WORK ENERGY & POWER

Abhilash Sharma

B.Tech - NIT Calicut

- Experience - 8+ years
- 700+ selections in JEE Advanced
- 6000+ selections in JEE Mains

BounceBack

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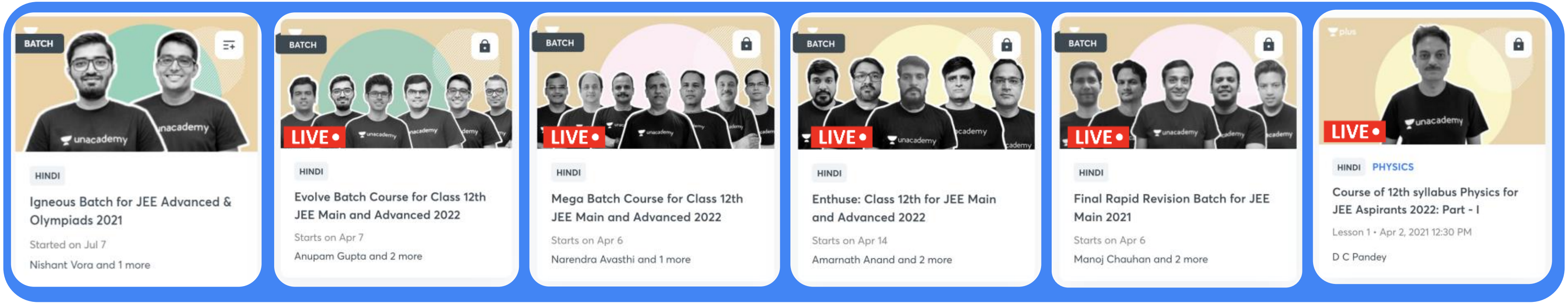

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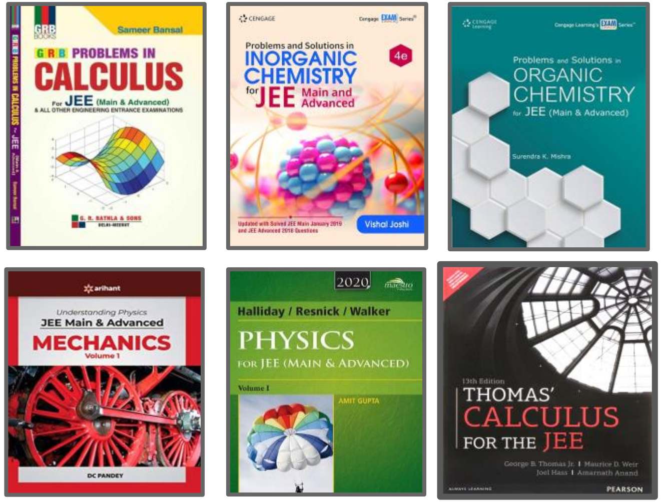

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for IIT JEE Main and Advanced 2024

Code: ABHILASH

Batch highlights:

- Curated by India's Top Educators
- Coverage of Class 11 JEE syllabus
- Enhance conceptual understanding of JEE Main & JEE Advanced subjects
- Systematically designed courses
- Strengthen JEE problem-solving ability

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Mathematics Maestro

Nishant Vora
Mathematics Maestro

Ajit Lulla
Physics Maestro

Abhilash Sharma
Physics Maestro

Sakshi Vora
Chemistry Maestro

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Chemistry Maestro

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- Learn from India's Top Educators
- Coverage of Class 11 & 12 syllabus of JEE
- Deep dive at a conceptual level for JEE Main and JEE Advanced
- Systematic course flow of subjects and related topics
- Strengthening the problem-solving ability of JEE level problems

For more details, contact **8585858585**

Nishant Vora
Mathematics Maestros

Prashant Jain
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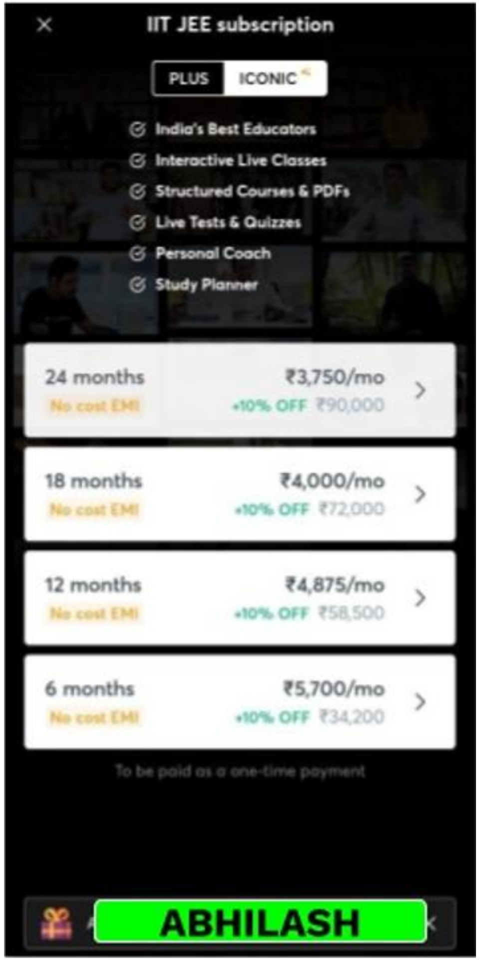
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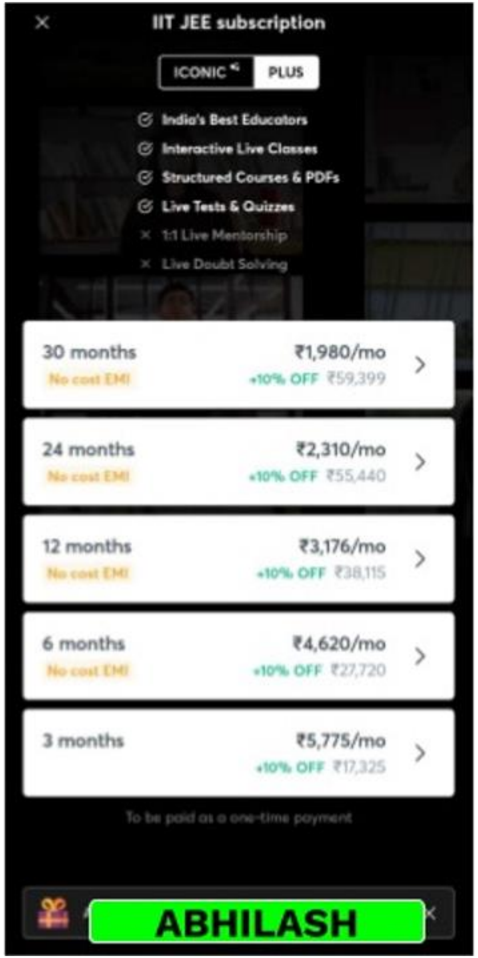
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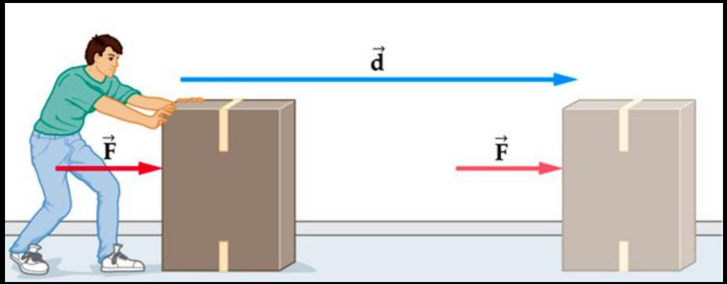
Work Energy & Power

Work

⇒ creating change

1) Shape ✓
2) Size ✓
3) Location ✓
4) Temperature ⇒ Thermodynamics



$$W = Fd$$ **

* $F = const$
* $\theta = 0^\circ$

Work

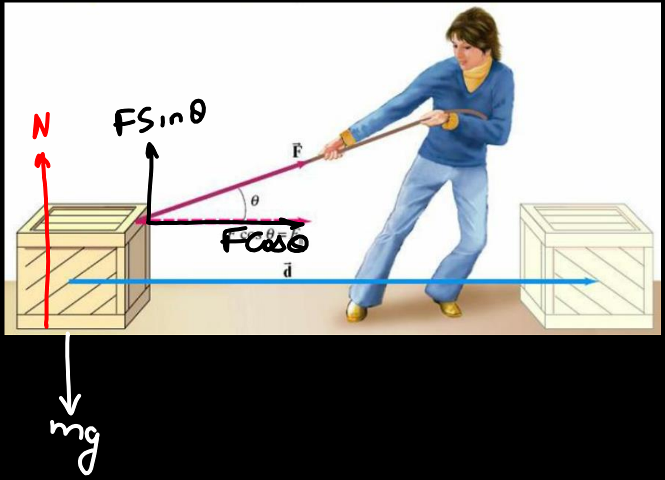
$N + F\sin\theta = mg$

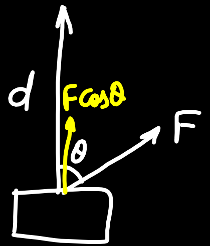
$W = (F\cos\theta)(d)$

$W = F d \cos\theta$

$W = \vec{F} \cdot \vec{d}$ *

* $F = \text{const}$





Work ⇒ Scalar

SI Unit ⇒ Nm = Joule

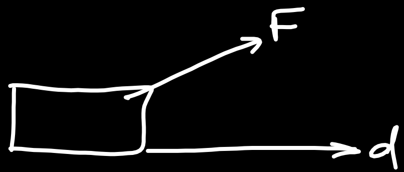
Work

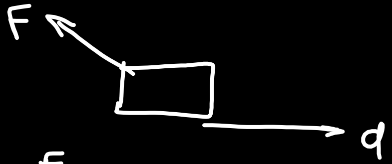
Special Cases

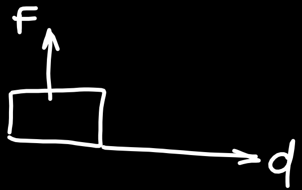
1) $\theta \Rightarrow$ acute $\Rightarrow \cos\theta = +ve \Rightarrow$ $\boxed{W = +ve}$

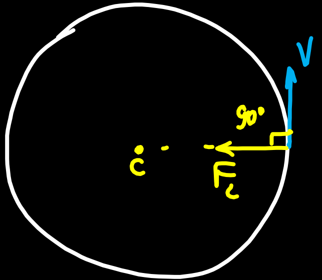
2) $\theta \Rightarrow$ obtuse $\Rightarrow \cos\theta = -ve \Rightarrow$ $\boxed{W = -ve}$

3) $\theta = 90^\circ \Rightarrow \cos\theta = 0 \Rightarrow$ $\boxed{W = 0}$







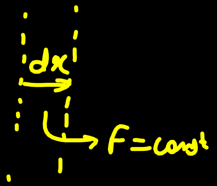


$W = 0$ $\quad \vec{F} = q(\vec{v} \times \vec{B})$

Work

Variable Force

$$dW = \vec{F}\, d\vec{x}$$

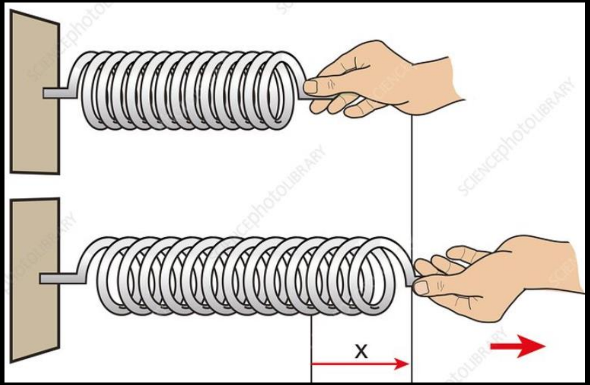


$$W = \int \vec{F}\, d\vec{x}$$

⇒ Valid For all Cases

1) F = const. ⇒ $W = \vec{F}\int d\vec{x} = \vec{F}\,\vec{x}$

2) F = const. , $\theta = 0^\circ$ ⇒ $W = Fx$



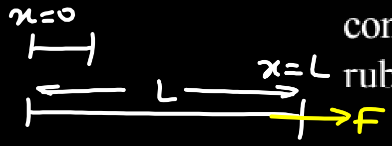
$F = kx$

$F \neq$ const

Q

When a rubber-band is stretched by a distance x, it exerts restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is:

