of tanθ is



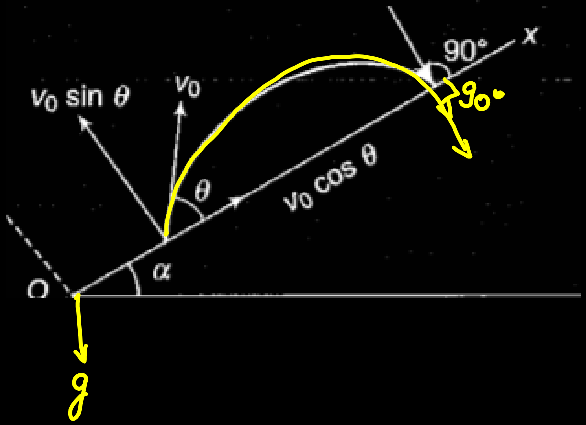
(A) tanα

(B) cotα

(C) tanα)/2

(D) (cotα)/2

$$T = \frac{2V_0 \sin\theta}{g\cos\alpha} \rightarrow (1)$$



$$u_x = V_0\cos\theta,\ a_x = -g\sin\alpha,\ V_x = 0,\ t = T$$

$$V_x = u_x + a_x t$$

$$0 = V_0\cos\theta + (-g\sin\alpha)T$$

$$T = \frac{V_0 \cos\theta}{g \sin\alpha} \rightarrow (2)$$

$$(1) = (2)$$

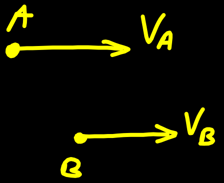
$$\frac{2V_0 \sin\theta}{g\cos\alpha} = \frac{V_0 \cos\theta}{g\sin\alpha}$$

$$2\frac{\sin\theta}{\cos\theta} = \frac{\cos\alpha}{\sin\alpha}$$

$$2\tan\theta = \cot\alpha$$

$$\tan\theta = \frac{\cot\alpha}{2}$$

Relative Motion in 2D



$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

Example

$\vec{V}_A = 2\hat{i} + 3\hat{j}$, $\vec{V}_B = -\hat{i} + 4\hat{j}$

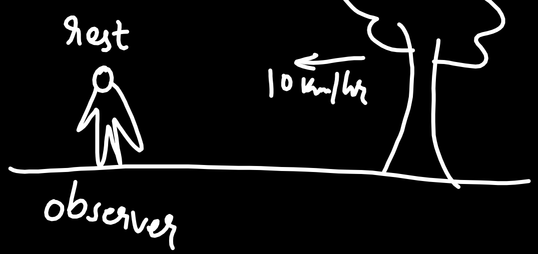
$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B = (2\hat{i} + 3\hat{j}) - (-\hat{i} + 4\hat{j})$$

$$\vec{V}_{AB} = 3\hat{i} - \hat{j}$$

Observer is always at Rest

Ground

Your frame

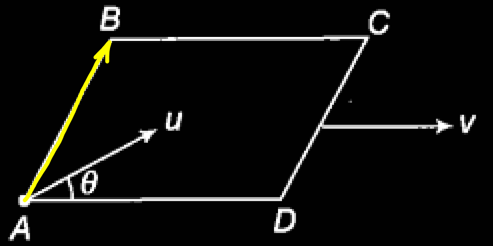


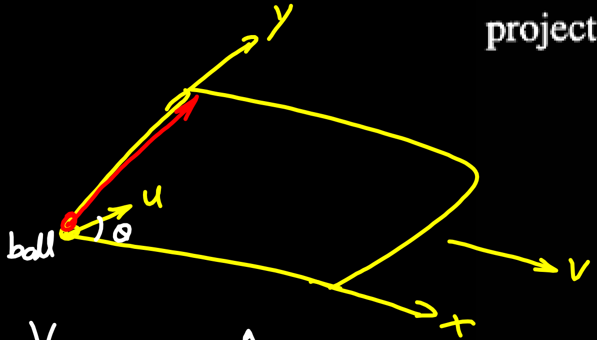
Relative Motion in 2D

Relative Motion in 2D

Q

A smooth square platform $ABCD$ is moving towards right with a uniform speed v. At what angle θ must a particle be projected from A with speed u so that it strikes the point B





(a) $\sin^{-1}\left(\frac{u}{v}\right)$

(b) $\cos^{-1}\left(\frac{v}{u}\right)$

(c) $\cos^{-1}\left(\frac{u}{v}\right)$

(d) $\sin^{-1}\left(\frac{v}{u}\right)$

$V_{ball} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

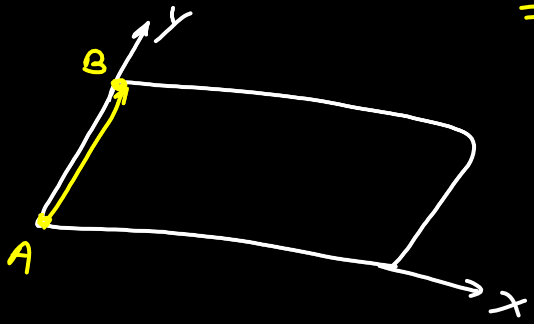
$V_{train} = v\,\hat{i}$

$(\vec{V}_{ball})_{train} = \vec{V}_{ball} - \vec{V}_{train}$

$= (u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}) - v\,\hat{i}$

$$(V_{ball})_{train} = (u\cos\theta - v)\hat{i} + (u\sin\theta)\hat{j}$$

For ball to hit B



$$\Rightarrow \left[(V_{ball})_{train}\right]_{x\ component} = 0$$

$$u\cos\theta - v = 0$$

$$\boxed{\cos\theta = \frac{v}{u}}$$

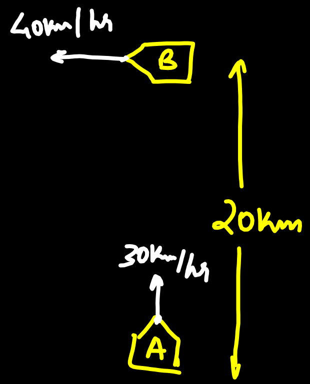
Two boats A and B are stationary in the middle of sea with a distance of 20 km between them. Boat A starts moving along North with a constant speed of 30 km/hr while boat B starts moving along West with a constant speed of 40 km/hr. What will be the minimum distance between the boats

(A) 5 km

(B) 12 km

(C) 16 km ✓

(D) 20 km

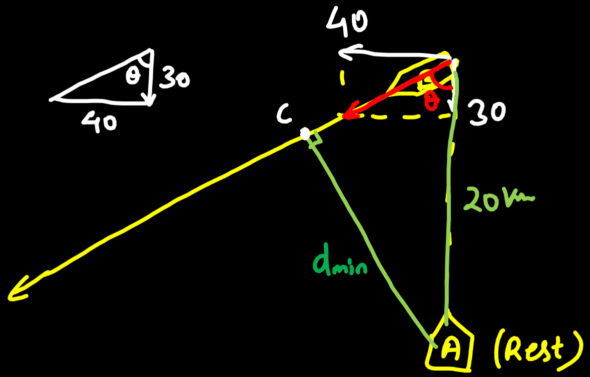


$V_A = 30\hat{j}$

$V_B = -40\hat{i}$

A ⇒ observer (Rest)

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A = -40\hat{i} - 30\hat{j}$$