[JEE Main 2014]



(a) $aL^2 + bL^3$

(b) $\frac{1}{2}\left(aL^2 + bL^3\right)$

(c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ ✓

(d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

$$W = \int \vec{F} \cdot \vec{dx}$$

$$= \int_0^L (ax + bx^2)\, dx \cos 0^\circ$$

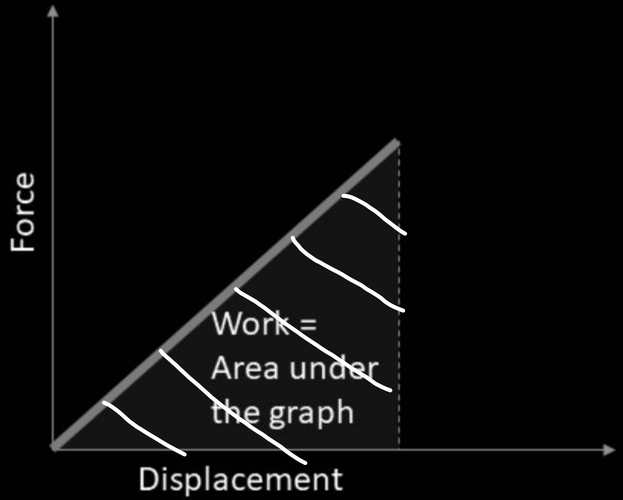
$$= a\int_0^L x\, dx + b\int_0^L x^2\, dx$$

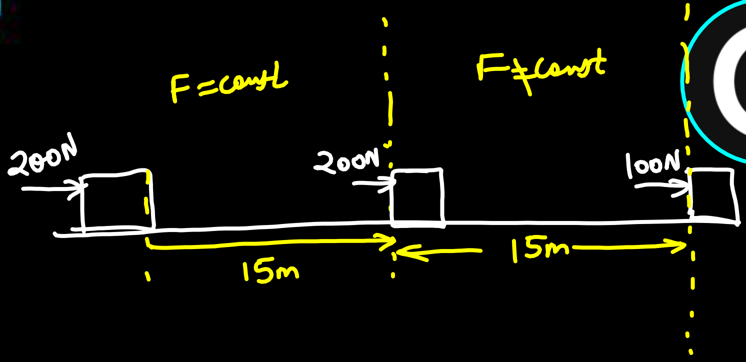
$$= \frac{aL^2}{2} + \frac{bL^3}{3}$$

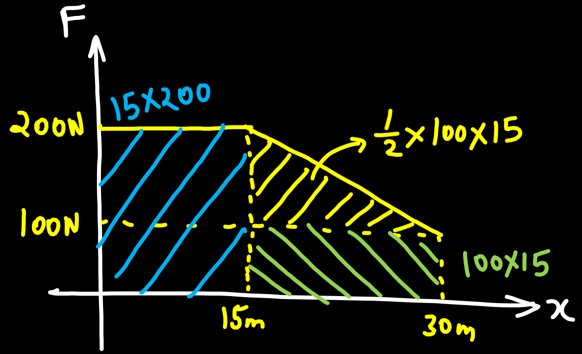
Force Displacement Graph

$$W = \int \vec{F}\, d\vec{x}$$

$$\boxed{W = \text{Area under } F\,x \text{ curve}}$$







A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box ?

(a) 3280 J (b) 2780 J (c) 5690 J (d) 5250 J

[JEE Main 2020]

$W = 3000 + 1500 + 750$

$W = 5250\,J$

Work by 3D/2D Force

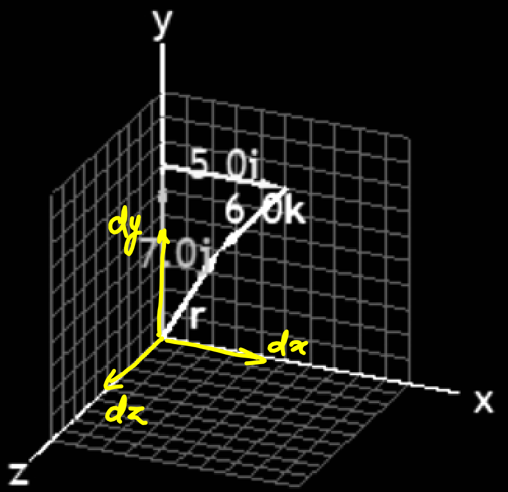
$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{dr} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\boxed{W = \int \vec{F}\,\vec{dr}}$$

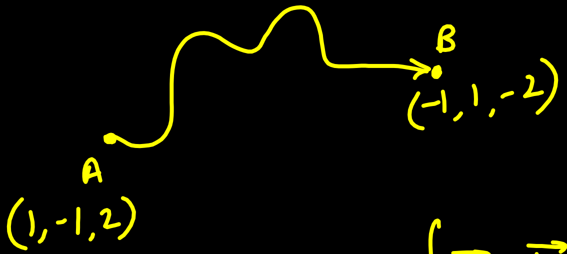
$$= \int (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$= \int F_x\,dx + \int F_y\,dy + \int F_z\,dz$$



A force F = (2x Î + 4yĴ - 3z k) is acted upon a particle which moves from (1,-1,2) to (-1,1,-2). Find the work done on this particle by the force.

(a) 0 (b) +3 J (c) -3 J (d) 3/2 J



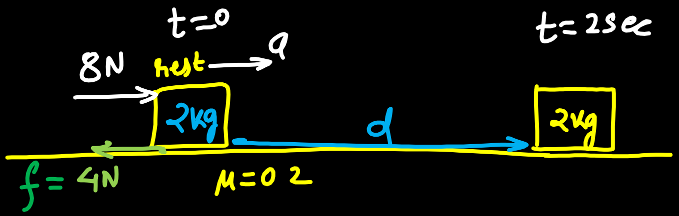
$$W = \int \vec{F} \cdot \vec{dr} = \int (2x\hat{i} + 4y\hat{j} - 3z\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= 2\int_{1}^{-1} x\,dx + 4\int_{-1}^{1} y\,dy - 3\int_{2}^{-2} z\,dz$$

$$= \frac{2}{2}x^2\Big|_{1}^{-1} + \frac{4}{2}y^2\Big|_{-1}^{1} - \frac{3}{2}z^2\Big|_{2}^{-2}$$

$$= 0$$

Work Done by Friction



Work done by the friction = ?

$(f_{max}) = \mu N = 0.2 \times mg = 0.2 \times 20 = 4N$

$a = \frac{8-4}{2} = 2\, m/s^2$

$s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2}(2)(2)^2$

$\boxed{S = 4m}$

$\text{Work} = \vec{F} \cdot \vec{d}$

$= f\, d \cos 180^\circ$

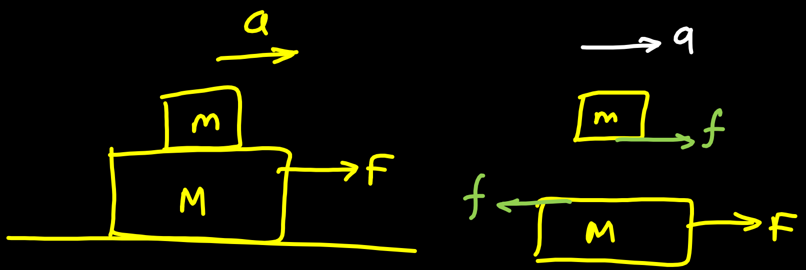
$= (4)(4)(-1)$

$\boxed{W = -16\ J}$ ⇒ heat generated

Work done by friction can be

A) Positive
B) Negative
C) Zero
D) All of the above

Work Done by Friction



Work Done by friction on

$m \Rightarrow +ve$

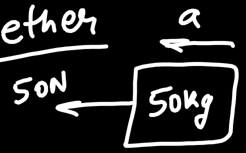
$M \Rightarrow -ve$

System $\Rightarrow 0$

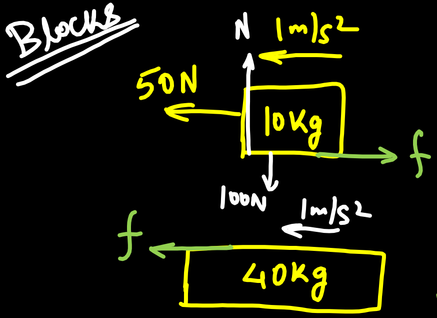
Q

A 40 kg slab rests on a frictionless floor. A 10 kg block rests on top of the slab as shown in the figure. The coefficient of static friction between the block and slab is 0.60 and coefficient of kinetic friction is 0.40. The 10 kg block is acted upon by a horizontal force of 50 N. Find the work done by the friction acting on the slab in 2 sec. (take g = 10 m/s²).

(a) 40 J ✓(b) 80 J (c) 120 J (d) 160 J



$a = 1\, m/s^2$

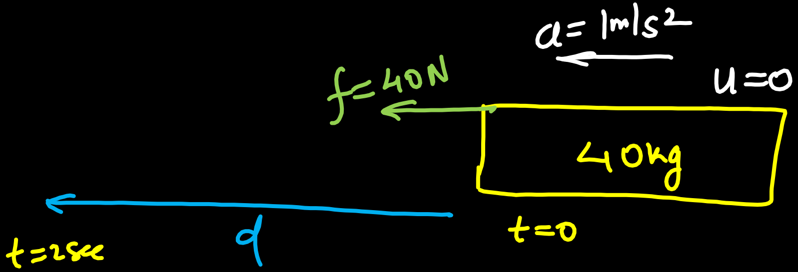


$f = ma = 40 \times 1$

$f = 40N$

$(f_s)_{max} = \mu_s N = (0.6)(100) = 60N$

$a_{max} = \frac{(f_s)_{max.}}{40} = \frac{60}{40} = 1.5\, m/s^2$



$$d = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2} \times 1 \times (2)^2$$

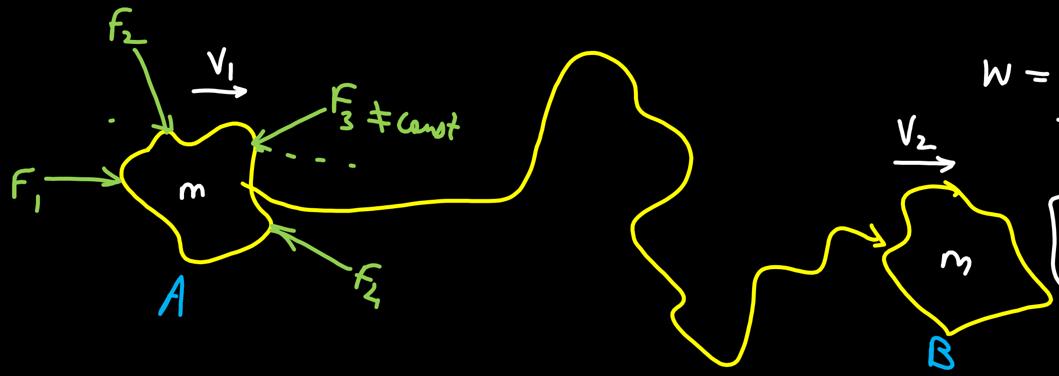
$$d = 2m$$

$$\text{Work} = fd = 40 \times 2 = 80\,J$$

Work Energy Theorem

Work ≡ Energy

Work Done by all the forces = Change in KE



$$W = \int \vec{F_1} \cdot \vec{dr} + \int \vec{F_2} \cdot \vec{dr} + \_ \_ \_ \_$$

$\_ \_ \_ \_$ 1000 times

$$\boxed{W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2}$$

Work Energy Theorem

A particle of mass 500 gm is moving in a straight line with velocity $\upsilon = b\,x^{5/2}$. The work done by the net force during its displacement from x = 0 to x = 4 m is : (Take $b = 0.25\ m^{-3/2}\ s^{-1}$).