ΔABC

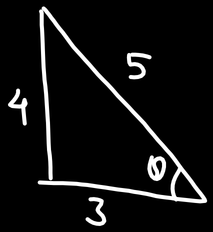
$$\sin\theta = \frac{d_{min}}{20}$$

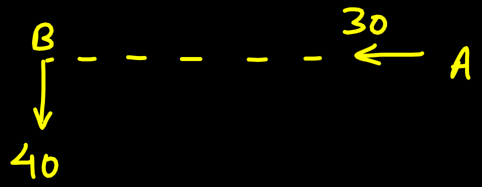
$$\frac{4}{5} = \frac{d_{min}}{20}$$

$$d_{min} = 16 \text{ Km}$$

$$\tan\theta = \frac{4}{3}$$

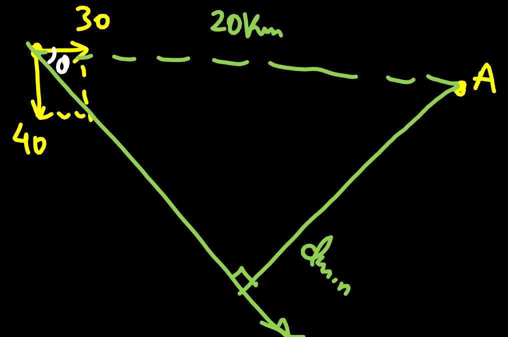
$$\theta = 53^\circ$$





$$\tan\theta = 4/3$$

$$\sin\theta = \frac{d_{min}}{20}$$



Q

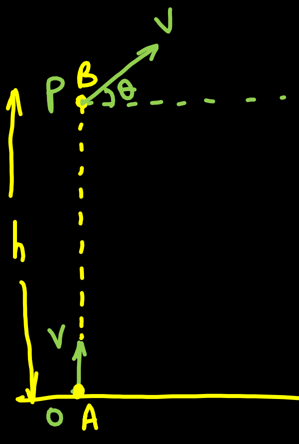
A particle is projected vertically upwards from O with velocity v and a second particle is projected at the same instant from P(at a height h above O) with velocity v at an angle of projection θ. The time when the distance between them is minimum is

(a) $\dfrac{h}{2v\sin\theta}$

(b) $\dfrac{h}{2v\cos\theta}$

(c) h/v

(d) $h/2v$



$\vec{V}_A = v\hat{j}$

$\vec{V}_B = (v\cos\theta)\hat{i} + (v\sin\theta)\hat{j}$

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

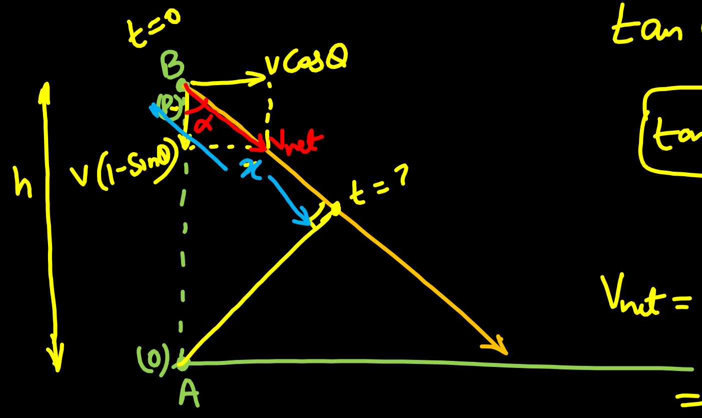
$= (v\cos\theta)\hat{i} + (v\sin\theta - v)\hat{j}$

$\vec{V}_{BA} = (v\cos\theta)\hat{i} - (v - v\sin\theta)\hat{j}$

$a_{BA} = a_B - a_A$

$= g - g$

$= 0$



$$\tan\alpha = \frac{v\cos\theta}{v(1-\sin\theta)}$$

$$\boxed{\tan\alpha = \frac{\cos\theta}{1-\sin\theta}}$$

$$V_{net} = \sqrt{(v\cos\theta)^2 + (v - v\sin\theta)^2}$$

$$= v\sqrt{\cos^2\theta + 1 + \sin^2\theta - 2\sin\theta}$$

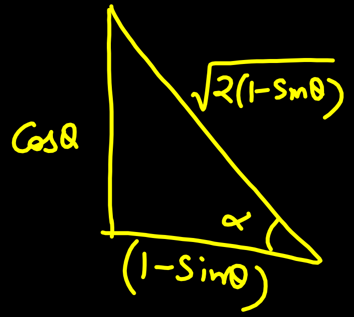
$$\boxed{V_{net} = v\sqrt{2(1-\sin\theta)}}$$

$$t = \frac{x}{V_{net}} = \frac{\frac{h}{\sqrt{2}}\left(\sqrt{1-\sin\theta}\right)}{v\sqrt{2(1-\sin\theta)}}$$