(A) 2 J

(B) 4 J

(C) 8 J

(D) 16 J

[JEE Main 2022]

$x = 0$

$v = bx^{5/2}$

$v_1 = 0$

$x = 4\,m$

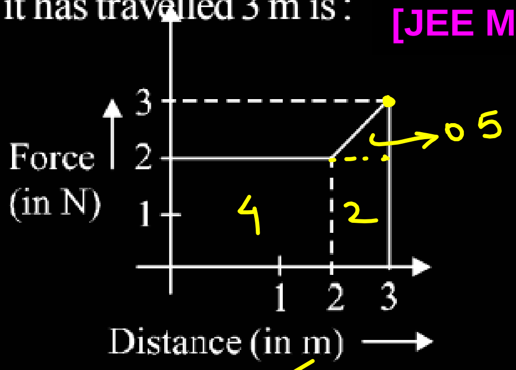
$v = bx^{5/2}$

$v_2 = \left(\frac{1}{4}\right)(4)^{5/2}$

$v_2 = \frac{2^5}{2^2} = 8\ m/s$

$Work = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2} \times 0.5 \times (8)^2 = \frac{64}{4} = 16\,J$

A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is : **[JEE Main 2019]**



(a) 4 J (b) 2.5 J (c) 6.5 J (d) 5 J

Work = 6.5 J

ΔKE = 6.5 J

$KE_f - 0 = 6.5$

$KE_f = 6.5$ J

Q

A body of mass starts moving from rest along x-axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant s and is the distance covered by the body. The total work done by all the forces acting on the body in the first second after the start of the motion is: [JEE Main 2018]

(a) $\frac{1}{8}ma^4t^2$ (b) $4ma^4t^2$ (c) $8ma^4t^2$ (d) $\frac{1}{4}ma^4t^2$

t=0 t=1 sec

$v = a\sqrt{s}$

$\frac{ds}{dt} = a\sqrt{s}$

$\int_0^x \frac{ds}{\sqrt{s}} = a\int_0^t dt$

$\left.\frac{s^{-1/2+1}}{-1/2+1}\right|_0^x = at$

$2\sqrt{x} = at$

$x = \frac{a^2t^2}{4}$

$v = \frac{dx}{dt} = \frac{a^2}{4}(2t) = \frac{a^2t}{2}$

$v = \frac{a^2t}{2}$

$t = 0$ $\quad\quad$ $t = 1$

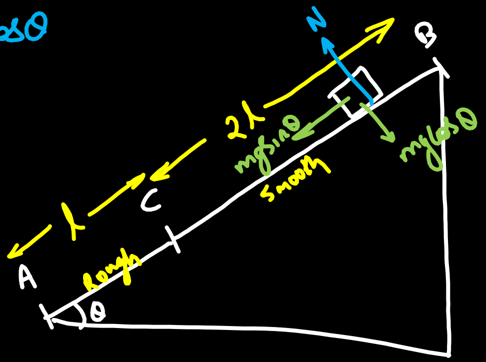
$v_1 = 0$ $\quad\quad$ $V_2 = a^2/2$

$$\text{Work} = \frac{1}{2}m\left(V_2^2 - V_1^2\right) = \frac{1}{2}m\frac{a^4}{4} = \frac{ma^4}{8}$$

Q

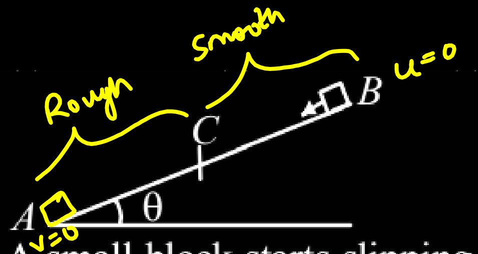
$W_{total} = \Delta KE = 0$

$N = mg\cos\theta$



$W_{gravity} = (mg\sin\theta)(3l)$

$W_{friction} = -f(l)$



A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If $BC = 2AC$, the coefficient of friction is given by $\mu = k\tan\theta$. The value of k is **[JEE Main 2020]**

(a) 1 (b) 2 (c) 3 ✓ (d) 4

$(mg\sin\theta)(3l) + (-fl) = 0$

$3mg\sin\theta - \mu N = 0$

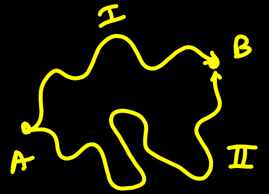
$3mg\sin\theta - \mu(mg\cos\theta) = 0$

$3\tan\theta = \mu$

Conservative & Non Conservative Forces

Conservative (Gravity, spring etc)

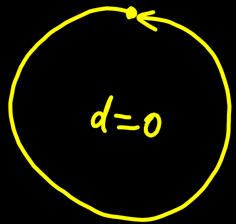
1) Work done is path independent



$W_I = W_{II}$

W = force × displacement

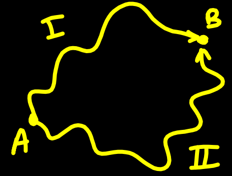
2)



$W = 0$

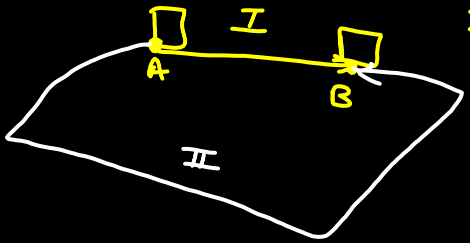
Non conservative (friction, Resistive Forces (Induced Electric Field))

1) work is path dependent

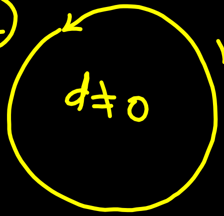


$W_I \neq W_{II}$

W = Force × distance



2)

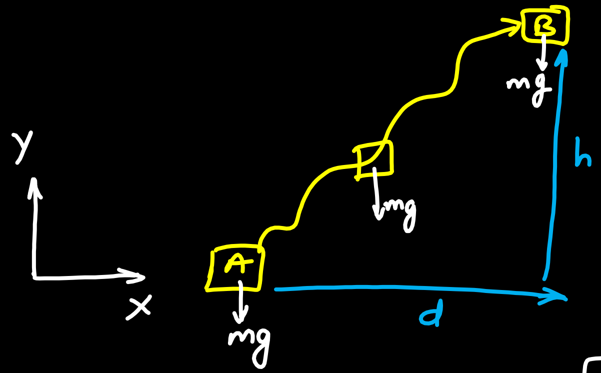


$W \neq 0$

Conservative & Non Conservative Forces

Potential Energy

⇒ hidden Energy



$$\vec{F} = mg(-\hat{j})$$

$$\vec{d} = d\hat{i} + h\hat{j}$$

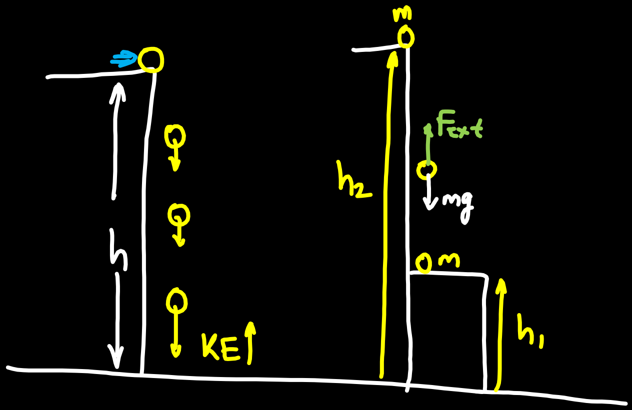
$$W_g = \vec{F}\cdot\vec{d} = (-mg\hat{j})\cdot(d\hat{i} + h\hat{j})$$

$$\boxed{W_g = -mgh}$$

$$\boxed{PE = -W_g = +mgh}$$

PE ⇒ Height change of COM

Potential Energy



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$$W_{ext} + W_g = 0$$

$$\boxed{W_{ext} = -W_g = \Delta PE}$$

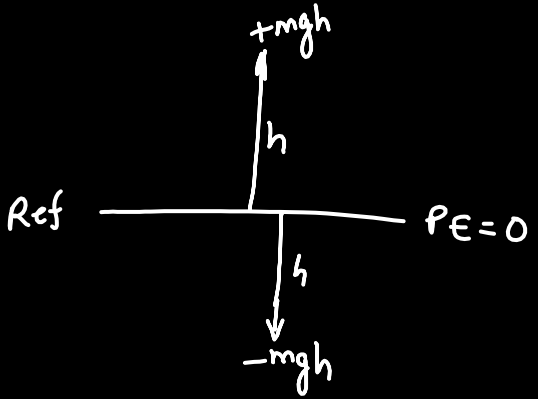
$$W_{ext} = F_{ext}(h_2 - h_1) = mg(h_2 - h_1) = \Delta PE$$

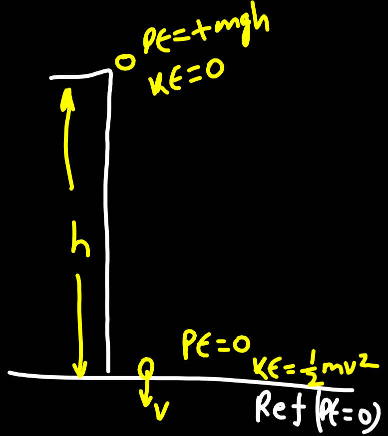
$$W_g = -mg(h_2 - h_1) = -\Delta PE$$

Potential Energy

Reference of PE ⇒ **P.E = 0**

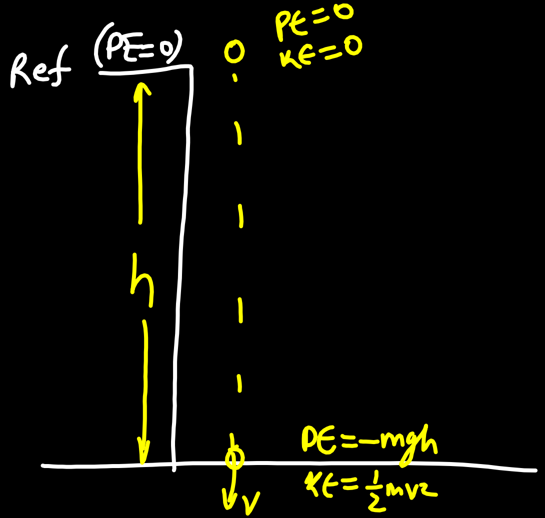
(free to choose anywhere)





$$mgh = \frac{1}{2}mv^2$$

$$\boxed{v = \sqrt{2gh}}$$



$$\frac{1}{2}mv^2 + (-mgh) = 0$$

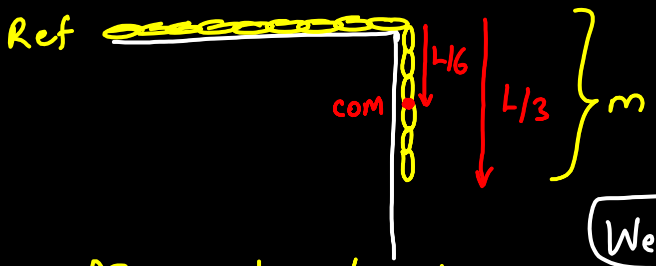
$$\boxed{v = \sqrt{2gh}}$$

Total mass = M

Q

A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is [1985 - 2 Marks]