$$\boxed{t = \frac{h}{2v}}$$

$$\cos\alpha = x/h$$

$$x = h\cos\alpha$$



$$x = \frac{h(1-\sin\theta)}{\sqrt{2(1-\sin\theta)}}$$

$$\boxed{x = \frac{h}{\sqrt{2}}\left(\sqrt{1-\sin\theta}\right)}$$

Q

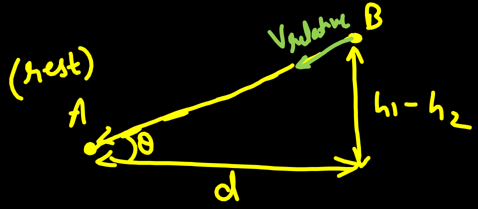
HW

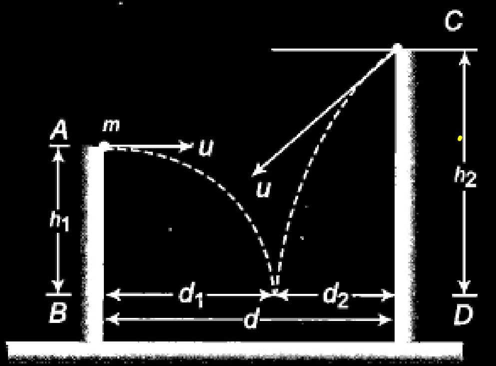
Two towers AB and CD are situated a distance d apart as shown in the figure. AB is 20 m high and CD is 30 m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity 10 m/s towards CD. Simultaneously another object of 2 m is thrown from the top of CD at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each other. Calculate the distance d between the towers.

(a) 10 m (b) 17m (c) 21 m (d) 25 m

Answer in Comment

$a_{relative} = 0$

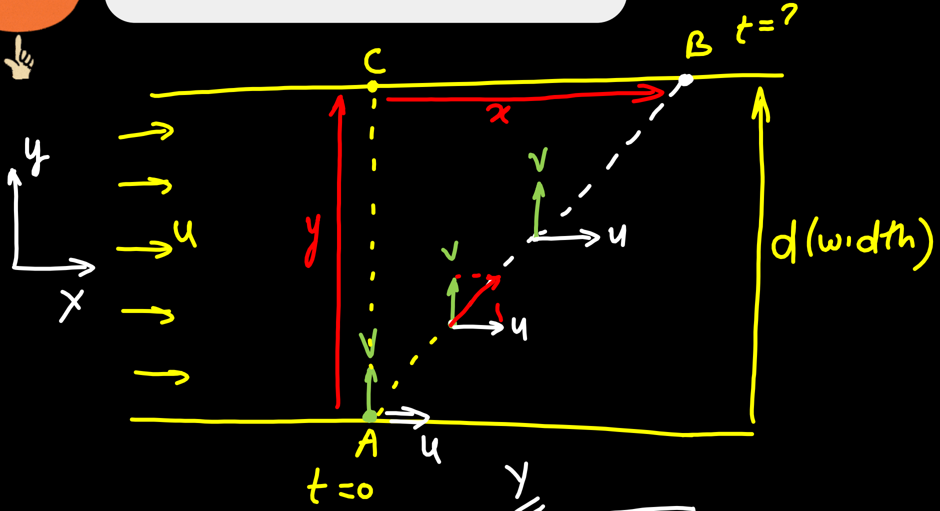




River Problems

Swimming speed in still water $= V$

River Problems



Y

$t = \frac{d}{v}$

X	Y
$a_x = 0$	$a_y = 0$
$u_x = u$	$u_y = v$
$S_x = x$	$S_y = d$
t	t

X

$x = ut$

$x = \frac{ud}{v}$

Q

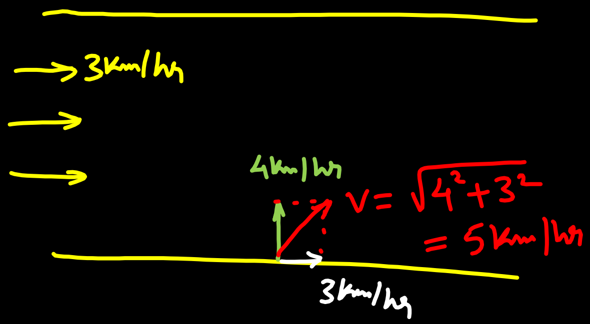
A swimmer can swim in still water at a rate 4.0 km/h. If he swims in a river flowing at 3.0 km/h and keeps his direction (with respect to water) perpendicular to the current, find his velocity with respect to the ground)