(a) MgL (b) $MgL/3$ (c) $MgL/9$ (d) $MgL/18$



$$PE = -mgh = -\left(\frac{M}{3}\right)g\left(\frac{L}{6}\right)$$

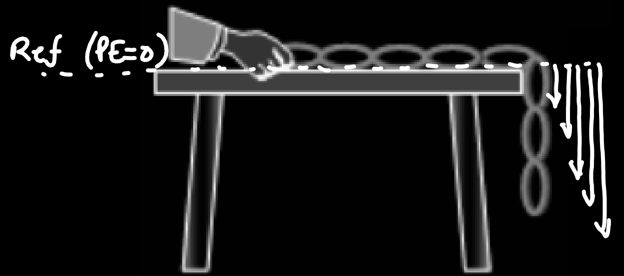
$$PE_1 = -\frac{MgL}{18}$$

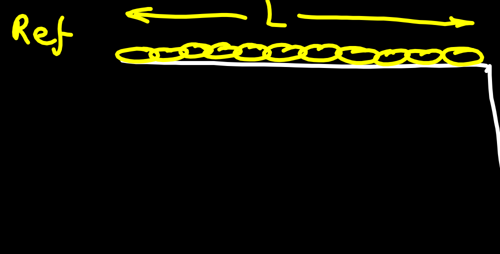
$$W_{ext} = \Delta PE$$

$$W_{ext} = PE_2 - PE_1$$

$$= 0 - \left(-\frac{MgL}{18}\right)$$

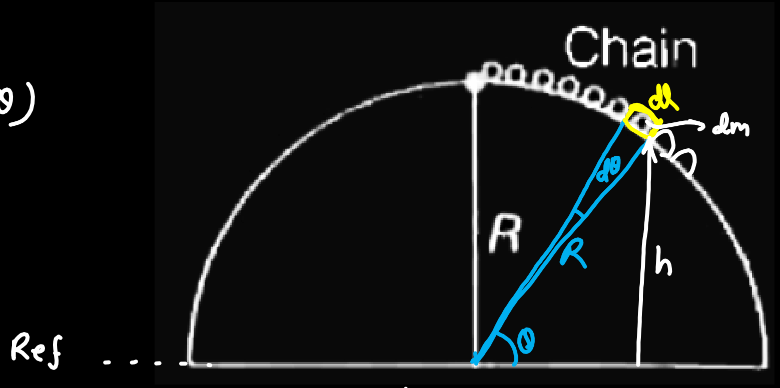
$$W_{ext} = \frac{MgL}{18}$$





$PE_2 = 0$

FInd the potential of a chain of mass m and length l lying on a hemisphere of Radius R as shown in the figure. (GIven that l < πr and the reference for potential energy is the base of the hemisphere)



$\sin\theta = h/R$

$dl = R\,d\theta$

Mass per unit Length $= \frac{m}{l}$

$dm = \left(\frac{m}{l}\right) dl$

$dU = (dm)(g)(h)$

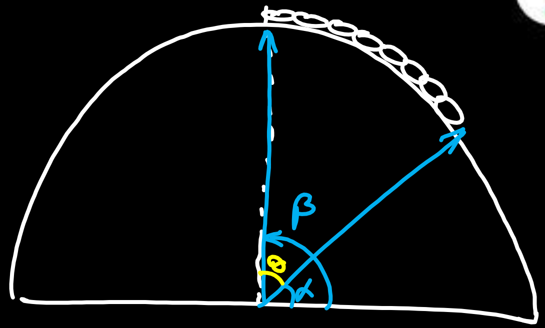
$dU = \left(\frac{m}{l} dl\right) g (R\sin\theta)$

$dU = \frac{mgR}{l}(R\,d\theta)\sin\theta$

$\int dU = \frac{mgR^2}{l}\int_\alpha^\beta \sin\theta\, d\theta$

$U = \frac{mgR^2}{l}\left[-\cos\theta\right]_\alpha^\beta$

$$U = \frac{mgR^2}{l}(\cos\alpha - \cos\beta)$$



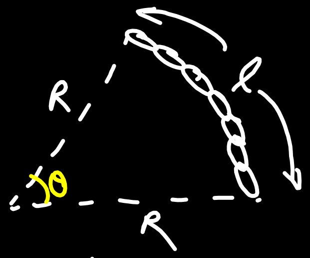
$\beta = 90^\circ$

$$U = \frac{mgR^2}{l}\left(\cos(90-\theta) - \cos 90^\circ\right)$$

$\alpha + \theta = \beta$

$\alpha + \theta = 90^\circ$

$$U = \frac{mgR^2}{l}\sin\theta$$



$$U = \frac{mgR^2}{l}\sin\left(\frac{l}{R}\right)$$

$\theta = l/R$

Types of Equilibrium

Related to PE

$$W_{ext} = \Delta PE$$

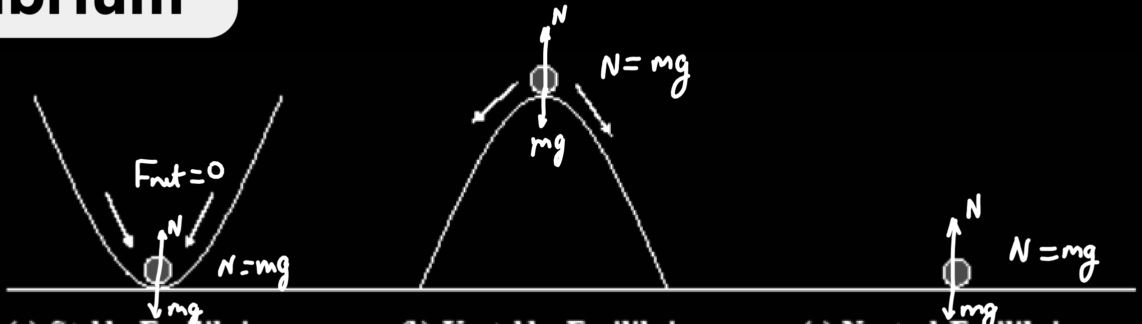
$$W_c = -\Delta PE$$

$$\int \vec{F}_c \cdot d\vec{r} = -\Delta U$$

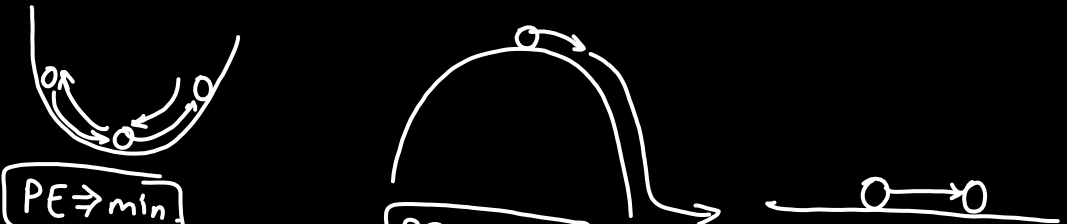
$$\vec{F}_c \cdot d\vec{r} = -dU$$

$$F_c = -\frac{dU}{dr}$$

$$\vec{F}_c = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)$$



(a) Stable Equilibrium (b) Unstable Equilibrium (c) Neutral Equilibrium



PE ⇒ min

$F_{net} = 0$

PE ⇒ max

$F_{net} = 0$

PE = const

$F_{net} = 0$

Q) $U = -x^2y + xyz + zy^2$

$\vec{F} = ?$

$$\frac{\partial U}{\partial x} = (-2x)y + (1)yz + 0$$

$$\frac{\partial U}{\partial y} = -x^2(1) + xz + z(2y)$$

$$\frac{\partial U}{\partial z} = 0 + xy(1) + (1)y^2$$

$$\vec{F}_c = -\left[(yz - 2xy)\hat{i} + (2zy + xz - x^2)\hat{j} + (xy + y^2)\hat{k}\right]$$

Potential energy as a function of r is given by $U = \frac{A}{r^{10}} - \frac{B}{r^5}$, where r is the interatomic distance, A and B are positive constants. The equilibrium distance between the two atoms will be :

(A) $\left(\frac{A}{B}\right)^{\frac{1}{5}}$

(B) $\left(\frac{B}{A}\right)^{\frac{1}{5}}$

(C) $\left(\frac{2A}{B}\right)^{\frac{1}{5}}$ ✓

(D) $\left(\frac{B}{2A}\right)^{\frac{1}{5}}$

[JEE Main 2022]

Equilibrium ⇒ $F = 0$

$$F = -\frac{dU}{dr} = 0$$

$$-\frac{d}{dr}\left(Ar^{-10} - Br^{-5}\right) = 0$$

$$-10Ar^{-11} + 5Br^{-6} = 0$$

$$Br^{-6} = 2Ar^{-11}$$

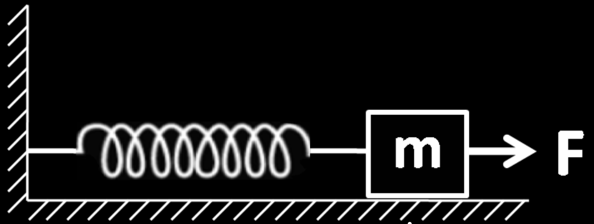
$$Br^5 = 2A$$

$$r = \left(\frac{2A}{B}\right)^{1/5}$$

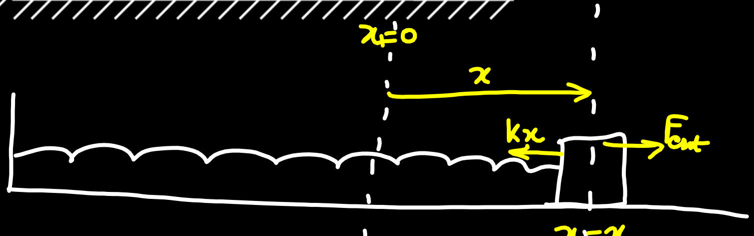
Spring's Energy

↳ Elastic PE.

$$W_{ext} = \text{change in Elastic PE } (\Delta E_s)$$



$$W_{ext} = \int_{x_1}^{x_2} \vec{F}_{ext} \cdot \vec{dx} = \int_{x_1}^{x_2} kx \, dx \cos 0°$$



$$|\vec{F}_{ext}| = kx$$

$$\Delta E_{spring} = k\frac{x^2}{2}\Big|_{x_1}^{x_2} = \frac{1}{2}k(x_2^2 - x_1^2)$$

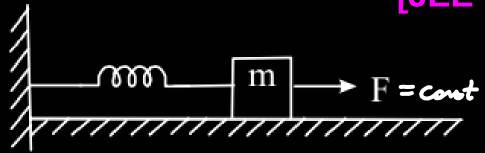
$$\boxed{E_s = \frac{1}{2}kx^2}$$ ⇒ For both Elongation & compression

Q

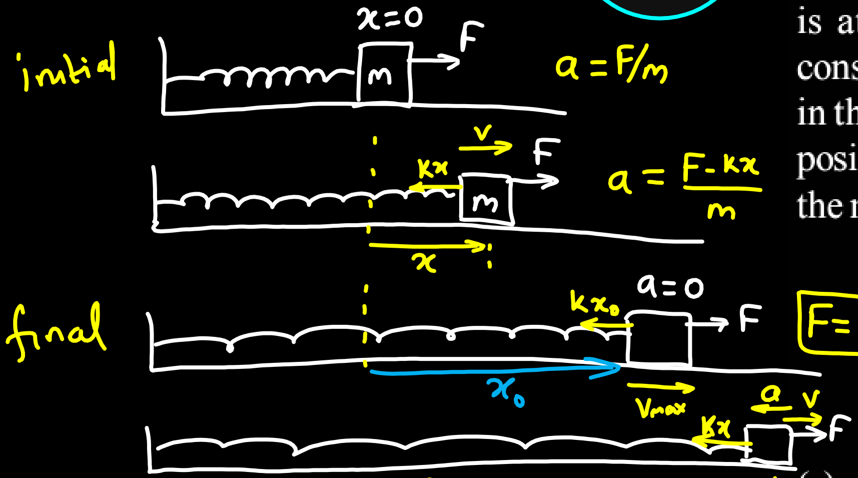
Unacademy Atoms

A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is:

[JEE Main 2019]



(a) $\frac{2F}{\sqrt{mk}}$ (b) $\frac{F}{\pi\sqrt{mk}}$ (c) $\frac{\pi F}{\sqrt{mk}}$ (d) $\frac{F}{\sqrt{mk}}$ ✓



$W_{ext} = KE + E_s$

$Fx_0 = \frac{1}{2}mV_{max}^2 + \frac{1}{2}Kx_0^2$

$2F\left(F/k\right) = mV_{max}^2 + k\left(\frac{F^2}{k^2}\right)$

$$\frac{F^2}{K} = mV_{max}^2$$

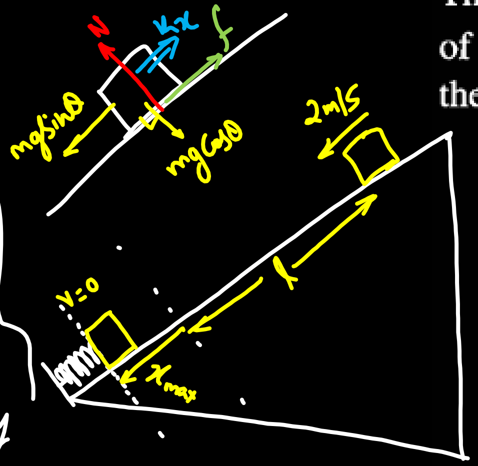
$$V_{max.} = \frac{F}{\sqrt{mK}}$$

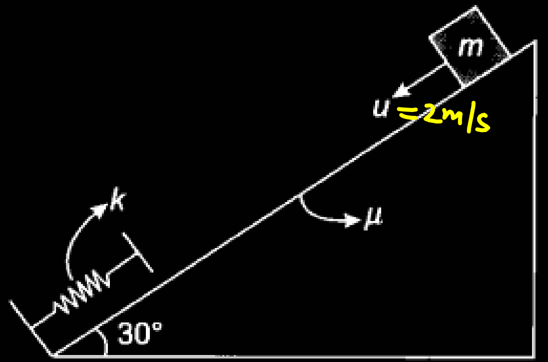
$W_{total} = \Delta KE.$

$W_f + W_g + W_{spring} = \Delta KE$

$-f(l+x) + (mg\sin\theta)(l+x) - \frac{1}{2}kx^2 = 0 - \frac{1}{2}mu^2$

A block of mass 1 kg is released from top of a rough incline having $\mu = \frac{1}{\sqrt{3}}$. The initial speed of block is 2 m/s. The incline plane is of unknown length and has a spring of constant $k = 1$ N/m connected at base as in figure. Find the maximum compression of spring (answer in meter).





Alternate way

|Loss| = |Gain|

$\frac{1}{2}mv^2 + mgh =$ Heat + Spring Energy

$mg(l+x)\sin 30° + \frac{1}{2}mv^2 = \mu N(l+x) + \frac{1}{2}kx^2$

$$(l+x)(mg\sin\theta - \mu N) - \frac{1}{2}kx^2 = -\frac{1}{2}mu^2$$

$$(l+x)\left(10\times\frac{1}{2} - \frac{1}{\sqrt{3}}\times 10\times\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\times 1\times x^2 = -\frac{1}{2}\times 1\times (2)^2$$

$$(l+x)(5-5) - \frac{x^2}{2} = -2$$

$$-\frac{x^2}{2} = -2$$

$$x^2 = 4$$

$$\boxed{x = 2m}$$

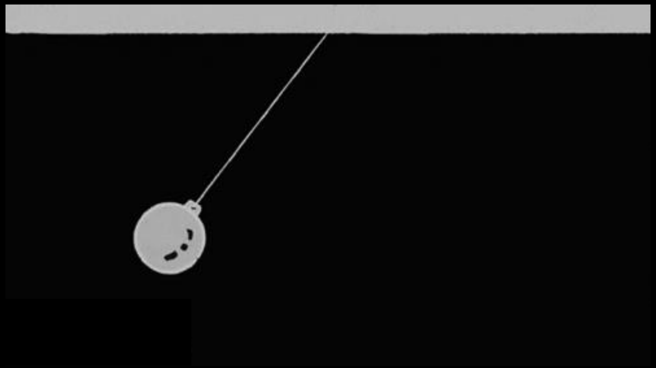
Mechanical Energy Conservation

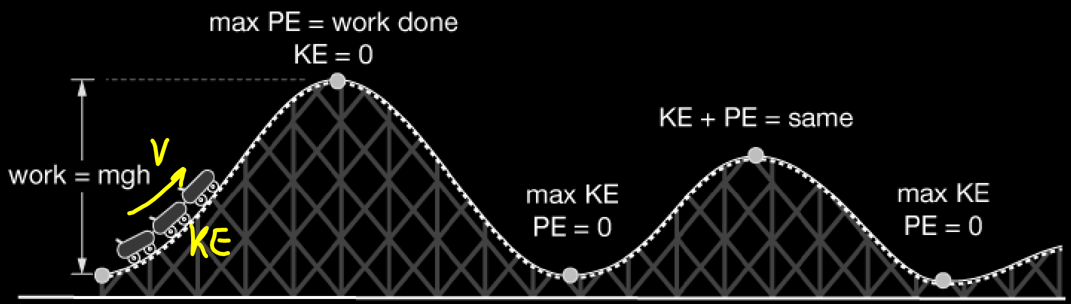
$ME = PE + KE$

For $ME = const$

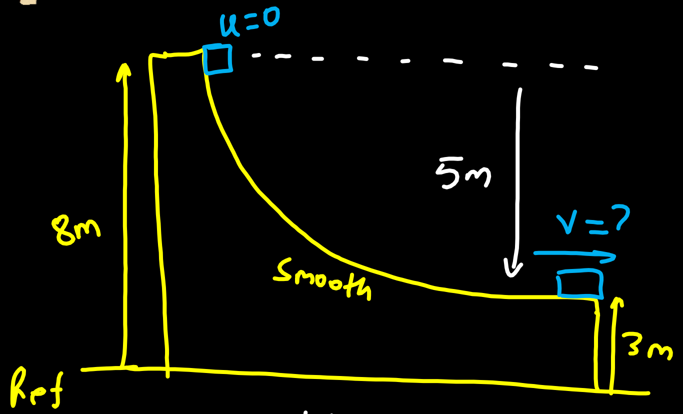
$\Rightarrow W_{friction} = 0$

$\Rightarrow W_{external} = 0$





Mechanical Energy Conservation



$|loss| = |Gain|$

$mg(5) = \frac{1}{2}mv^2$

$v = 10 m/s$

$PE_1 + KE_1 = PE_2 + KE_2$

$mg(8) + 0 = mg(3) + \frac{1}{2}mv^2$

$8g - 3g = \frac{1}{2}v^2$

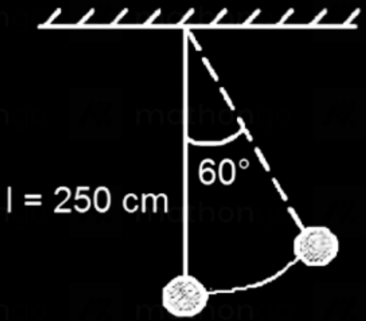
$10g = v^2$

$\sqrt{100} = v$

$\boxed{v = 10 m/s}$

Q

A pendulum is suspended by a string of length 250 cm. The mass of the bob of the pendulum is 200 g. The bob is pulled aside until the string is at 60° with vertical as shown in the figure. After releasing the bob. the maximum velocity attained by the bob will be ________ ms^{-1}. (if $g = 10\ m/s^2$)



[JEE Main 2022]

$mgh = \frac{1}{2}mv^2$

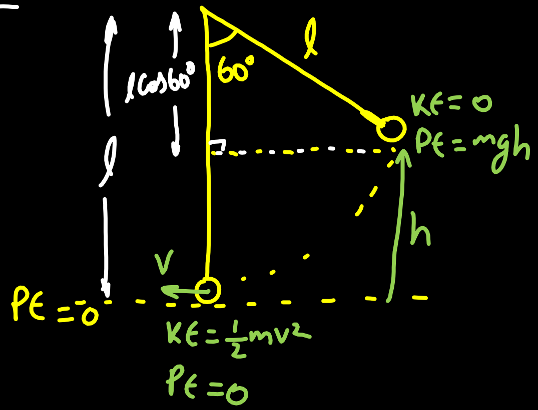
$v = \sqrt{2gh}$

$v = \sqrt{2g(l - l/2)}$

$v = \sqrt{gl}$

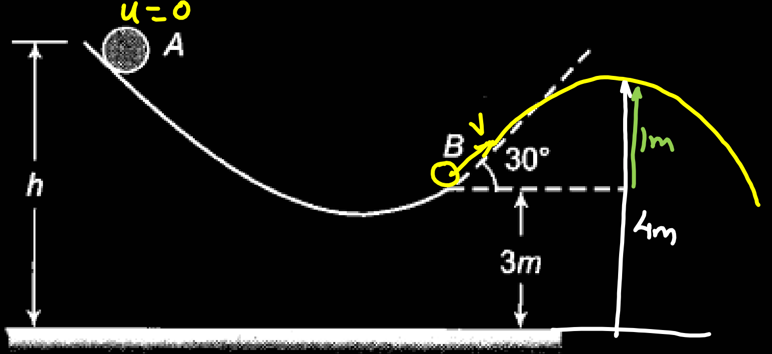
$v = \sqrt{10\times 25}$

$v = 5\ m/s$



Q

A ball released at A leaves the frictionless track at B, which is at a height of 3 m from the ground. The ball further rises maximum up to 4 m above the ground before falling down. Find h (in m) if the track at B makes an angle of 30° with the horizontal.



$H_{max.} = 1$

$\frac{v^2 \sin^2\theta}{2g} = 1$

$v^2 (1/2)^2 = 20$

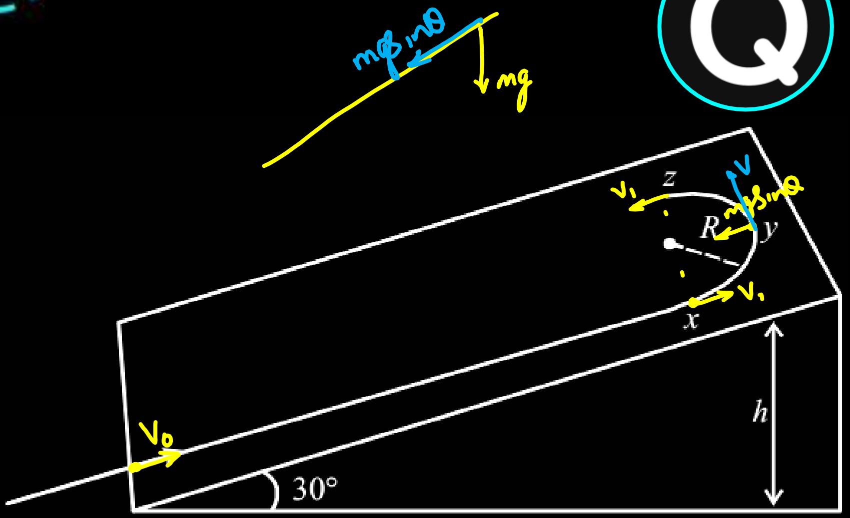
$v = \sqrt{80}\ m/s$

$\cancel{m}gh = \cancel{m}g(3) + \frac{1}{2}\cancel{m}v^2$

$gh = 30 + \frac{80}{2}$

$10h = 70$

$h = 7m$



Q

A student skates up a ramp that makes an angle 30° with the horizontal. He/she starts (as shown in the figure) at the bottom of the ramp with speed v_0 and wants to turn around over a semicircular path xyz of radius R during which he/she reaches a maximum height h (at point y) from the ground as shown in the figure. Assume that the energy loss is negligible and the force required for this turn at the highest point is provided by his/her weight only. Then (g is the acceleration due to gravity)