(A) 3 km/hr

(B) 4 km/hr

(C) 5 km/hr

(D) 7 km/hr

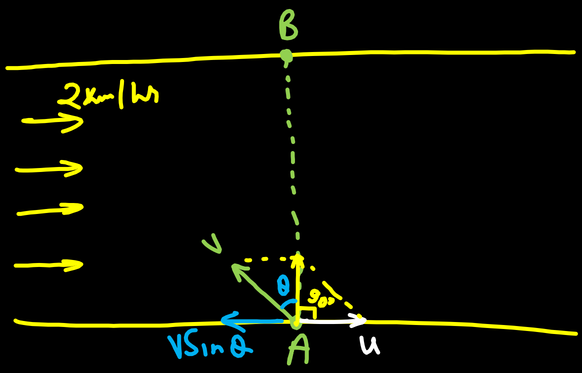


Q

The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?

[Main 9 April 2019 (I)]

(a) 90° (b) 150° (c) 120° (d) 60°



$v\sin\theta = u$

$4\sin\theta = 2$

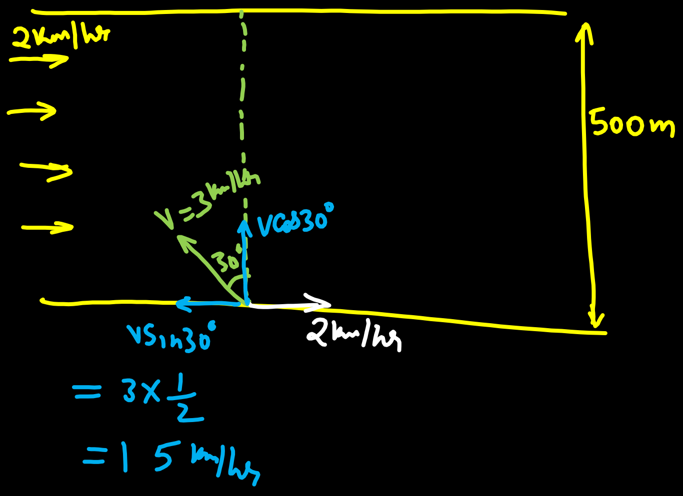
$\sin\theta = 1/2$

$\theta = 30°$

Total angle from the flow $= 90° + \theta$

$= 120°$

A man can swim at a speed of 3 km/h in still water. He wants to cross a 500 m wide river flowing at 2 km/h. He keeps himself always at an angle of 120° with the river flow while swimming. At what point on the opposite bank will he arrive ?