[JEE Advanced 2020]

(a) $v_0^2 - 2gh = \frac{1}{2} gR$

(b) $v_0^2 - 2gh = \frac{\sqrt{3}}{2} gR$

(c) the centripetal force required at points x and z is zero

(d) the centripetal force required is maximum at points x and z

At y

$mg\sin\theta = \frac{mv^2}{R}$

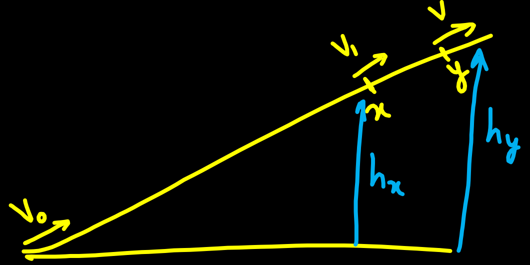
$V = \sqrt{gR\sin\theta}$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgh$$

$$v_0^2 = v^2 + 2gh$$

$$v_0^2 = gR\sin\theta + 2gh$$

$$\boxed{v_0^2 = \frac{gR}{2} + 2gh}$$



(d) $F_c = \frac{mv^2}{R}$

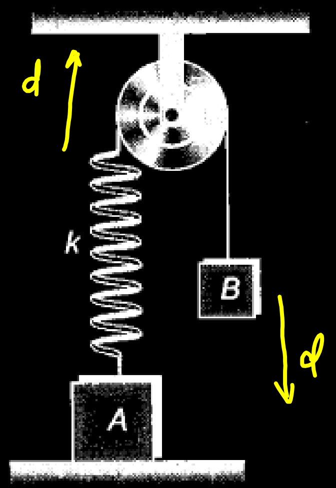
$$\frac{1}{2}mv_0^2 = mgh\downarrow + \frac{1}{2}mv^2\uparrow$$

$$v < v_1$$

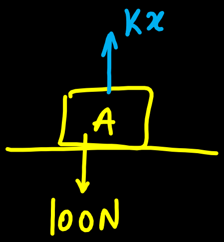
Very Good Question

For the situation shown in the figure, block A of mass 10 kg is resting on the ground. The block B is released from rest when the light spring is unstretched. If the extension in the spring when the block A just lifts off the ground is x, then find the velocity (in m/s) of block B when the extension in the spring is x/2. (k= 50 N/m)

(a) $\sqrt{10}$ (b) $\sqrt{15}$ (c) $\sqrt{20}$ (d) $\sqrt{25}$



Just lift A (N=0)



$kx = 100$

$(50)x = 100$

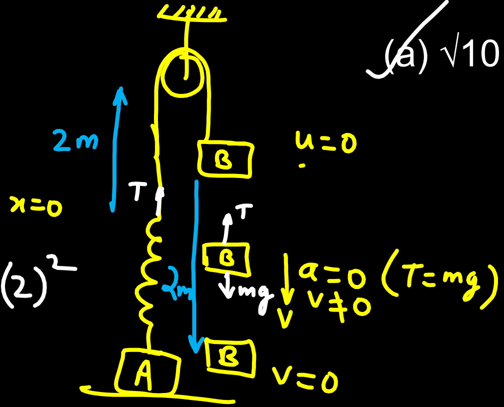
$x = 2m$

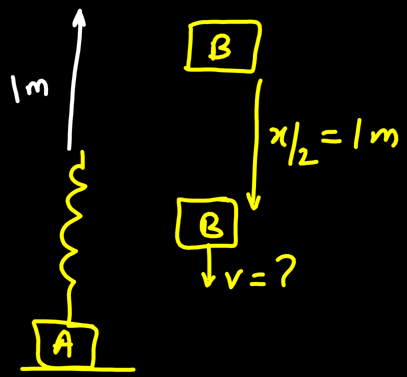
|loss| = |Gain|

$mgh = \frac{1}{2}kx^2$

$m \times 10 \times 2 = \frac{1}{2} \times 50 \times (2)^2$

$m = 5\,kg$





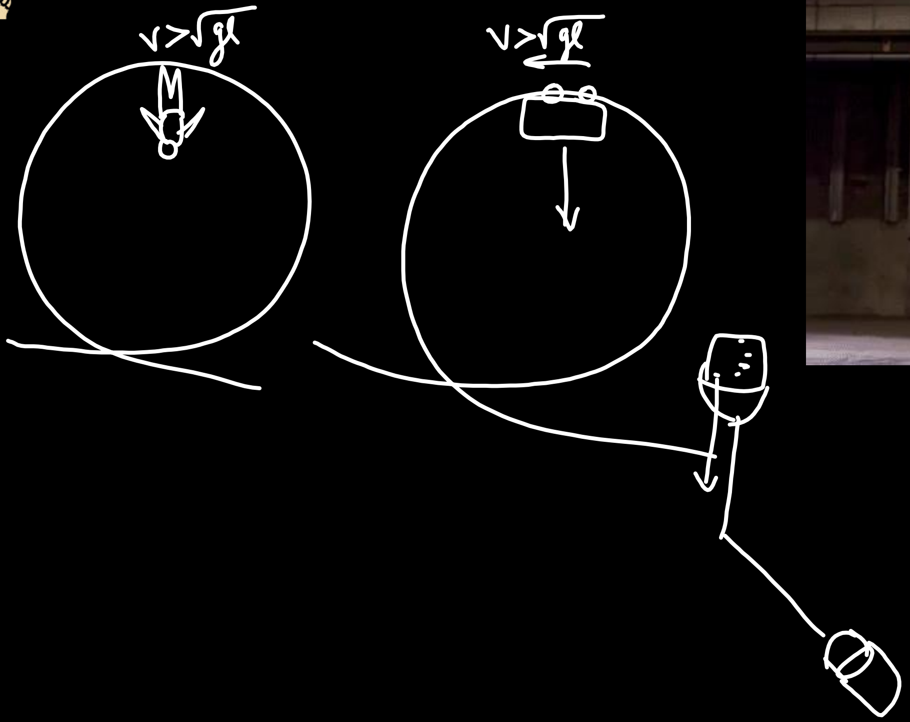
$$|loss| = |gain|$$

$$mg(1) = \frac{1}{2}K(1)^2 + \frac{1}{2}mv^2$$

$$50 = \frac{50}{2} + \frac{5}{2}v^2$$

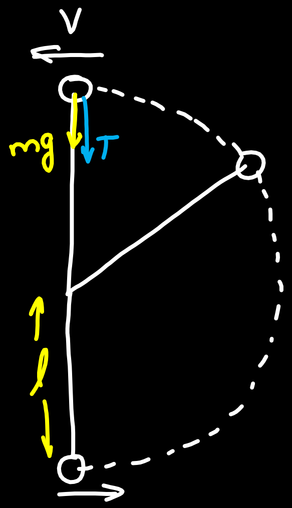
$$\boxed{V = \sqrt{10}\ m/s}$$

Vertical Circular Motion

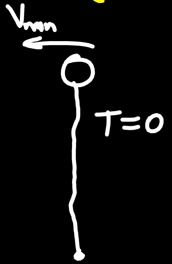


Vertical Circular Motion

$$mg + T = \frac{mv^2}{l}$$



Minimum Case
(Just saved)



$$mg = \frac{mV_{min}^2}{l}$$

$$\boxed{V_{min} = \sqrt{gl}} \Rightarrow \text{Topmost Point}$$

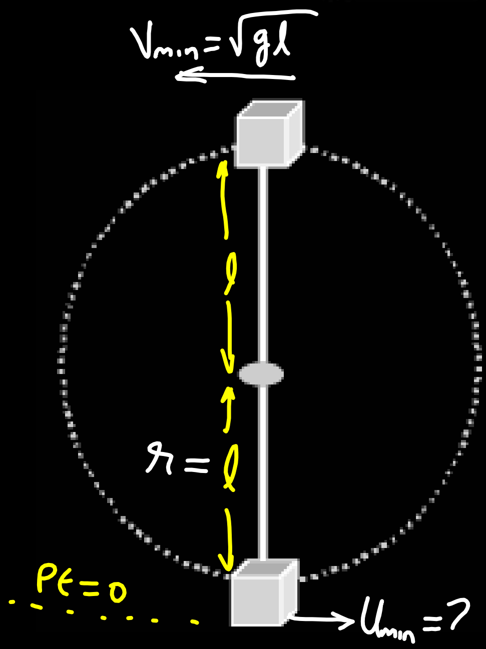
Vertical Circular Motion

$$\frac{1}{2}mU_{min}^2 = \frac{1}{2}mV_{min}^2 + mg(2l)$$

$$U_{min}^2 = V_{min}^2 + 4gl$$

$$U_{min}^2 = gl + 4gl$$

$$\boxed{U_{min} = \sqrt{5gl}}$$



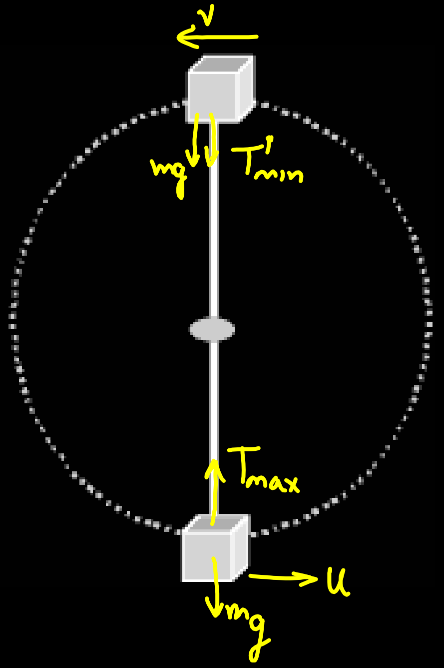
Vertical Circular Motion

$$T' + mg = \frac{mv^2}{r}$$

$$T' = \frac{mv^2}{r} - mg$$

$$T - mg = \frac{mu^2}{r}$$

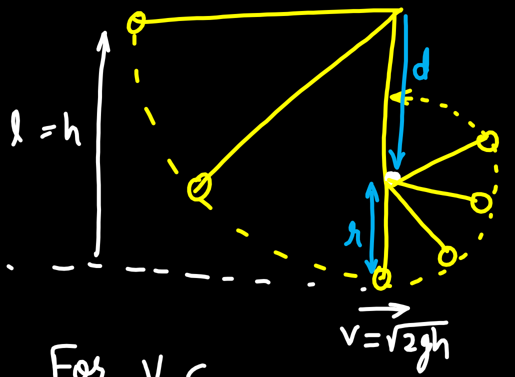
$$T = mg + \frac{mu^2}{r}$$

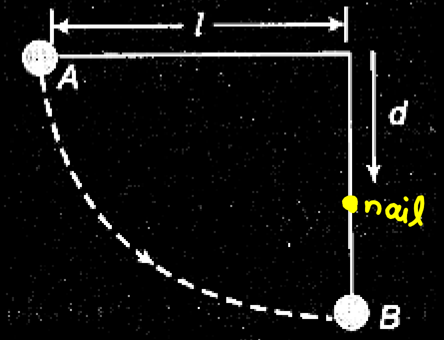


Q

A pendulum bob connected with a string of length l=10 cm is released from the horizontal position. A nail is located 'd' distance below the point of suspension. Find the value of d so that the bob swing completely around a circle centred along the nail.