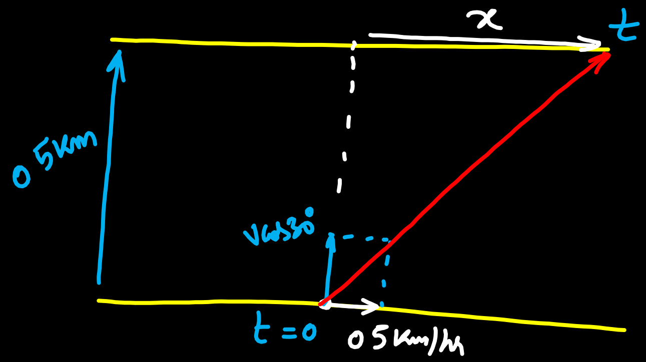


(A) 1/3√3 km

(B) 1/2√3 km

(C) 1/6√3 km ✓

(D) 1/4√3 km



$$t = \frac{0\ 5\ km}{v \cos 30^\circ}$$

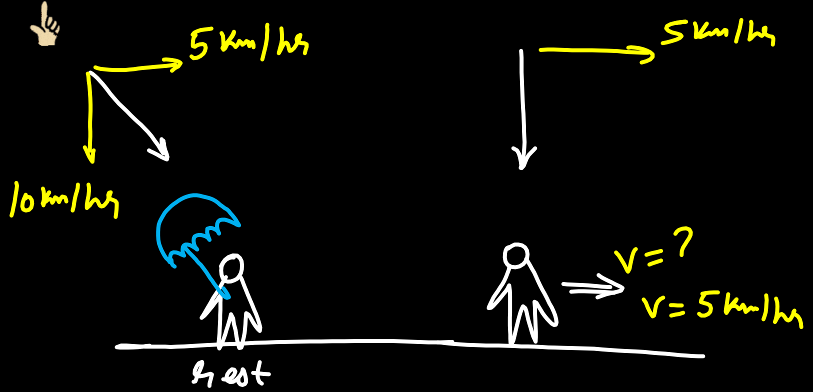
$$t = \frac{0\ 5}{3 \times \sqrt{3}/2}$$

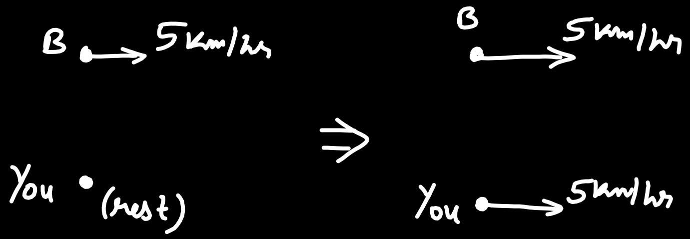
$$t = \frac{1}{3\sqrt{3}}\ hr$$

$$x = \left(0\ 5\ \frac{km}{hr}\right)\left(\frac{1}{3\sqrt{3}}\ hr\right)$$

$$x = \frac{1}{6\sqrt{3}}\ km$$

Rain and the Man Problems





Q

A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that raindrops are hitting his head vertically. Find the speed of raindrops with respect to the road

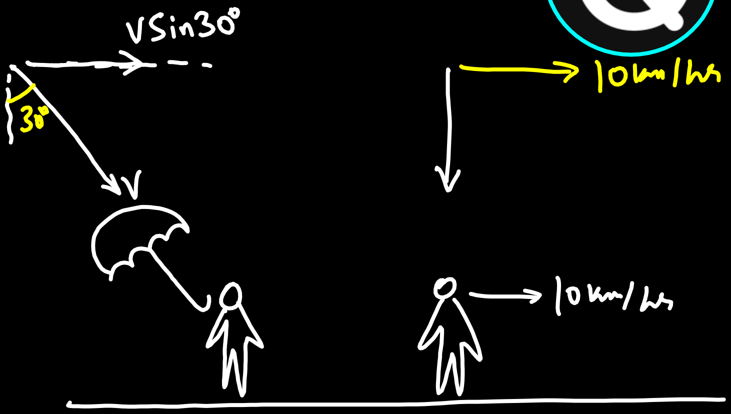
(A) $10\sqrt{3}$ km/hr

(B) $20\sqrt{3}$ km/hr

(C) 10 km/hr

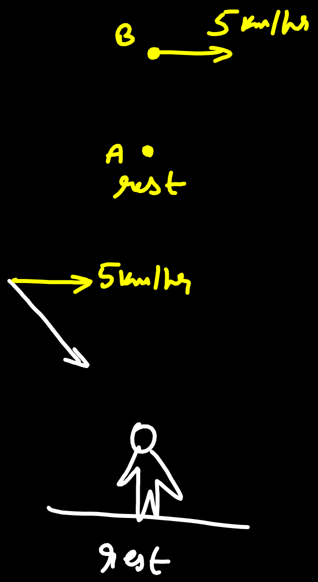
(D)