(a) 3 cm (b) 4 cm (c) 6 cm (d) 8





$l = r + d$

$r = l - d$

For V C

$V \geq \sqrt{5gr} = \sqrt{5g(l-d)}$

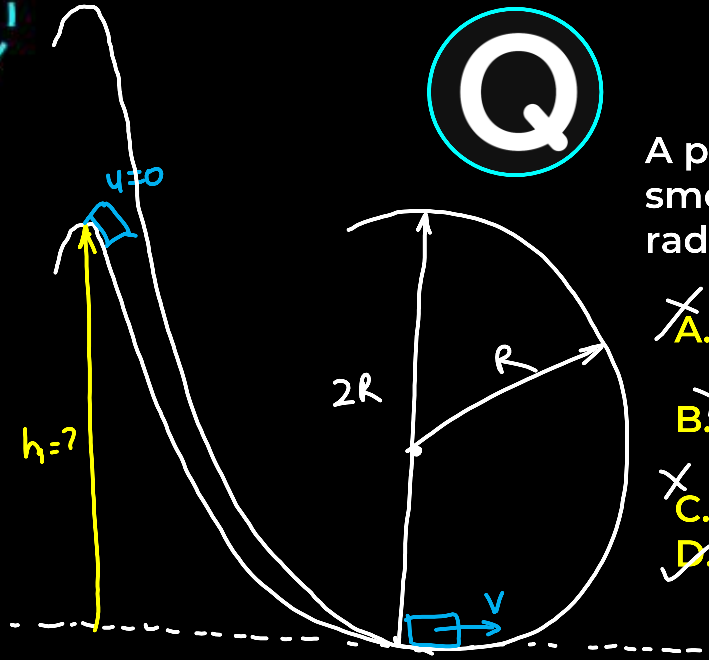
$\sqrt{2gh} = \sqrt{5g(l-d)}$

$2gl = 5g(l-d)$

$d = \frac{3}{5} l$

$d = \frac{3}{5}(10)$

$d = 6\,cm$



Q

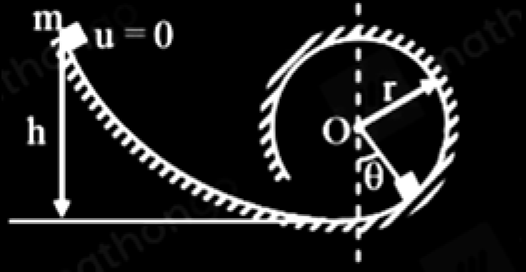
A particle of mass m is released from height h on a smooth curved surface which ends into a vertical loop of radius r, as shown. If h = 2r, then

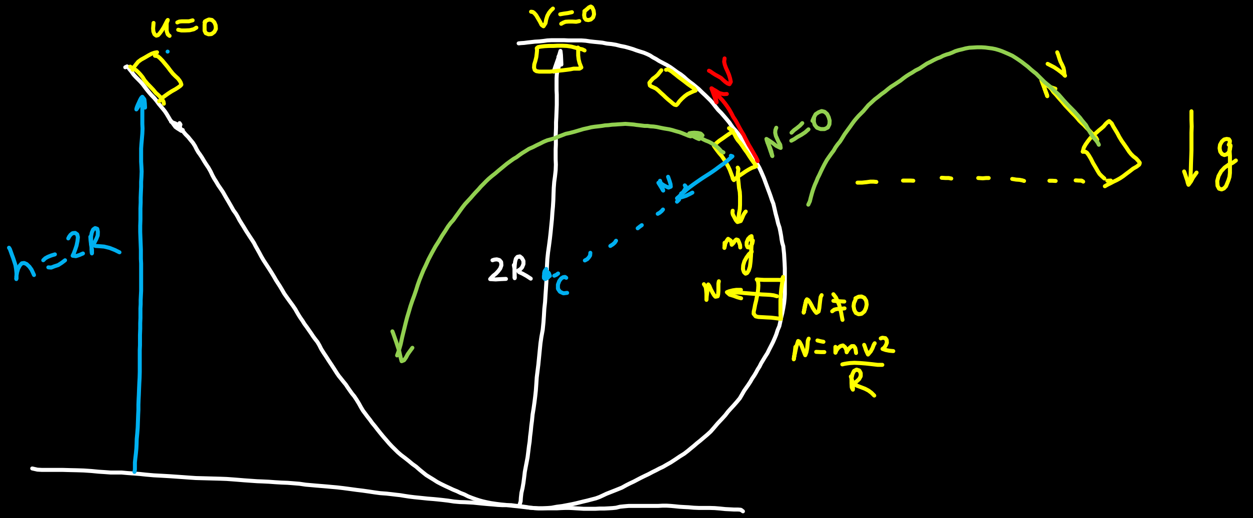
A. The particle reaches the top of the loop with zero velocity
B. The particle reaches the top of the loop with a non-zero velocity
C. The particle breaks off at a height h = r from base
D. The particle breaks off at a height r < h < 2r

$V_{min} = \sqrt{5gR}$

$\sqrt{2gh} = \sqrt{5gR}$

$h_{min} = \frac{5}{2}R = 2.5R$

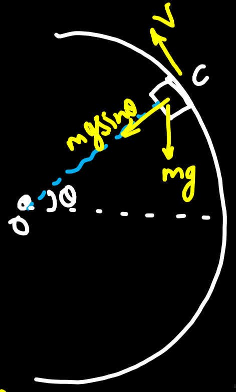


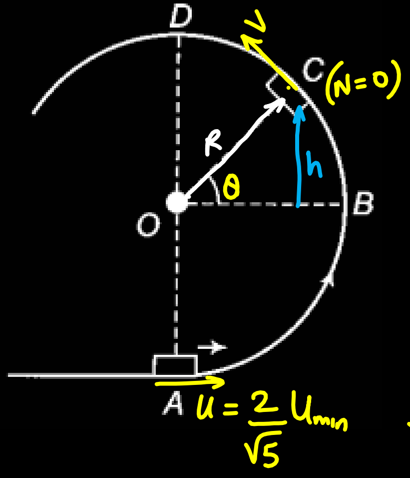


u=0
v=0
V
N=0
V
g
h=2R
N
mg
2R
C
N
N≠0
N=mv2/R

Q

$\sin\theta = h/R$





At C

N = 0

$$mg\sin\theta = \frac{mv^2}{R}$$

$$v = \sqrt{gR\sin\theta}$$

A small block of mass m is pushed on a smooth track from position A with a velocity $\frac{2}{\sqrt{5}}$ times the minimum velocity required to reach point D. The block will leave the contact with track at the point where normal force between them becomes zero.

At what angle θ with horizontal does the block gets separated from the track?

(a) $\sin^{-1}(1/3)$

(b) $\sin^{-1}(3/4)$

(c) $\sin^{-1}(2/3)$

(d) never leaves contact with the track

$$(TE)_A = (TE)_C$$

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(R+h)$$

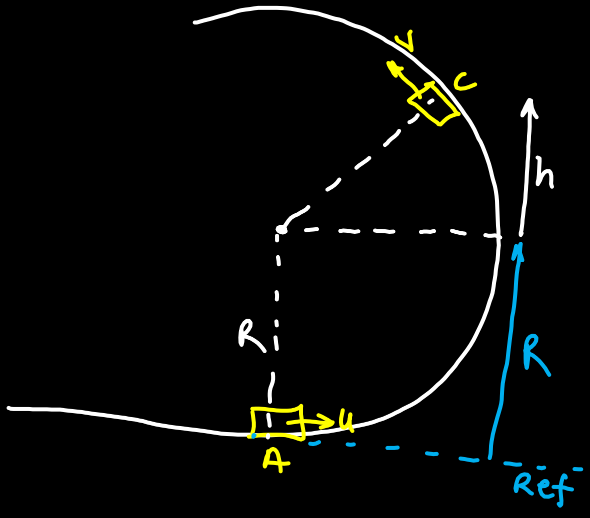
$$u^2 = v^2 + 2g(R + R\sin\theta)$$

$$\left(\frac{2}{\sqrt{5}}u_{min}\right)^2 = \left(\sqrt{gR\sin\theta}\right)^2 + 2gR + 2gR\sin\theta$$

$$\frac{4}{5}(5gR) = gR\sin\theta + 2gR + 2gR\sin\theta$$

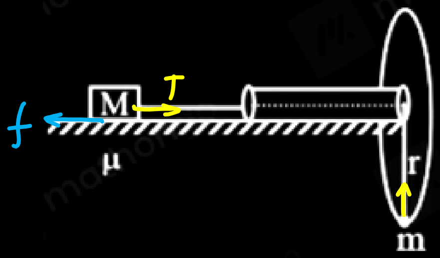
$$2gR = 3gR\sin\theta$$

$$\boxed{\sin\theta = \frac{2}{3}}$$



The figure below shows a block of mass M connected to an ideal string which passes through a thin fixed smooth pipe. On the other end, a particle of mass m is connected which revolves in a vertical circle of radius r. If the coefficient of friction between M and the surface is $\mu = 2/3$, then for what minimum value of M, the block of mass m can undergo complete vertical circular motion ?

A. $M_{min} = 6m$
B. $M_{min} = 9m$ ✓
C. $M_{min} = 3m$
D. $M_{min} = 15m$



$T_{max} = (f_s)_{max}$

$T_{max} = mg + \frac{m u_{min}^2}{r}$

$= mg + \frac{m}{r}(\sqrt{5gr})^2$

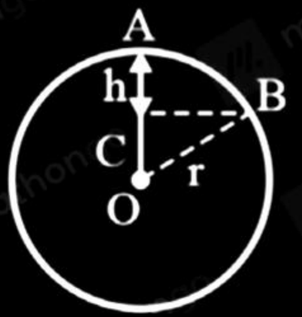
$T_{max} = 6mg$

$6mg = \mu N = \frac{2}{3}(Mg)$

$M = 9m$

HW Comment

In figure, a particle is placed at the highest point A of a smooth of radius r. It is given slight push, and it leaves the sphere at B, at a depth h vertically below A. The value of h is

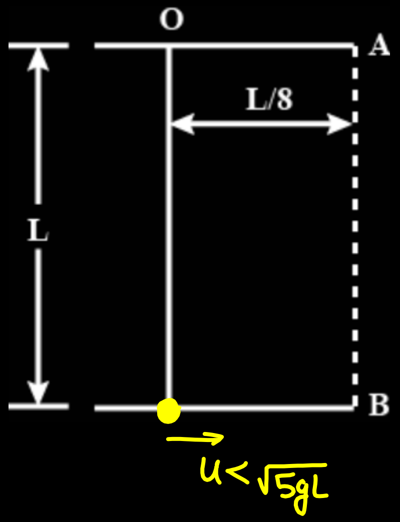


A. r/6
B. 1/4 r
C. 1/3 r
D. 1/2 r

Q

IPhO

A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance L/8 from O as shown in figure. The object is given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB. Its velocity is horizontal. Find u.



A $\sqrt{\left(4+\frac{3\sqrt{3}}{2}\right)Lg}$

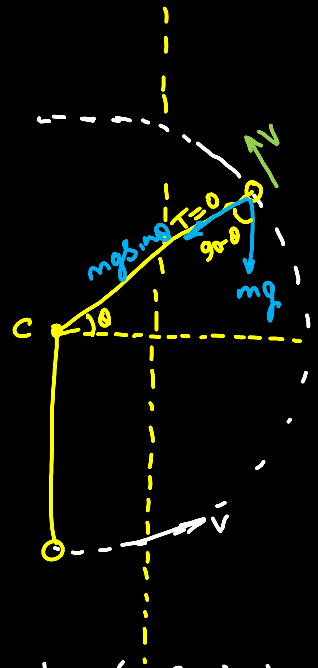
B $\sqrt{\left(12+\frac{3\sqrt{3}}{2}\right)Lg}$

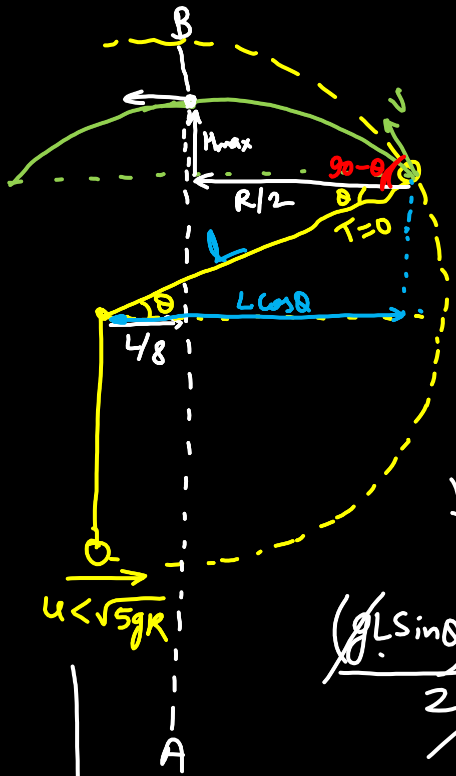
C $\sqrt{\left(3+\frac{3\sqrt{3}}{2}\right)Lg}$

D ✓ $\sqrt{\left(2+\frac{3\sqrt{3}}{2}\right)Lg}$

$$mg\sin\theta = \frac{mv^2}{L}$$

$$\boxed{v = \sqrt{gL\sin\theta}}$$





$$\frac{R}{2} + \frac{L}{8} = L\cos\theta$$

$$\frac{v^2\sin 2(90-\theta)}{2g} + \frac{L}{8} = L\cos\theta$$

$$\frac{v^2\sin(180-2\theta)}{2g} + \frac{L}{8} = L\cos\theta$$

$$\frac{(gL\sin\theta)\sin(2\theta)}{2g} + \frac{L}{8} = L\cos\theta$$

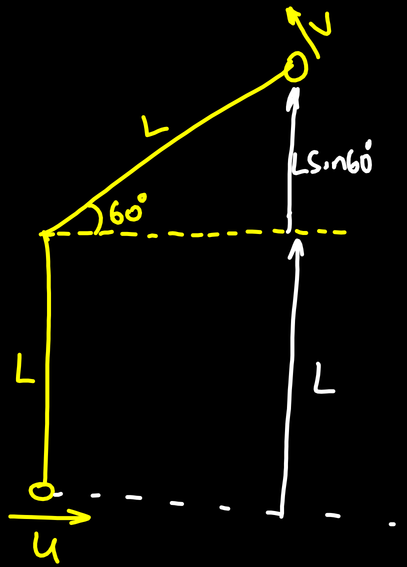
$$\sin^2\theta\cos\theta + \frac{1}{8} = \cos\theta$$

$$\frac{1}{8} = \cos\theta - \sin^2\theta\cos\theta$$

$$\frac{1}{8} = (1-\sin^2\theta)\cos\theta$$

$$\frac{1}{8} = \cos^3\theta$$

$$\cos\theta = \frac{1}{2} \Rightarrow \boxed{\theta = 60^\circ}$$



$$v = \sqrt{gL\sin 60^\circ}$$

$$v = \sqrt{\frac{\sqrt{3}}{2} gL}$$

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(L + L\sin 60^\circ)$$

$$u^2 = \frac{\sqrt{3}}{2}gL + \left(gL + \frac{\sqrt{3}}{2}gL\right) \times 2$$

$$u^2 = \left(\frac{\sqrt{3}}{2} + \sqrt{3}\right)gL + 2gL$$

$$u = \sqrt{\left(\frac{3\sqrt{3}}{2} + 2\right)gL}$$

Power

Power = Rate of doing work

$$P_{avg} = \frac{W}{t}$$

$$P = \frac{dW}{dt}$$

SI Unit $1 \frac{J}{sec} = 1$ Watt

1 HP = 746 Watts

A

W = 50J in 2 mins

B

W = 200J in 2 hr

Power

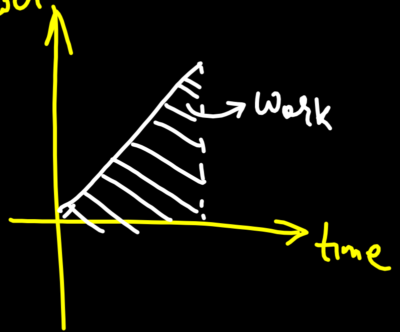
$$P = \frac{dW}{dt} = \frac{\vec{F}\cdot d\vec{r}}{dt} = \vec{F}\cdot\left(\frac{d\vec{r}}{dt}\right)$$

$$\boxed{P = \vec{F}\cdot\vec{v}} \Rightarrow \text{Scalar}$$

$F \perp v \Rightarrow \text{Power} = 0$

$dW = P\,dt$

$$\boxed{W = \int P\,dt}$$



$1\,J = 1\ \text{Watt} \times \text{sec}$

Energy $\Rightarrow$ 1 kWhr

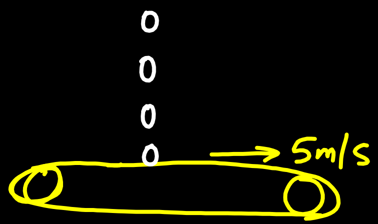
Q

Sand is being dropped from a stationary dropper at a rate of 0.5 kgs^{-1} on a conveyor belt moving with a velocity of 5 ms^{-1}. The power needed to keep belt moving with the same velocity will be :

(A) 1.25 W (B) 2.5 W

(C) 6.25 W (D) 12.5 W

[JEE Main 2022]



$V_1 = 0$, $V_2 = 5 m/s$

$F = \frac{\Delta p}{\Delta t} = \frac{(0.5)(5-0)}{1} = 2.5 N$

Power $= Fv = 2.5 \times 5 = 12.5$ Watts

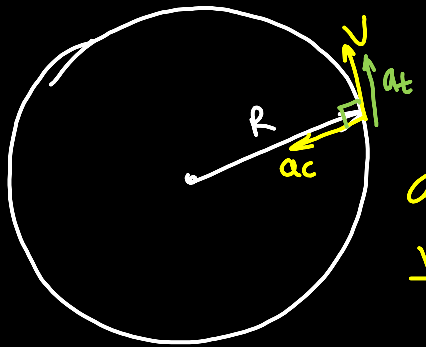
Q

A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by $n^2 R t^2$ where n is a constant. The power delivered to the particle by the force acting on it, is :

[JEE Main 2016]

(a) $\frac{1}{2} M n^2 R^2 t^2$ (b) $M n^2 R^2 t$

(c) $M n R^2 t^2$ (d) $M n R^2 t$



$a_c = n^2 R t^2$

$\frac{v^2}{R} = n^2 R t^2$

$v = nRt$

$a_t = \frac{dv}{dt} = nR$

$F_t = M a_t = MnR$

$\text{Power} = \vec{F_t} \cdot \vec{v} = F_t v \cos 0°$

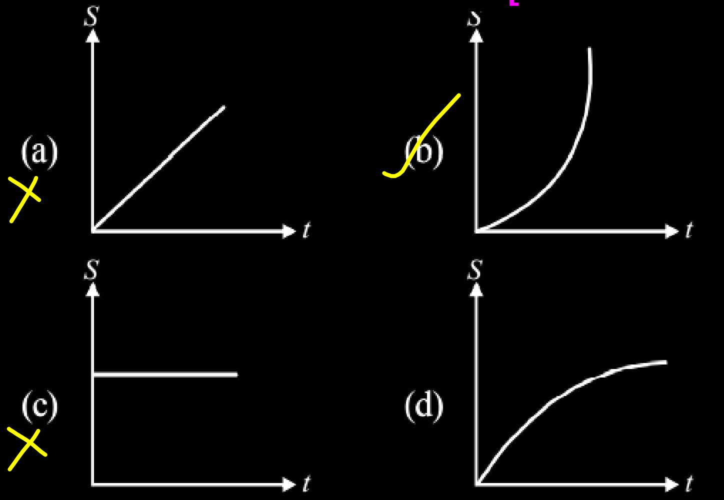
$= (MnR)(nRt)$

$= M n^2 R^2 t$

Q

A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) - time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) :

[JEE Main 2020]



$$P = \text{const}$$

$$Fv = K$$

$$\left(m\frac{dv}{dt}\right)v = K$$

$$\int v\,dv = K'\int dt$$

$$\frac{v^2}{2} = K't$$

$$v = \sqrt{2K'}\sqrt{t}$$

$$\boxed{v = b\sqrt{t}}$$

$$\frac{ds}{dt} = b\sqrt{t}$$

$$\int ds = b\int\sqrt{t}\cdot dt$$

$$s = b\frac{t^{3/2}}{3/2}$$

$$\boxed{s \propto t^{3/2}}$$

NOTES

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For more details, contact **8585858585**

Nishant Vora
Mathematics Maestros

Prashant Jain
Mathematics Maestros

Ajit Lulla
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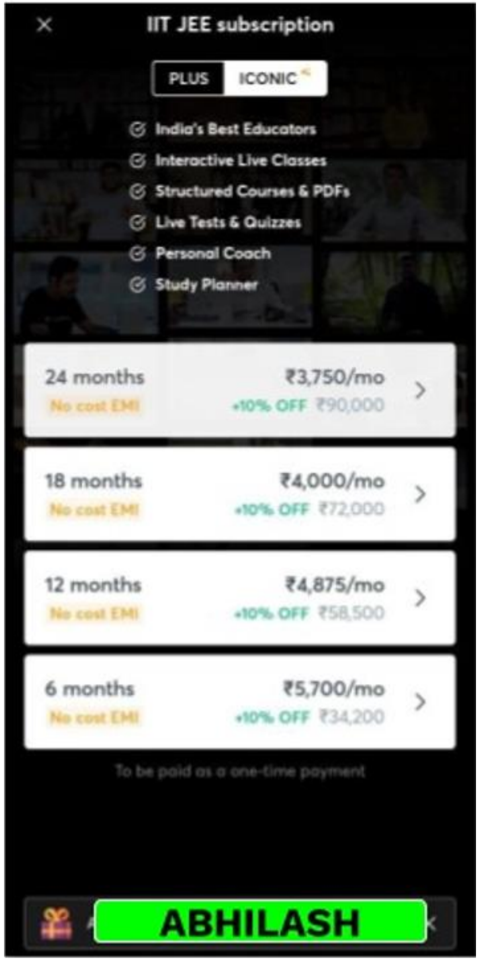
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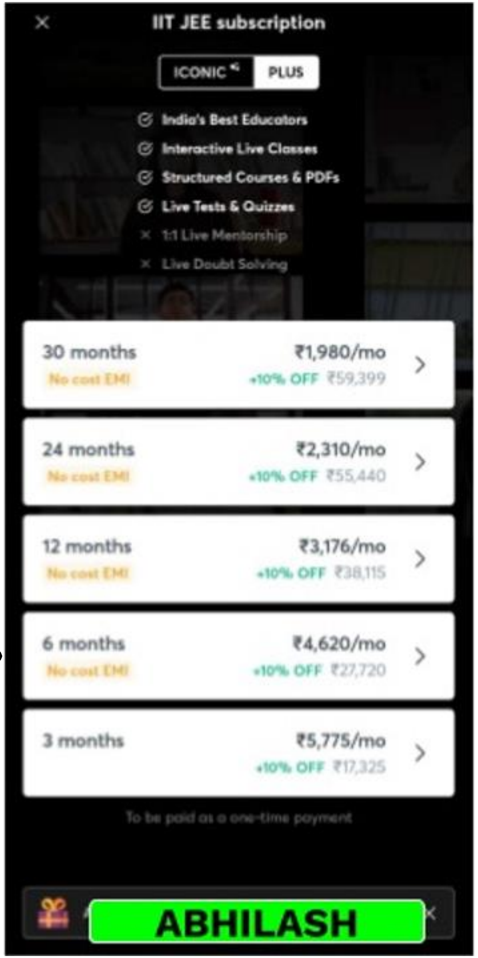
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