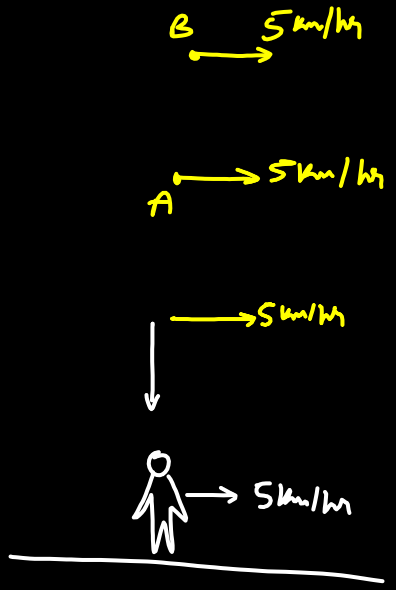
20 km/hr ✓

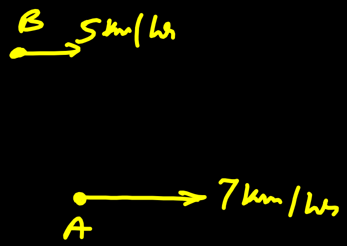


$v \sin 30^\circ = 10$

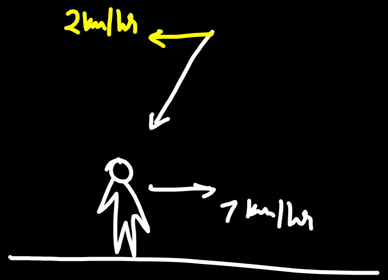
$v = 20$ km/hr







$$V_{BA} = V_B - V_A = 5 - 7 = -2\ \text{km/hr}$$



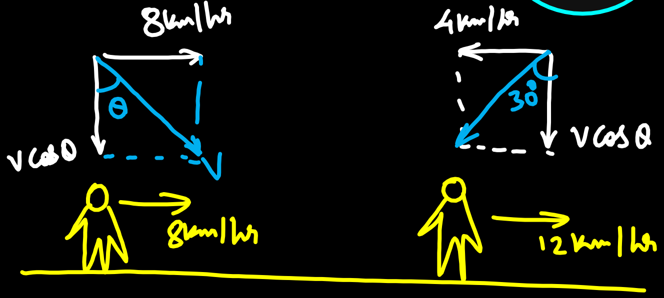
A man running on a horizontal road at 8 km/h finds the rain falling vertically. He increases his speed to 12 km/h and finds that the drops make angle 30° with the vertical. Find the speed and direction of the rain with respect to the road.

(A) $4\sqrt{7}$ km/hr

(B) $2\sqrt{7}$ km/hr

(C) $\sqrt{7}$ km/hr

(D) $8\sqrt{7}$ km/hr



$$V\sin\theta = 8$$

$$\tan 30^\circ = \frac{4}{V\cos\theta}$$

$$V\cos\theta = 4\sqrt{3}$$

$$V^2(\sin^2\theta + \cos^2\theta) = 8^2 + (4\sqrt{3})^2$$

$$V^2 = 64 + 48$$

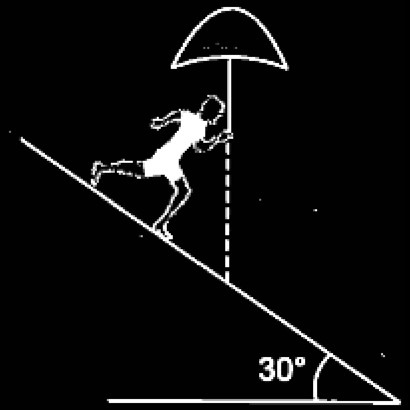
$$V = 4\sqrt{7} \text{ km/hr}$$

HW

Comment Ans

A man is coming down an incline of angle 30°.
When he walks with speed $2\sqrt{3}$ m/s he has to keep his umbrella vertical to protect himself from rain. The actual speed of rain is 5 m/s. At what angle with vertical should he keep his umbrella when he is at rest so that he does not get drenched?

(A) 30° (B) 37° (C) 45° (D) 60°

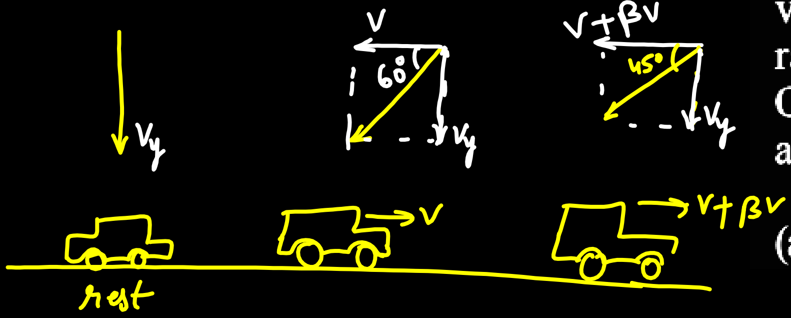


Q

When a car sit at rest, its driver sees raindrops falling on it vertically. When driving the car with speed v, he sees that raindrops are coming at an angle 60° from the horizontal. On furter increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45°. The value of β is close to:

[Main Sep. 06, 2020 (II)]

(a) 0.50 (b) 0.41 (c) 0.37 (d) 0.73



$$\tan 60^\circ = \frac{V_y}{V}$$

$$V_y = \sqrt{3}V$$

$$\tan 45^\circ = \frac{V_y}{V+\beta V}$$

$$1 = \frac{\sqrt{3}V}{V(1+\beta)}$$

$$1+\beta = \sqrt{3}$$

$$\beta = \sqrt{3}-1$$

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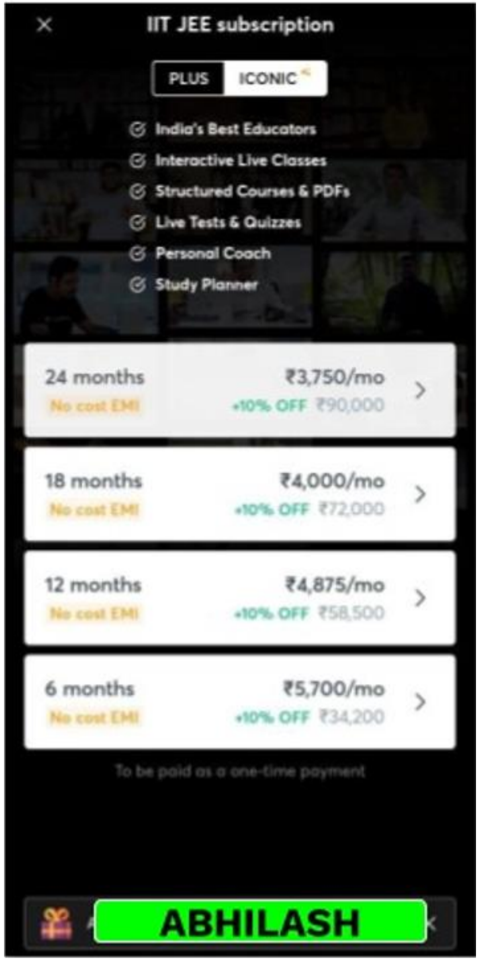
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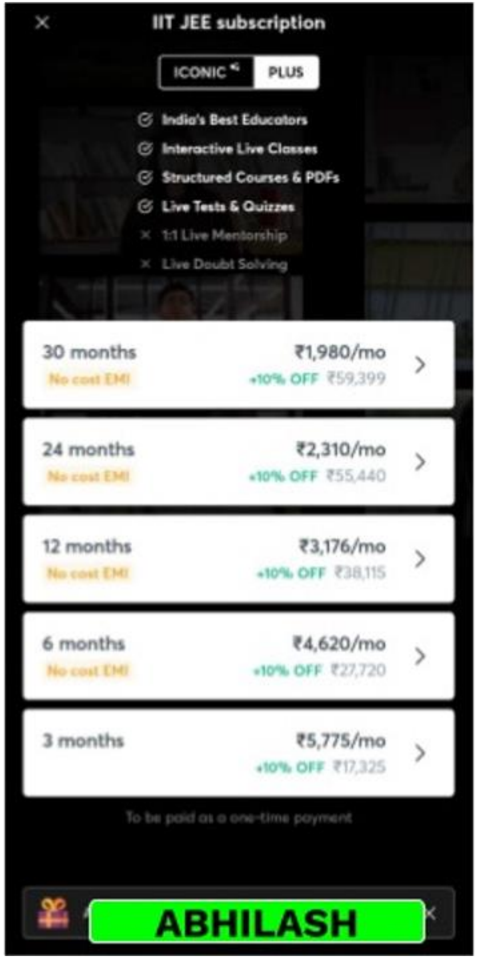
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